有限核の α 凝縮 （特に ^{16}O）

Yasuro Funaki (RIKEN)

Taiichi Yamada (Kanto Gakuin Univ.)

Peter Schuck (IPN, Orsay, Paris-Sud Univ.)

Hisashi Horiuchi (RCNP)

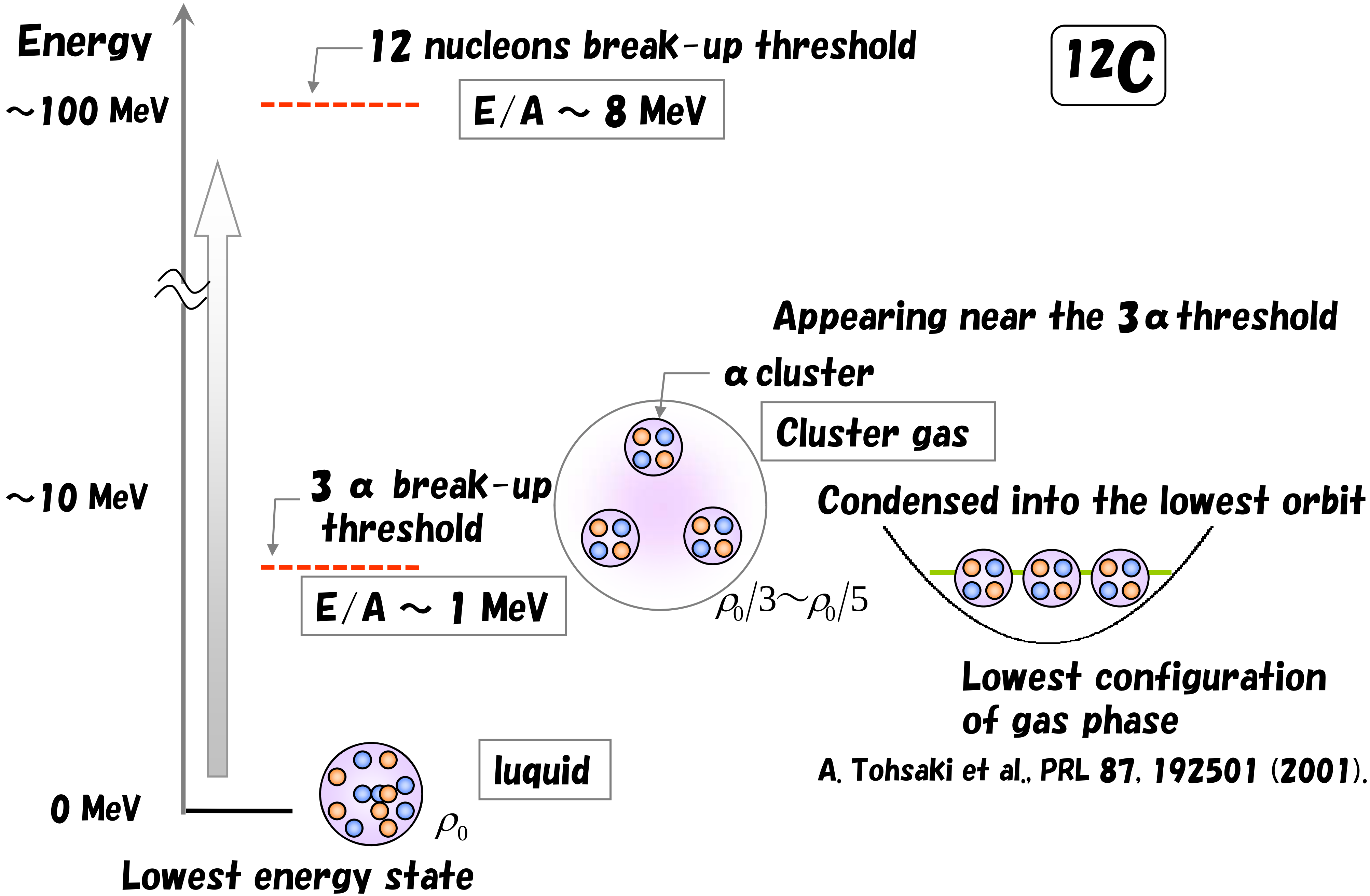
Akihiro Tohsaki (RCNP)

Gerd Röpke (Rostock Univ.)

and with T. Wakasa, M. Takashina, etc.

RCNP ワークショップ、「核子と中間子の多体問題の統一的描像に向けて 」
2007年、12月14日―15日

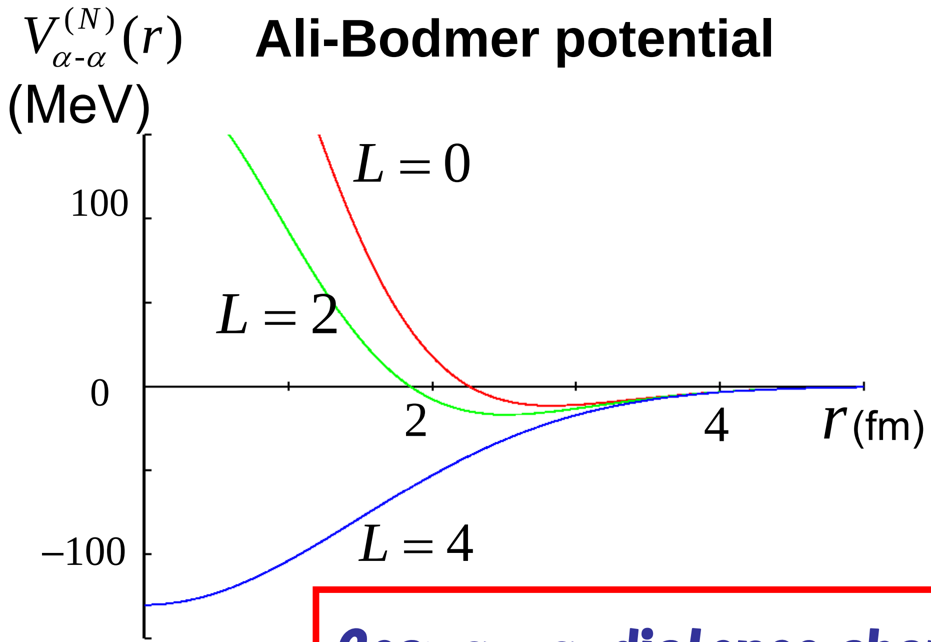
Appearing of cluster gas state and "BEC" state in finite nuclei



A. Tohsaki et al., PRL 87, 192501 (2001).

α－α interaction range and potentials α particles feel in the gas states

Ali-Bodmer potential

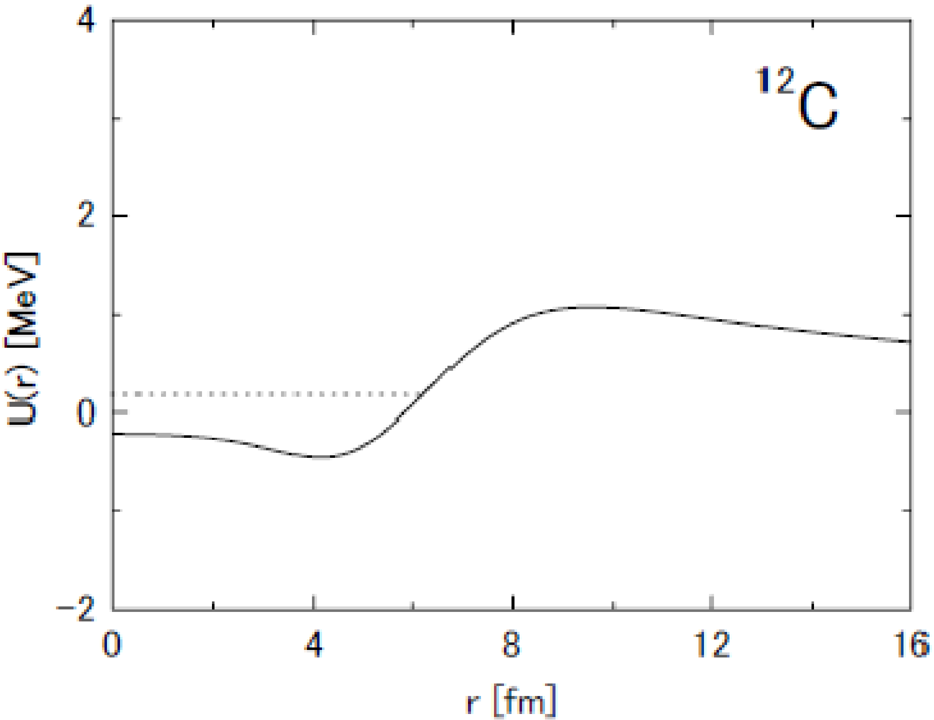


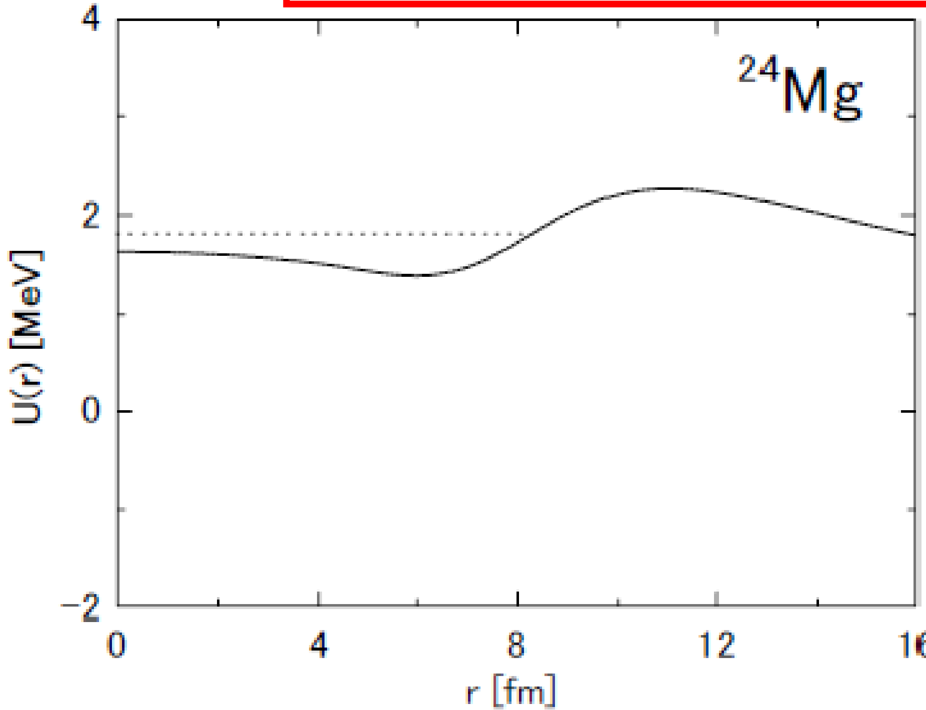
Coulomb barrier

Position is outside the α－α interaction range(～4 fm)

Stabilization of α condensate state

Gas: α－α distance should be more than 4 fm





Gross-Pitaevsky eq.

T. Yamada and P. Schuck, PRC 69, 024309 (2004)

$n\alpha$ condensate wave function (THSR-w.f.)

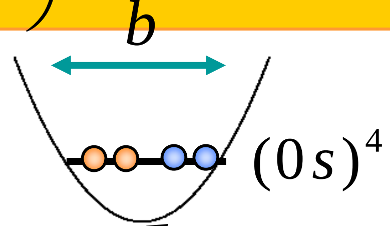
$$\Phi_{n\alpha}(\beta, b) = \mathcal{A}\left\{\prod_{i=1}^{n}\left(\exp\left(-\frac{2}{B^2}\vec{X}_i^2\right)\phi(\alpha_i)\right)\right\} \quad (B^2 = b^2 + 2R_0^2)$$

$$\propto \left\langle \vec{r}_1 i_1, \cdots, \vec{r}_{4n} i_{4n} \middle| \left(C_\alpha^\dagger\right)^n \middle| \mathrm{VAC} \right\rangle$$

$$C_\alpha^\dagger = \int d^3\vec{R}\, \exp\left(-\frac{R^2}{R_0^2}\right) B_\alpha^\dagger(\vec{R})$$

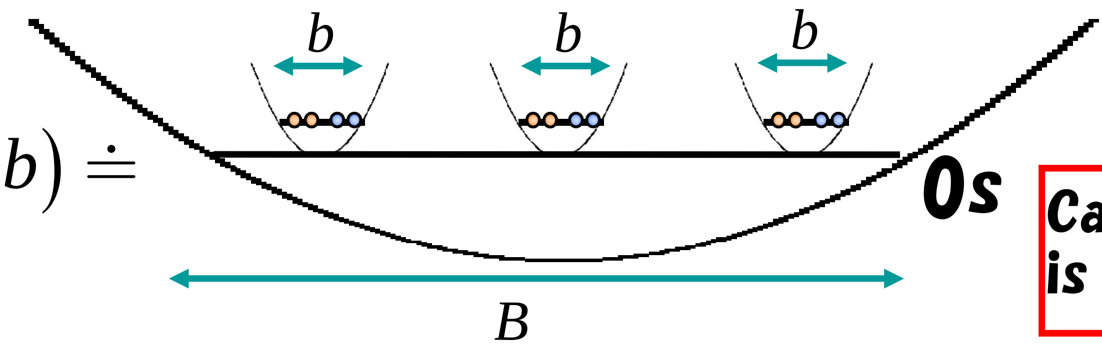
Brink's wave function

$$\phi(\alpha) \propto \left\langle \vec{r}_1 i_1, \cdots, \vec{r}_4 i_4 \middle| B_\alpha^\dagger(\vec{R}) \middle| \mathrm{VAC} \right\rangle \doteq$$



$(0s)^4$ configuration around $\vec{R}$

$$\Phi_{n\alpha}(R_0, b) \doteq$$



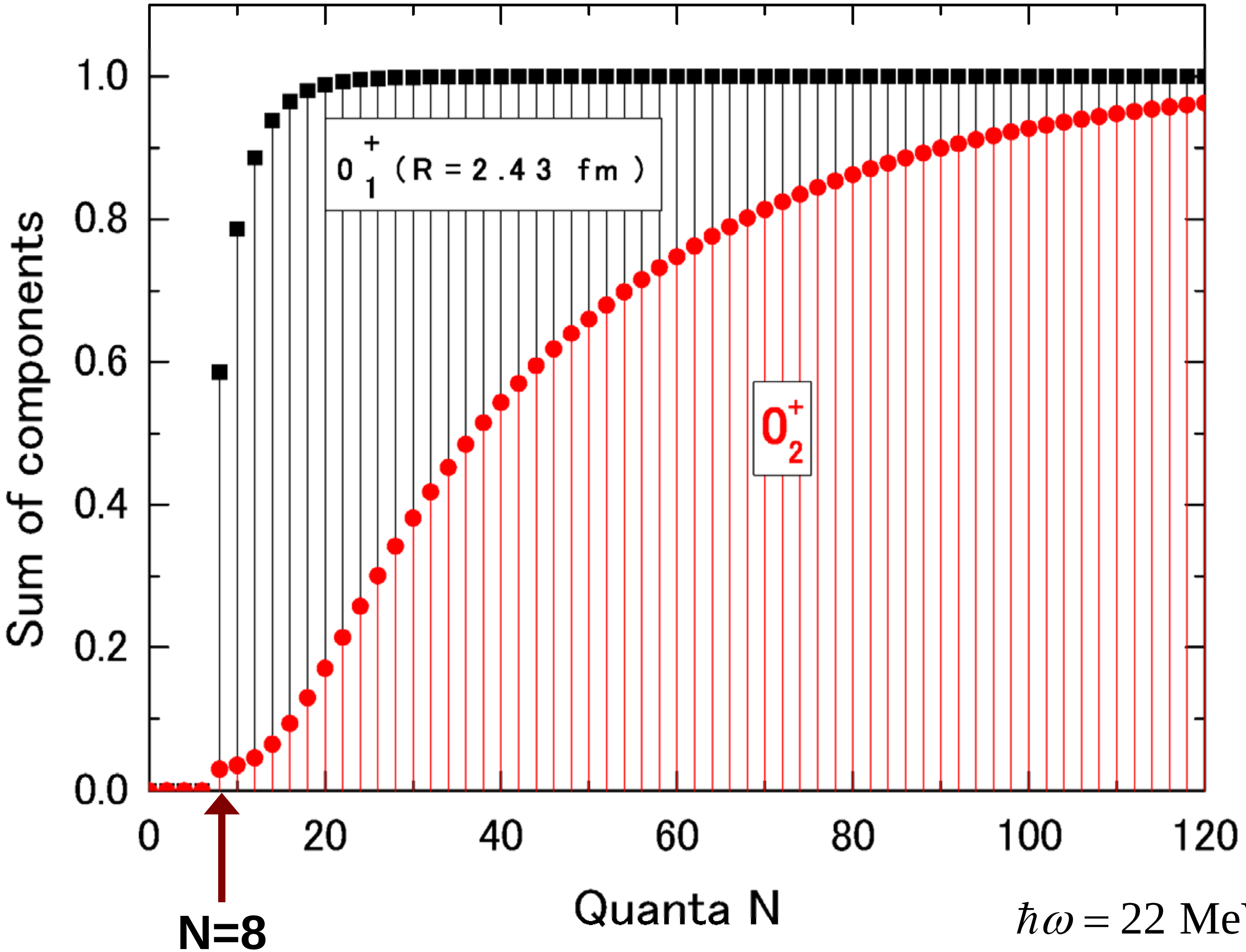
Calculation of matrix elements is owing to Tohsaki's technique

Hill-Wheeler equation (R_0: generator coordinate, b: fixed)

$$\sum_{R_0'} \left\langle \Phi_{n\alpha}(R_0, b) \middle| H - E^\lambda \middle| \Phi_{n\alpha}(R_0', b) \right\rangle f_{R_0'}^\lambda = 0 \qquad \Psi_{n\alpha}^\lambda = \sum_{R_0} f_{R_0}^\lambda \Phi_{n\alpha}(R_0, b)$$

^{12}C

Expansion of 0^+_1 and 0^+_2 wfs with H.O. basis



N=8

$\hbar\omega = 22$ MeV

The expansion was done wrt relative motions of α's.

Calculated by T. Yamada

First example of α condensate state in finite nuclei

3α break-up threshold : 7.27 MeV

Hoyle state (0_2^+ state in ^{12}C (excitation energy : 7.65 MeV))

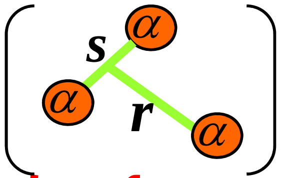
Indicating 3α condensate character

Microscopic approach (3α cond. model w.f.)

The Solution of 3α RGM eq. of motion, RGM

$$\left\langle \phi^3(\alpha) \middle| H - E \middle| \mathcal{A}\left[\chi(\mathbf{s},\mathbf{r})\phi^3(\alpha)\right]\right\rangle = 0$$

M. Kamimura, NPA 351, 456 (1981).

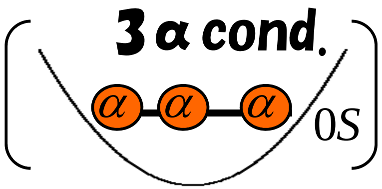


is almost equivalent to the 3α cond. w.f.

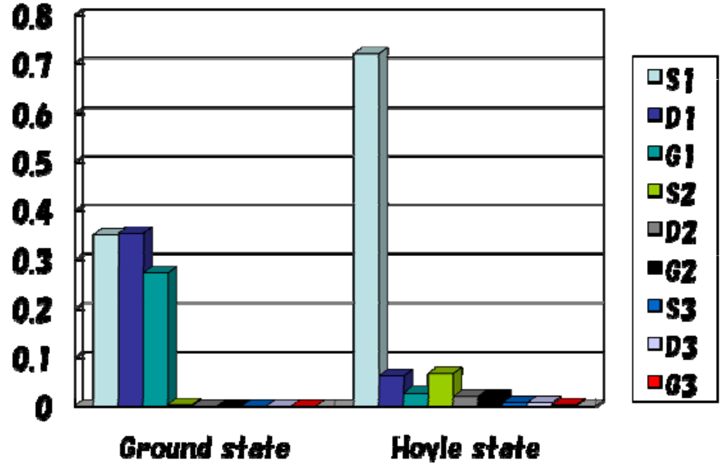
$$\chi(\mathbf{s},\mathbf{r}) = \exp\left(-\frac{2}{B^2}\sum_{i=1}^{3}(X_i - X_G)^2\right)\mathcal{A}$$

X_i : c.o.m of α-particle

Y. F et al., PRC 67, 051306(R) (2003).

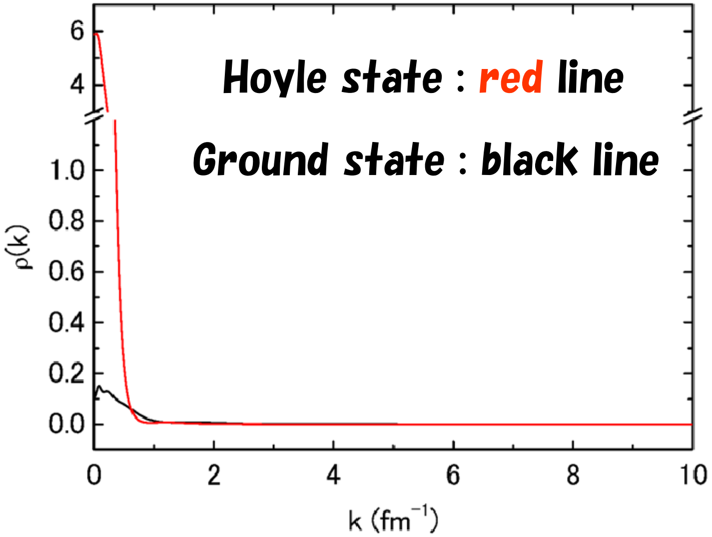


Occupation probability of α-particle orbit



- Occupation probability of α-particle orbit
  Huge 0S occupancy (>70 %)
- Momentum distribution
  Delta-function-like behavior

Momentum distribution of α-particle

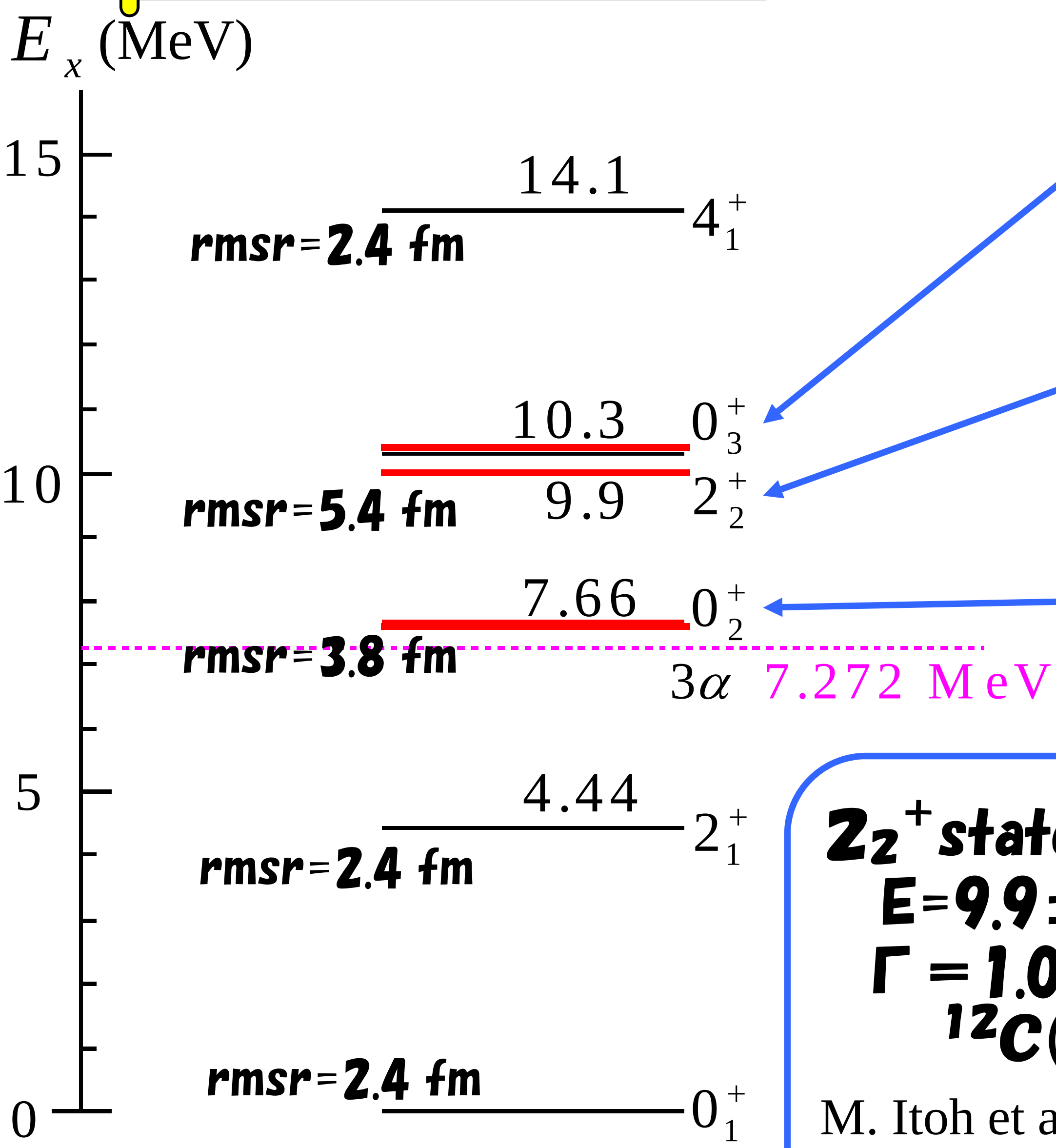


H. Matsumura and Y. Suzuki, NPA 739, 238 (2004).

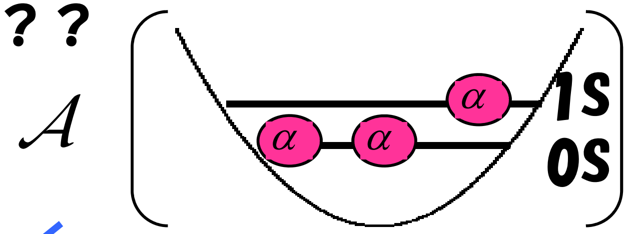
T. Yamada and P. Schuck EPJA 26, 185 (2005).

"BEC" in ^{12}C

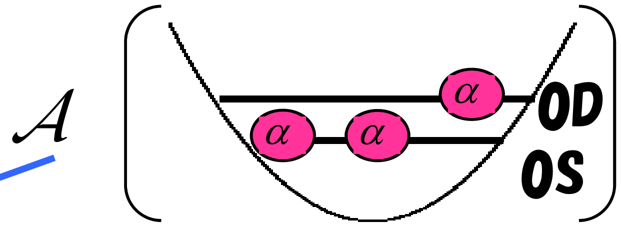
Observed levels of ^{12}C

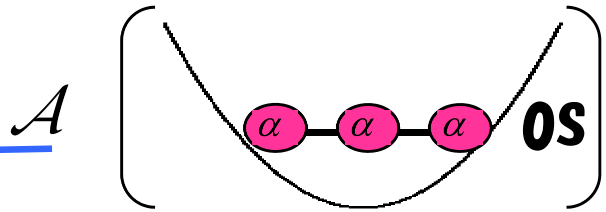


??



C. Kurokawa and K. Katō, PRC 71, 021301 (2005).





2_2^+ state :
E=9.9±0.3 MeV
Γ=1.0±0.3 MeV
$^{12}C(\alpha, \alpha')$

M. Itoh et al., Nucl. Phys. A 738 (2004) 268-272

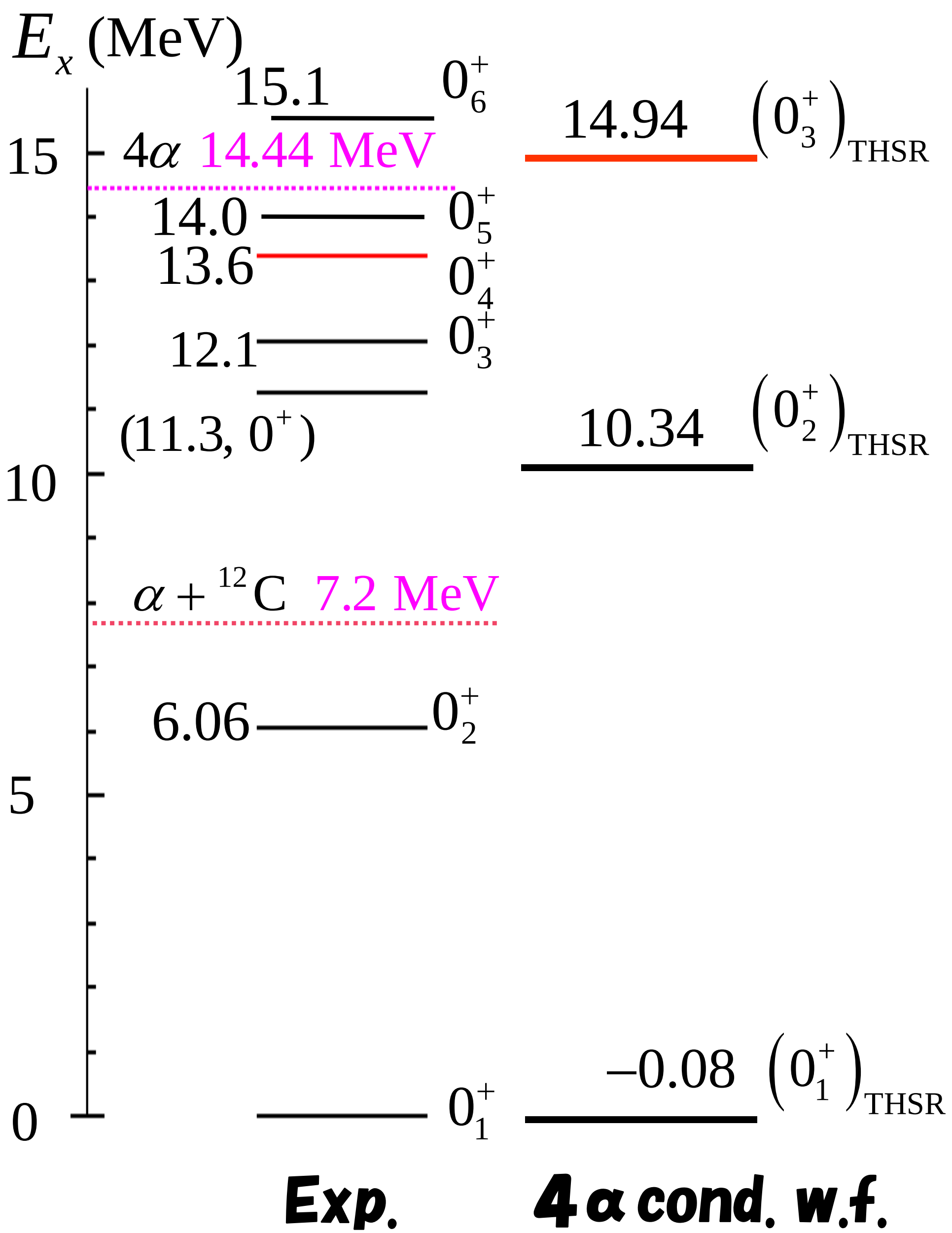
α cond. +ACCC
E=9.38 MeV
Γ=0.64 MeV

Volkov No. 1 force is adopted

Y. F. et al., EPJA 24, 321 (2005).

First attempt to explore 4α condensate state in ^{16}O

Low lying 0^+ levels of ^{16}O

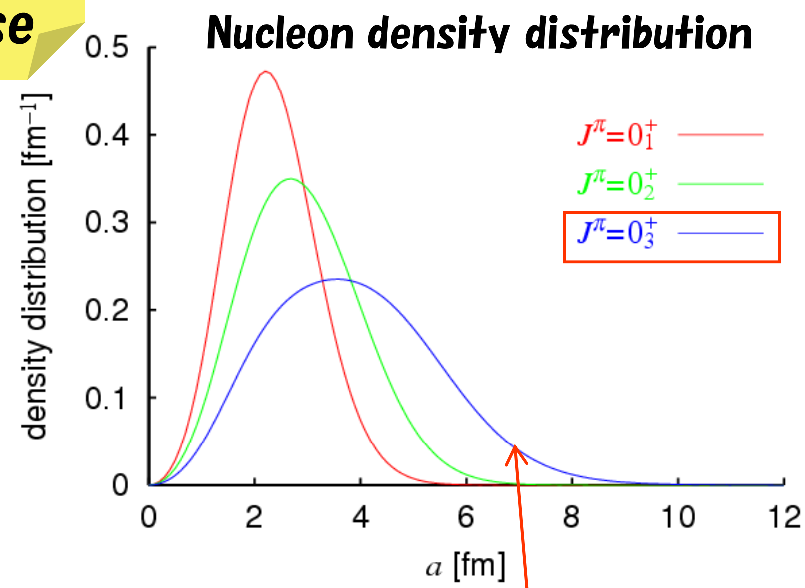


nα condensate model w.f. (THSR-w.f.)

$$\Phi_{n\alpha} = \mathcal{A}\left\{\prod_{i=1}^{n}\left(\exp\left(-\frac{2}{B^2}X_i^2\right)\phi(\alpha_i)\right)\right\}$$

A. Tohsaki, H. Horiuchi, P. Schuck and G. Röpke, PRL 87, 192501 (2001).

n=4 case



$(0_3^+)_{\mathrm{THSR}}$: **Very dilute density.**
~~4 α condensate state~~

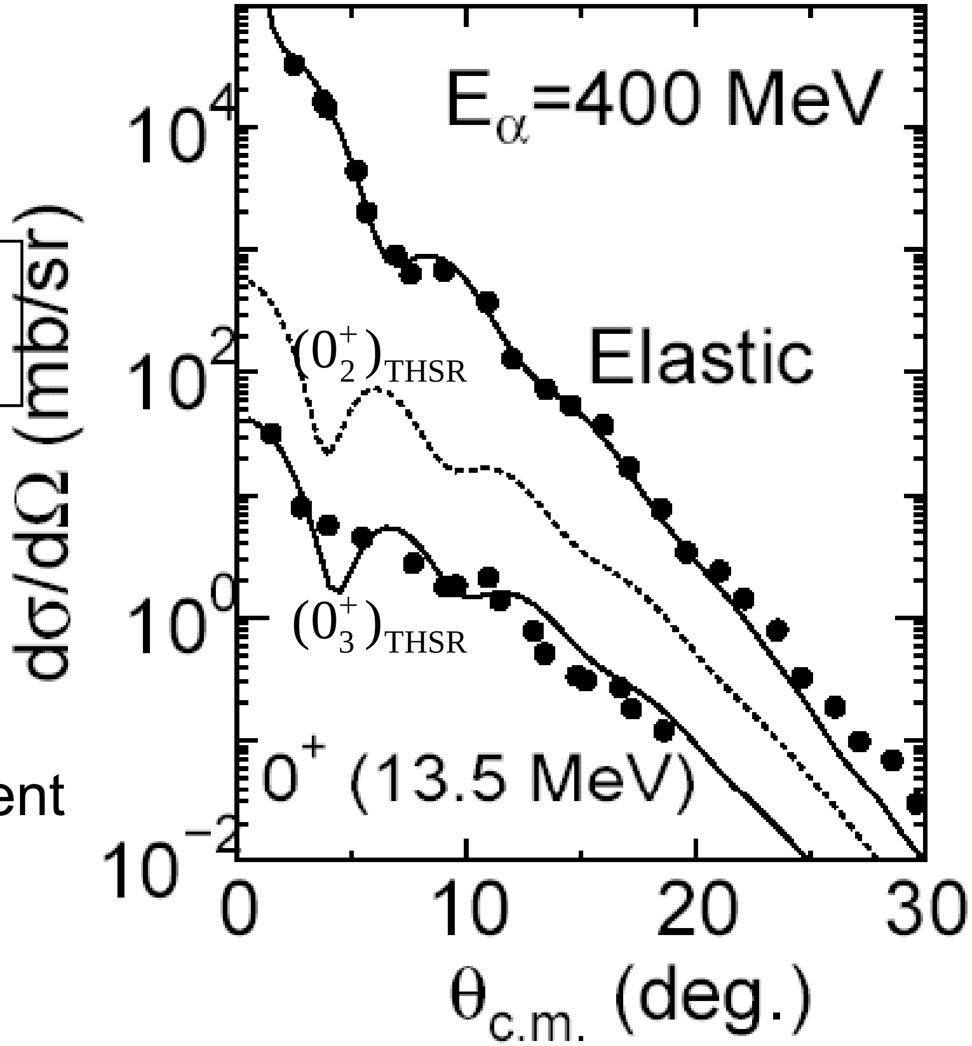
α+^{16}O inelastic scattering

0_5^+ state:
A candidate of 4α condensate
E = 13. 6MeV
Γ = 0. 8MeV
^{16}O(α, α') Wakasa et al.

The result of the calculation is consistent with the experimental data.

The 0^+ state at E_x =13.5 MeV can be assigned to the four-α condensed state.

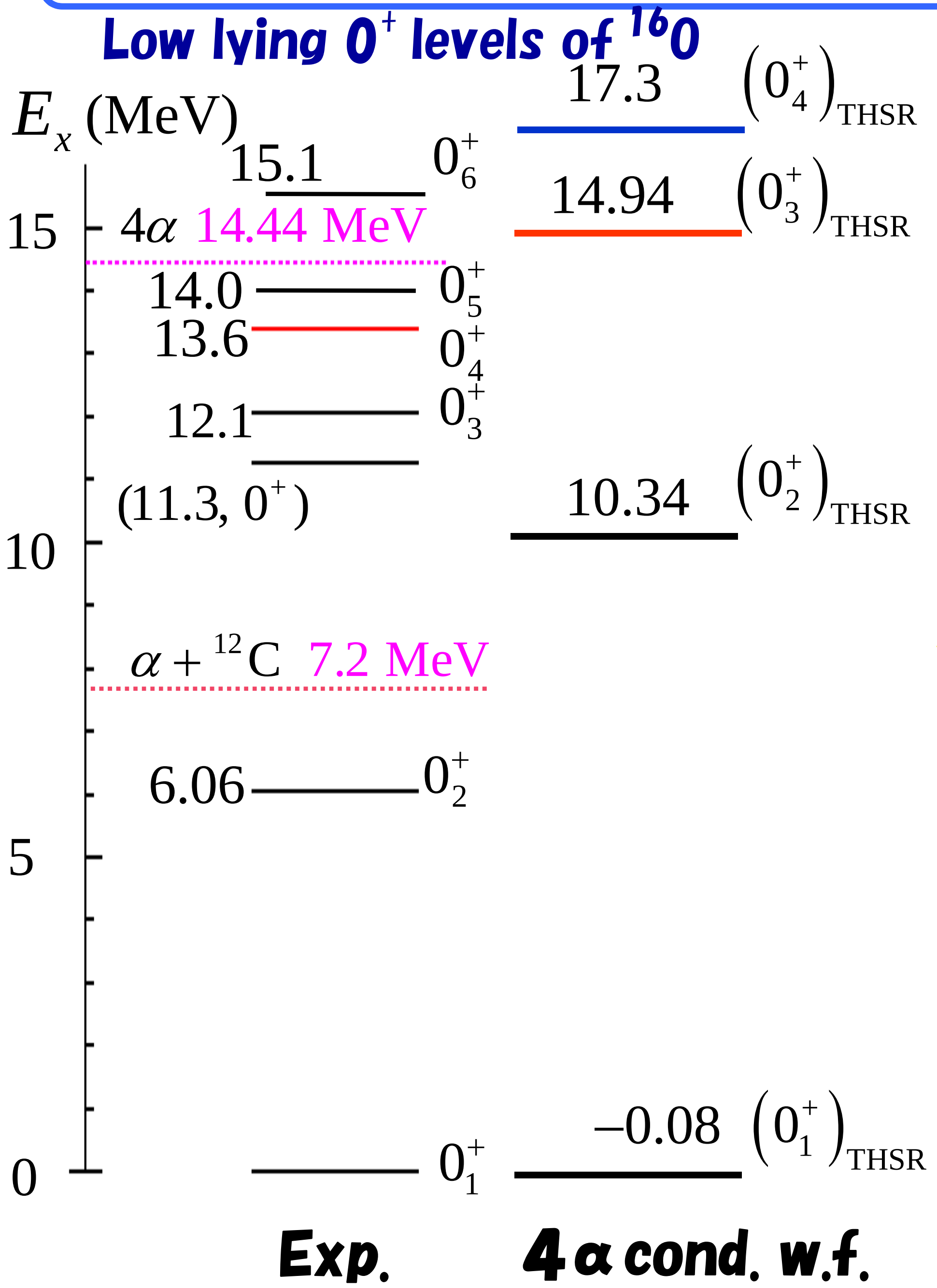
The 0^+ state wave function obtained at E_x=10.3 MeV leads to a largely different absolute value.



$(0_3^+)_{THSR}$:
E = 14.9 MeV
Γ = 1.5 MeV
(based on R-matrix theory)

T. Wakasa, E. Ihara, M. Takashina and Y. F. et al, PLB 653, 173 (2007).

First attempt to explore 4α condensate state in ^{16}O



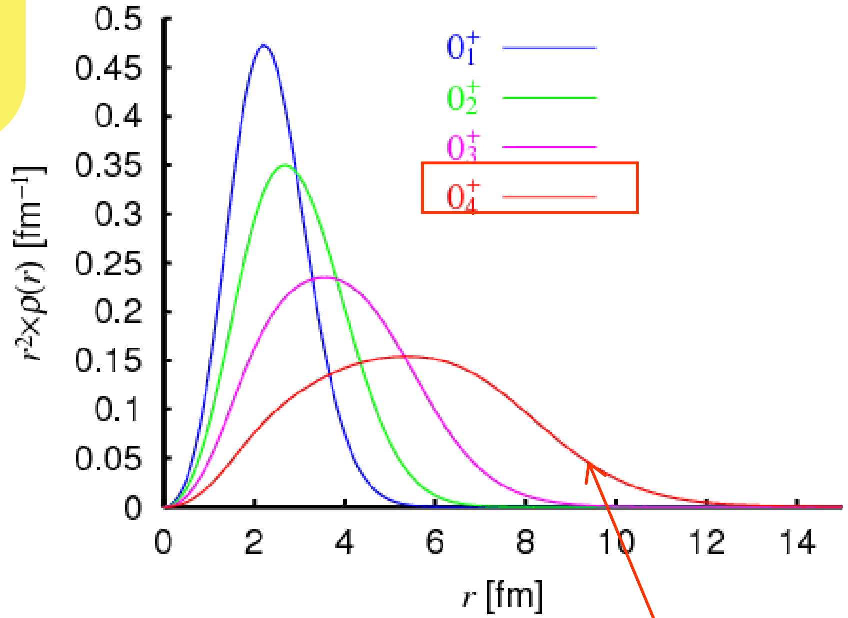
nα condensate model w.f. (THSR-w.f.)

$$\Phi_{n\alpha} = \mathcal{A}\left\{\prod_{i=1}^{n}\left(\exp\left(-\frac{2}{B^2}X_i^2\right)\phi(\alpha_i)\right)\right\}$$

A. Tohsaki, H. Horiuchi, P. Schuck and G. Röpke, PRL 87, 192501 (2001).

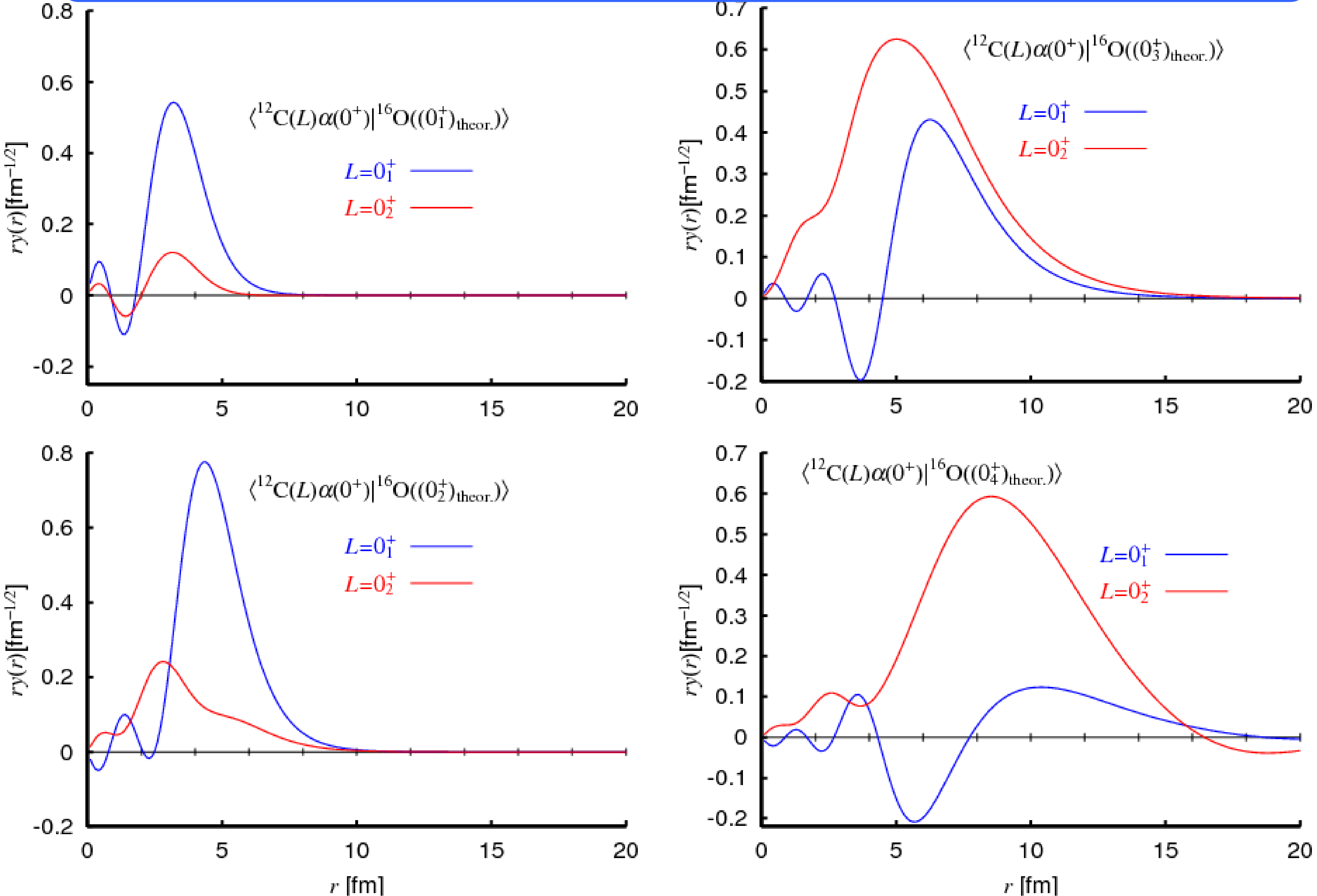
n=4 case

Nucleon density distribution

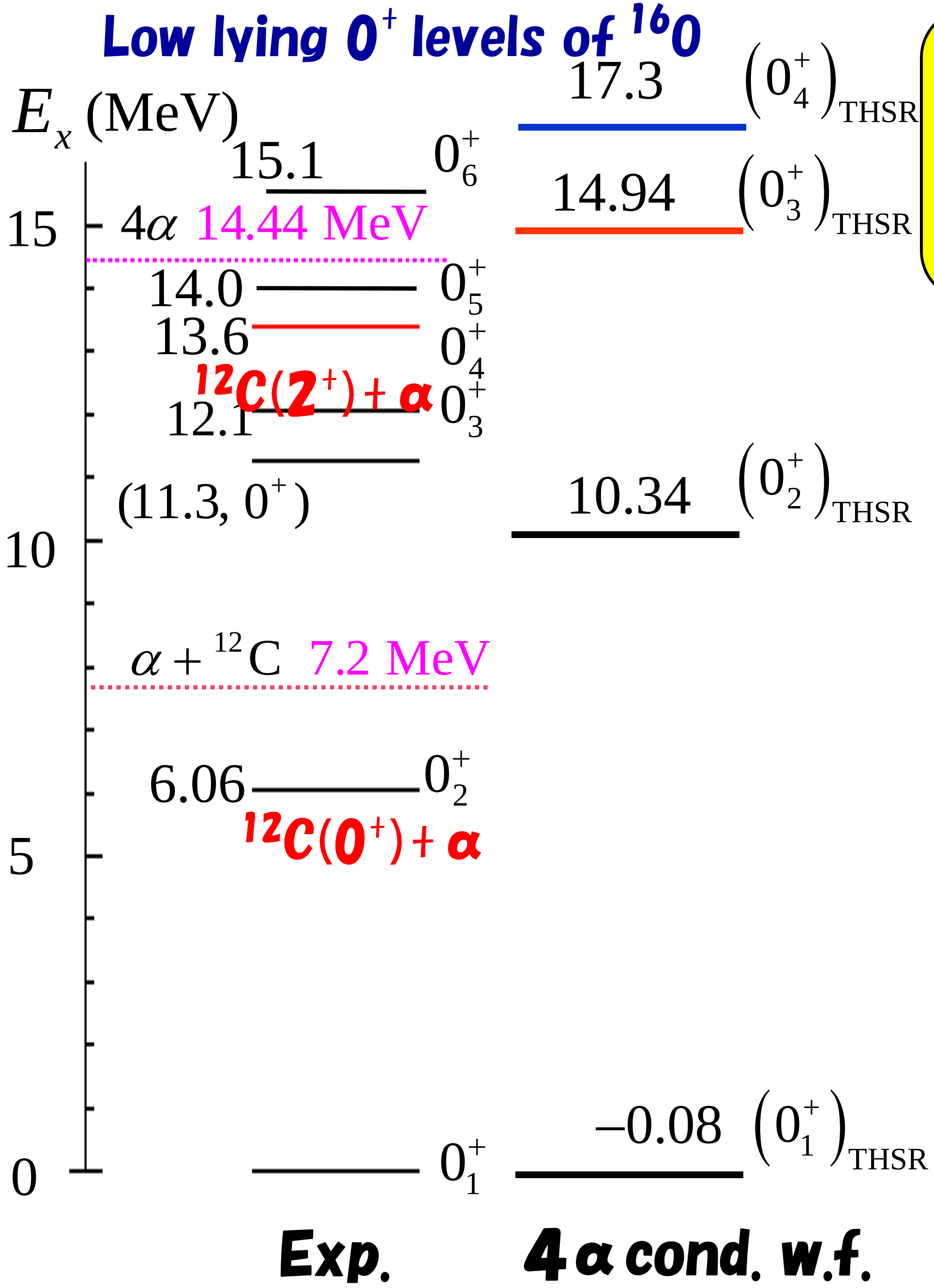


$(0_4^+)_{THSR}$: **Very much dilute density. 4 α condensate state**

Reduced width amplitudes of 0_1^+ – 0_4^+ states obtained with THSR w.f.(overlap amplitude between α plus $^{12}C(0_1^+$ or $0_2^+)$ and ^{16}O wfs)



Motivation for 4α OCM (Orthogonality Condition Model)



n α cond. w. f. (microscopic)

$$\Phi_{n\alpha}(R_0,b)=\mathcal{A}\left\{\prod_{i=1}^{n}\left(\exp\left(-\frac{2}{B^2}\vec{X}_i^2\right)\phi(\alpha_i)\right)\right\}$$

- **Need to check the existence of 4 α cond. state in larger (4α) model space.**
- **4α cond. w. f. may have a difficulty to represent the ^{12}C+α structure.**

4 α cond. w.f. hardly describes these states

- $0_2^+(\alpha+{}^{12}\mathrm{C}(0^+))$ △
- (0_3^+) ?
- $0_4^+(\alpha+{}^{12}\mathrm{C}(2^+))$ ✕

Need to solve full 4 α problem, 4 α OCM (semi microscopic)

Fully solving 4 α-particles relative motions (4α OCM)

Present: Larger model space

$$\varphi_{\ell m}(\mathbf{r},\nu) = N_\ell(\nu) r^\ell \exp\left(-\nu r^2\right) Y_{\ell m}(\mathbf{r})$$

Gaussian basis (GEM)

E. Hiyama et al. Prog. Part. Phys. 51, 223(2003).

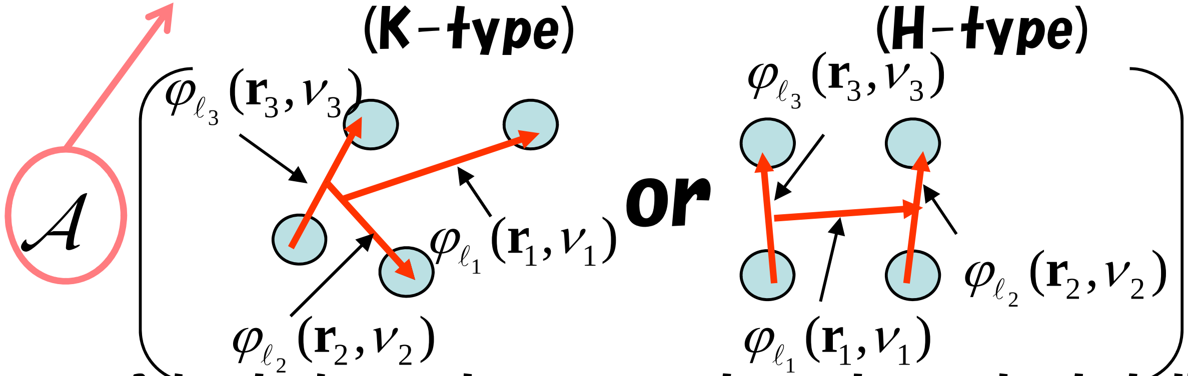
↔

^{12}C+α : succeeded
dilute 4α : not reproduced

4α OCM(H. O. basis)
K. Fukatsu and K. Kato, PTP 87, 151 (1992).

^{12}C+α coupled channel OCM (H. O. basis)
Y. Suzuki, PTP 55, 1751 (1976);
56, 111 (1976).

Approximately taken into account



Adopted angular momentum channels, totally 10 channels

K-type and H-type, $[[l_1,l_2]_{l_{12}},l_3]_L$: $[[0,0]_0,0]_0\ [[2,0]_2,2]_0 [[0,2]_2,2]_0\ [[2,2]_0,0]_0$

$[[0,1]_1,1]_0\ [[2,1]_1,1]_0$ **Added in only K-type**

Hamiltonian

$$H = T + \sum_{i<j}\left[V_{2\alpha}(r_{ij}) + V_{2\alpha}^{Coul}(r_{ij})\right] + V_{3\alpha} + V_{4\alpha} + V_{Pauli}$$

$V_{2\alpha}(r)$: 2-body force (folding MHN), $V_{2\alpha}^{Coul}$: Coulomb force

$V_{3\alpha}$ & $V_{4\alpha}$: phenomenological 3-body and 4-body forces (repulsive)

V_{pauli} : pauli operator (to eliminate 0s, 1s, 0d w.r.t. $\alpha-\alpha$ rel. motion)

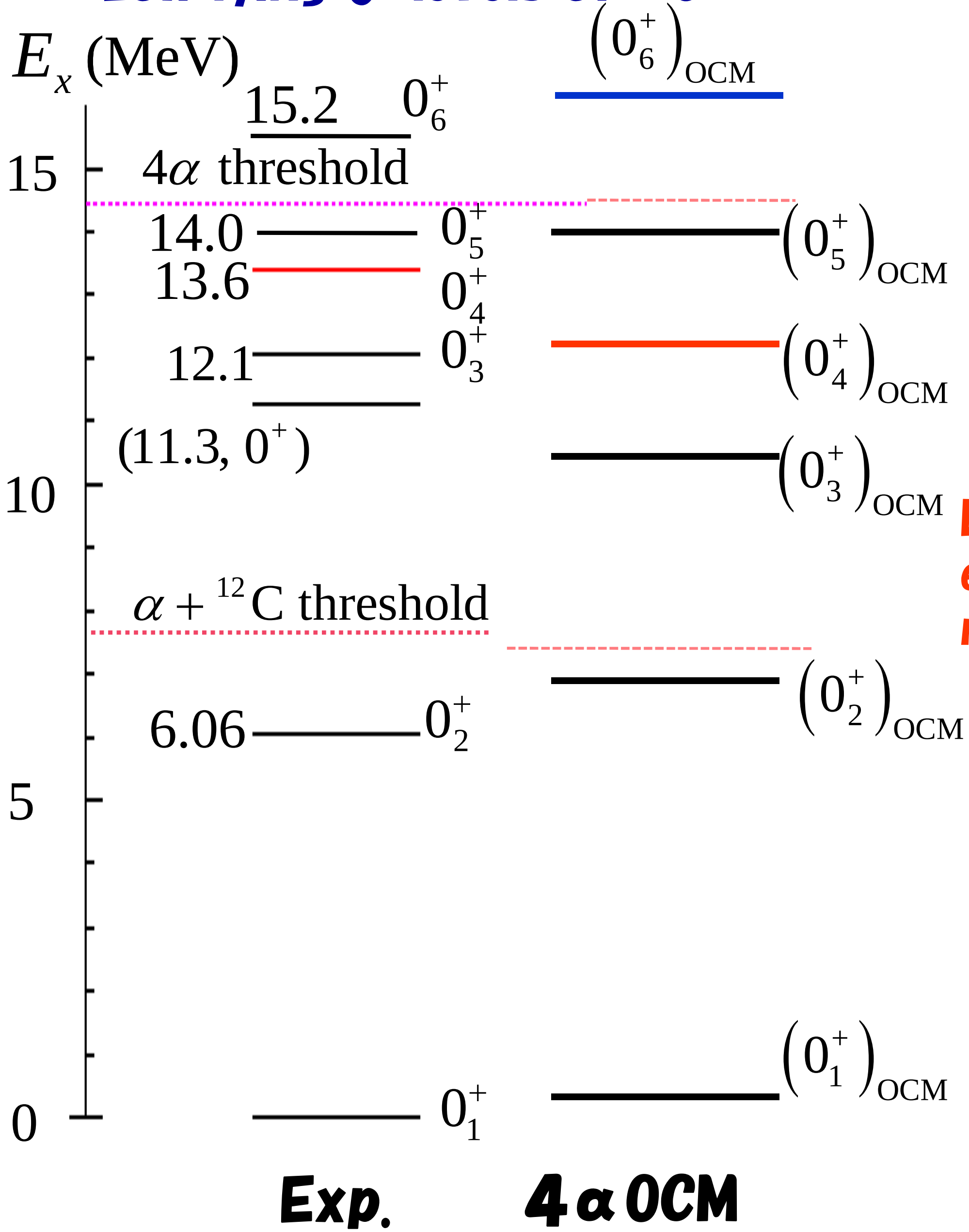
Energies from 4α threshold

	Cal. (MeV)	Exp. (MeV)
^{12}C(g.s.)	−7.32	−7.27
^{16}O(g.s.)	−14.2	−14.44

Correct ^{12}C(g.s.)+α and 4α threshold energies

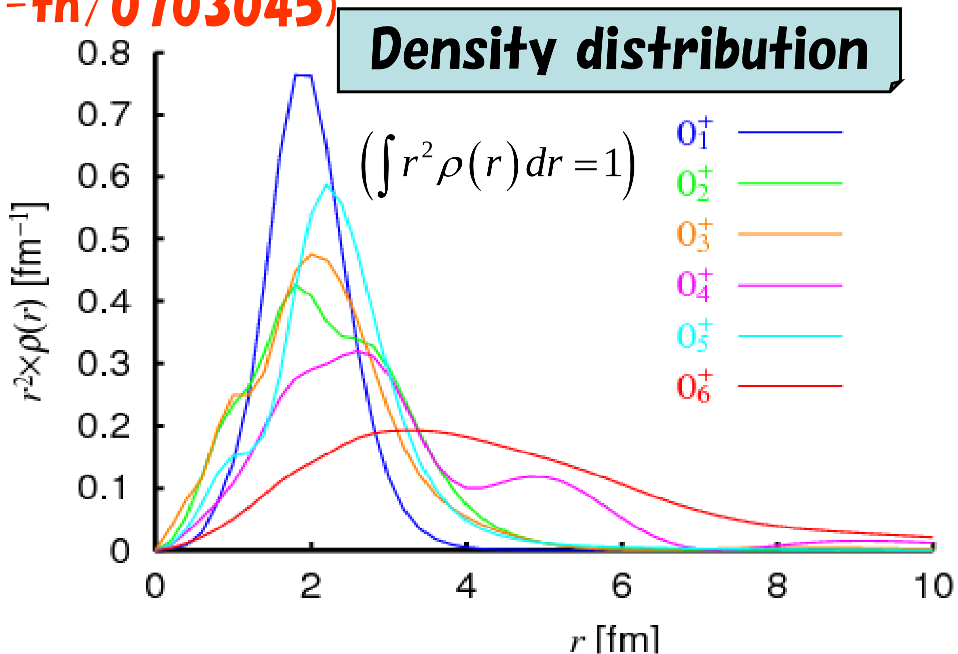
Energy levels, rms radii, monopole matrix elements and density distribution.

Low lying 0^+ levels of ^{16}O



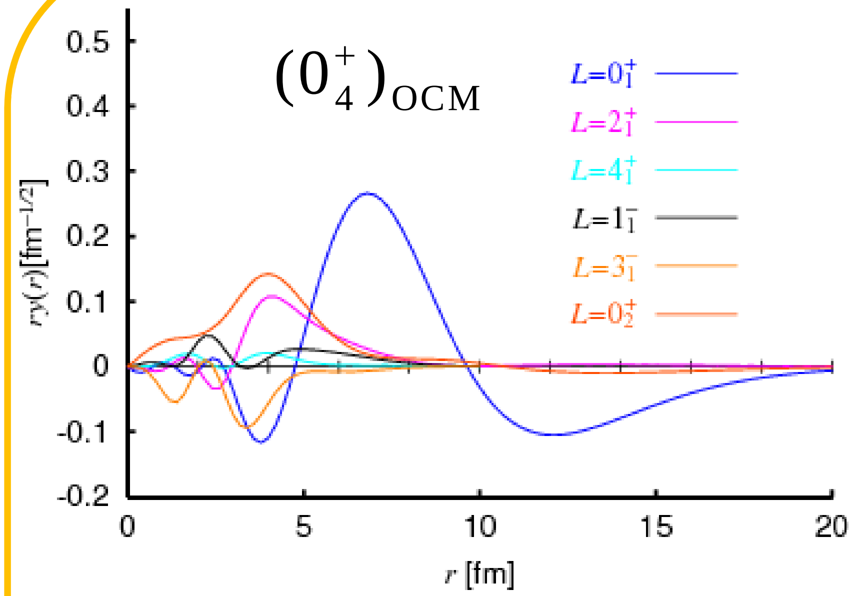
	R_{rms} (fm)	M(E0) (fm²)	M(E0) (fm²) Exp.
$(0_1^+)_{OCM}$	2.7		
$(0_2^+)_{OCM}$	3.1	4.2	0_2^+: 3.55
$(0_3^+)_{OCM}$	2.9	4.1	0_3^+: 4.03
$(0_4^+)_{OCM}$	3.9	2.4	0_4^+: no data
$(0_5^+)_{OCM}$	3.1	2.0	0_5^+: 3.3
$(0_6^+)_{OCM}$	5.4	1.4	

Large monopole matrix element can be the evidence of cluster states (Yamada, Y.F. et el., nucl-th/0703045)

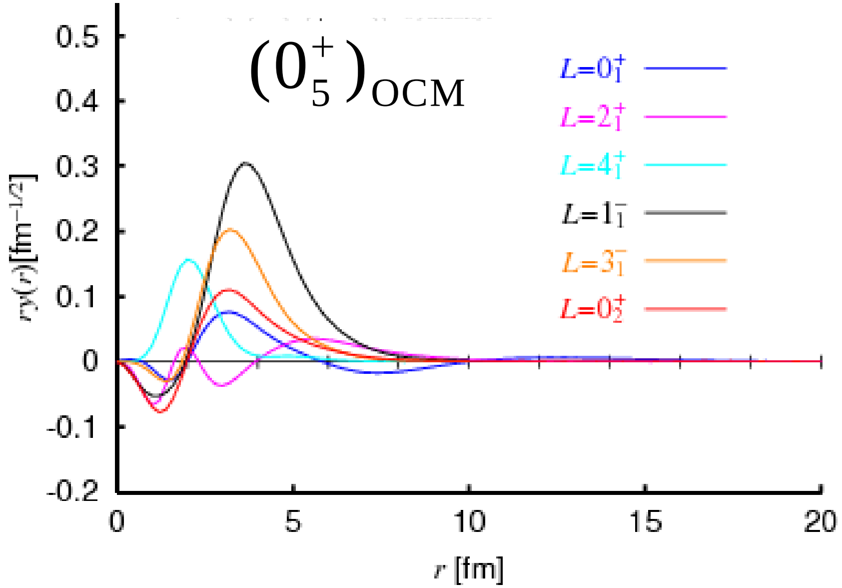


Reduced width amplitudes of 0_4^+ and 0_5^+ states obtained with 4α OCM

Defined as $$r \times \mathcal{Y}_{L,J_k=0_k}(r) = r \times \left\langle \left[Y_L(\mathbf{r}) \Phi_L(^{12}\mathrm{C}) \right]_0 \middle| \Phi_{J_k=0_k}(^{16}\mathrm{O}) \right\rangle$$



- Very well developed α cluster structure
- $^{12}C(g.s.) + \alpha$ component is dominant. higher nodal structure

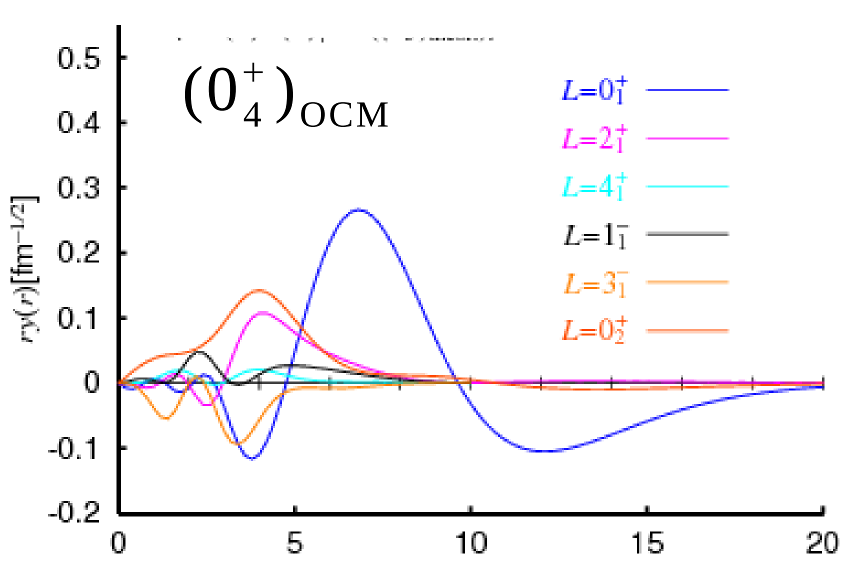


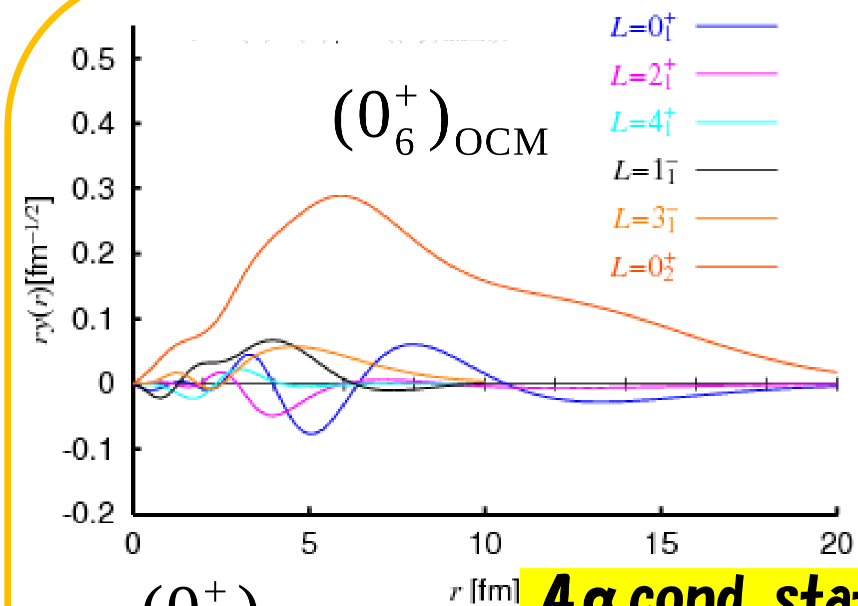
- Large amplitude in surface region(～5 fm)
- developed α cluster structure
- mixing of $^{12}C(1^-) + \alpha$, $^{12}C(3^-) + \alpha$ and $^{12}C(0_2^+) + \alpha$ structures

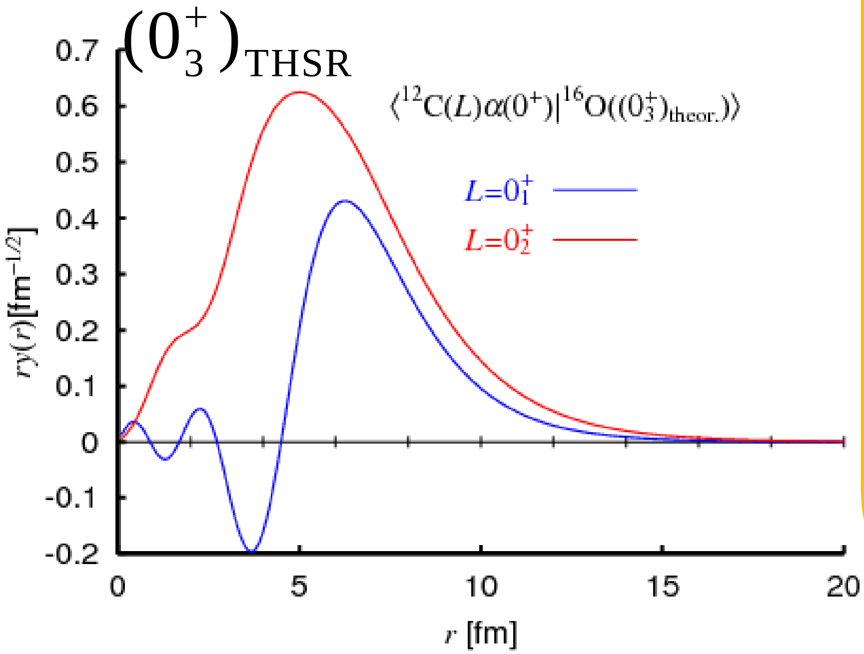
- New (not discussed so far) $\alpha + {}^{12}C$ cluster states.
- $\alpha + {}^{12}C$ dynamics survives up to around the 4α threshold.

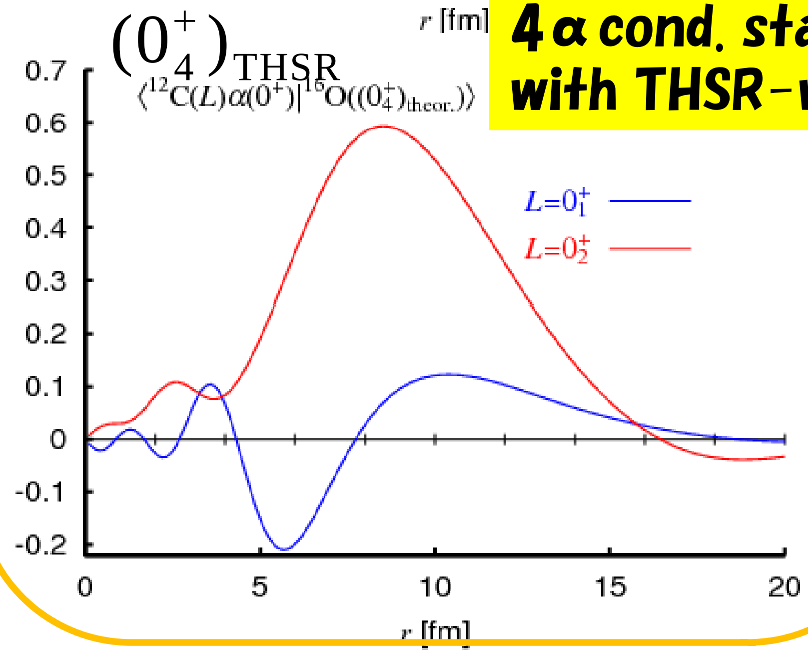
Reduced width amplitudes of 0_4^+ and 0_6^+ states obtained with 4α OCM

Defined as $r \times \mathcal{Y}_{L,J_k=0_k}(r) = r \times \left\langle \left[Y_L(\mathbf{r}) \Phi_L(^{12}\mathrm{C}) \right]_0 \middle| \Phi_{J_k=0_k}(^{16}\mathrm{O}) \right\rangle$



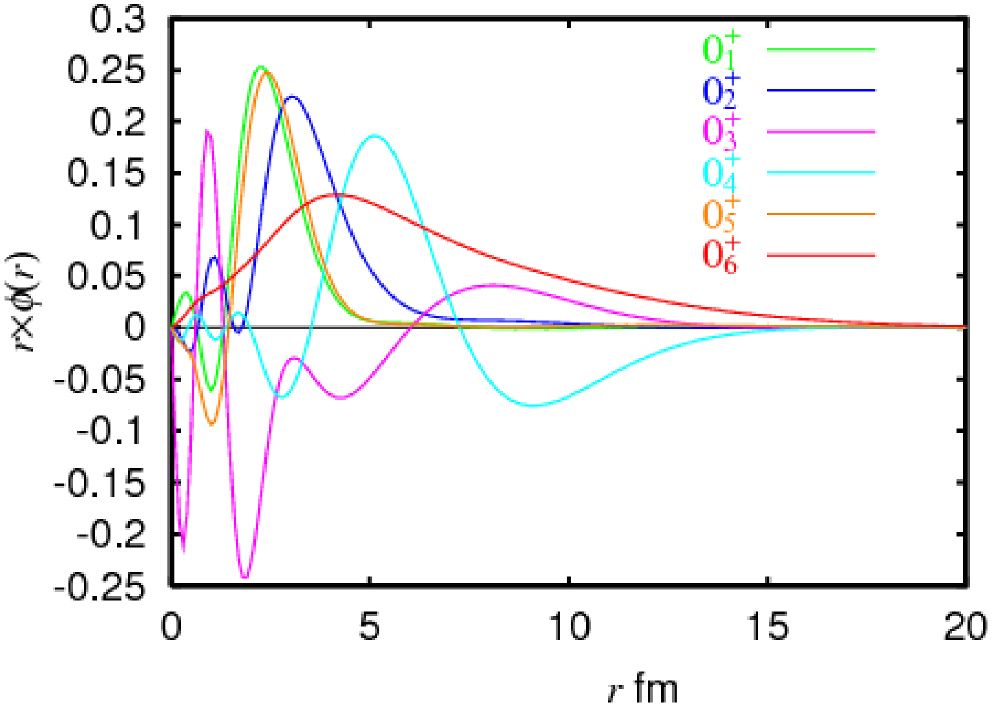






4α cond. state obtained with THSR-w.f.

Single particle occupancy and single particle orbit for the 0_1^+ – 0_6^+ states obtained with 4α OCM (Only the S orbit (L=0))



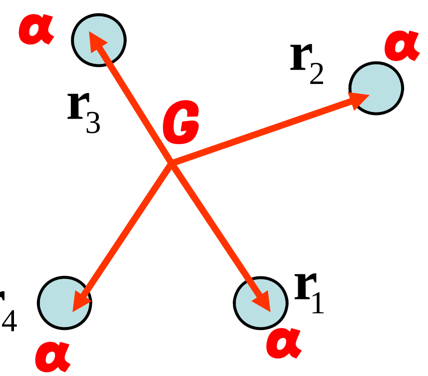
The largest (for $L = 0$) $\mu^{\lambda} = 2.33$

0_6^+ state : 2.33/4 = 58 %

Large 0S occupancy !

Strongly evidence that the 0_6^+ state is the 4α condensate

The largest occupancies for the other states are less than 20 %.



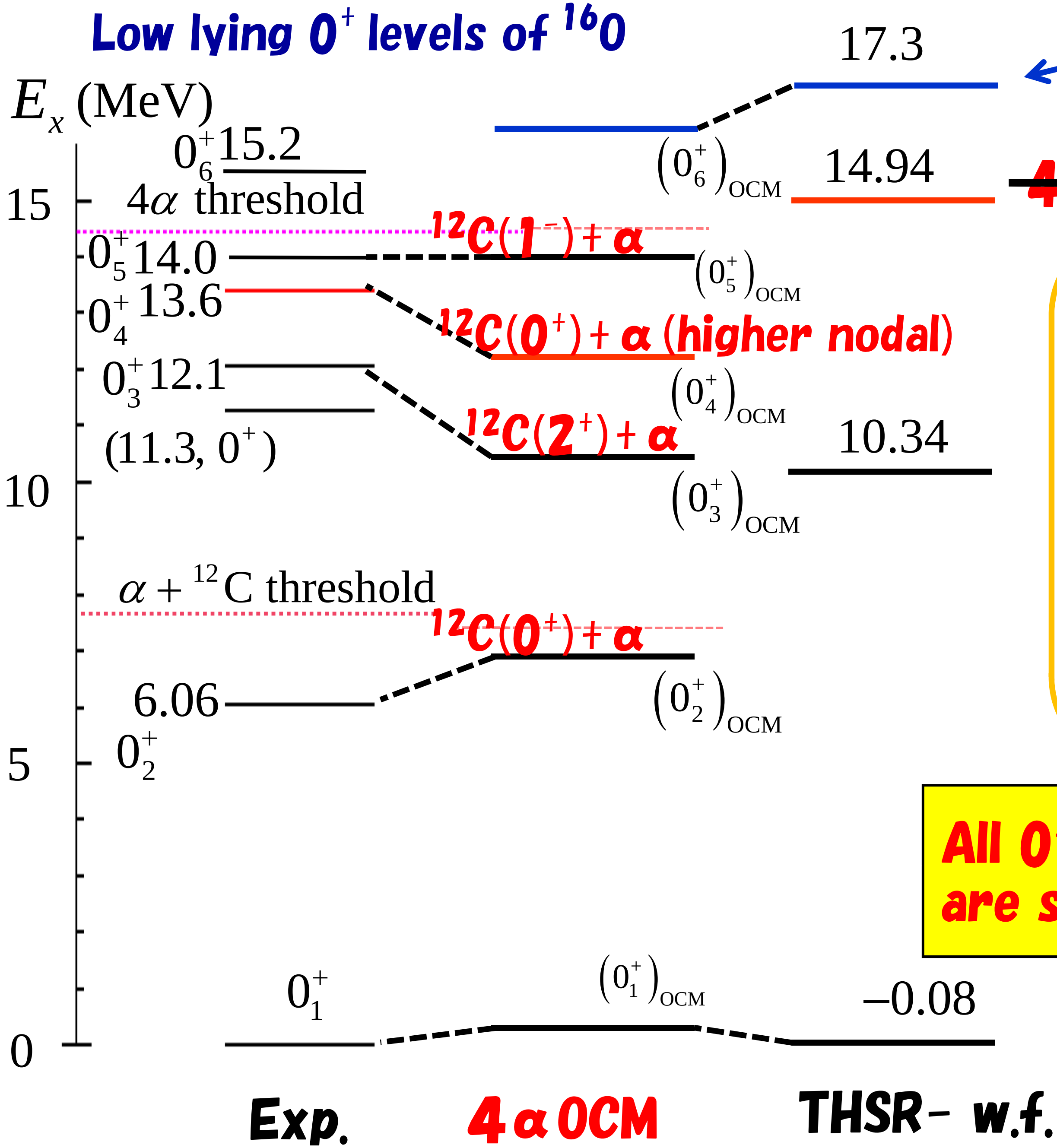
$$\int dr' \, \rho(r,r') \, f^{\lambda}(r') = \mu^{\lambda} f^{\lambda}(r)$$

$$\rho(r,r') = 4\int \Phi_{0_k^+}^{\;*}(r, r_2, r_3, r_4)\Phi_{0_k^+}(r', r_2, r_3, r_4) dr_2\, dr_3\, dr_4$$

$\lambda = (n, L)$, μ^{λ} : single particle occupancy, $f^{\lambda}(r)$: single particle orbit

$$\sum_{\lambda} \mu^{\lambda} = 4$$

Possible assignment of the two calculations and observations



$\Gamma(0_4^+)_{theor} \sim 0.2$ MeV
$\Gamma(0_5^+)_{theor} < 0.05$ MeV
(calculated based on R-matrix theory)

$\Gamma(0_4^+$ at 13.6 MeV$) = 0.6$ MeV
$\Gamma(0_5^+$ at 14.0 MeV$) = 0.17$ MeV

All 0^+ states up to 4 α threshold are simultaneously reproduced !

Summary

- Beyond doubt the Hoyle state is the 3α condensate state. (THSR and OCM)

- 4α condensate w. f. (4α THSR-w.f.) predicts the existence of 4α condensate state. (not as the third 0^+ state but as the fourth 0^+ state)

Analysis by using 4α OCM(orthogonality condition model) in order to describe both $^{12}C+\alpha$, 4α gas states and others, if any, in larger model space.

4α condensate state (0_6^+) and other cluster states are simultaneously obtained. Large 0S-occupancey, 60 %

- Two new resonance states are obtained near the 4α threshold.
  One has a developed α cluster structure (R_{rms}~3.0 fm) in which $^{12}C(1^-)+\alpha$, $^{12}C(3^-)+\alpha$ and $^{12}C(0_2^+)+\alpha$ components are mixed.
  The other has a very well developed α cluster structure (R_{rms}~4.0 fm).
  $^{12}C(0^+)+\alpha$ (higher nodal)
  ⇒ corresponding to the observed 0_4^+ and 0_5^+ states, respectively
- Successfully reproducing the well known $^{12}C(g.s.)+\alpha$ (6.05 MeV) (0_2^+) and $^{12}C(2^+)+\alpha$ (12.05 MeV) (0_3^+) structures,

For future work,

- Analyses of condensate fraction and 4α CSM are necessary for more reliable conclusion.
- 4α linear chain structure (the band head is estimated at 16.7 MeV)
  Another type of 4α structure !
  We are able to discuss it simultaneously with the lower structures !