Revisiting Neutron EDM in the Standard Model

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T. Fukuyama (RCNP)
in collaboration with
D. Nomura (KEK) and T. Sato (RCNP)

We consider the neutron EDM in the SM.

Peculiar problem of neutron is that it is a hadron suffering from non-perturbative QCD corrections. We use both perturbation wrt weak-electromagnetism and effective chiral Lagrangian wrt strong interactions.The different approaches roughly converge to $d_n^{SM} = O(10^{-32})$ ecm. However these calculation have not considered the weak interactions among the valence quarks. We show that this yields rather small EDM contrary to the naive estimation. As a result, various stimations of EDM are around 10^{-32} e cm but their contents are very different.

Sec. 2. Penguin + Chiral PT
Sec. 3. Heavy Baryonic chiral PT
Sec. 4. Heavy quark bound state
Sec. 5. Transition quark EDM

We do not discuss, at least explicitly, on strong CPV like θ due to Peccei-Quinn symmetry.

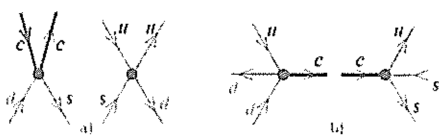
We reconsider the neutron EDM in the SM.

$$\mathscr{L}_W = \frac{G_F}{\sqrt{2}} J_\mu^\dagger J^\mu, \qquad J^\mu = (\bar{u}, \bar{c}, \bar{t})\gamma^\mu P_L V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.$$

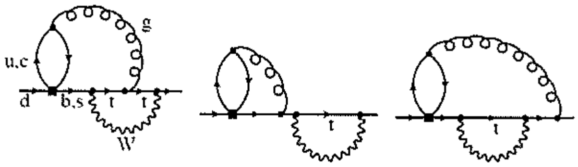
$$J_{CP} \equiv \left|\Im(V_{\alpha j}V_{\beta j}^* V_{\alpha k}^* V_{\beta k})\right| = \left|s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin\delta\right|$$

There are two types of vertexes



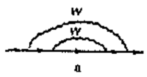
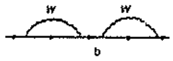
Quark EDM

three loop



two loop

etc.

 and 

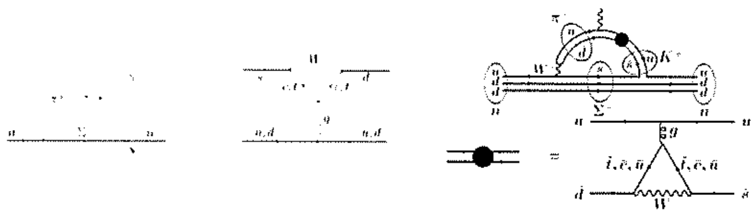
$\rightarrow 1 \times 10^{-34}$ e cm

are zero contribution

$$
\begin{aligned}
L &= \frac{e}{4m}\overline{\psi}\gamma_5\sigma^{\mu\nu}\psi F_{\mu\nu} = \frac{e}{2m}\overline{\psi}(\vec{\Sigma}\cdot\vec{E} - i\begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}\cdot\vec{H})\psi \\
&= \vec{d}\cdot\vec{E} + \text{off-diagonal Magnetic moment}
\end{aligned}
$$

Sec.2 Penguin+Chiral PT

Khriplovich-Zhitnitsky P.L.B109 (1981)



$H = iG_F m_\pi^2 \overline{u}_n (A + B\gamma_5) u_\Sigma \varphi_\pi$ with $A = -1.93$, $B = -0.65$

$$
\begin{aligned}
H_I &= i\frac{G_F}{\sqrt{2}}\frac{\alpha_s(\overline{m})}{12\pi}\Delta\ln(m_t^2/m_c^2)\bar{s}\gamma_\mu(1-\gamma_5)\lambda^a d \sum_{q=u,d}\bar{q}\gamma^\mu\lambda^a q \\
&\equiv iX\bar{s}\gamma_\mu(1-\gamma_5)\lambda^a d \sum_{q=u,d}\bar{q}\gamma^\mu\lambda^a q
\end{aligned}
$$

A and B are determined from the reaction rate of $\Sigma \rightarrow n\pi$ and therefore overall signature is undetermined.

$$H_I = i\frac{G_F}{\sqrt{2}}\frac{\alpha_s(\bar{m})}{12\pi}\Delta \ln(m_t^2/m_c^2)\bar{s}\gamma_\mu(1-\gamma_5)\lambda^a d \sum_{q=u,d}\bar{q}\gamma^\mu\lambda^a q$$

under the assumption of all $m_q < M_W$

Top quark mass was unexpectedly heavy 173 Gev (1995, FNAL) $m_t > M_W$ (80 GeV).

$$H_I = -i\frac{G_F}{\sqrt{2}}\alpha_s\frac{1}{8\pi}c_{23}s_{23}s_{13}\sin\delta\left(A_t - \frac{2}{3}\ln\left(\frac{m_c^2}{M_W^2}\right)\right)\bar{s}\gamma^\mu(1-\gamma_5)\lambda^a d\bar{q}\gamma_\mu\lambda^a q.$$

Here

$$A_j = \frac{2}{3}\left(1+a_j\right)\left(1-a_j-\frac{11}{4}a_j^2-\frac{3}{4}a_j^3\right)\ln(m_j^2/M_W^2) - \frac{3}{2}a_j\left(1+\frac{25}{18}a_j+\frac{1}{3}a_j^2\right)$$

with $a_j \equiv m_j^2/(M_W^2 - m_j^2)$.

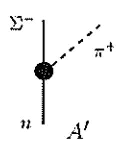
The results of KZ, which is still believed are :

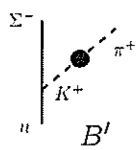
$H = iG_F m_\pi^2 \overline{u}_n(A+B\gamma_5)u_\Sigma\varphi_\pi$ with $A = -1.93$, $B = -0.65$

$$\langle\Sigma^-\pi^+|H_I|n\rangle = -X\overline{u}_\Sigma(A' + B'\gamma_5)u_n.$$

$$A' = \frac{4}{9}f_\pi\frac{m_\pi^2}{m_u+m_d}\frac{m_\Sigma - m_n}{m_s},$$

$$B' = \frac{4}{9}f_\pi\frac{m_\pi^2}{m_u+m_d}\frac{m_K^2}{m_s}(m_\Sigma+m_n)\frac{1}{m_K^2-q^2}(2\alpha-1)g_A$$





$$\begin{aligned} d_n &= -eG_F^2\sin\delta s_{12}s_{23}s_{31}c_{23}[\alpha_s\Delta/(\sqrt{2}27\pi^3) \\ &\times \ln\frac{m_t^2}{m_c^2}\frac{f_\pi m_\pi^4}{m_s(m_u+m_d)}A(2\alpha-1)g_A\ln\frac{m_K}{m_\pi}. \end{aligned}$$

$\frac{D}{F} = \frac{\alpha}{1-\alpha}$, $D + F = g_A$ $\rightarrow 1 \times 10^{-32}$ e cm

Fiertz transformation with color contraction

$$
\begin{aligned}
H_I &= i\frac{G_F}{\sqrt{2}}\frac{\alpha_s(\tilde{m})}{12\pi}\Delta \ln(m_t^2/m_c^2)\bar{s}\gamma_\mu(1-\gamma_5)\lambda^a d \sum_{q=u,d} \bar{q}\gamma^\mu\lambda^a q \\
&\equiv iX\bar{s}\gamma_\mu(1-\gamma_5)\lambda^a d \sum_{q=u,d} \bar{q}\gamma^\mu\lambda^a q
\end{aligned}
$$

$$
\mathcal{O} \equiv \sum_a (\bar{s}\gamma_\mu(1-\gamma_5)\lambda^a d)(\sum_{q=u,d} \bar{q}\gamma^\mu\lambda^a q)
$$

$$
= \mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3 + \mathcal{O}_4,
$$

$$
\mathcal{O}_1 = \frac{8}{9}\sum_{u,d}(\bar{q}\gamma_\mu(1-\gamma_5)d)(\bar{s}\gamma^\mu(1-\gamma_5)q), \qquad \mathcal{O}_2 = -\frac{16}{9}\sum_{u,d}(\bar{q}(1-\gamma_5)d)(\bar{s}(1+\gamma_5)q),
$$

$$
\mathcal{O}_3 = -\frac{1}{6}\sum(\bar{q}\gamma_\mu(1-\gamma_5)\lambda^a d)(\bar{s}\gamma^\mu(1-\gamma_5)\lambda^a q), \qquad \mathcal{O}_4 = \frac{1}{3}\sum_{u,d}(\bar{q}(1-\gamma_5)\lambda^a d)(\bar{s}(1+\gamma_5)\lambda^a q).
$$

We use chiral Lagrangian to evaluate the expectation value

chiral symmetry breaking

$$
\mathcal{L}_{SB} = \frac{F_\pi^2 B_0}{2} Tr[MU^\dagger + UM^\dagger] \to -\frac{B_0}{2} Tr(\phi^2 M)
$$

with

$$
U = exp\left(i\frac{\lambda_a\phi_a}{F_\pi}\right), \quad M = \mathrm{diag}(m_u, m_d, m_s), \quad F_\pi = \frac{f_\pi}{\sqrt{2}} = 93\mathrm{MeV},
$$

$$
\langle\mathcal{H}\rangle = -F_\pi^2 B_0(m_u+m_d+m_s), \quad \frac{\partial\langle 0|\mathcal{H}_{QCD}|0\rangle}{\partial m_q} = \frac{1}{3}\langle\bar{q}q\rangle
$$

$$
\langle\pi^+|\bar{u}\gamma_5 d|0\rangle = -iB_0 F_\pi = \frac{m_\pi^2}{m_u + m_d}(-iF_\pi),
$$

$$
\langle\Sigma^-|\bar{s}u|n\rangle = -\frac{2}{\sqrt{3}}\left[\langle n|\bar{q}\frac{\lambda_8}{2}q|n\rangle - \langle\Sigma^-|\bar{q}\frac{\lambda_8}{2}q|\Sigma^-\rangle\right] \approx -\frac{m_\Sigma - m_n}{m_s}.
$$

$$\langle h_1' h_2' | \hat{O}_i | h_1 h_2 \rangle \approx \langle h_1' | \overline{q} \Gamma_i d | h_1 \rangle \langle h_2' | \overline{s} \Gamma_i' q | h_2 \rangle.$$

Then the color singlet matrix elements of $\hat{O}_3$ and $\hat{O}_4$ terms including λ^a vanish

$$\begin{aligned}
\langle \Sigma^- \pi^+ | \hat{O}_2 | n \rangle &= \frac{16}{9} \langle \pi^+ | \overline{u}(1-\gamma_5) d | 0 \rangle \langle \Sigma^- | \overline{s}(1+\gamma_5) u | n \rangle \\
&= \frac{16}{9} \langle \pi^+ | \overline{u} \gamma_5 d | 0 \rangle \langle \Sigma^- | \overline{s} u | u \rangle \\
&= \frac{16}{9} \sqrt{2} F_\pi \frac{m_\pi^2}{m_u + m_d} \frac{m_\Sigma - m_n}{m_S},
\end{aligned}$$

which correspond to A of KZ but larger than that by factor 4.

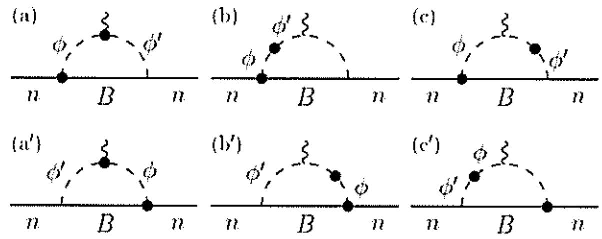
$$\begin{aligned}
|n\rangle &= \frac{1}{\sqrt{2}} (\phi_6 + i\phi_7) |0\rangle, \ |\Sigma\rangle = \frac{1}{2} (\phi_1 + i\phi_2) |0\rangle \\
\frac{\sqrt{3}}{2} \overline{s} u &= [V_-, V_8], \quad V_\pm \equiv V_4 \pm i D_5, \ V_i = \overline{q} \frac{\lambda_i}{2} q \\
\langle \Sigma^- | \overline{s} u | n \rangle &= -\frac{2}{\sqrt{3}} \left\{ \langle n | \overline{q} \frac{\lambda_8}{2} q | n \rangle - \langle \Sigma^- | \overline{q} \frac{\lambda_8}{2} q | \Sigma^- \rangle \right\} \\
&= -\frac{m_\sigma - m_n}{m_s} \\
\langle \Sigma^- \pi^+ | \hat{O}_2 | n \rangle |_{B'} &= -\frac{8}{9} i F_\pi \frac{m_\pi^2}{m_u + m_d} \frac{m_K^2}{m_s} \left(-\frac{g_A}{g_V} \right) (m_n + m_\Sigma) \frac{D-F}{D+F} \\
&\quad \times \frac{1}{q^2 - m_K^2} \overline{u}_\Sigma \gamma_5 u_n . \gg \ \langle \Sigma^- \pi^+ | \hat{O}_1 | n \rangle
\end{aligned}$$

This term corresponds to B' of KZ and larger than theirs by factor 2

Sec.3 Heavy baryonic χ PT

In this section we treat hadrons more systematically in the framework of HBχPT

C-Y.Seng P.R.C91 (2015)



$$H \longrightarrow \mathcal{L}_{BBM} = h_D e^{i\phi_D} Tr\left[\overline{B}\{\xi^\dagger\lambda_+\xi, B\}\right] + h_F e^{i\phi_F} Tr\left[\overline{B}[\xi^\dagger\lambda_+\xi, B]\right] + H.c.$$

$$\mathcal{L}_8 = g_8 e^{i\phi} Tr[\lambda_+ D_\mu U D^\mu U^\dagger] + H.c.$$

$$U = \exp(i\phi/F_\pi),\ \lambda_+ = (\lambda_6 + i\lambda_7)/2$$

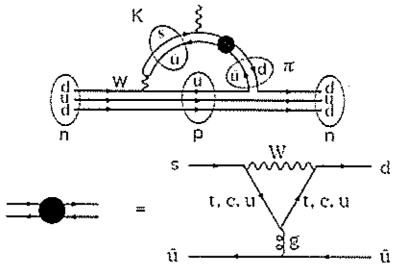
where $g_8, \varphi, h_D, \varphi_D$ etc are LECs.

$$\mathcal{L}_w^{(s)} = h_D e^{i\varphi_D} Tr[\overline{B}\left\{\xi^\dagger\lambda_+\xi, B\right\}] + h_F e^{i\varphi_F} Tr[\overline{B}\left[\xi^\dagger\lambda_+\xi, B\right]] + H.c.\ ,$$

$$\begin{aligned}
\mathcal{L}_w^{(s)} = \ & h_D e^{i\varphi_D}\Big[\Big(\overline{p}\Sigma^+ - \frac{1}{\sqrt{2}}\bar{n}\Sigma^0 - \frac{1}{\sqrt{6}}\bar{n}\Lambda + \overline{\Sigma^-}\Xi^- - \frac{1}{\sqrt{2}}\overline{\Sigma^0}\Xi^0 - \frac{1}{\sqrt{6}}\overline{\Lambda}\Xi^0\Big) \\
& + \frac{i}{2F_\pi}(\sqrt{2}\bar{n}pK^- - \sqrt{2}\bar{n}\Sigma^-\pi^+ - \frac{1}{\sqrt{2}}\bar{n}\Sigma^0\pi^0 - \frac{1}{\sqrt{6}}\bar{n}\Lambda\pi^0 + \sqrt{\frac{3}{2}}\bar{n}\Sigma^0\eta_8 + \frac{1}{\sqrt{2}}\bar{n}\Lambda\eta_8 \\
& + (\text{non } n\text{-terms}))\Big] + H.c. \\
& + h_F e^{i\varphi_F}\Big[\Big(-\overline{p}\Sigma^+ + \frac{1}{\sqrt{2}}\bar{n}\Sigma^0 - \sqrt{\frac{3}{2}}\bar{n}\Lambda + \overline{\Sigma^-}\Xi^- - \frac{1}{\sqrt{2}}\overline{\Sigma^0}\Xi^0 + \sqrt{\frac{3}{2}}\overline{\Lambda}\Xi^0\Big) \\
& + \frac{i}{2F_\pi}(\sqrt{2}\bar{n}pK^- + \sqrt{2}\bar{n}\Sigma^-\pi^+ + \frac{1}{\sqrt{2}}\bar{n}\Sigma^0\pi^0 - \sqrt{\frac{3}{2}}\bar{n}\Lambda\pi^0 - \sqrt{\frac{3}{2}}\bar{n}\Sigma^0\eta_8 + \frac{3}{\sqrt{2}}\bar{n}\Lambda\eta_8 + 2\sqrt{2}\bar{n}n\overline{K}^0 \\
& + (\text{non } n\text{-terms}))\Big] + H.c. + \mathcal{O}(1/F_\pi^3)\ .
\end{aligned}$$

which is obtained wrt $1/F_\pi$. $\xi = \sqrt{U} = \exp(i\phi/(2F_\pi))$.

$$\phi = \sum_{a=1}^{8}\phi_a\lambda_a = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 \end{pmatrix} \quad B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$



$$
\begin{aligned}
\mathcal{L}_8 &= g_8 e^{i\varphi} Tr[\lambda_+ D_\mu U D^\mu U^\dagger] + H.c. \\
&= \frac{g_8 e^{i\varphi}}{F_\pi^2}\left(2\partial_\mu K^- \partial^\mu \pi^+ - \sqrt{2}\partial_\mu \bar{K}^0 \partial^\mu \pi^0 - \sqrt{\frac{2}{3}}\partial_\mu \bar{K}^0 \partial^\mu \eta_8\right) + H.c. \\
&\quad + \frac{2ieg_8 e^{i\varphi}}{F_\pi^2} A_\mu \left\{(\partial^\mu K^-)\pi^+ - K^-(\partial^\mu \pi^+)\right\} + H.c. \\
&\quad + \mathcal{O}(e^2 g_8 e^{i\varphi}/F_\pi^2) + \mathcal{O}(1/F_\pi^3) \ .
\end{aligned}
$$

$$
\begin{aligned}
i\mathcal{M}_{(a)} &= \frac{2eg_8 e^{i\phi}}{F_\pi^4}(D+F)(h_D e^{-i\phi_D} + h_F e^{-i\phi_F}) \\
&\times \bar{u}(p+q)\int \frac{d^4k}{(2\pi)^4} S^\mu \frac{(p'-k+q)_\mu}{(p'-k+q)^2 - m_\pi^2} \frac{(2p'-2k+q)_\nu}{(p'-k)^2 - m_K^2} \frac{P_v^+}{v\cdot k - \delta_p + i\epsilon} u(p)\epsilon^\nu(q) \ ,
\end{aligned}
$$

LECs φ, φ_D, φ_F in place of KM phase (and the other D, F, h_D, h_F) appear, which are determined phenomenologically.

$$
\mathcal{M}_{(a)}^{(\Sigma)} = \mathcal{M}_{(a)}|_{(h_D, F, e, m_\pi, m_K, \delta_p) \to (-h_D, -F, -e, m_K, m_\pi, \delta_\Sigma)} \ .
$$

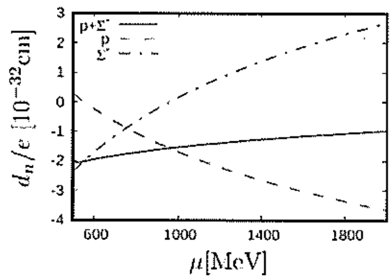
$$
(D+F)\langle B_i^m | j_k^i | B_m^k \rangle + (D-F)\langle B_m^k | j_k^i | B_i^m \rangle
$$

for instance, $j_2^1 = \overline{u}d$ and $j_1^3 = \overline{s}u$

Sum of all the p and Σ contributions

$$
\begin{aligned}
d_n = & -\frac{eg_8}{4\pi^2 F_\pi^4}\frac{Dh_D\sin(\phi-\phi_D)+Fh_F\sin(\phi-\phi_F)}{m_\pi^2-m_K^2}\left(m_\pi^2\ln\frac{m_\pi^2}{\mu^2}-m_K^2\ln\frac{m_K^2}{\mu^2}\right) \\
& -\frac{\delta_\Sigma eg_8}{4\pi^2 F_\pi^4}\frac{(D-F)(h_D\sin(\phi-\phi_D)-h_F\sin(\phi-\phi_F))}{m_\pi^2-m_K^2} \\
& \times\left\{\frac{m_\pi^2}{\sqrt{\delta_\Sigma^2-m_\pi^2}}\mathrm{arccosh}\left(\frac{\delta_\Sigma}{m_\pi}\right)-\frac{m_K^2}{\sqrt{m_K^2-\delta_\Sigma^2}}\arctan\left(\frac{\sqrt{m_K^2-\delta_\Sigma^2}}{\delta_\Sigma}\right)\right\}.
\end{aligned}
$$

$$d_n = (-1.52(p) - 0.07(\Sigma)) \times 10^{-32} \text{e cm}$$



Sec.4 Heavy quark bound state

Mannel-Uraltsev P.R.D85 (2002)

The diagram (b) in page 4 seems to give the additional $c\bar{c}$ bound state.

$$\mathcal{L} = \frac{G_F^2}{2}V_{cs}V_{cd}^*V_{ud}V_{us}^*\int d^4x iT\{(\bar{d}\Gamma_\mu c)(\bar{u}\Gamma^\mu d)_0\cdot(\bar{c}\Gamma_\nu s)(\bar{s}\Gamma^\nu u)_x\}+H.c.$$

$$c(0)\bar{c}(x) = \left(\frac{1}{m_c - i\hat{D}}\right) = \frac{1}{m_c}\delta^4(x) + \frac{1}{m_c^2}\delta^4(x)i\hat{D} + \frac{1}{m_c^3}\delta^4(x)(i\hat{D})^2 + \ldots$$

$$S_{CPV} = -iJ_{CP}\frac{G_F^2}{2m_c^2}O_{uds}$$

$$O_{uds} \equiv (\bar{u}\Gamma^\mu d)(\bar{d}\gamma_\mu i\hat{D}\Gamma_\nu s)(\bar{s}\Gamma^\nu u) - \{d \leftrightarrow u\}.$$

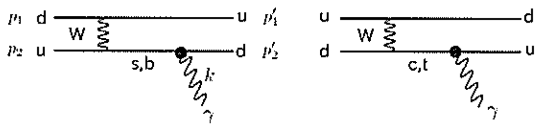
$$\langle n|O_{uds}|n\rangle = -2iK_{uds}q_\nu\bar{u}(p+q)\sigma^{\mu\nu}\gamma_5 u(p)$$

From the dimensional analyses, $|K_{uds}| = (250\text{MeV})^5$ and $|d_n| \approx O(10^{-31})$ e cm

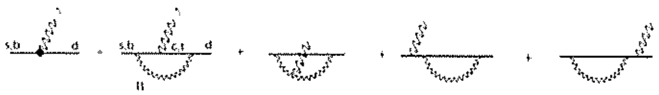
(We will see this vertex from the different point of view in the next section.)

Sec. 5 Transition quark EDM

2-quark EDM



$\Gamma \cdot E =$



$$\mathrm{Amp(a)} = \frac{G_F^2}{32\pi^2} J_{CP} F' \left[\overline{u}(p_1')\gamma^\mu(1-\gamma_5)d(p_1)\right] \left[\overline{d}(p_2')\Gamma\cdot E \frac{\acute{\overline{p}}+m_s}{\overline{p}^2 - m_s^2}\gamma_\mu(1-\gamma_5)u(p_2)\right]$$

$$\mathrm{Amp(a)} = \frac{G_F^2}{32\pi^2} J_{CP} F' \left[\overline{u}(p_1')\gamma^\mu(1-\gamma_5)d(p_1)\right] \left[\overline{d}(p_2')\Gamma\cdot E \frac{\acute{\overline{p}}+m_s}{\overline{p}^2 - m_s^2}\gamma_\mu(1-\gamma_5)u(p_2)\right]$$

$$F' = -\frac{8x_j + 5x_j^2 - 7x_j}{12(1-x_j)^3} + \frac{x_j^2(2-3x_j)}{2(1-x_j)^4}\ln x_j, \quad (x_j \equiv m_j^2/M_W^2)$$

$$\Gamma_\mu = i\sigma_{\mu\nu}k^\nu\left[(m_d+m_s)+(m_s-m_d)\gamma_5\right]$$

$$\psi_B = \psi_{\text{spin-flavour}} \times \psi_{\text{orbital}} \times \psi_{\text{color}}$$

$$\psi_{\text{spin-flavour}} = \frac{1}{\sqrt{2}}(\chi_{MS}\eta_{MS} + \chi_{MA}\eta_{MA})$$

$$\chi_{MS} = \frac{1}{\sqrt{6}}[2\alpha\alpha\beta - (\alpha\beta+\beta\alpha)\alpha] \qquad \eta_{MS} = -\frac{1}{\sqrt{6}}[(ud+du)d - 2ddu]$$

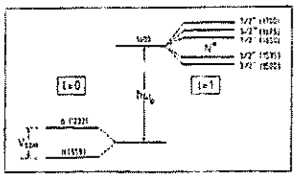
$$\psi(\vec{r},\vec{\rho}) = \phi_r(\vec{r})\phi_\rho(\vec{\rho}), \text{ with } \int |\phi_r|^2 d\vec{r} = \int |\phi_\rho|^2 d\vec{\rho} = 1.$$

$$H_0^{int} = \frac{(\vec{p})^2}{2m_0} + \frac{(\vec{q})^2}{2m_0} + \frac{3K}{2}(\vec{r})^2 + \frac{3K}{2}(\vec{\rho})^2.$$

$$\psi(\vec{r},\vec{\rho}) = \left(\frac{m_0\omega_0}{\pi}\right)^{3/2} e^{-\frac{m_0\omega_0}{2}(\rho^2+r^2)}$$

$$\langle n\ 1/2|(\sigma_1+\sigma_2)_z[\tau_{1+}\cdot\tau_{2-}+\tau_{1-}\cdot\tau_{2-}]|n\ 1/2\rangle = \frac{2}{9}$$

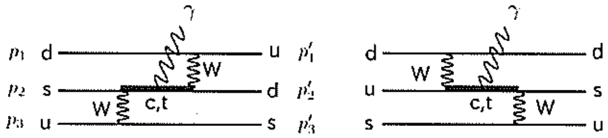
$$\int \psi^*(\vec{r},\vec{\rho})\delta(\vec{r})\psi(\vec{r},\vec{\rho}) = |\phi_r(0)|^2 = \left(\frac{m\omega_0}{\pi}\right)^{3/2}$$



$\omega_0 = \overline{M}^* - \overline{M} \approx 0.5$ GeV
$m_0 = 0.34$ GeV

$$d_n/Qe = -\frac{G_F^2}{16\pi^2} J_{CP} F'(x_t) \frac{2}{9}\left(\frac{m_0\omega_0}{\pi}\right)^{3/2} \approx -5.4\times10^{-34}\ \text{cm}$$

3-quark EDM



$$\begin{aligned} J_{3q}^\rho &= (Qe)\left(\frac{G_F}{\sqrt{2}}\right)^2 V_{1'1}V_{c2}V_{2'c}V_{3'3}\frac{1}{(2\pi)^6} \\ &\times \left[\overline{u}(p_1')\gamma^\mu(1-\gamma_5)u(p_1)\right]\left[\overline{u}(p_3')\gamma^\mu(1-\gamma_5)u(p_3)\right] \\ &\times \left[\overline{u}(p_2')\gamma^\mu(1-\gamma_5)\frac{1}{\not{q}+\not{k}-m_c}\gamma^\rho\frac{1}{\not{q}-m_c}\gamma_\mu(1-\gamma_5)u(p_2)\right] \end{aligned}$$

This is the same operator discussed in Sec.4.

$$\eta_{MS} = \frac{1}{2}\left[usd - dsu + s(ud - du)\right].$$

$$\eta_{M\Lambda} = \frac{1}{2\sqrt{3}}\left[s(du - ud) + usd - dsu - 2(du - ud)s\right]$$

$$d_\Lambda = -\sum_{Q=c,t} \frac{2e}{27}\frac{G_F^2}{m_Q^2 m}\Im(V_{Qd}V_{us}(V_{ud}V_{Qs})^*)|\psi(\vec{0},\vec{0})|^2 \times 6$$

$$d_\Lambda/e = -\frac{4}{9}\frac{G_F^2}{m_c^2}\left(\frac{1}{m} - \frac{1}{m_s}\right) J_{CP}\left(\frac{m_0\omega_0}{\pi}\right)^3 = -9.6 \times 10^{-31}\text{cm}.$$

$$|n\rangle = |udd\rangle + \epsilon|udds\overline{s}\rangle$$

So tri-quark neutron EDM is suppressed by ϵ^2 relative to d_Λ.

Summary

Both Penguin+Chiral PT (due to KM phase)and HBχPT gives the same order of $d_n \approx 10^{-32}$ e cm though the physical their contents are different. Heavy quark bound state gives some possibility of larger estimate than χPT approach. However, further survey seems to reject such optimistic possibility. Lattice QCD may reduce these rather small ambiguities in the near future. The transition quark EDM are evaluated semi-qualitatively by taking into account the structure of baryon within the constituent quark model.