Shell-model description for beta decays of pfg-shell nuclei

Workshop on
New Era of Nuclear Physics in the Cosmos
– the r-process nucleosynthesis
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Introduction

• Precise information of nuclear properties is needed for the analysis of the r-process nucleosynthesis which occur under extreme conditions which are not accessible by current experiments. Therefore, theoretical estimations are useful, and predictions with high accuracy are desired.

• Nuclear structure models based on the mean-field approximation such as RPA are widely used, which are applicable to any nuclei in the nuclear chart by taking sufficiently large model space and provide us reasonable description of gross properties. However, the results are not necessarily accurate because only limited correlations can be treated.

• The shell model can treat any two-body correlations and give accurate descriptions of nuclear structure, but its applicability is limited to relatively light nuclei or semi-magic nuclei because of too heavy numerical tasks. We have to take a small model space and introduce an effective interaction.

• To what extent can shell model predict nuclear properties which are important for the study of the r-process?
shell model for r-processes
f5pg9-shell

- f5pg9-shell
  - $^{56}\text{Ni}$ inert core
  - Valence orbits: $p_{3/2}$, $f_{5/2}$, $p_{1/2}$, $g_{9/2}$
  - No spurious center-of-mass motion

- Interests
  - Neutron-rich
  - Isomer
  - Shape-coexistence
  - Astrophysics

- Recent shell-model studies
    - Second order correction to the Sussex matrix elements
    - $N=50,49,48$ with severe truncation (weak coupling assumption)
    - Modify G-matrix interaction by least squares fit
    - $T=1$ part for proton and neutron ($\text{Ni}$ isotopes and $N=50$ isotones)
Effective interaction

- **JUN45 interaction**
  - Keep isospin symmetry
  - Modify microscopic interaction $G\cdot f5pg9$
    - M. Hjorth-Jensen, unpublished
    - Bonn-C potential
    - 3rd order Q-box and folded diagram
  - Vary 45 LC’s of 133 TBME and 4 SPE
  - Fit to 400 energy data out of 87 nuclei of $A=63\sim96$
    - Include low-lying states of
      - even-Z nuclei
      - odd-A nuclei
    - Exclude
      - $N<46$ for $Z>33$… large quadrupole collectivity (needs for d5/2)
      - Ni, Cu isotopes … large effects of f7/2 core-excitations
  - Assume $A^{-0.3}$ mass dependence
  - Rms error of 185 keV
Binding energy

- Empirical Coulomb energy

\[ E_C(\pi, \nu) = \epsilon_C \pi + V_C \frac{\pi(\pi - 1)}{2} + b_c \left[ \frac{1}{2} \pi \right] - \Delta_{\pi, \pi \nu} \]

- f7/2 effects?
  - Underbinding in Ni, Cu with N<40
  - Overbinding in neutron-rich Ni, Cu, Zn, Ga

B.J. Cole, PRC59(1999)726
Magnetic moments

- Effective spin g-factors
  - Free-nucleon values are already good (also for sd- and pf-shell)
  - Slight improvement by effective $g_s(\text{eff})/g_s(\text{free}) \sim 0.7$
  - Note: $\sim 0.85 \ldots$ GXPF1 for pf-shell
### β⁻ decay

- **Q_β⁻** and **T_{1/2}**
- **N=50**
  - Waiting point
  - T=1 TBME contributes
  - ⁷⁸Ni ... No valence particles/holes
- **N=49**
  - Low-lying isomer

NNDC Chart of Nucleids
Neutron-rich nuclei

• f5pg9-shell
  Spin-flip decay
  \[ \nu_{g9/2} \rightarrow \pi g7/2 \]
  \[ \nu_{f5/2} \rightarrow \pi f7/2 \]
  are out of the model space

• Large Q-value
  Higher excited states may be important

• Phase space factor
  Low-lying states mainly contribute to \( T_{1/2} \)
Single-particle levels around $N=50$

- Low-lying states in odd-$A$ nuclei

**Proton Orbits**

- $N=50$
- Ex (MeV)

**Neutron Orbits**

- $N=49$
- Ex (MeV)
$\beta$-decay

- **Q-value**
  - $\beta^-$: $(A, Z) \rightarrow (A, Z+1) + e^- + \bar{\nu}_e$ \hspace{1cm} $Q = (\frac{A}{Z}M - \frac{A}{Z+1}M)c^2$
  - $\beta^+$: $(A, Z) \rightarrow (A, Z-1) + e^+ + \nu_e$ \hspace{1cm} $Q = (\frac{A}{Z}M - \frac{A}{Z-1}M)c^2 - 2mc^2$
  - EC: $(A, Z) + e^- \rightarrow (A, Z-1) + \nu_e$ \hspace{1cm} $Q = (\frac{A}{Z}M - \frac{A}{Z-1}M)c^2 - E_K$

- **Allowed transition matrix elements**
  - Fermi
    \hspace{1cm} $B(F) = \frac{\left| \left\langle f \right| \Sigma_k t^k_{\pm} \left| i \right\rangle \right|^2}{2J_i + 1}$ \hspace{1cm} $\Delta J=0$, $\Delta \pi=$No
  - Gamow-Teller
    \hspace{1cm} $B(GT) = \frac{\left| \left\langle f \right| \Sigma_k \sigma^{k} t^k_{\pm} \left| i \right\rangle \right|^2}{2J_i + 1}$ \hspace{1cm} $\Delta J=0$, $\pm 1$, $\Delta \pi=$No

- **Lifetime**
  \hspace{1cm} $ft = \frac{6144.4 \pm 1.6 \text{ sec}}{B(F) + \left( g_A/g_V \right)^2 B(GT)}$
  - $g_A/g_V = -1.2720(18)$ … ratio of axial-vector and vector c.c.
  - $f$ … phase space factor
GT quenching

- **Quenching** of GT operator: $O_{GT} \rightarrow qO_{GT}$
  - Take into account the configurations outside the model space
  - p-shell ... $q = 0.82$ W.T.Chou et al, PRC47 (1993) 163
  - sd-shell ... $q = 0.77$ B.H.Wildenthal et al., PRC28 (1983) 1343
  - pf-shell ... $q = 0.74$ G.Martinez-Pinedo et al., PRC53 (1996) R2602

- f5pg9-shell
  - Violate Ikeda sum-rule: $S_- - S_+ = 3(N-Z)$
  - Low-lying transitions ... spin-flip contribution may be minor

- Fitting to $\beta$-decay data
  - Effective GT matrix element $M_{\text{eff}} = \sqrt{(2J_i + 1)B(GT)}$
  - Sum over final states relative to the sum-rule

\[ T = \sqrt{\Sigma_i \left( M_{\text{eff},i} \right)^2 / W} \]

\[ W = \begin{cases} 
  \left| g_A / g_V \right| \sqrt{(2J_i + 1)3 |N_i - Z_i|} & \text{(if } N_i \neq Z_i) \\
  \left| g_A / g_V \right| \sqrt{(2J_f + 1)3 |N_f - Z_f|} & \text{(if } N_i = Z_i) 
\end{cases} \]
Summed strength (f5pg9-shell)

- Exp. Data
  setA … all data
  setB … spin assigned
  setC … error in Ex < 0.1MeV
- Fit to setC
  ⇒ $q \sim 0.65$
  (with $^{64}\text{Ge}$, $^{78}\text{Ge}$)
  ⇒ $q \sim 0.59$
  (without $^{64}\text{Ge}$, $^{78}\text{Ge}$)

Adopt
$q = 0.6$
(large uncertainty)
Q-value

Underestimation by ~1MeV for neutron-rich nuclei

Good description around stable nuclei
Half life

- No allowed transition in $^{86}\text{Rb}$
- $^{81}\text{Ge}$ isomer at 679keV ($\frac{1}{2}^+$) assumed which decays mainly to the ($\frac{3}{2}^-$) ground state with log $ft > 5.9$
- Shell model suggests $\frac{1}{2}^-$

- $^{84}\text{Br}$
  - Dominant (2-) to 0+ 1st unique forbidden decay

- $^{85}\text{Kr}$
  - Dominant $\frac{9}{2}^+$ to $\frac{5}{2}^-$ 1st unique forbidden decay
  - Very weak $\frac{9}{2}^+$ to $\frac{9}{2}^+$ allowed decay ($\log ft = 9.5$)

Systematic underestimation for neutron-rich nuclei
79Cu

- No experimental level scheme

Shell model
5/2- ground state
Q(cal) = 9.67MeV
T_{1/2}(cal.) = 0.028 s

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79Cu β- Decay (188 ms) 1991Kr15
Published: 2002 Nuclear Data Sheets.

79Cu Parent: E_x = 0.0; T_{1/2} = 188 ms 25; Q_{g.s.->g.s.} = 11742 MeV

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<td>Full evaluation Balraj Singh Nuclear Data Sheets 96, 1</td>
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Measured T_{1/2}, %β⁻n.

Q(β⁻)(79Cu)=11742 943 (syst, 1995Au04); %β⁻n=55 17 (1991Kr15)

The isotope produced by 238U(p,X) E=600 MeV reaction followed by
and mass separation techniques. 1995En07 use 9Be(238U,F) reaction i
and magnetic methods to identify 79Cu.

The details of this decay mode are not known.
Beta strengths are predicted at around \(~4\text{MeV}\)
$^80_{30}\text{Zn}$

Shell model

$Q(\text{cal}) = 6.64\text{MeV}$

$T_{1/2}(\text{cal.}) = 0.22\text{ s}$
Ex of predicted strength is too low by \(~0.5\text{MeV}\)
Shell model

$^5/2$- ground state

$Q_{\text{cal}} = 7.41 \text{MeV}$

$T_{1/2_{\text{(cal.)}}} = 0.39 \text{ s}$
$^{81}$Ga

Low-lying strength is too large for forbidden.
Calculation predicts 2- or 5- ground state for $^{82}\text{As}$.

Predicted 1+ state at 0.519MeV almost explains the half life.
78\textsuperscript{Cu}

Shell model
6- ground state
Q(cal) = 11.76 MeV
\(T_{1/2}(\text{cal.}) = 0.074 \text{ s}\)

Adopted Levels

\begin{align*}
Q_p &= 13270 \text{ SY} \\
S_n &= 3540 \text{ SY} \\
S_p &= 14760 \text{ SY}
\end{align*}

1995Au04

History

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measured \(T_{1/2}\) (1987LuZX, 1997Au04).

\textbf{78\textsuperscript{Cu} levels}

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<tr>
<th>(E_{\text{level}})</th>
<th>(T_{1/2})</th>
<th>Comments</th>
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<td>0.0</td>
<td>342 ms (11%\beta^-=100)</td>
<td>(T_{1/2}): from 1987LuZX</td>
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Beta strengths are predicted at around \( \sim 6 \text{MeV} \)
Shell model

9/2+ ground state

$Q_{\text{cal}} = 8.49\text{MeV}$

$T_{1/2}(\text{cal.}) = 0.54\text{s}$

Forbidden ?
Shell model predicts a $1/2^-$ isomer at 65 keV which decays with $T_{1/2}=0.66$ s.

Low-lying strength?
**84Br, 85Kr, 86Rb**

**Shell model**

2- ground state

Q(cal) = 4.59MeV

T_{1/2}(cal.) = 174min

Significantly hindered

B(GT)_{exp} = 0.0000019

B(GT)_{cal} = 0.00038

**Shell model**

9/2+ ground state

Q(cal) = 1.09MeV

T_{1/2}(cal.) = 0.38y
Unique forbidden decay

- $^{84}$Br  $(2^-) \rightarrow 0^+$  33%  \( Q=4655(25) \text{keV} \)
  - Exp.  \( \log f_{\text{lut}} = 9.47 \)  \( T_{1/2}=31.80(8) \text{min} \)
  - Cal.  8.7  174min \( \rightarrow 14.2 \text{min} \)

- $^{85}$Kr  $\frac{9}{2}^+ \rightarrow \frac{5}{2}^-  \ 99.563\% \  Q=687.4(20) \text{keV} \)
  - Exp.  \( \log f_{\text{lut}} = 9.4475 \)  \( T_{1/2}=10.756(18) \text{y} \)
  - Cal.  8.5  0.38y \( \rightarrow 0.28 \text{y} \)

- $^{86}$Rb  $2^- \rightarrow 0^+  \ 91.36\% \  Q=1774.2(14) \text{keV} \)
  - Exp.  \( \log f_{\text{lut}} = 9.4399 \)  \( T_{1/2}=18.642(18) \text{d} \)
  - Cal.  8.5  \( \infty \rightarrow 2.0 \text{d} \)

Calculation underestimates the half-life
Summary

• Beta-decay properties for N=50 and 49 nuclei can be described reasonably well by the shell model
• f5pg9-shell with JUN45 interaction
• Quenching factor q=0.6 for Gamow-Teller operator
• Underestimate Q-values by ~1 MeV for neutron-rich nuclei
• Predicted half-life agrees with experimental data within a factor of 2 ~ 3 near the stable nuclei, while it systematically underestimates for neutron-rich nuclei probably because of insufficient correlations
• Allowed transitions already give reasonable predictions for N=50 cases, but forbidden decays sometimes play a crucial role for N=49 cases near the stable nuclei.
• Enlarged model space is desired for better description