Momentum Distributions and Related Observables in Light Nuclei

J. Carlson - LANL

- Introduction
- Momentum Distribution
- Back-to-Back Nucleons
- Inclusive lepton scattering
- Low-momentum processes
  (weak decays, …)
- Conclusion

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Momentum Distribution (1- and 2-body)

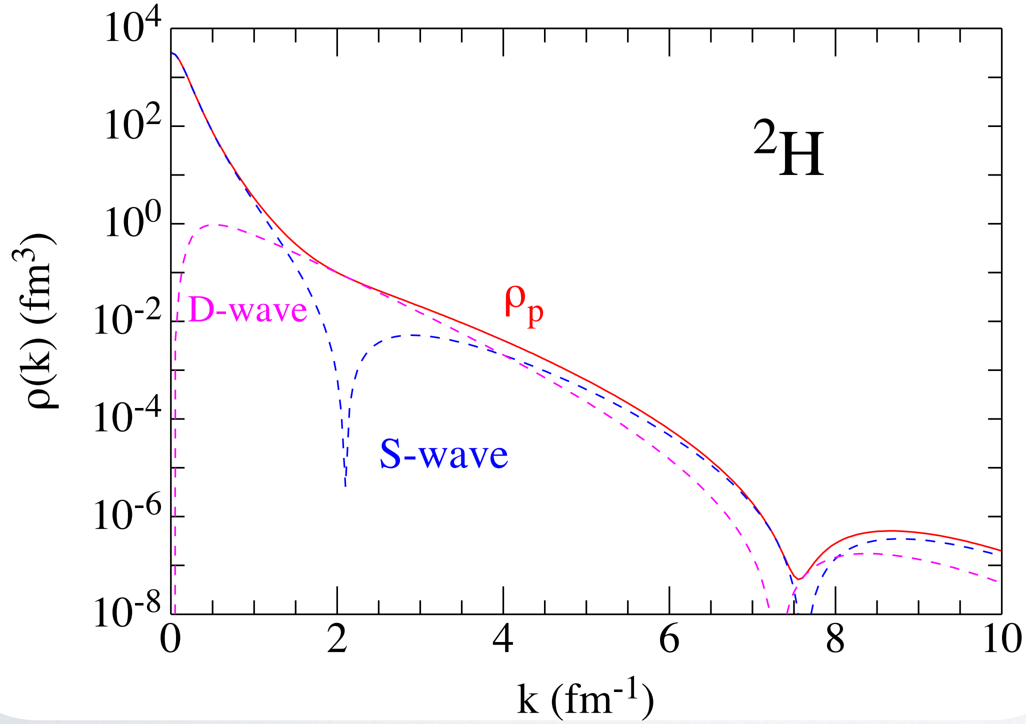
Not directly observable but influence many observables

1-body momentum distribution from 1-body off-diagonal density matrix

$$n(k) \;=\; \langle \Psi_0(r', ...)| \;\exp[i\mathbf{k}\cdot(\mathbf{r}' - \mathbf{r})]\; |\Psi_0(r, ...)\rangle$$

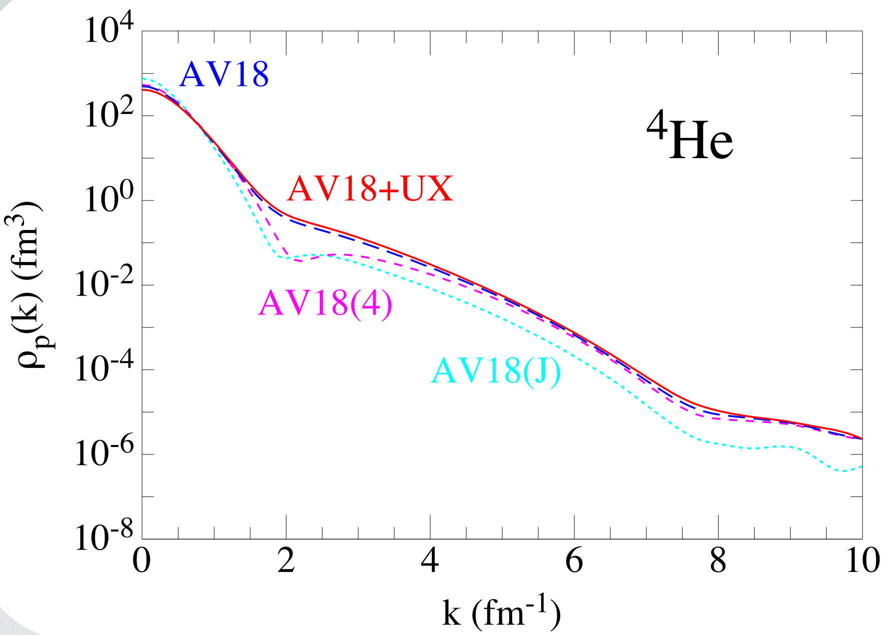
2-body momentum distribution

$$n(k_r, P) \;=\; \langle \Psi_0(r'_1, r'_2 ...)| \;\exp[i\mathbf{k_r}\cdot(\mathbf{r'_{12}} - \mathbf{r_{12}})]\exp[i\mathbf{P}\cdot(\mathbf{R'_{12}} - \mathbf{R_{12}})]\; |\Psi_0(r_1, r_2, ...)\rangle$$



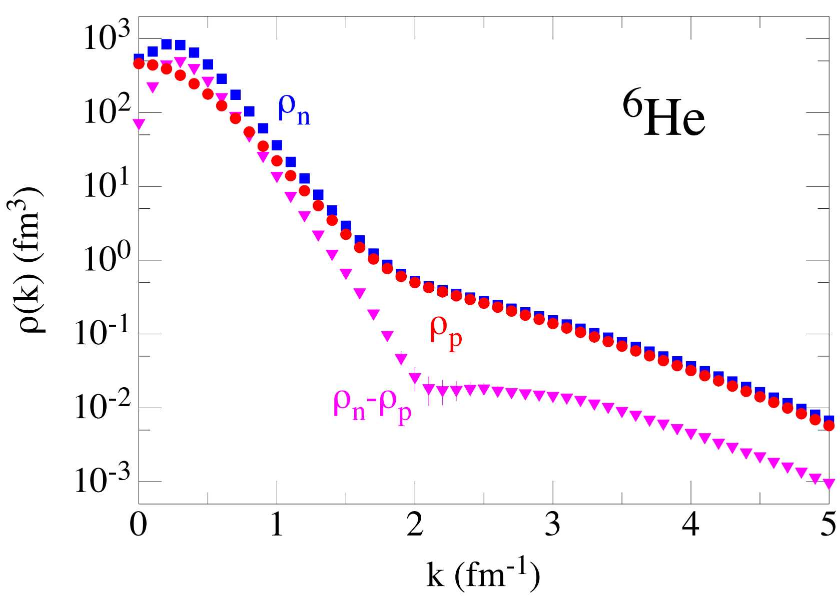
- n(k) near k=0 governed by large size of deuteron
- D-wave contributes at k ≳ 2 fm^{-1}
- Dominated by Pion (tensor) correlations

A=4,6 momentum distributions



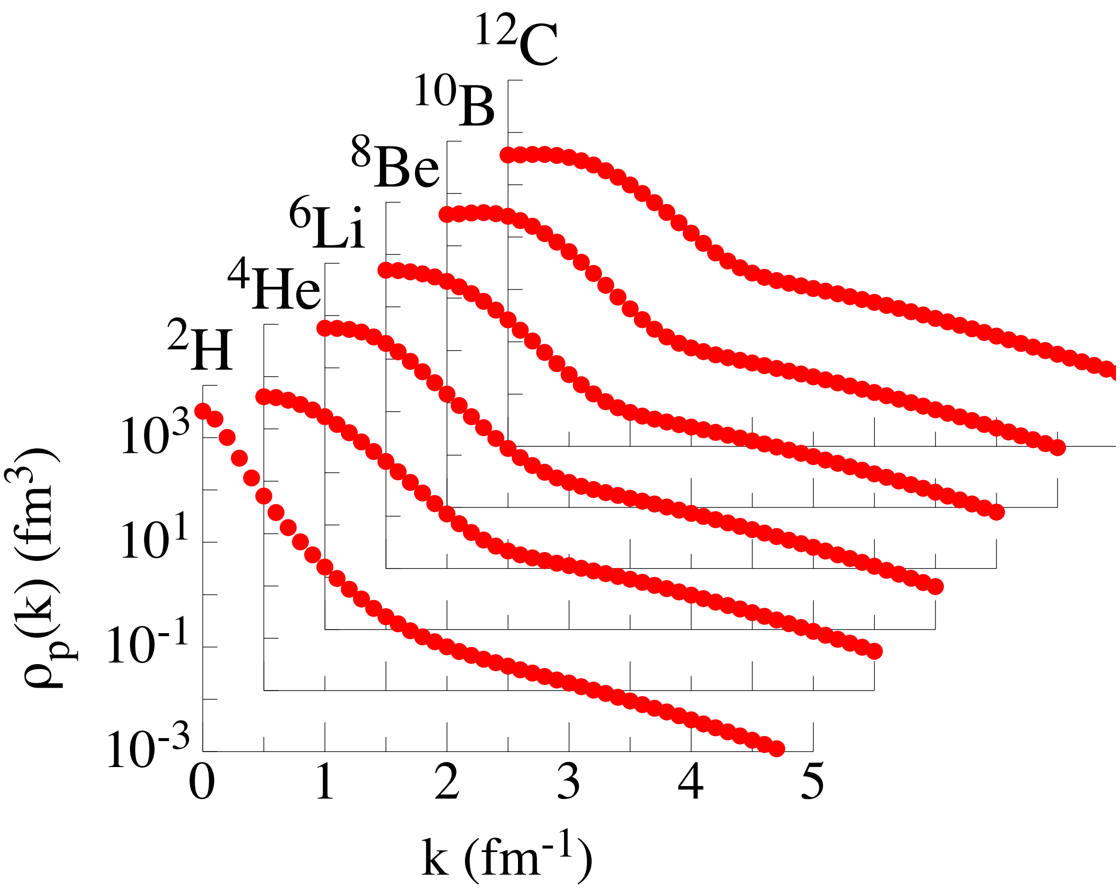
^{4}He with different interactions

Tensor interaction (pion) very important around 2 fm^{-1}

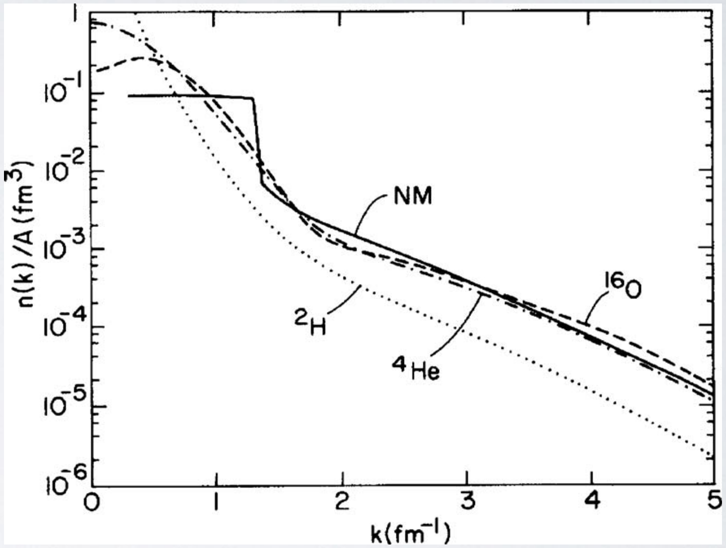


^{6}He at large momenta proton and neutron distributions are similar (alpha core)

Proton (1-body) momentum distributions from A=2..12, 16 …



Nuclear Matter

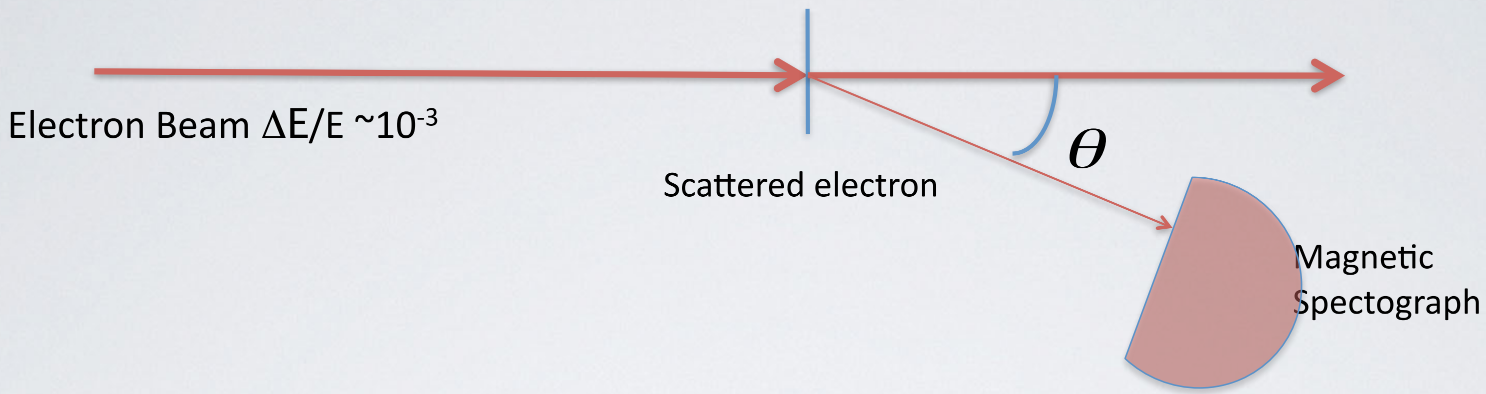


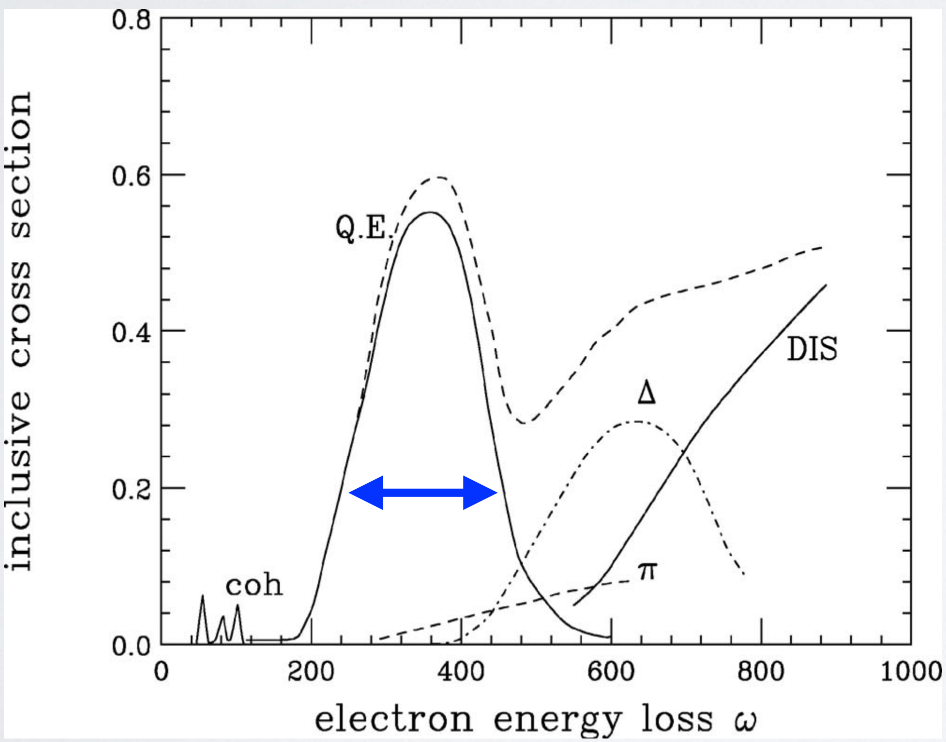
Schiavilla, et al 1986, Benhar, et al 1993

$k_F \sim 1.35$ fm^{-1} in large nuclei

Single Nucleon n(k) is a property of the 'average' nuclear medium

Inclusive Electron Scattering





$$(E, 0, 0, p), \quad (E', p'\sin\theta, 0, p'\cos\theta)$$

$$\omega \equiv E - E'$$

$$\vec{q} = \vec{p} - \vec{p}\,'$$

Thus q and ω are precisely known <u>without any reference to the nuclear final state</u>

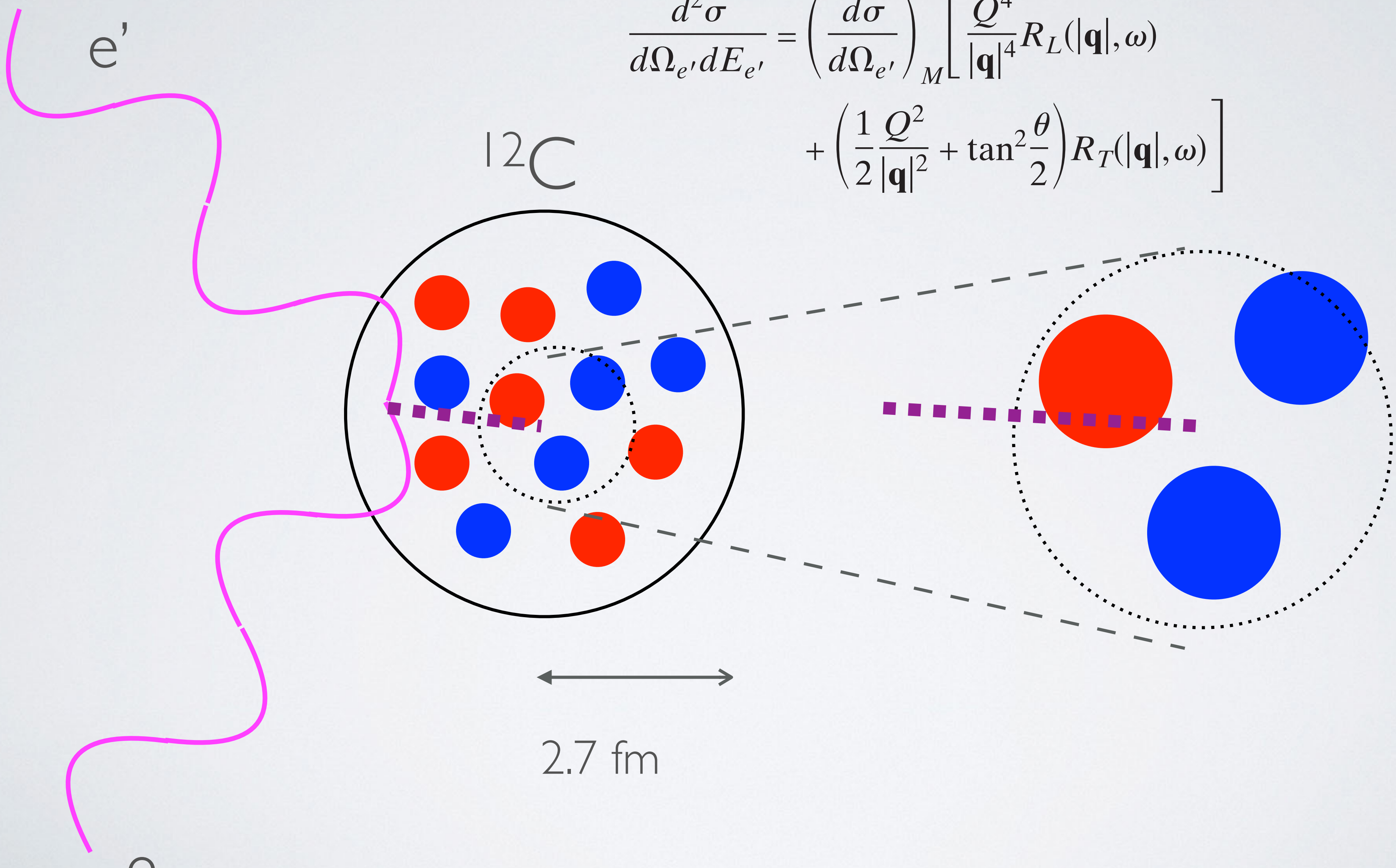
from Benhar, Day, Sick, RMP 2008

in PWIA width governed by momentum distribution

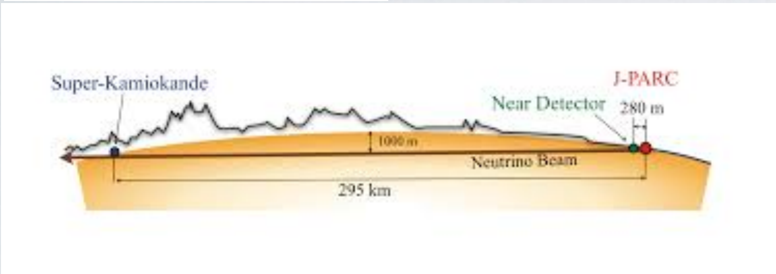
Inclusive electron scattering,

measure electron kinematics only

$$\frac{d^2\sigma}{d\Omega_{e'}dE_{e'}} = \left(\frac{d\sigma}{d\Omega_{e'}}\right)_M \left[\frac{Q^4}{|\mathbf{q}|^4} R_L(|\mathbf{q}|,\omega) + \left(\frac{1}{2}\frac{Q^2}{|\mathbf{q}|^2} + \tan^2\frac{\theta}{2}\right) R_T(|\mathbf{q}|,\omega)\right]$$

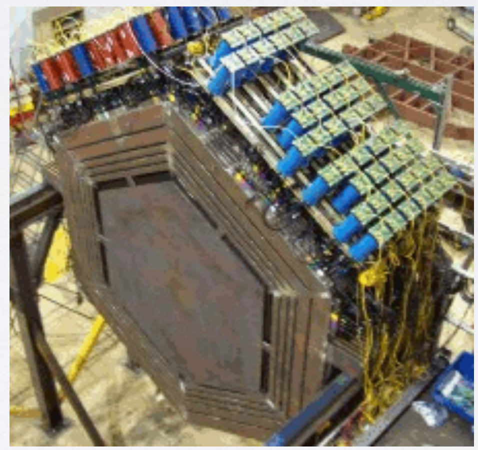


Accelerator Neutrinos

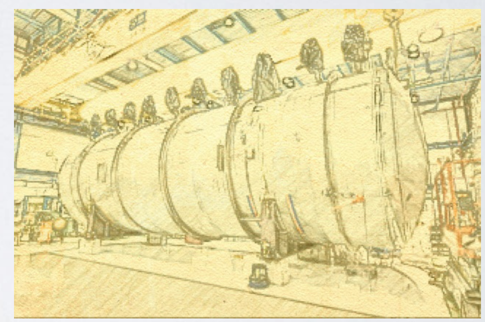


MINOS

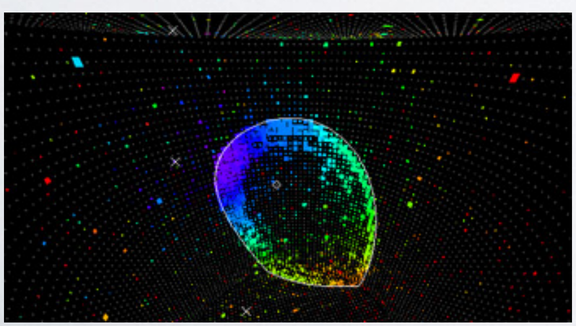
SuperK



MINERva



MicroBooNE



Advantages: Control over Energy, flux

neutrino 'beams' can be sent over long distances

Why are 'local' properties enough?
Simple view of Nuclei: inclusive scattering

Charge distributions of different Nuclei:

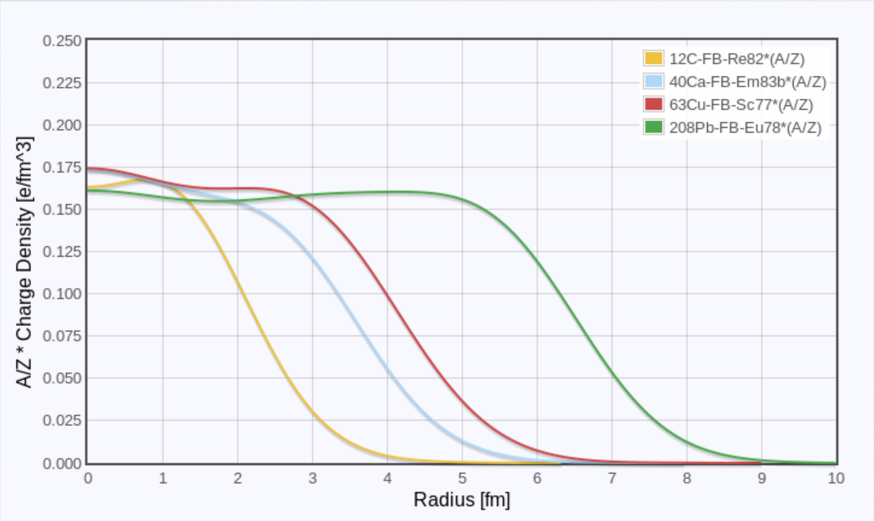
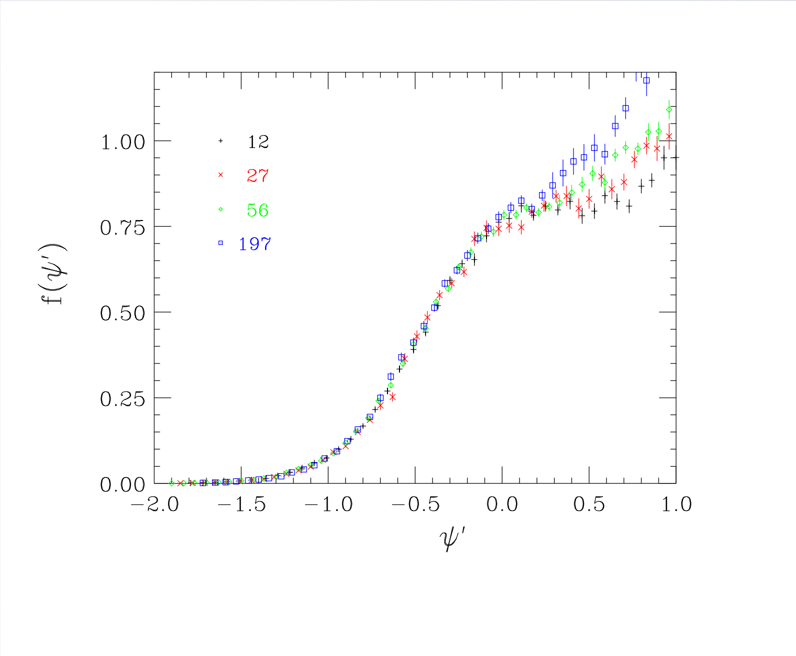


figure from faculty.virginia.edu/ncd
based on work of Hofstadter, et al.: Nobel Prize 1961

Scaling (2nd kind) different nuclei



Donnelly and Sick, 1999

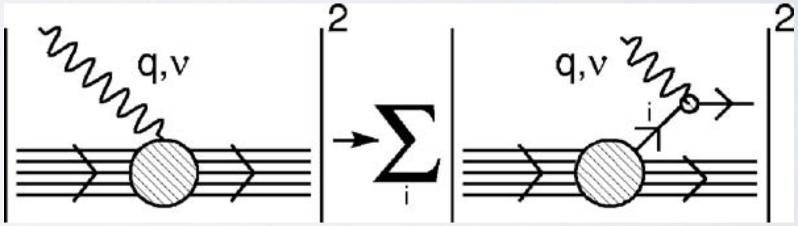
Inclusive scattering measures properties at distances ~ π / q ≲ 1 fm

Response in PWIA

$$R\ (q,\omega)\ =\ \sum_i \langle\ 0\ |\rho_i^\dagger(q;r')\ \rho_i(q;r)|0\rangle\ \delta(E_F - E_I - \omega)$$

Requires one-body momentum distribution

$E_F = (q+k)^2/(2m) + \Delta$ can include a mean-field shift



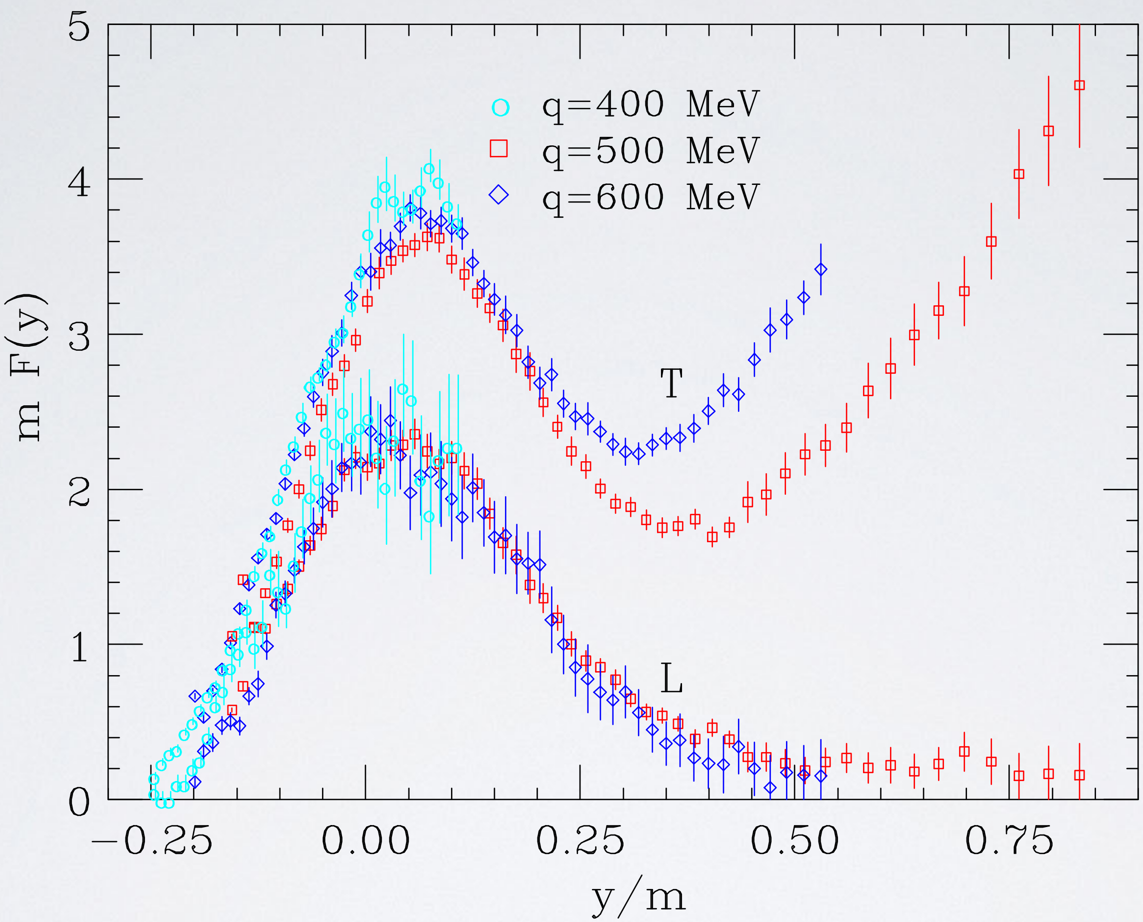
Spectral function:

includes energy of A-1 particles not interacting with the probe

$$R\ (q,\omega)\ =\ \sum_i \sum_f \langle 0\ |a_i^\dagger(q;r')|f_{A-1}\rangle\ \langle f_{A-1}|\ a_i(q;r)|0\rangle\ \delta(E_F - E_I - \omega)$$

$E_F = (q+k)^2/(2m) + \Delta + E_{f,A-1}$

Longitudinal/Transverse separation in electron scattering: ^{12}C

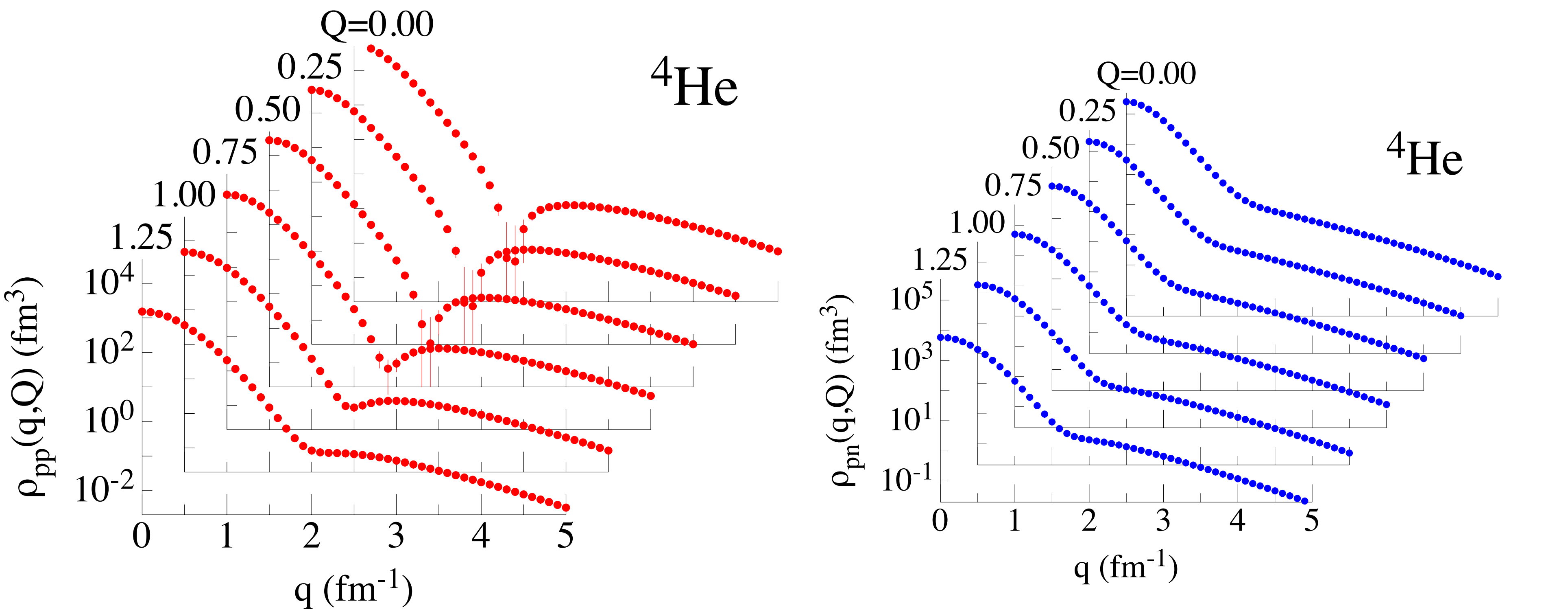


from Benhar, Day, Sick, RMP 2008
Benhar, arXiv: 1501.06448
data Finn, et al 1984

Single nucleon FF divided out;
T/L > 1 implies more than 1-body physics
PWIA or spectral fn not sufficient

Two-nucleon momentum distributions

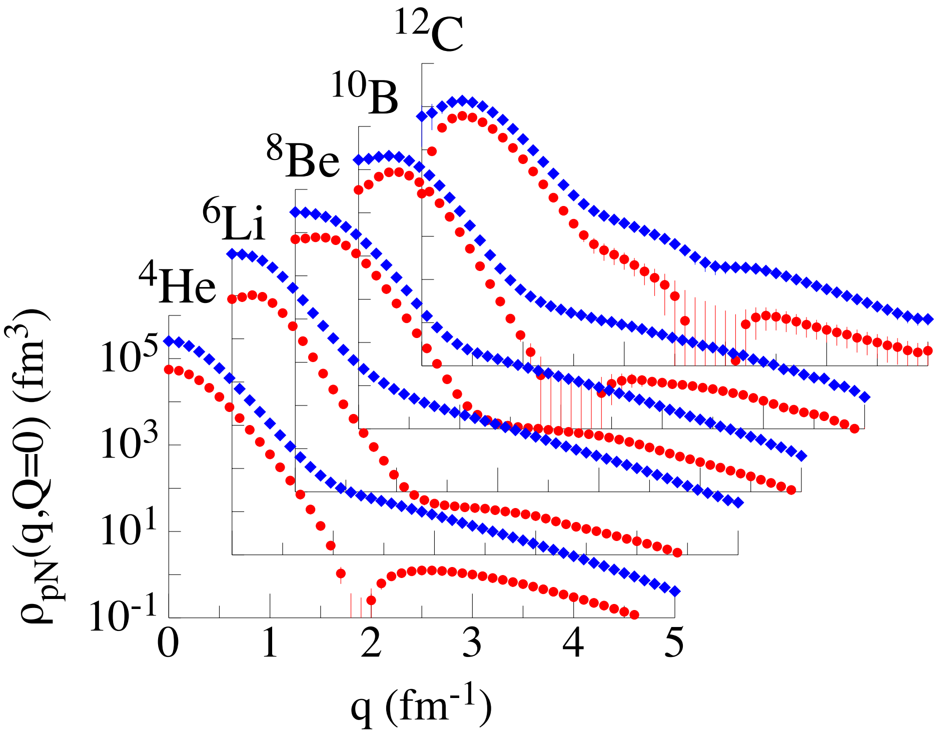
pp versus np 2-body momentum distributions in 4He



CM momentum near 0 emphasizes back-to-back (nearby) pairs

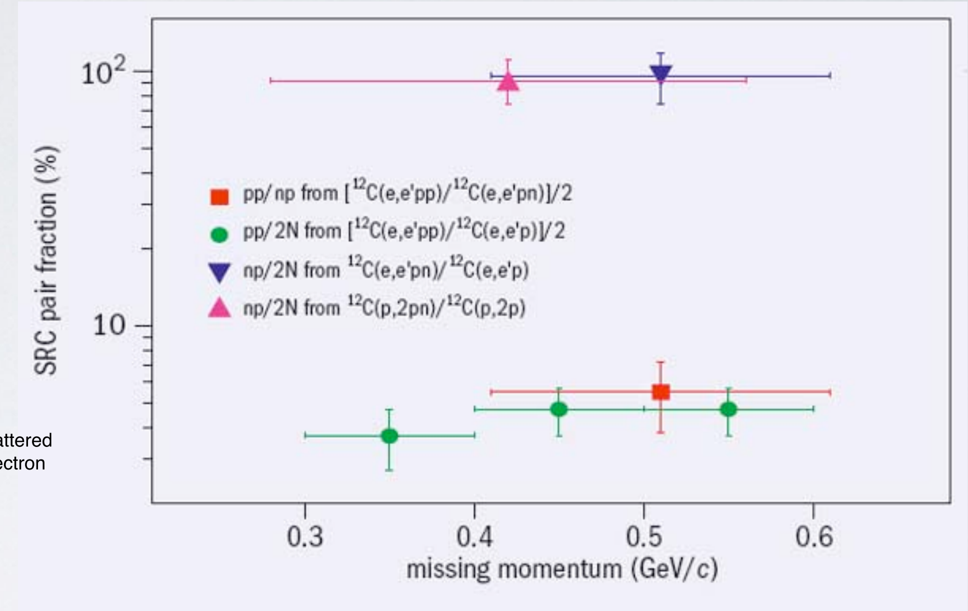
np dominates near q ~ 2 fm^{-1}

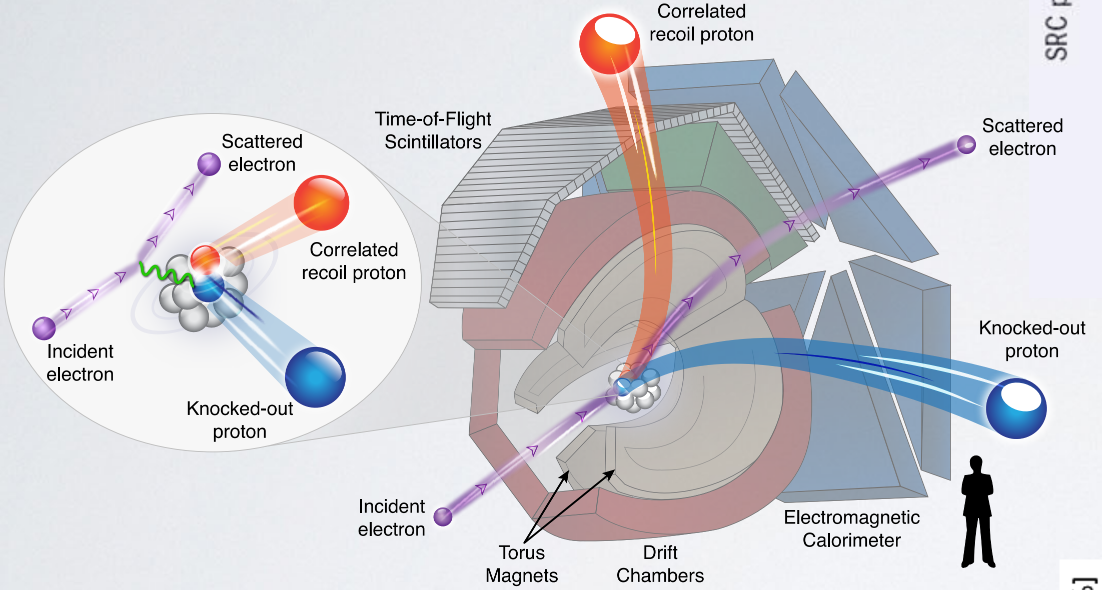
2-body momentum distributions in light nuclei



Some enhancement due to counting, but
np momentum distribution >> nn or pp at $q > k_F$

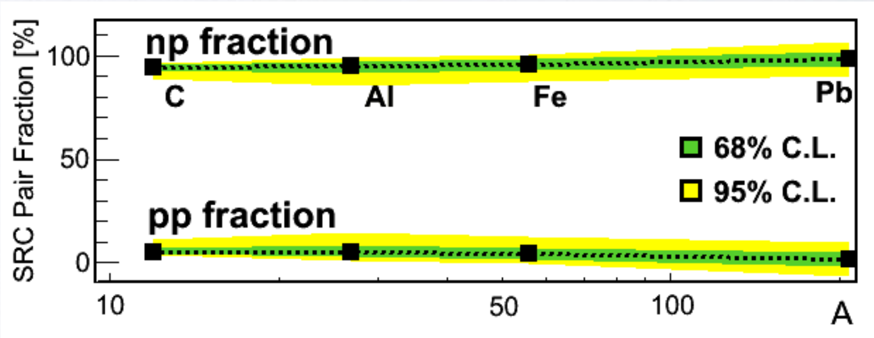
JLAB, BNL back-to-back pairs in ^{12}C np pairs dominate over nn and pp





Subedi, et al, Science (2008)

Large Nuclei



E Piasetzky *et al.* 2006 **Phys. Rev. Lett. 97** 162504.
M Sargsian *et al.* 2005 **Phys. Rev. C 71** 044615.
R Schiavilla *et al.* 2007 **Phys. Rev. Lett. 98** 132501.
R Subedi *et al.* 2008 **Science 320** 1475.

Hen, et al, Science (2014)

‘Complete’ Electron, Neutrino Scattering

$$R_{L,T}\ (q,\omega)\ = \sum_f\ \delta(\omega\ +\ E_0\ + E_f)\ |\ \langle\ f\ |\mathcal{O}_{\mathcal{L},\mathcal{T}}\ |\ 0\ \rangle\ |^2$$

Easy to calculate Sum Rules: ground-state observable

$$S(q)\ =\ \int d\omega\ R(q,\omega) = \langle 0|O^\dagger(q)\ O(q)|0\rangle$$

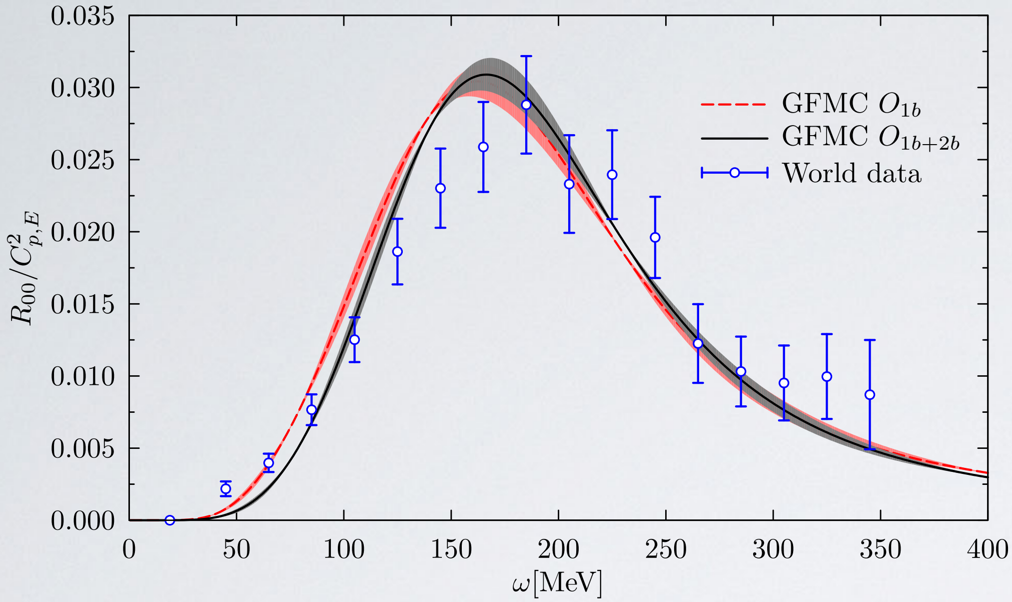
Imaginary Time (Euclidean Response)
statistical mechanics
inversion with Maximum Entropy

$$\tilde{R}(q,\tau)\ =\ \langle 0|\ \mathbf{j}^\dagger\ \exp[-(\mathbf{H}-\mathbf{E_0}-\mathbf{q^2}/(\mathbf{2m}))\tau]\ \ \mathbf{j}\ |\mathbf{0}\rangle >$$

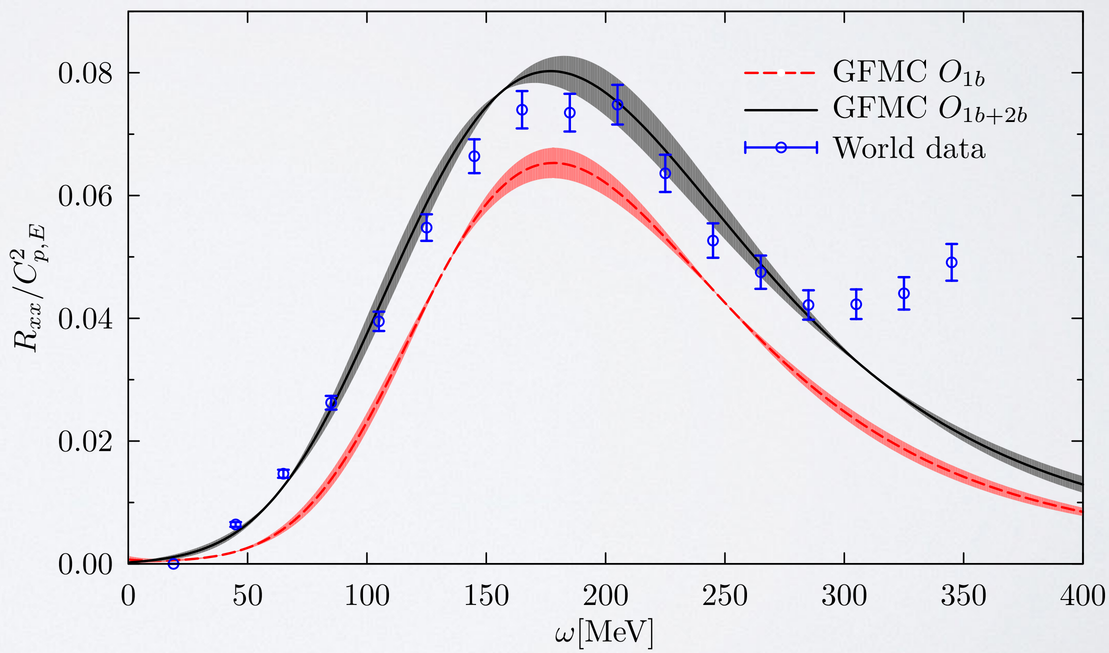
$$H\ = \sum_i \frac{p_i^2}{2m}\ + \sum_{i<j} V_{ij}\ +\ \sum_{i<j<k} V_{ijk}$$

$$\mathbf{j}\ = \sum_i\ \mathbf{j}_i\ +\ \sum_{i<j}\ \mathbf{j}_{ij} + ...$$

^{12}C electron scattering inverting Euclidean Response - Lovato



Longitudinal



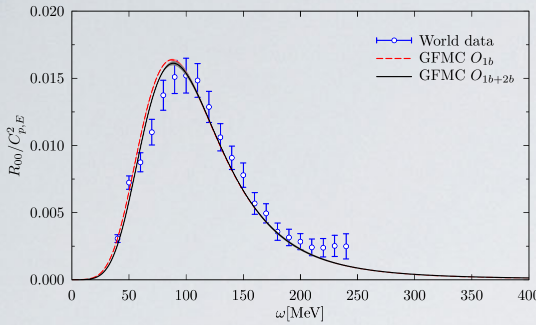
Transverse

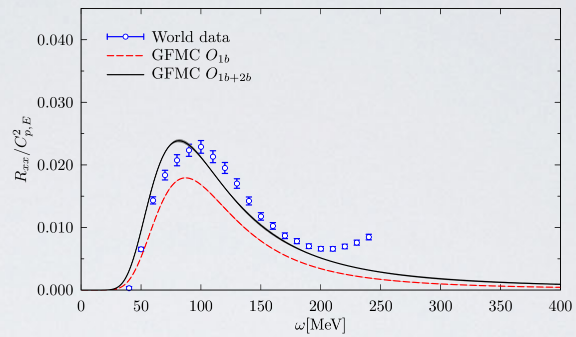
^4He

Longitudinal

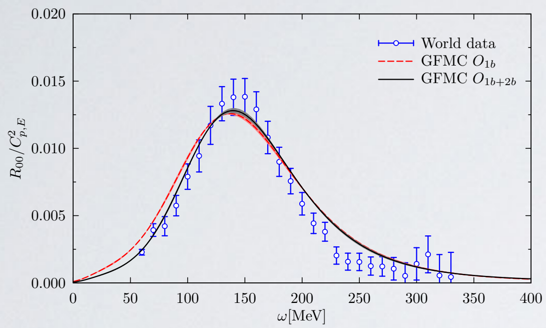
Transverse

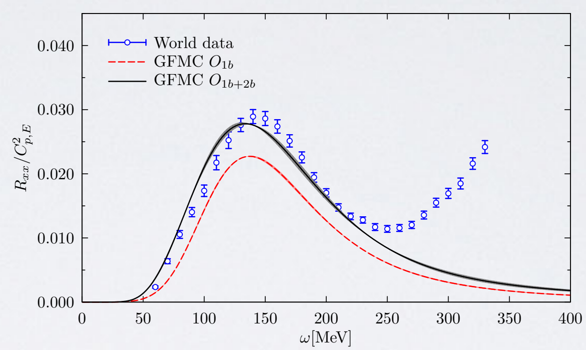
q=400



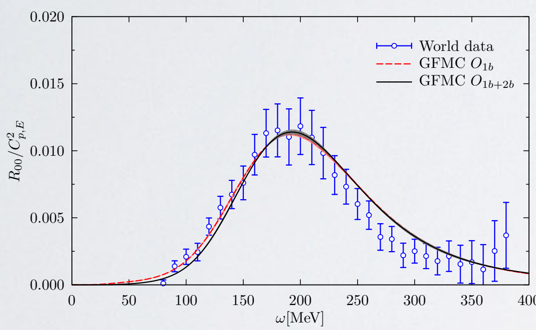


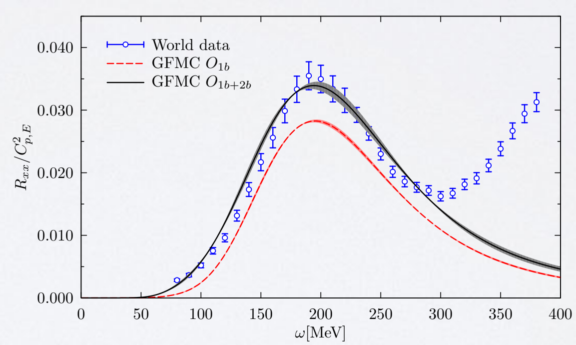
500



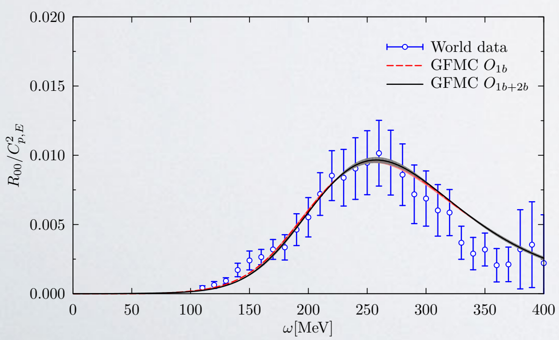


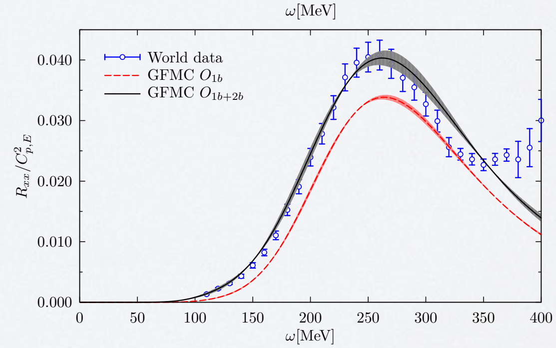
600





700





Large Transverse Enhancement in Electron Scattering

Single-Nucleon Currents

1+2 Nucleon Currents

Lovato, et al: arXiv:1501.01981

Neutrino Scattering Involves 5 response functions

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\epsilon'\mathrm{d}\Omega}\right)_{\nu/\overline{\nu}} = \frac{G_F^2}{2\pi^2}\,k'\epsilon'\cos^2\frac{\theta}{2}\left[R_{00} + \frac{\omega^2}{q^2}\,R_{zz} - \frac{\omega}{q}R_{0z}\right.$$

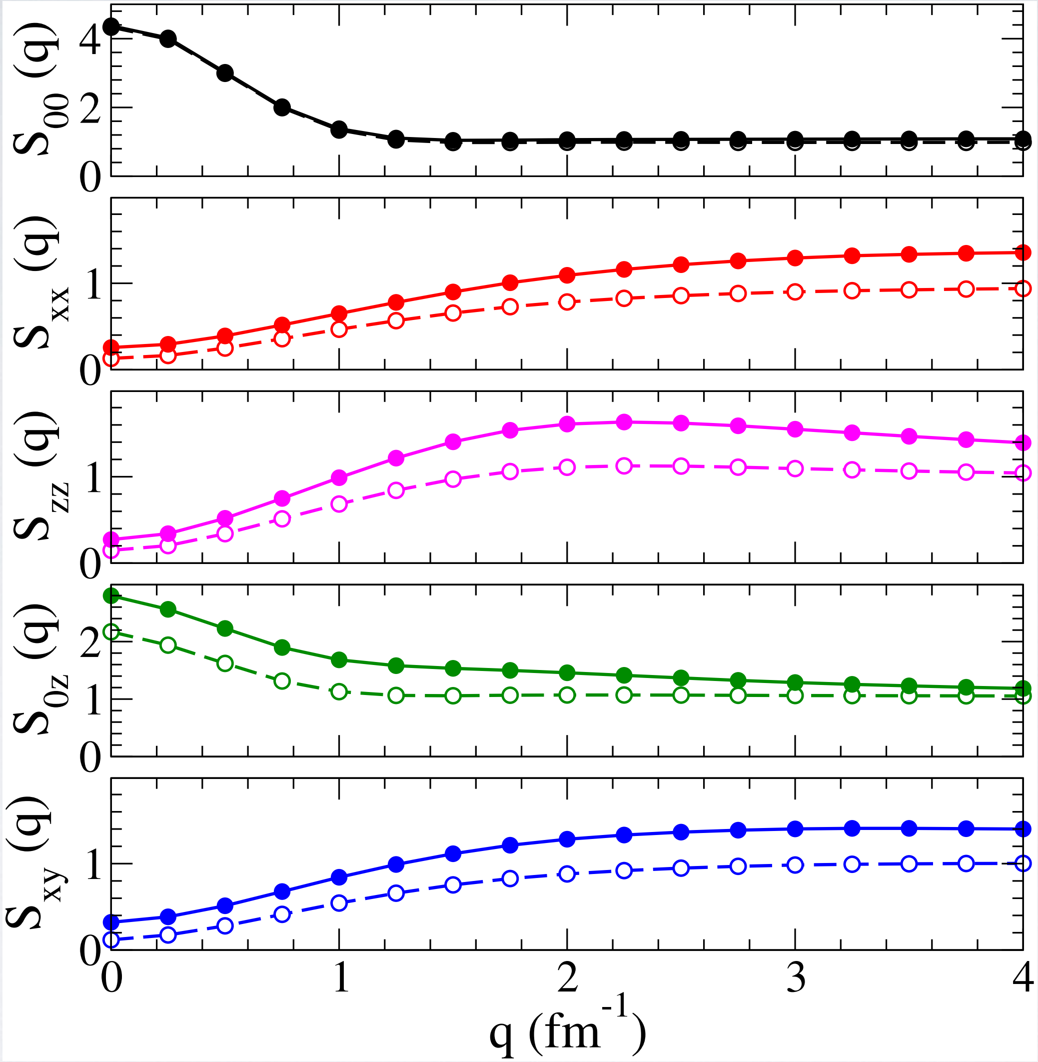
$$\left. + \left(\tan^2\frac{\theta}{2} + \frac{Q^2}{2\,q^2}\right)R_{xx} \mp \tan\frac{\theta}{2}\sqrt{\tan^2\frac{\theta}{2} + \frac{Q^2}{q^2}}\,R_{xy}\right] ,$$

$$R_{\alpha\beta}(q,\omega) \sim \overline{\sum_i}\sum_f \delta(\omega+m_A-E_f)\langle f\,|\,j^\alpha(\mathbf{q},\omega)\,|i\rangle$$

$$\times\langle f\,|\,j^\beta(\mathbf{q},\omega)\,|i\rangle^* ,$$

Vector - Axial Vector Interference determines the difference between neutrino and antineutrino scattering

Sum rules in 12C

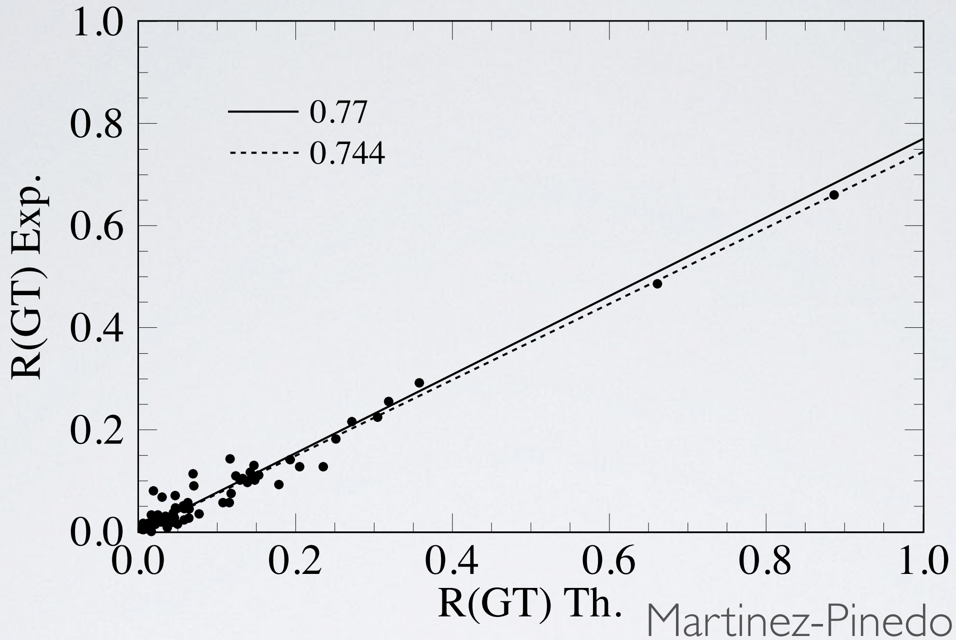


EM

Lovato, et. al PRL 2014

Single Nucleon currents (open symbols) versus
Full currents (filled symbols)

Low Momentum Observables: Beta Decay



Martinez-Pinedo and Poves, PRC 1996

g_A "quenched" by factor of ~ 0.75 in all heavy nuclei
small quenching in tritium, about 0.9 in A=6,7
role of pion-range correlations and currents
Similar questions arise in double beta decay,
even more important as rate $\propto g_A^4$

Conclusions

Measured large enhancement in back-to-back np vs. pp pairs
due to tensor (pion) correlations
In general need treatment of both correlations and currents
Very important in understanding quasi-elastic scattering
(neutrino and electron) scattering from nuclei

Outlook

More data needed for many observables and many
ranges of momentum transfer, including:

Lower energy (astrophysical) neutrinos
Strength distributions of isovector response
Beta decay and low-energy weak transitions
Neutrinoless double-beta decay