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Dispersion Matching of Stable and Radioactive Beams

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Outline

- Dispersion matching in a nutshell
- Brief summary of long history of dispersion matching
- Dispersion matching at stable beam facilities
- Dispersion matching at RI facilities

Why do we dispersion match beam lines and spectrometers?

- Resolution better than energy spread of accelerator, limited by resolving power of spectrometer $D/(M*2x_0)$
- Reconstruction of scattering angle Θ_{target} (Θ_{fp}) in dispersive plane (x); non-dispersive plane, angle $\phi(y)$, out-of-focus mode

What ion-optical parameters on target need to be "matched" to the spectrometer?

- Spacial Dispersion b16, for resolution
- Angular dispersion b26, for Θ_{target} (Θ_{fp}) reconstruction
- Focus on target b12=0, for $k = dp/(d\Theta*p) = 0$

Spacial and Angular Dispersion Matching
A Cartoon to Remember

Achromatic Beam on Target

Dispersive Beam on Target

Angular dispersion on Target

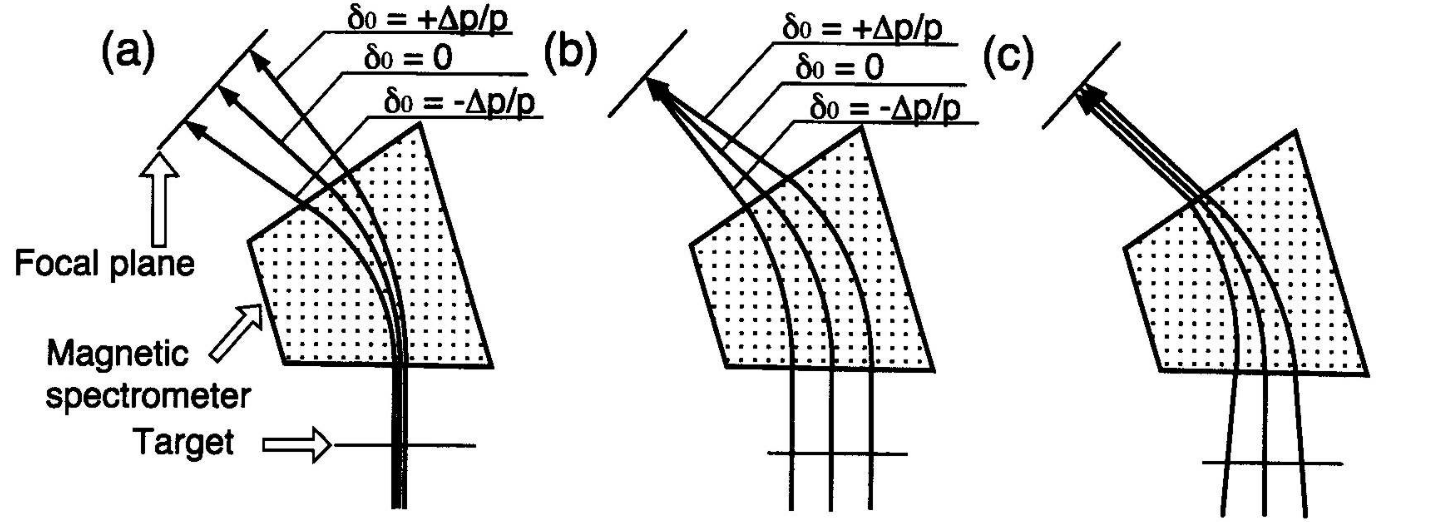


Figure 2.2: Schematic ion trajectories under different matching conditions of a beam line

Great diagnostic for beam momentum distribution

$$b_{16} = -\frac{s_{16}}{s_{11}}(1 + s_{11}\, s_{26}\, K - s_{21}\, s_{16} K)\ \frac{C}{T}$$

$$b_{26} = (s_{21}\, s_{16} - s_{11}\, s_{26})\ C$$

Defining a RAY

Code TRANSPORT:

(x, Θ, y, Φ, l, dp/p)

(1, 2, 3, 4, 5, 6)

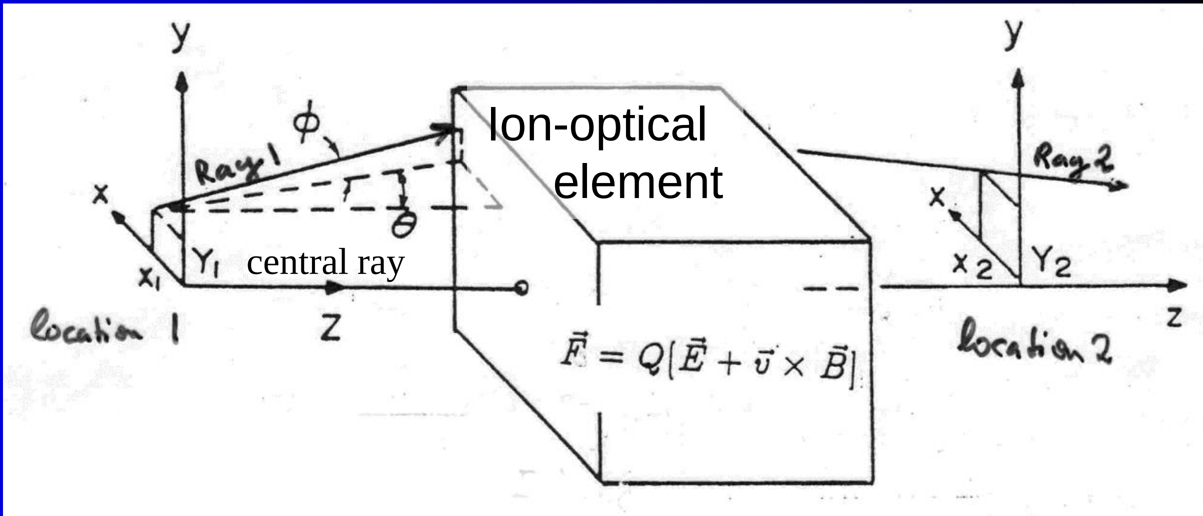
Convenient "easy to use" program for beam lines with paraxial beams

Not defined in the figure are:

dp/p = rel. momentum

l = beam pulse length

All parameters are relative to "central ray"



Code: COSY Infinity:

$(x, a, y, b, l, \delta_K, \delta_m, \delta_z)$

Needed for complex ion-optical systems including several
- charge states
- different masses
- velocities (e.g. Wien Filter)
- higher order corrections

Not defined in the figure are:

δ_K = dK/K = rel. energy

δ_m = dm/m = rel. mass

δ_z = dq/q = rel. charge change

$a = p_x/p_0$

$b = p_y/p_0$

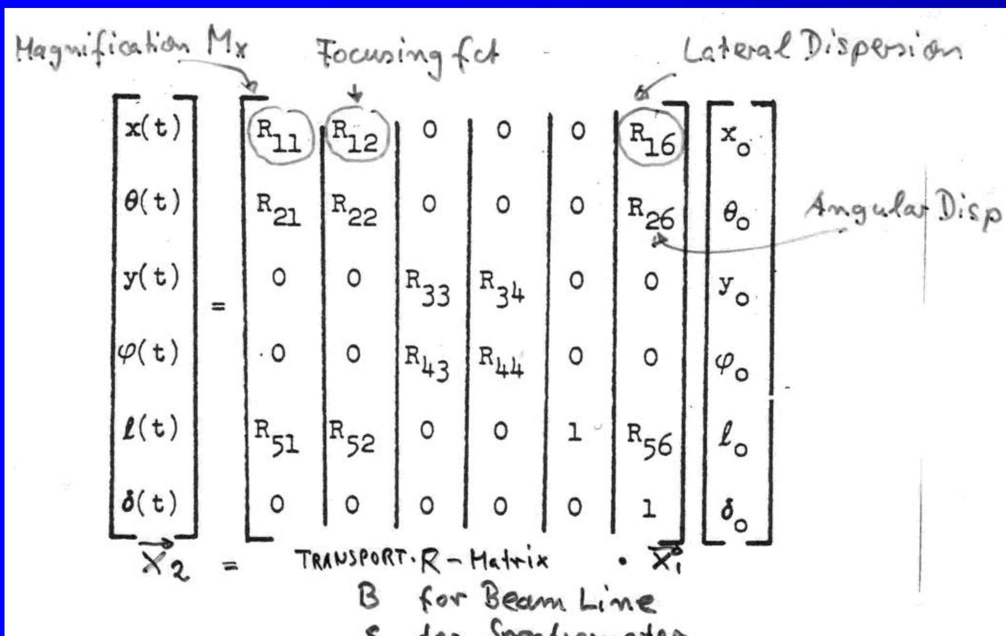
All parameters are relative to "central ray" properties

Note: Notations in the Literature are not consistent!

Transport of a ray

6x6 Matrix
representing
optic element
(first order)

↓



↑ Ray after element at Location t

↑ Ray at initial Location 0

Note: We are not building "random" optical elements. Many matrix elements = 0 because of symmetries, e.g. mid-plane symmetry

Transport of a ray through a system of beam line elements

6x6 Matrix representing first optic element (usually a Drift)

$$x_n = R_n R_{n-1} \ldots R_0 x_0$$

Ray at final Location n

Ray at initial Location 0 (e.g. a target)

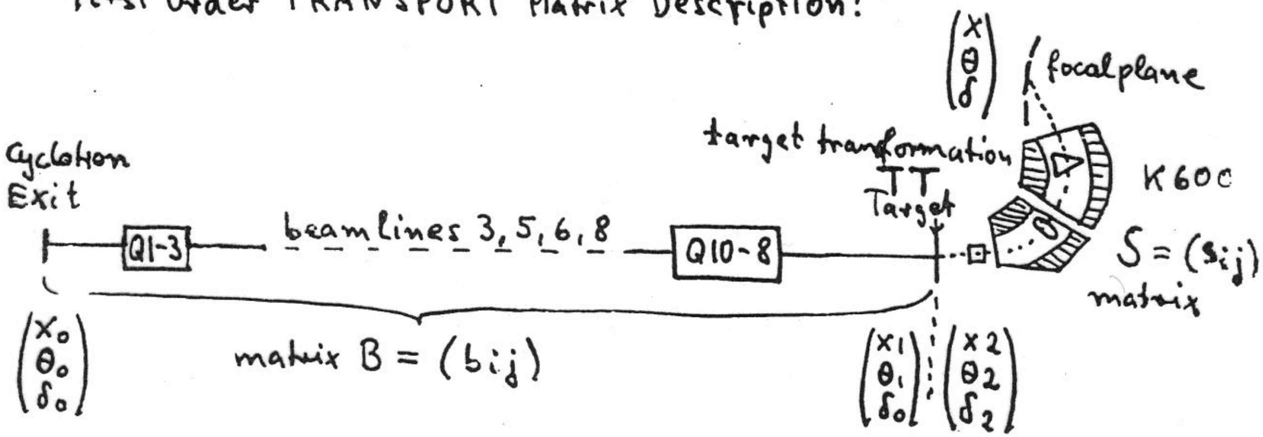
Complete system is represented by one Matrix $R_{system} = R_n R_{n-1} \ldots R_0$

Dispersion Matching

- High resolution experiments
- Secondary beam (large dp/p)

Matching between beam line and spectrometer

First Order TRANSPORT Matrix Description:



$$B = \begin{pmatrix} b_{11} & b_{12} & b_{16} \\ b_{21} & b_{22} & b_{26} \\ 0 & 0 & 1 \end{pmatrix}$$

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{16} \\ s_{21} & s_{22} & s_{26} \\ 0 & 0 & 1 \end{pmatrix}$$

TT:

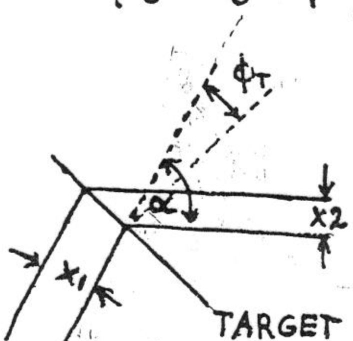
$x_2 = T\, x_1$

$\theta_2 = \Theta + \theta_1$

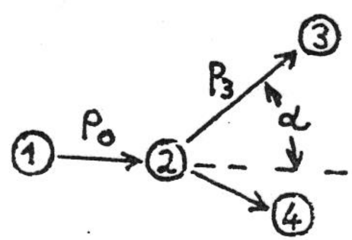
$\theta_2 =$ ~~random~~ angle within acceptance of spectrom.

$\delta_2 = K \cdot (\theta_2 - \theta_1) + C\, \delta_0$

Θ is random



$$T = \frac{\cos(\alpha - \phi_T)}{\cos \phi_T}$$



Reaction Kinematics

$$P_3 = P_3(P_0, \alpha, Q)$$

$$K = \frac{\partial P_3}{\partial \alpha} \frac{1}{P_3} \qquad C = \frac{\partial P_3}{\partial P_0} \frac{P_0}{P_3}$$

$$\begin{pmatrix} x \\ \theta \\ \delta \end{pmatrix} = S \cdot TT \cdot B \cdot \begin{pmatrix} x_0 \\ \theta_0 \\ \delta_0 \end{pmatrix}$$

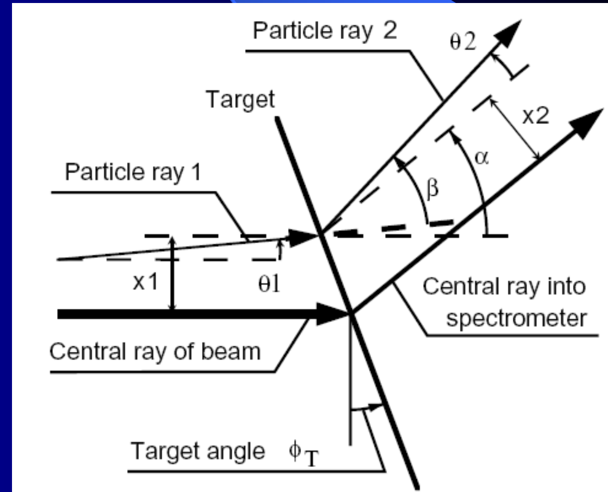


Fig. 1. Schematic layout of the incident particle 1 and the outgoing particle 2 relative to the beam and spectrometer.

Solution of first order Transport and Complete Matching

The transformation (without assuming $s_{12} = -s_{16}K$) in the bending plane from the cyclotron exit to the focal plane is given as:

$$x_{f.p.} = x_0 (s_{11} b_{11} T + s_{12} b_{21})$$
$$+\ \theta_0 (s_{11} b_{12} T + s_{12} b_{22})$$ → kin. defoc. equ. (1) **(1)**
$$+\ \delta_0 (s_{11} b_{16} T + s_{12} b_{26} + s_{16} C)$$ → disp. matching
$$+\ \theta (s_{12} + s_{16} K)$$ → kin. correction (kin. displac.)

$$\theta_{f.p.} = x_0 (s_{21} b_{11} T + s_{22} b_{21})$$
$$+\ \theta_0 (s_{21} b_{12} T + s_{22} b_{22})$$ equ. (2) **(2)**
$$+\ \delta_0 (s_{21} b_{16} T + s_{22} b_{26} + s_{26} C)$$ → angular disp. matching
$$+\ \theta (s_{22} + s_{26} K)$$

$$\delta_{f.p.} = K \cdot \theta + C \delta_0$$

For details see: Y. Fujita et al., NIM B 126 (1997) 274

Complete Matching

For best **Resolution** in the focal plane, minimize the coefficients of all terms in the expression of **x f.p.**

For best **Angle Resolution** Minimize Coefficients of δ_0 in expression of **Y** f.p.

Note: Also the beam focus **b12** on target is important (**b12** = 0 for kinem. k = 0)

Spacial Dispersion Matching:
D.L. Hendrie In: J. Cerny, Editor, *Nuclear Spectroscopy and Reactions, Part A*, Academic Press, New York (1974), p. 365.

Hendrie, Dispersion Matching $b_{16} = -\frac{D}{M} * \frac{C}{T}$

D = s_{16} = Spectrometer dispersion

M = s_{11} = Spectrometer magnification

Spacial and Angular Dispersion Matching

Solutions for b_{16} and b_{26} under conditions that both δ_0-coefficients = 0 in **(1)** and **(2)**

$$s_{11}\, b_{16}\, T + s_{12}\, b_{26} + s_{16}\, C = 0$$

$$s_{21}\, b_{16}\, T + s_{22}\, b_{26} + s_{26}\, C = 0$$

Solutions:

$$b_{16} = -\frac{s_{16}}{s_{11}}(1 + s_{11}\, s_{26}\, K - s_{21}\, s_{16} K)\, \frac{C}{T} \quad \textbf{(19)}$$

Spacial Dispersion Matching

$$b_{26} = (s_{21}\, s_{16} - s_{11}\, s_{26})\, C \quad \textbf{(20)}$$

Angular Dispersion Matching

$$b_{12} = -\frac{s_{12}\, b_{22}}{s_{11}\, T} = \frac{s_{16}\, b_{22}\, K}{s_{11}\, T} \quad \textbf{(21)}$$

Focusing Condition

Brief History of Dispersion matching

- 1956 Early spectrometers, MIT, ND (Browne-Buechner), effects on resolution
- 1974 D.L. Hendrie, - D*C/(M*T), target functions T,C, k defined and discussed
- 1978 Big Karl, disp. matched BL,ion-optics,insufficient diagn.,S. Martin, K. Brown
- 1986 K600, IUCF, Disp. Matching incl. angular dispersion, improved diagnostics, k>0 matching, 0 deg measurements, angle reconstruction.
- 1994, 1996 Study group to develop disp. Matching for GRAND RAIDEN (M. Fujiwara), lead: Y. Fujita, K. Hatanaka, T. Wakasa, T.Kawabata et al., H. Ejiri secured funds from Japanese government for fully dispersion matched WS course.
- 2000 Grand Raiden, developm. WS incl. all known effects and diagnostics, k=0 disp. matching. Resolv. Power limit of about p/dp =37000 at 300 – 400 MeV (p,p')
- Grand Raiden unique (one on this planet) high Resol. facility to study (GT fine structure with 20- 30 keV at 140 MeV/u, Yoshi Fujita, (K600 E(3He) ~ 70Mev/u)
- 2008 K600, iThembaLABS (Ricky Smit, R. Neveling): Successful Int'l initiative (Japan (Hiro Fujita, Yoshi Fujita), Germany (P. von Neumann-Cosel, USA(GB) to implement dispersion matching incl. 0 deg measurements.
- 2006 T. Kawabata design of Matching for RI beam at BigRIPS/SHARAQ system.
- > 2015 Future developments of High Energy Spectrometers at RI beam facilities, e.g. FAIR, LEBS, H. Geissel, H.Weick, J. Winfield; FRIB, HRS, Remco, GB.

BIG KARL Spectrometer (Juelich, KFZ)

Bending radius ρ_0 = 1.98 m
B_{max} = 1.7 T
Gap = 6cm
Weight = ~ 50 tons (D1)
~ 70 tons (D2)

Resolv. power: p/Δp = 0 - 20600
Dispersion = -2.0 to 26 cm/%
Magnification M_x = 0.63 – 1.26
Magnification M_y = 25.4 – 1.94
Large range: E_{min} /E_{max} = 1.14
Solid angle: < 12.5 msr

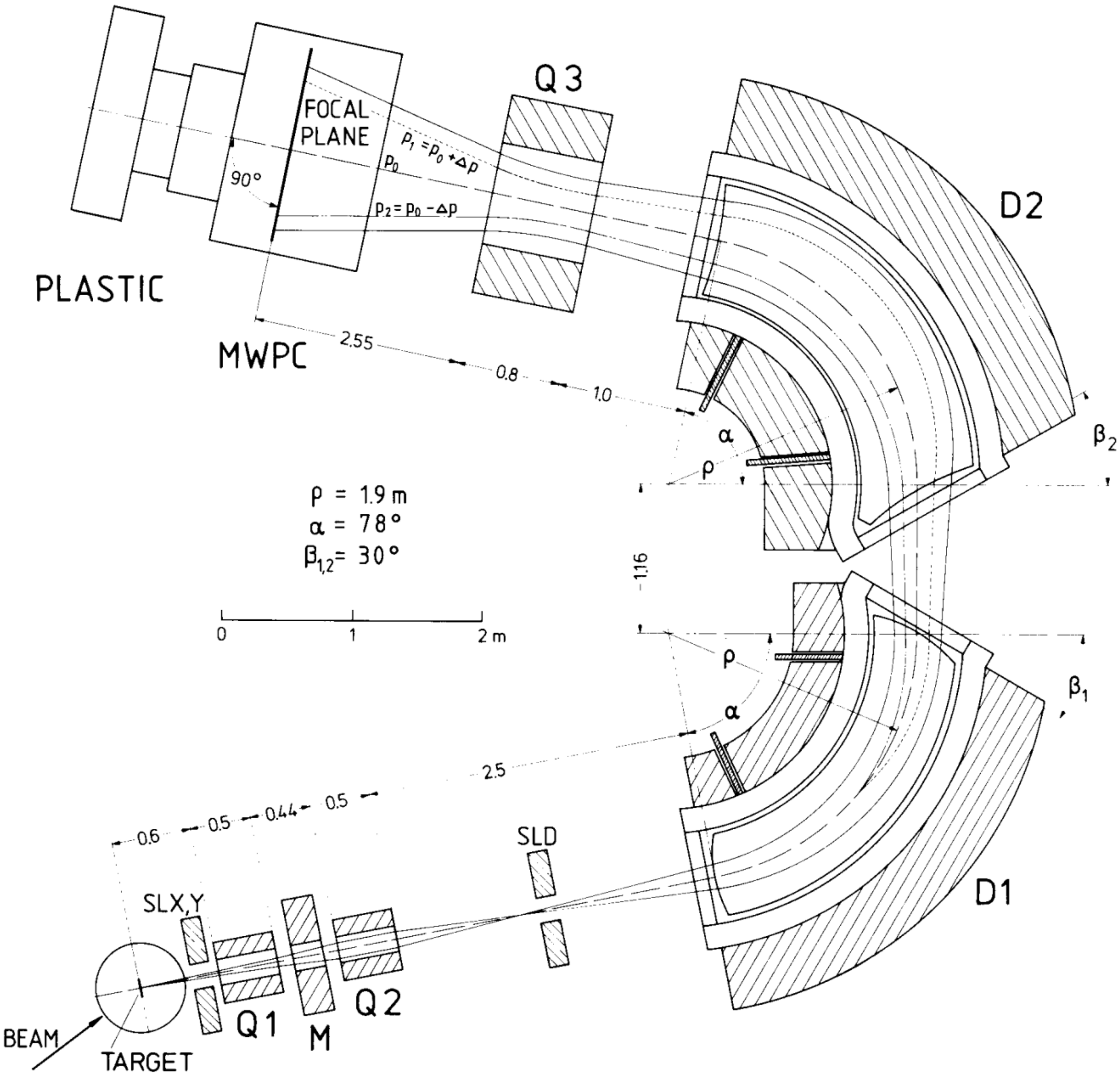


Fig. 9. Arrangement of the magnetic elements of the QQDDQ spectrometer BIG KARL. The central ray (optical axis) is shown as dashed curve. The outermost rays with the extreme radial distances are drawn as full lines. Four channels in the inner yokes allow NMR probes to be moved into the gaps of the dipoles for radial field measurements. The multipole element between Q1 and Q2 allows the correlation of vertical aberration.

BIG KARL Sample Spectra

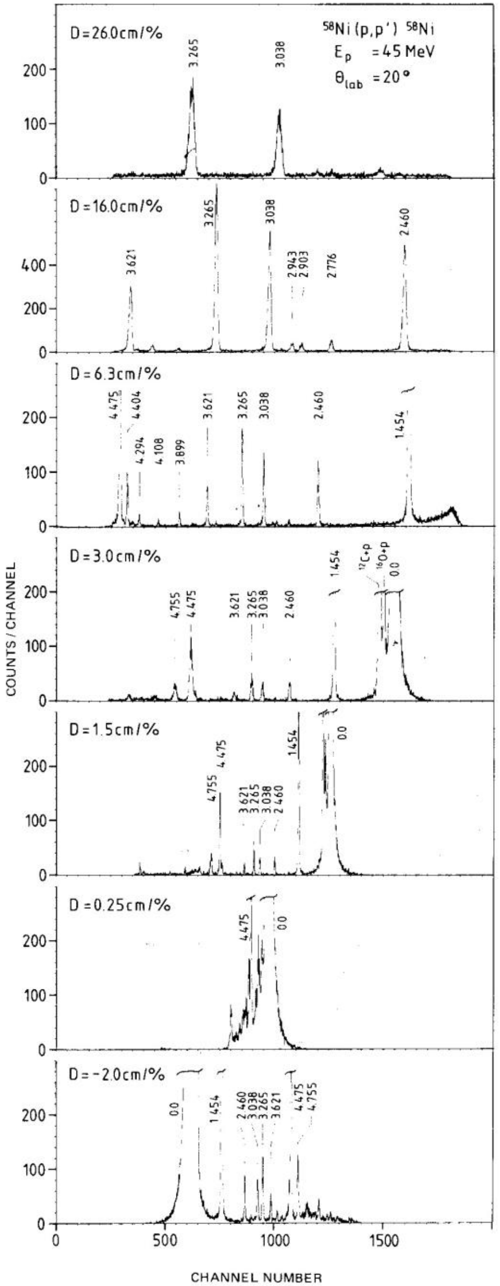


Fig. 4. Spectra of ^{58}Ni(p, p′) measured for different dispersions $D = 26$, 16, 6.3, 3, 1.5, ~ 0.25 and -2 cm/%. The spectrograph was optimized for $D = 16$ cm/%.

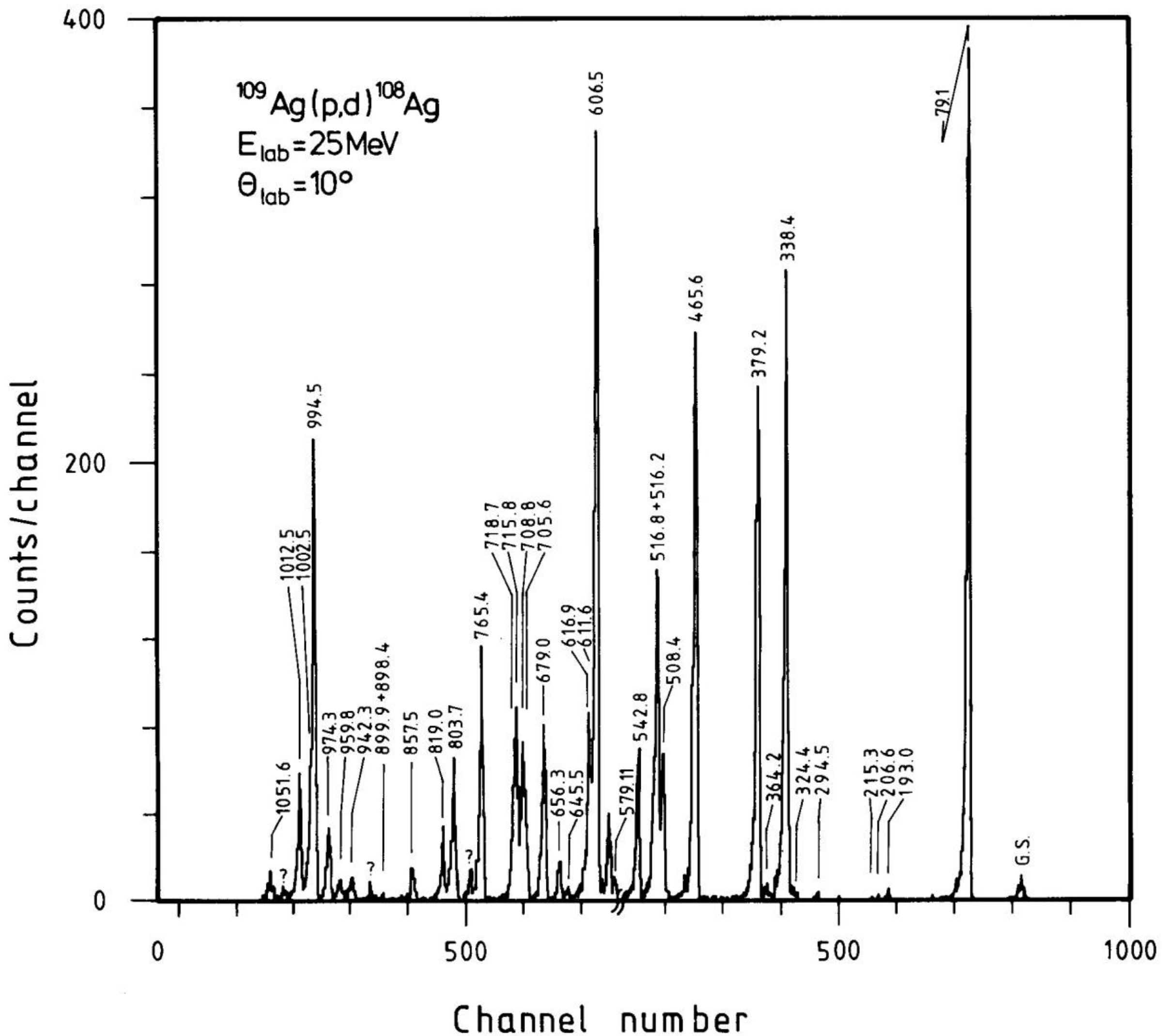
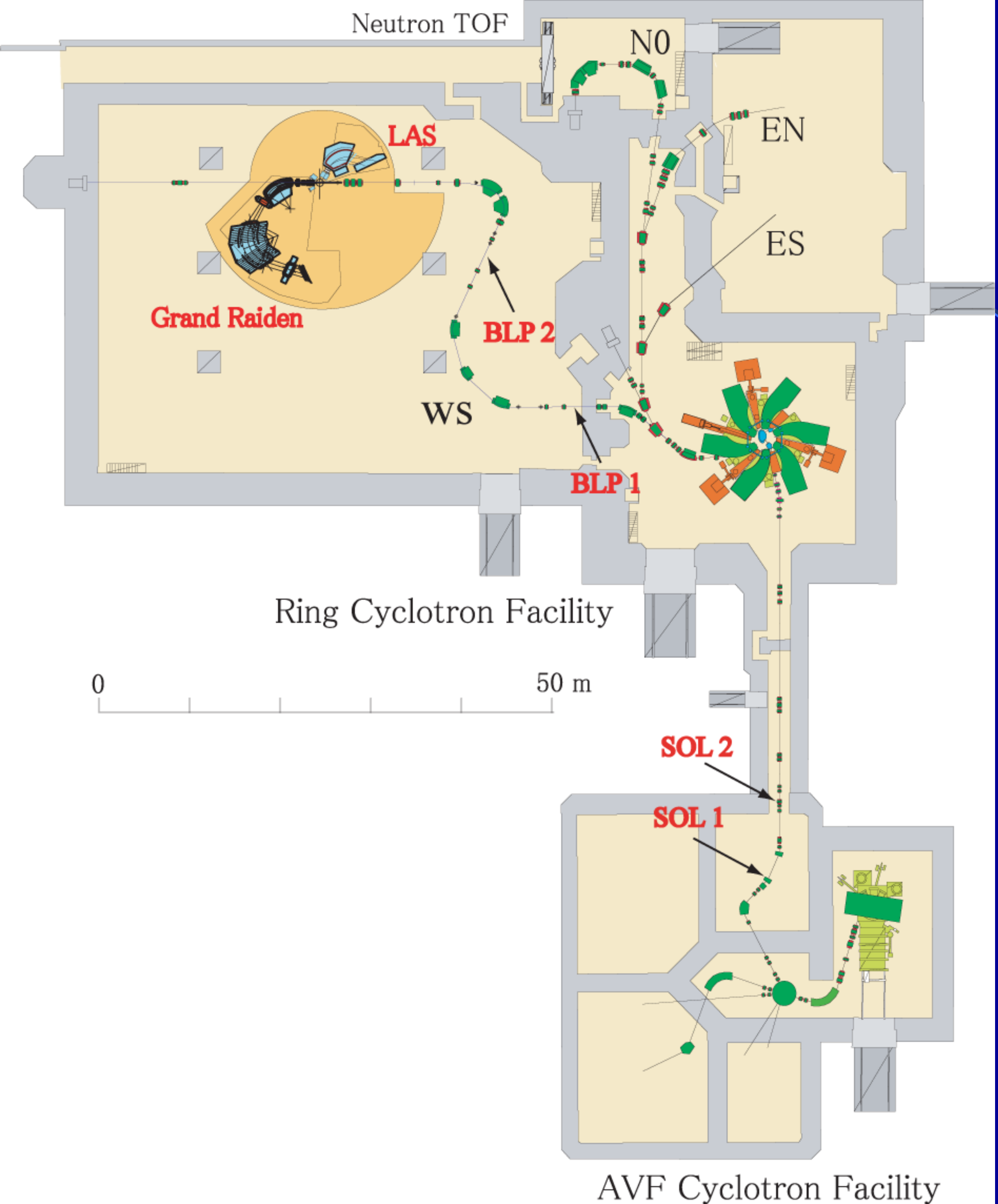


Fig. 19. High resolution spectrum of the (p, d) neutron pick up reaction on ^{109}Ag at 25 MeV incident energy and a solid angle of 1.2 msr. The resolution was 4 keV.



RCNP Facility Layout Osaka, Japan

$D = S_{16} = 17$ cm/% = 17 m

$M = S_{11} \sim -0.45$

Dispersion on target:

$B_{16} = D/M = -37$ m

Resolving power:

$2x_0 = 1$ mm

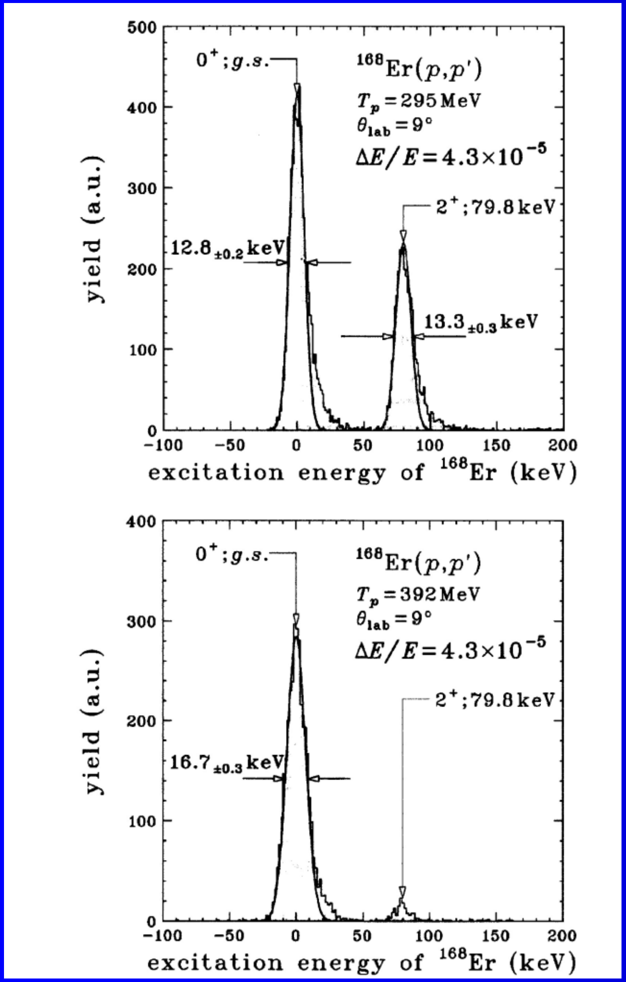
$R = p/\Delta p = 37000$

Dispersion matched beam line WS to the high resolution spectrometer Grand Raiden

Momentum and Angular Resolution

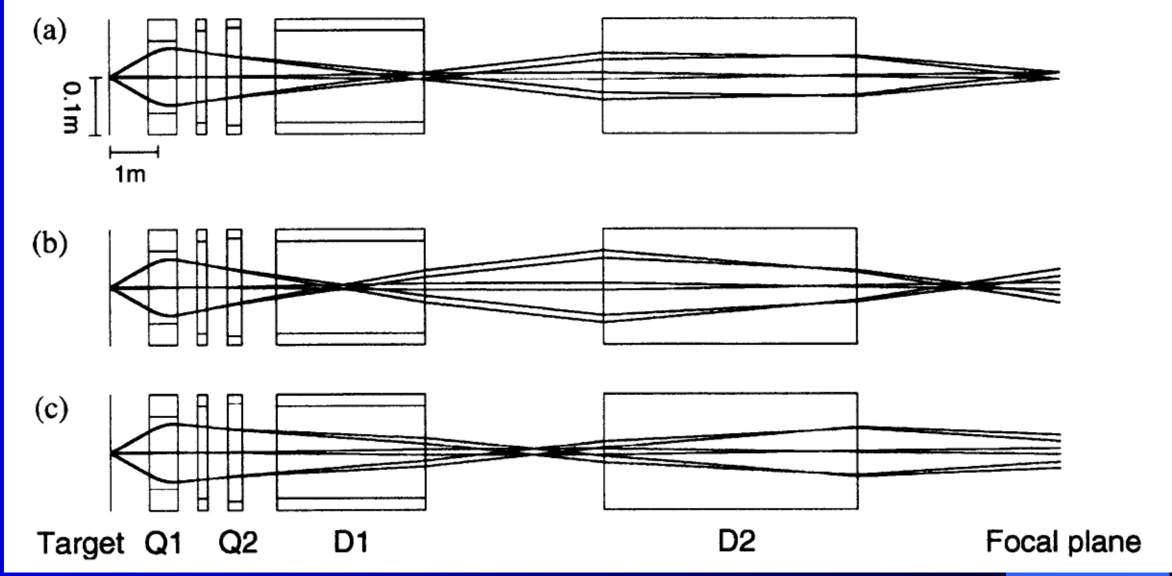
Spacial & Angular Dispersion Matching & Focus Condition allows

Energy Resolution: $E/\Delta E=23000$, $p/\Delta p = 40000$, despite beam spread: $E/\Delta E = 1700 - 2500$



Angular resolution: $\Delta Y_{scatt} = SQRT(\Delta Y^2_{hor}+\Delta\Phi^2) = 4 - 8$ msr

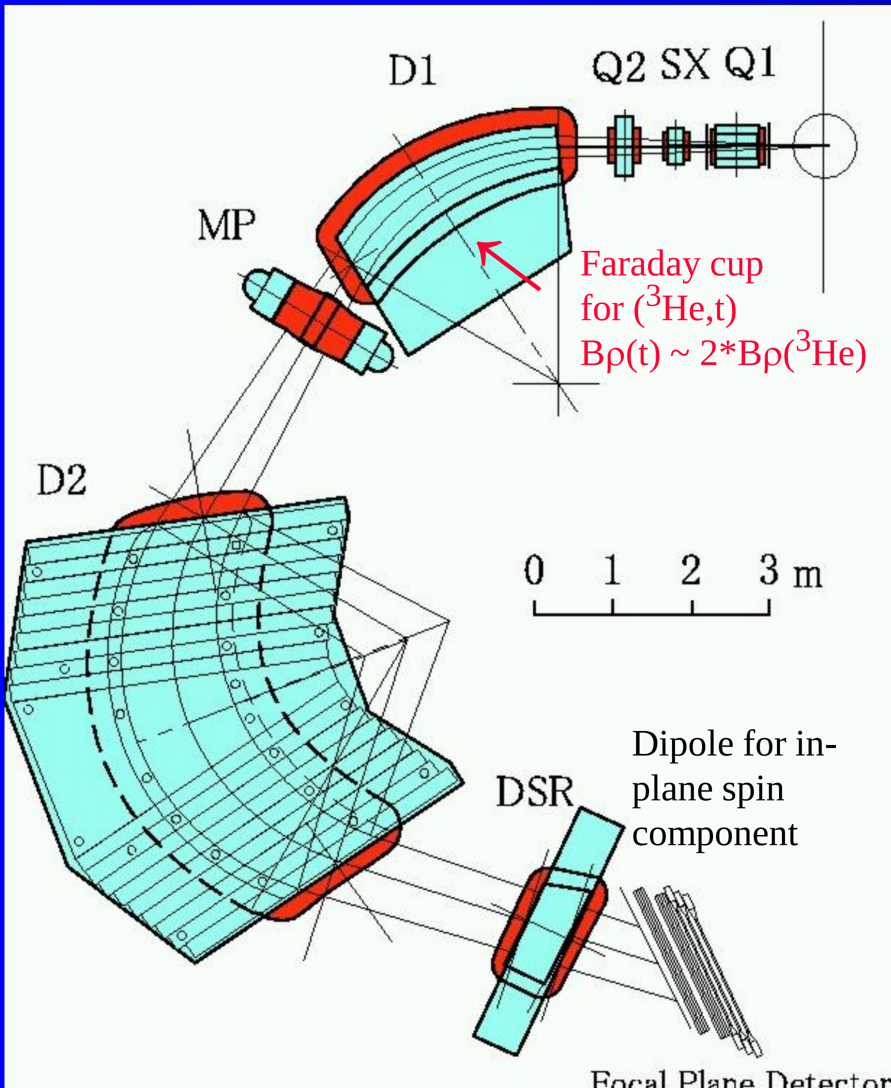
At angles close to beam (e.g. 0 deg) vert. angle component is needed → Overfocus mode, small target dimension, because (y|y) is large, Limitation: multiple scattering in detector



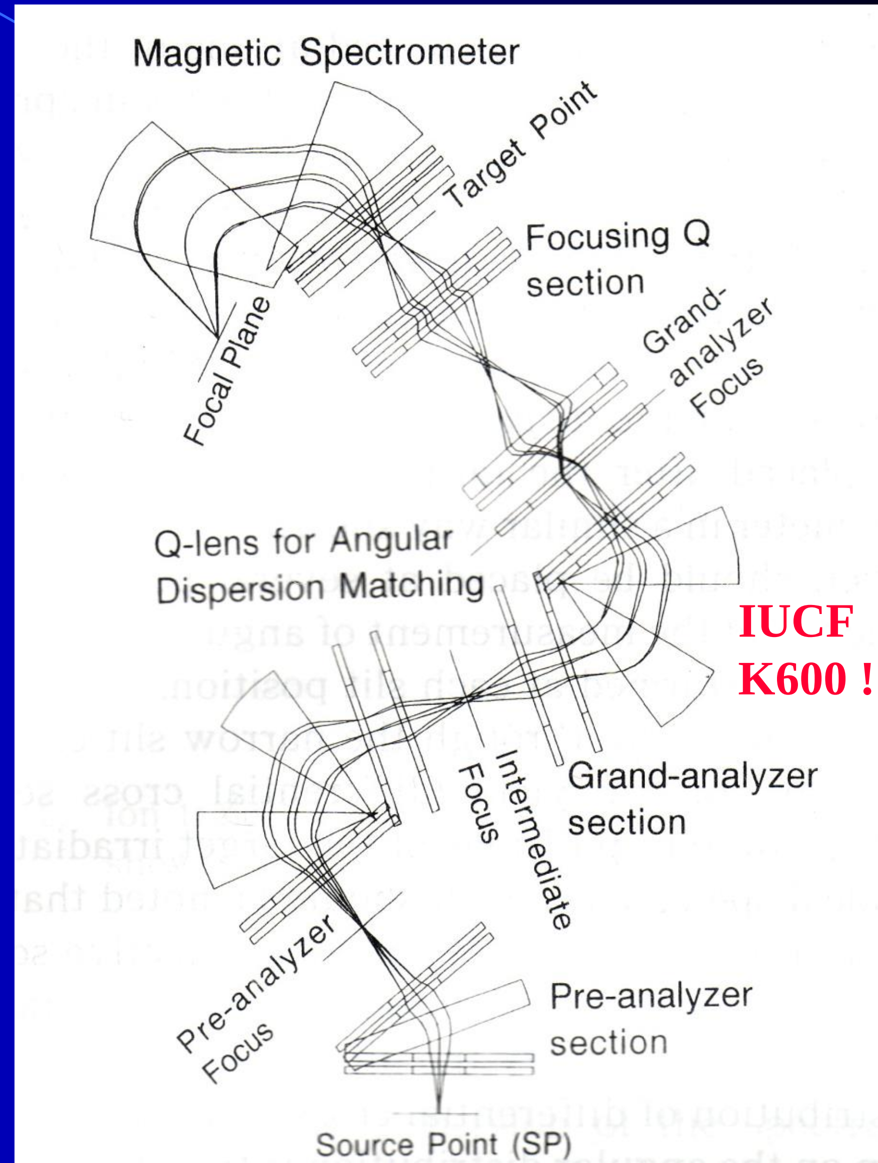
Refs.: Y.Fujita et al, NIM B126(1997)274, H.Fujita et al. NIM A 469(2001)55, T.Wakasa et al, NIM A482(2002)79

Grand Raiden High Resolution Spectrometer

Max. Magn. Rigidity:	5.1 Tm
Bending Radius ρ_0:	3.0 m
Solid Angle:	3 msr
Resolv. Power p/dp	37000

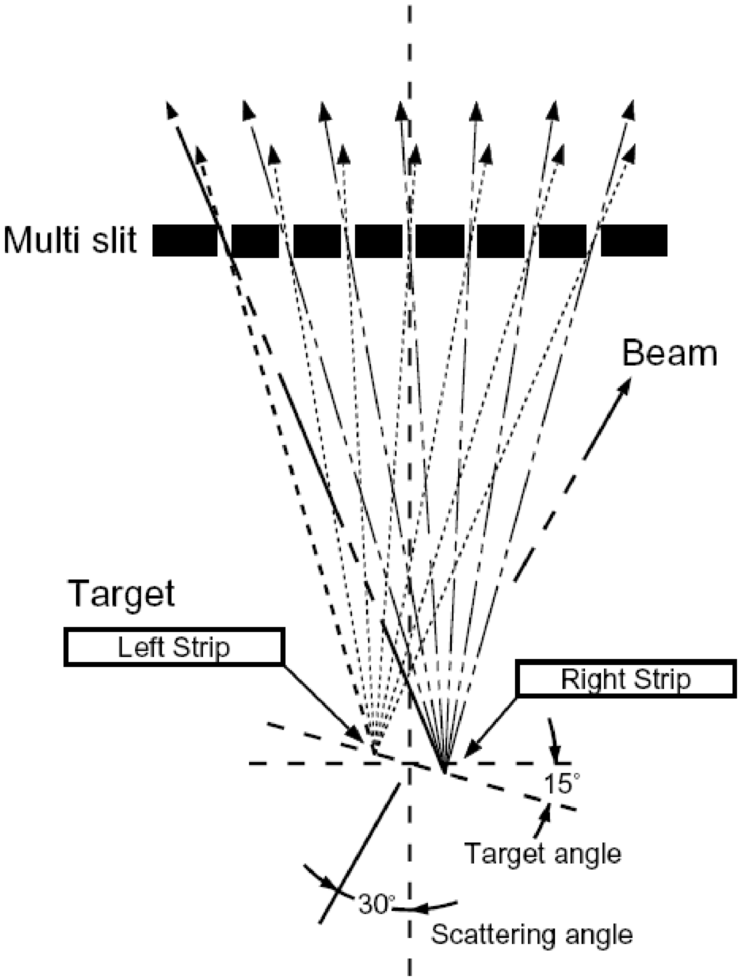


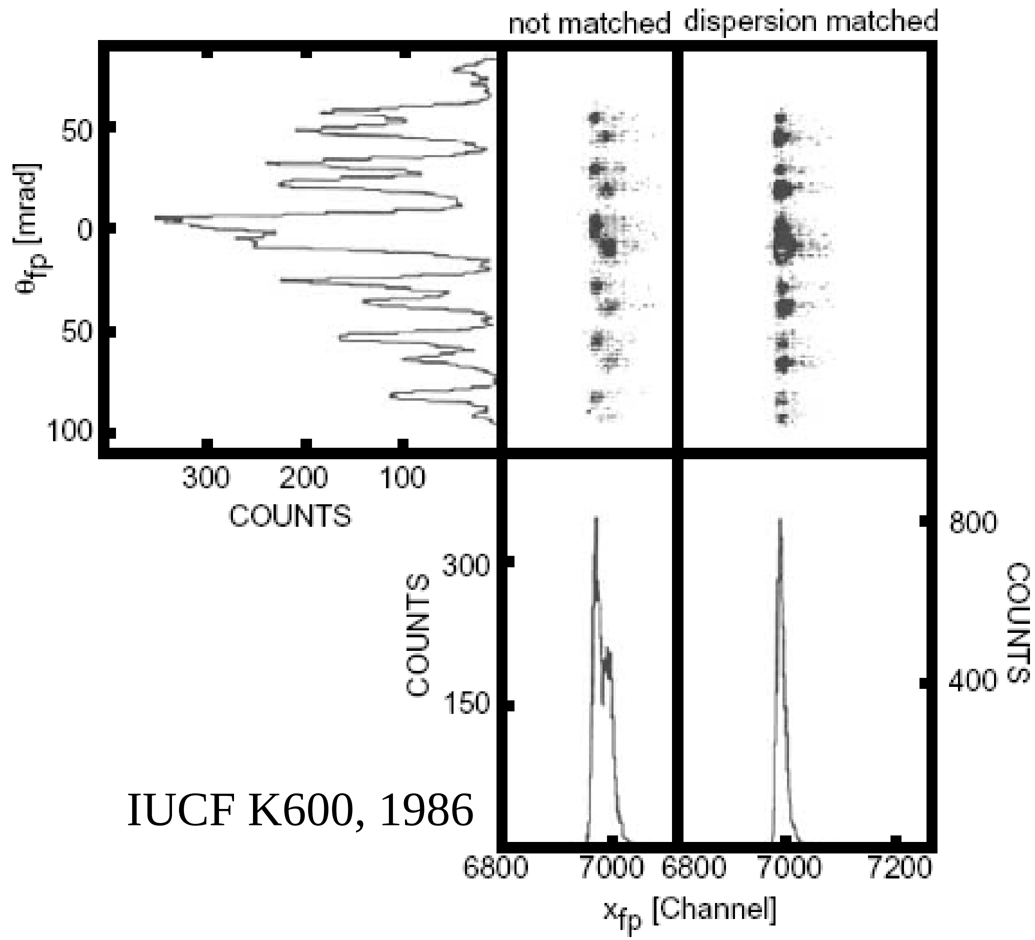
Beam Line/Spectrometer fully matched



Diagnostic of Dispersion Matching

of beam line & spectrometer using a double strip target & multi slit





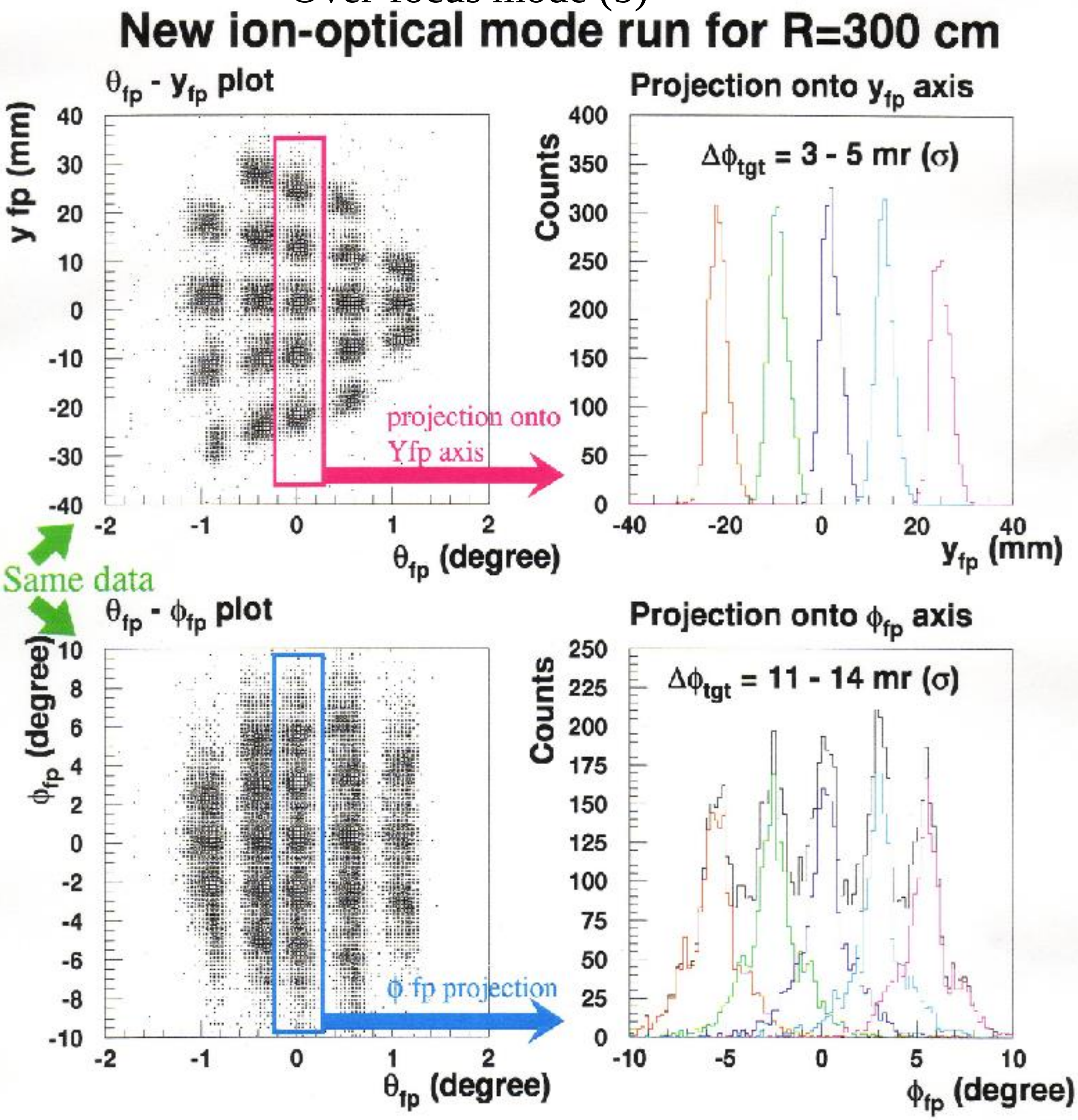
IUCF K600, 1986

Fig. 4. Scatterplots of horizontal position x_{fp} versus angle θ_{fp} and projections measured in the focal plane of the K600 using the "multi-slit system". For details, see text.

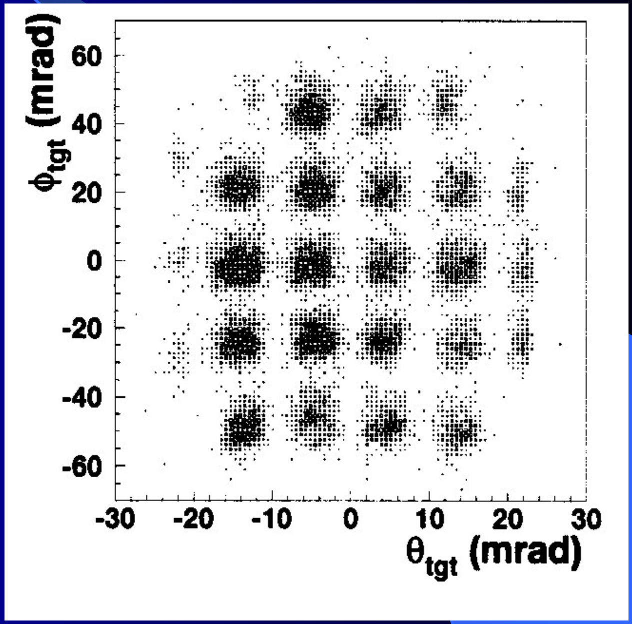
Grand Raiden Angle Calibration

Data suggest: Use y_{fp} not Φ_{fp} to calibrate angle!

Over-focus mode (b)



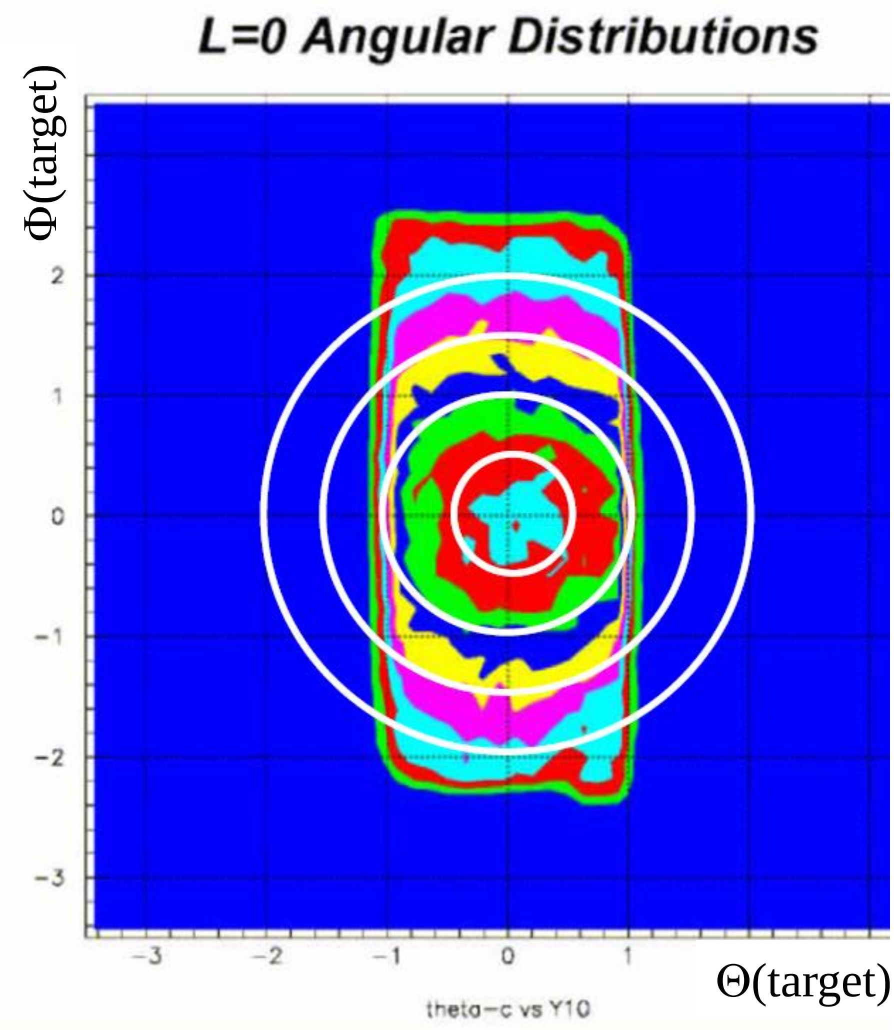
Calibrated!



Scattering Angle

reconstructed from focal plane measurements using complete dispersion matching techniques

$E(^3He) = 420$ MeV



Scatt. Angle reconstruction near 0 deg using Overfocus Mode

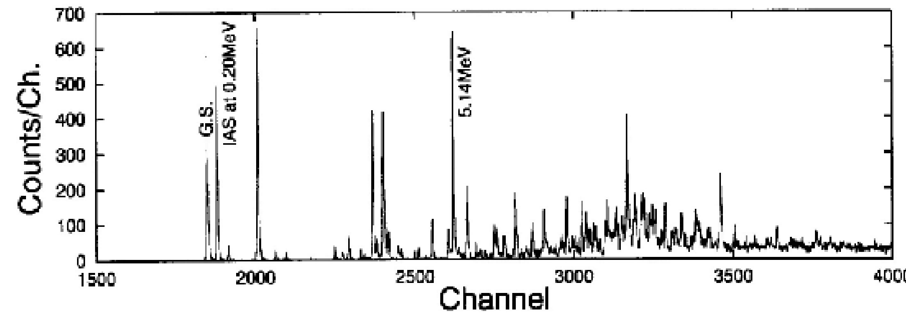


Figure 4.4: Spectrum of $^{58}Ni(^3He,t)$ reaction. The *lateral* and *angular dispersion matching* technique and *over-focus mode* were applied in this experiment for high energy and scattering angle resolution. Energy resolution of about 30 keV (FWHM) was realized.

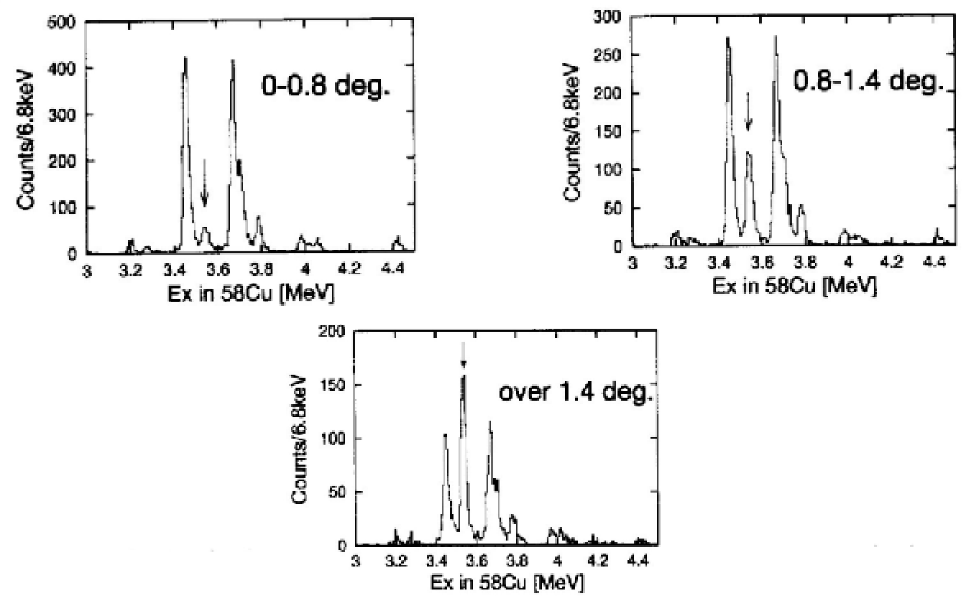


Figure 4.5: Example of angle dependence in the $^{58}Ni(^3He,t)$ spectra near 0°. Three spectra are shown for the angle ranges 0-0.8° (left), 0.8-1.4° (middle) and over 1.4° (right), respectively. The 3.54 MeV state show clearly different angular distribution from the adjacent 1^+ states which are dominated at forward angle.

Horizontal Beam Profiles in the Focal Plane of Grand Raiden

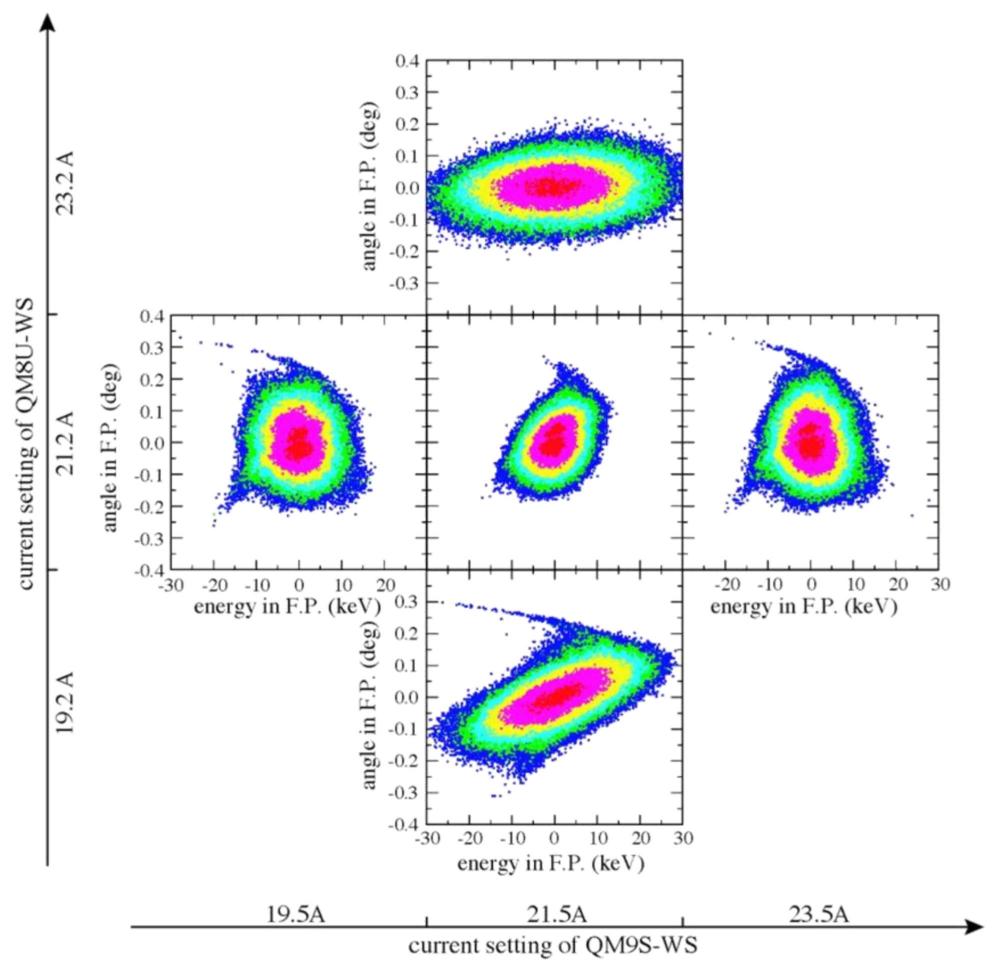
- QM8U
  →Control lateral
  dispersion
- QM9S
  →Control angular
  dispersion
- Lateral and angular dispersions can be controlled independently
- References

Y. Fujita at al., NIMB 126(1997)274

H. Fujita et al., NIMA 469(2001)55

T. Wakasa et al., NIMA 482(2002)79

Dispersion matching for K = 0 with faint beam



High Resolution Spectrometers
Momentum Analysis

- Momentum Resolving Power

$$R_p = \frac{(x|dp)}{(x'|x) * 2x_0}$$

Image size

$(x|dp)$ = M_{16} = Momentum (p) dispersion

$(x'|x)$ = M_{11} = Magnification

$2x_0$ = Target spot size

- Momentum Resolution:

$$R_p^{HO} = \frac{(x|dp)}{x_{HO}} \qquad x_{HO} = (x'|x) * 2x_0 + Higher\ Orders$$

- For **High Resolution** using Spectrometers (no physical separation) consider the following
- Momentum resolving power R_p has to meet the design goal (e.g. Grand Raiden: 37000, SHARAQ: 15000 for $2x_0$ = 1 mm), given by science requirements.
- If beam momentum spread $\delta p/p > 1/R_p$ need Dispersion Matching or Beam Tracking, count rate limit ~10^6 p/sec, not suitable for high intensity stable beams.
- RI beam with $\delta p/p$ ~ 1- 3 % dispersion matched beam ($-S_{16}/S_{11}$) on target too large (50 –100 cm). Therefore, SHARAQ has several modes (achromatic, high resol. achromatic, dispersive)
- RI beams, high energies, 100 – 300 MeV/A, tracking detectors in beam line (BigRips, SHARAQ)
- Within limits (multiple-scattering in focal plane (FP) detectors) HO can be corrected using standard FP detectors (x,x',y,y').

Dispersion matching modes

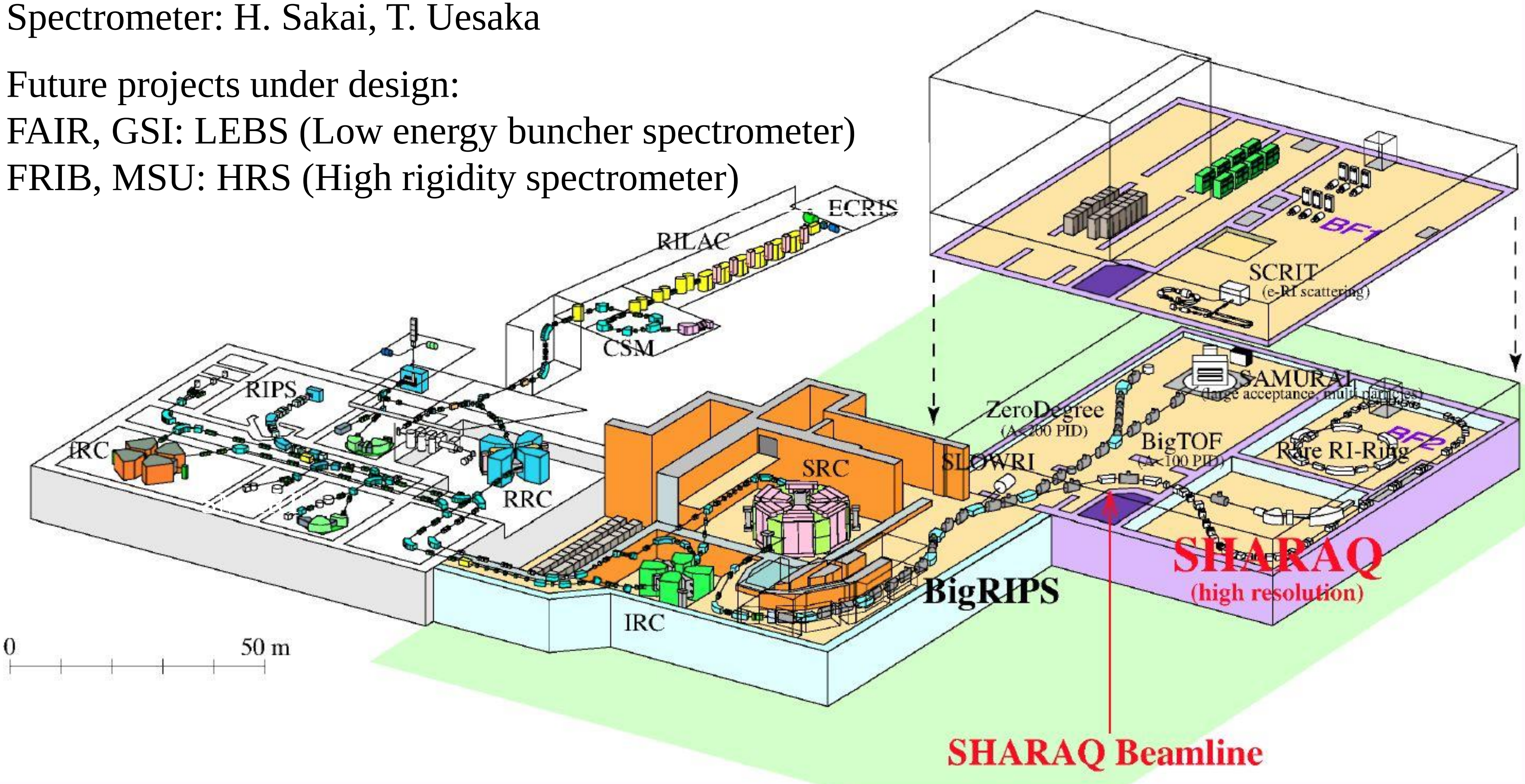
- Beam momentum spread p/dp < Resolving power R_p: Full resolution without dispersion matching, beam line achromatic mode sufficient.
- Beam momentum spread p/dp ~ (1- 10)* R_p: Full resolution requires dispersion matching, e.g. Grand Raiden: 300 MeV p: beam ~150 keV, resolution 13 keV, 400 MeV p: beam ~ 150 keV, resol. 17 keV
- Secondary Radioactive Beam (RI) : Beam momentum spread p/dp > 10* R_p: Dispersion matching with full beam is possible but typically dispersed beam on target impractically large, e.g. SHARAQ: > 10 cm). Mitigation: Intermediate modes with reduced beam momentum spread/intensity or reduced resolution.

Beam Factory RIBF at the RIKEN Accelerator Research Facility (RARF)

SHARAQ: Pionieering spectrometer in high resolution Dispersion Matching with RI beam.

BigRIPS: T. Kubo
Ion-optical design: T. Kawabata
Spectrometer: H. Sakai, T. Uesaka

Future projects under design:
FAIR, GSI: LEBS (Low energy buncher spectrometer)
FRIB, MSU: HRS (High rigidity spectrometer)



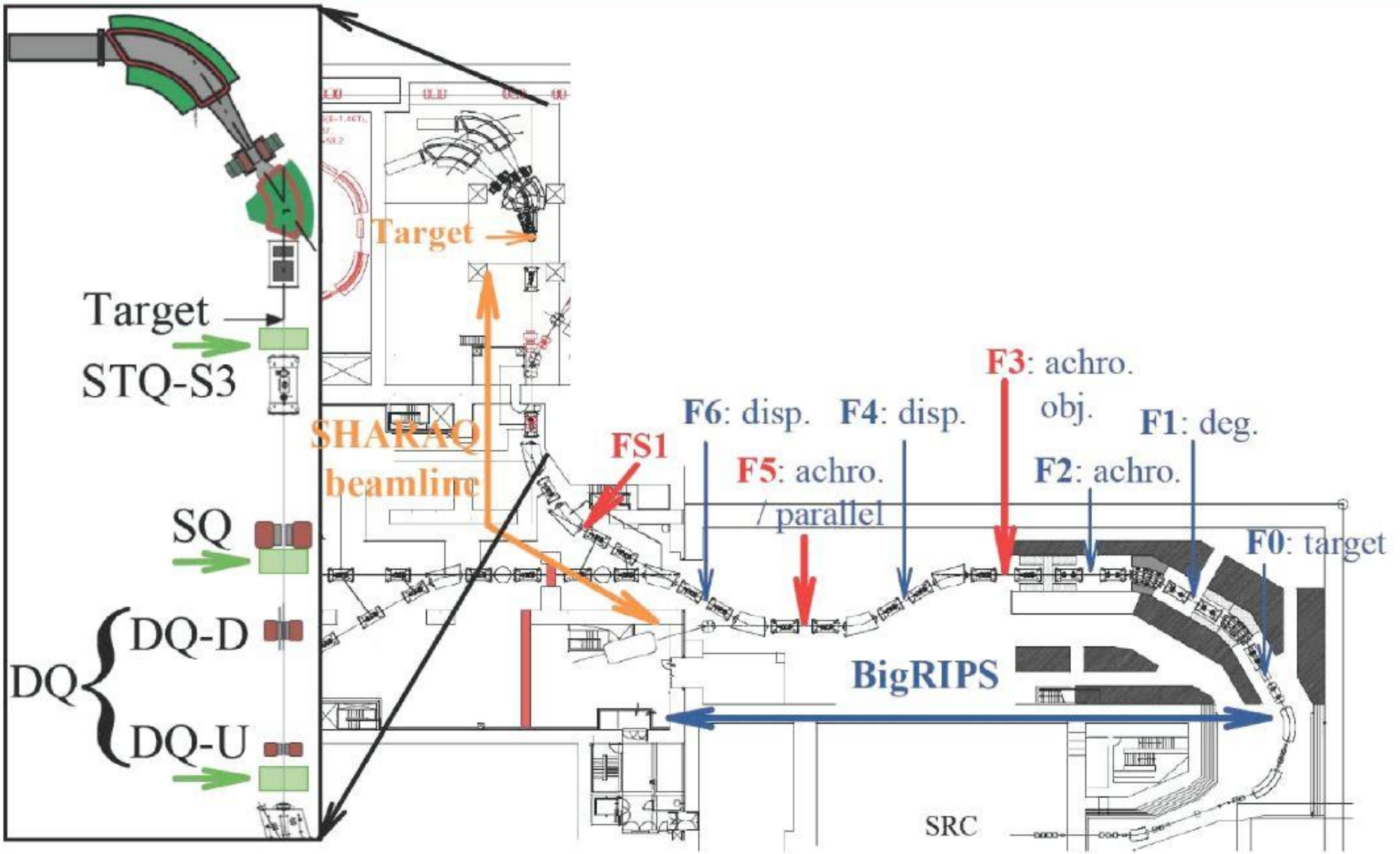
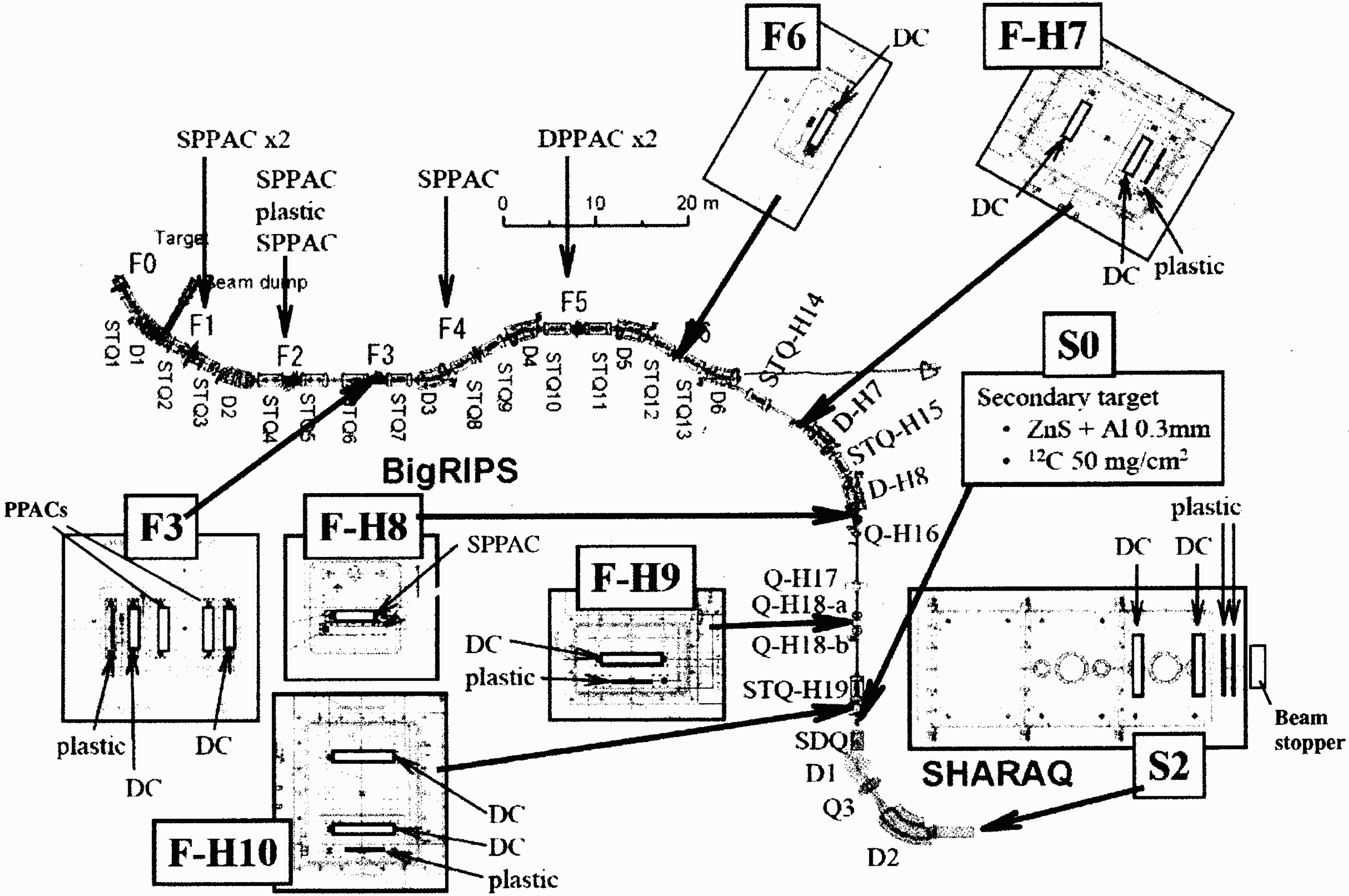


FIGURE 3. Dispersion-matched beam-line for the SHARAQ spectrometer. The RI beams produced at F0 are transported to the SHARAQ target through the BigRIPS beam-line and the SHARAQ beam-line. The dispersive beam transport starts from the achromatic focus at F3.

4. Check and calibration of F4–FH10 detectors

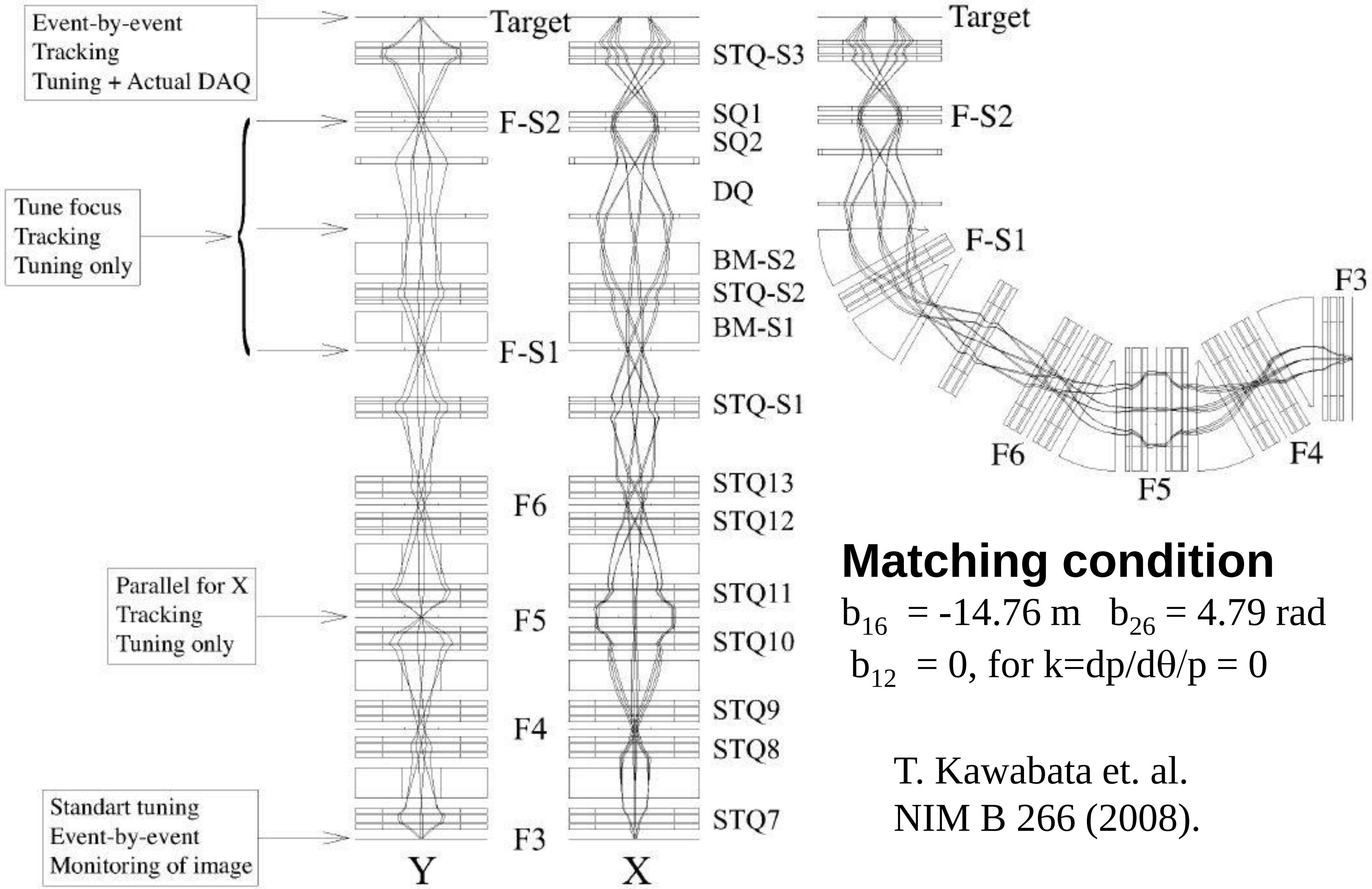


Pair of Drift Chambers at Location H10

SHARAQ, Modes of Operation

- Beam line requires special design for high-resolution spectrometer measurements with RI-beams.
- To achieve the high-resolution measurement with SHARAQ both dispersion matching and beam tracking methods are used.
- Depending on experiments, the following modes are available

Mode	Dispersive	Achromatic (high resolution)	Achromatic (large acceptance)
Resolution	**Δp/p=1/15000** at target	**Δp/p=1/7500** Tracking at F6	**Δp/p=1/1500** Tracking at F5
Acceptance	**Δp/p = +/- 0.3 %** Δθx= +/- 10 mr, Δθy= +/- 30 mr	**Δp/p = +/- 0.3 %** Δθx= +/- 10 mr, Δθy= +/- 30 mr	**Δp/p = +/- 2 %** Δθx= +/- 20 mr, Δθy= +/- 20 mr
Hor. Target spot :	~100 mm	~30 mm	~30 mm



Matching condition

$b_{16} = -14.76$ m $b_{26} = 4.79$ rad

$b_{12} = 0$, for $k = dp/d\theta/p = 0$

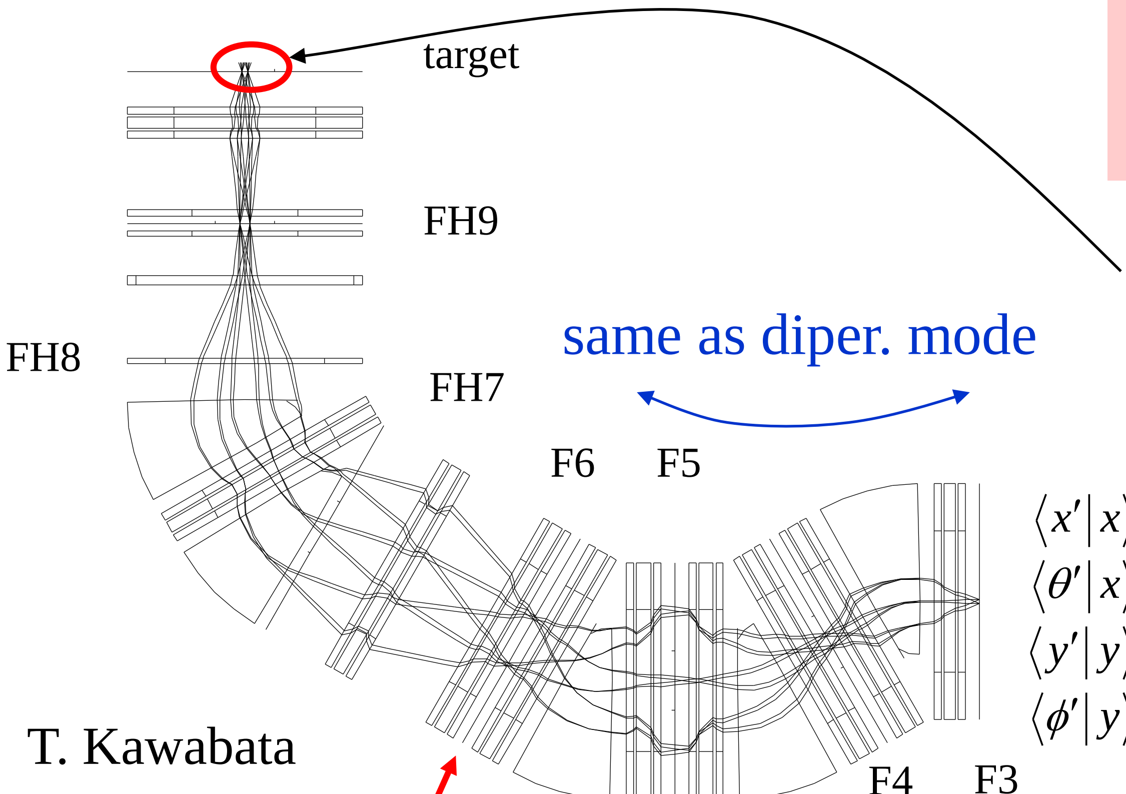
T. Kawabata et. al.
NIM B 266 (2008).

FIGURE 4. Calculated optics of the dispersive beam transport from F3 to the SHARAQ target.

“High resolution” achromatic mode

$\Delta\theta_x$= +/– 10 mr, $\Delta\theta_y$= +/– 30 mr,
Δx = +/– 3 mm, Δy = +/– 3 mm,
ΔP= +/– 0.3 %

Beam size < a few cm



$\langle x'|x\rangle = 1.56$ $\langle x'|\theta\rangle = 0.00$ $\langle x'|\delta\rangle = 0.00$

$\langle \theta'|x\rangle = 0.00$ $\langle \theta'|\theta\rangle = 0.64$ $\langle \theta'|\delta\rangle = 0.36$

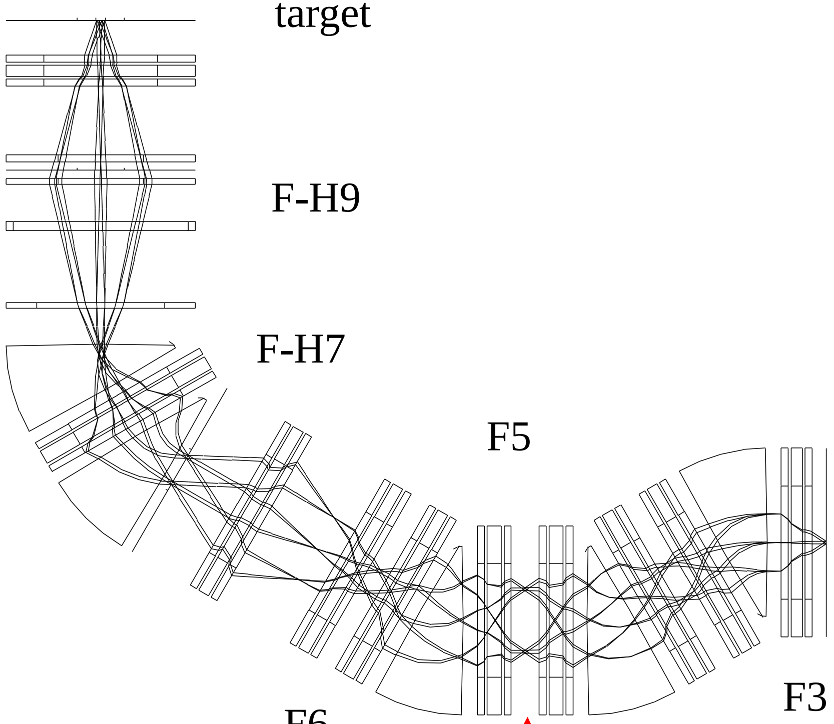
$\langle y'|y\rangle = 1.36$ $\langle y'|\phi\rangle = 0.00$

$\langle \phi'|y\rangle = 0.00$ $\langle \phi'|\phi\rangle = 0.74$

Momentum acceptance is ± 0.3 %, keeping Δp/p of ~ 1/7500.
(F6)

“Large acceptance” achromatic mode

$\Delta\theta_x$= +/– 20 mr, $\Delta\theta_y$= +/– 20 mr,
Δx = +/– 3 mm, Δy = +/– 3 mm,
ΔP= +/– 2 %



Momentum acceptance
can be increase up to ± 2 %.

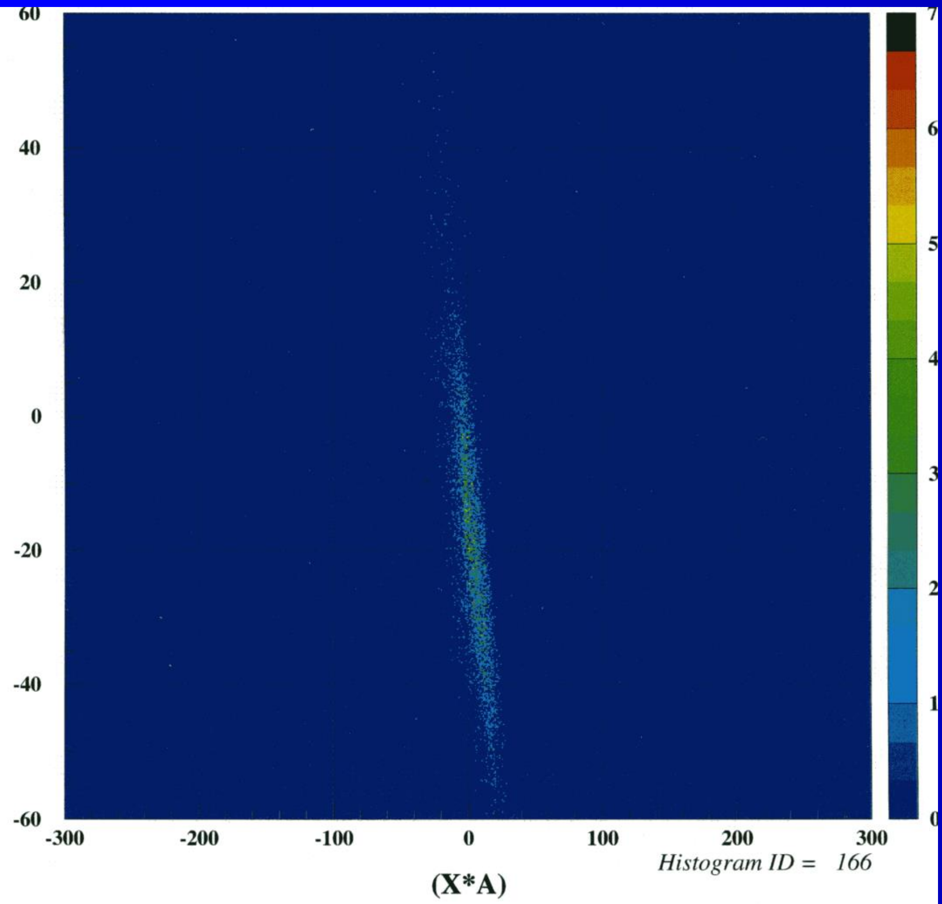
Beam transport is different
from dispersion matching mode.

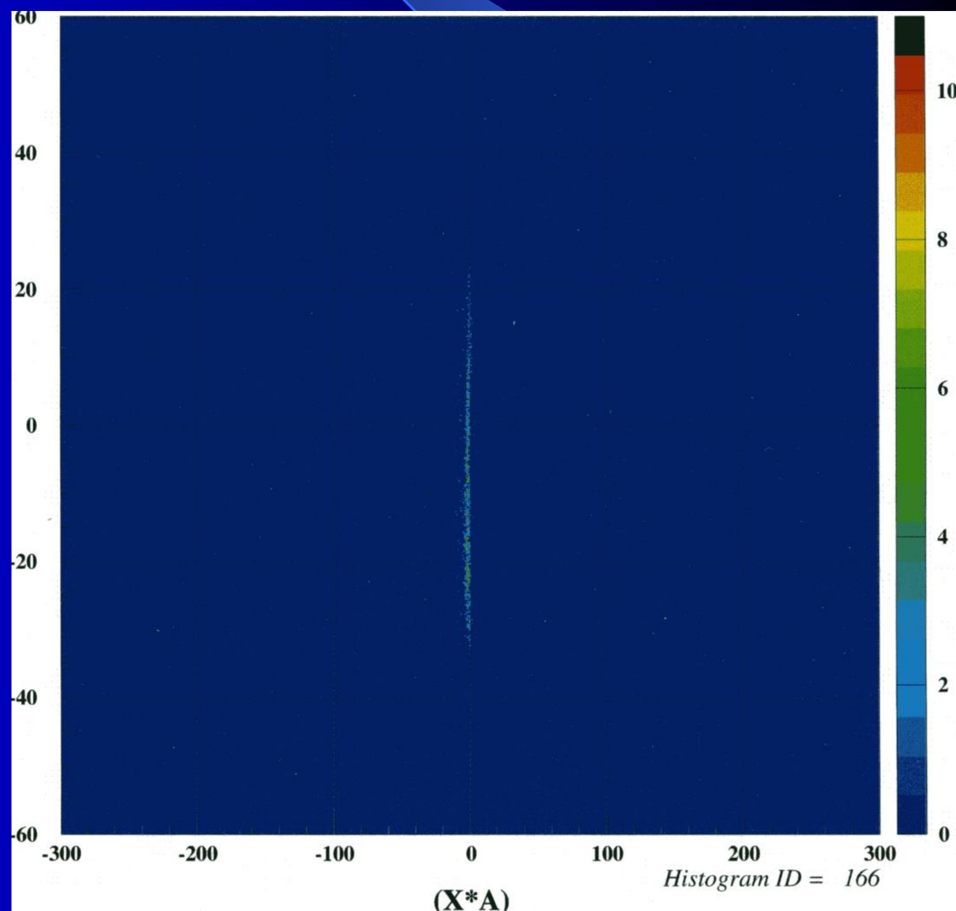
same as the standard BigRIPS
transport up to F5.

resolution $\Delta p/p \sim 1/1500$ (at F5)

Dispersion Matching of Beam Line and SHARAQ Spectrometer

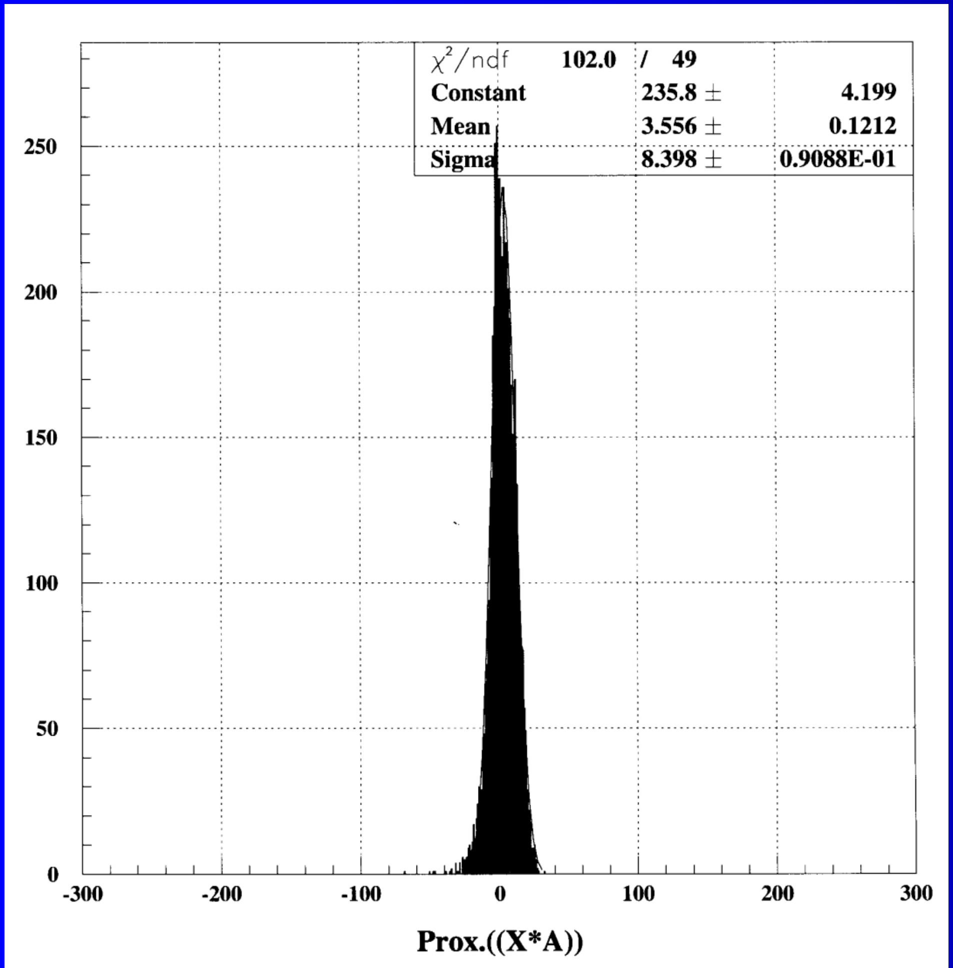
250 MeV/u ^{14}N beam, approx. 1000 events/s





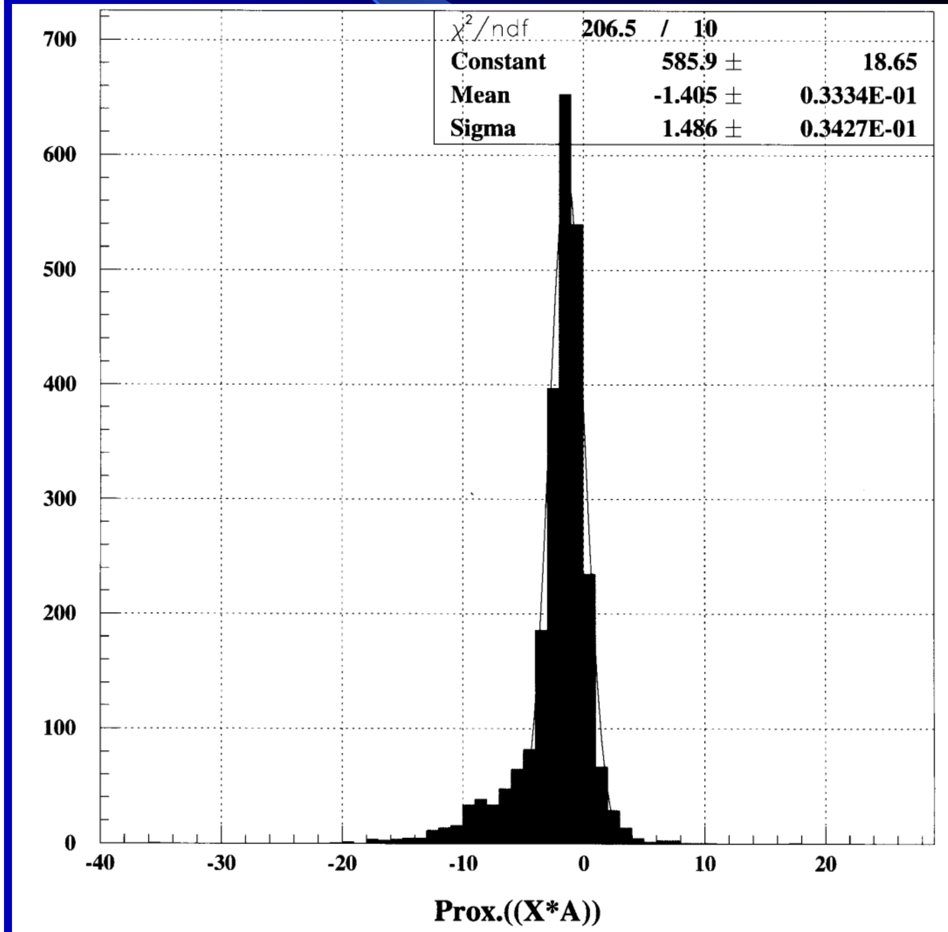
Resolution

Resolution: approx. 3.5 MeV



Resolution: approx. 0.65 MeV

Resolv. Power: 0.43 MeV



End

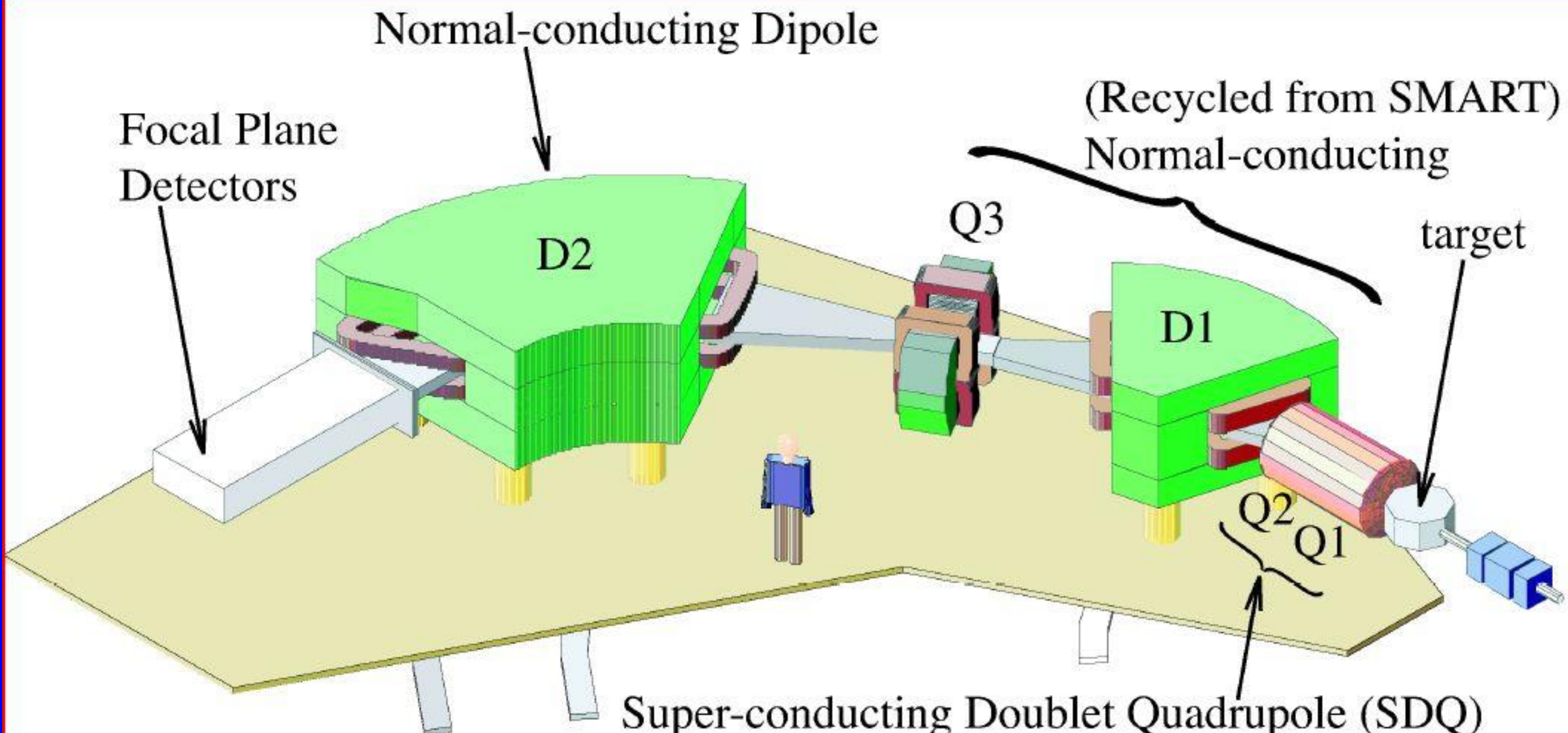


FIGURE 2. The SHARAQ spectrometer.

TABLE 1. Specifications of the SHARAQ spectrometer.

Maximum rigidity	6.8 Tm
	(c.f. 9 Tm in BigRIPS)
Momentum resolution	$p/\delta p \sim 1.5 \times 10^4$
Angular resolution	$\sim$1 mrad
Momentum acceptance	$\pm 1\%$
Angular acceptance	$>$ a few msr
Rotating angle	from -2 deg. to $+15$ deg.