Tensor optimized antisymmetrized molecular dynamics (TOAMD) for relativistic nuclear matter

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Tensor Optimized Antisymmetrized Molecular Dynamics (TOAMD)

Myo Toki Ikeda

Tensor optimized shell model (TOSM)

1. We include tensor interaction most effectively to shell model
2. Difficult to treat cluster structure

+

Horiuchi Enyo Kimura..

Antisymmetrized molecular dynamics (AMD)

1. Cluster+shell structure is handled on the same footing with effective interaction
2. Difficult to treat bare nucleon-nucleon interaction

Study nuclear structure based on nuclear interaction

TOAMD wave function (variational wave function)

$$\Psi = (1+F_S)(1+F_D)\Phi(AMD)$$

$$\Phi(AMD) = A\Pi_i e^{-\nu(x_i - D_i)^2}\chi_i(\sigma)\xi_i(\tau)$$

$$F_D = f_D(r:\alpha)(3\sigma_1\cdot\hat{r}\sigma_2\cdot\hat{r} - \sigma_1\cdot\sigma_2)$$ Tensor correlation

$$F_S = f_S(r:\beta)$$ Short range correlation

1. Gauss expansion
2. Momentum space
3. Anti-symmetrization

Matrix elements ↓ Gauss integration(analytical) ↓ Anti-symmetrization ↓

$$\langle AMD | FVF | AMD \rangle = \sum_{ij..}^{A} \sum_{gauss} F_g V_g F_g I^{(ij..)}(space) M(spin) C(ij..)$$

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Tensor-optimized antisymmetrized molecular dynamics in nuclear physics

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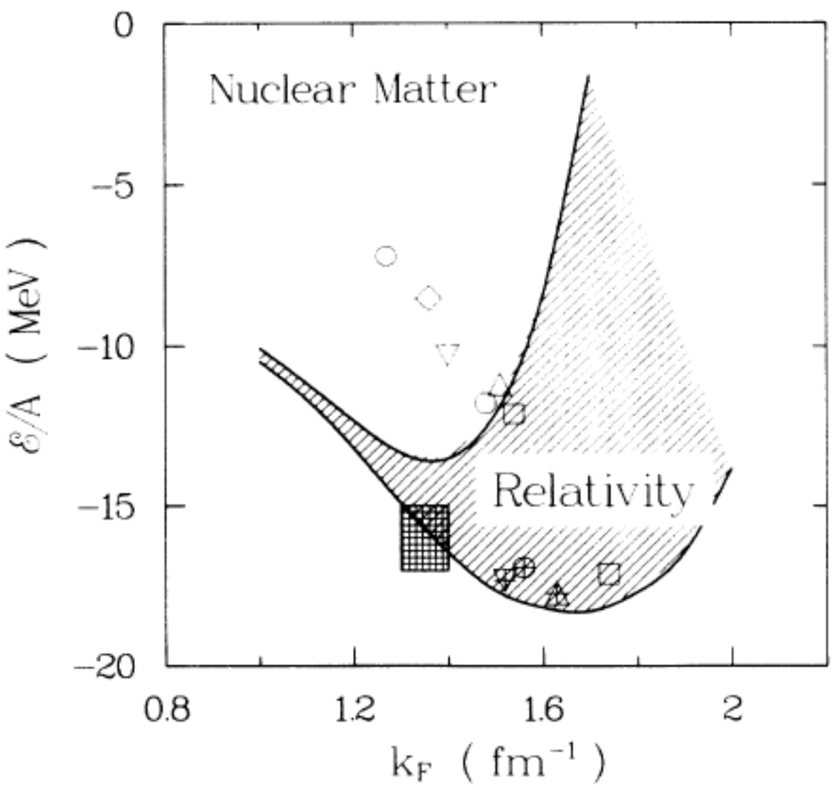
TOAMD project

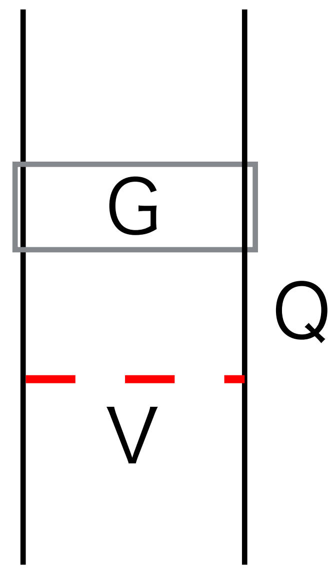
1. T. Myo : S-shell nuclei (demonstrated)
   Make fundamental programs and establish the TOAMD concept
2. T. Suhara : P-shell nuclei
   Establish the treatment of shell structure
3. H. Toki, T. Yamada : Nuclear matter
   Study infinite matter
4. Many collaborations : China, Korea

Nuclear Matter（Relativistic effect）

Brockmann Machleidt : PRC42(1990)1965

Relativistic Brueckner-Hartree-Fock with Bonn-potential





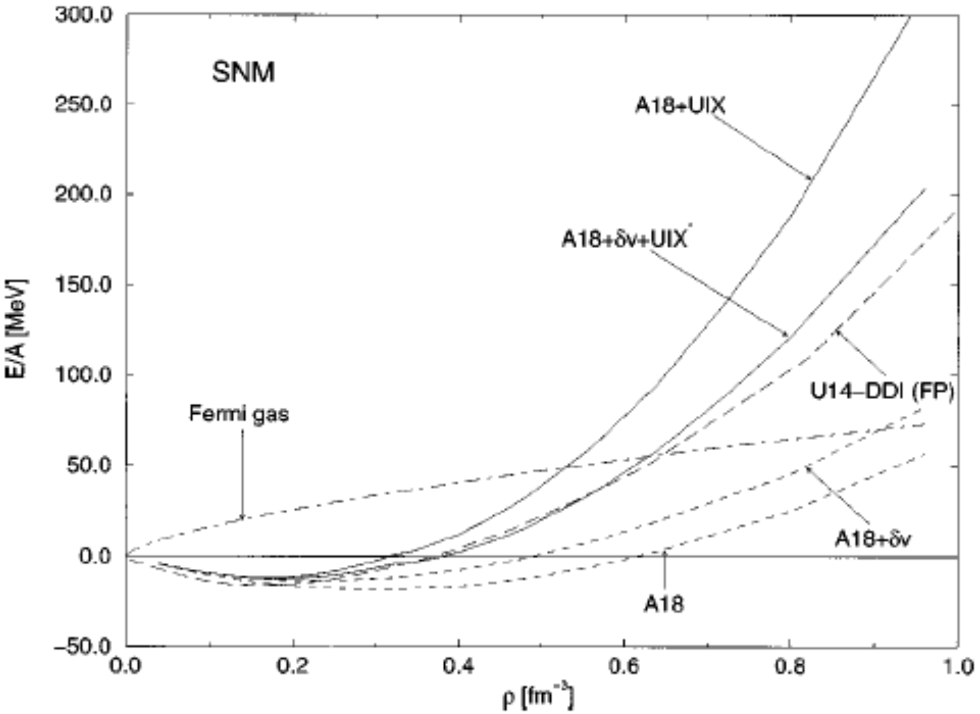
$$(\alpha \cdot p + \beta m + U)\tilde{\psi} = E\tilde{\psi}$$

$$U_k(\tilde{m}) = \sum_p \langle kp | G(\tilde{m}) | kp - pk \rangle$$

$$U = \beta U_S + U_V \qquad \tilde{m} = m + U_S(\tilde{m})$$

Infinite matter（non-relativistic framework：3 body repulsion +Boost corrections)

Akmal Pandhyaripande Ravenhall : PRC58(1998)1804



Relativistic effect

Effective mass

＝3 body repulsion

C.M. boost effect

＝C.M. boost interaction

＋

3 body attraction（Δ）

Variational chain summation (VCS)

$$\Psi = \prod_{ij} \left(1 + F_{ij}\right) \Phi$$

F_{ij}^{p} correlation function

Extension of Hartree–Fock theory including tensor correlation in nuclear matter

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Jinniu Hu[1,2,*,†], Hiroshi Toki[1,*,†], and Yoko Ogawa[1,*,†]

TOSM for relativistic matter

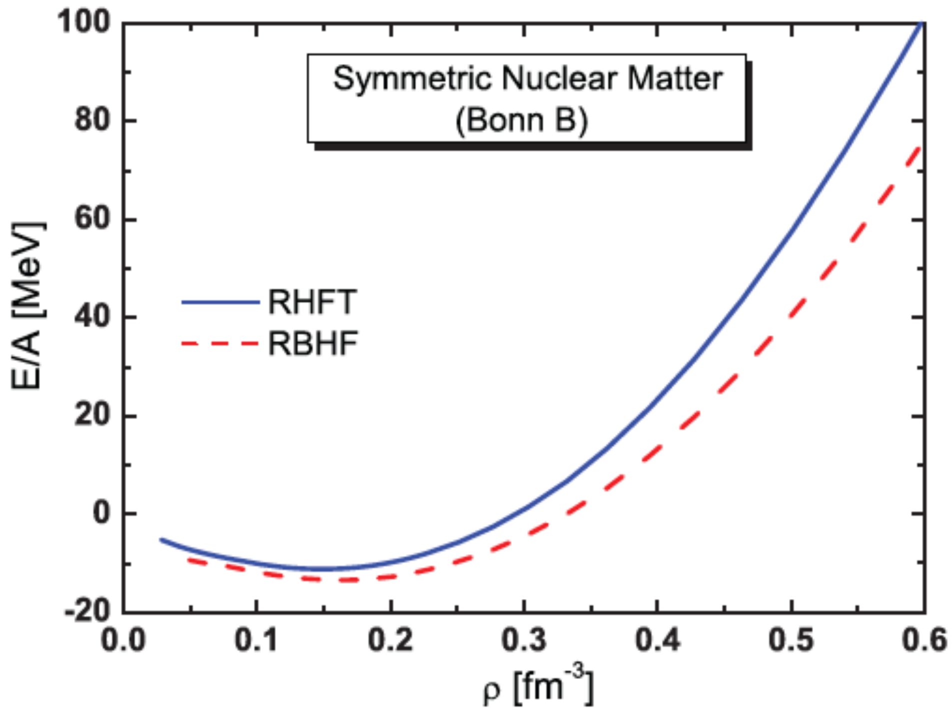
$$\Psi = C_0|0\rangle + \sum_\alpha C_\alpha |2p2h:\alpha\rangle \qquad |C_0|^2 + \sum_\alpha |C_\alpha|^2 = 1$$

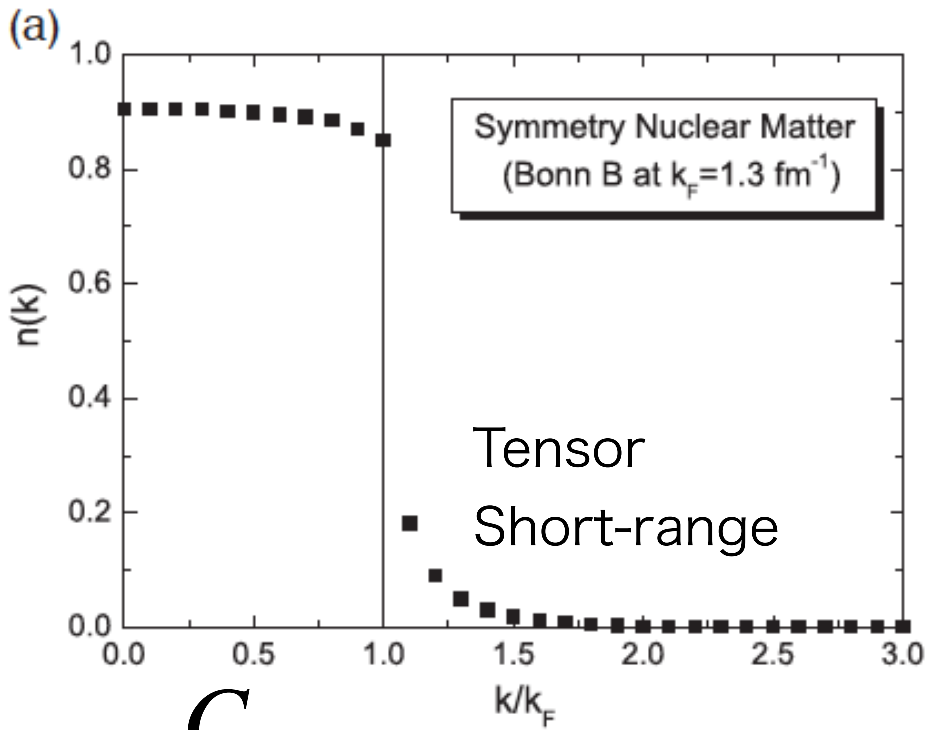
$$\langle\Psi|H|\Psi\rangle = |C_0|^2\langle 0|H|0\rangle + \sum_\alpha C_0^* C_\alpha \langle 0|H|\alpha\rangle + \sum_\beta C_\beta^* C_0 \langle\beta|H|0\rangle + \sum_{\alpha,\beta} C_\beta^* C_\alpha \langle\beta|H|\alpha\rangle$$

$$\langle 0|H_{\text{eff}}|0\rangle = |C_0|^2\langle 0|T+V|0\rangle - |C_0|^2 \sum_{\alpha,\beta}\langle 0|V|\alpha\rangle\langle\alpha|\frac{1}{H-E}|\beta\rangle\langle\beta|V|0\rangle$$

Brueckner-Hartree-Fock type equation

Numerical results of TOSM and comments

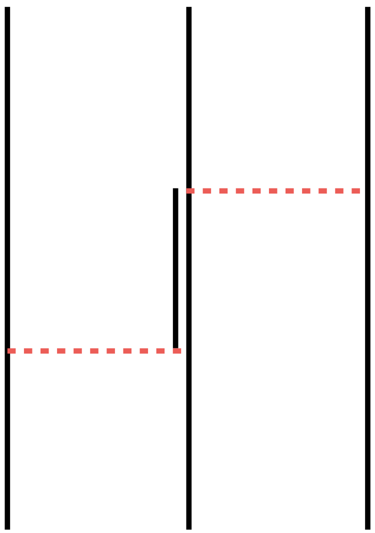




C_0
(low momentum)

C_α
(high momentum)

5MeV/A short



3 body interaction
(Fujita-Miyazawa delta term)

It is difficult to include 3 body interaction in TOSM

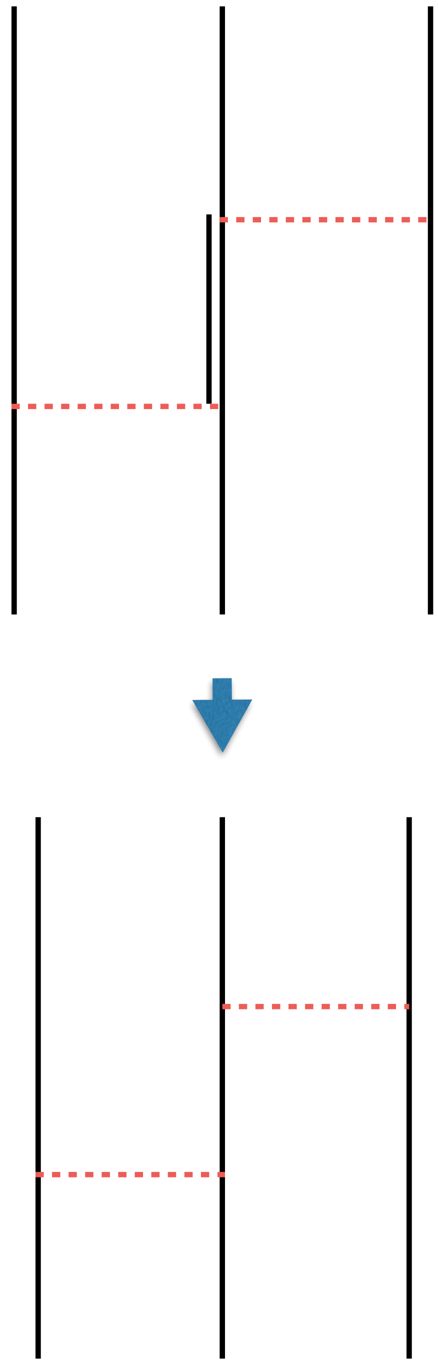
TOAMD vs TOSM

$$\Psi_{TOSM} = C_0|0\rangle + \sum_{\alpha} C_\alpha |2p2h:\alpha\rangle$$

$$\Psi_{TOAMD} = C_0|0\rangle + F_D|0\rangle$$

$$= C_0|0\rangle + \sum_{\alpha} |2p2h:\alpha\rangle\langle 2p2h:\alpha|F_D|0\rangle$$

$$C_\alpha$$



Concept is same, but TOAMD is flexible

$$VF_D = \frac{1}{2}\sum_{i\neq j} V_{ij} \frac{1}{2}\sum_{k\neq l} F_{Dkl}$$

$$= \frac{1}{2}\sum_{i\neq j} V_{ij}F_{Dij} + \sum_{i\neq j\neq k} V_{ij}F_{Djk} + \frac{1}{4}\sum_{i\neq j\neq k\neq l} V_{ij}F_{Dkl}$$

2 body term 3 body term 4 body term

We can include naturally the 3 body interaction

TOAMD for nuclear matter

$$\Psi = (1 + F_S)(1 + F_D)\Phi(RNM)$$

$$\Phi(RNM) = \prod_p^A \left| \psi_p(r,s)\xi_p(t) \right|$$

$$\psi_p(r,s) = \sqrt{\frac{E_p + \tilde{m}}{2\tilde{m}}} \begin{pmatrix} \chi_p(s) \\ \dfrac{\sigma \cdot p}{E_p + \tilde{m}} \chi_p(s) \end{pmatrix} \frac{1}{\sqrt{V}} e^{ipr}$$

$$F_D = f_D(r_{ij})(3(2m)^2 \gamma_{5i}\gamma_{5j} - k^2 \sum_x \gamma_{5i}\gamma_i^x \gamma_{5j}\gamma_j^x)\tau_i \cdot \tau_j \quad \rightarrow 3\sigma_1 \cdot k\sigma_2 \cdot k - k^2 \sigma_1 \cdot \sigma_2$$

$$F_S = f_S(r_{ij})\gamma_i^0 \gamma_j^0 \quad \rightarrow 1$$

$$H = T + V_{Bonn} + U_\Delta$$ (Three body interaction)

Formulation is simple（2 body＋3 body..)

2 body term

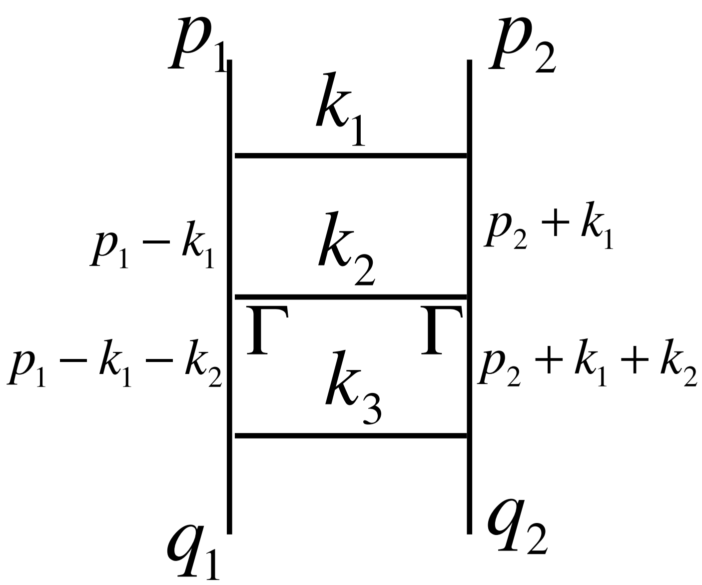
$$\langle RNM | F_S V F_S | RNM \rangle = \frac{1}{2} \sum_{p_1 p_2 : q_1 q_2} C(p_1 p_2 : q_1 q_2) \sum_{\mu_1 \mu_2 \mu_3} C_{\mu_1} C_{\mu_2} C_{\mu_3}$$

$$\sum_{k_1 k_2} e^{-k_1^2/k_{\mu_1}^2} e^{-k_2^2/k_{\mu_2}^2} e^{-(p_1 - q_1 - k_1 - k_2)^2/k_{\mu_3}^2} M(p_1 - k_1 | \Gamma | p_1 - k_1 - k_2) M(p_2 + k_1 | \Gamma | p_2 + k_1 + k_2)$$

$$M(p|1|q) = \sqrt{\frac{E_p + m}{2E_p}} \sqrt{\frac{E_q + m}{2E_q}} \chi_p^\dagger \left(1 - \frac{\sigma \cdot p}{E_p + m} \frac{\sigma \cdot q}{E_q + m} \right) \chi_q$$

$$M(p|\gamma_5|q) = \sqrt{\frac{E_p + m}{2E_p}} \sqrt{\frac{E_q + m}{2E_q}} \chi_p^\dagger \left(-\frac{\sigma \cdot p}{E_p + m} + \frac{\sigma \cdot q}{E_q + m} \right) \chi_q$$

$$C(p_1 p_2 : q_1 q_2) = \delta_{p_1 q_1} \delta_{p_2 q_2} - \delta_{p_1 q_2} \delta_{p_2 q_1}$$



MC(Metropolis) method for integration

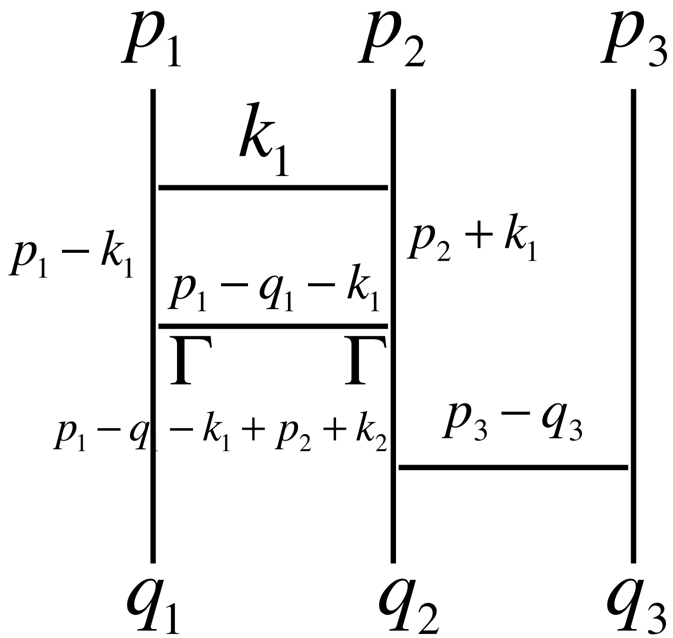
$$\langle f \rangle = \int dp_1 dp_2 dk_1 dk_2 f(p_1 p_2 k_1 k_2) \theta(p_1 - k_F) \theta(p_2 - k_F) e^{-k_1^2/k_{\mu 1}^2} e^{-k_1^2/k_{\mu 2}^2}$$

3 body term

$$\langle RNM | F_S V F_S | RNM \rangle = \sum_{p_1 p_2 p_3 : q_1 q_2 q_3} C(p_1 p_2 p_3 : q_1 q_2 q_3) \sum_{\mu_1 \mu_2 \mu_3} C_{\mu_1} C_{\mu_2} C_{\mu_3}$$

$$\sum_{k_1} e^{-k_1^2/k_{\mu 1}^2} e^{-(p_1 - q_1 - k_1)^2/k_{\mu 2}^2} e^{-(p_3 - q_3)^2/k_{\mu 3}^2} M(p_1 - k_1 | \Gamma | q_1) M(p_2 + k_1 | \Gamma | p_1 - q_1 - k_1 + p_2 + k_2)$$

$$C(p_1 p_2 p_3 : q_1 q_2 q_3) = \begin{vmatrix} \delta_{p_1 q_1} & \delta_{p_1 q_2} & \delta_{p_1 q_3} \\ \delta_{p_2 q_1} & \delta_{p_2 q_2} & \delta_{p_2 q_3} \\ \delta_{p_3 q_1} & \delta_{p_3 q_2} & \delta_{p_3 q_3} \end{vmatrix}$$



MC(Metropolis) method for integration

$$\langle f \rangle = \int dp_1 dp_2 p_3 dk_1 f(p_1 p_2 p_3 k_1) \theta(p_1 - k_F) \theta(p_2 - k_F) \theta(p_3 - k_F) e^{-k_1^2/k_{\mu 1}^2}$$

Non-relativistic nuclear matter (1/m expansion)

$$M(p|1|q)=\sqrt{\frac{E_p+m}{2E_p}}\sqrt{\frac{E_q+m}{2E_q}}\chi_p^{\dagger}\left(1-\frac{\sigma\cdot p}{E_p+m}\frac{\sigma\cdot q}{E_q+m}\right)\chi_q$$

$$M(p|1|q)=1-\frac{1}{8}\frac{(p+q)^2}{m^2}-\frac{1}{(2m)^2}\sum_{xyz}\varepsilon_{xyz}i\sigma_x p_y q_z$$

c.m. correction spin-orbit

$$M(p_1|1|q_1)M(p_2|1|q_2)=1-\frac{1}{8}\frac{(p_1+q_1)^2+(p_2+q)^2}{m^2}-\frac{1}{(2m)^2}\sum_{xyz}\varepsilon_{xyz}i\sigma_{1x}p_{1y}q_{1z}$$

$$-\frac{1}{(2m)^2}\sum_{xyz}\varepsilon_{xyz}i\sigma_{2x}p_{2y}q_{2z}+\frac{1}{(2m)^4}\sum_{xyz}\varepsilon_{xyz}i\sigma_{1x}p_{1y}q_{1z}\sum_{x'y'z'}\varepsilon_{x'y'z'}i\sigma_{2x'}p_{2y'}q_{2z'}$$

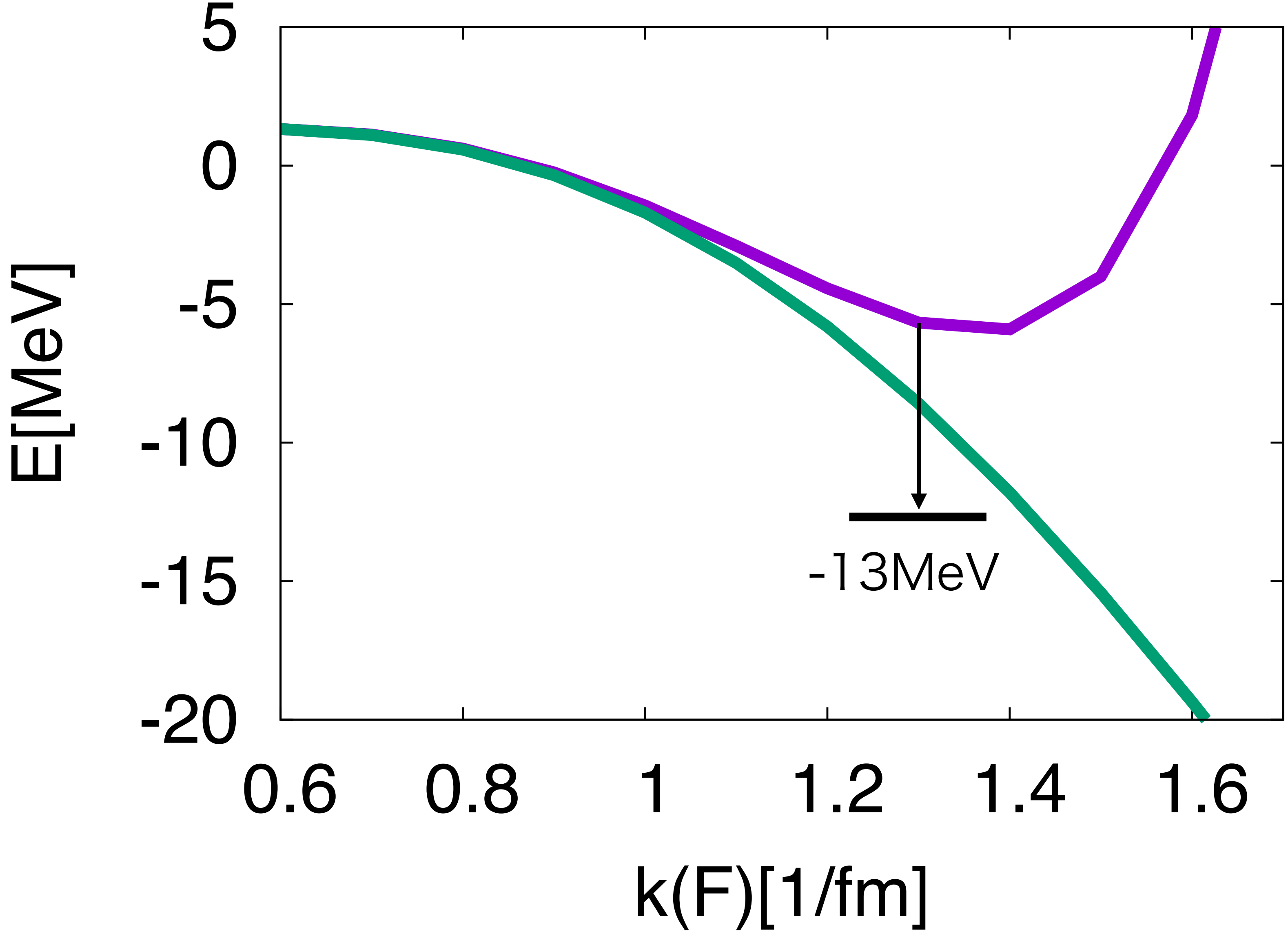
spin-orbit square

Analytical gauss integration and differentiation

Numerical results

Hartree-Fock with sigma+omega exchange

+pion with short and tensor correlation



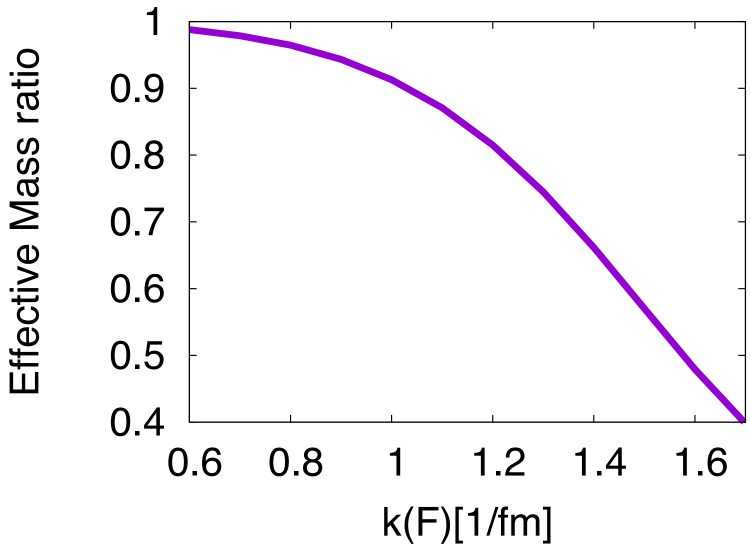
Present status (limitation)

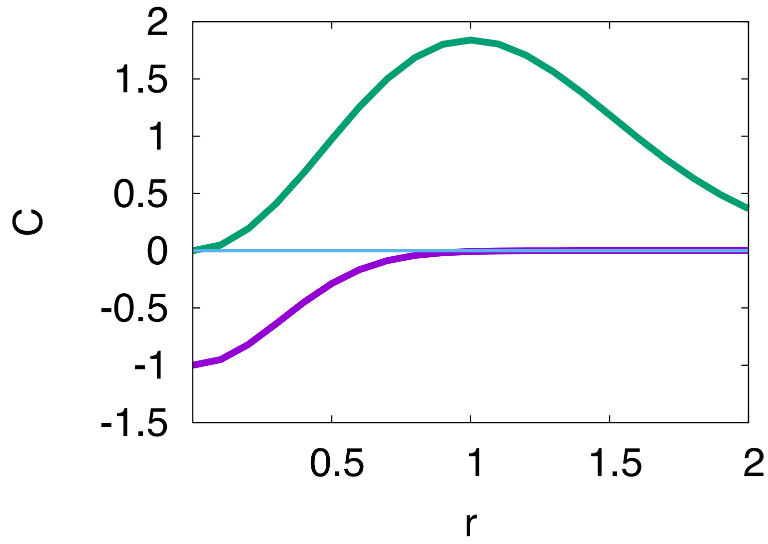
$\sigma + \omega + \pi + (\rho + \delta + \eta)$ (Bonn potential)

One gaussian -> many gaussians

Two body term + (many body term)

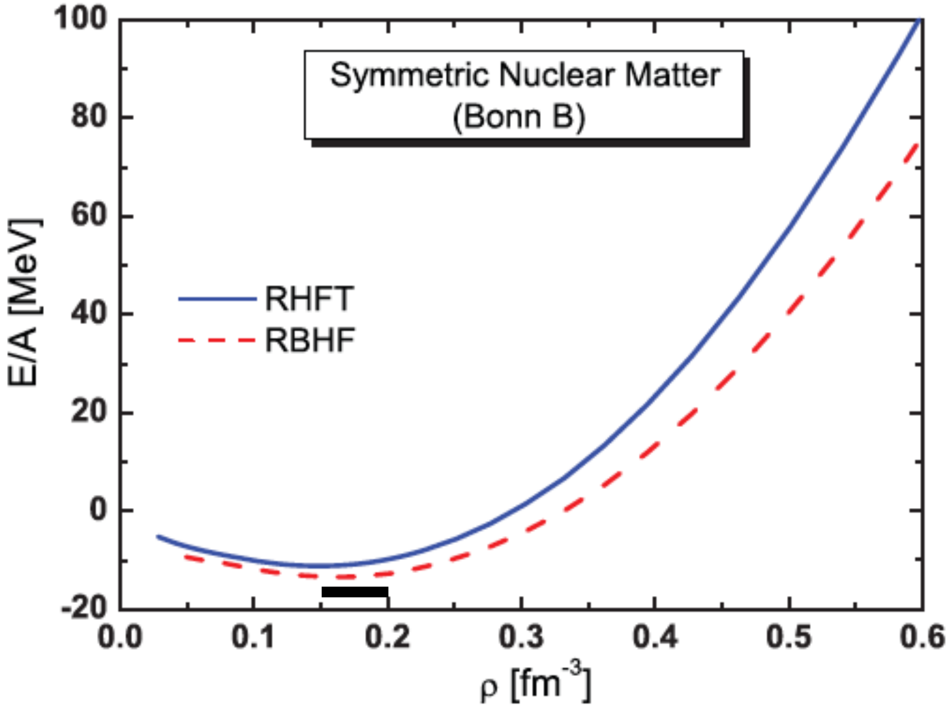
Two -> Three body interaction





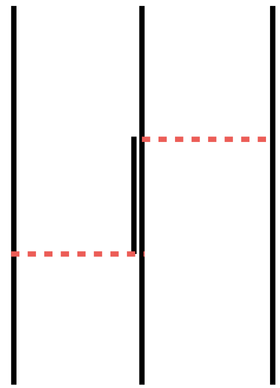
E(kin)=18MeV+5MeV+10MeV=33MeV

Fermi Short Tensor



TOSM

Hu Toki Ogawa



1. Reproduce TOSM results by TOAMD
2. Add three body interaction in TOAMD
3. Complete EOS in nuclear matter
4. Hyper-nuclear matter

Conclusion

1. We formulated relativistic nuclear matter using TOAMD
2. We formulated non-relativistic nuclear matter using TOAMD
3. We calculated various terms using Bonn potential
4. We express 3 body term and 3 body interaction
5. We get first (preliminary) results with correlations
6. We shall get relativistic EOS soon using Bonn potential