Study of resonances in nuclei via CDCC analyses with CSM

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Neutrinos Electro-Weak interactions and Symmetries
(NEWS)

August 23rd, 2021

Outline

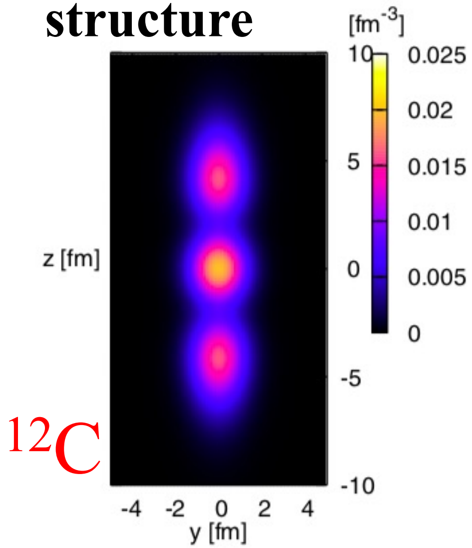
- Introduction
- Method
  - Continuum-discretized coupled-channels method (CDCC)
  - Complex-scaling method (CSM)
- Results
  - ^{11}Li(p,p') T.M., J. Tanaka, K. Ogata PTEP 123D02 (2019)
  - ^{12}C(α, α')
- Summary

Introduction

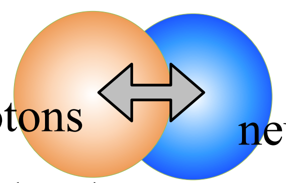
- Study of resonances is one of the most important subject in quantum systems (Atomic, Nuclear, Hadron).

- In nuclear physics, various types of resonances have been discovered, e.g., giant dipole, soft dipole, cluster resonances.

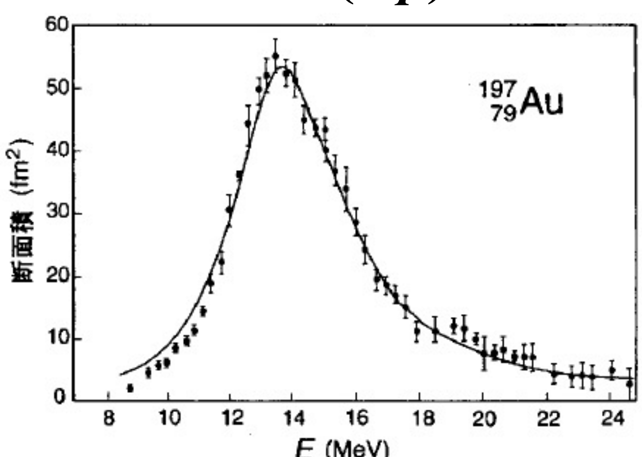
Linear chain structure

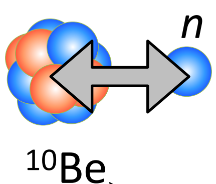


Y. Funaki, Phys. Rev. C **94**, 024344 (2016).

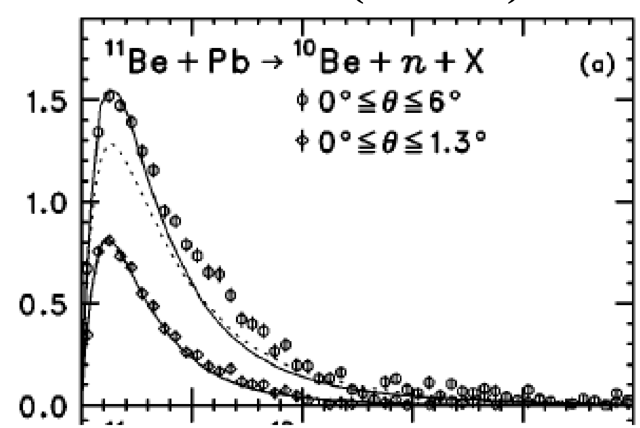


GDR (*n*-*p*)





SDR (*n*-core)

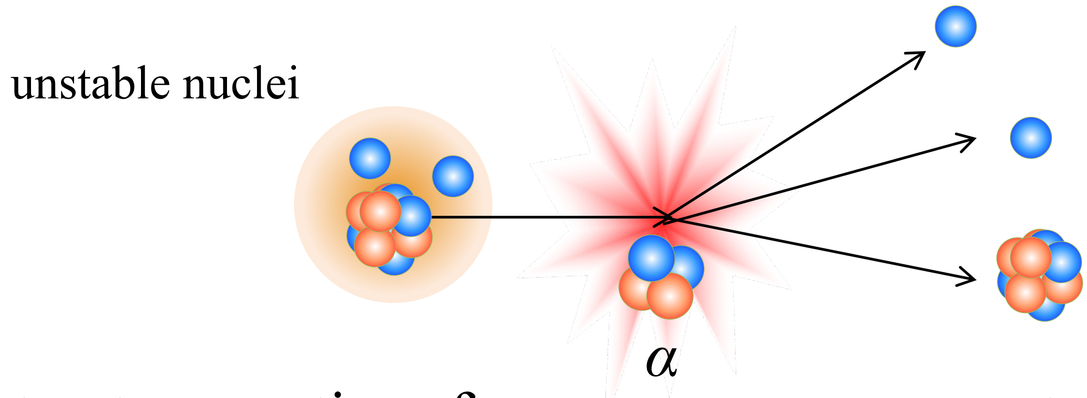


N. Fukuda et al., PRC 70 054606 (2004)

(p, p') and (α, α') reactions

- In order to investigate resonances in nuclei, (p,p') and (α,α') inelastic reactions are widely used.

- From the view of inverse kinematics, the inelastic reaction is regarded as *the breakup reaction*.



- To extract properties of resonances, an accurate method of treating breakup processes is highly desired

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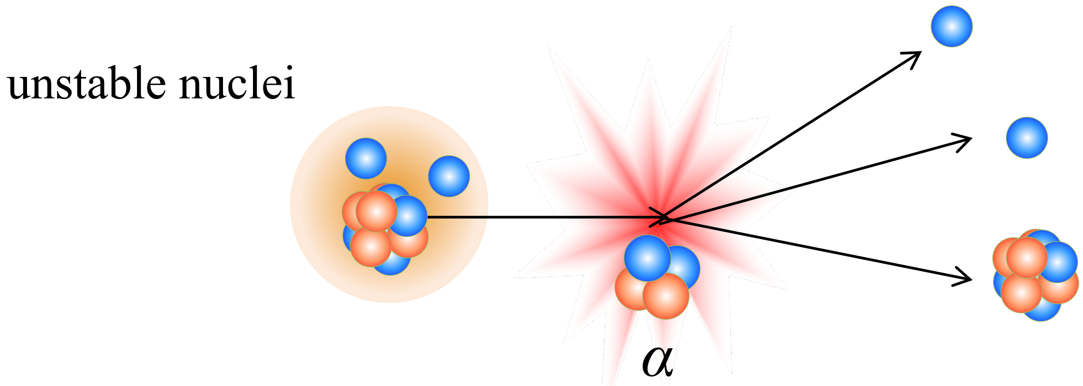
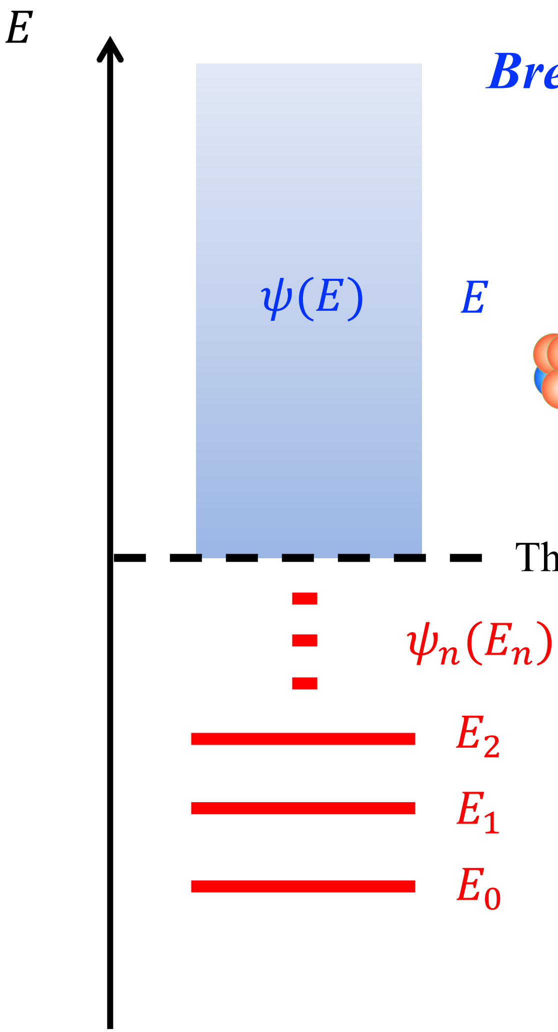


- To extract properties of resonances, an accurate method of treating breakup processes is highly desired

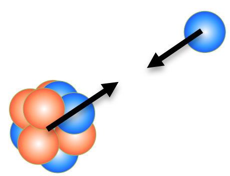
→ Continuum-discretized coupled-channels (CDCC) with complex-scaling method (CSM)

Breakup (continuum) states



Breakup (continuum) state

Scattering state



$$\psi(E) \underset{r \to \infty}{\to} e^{-ikr} - S e^{ikr}$$ (2-body system)

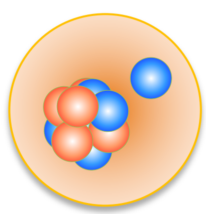
$$\langle \psi(E) | \psi(E') \rangle = \delta(E - E')$$

Threshold (particle decay)

Bound state

$$\langle \psi_n(E_n) | \psi_m(E_m) \rangle = \delta_{nm}$$

$$\psi_n(E_n) \underset{r \to \infty}{\to} 0$$



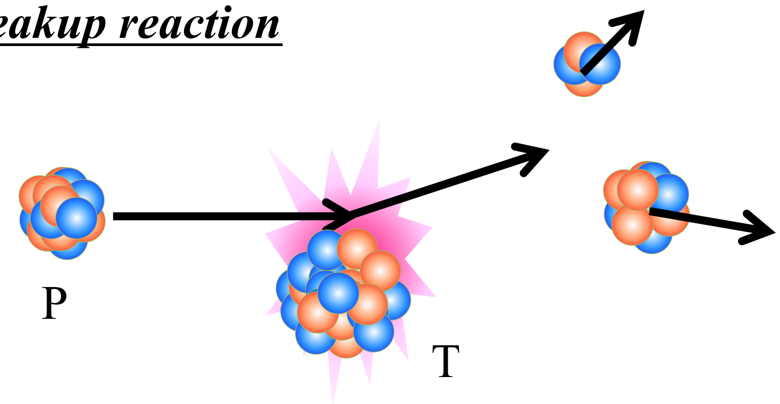
Completeness

$$1 = \sum_n |\psi_n\rangle\langle\psi_n| + \int dE |\psi(E)\rangle\langle\psi(E)|$$

Continuum-Discretized Coupled-Channels (CDCC) method

(Review) Yahiro, Ogata, TM, Minomo, PTEP01A206, (2012).

Breakup reaction



$$1 = |\psi_0\rangle\langle\psi_0| + \int dE|\psi(E)\rangle\langle\psi(E)|$$

Total scattering wave function

$$\Psi = \psi_0\chi_0 + \int_0^\infty dE\psi(E)\chi(E)$$

χ : Relative w.f. between P and T

Difficult to solve CC equation for χ

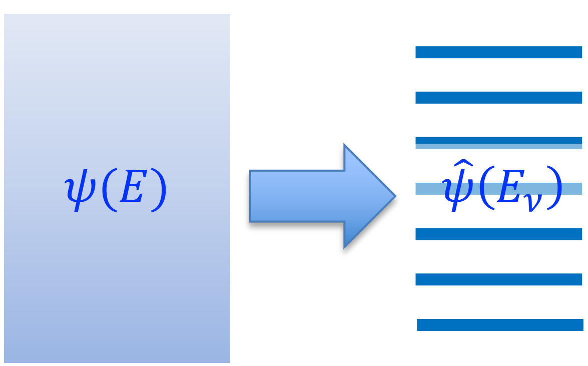
✓ $\psi(E) \underset{r\to\infty}{\to}$ *oscillating*

✓ *E is continuous*

Discretization for breakup (continuum) state

$$\psi(E) \to \{\hat{\psi}(E_\nu), \nu = 1, \cdots, N\}, \hat{\psi}(E_\nu) \underset{r\to\infty}{\to} 0$$

$$\langle\hat{\psi}(E_\nu)|\hat{\psi}(E_{\nu'})\rangle = \delta_{\nu\nu'}$$



Completeness (within a model space)

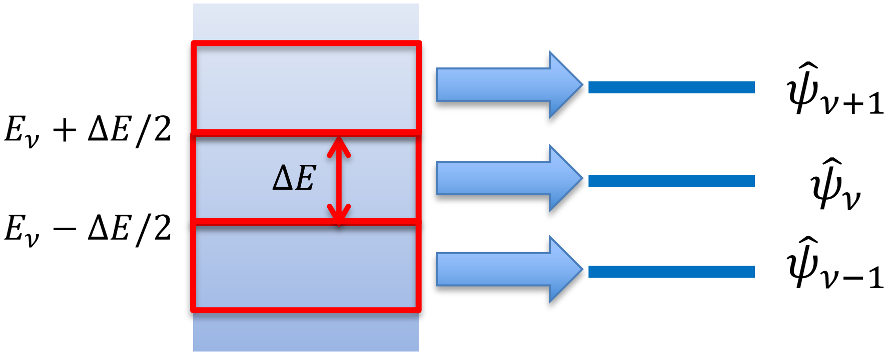
$$1 \approx |\psi_0\rangle\langle\psi_0| + \sum_\nu |\hat{\psi}_\nu\rangle\langle\hat{\psi}_\nu|$$

Discretizing Method

Momentum (energy) -bin method (Average method)

$$\hat{\psi}_\nu(E_\nu) = \frac{1}{\sqrt{\Delta E}} \int_{E_\nu - \Delta E/2}^{E_\nu + \Delta E/2} \psi(E)\, dE$$

$$\xrightarrow[\Delta E \to 0]{} \quad \hat{\psi}_\nu(E_\nu) = \psi(E_\nu)$$



- Successful for describing scattering of two-body projectile
- Need the exact continuum wave function for E
- Difficult to apply to many-body scattering system

Pseudostate method

$$\hat{\psi}_\nu(E_\nu) = \sum_{i=1}^{N} c_i^{(\nu)} \varphi_i$$

$c_i^{(\nu)}$ is calculated by diagonalizing $H_{ij} = \langle \varphi_i | H | \varphi_j \rangle$

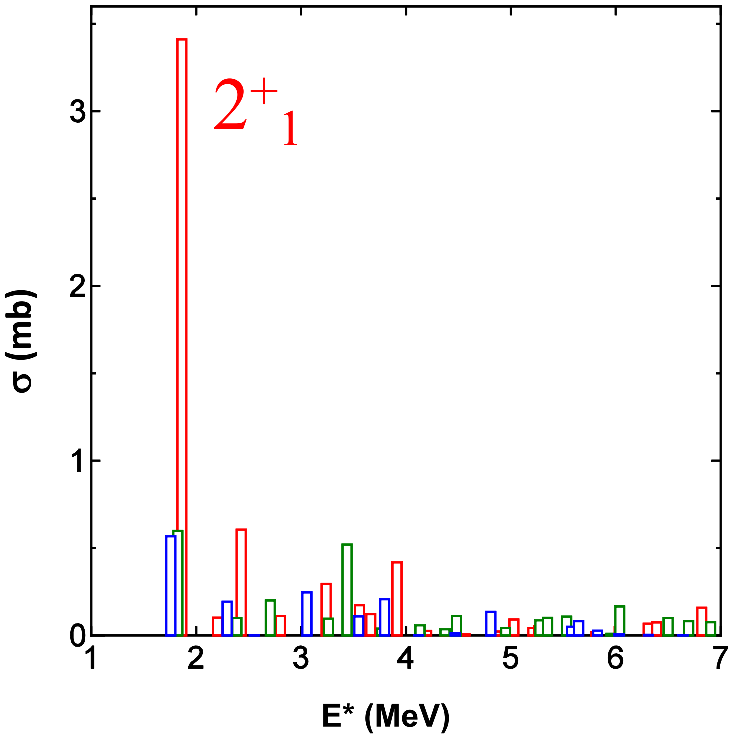
- Expand in terms of a L^2-type basis function
- Applicable to many-body scattering system

Discretized Cross Section

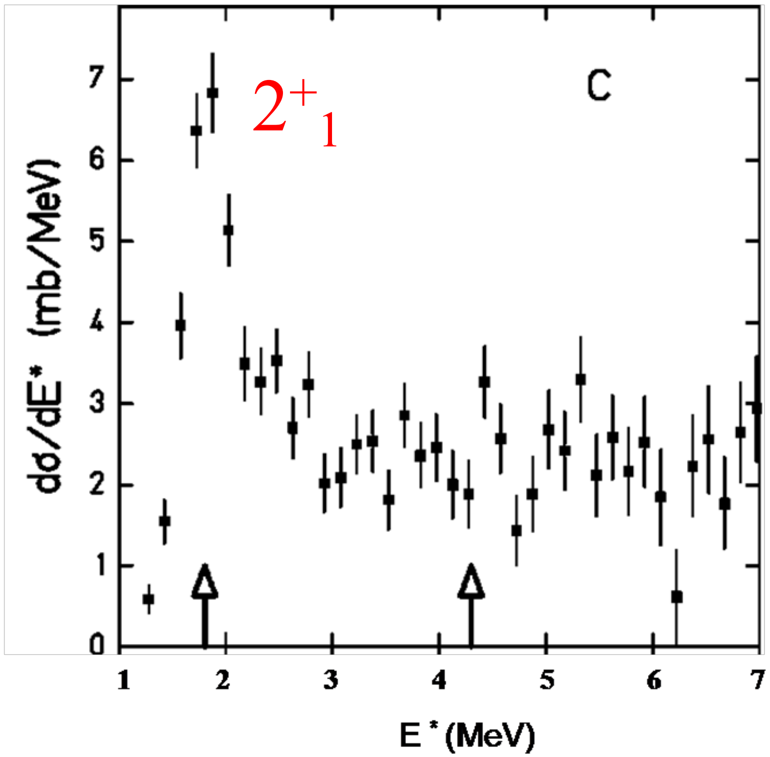
Breakup cross sections calculated by CDCC are discrete in the internal energy of the projectile.

$^{6}He+^{12}C$ scattering at 240 MeV/nucl.

4-body CDCC calc.



PRC59, 1252(1999), T. Aumann *et al*.



How to calculate the continuuous breakup cross section

Smoothing Method

- Expansion of $\psi(E)$ in terms of a set of $\hat{\psi}_\nu(E_\nu)$

$$\psi(E) = \sum_\nu f_\nu(E)\hat{\psi}_\nu(E_\nu)$$

Smoothing factor

$$f_\nu(E) = \langle \hat{\psi}_\nu(E_\nu) | \psi(E) \rangle$$

Orthonormality of $\hat{\psi}_\nu$

$$\langle \hat{\psi}(E_\nu) | \hat{\psi}(E_{\nu'}) \rangle = \delta_{\nu\nu'}$$

- T-matrix to continuum state

$$1 \approx |\psi_0\rangle\langle\psi_0| + \sum_\nu |\hat{\psi}_\nu\rangle\langle\hat{\psi}_\nu|$$

$$T(E) = \langle \psi(E)\chi^{(-)} | V | \Psi^{(+)} \rangle$$

$$= \sum_\nu \langle \psi(E) | \hat{\psi}_\nu(E_\nu) \rangle \langle \hat{\psi}_\nu(E_\nu)\chi^{(-)} | V | \Psi^{(+)} \rangle$$

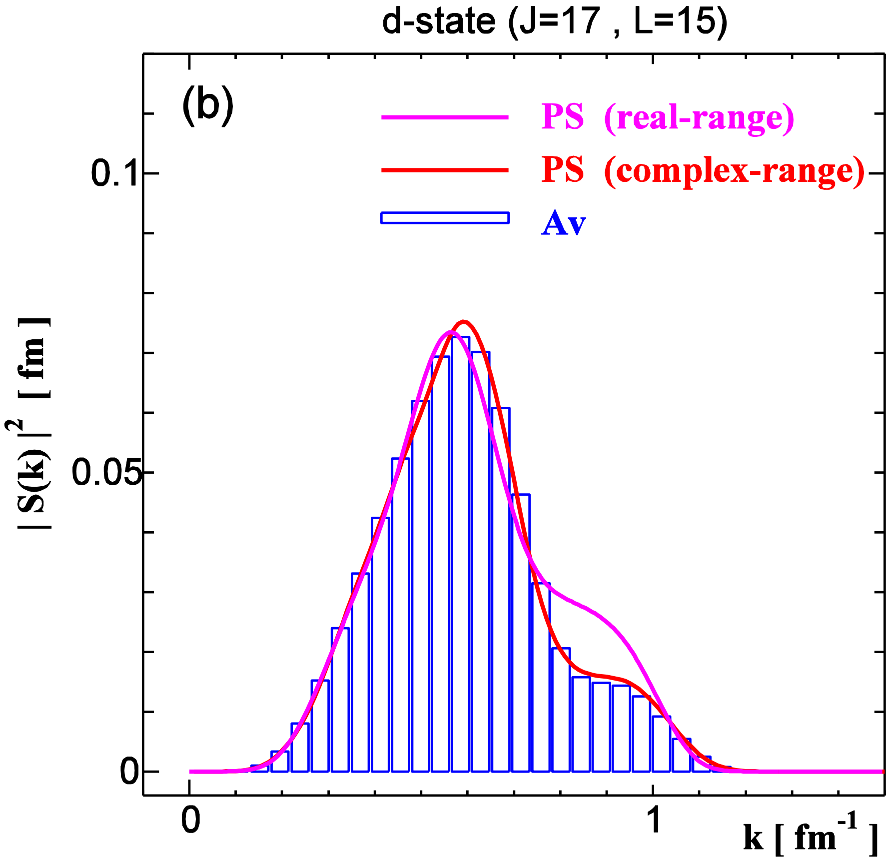
Smoothing factor

T-matrix to discretized states $\approx \hat{T}_\nu(E_\nu)$

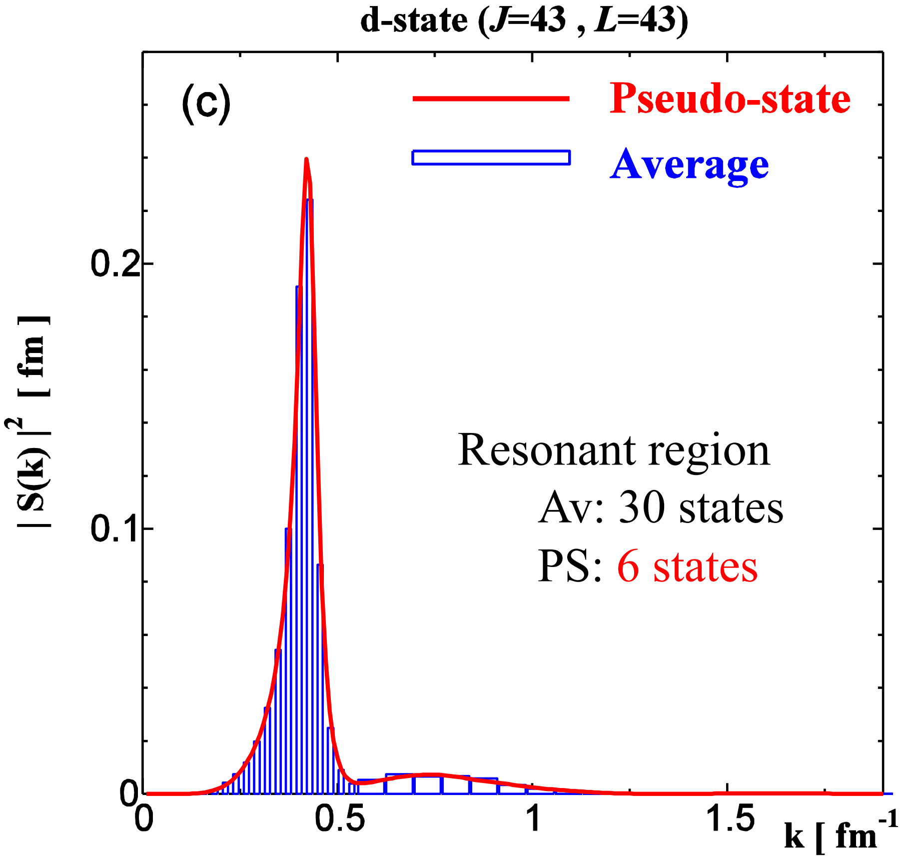
$$\approx \sum_\nu f_\nu^*(E)\hat{T}_\nu(E_\nu)$$

- Need the exact continuum wave function for E
- Difficult to apply to many-body scattering system

Deuteron breakup into *n* and *p*



^{6}Li breakup into *d* and *α*



Average method

$$f_\nu(E) = \begin{cases} \sqrt{\Delta E} & E_\nu - \frac{\Delta E}{2} \le E \le E_\nu + \frac{\Delta E}{2} \\ 0 & otherwise \end{cases}$$

Pseudostate method

- ✓ Consistent with Average method
- ✓ Useful for discretization including resonance

(review) S. Aoyama, T. Myo, K. Kato, and K. Ikeda, Prog. Theor. Phys. 116, 1 (2006)

Complex-scaling method

Resonant state

$\psi_{\rm res}(E_{\rm res}) \qquad E_{\rm res} = E_R - i\frac{\Gamma}{2}$

$\psi_{\rm res}(E_{\rm res}) \underset{r\to\infty}{\to} e^{ikr}$

(Only outgoing wave)

Scattering state

$\psi(E) \qquad E$ is real.

$\psi(E) \underset{r\to\infty}{\to} e^{-ikr} - S e^{ikr}$

(Incoming wave + outgoing wave)

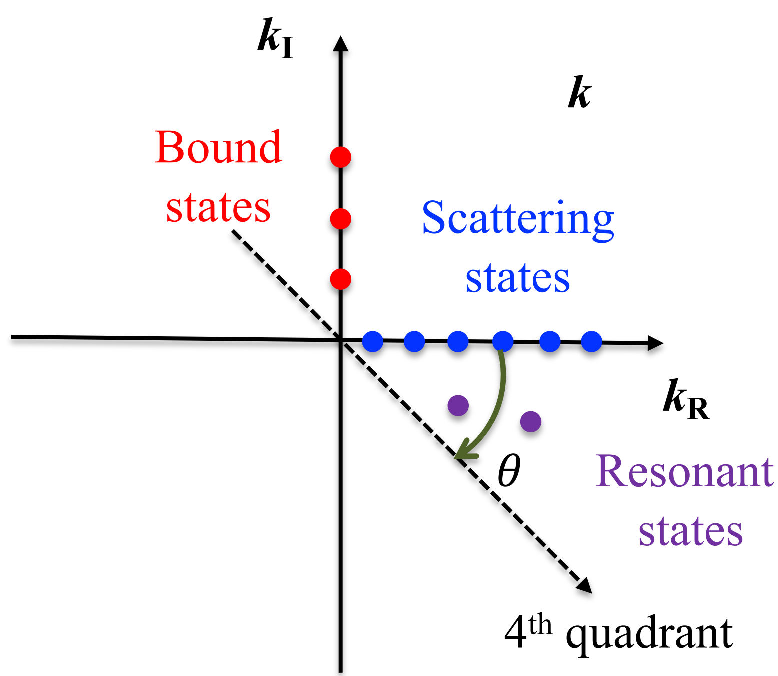
***Complex-scaling method* (CSM)**

Complex-scaling operator $U(\theta)$

$$U(\theta) f(r) = e^{i3/2\theta} f\left(re^{i\theta}\right)$$

$$\psi^{\theta}_{\rm res}(E_{\rm res}) \underset{r\to\infty}{\to} e^{ikr\cos\theta} e^{-kr\sin\theta} \to 0$$

$$\left(\tan\theta > \frac{k_I}{k_R}\right)$$



Resonant states can be treated the same way as **a bound state**.

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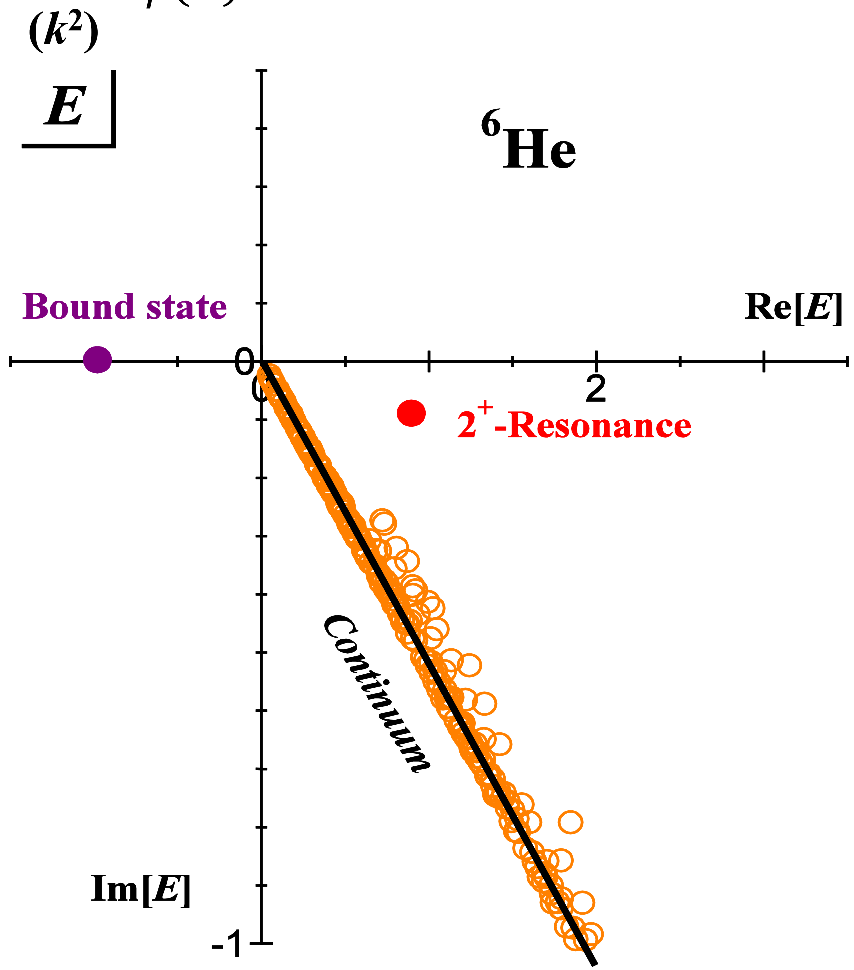

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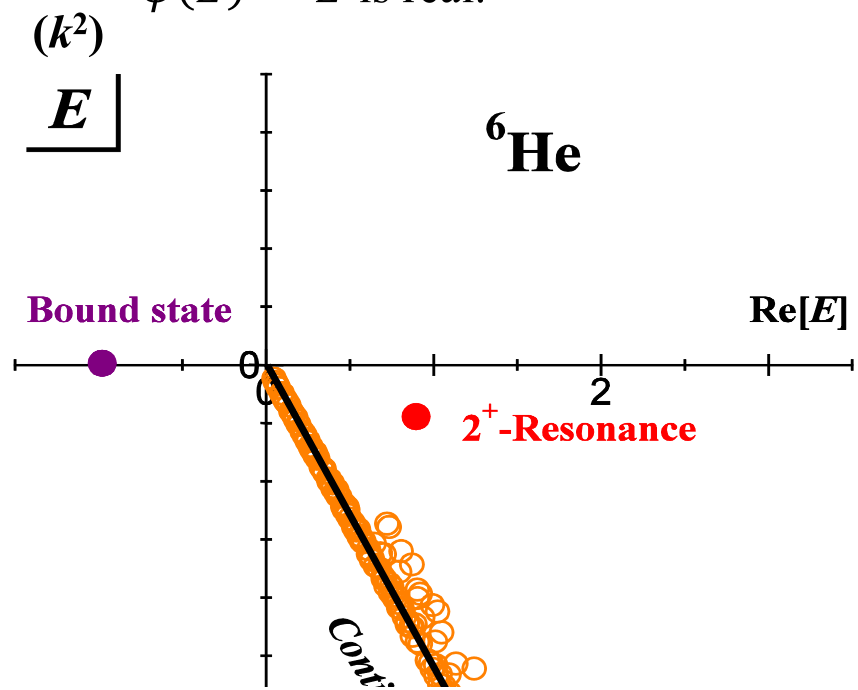


Extended completeness relation (ECR)

$$1 \approx |\phi^{\theta}_0\rangle\langle\tilde{\phi}^{\theta}_0| + |\phi^{\theta}_{res}\rangle\langle\tilde{\phi}^{\theta}_{res}| + \sum_i |\phi^{\theta}_i\rangle\langle\tilde{\phi}^{\theta}_i|$$

T. Berggren and P. Lind, Phys. Rev. C 47, 768 (1993).

New Smoothing Method

ECR

$$1 \approx |\phi_0^\theta\rangle\langle\tilde{\phi}_0^\theta| + |\phi_{res}^\theta\rangle\langle\tilde{\phi}_{res}^\theta| + \sum_i |\phi_i^\theta\rangle\langle\tilde{\phi}_i^\theta|$$

Complex-scaled Green's function (CSGF)

$$\frac{1}{E-H+i\epsilon} = U^{-\theta}\frac{1}{E-H^\theta+i\epsilon}U^\theta = \sum_\nu U^{-\theta}|\phi_\nu^\theta\rangle\frac{1}{E-E_\nu^\theta}\langle\tilde{\phi}_\nu^\theta|U^\theta$$

Complex values

Differential cross section

$$\frac{d\sigma}{dE} = \int |T(E')|^2\delta(E-E')dE'$$

$$T(E) = \langle\psi(E)\chi|V|\Psi^{(+)}\rangle$$

$$\delta(E-E') = -\frac{1}{\pi}\mathrm{Im}\frac{1}{E-E'+i\epsilon}$$

$$= -\frac{1}{\pi}\mathrm{Im}\langle\Psi^{(+)}|V|\chi\rangle\frac{1}{E-H+i\epsilon}\langle\chi|V|\Psi^{(+)}\rangle$$

$$= -\frac{1}{\pi}\mathrm{Im}\sum_{n,n'}\langle\Psi^{(+)}|V|\chi\hat{\psi}_n\rangle\langle\hat{\psi}_n|\frac{1}{E-H+i\epsilon}|\hat{\psi}_{n'}\rangle\langle\hat{\psi}_{n'}\chi|V|\Psi^{(+)}\rangle$$

Discretized T matrix

$$1 \approx \sum_n |\hat{\psi}_n\rangle\langle\hat{\psi}_n|$$

$$= -\frac{1}{\pi}\mathrm{Im}\sum_\nu\sum_{n,n'} T_n^\dagger\langle\hat{\psi}_n|U^{-\theta}|\phi_\nu^\theta\rangle\frac{1}{E-E_\nu^\theta}\langle\tilde{\phi}_\nu^\theta|U^\theta|\hat{\psi}_{n'}\rangle T_{n'}$$

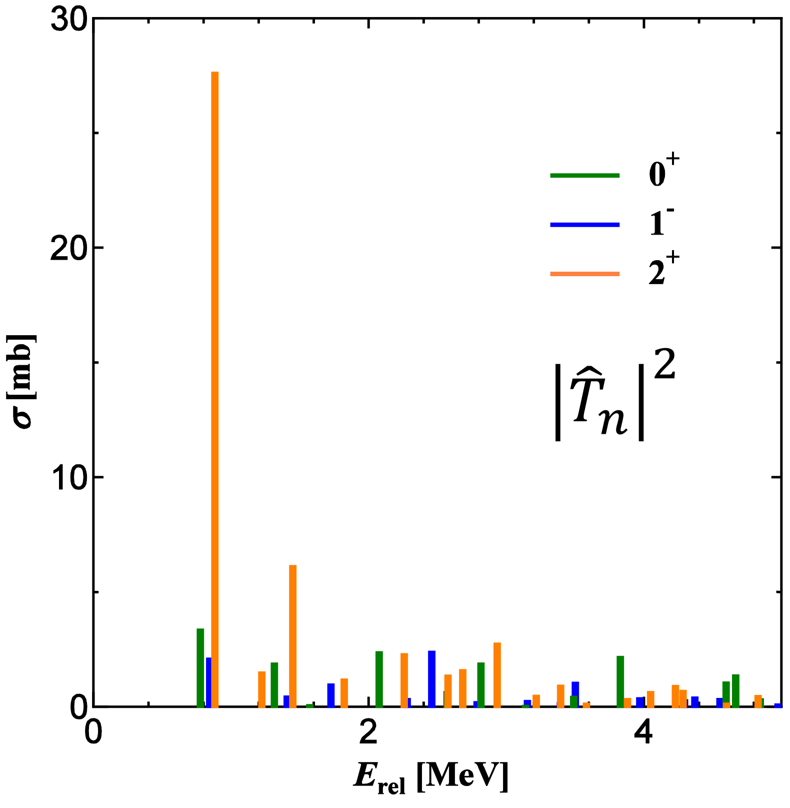
CSGF

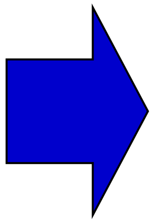
Discretized states

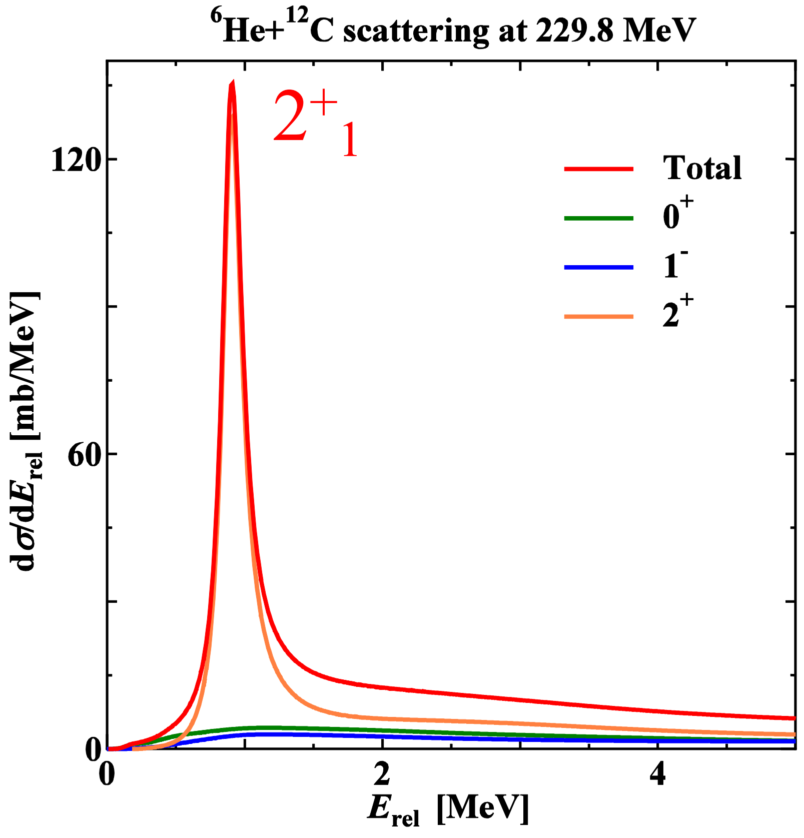
$$|\phi_\nu^\theta\rangle, |\hat{\psi}_n\rangle$$

Differential cross section

$$\frac{d\sigma}{dE} = -\frac{1}{\pi}\mathrm{Im}\sum_{\nu}\sum_{n,n'}\hat{T}_n^{\dagger}\langle\hat{\psi}_n|U^{-\theta}|\phi_\nu^\theta\rangle\frac{1}{E-E_\nu^\theta}\langle\phi_\nu^\theta|U^\theta|\hat{\psi}_{n'}\rangle T_{n'}$$







New smoothing method is easily applicable to many-body scattering systems.

Resonant Contribution

Differential cross section

$$\frac{d\sigma}{dE} = -\frac{1}{\pi}\mathrm{Im}\sum_{\nu}\sum_{n,n'}\hat{T}_n^{\dagger}\langle\hat{\psi}_n|U^{-\theta}|\phi_\nu^\theta\rangle\frac{1}{E-E_\nu^\theta}\langle\phi_\nu^\theta|U^\theta|\hat{\psi}_{n'}\rangle T_{n'}$$

Extended completeness

Non-resonant states

$$1 \approx |\phi_0^\theta\rangle\langle\tilde{\phi}_0^\theta| + |\phi_{res}^\theta\rangle\langle\tilde{\phi}_{res}^\theta| + \sum_i|\phi_i^\theta\rangle\langle\tilde{\phi}_i^\theta|$$

Resonant state

Separate contribution for resonant state and non-resonant states

$$\frac{d\sigma}{dE} = -\frac{1}{\pi}\mathrm{Im}\sum_{n,n'}\hat{T}_n^{\dagger}\langle\hat{\psi}_n|U^{-\theta}|\phi_{res}^\theta\rangle\frac{1}{E-E_{res}^\theta}\langle\tilde{\phi}_{res}^\theta|U^\theta|\hat{\psi}_{n'}\rangle T_{n'}$$

Resonant contribution

$$+\sum_{i}\sum_{n,n'}\hat{T}_n^{\dagger}\langle\hat{\psi}_n|U^{-\theta}|\phi_\nu^\theta\rangle\frac{1}{E-E_\nu^\theta}\langle\tilde{\phi}_\nu^\theta|U^\theta|\hat{\psi}_{n'}\rangle T_{n'}$$

Non-resonant contribution

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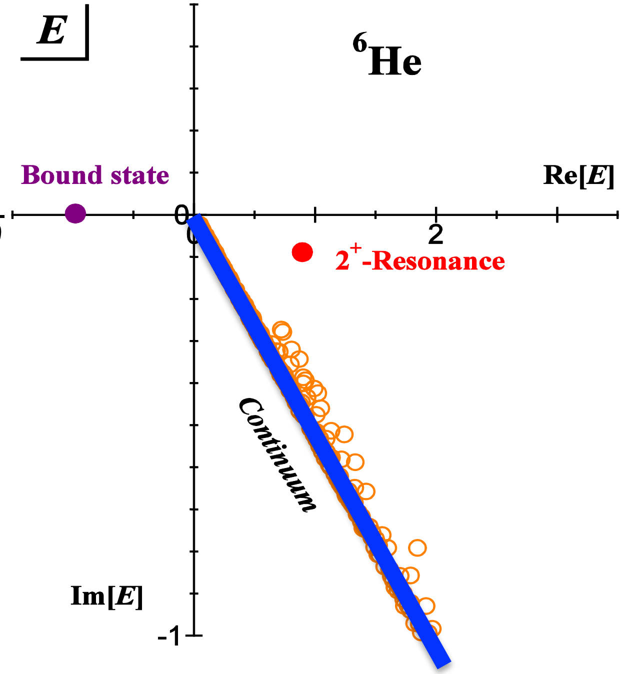

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Non-resonant contribution

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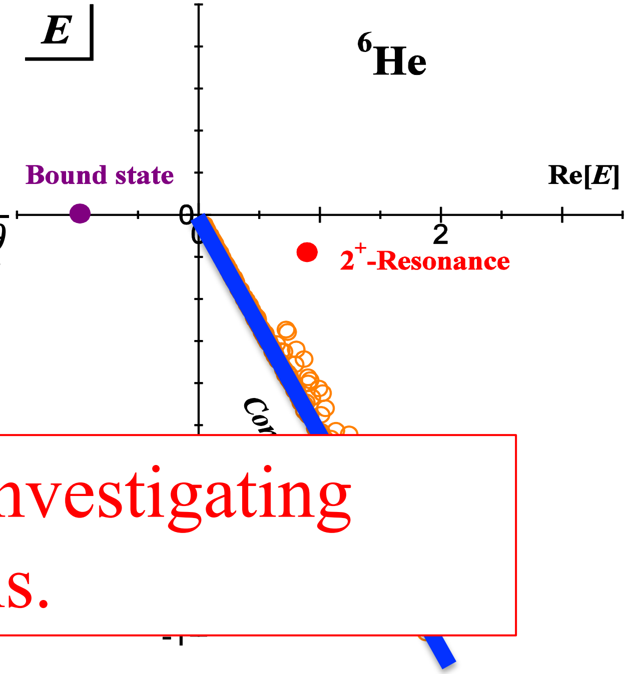

Extended completeness

Non-resonant states

The CDCC with CSM is useful for investigating resonances via breakup cross sections.

Separate contribution for resonant state and non-resonant states

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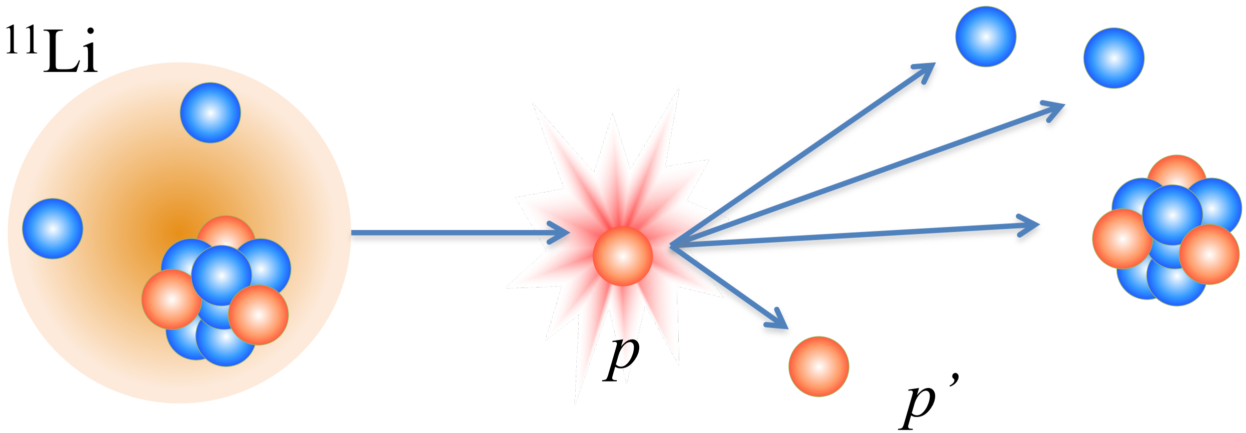
Non-resonant contribution

Analysis of ^{11}Li(p, p')@6MeV

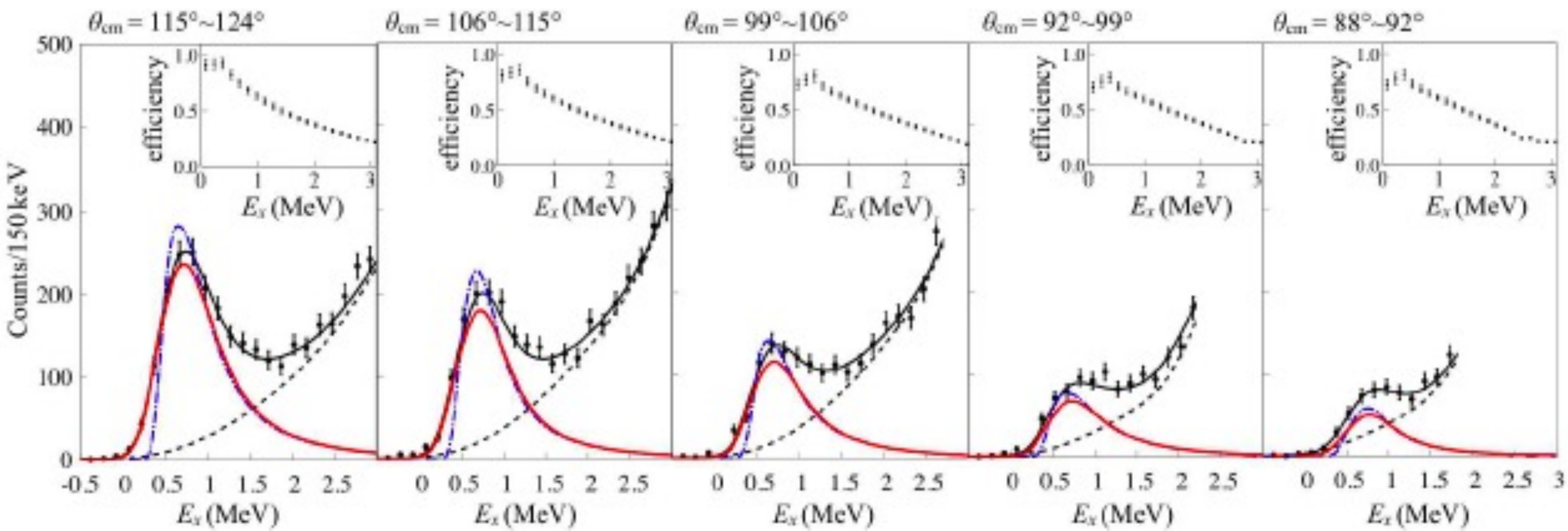


T.M., J. Tanaka, K. Ogata PTEP 123D02 (2019)

Recent experiment of ^{11}Li

◆ Measurement of the ^{11}Li(p, p') reaction at 6 MeV/nucleon with high static and high resolution has been performed, and a low-lying excited state of ^{11}Li has clearly been identified.

J. Tanaka et al., Phys. Lett. B774, 268 (2017).

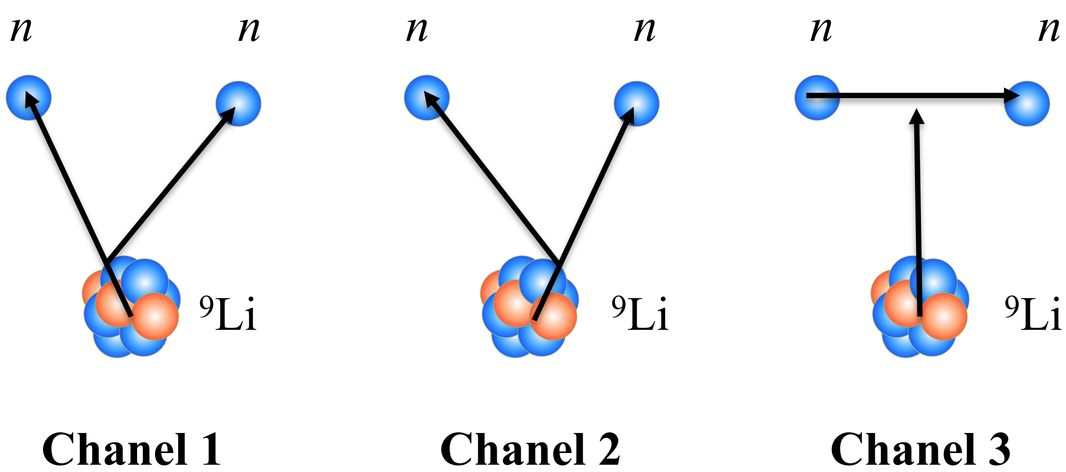


◆ The purpose of this work is to analyse the reaction by using the CDCC with CSM.

^{11}Li Three-body model

Gaussian Expansion Method

E. Hiyama, Y. Kino, and M. Kamimura, Prog. Part. Nucl. Phys. 51, 223 (2003)



V_{NN} : Minnesota interaction
V_{NC} : Esbensen and Bertsch, NPA 542, 310
V_{NNC} : S_{2n}~ 0.37 MeV
^{9}Li : spinless

n-^{9}Li forbidden → OCM

S. Saito, Prog. Theor. Phys. **41** (1969), 705

Three-body Hamiltonian

$$H = T_r + T_y + V_{nn} + V_{nc} + V_{nc} + V_{nnc}$$

Gaussian basis functions

$$\phi_{\mathrm{Im}}(r,y) = \sum_{c=1}^{3} \sum_{nj\ell\lambda} A^{(c)}_{nj\ell\lambda} \varphi_{n\ell}(r_c) \varphi_{j\lambda}(y_c) \left[\left[Y_\ell(\Omega_r) \otimes Y_\lambda(\Omega_y) \right] \otimes S \right]_{\mathrm{Im}}$$

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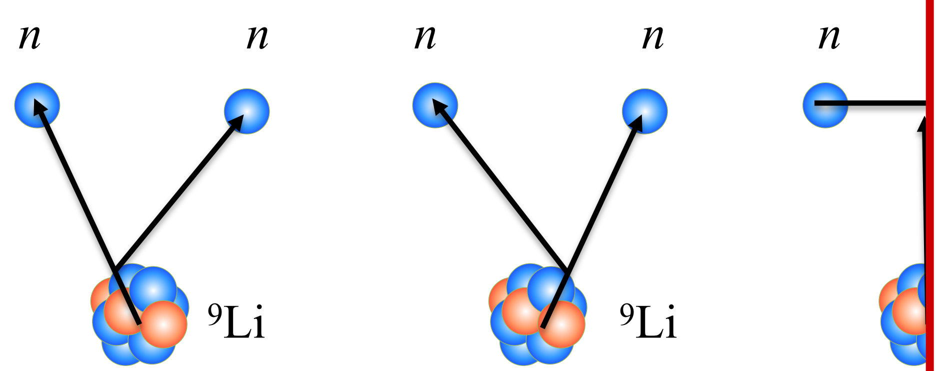
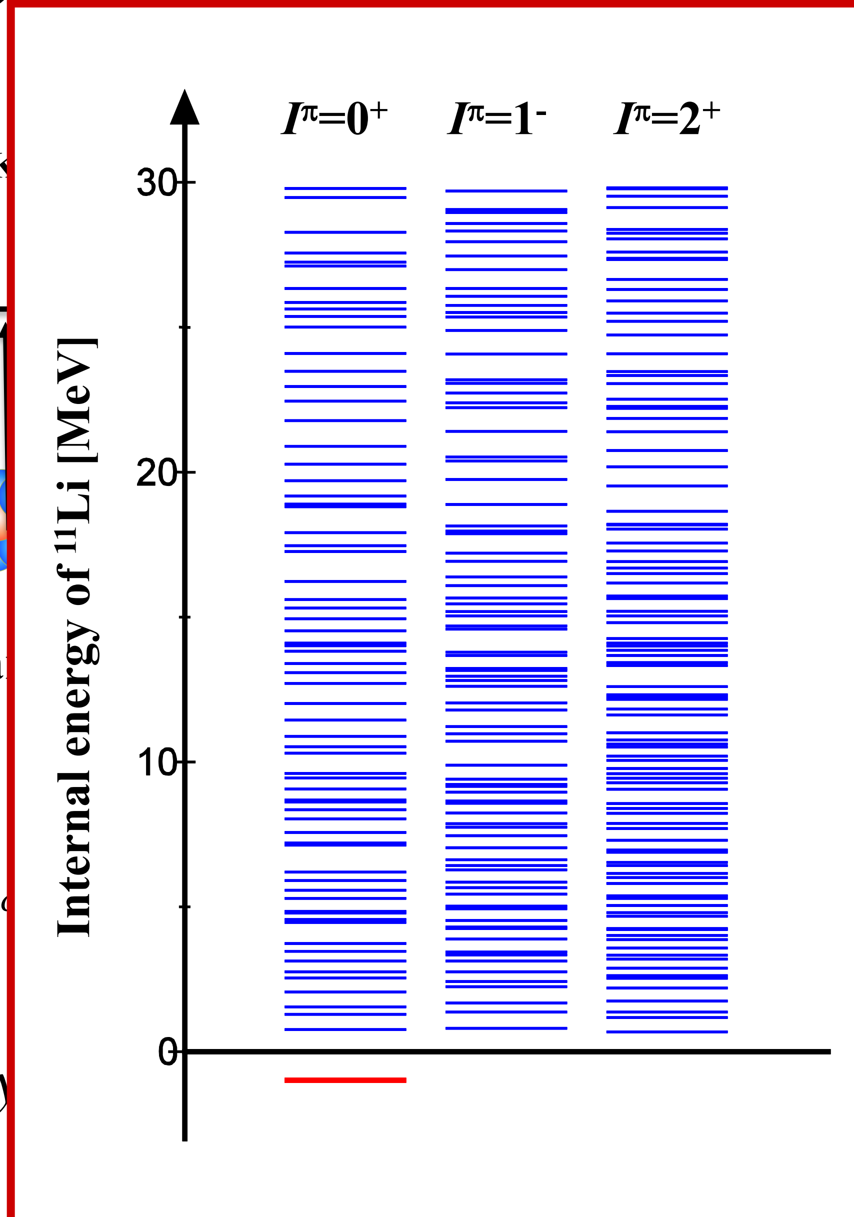


Chanel 1 **Chanel 2** **Cha**



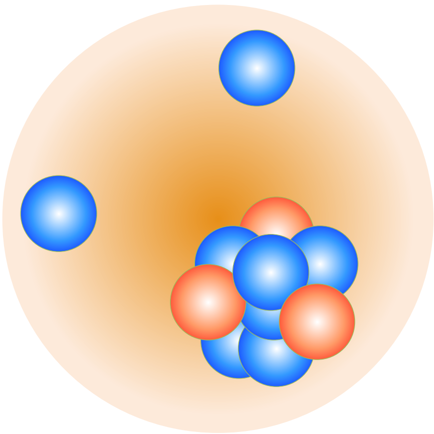
Three-body Hamiltonian

$$H = T_r + T_y + V_{nn} + V_{nc} + V_{n}$$

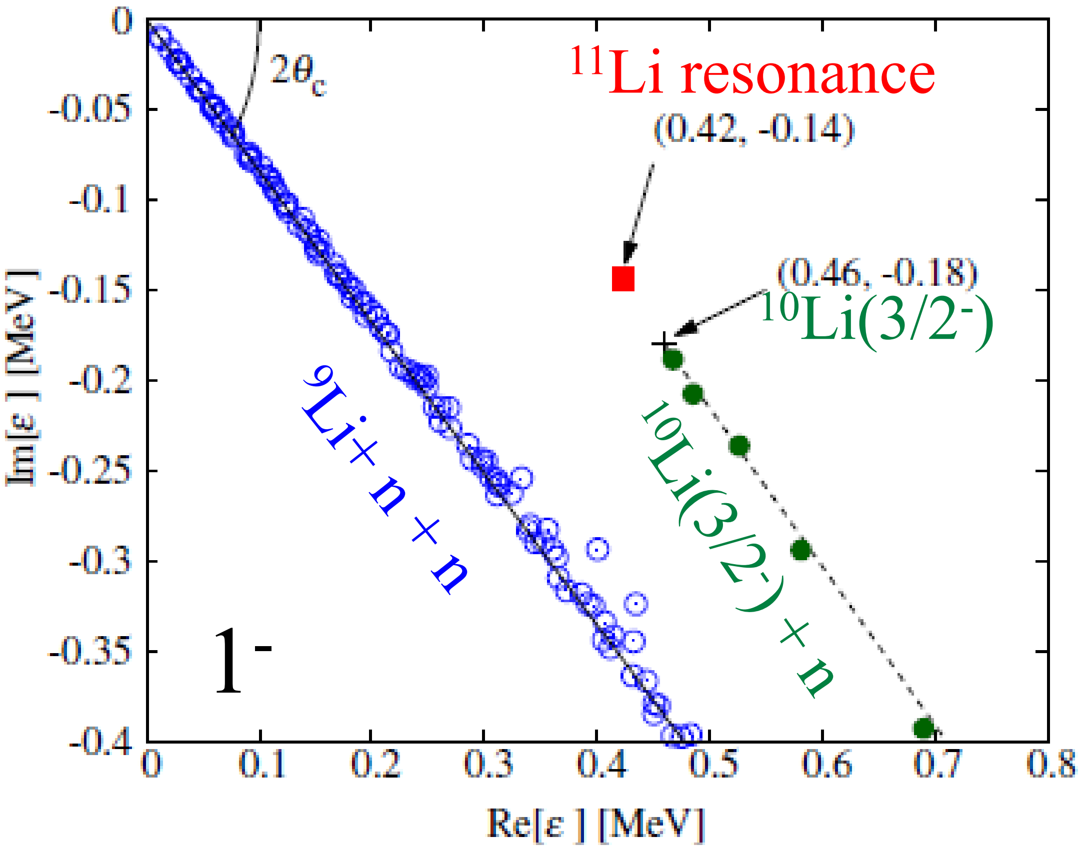
Gaussian basis functions

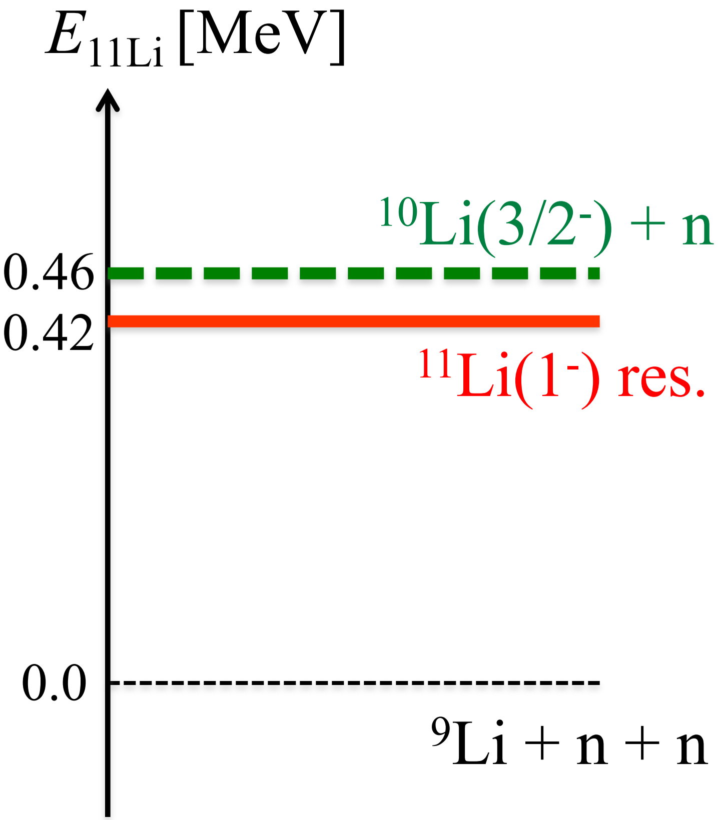
$$\phi_{\mathrm{Im}}(r,y) = \sum_{c=1}^{3} \sum_{nj\ell\lambda} A^{(c)}_{nj\ell\lambda} \varphi_{n\ell}(r_c) \varphi_{j\lambda}(y$$

Resonance in ^{11}Li



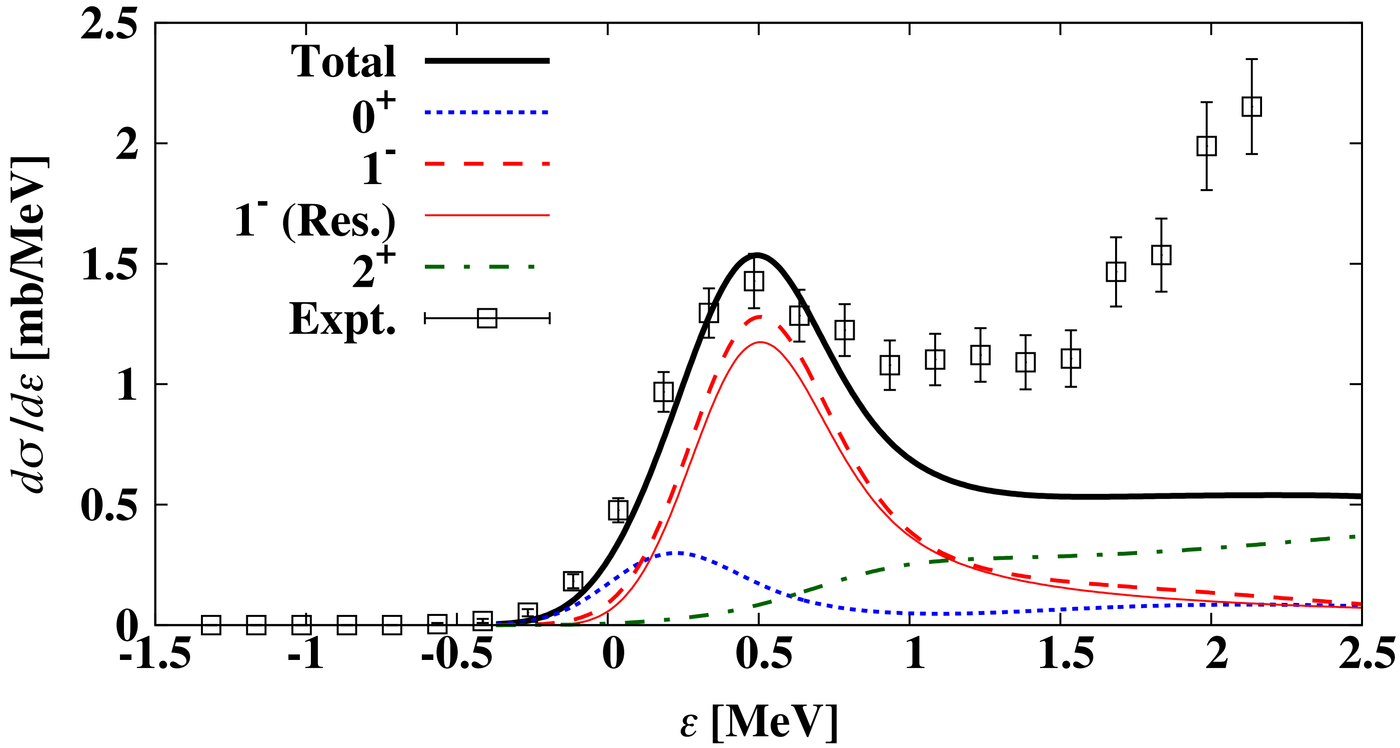
- ✓ Two-neutron halo nuclei (S_{2n}=0.37 MeV)
- ✓ Borromean structure





J. Tanaka et al., Phys. Lett. B 774, 268 (2017).

Energy Spectrum of ^{11}Li(p, p')



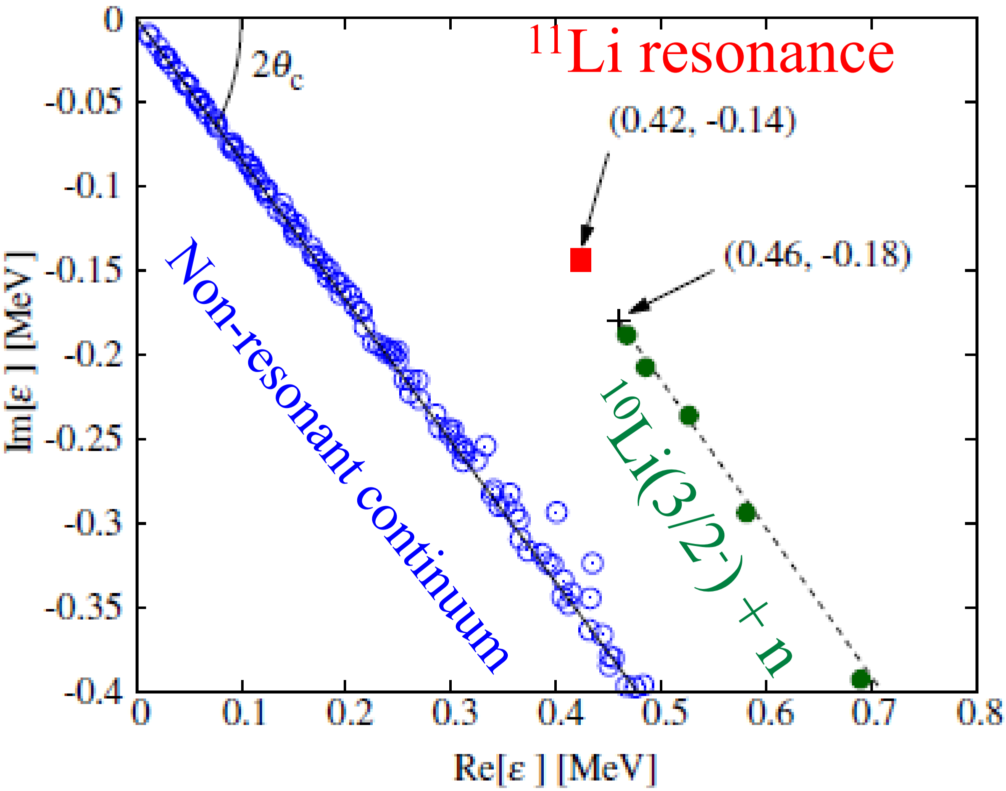
- **The contribution of the dipole resonance dominates the low-lying peak. →possibility of the resonance**
- **The width of the low-lying peak is reproduced by taking into account non-resonant components.**

Wave Function of ^{11}Li Resonance

- Probability of ^{10}Li resonance in ^{11}Li

$$\alpha_n^\theta = 2\langle\Phi_n^\theta|\underline{\phi_{res}^\theta\rangle\langle\tilde{\phi}_{res}^\theta}|\Phi_n^\theta\rangle$$

^{10}Li res. w.f.



- Non-resonant continuum

Re[α_n] ~ 0.0

- ^{10}Li(3/2$^-$) + n continuum

Re[α_n] ~ 0.9

- ^{11}Li resonance

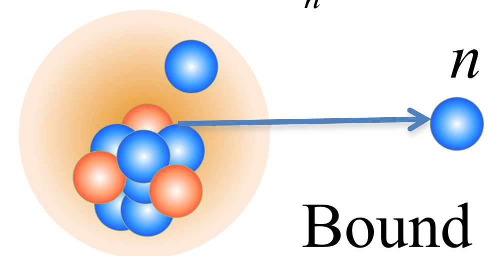
Re[α_n] ~ 0.9

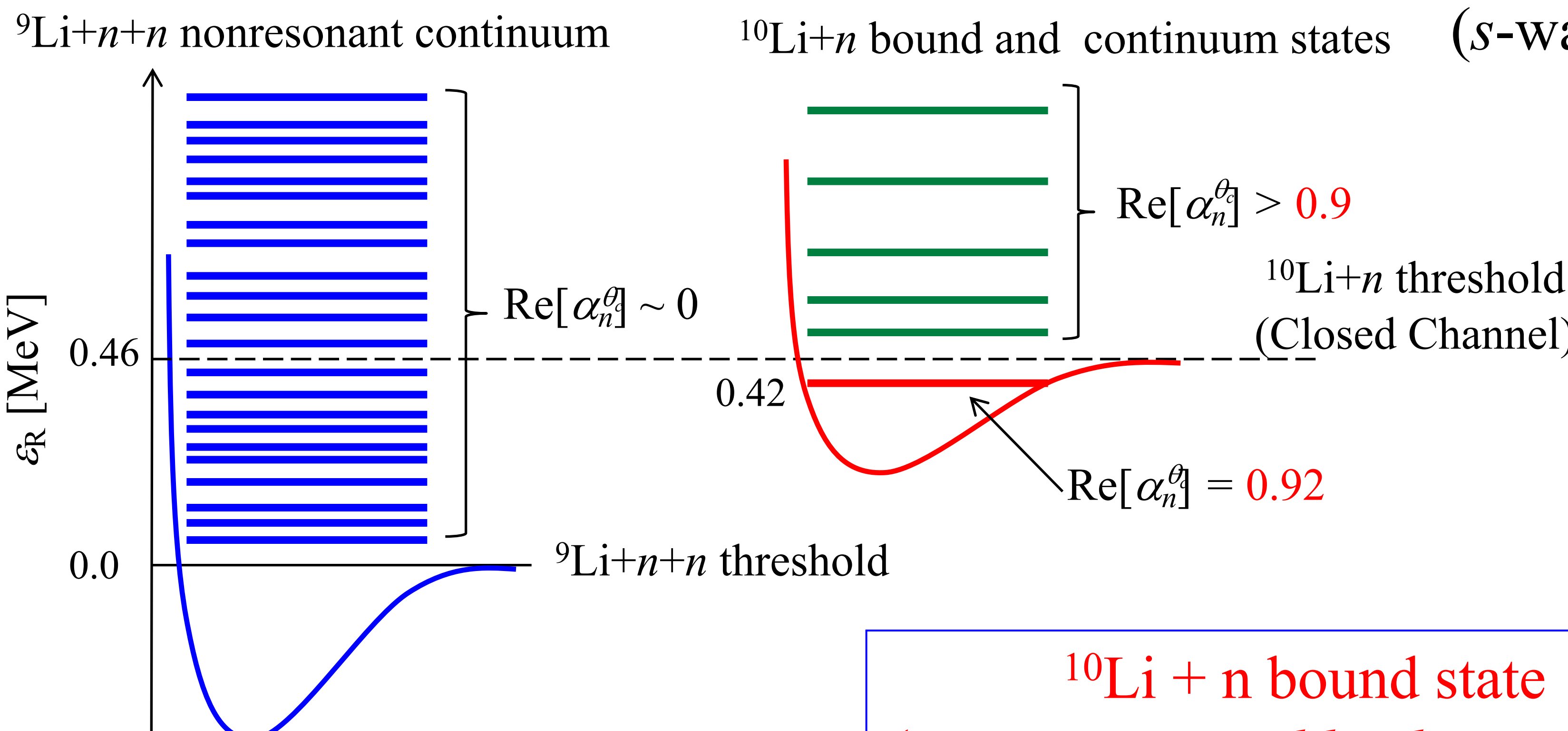
TM, J. Tanaka, and K. Ogata, Prog. Theor. Exp. Phys. Vol. 2019, Issue 12, 123D02

Borromean Feshbach Resonance

$$\psi_{res}({}^{11}Li) = \left[\phi_{res}({}^{10}Li) \otimes \chi_n(\ell = 0)\right]$$

0.42 MeV 0.46 MeV -0.04 MeV

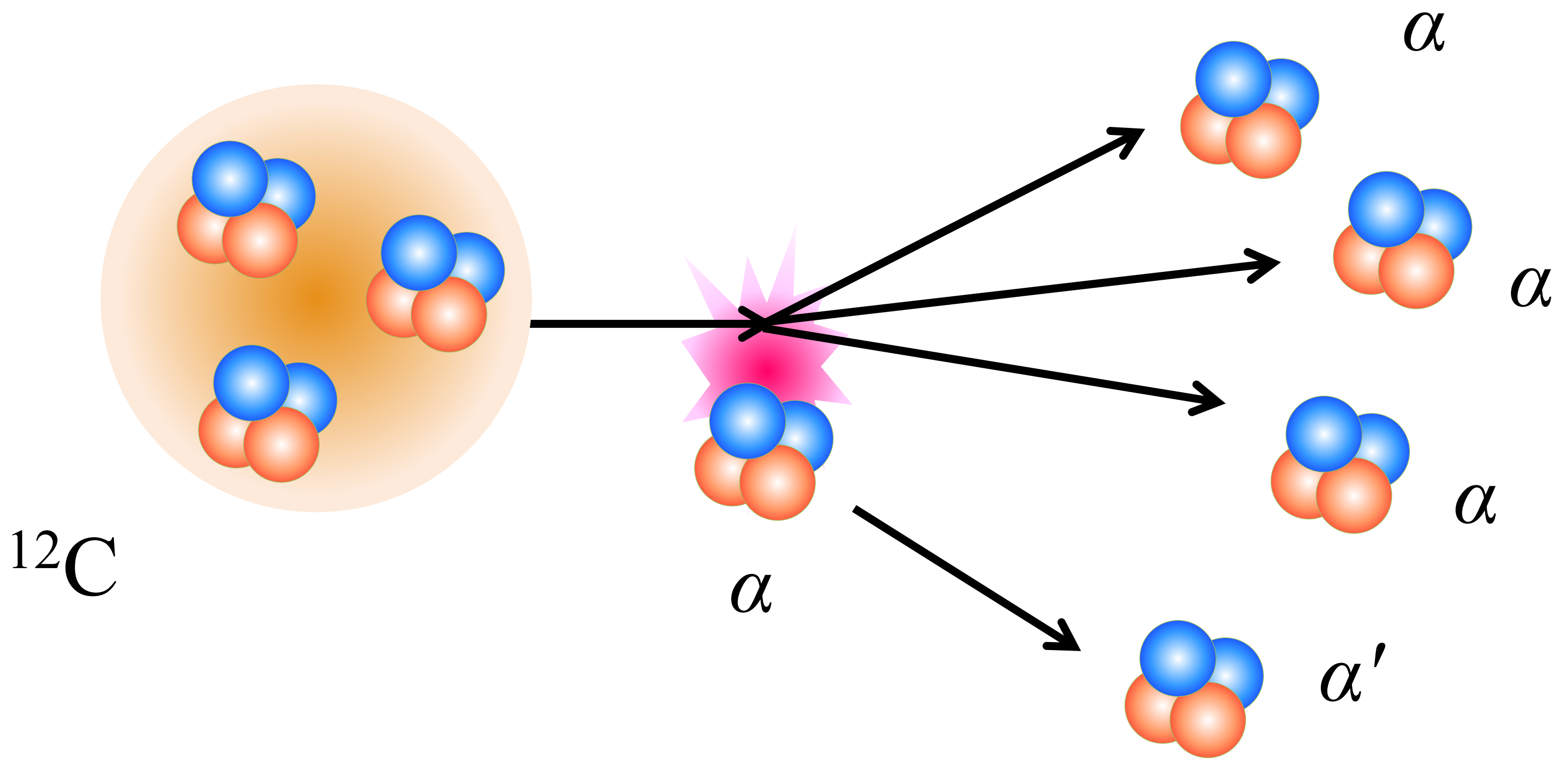




^{10}Li + n bound state

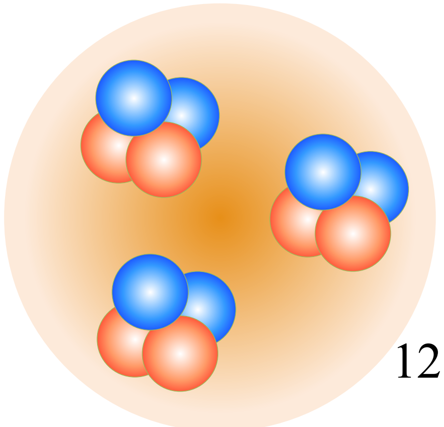
(*Borromean Feshbach resonance*)

Analysis of $^{12}C(\alpha, \alpha')$

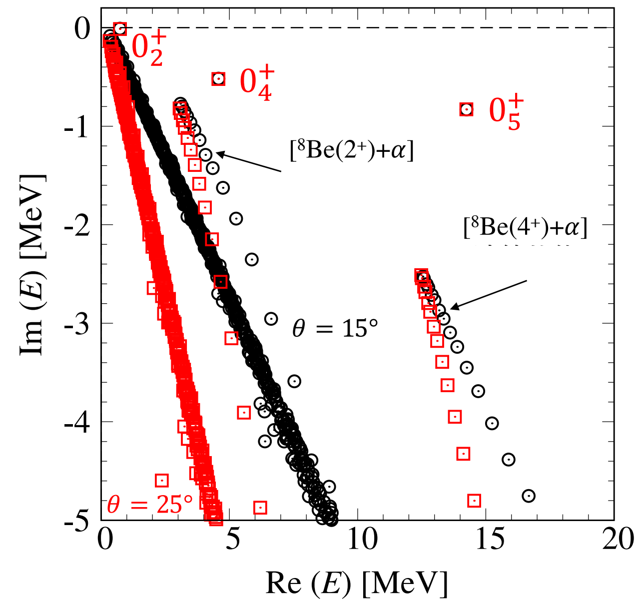


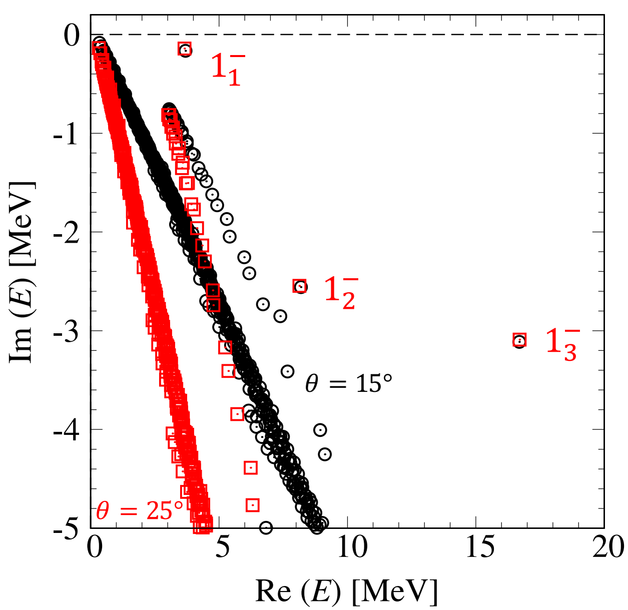
- ✓ ^{12}C has many resonances
- ✓ Validity of the CDCC analysis with CSM for a system with many resonances

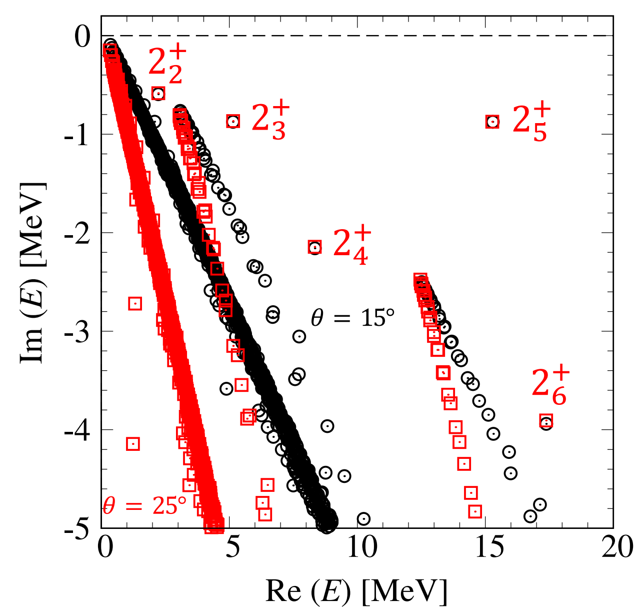
Resonances in ^{12}C



$^{12}C = \alpha + \alpha + \alpha$

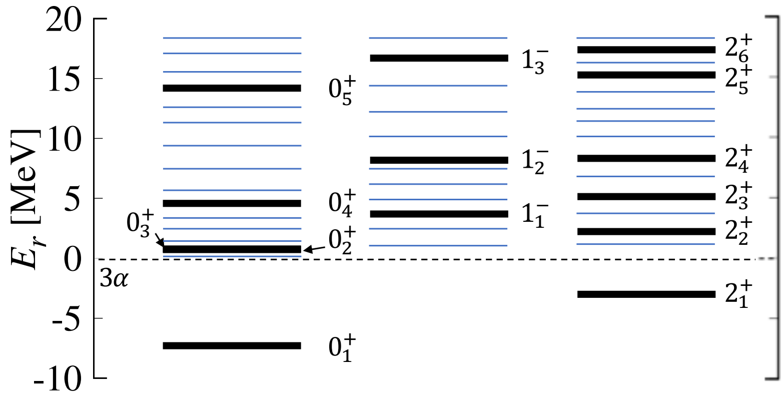




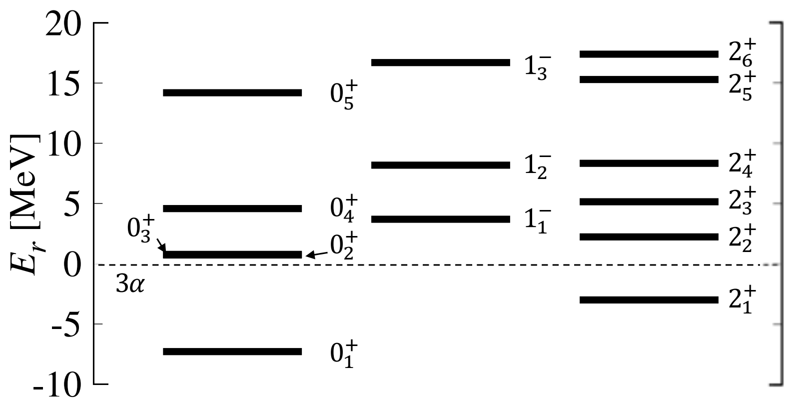


$^{12}\mathrm{C}(\alpha,\alpha')@E_\alpha = 386\ \mathrm{MeV}$

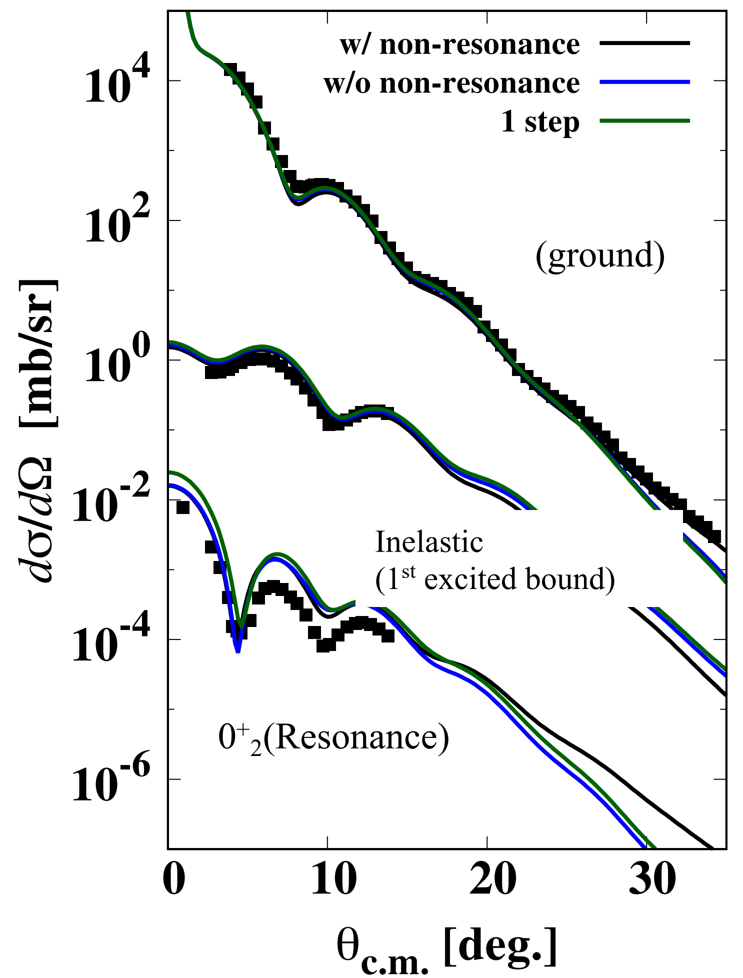
- CDCC



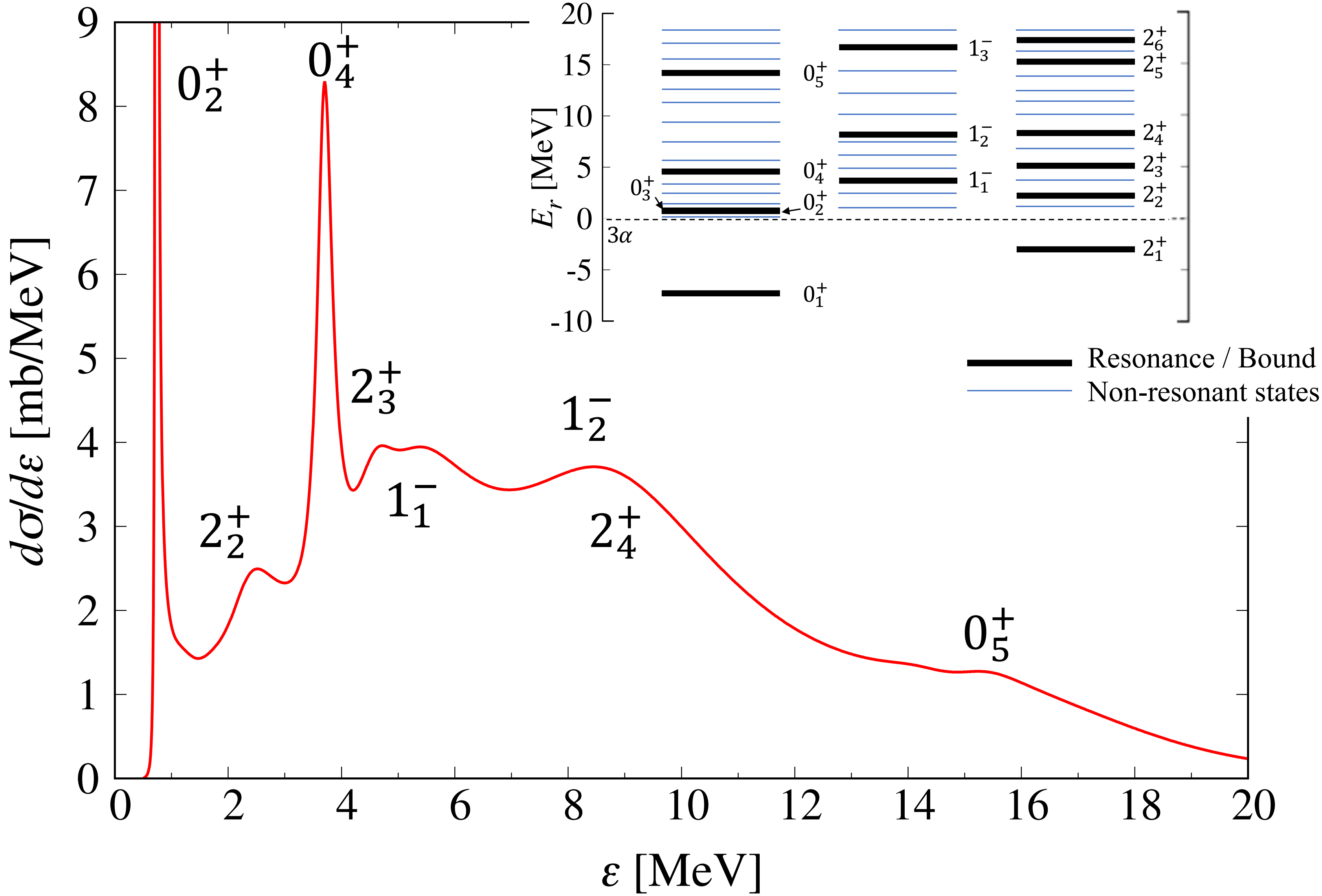
- w/o non-resonance



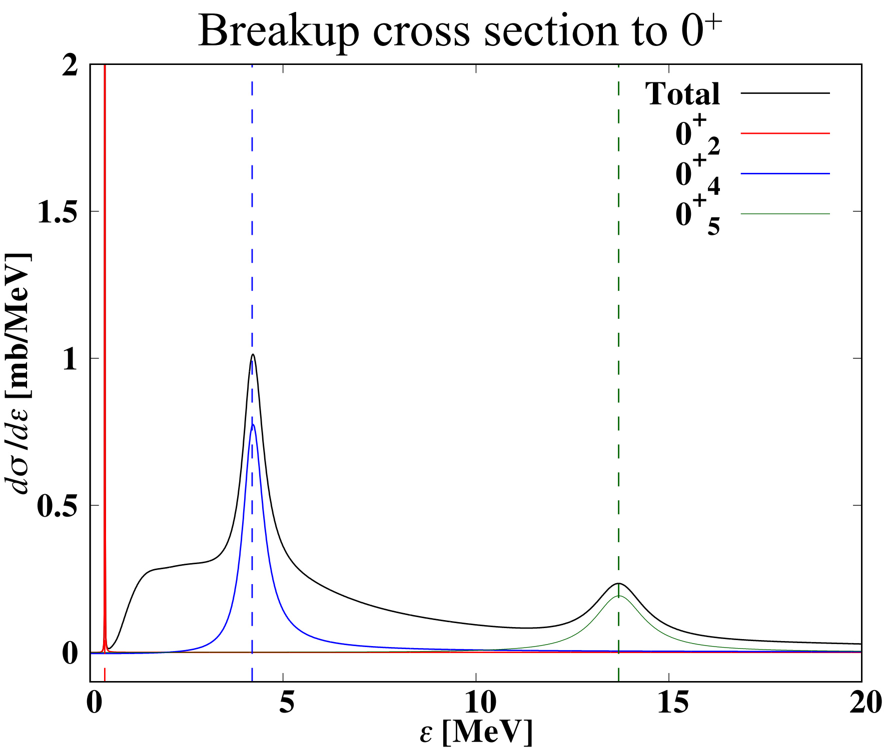
M. Itoh *et al.*, Phys. Rev. C **84**, 054308 (2011).

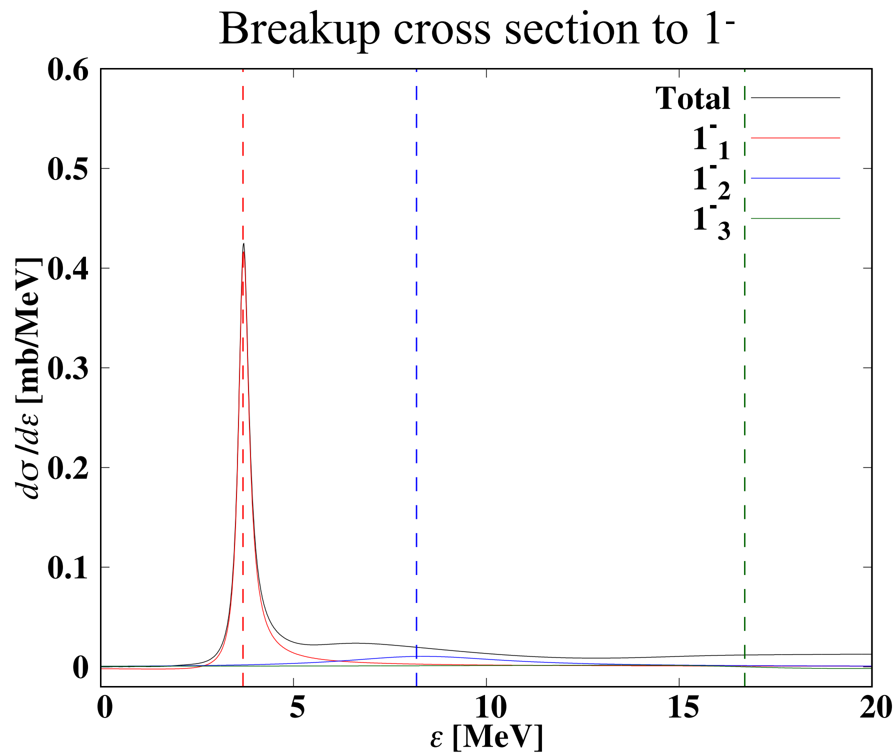


Energy spectrum



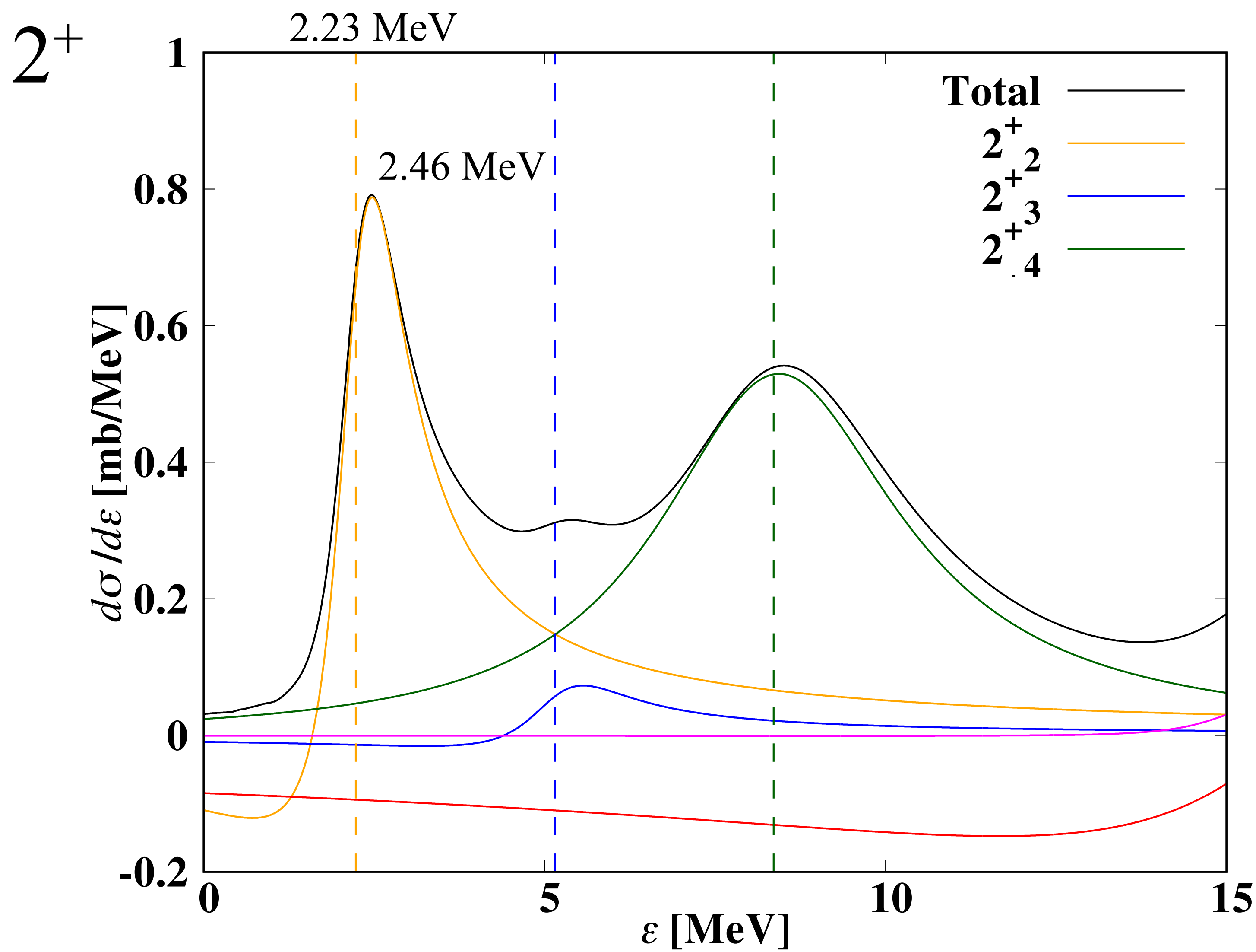
Breakup cross section to 0^+ & 1^-





$$\frac{d\sigma_{res}}{dE} = -\frac{1}{\pi}\mathrm{Im}\sum_{n,n'}\hat{T}_n^\dagger\langle\hat{\psi}_n|U^{-\theta}|\phi_{res}^\theta\rangle\frac{1}{E-E_{res}^\theta}\langle\tilde{\phi}_{res}^\theta|U^\theta|\hat{\psi}_{n'}\rangle T_{n'}$$

- ✓ 0^+_3 , 1^-_2 and 1^-_3 have a large decay width
- ✓ Peak positions of energy spectrum are consistent with the resonant energy.



- ✓ The peak positions of the cross section are higher than the corresponding resonant energies about 200 keV.
- ✓ The peak position of the cross section is not always the same value as the resonant energy.

Summary

- CDCC with CSM is useful for investigating resonances via breakup cross sections.
- We analyze $^{12}C(\alpha, \alpha')$ and $^{11}Li(p, p')$ reactions by CDCC with CSM.
- The resonance of ^{11}Li is interpreted as a bound state of ^{10}Li + n system, that is, a *Borromean Feshbach resonance*.
- For the analysis of ^{12}C, the peak position of the cross section is not always the same value as the resonant energy.