Alpha particle scattering and alpha clustering in the MCAS

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MCAS Collaboration

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Introduction

Low-energy α-nucleus interactions play a central role in nucleosynthesis. Beyond the *p*-*p* chain, the triple-α process forms ^{12}C, beyond which the star enters the CNO cycle.

CN:

$$
\begin{aligned}
{}^{12}\mathrm{C} + p &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + e^{+} + \nu \\
{}^{13}\mathrm{C} + p &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + p &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + e^{+} + \nu \\
{}^{15}\mathrm{N} + p &\rightarrow {}^{12}\mathrm{C} + \alpha \\
{}^{15}\mathrm{N} + p &\rightarrow {}^{16}\mathrm{O} + \gamma
\end{aligned}
$$

NO:

$$
\begin{aligned}
{}^{16}\mathrm{O} + p &\rightarrow {}^{17}\mathrm{F} + \gamma \\
{}^{17}\mathrm{F} &\rightarrow {}^{17}\mathrm{O} + e^{+} + \nu \\
{}^{17}\mathrm{O} + p &\rightarrow {}^{14}\mathrm{N} + \alpha
\end{aligned}
$$

OF:

$$
\begin{aligned}
{}^{17}\mathrm{O} + p &\rightarrow {}^{18}\mathrm{F} + \gamma \\
{}^{18}\mathrm{F} + e^{-} &\rightarrow {}^{18}\mathrm{O} + \nu \\
{}^{18}\mathrm{O} + p &\rightarrow {}^{19}\mathrm{F} + \gamma \\
{}^{19}\mathrm{F} + p &\rightarrow {}^{16}\mathrm{O} + \alpha
\end{aligned}
$$

Alpha capture reactions then play a central role beyond the CNO cycle forming the α-nuclei up to ^{40}Ca.

Studying alpha scattering yields knowledge of those interactions.

MCAS

We seek to obtain *S* matrices and evaluate:

Total elastic scattering cross sections:

$$\sigma_{\rm el} = \frac{\pi}{k^2} \sum_{l=0}^{\infty} \left\{ (l+1) \left| S_{ll+\frac{1}{2}}(k) - 1 \right|^2 + l \left| S_{ll-\frac{1}{2}}(k) - 1 \right|^2 \right\}$$

Total reaction cross sections:

$$\sigma_{\rm R} = \frac{\pi}{k^2} \sum_{l=0}^{\infty} \left\{ (l+1) \left[1 - \left| S_{ll+\frac{1}{2}}(k) \right|^2 \right] + l \left[1 - \left| S_{ll-\frac{1}{2}}(k) \right|^2 \right] \right\}$$

The MCAS approach is built upon:

1. Finite-rank separable representations of realistic interactions;
2. Scattering matrices for separable Schrödinger interactions;
3. Sturmian functions (Weinberg states) to define form factors.

Multi-channel T matrices

Solution of coupled Lippmann-Schwinger equations:

$$T_{cc'}(p,q;E) = V_{cc'}(p,q) - \mu \sum_{c'=1}^{\text{closed}} \int_0^\infty V_{cc''}(p,x) \frac{1}{h_{c''}^2 + x^2} T_{c''c'}(x,q;E)\, x^2 dx$$

$$+ \mu \sum_{c'=1}^{\text{open}} \int_0^\infty V_{cc''}(p,x) \frac{1}{k_{c''}^2 - x^2 + i\varepsilon} T_{c''c'}(x,q;E)\, x^2 dx$$

Expand the potential matrix:

$$V_{cc'}(p,q) \sim V_{cc'}^{(N)}(p,q) = \sum_{n=1}^{N} \hat{\chi}_{cn}(p) \eta_n^{-1} \hat{\chi}_{c'n}(q)$$

Optimal functions, $\hat{\chi}_{cn}(q)$, involve Sturmians $|\Phi_{c'n}\rangle$:

$$|\hat{\chi}_{cn}\rangle = \sum_{c'} V_{cc'} |\Phi_{c'n}\rangle$$

$$\sum_{c'} G_c^{(0)} V_{cc'} |\Phi_{c'n}\rangle = -\eta_n |\Phi_{cn}\rangle$$

Multi-channel S matrices

Separable expansion of multi-channel $V_{cc'}$ $\Rightarrow$ multi-channel S matrix (c,c' are open channels, specified by J^π):

$$S_{cc'} = \delta_{cc'} - i\pi\mu\sqrt{k_c k_{c'}}T_{cc'}$$
$$= \delta_{cc'} - i\pi\mu \sum_{n,n'=1}^{N} \sqrt{k_c}\chi_{cn}(k_c)\left([\boldsymbol{\eta} - \mathbf{G}_0]^{-1}\right)_{nn'} \chi_{c'n'}(k_{c'})\sqrt{k_{c'}}$$

Matrix elements (Sturmian basis) $[\boldsymbol{\eta}]_{nn'} = \eta_n \delta_{nn'}$, with:

$$[\mathbf{G}_0]_{nn'} = \mu \sum_{c=1}^{\text{open}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{k_c^2 - x^2 + i\varepsilon} \hat{\chi}_{cn'}(x)\, dx$$
$$- \mu \sum_{c=1}^{\text{closed}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{h_c^2 + x^2} \hat{\chi}_{cn'}(x)\, dx$$

Alpha-nucleus potentials

Consider a basis of channel states defined by the coupling

$$\begin{aligned} |c\rangle &= |lI; J^{\pi}\rangle \\ &= \left[|l\rangle \otimes |\psi_I\rangle\right]_J^{M,\pi} \end{aligned}$$

$l:$ relative orbital angular momentum for spin-0 projectile on the target

$\left|\psi_I^{(N)}\right\rangle$: target state

The potential may be written as

$$\begin{aligned} V_{cc'} &= \langle lI\,|W(r)|\,l'I'\rangle \\ &= \left[V_0 f(r) + V_{ll} f(r)\,[l\cdot l] + V_{II} f(r)\,[\mathbf{I}\cdot\mathbf{I}] + V_{lI} g(r)\,[l\cdot\mathbf{I}]\right]_{cc'} \\ &\quad + V_{\mathrm{mono}}\delta_{cc'}\delta_{I0^+_{gs}} f(r); \end{aligned}$$

$$f(r) = \left[1 + \exp\left(\frac{r-R}{a}\right)\right]^{-1}$$

$$g(r) = \frac{1}{r}\frac{df}{dr}.$$

Monopole term: for *N*=*Z* systems in which pairing leads to extra binding.

Effects of deformation are included to second order, *viz*.

$$
\begin{aligned}
f(r) &= f(r - R(\theta,\varphi)) \\
R(\theta,\varphi) &= R_0(1+\varepsilon) \\
\Rightarrow f(r) &= f_0(r) + \varepsilon\left[\frac{df}{d\varepsilon}\right]_0 + \frac{1}{2}\varepsilon^2\left[\frac{d^2f}{dr^2}\right]_0 \\
&= f_0(r) - R_0\varepsilon\frac{df_0}{dr} + \frac{1}{2}R_0^2\varepsilon^2\frac{d^2f_0}{dr^2}
\end{aligned}
$$

0: indicates spherical Woods-Saxon functional form, R=R_0.

Pauli principle considerations

No natural inclusion of the Pauli principle in a collective model. MCAS accounts for Pauli among the nucleons by the inclusion of an orthogonalising pseudo-potential (OPP). The potential becomes

$$\mathcal{V}_{oc'} = V_{oc'}\delta(r - r') + \lambda_c A_c(r) A_{c'}(r')\delta_{cc'}$$

$$V_{oc'} : \text{nuclear interaction potential}$$
$$\lambda_c : \text{scale for Pauli blocking}$$
$$A_c(r) : \text{bound state wave functions of the } \alpha$$

$$\lambda \sim 10^6 \text{ MeV - Pauli blocking}$$

Test cases: $\alpha+^{3}H$, $\alpha+^{3}He$, $\alpha+\alpha$

NB: All results presented are **preliminary.**

		^{7}Li			^{7}Be	
J^{π}	$^{3}H(\alpha,n)$	$^{4}He(^{3}He,\pi^{+})$	$^{4}He(\alpha,p)$	$^{4}He(^{3}He,\gamma)$	$^{4}He(^{3}He,^{3}He),(^{3}He,p)$	$^{4}He(\alpha,n)$
$3/2^{-}$	✓	✓	✓	✓		✓
$1/2^{-}$		✓	✓	✓		✓
$7/2^{-}$			✓		✓	
$5/2^{-}$		✓			✓	

Spectra of ^{7}Li and ^{7}Be

Energies in MeV, widths in keV

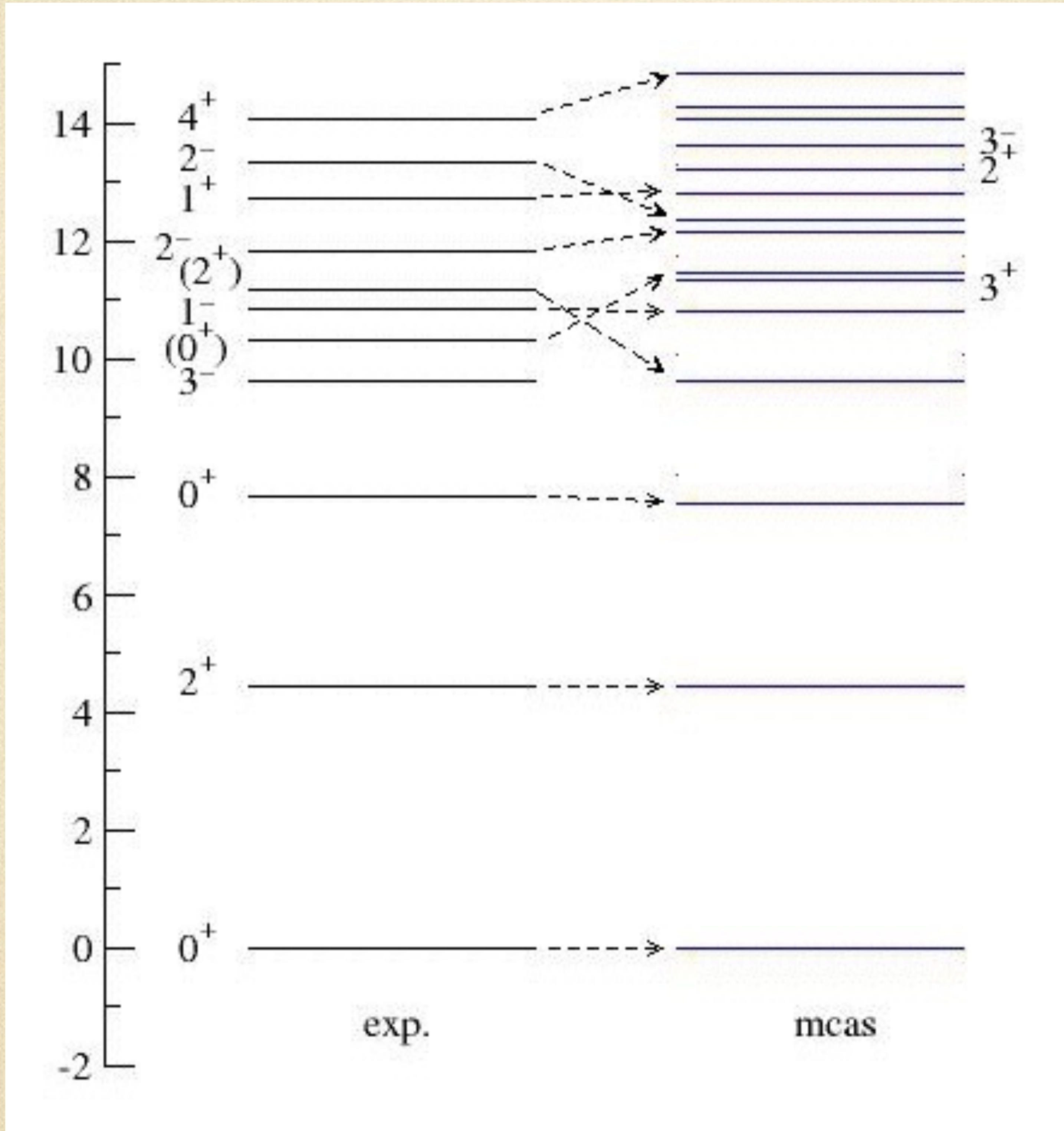
		^{7}Li			^{7}Be	
J^π	Exp.	present	previous	Exp.	present	previous
$3/2^-$	spurious	-29.6	-29.4	spurious	-27.8	-28.0
$1/2^-$	spurious	-28.0	-27.8	spurious	-26.3	-26.4
$3/2^-$	-2.47	-2.59	-2.47	-1.59	-1.53	-1.53
$1/2^-$	-1.99	-1.87	-1.75	-1.16	-0.85	-0.84
$7/2^-$	2.18(69)	2.09(80)	2.12(83)	2.98(175)	3.14(204)	3.07(180)
$5/2^-$	4.13(918)	4.05(800)	4.12(834)	5.14(1200)	5.13(1250)	5.09(1194)

$^{12}C: \alpha+^{8}Be$

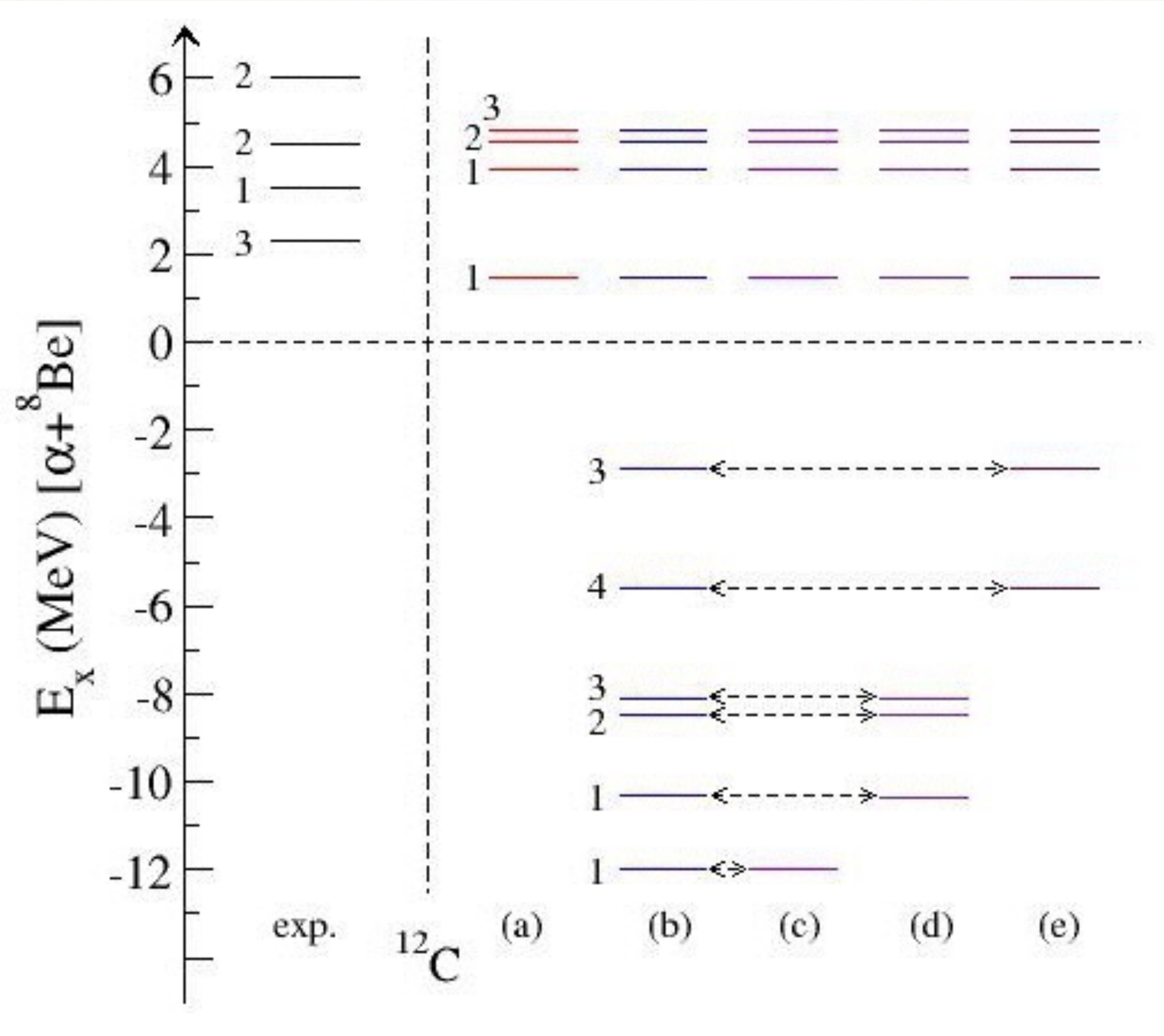
^{8}Be is unstable to emission of α particles. Ground state (0^{+}) is 0.092 MeV above the breakup threshold, but very narrow, allowing formation in the stellar environment of ^{12}C by the capture of a third α. The state formed is the Hoyle state at 7.65 MeV.

Potential		Geometry / OPP	
V_0	-39.5	$R_0=R_c$	2.8
V_{ll}	1.5	$a_0=a_c$	0.65
V_{lI}	-1.8	β_2	-0.7
V_{II}	2.0	β_4	0.2
V_{mono}	-2.7	$\lambda_s=\lambda_p$	10^6

Energies in MeV, lengths in fm.

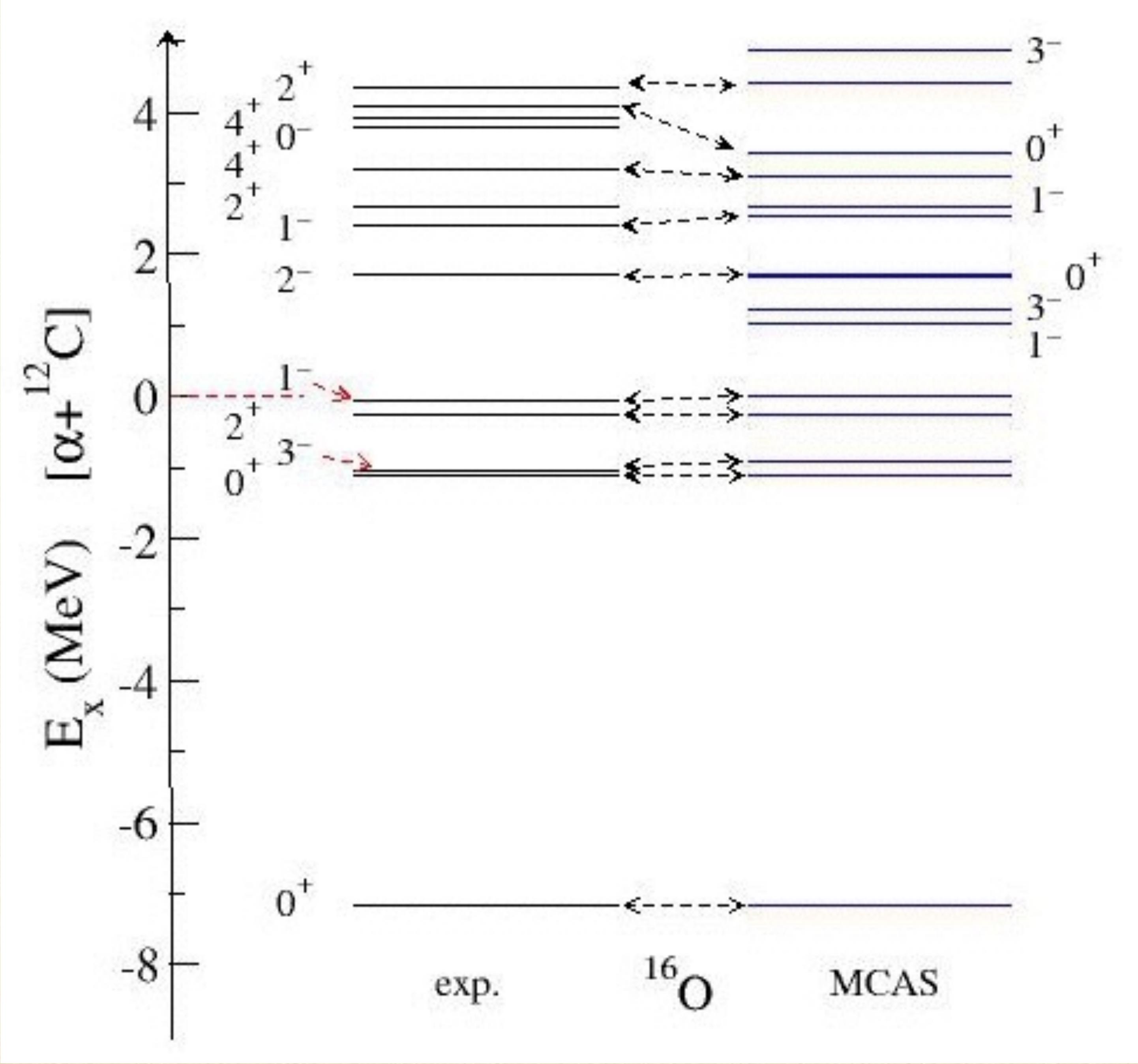


Spectrum of ^{12}C.

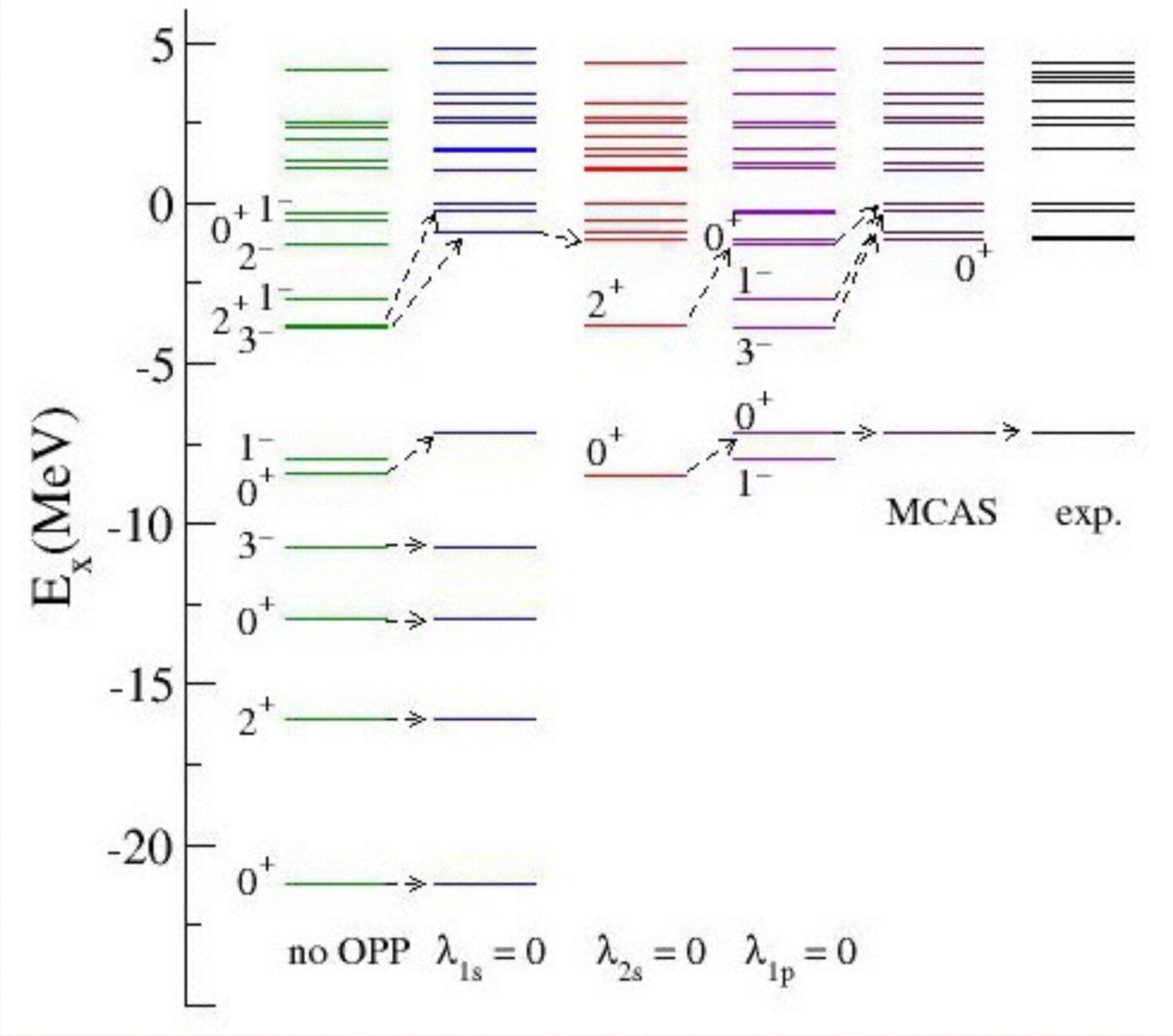


Effects of varying Pauli blocking.

^{16}O: $\alpha + {}^{12}C$

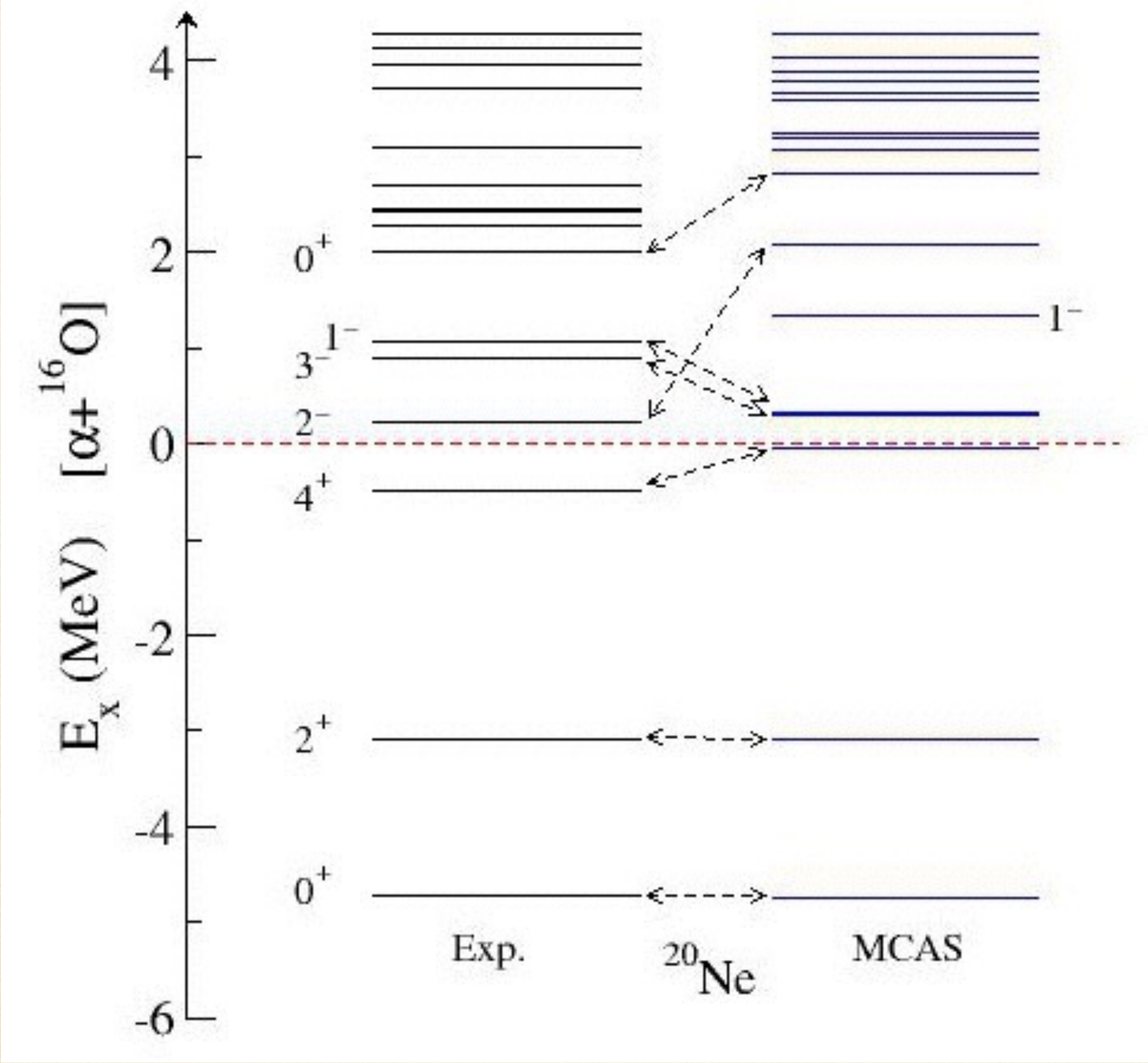


Spectrum of ^{16}O.



Effects of OPP on spectrum of ^{16}O.

^{20}Ne: $\alpha + ^{16}O$



Spectrum of ^{20}Ne.

Conclusions

We have an MCAS which handles the scattering of alphas from nuclei. The version allows:

- Considers spin-zero projectiles on spin-zero targets.
- Collective model specification of the target with up to octupole deformation when required.
- Known states of ^{8}Be are reproduced.
- Spectra of ^{12}C, ^{16}O, and ^{20}Ne are reasonably well reproduced, but more work is required to define the nuclear potentials.