2022 / 01 / 20 @ NEWS seminar

Models of the muonium to antimuonium transition

based on T. Fukuyama, Y. Mimura, & Y. Uesaka,
PRD**105**, 015026 (2022). [arXiv:2108.10736]

Contents

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Charged Lepton Flavor Violation (CLFV)

- lepton flavor numbers (L_e, L_μ, L_τ) cf. Lepton number, $L = L_e + L_\mu + L_\tau$

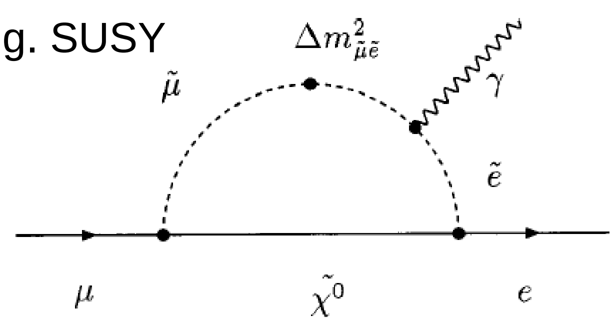
	e^-	μ^-	τ^-	ν_e	ν_μ	ν_τ	e^+	μ^+	τ^+	$\overline{\nu_e}$	$\overline{\nu_\mu}$	$\overline{\nu_\tau}$	others
L_e	+1	0	0	+1	0	0	-1	0	0	-1	0	0	0
L_μ	0	+1	0	0	+1	0	0	-1	0	0	-1	0	0
L_τ	0	0	+1	0	0	+1	0	0	-1	0	0	-1	0

◆ lepton flavor violation in charged lepton sector = CLFV (e.g. $\mu^+ \to e^+\gamma$)

CLFV is a good probe for new physics because

- forbidden in SM
- predicted in many theories beyond SM
- tiny contribution of neutrino oscillation:

e.g. SUSY

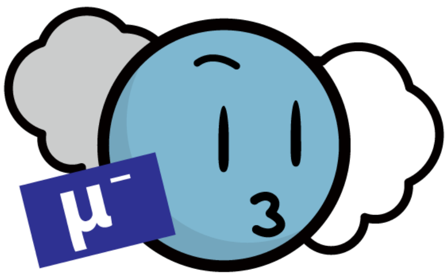


$$\mathrm{Br}(\mu \to e\gamma) < 10^{-54}$$

✓ cannot be observed by current technology (does not contaminate the new physics search)

⇨ If found, it must be an evidence of new physics !! (not ν osci.)

Muon rare decays



- Many muons can be produced ($\sim 10^9$/s).
- Long lifetime: easy to treat

- LFV processes with $\Delta L_\mu = -\Delta L_e = \pm 1$

  e.g. $\mu^+ \to e^+\gamma$ $\mu^+ \to e^+e^+e^-$ $\mu^- N \to e^- N$

➢ Current upper limits

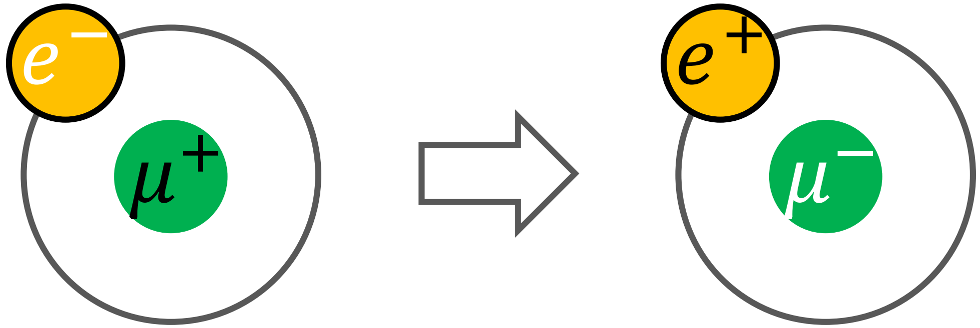
L. Calibbi & G. Signorelli, Riv. Nuovo Cim. **41**, 1 (2018).

Reaction	Present limit	C.L.	Experiment	Year
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	90%	MEG at PSI	2016
$\mu^+ \to e^+e^-e^+$	$< 1.0 \times 10^{-12}$	90%	SINDRUM	1988
$\mu^-\mathrm{Ti} \to e^-\mathrm{Ti}$	$< 6.1 \times 10^{-13}$	90%	SINDRUM II	1998
$\mu^-\mathrm{Pb} \to e^-\mathrm{Pb}$	$< 4.6 \times 10^{-11}$	90%	SINDRUM II	1996
$\mu^-\mathrm{Au} \to e^-\mathrm{Au}$	$< 7.0 \times 10^{-13}$	90%	SINDRUM II	2006

⇧ strong experimental constraints

Muonium(Mu)-to-Antimuonium($\overline{\text{Mu}}$)

Pontecorvo (1957), Weinberg & Feinberg (1961).



Mu : $\mu^+ e^-$ $\overline{\textbf{Mu}}$: $\mu^- e^+$

- CLFV process with $\Delta L_\mu = -\Delta L_e = 2$

  CLFV with $\Delta L_\mu = -\Delta L_e = \pm 1$ are constrained by $\mu \to e\gamma$, $\mu \to 3e$, …

  Mu-to-$\overline{\text{Mu}}$ can be sizable if a new particle carries **2** units of flavors.

- pure leptonic system (no hadronic umbiguities)
- J-PARC (Japan, N.Kawamura *et al.*, JPS Conf. Proc. 33, 011120 (2021)) and CSNS (China、MACE collab.) plan future experiments.

$P < 8.3 \times 10^{-11}$ (PSI) ⇨ $\mathcal{O}(10^{-14})$ (CSNS)

Mu-to-$\overline{\text{Mu}}$ transition

cf. K-$\overline{\text{K}}$ mixing

$$|\psi(t)\rangle = \alpha(t)|\text{Mu}\rangle + \beta(t)\left|\overline{\text{Mu}}\right\rangle$$

$$i\frac{\partial}{\partial t}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\mathcal{M}_{11} & \mathcal{M}_{12}\\ \mathcal{M}_{21} & \mathcal{M}_{11}\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

New physics !!

$$\mathcal{M}_{ij} = M_{ij} - i\Gamma_{ij}/2$$

$$M = M^\dagger, \Gamma = \Gamma^\dagger$$

Transition probability

$$P\left(\text{Mu} \to \overline{\text{Mu}}\right) \simeq 2\tau^2|\mathcal{M}|^2$$

τ : Mu lifetime ≃ 2.2 μs

$$\mathcal{M} \equiv \sqrt{\mathcal{M}_{12}\mathcal{M}_{21}}$$

If the initial state ($t = 0$) is a pure muonium state $|\mathrm{Mu}\rangle$,
the state at time t is given by

$$|\mathrm{Mu}(t)\rangle = f_+(t)|\mathrm{Mu}\rangle + \sqrt{\frac{\mathcal{M}_{21}}{\mathcal{M}_{12}}} f_-(t)\left|\overline{\mathrm{Mu}}\right\rangle$$

where

$$f_\pm(t) = \frac{e^{-i\lambda_+} \pm e^{-i\lambda_-}}{2}$$

$$\lambda_\pm = M - i\frac{\Gamma}{2} \pm \frac{1}{2}\left(\Delta M - i\frac{\Delta\Gamma}{2}\right)$$

$$\Delta M - i\frac{\Delta\Gamma}{2} = 2\sqrt{\mathcal{M}_{12}\mathcal{M}_{21}}$$

Mu-to-$\overline{\mathrm{Mu}}$ transition probability at time t :

$$P\left(\mathrm{Mu} \to \overline{\mathrm{Mu}}; t\right) \sim \left|\left\langle \overline{\mathrm{Mu}} \middle| \mathrm{Mu}(t) \right\rangle\right|^2 \simeq e^{-\Gamma t} \sin^2 \frac{\Delta M}{2} t$$

(assuming CP conservation or $|\Gamma_{12}/M_{12}| \ll 1$, $\left|\frac{\mathcal{M}_{21}}{\mathcal{M}_{12}}\right| = 1$)

Integrated probability of Mu-to-$\overline{\mathrm{Mu}}$ transition :

$$P\left(\mathrm{Mu} \to \overline{\mathrm{Mu}}\right) = \int_0^\infty dt\, \Gamma P\left(\mathrm{Mu} \to \overline{\mathrm{Mu}}; t\right)$$

$$= \int_0^\infty dt\, \Gamma e^{-\Gamma t} \sin^2 \frac{\Delta M}{2} t = \frac{1}{2} \frac{(\Delta M)^2}{(\Delta M)^2 + \Gamma^2}$$

$\Delta M \ll \Gamma$

$$\simeq \frac{(\Delta M)^2}{2\Gamma^2} = \boxed{2\tau^2 |\mathcal{M}|^2}$$

Γ : Mu decay width

τ : muonium lifetime $\simeq$ 2.2 μs

Effective interactions

Cf. R. Conlin & A. A. Petrov, PRD**102**, 095001 (2020).

$$-\mathcal{L}_{\mathrm{Mu}-\overline{\mathrm{Mu}}} = \sum_i \frac{G_i}{\sqrt{2}} Q_i$$

G_i : coupling constants

$$Q_1 = [\bar{\mu}\gamma_\alpha(1-\gamma_5)e][\bar{\mu}\gamma^\alpha(1-\gamma_5)e]$$ LL vector

$$Q_2 = [\bar{\mu}\gamma_\alpha(1+\gamma_5)e][\bar{\mu}\gamma^\alpha(1+\gamma_5)e]$$ RR vector

$$Q_3 = [\bar{\mu}\gamma_\alpha(1-\gamma_5)e][\bar{\mu}\gamma^\alpha(1+\gamma_5)e]$$ LR vector

$$Q_4 = [\bar{\mu}(1-\gamma_5)e][\bar{\mu}(1-\gamma_5)e]$$ LL scalar

$$Q_5 = [\bar{\mu}(1+\gamma_5)e][\bar{\mu}(1+\gamma_5)e]$$ RR scalar

※ Any 4-Fermi type operators can be written by the linear combination of the five operators (∵ Fierz identity)

Four muonium states

➢ Muonium has four hyperfine 1S states by combination of spins of the bound muon and electron

$$\mathrm{Mu}(F, m)$$

magnitude of total angular mom. (F)

z component of total angular mom. (m)

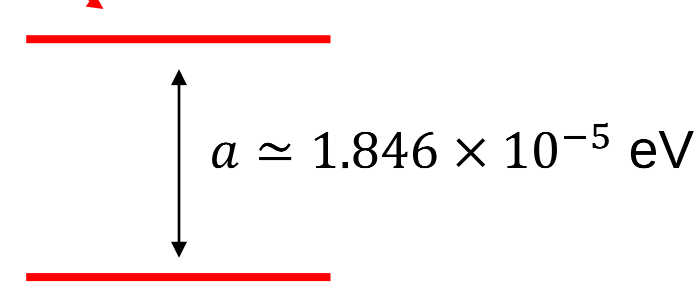
$$(F, m) = \begin{cases} (1, +1) \\ (1,0) \\ (1, -1) \\ (0,0) \end{cases}$$

$(1,+1)$, $(1,0)$, $(1,-1)$: triplet (ortho)

$(0,0)$: singlet (para)

$$E(\mathrm{Mu}; 1,0) = E(\mathrm{Mu}; 1, \pm 1) = E_0 + \frac{a}{4}$$

$$E(\mathrm{Mu}; 0,0) = E_0 - \frac{3}{4} a$$



➢ Mu-to-$\overline{\mathrm{Mu}}$ transition rate depends on the initial Mu states.

For the triplet ($F = 1$) states,

$$\mathcal{M}_{1,0} = \mathcal{M}_{1,\pm 1} = \frac{8|\varphi(0)|^2}{\sqrt{2}}\left(G_1 + G_2 + \frac{1}{2}G_3 - \frac{1}{4}G_4 - \frac{1}{4}G_5\right)$$

For the singlet ($F = 0$) state,

$$\mathcal{M}_{0,0} = \frac{8|\varphi(0)|^2}{\sqrt{2}}\left(G_1 + G_2 - \frac{3}{2}G_3 - \frac{1}{4}G_4 - \frac{1}{4}G_5\right)$$

$\varphi(0) = \sqrt{\frac{(m_{\mathrm{red}}\alpha)^3}{\pi}}$: overlap of lepton wave function

➢ Mu-to-$\overline{\mathrm{Mu}}$ transition rate depends on the magnetic field.

The magnetic field makes… ⇩

① nonzero energy gap of $\mathrm{Mu}(1,\pm 1)$ and $\overline{\mathrm{Mu}}(1,\pm 1)$

② mixing of $\mathrm{Mu}(1,0)$ and $\mathrm{Mu}(0,0)$

K.Horikawa & K.Sasaki, PRD53, 560 (1996); W.S.Hou & G.G.Wong. PLB357, 145 (1995).

① nonzero energy gap of $\mathrm{Mu}(1, \pm 1)$ と $\overline{\mathrm{Mu}}(1, \pm 1)$

Considering energy gap ΔE of Mu and $\overline{\mathrm{Mu}}$,
the transition rate is

$$P\left(\mathrm{Mu}(1, \pm 1) \to \overline{\mathrm{Mu}}\right) = \frac{2\tau^2 \left|\mathcal{M}_{1,\pm 1}\right|^2}{1 + (\tau \Delta E)^2}$$

$$\tau \Delta E = 3.8 \times 10^5 \times \frac{B}{\mathrm{Tesla}}$$

∴ The contribution of $m = \pm 1$ is suppressed
in a magnetic field stronger than μT.

cf. terrestrial magnetism 30-60 μT

② mixing of $\mathrm{Mu}(1,0)$ and $\mathrm{Mu}(0,0)$

transition amplitude of $m = 0$ states in a magnetic field B

$$\mathcal{M}^B_{0,0} \simeq \frac{1}{2}\left(\mathcal{M}_{0,0} - \mathcal{M}_{1,0} + \frac{\mathcal{M}_{0,0} + \mathcal{M}_{1,0}}{\sqrt{1+X^2}}\right)$$

$$\mathcal{M}^B_{1,0} \simeq \frac{1}{2}\left(-\mathcal{M}_{0,0} + \mathcal{M}_{1,0} + \frac{\mathcal{M}_{0,0} + \mathcal{M}_{1,0}}{\sqrt{1+X^2}}\right)$$

$$X = \frac{\mu_B B}{a}\left(g_e + \frac{m_e}{m_\mu} g_\mu\right) \simeq 6.31 \frac{B}{\mathrm{Tesla}}$$

Thus, the transition probability in a magnetic field is

$$P = 2\tau^2 \left(\left|c_{0,0}\right|^2 \left|\mathcal{M}^B_{0,0}\right|^2 + \left|c_{1,0}\right|^2 \left|\mathcal{M}^B_{1,0}\right|^2 + \sum_{m=\pm 1} \left|c_{1,m}\right|^2 \frac{\left|\mathcal{M}_{1,\pm 1}\right|^2}{1 + (\tau \Delta E)^2} \right)$$

$\left|c_{F,m}\right|^2$: population of the state (F, m)

Current experimental constraints by PSI

In a magnetic field $B = 0.1$ Tesla,

$$P < 8.3 \times 10^{-11}$$

L. Willmann *et al.*, PRL**82**, 49 (1999).

(considering the magnetic effects)

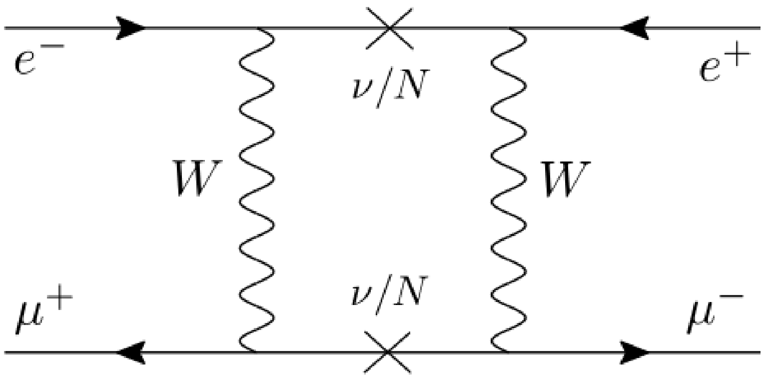
cf. K. Horikawa & K. Sasaki, PRD**53**, 560 (1996),
W. S. Hou & G. G. Wong, PLB**357**, 145 (1995).

$$|G_i| \lesssim 3.0 \times 10^{-3} G_F$$

Classification of new particles & interactions to generate Mu-to-$\overline{\text{Mu}}$ transition

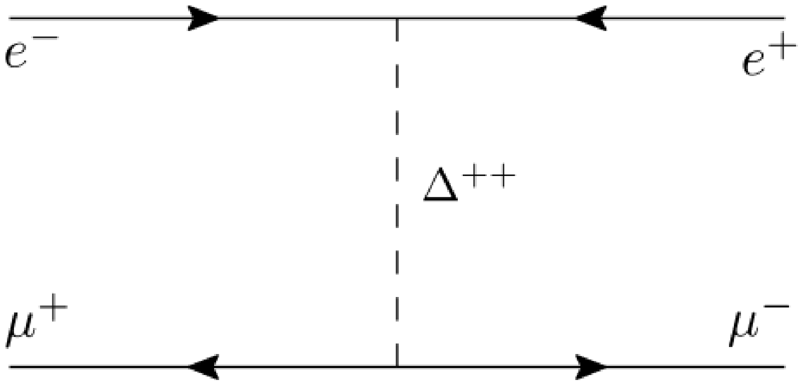
1. $\Delta L_e = \Delta L_\mu = 0$

- mass term of SM singlet which violates lepton number
- loop
- e.g. Majorana ν mass



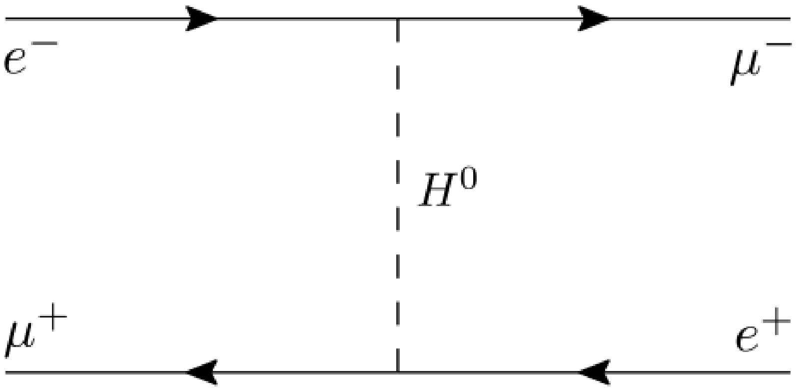
2. $(\Delta L_e, \Delta L_\mu) = (\pm 2, 0), (0, \pm 2)$

- doubly-charged mediator
- tree
- LNV is not needed.
- If mediator couples to W^+W^+, $0\nu 2\beta$ is induced.



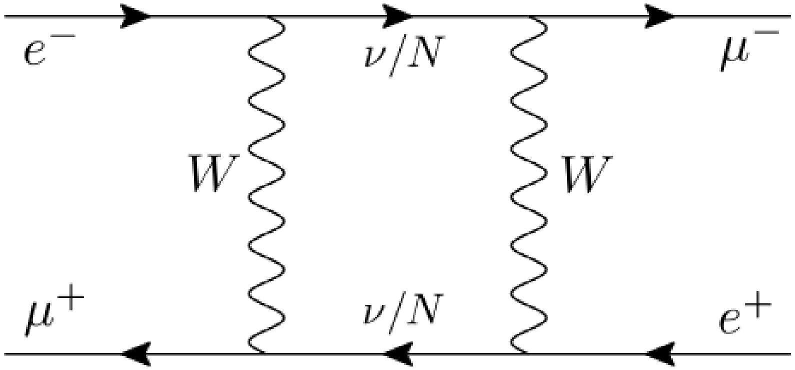
3. $\Delta L_e = -\Delta L_\mu = \pm 1$

- neutral mediator
- tree
- LNV is not needed.



4. $(\Delta L_e, \Delta L_\mu) = (\pm 1,0), (0, \pm 1)$

- loop
- model-independently, strongly constrained by $\mu \to e\gamma$ or $\mu \to 3e$



- Tree level

Model	G_1	G_2	G_3	G_4	G_5
Type I + II hybrid seesaw	✓	–	–	–	–
Left-right model with $SU(2)_R$ triplet	–	✓	–	–	–
Inert Higgs doublet	–	–	✓	△	△
R-parity violating SUSY	–	–	✓	–	–
Dilepton gauge boson	–	–	✓	–	–
Neutral flavor gauge boson	✓	✓	✓	–	–

✓ : $G_i/G_F \sim O(10^{-3})$ is allowed　　△ : suppressed by LFV bounds

- $\Delta L_e - \Delta L_\mu = \pm 1$

Model	$\lvert G_1\rvert/G_F$	$\lvert G_2\rvert/G_F$	$\lvert G_3\rvert/G_F$
Heavy singlet neutrino	$\lesssim O(10^{-8})$	–	–
Left-right model without $SU(2)_R$ triplet	$\lesssim O(10^{-8})$	$\lesssim O(10^{-8})$	$\lesssim O(10^{-10})$
SUSY (Gaugino loop)	$\lesssim O(10^{-8})$	–	–
Leptoquark	$\lesssim O(10^{-8})$	$\lesssim O(10^{-8})$	$\lesssim O(10^{-8})$

- radiative neutrino mass model (loop induced by LNV)

Model	$\|G_1\|/G_F$	$\|G_2\|/G_F$
Charged Higgs(ino)	$\lesssim O(10^{-5})$	–
KNT model	–	$\lesssim O(10^{-5})^{(*)}$
AKS model	–	$\lesssim O(10^{-6})$

- neutrino mass model (tree by doubly charged scalar)

Model	$\|G_1\|/G_F$	$\|G_2\|/G_F$
Type-II seesaw	$\lesssim O(10^{-5})^{(\#)}$	–
Zee-Babu model	–	$\lesssim O(10^{-3})$
Cocktail model	–	$\lesssim O(10^{-5})$

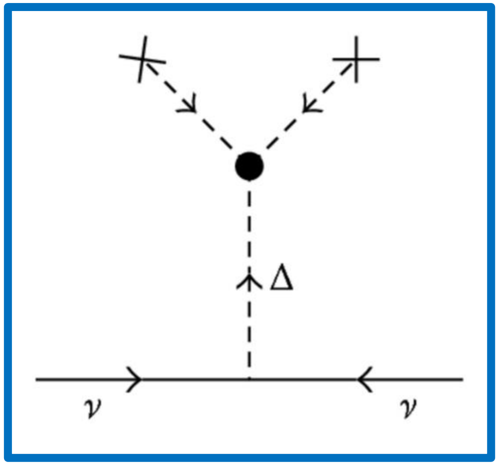
Example 1: Type-II seesaw

- SU(2) triplet scalar $\Delta \rightarrow \nu$ mass

$$-\mathcal{L} = \left(\frac{1}{2}\kappa_{ij}^{L}\,\overline{(\ell_{iL})^{c}}\ell_{jL}\Delta_{L} + \mu_{\Delta}HH\Delta_{L}^{*} + h.c.\right) + M_{\Delta}^{2}|\Delta_{L}|^{2}$$

$$(\Delta_L)_{ab} = \begin{pmatrix} \Delta_L^{++} & \Delta_L^{+}/\sqrt{2} \\ \Delta_L^{+}/\sqrt{2} & \Delta_L^{0} \end{pmatrix}$$

$$\overline{(\ell_{iL})^{c}}\ell_{jL}\Delta_{L} = \overline{\nu_{iL}^{c}}\nu_{jL}\Delta_{\mathrm{L}}^{0} - \frac{1}{\sqrt{2}}\overline{\nu_{iL}^{c}}e_{jL}\Delta_{L}^{+} - \frac{1}{\sqrt{2}}\overline{e_{iL}^{c}}\nu_{jL}\Delta_{L}^{+} + \overline{e_{iL}^{c}}e_{jL}\Delta_{L}^{++}$$

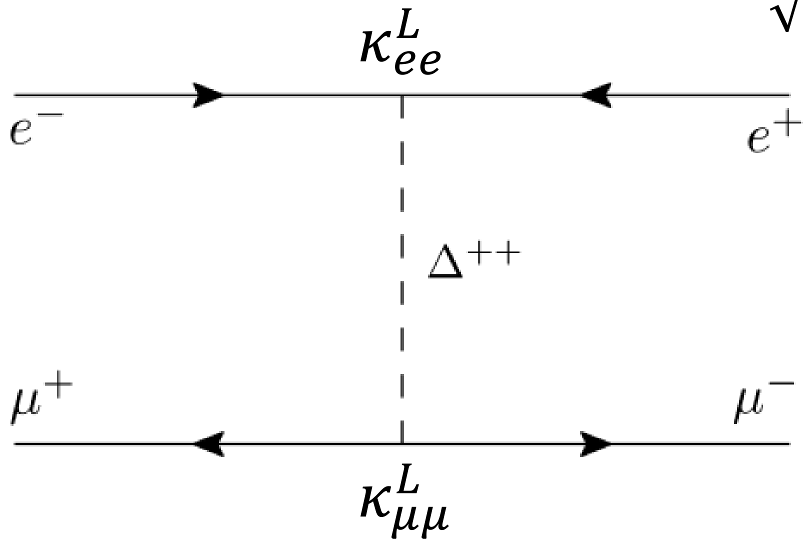


$$M_{\nu}^{\mathrm{II}} = \kappa^{L}\langle\Delta_{L}^{0}\rangle = -\kappa^{L}\mu_{\Delta}\langle H^{0}\rangle^{2}/M_{\Delta}^{2}$$

$\overline{e^c_{iL}} e_{jL} \Delta_L^{++}$

$$\frac{G_1}{\sqrt{2}} = -\frac{\kappa^L_{ee}\kappa^{L*}_{\mu\mu}}{32 M_\Delta^2} = -\frac{\left(M_\nu^{\rm II}\right)_{ee}\left(M_\nu^{\rm II}\right)^*_{\mu\mu}}{32 v_L^2 M_\Delta^2}$$

- Mu-$\overline{\rm Mu}$ (tree)



- constrained by $\mu \to 3e$

$${\rm Br}(\mu \to 3e) < 1.0 \times 10^{-12}$$

$$\frac{G_1}{G_F} = \frac{\sqrt{{\rm Br}(\mu \to 3e)}}{2\sqrt{2}} \left|\frac{\kappa^L_{\mu\mu}}{\kappa^L_{e\mu}}\right| \lesssim 3.5 \times 10^{-7} \left|\frac{\kappa^L_{\mu\mu}}{\kappa^L_{e\mu}}\right|$$

⇨ The off-diagonal components of $M_\nu^{\rm II}$ should be small (to obtain the sizable Mu-$\overline{\rm Mu}$ rate).

- If type-II contribution dominates neutrino masses,

$$M_\nu^{\mathrm{II}} = U_{\mathrm{PMNS}}^* \mathrm{diag}\left(m_1 e^{i\alpha_1}, m_2 e^{i\alpha_2}, m_3\right) U_{\mathrm{PMNS}}^\dagger$$

⇨ $$\frac{\kappa_{e\mu}^L}{\kappa_{\mu\mu}^L} \sim \mathcal{O}\left(\theta_{13}, \sqrt{\Delta m_{\mathrm{sol}}^2 / \Delta m_{\mathrm{atm}}^2}\right)$$

$$\sim 10\text{-}20\ \%$$

⇨ $$\frac{G_1}{G_F} < \mathcal{O}(10^{-6})$$

However, the neutrino mass can be reproduced even if $\kappa_{e\mu}^L \to 0$.

⇨ Next, let's consider such a case that $\kappa_{e\mu}^L$ is tiny.

$$\kappa^L \langle \Delta_L^0 \rangle = U_{\mathrm{PMNS}}^* \mathrm{diag}\left(m_1 e^{i\alpha_1}, m_2 e^{i\alpha_2}, m_3\right) U_{\mathrm{PMNS}}^\dagger$$

In case $\kappa_{e\mu}^L \to 0$,

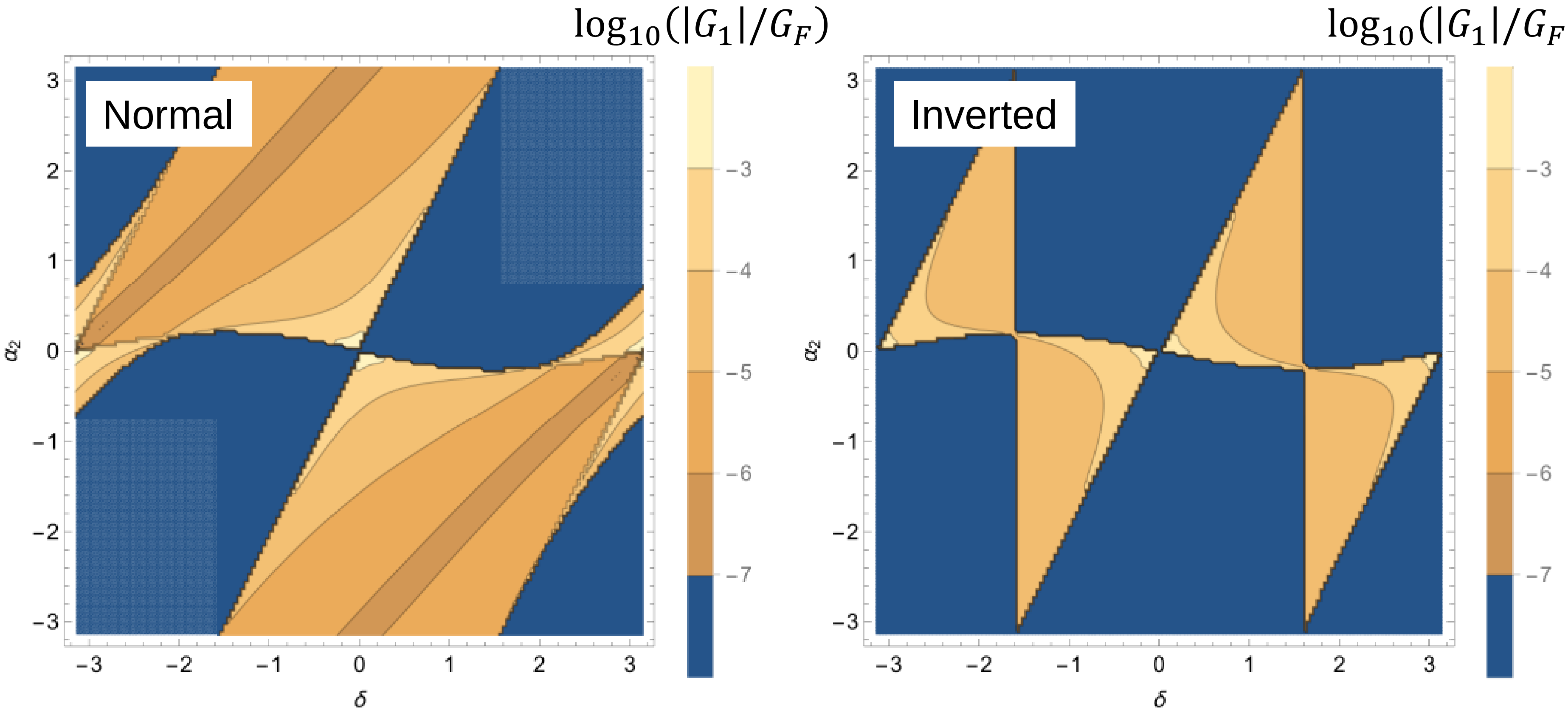
1. $m_1 e^{i\alpha_1} \simeq m_2 e^{i\alpha_2} (\simeq m_3)$
2. $\sum U_{ei}^* U_{\mu i}^* m_i e^{i\alpha_i}$ is accidentally canceled

Choosing $m_1 e^{i\alpha_1}$ to realize $\kappa_{e\mu}^L = 0$,

M_ν can be represented by a function of Dirac phase δ & Majorana phase α_2.

Considering LFV constraints from μ decay & τ decay,

we calculate the upper limit of G_1.

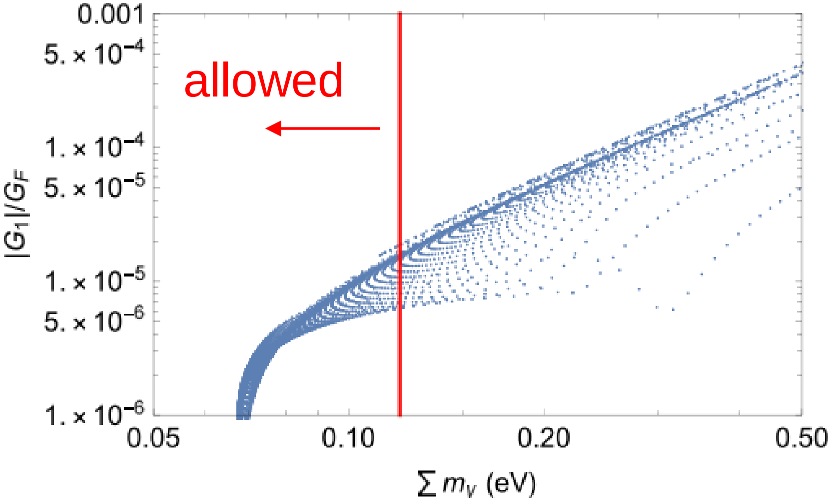


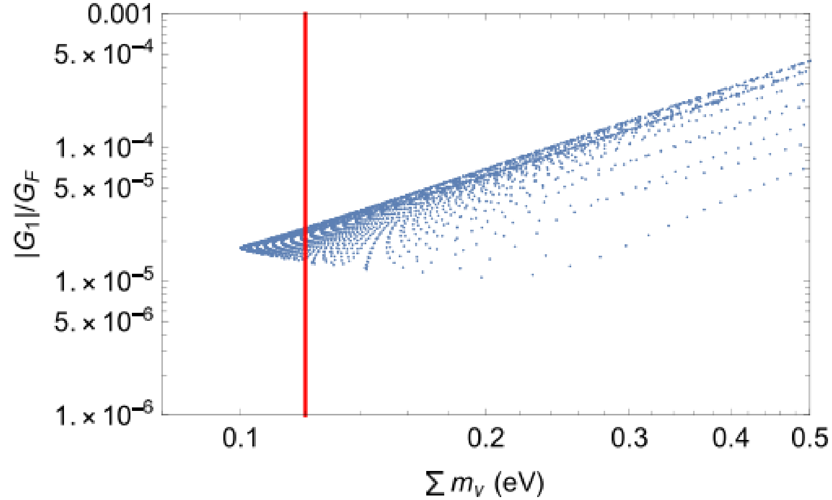
$\alpha_2 \sim 0, \delta \sim 0$ or π ⇨ G_1 →large (mass degenerate)

Inverted : When $m_1 \sim m_2$, G_1 can be large if the solution of $\kappa^L_{e\mu} = 0$ exists.

Normal : $\kappa^L_{e\mu} = 0$ can be realized in a wide region without mass degenerate.

- cosmological bound $\sum m_\nu < 0.12$ eV





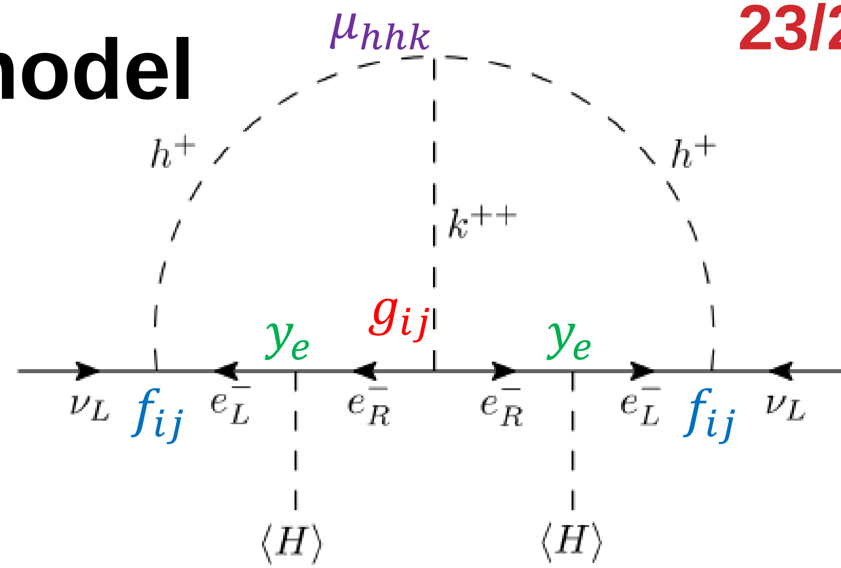
$\Longrightarrow \quad |G_1|/G_F \lesssim \mathcal{O}(10^{-5})$

- If the contribution from type-I cancels $\kappa^L_{e\mu}$,

  an arbitrary size of Mu-$\overline{\mathrm{Mu}}$ can be generated.

e.g. $\kappa^L_{ee} \sim \kappa^L_{\mu\mu} \sim 0.3, M_\Delta = 600$ GeV $\Longrightarrow \quad \frac{|G_1|}{G_F} \sim 1 \times 10^{-3}$

Example2: Zee-Babu model



one of radiative neutrino models

neutrino mass is generated by two-loop

$$-\mathcal{L} \supset \left(f_{ij} \overline{\ell_i^c} \cdot \ell_j h^+ + g_{ij} \overline{e_i} e_j^c k^{--} + \mu_{hhk} h^+ h^+ k^{--} + h.c. \right)$$
$$+ m_h^2 h^- h^+ + m_k^2 k^{--} k^{++}$$

f : anti-symmetric for i, j $\qquad$ g : symmetric for i, j

$$M_\nu = \frac{1}{M_0} f M_e g M_e f^T$$

$$\frac{1}{M_0} = \frac{\mu_{hhk}}{48\pi^2 \max(m_h^2, m_k^2)} \tilde{I}$$

$$M_e = \mathrm{diag}\left(m_e, m_\mu, m_\tau\right)$$

rank-2 ($m_{\text{lightest}} = 0$) $\because$ f is anti-symmetric

Normal ordering case

$$M_\nu = U^* \mathrm{diag}(0, m_2, m_3) U^\dagger$$
$$= m_2 u_2^* u_2^\dagger + m_3 u_3^* u_3^\dagger$$

$U = (u_1, u_2, u_3)$: PMNS matrix

where

$$u_1 = \begin{pmatrix} c_{12}c_{13} \\ -s_{12}c_{23} - e^{i\delta}c_{12}s_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}s_{13}c_{23} \end{pmatrix}, \quad u_2 = \begin{pmatrix} s_{12}c_{13} \\ c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} \\ -c_{12}s_{23} - e^{i\delta}s_{12}s_{13}c_{23} \end{pmatrix}, \quad u_3 = \begin{pmatrix} e^{-i\delta}s_{13} \\ c_{13}s_{23} \\ c_{13}c_{23} \end{pmatrix}$$

⇩ The structure of g is partly determined to reproduce the PMNS matrix.

$$M_\nu = \frac{1}{M_0} f M_e g M_e f^T \qquad f = f_0 \begin{pmatrix} 0 & U_{\tau 1} & -U_{\mu 1} \\ -U_{\tau 1} & 0 & U_{e1} \\ U_{\mu 1} & -U_{e1} & 0 \end{pmatrix}$$

f_0 : proportionality coefficient

$$\frac{f_0^2}{M_0} M_e g M_e = m_2\, u_3 u_3^T + m_3\, u_2 u_2^T + a_1 u_1 u_1^T + a_2 (u_1 u_2^T + u_2 u_1^T) + a_3 (u_1 u_3^T + u_3 u_1^T)$$

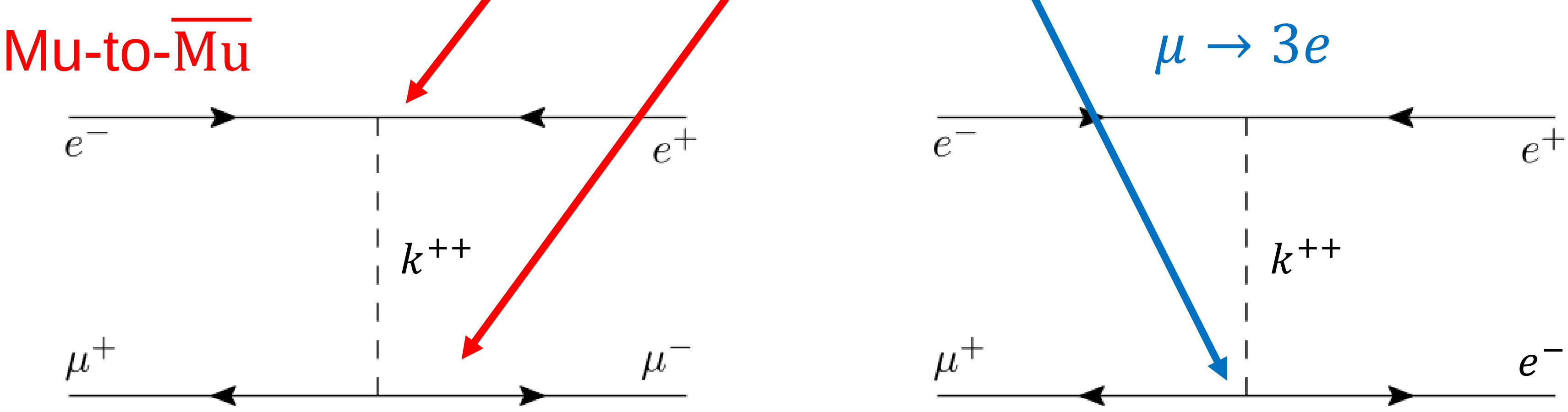
a_1, a_2, a_3 : free parameters

$$-\mathcal{L} \supset g_{ij}\overline{e_i}e_j^c k^{--}$$

$$g = \begin{pmatrix} g_{ee} & g_{e\mu} & g_{e\tau} \\ g_{e\mu} & g_{\mu\mu} & g_{\mu\tau} \\ g_{e\tau} & g_{\mu\tau} & g_{\tau\tau} \end{pmatrix}$$

$g_{e\tau} \rightarrow \tau^- \rightarrow e^+\ell\ell'$

$g_{\mu\tau} \rightarrow \tau^- \rightarrow \mu^+\ell\ell'$



In analysis, we set a_1, a_2, a_3 as follows:

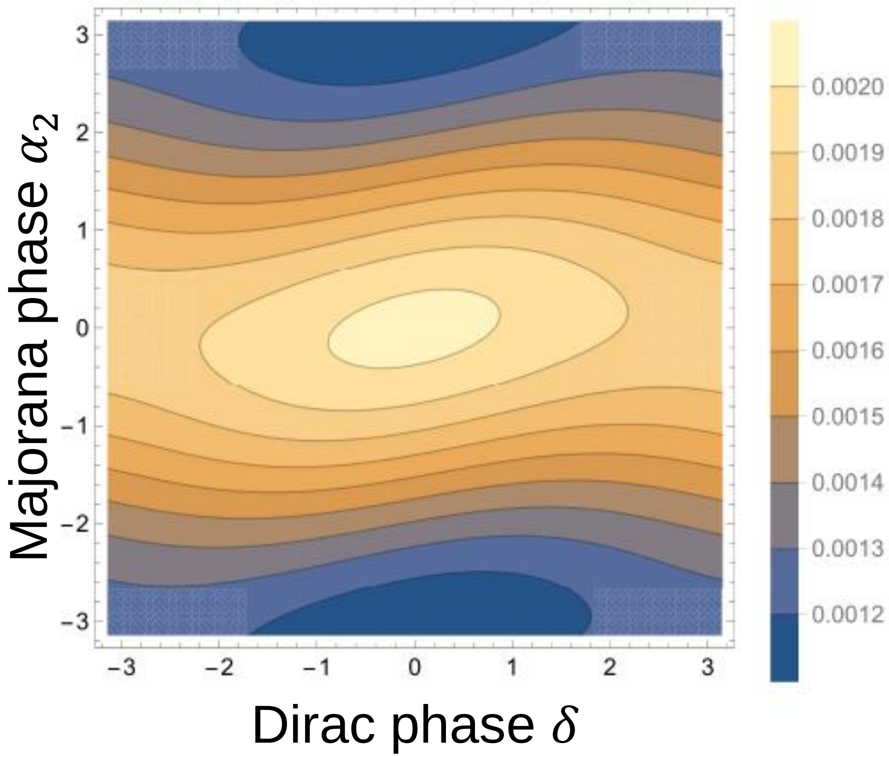
- adjust g_{ee} using one degrees of freedom of a_1, a_2, a_3
  (g_{ee} tends to be larger than 1 due to $g_{ee}m_e^2 \sim g_{\mu\mu}m_\mu^2$.)
- eliminate $g_{e\mu}$ & $g_{e\tau}$ using the rest two degrees

⇨ $g_{\mu\tau}$ cannot be eliminated to expect $\tau^- \rightarrow \mu^+e^-e^-, \tau \rightarrow 3\mu$

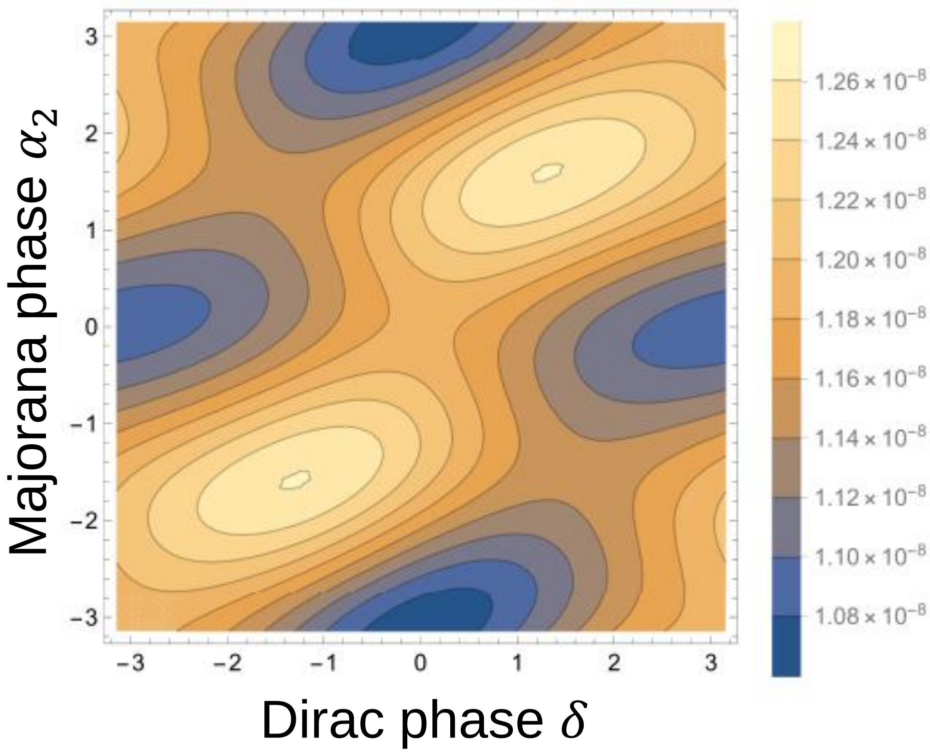
$$g_{ee} = g_{\mu\mu}, g_{e\mu} = g_{e\tau} = 0$$
$$f_0^2 = 0.002, m_k = 1.2\ \mathrm{TeV}, \mathrm{M}_0/(48\pi^2) = 500\ \mathrm{GeV}$$

$$-\mathcal{L} \supset \frac{G_2}{\sqrt{2}} [\overline{\mu}\gamma_\alpha(1+\gamma_5)e][\overline{\mu}\gamma^\alpha(1+\gamma_5)e]$$

$|G_2|/G_F$



$\mathrm{Br}(\tau \to 3\mu) = \mathrm{Br}(\tau^- \to \mu^+ e^- e^-)$



⇨ $\frac{G_2}{G_F} \sim \mathcal{O}(10^{-3})$

⇨ $\mathrm{Br}(\tau \to 3\mu) \sim \mathcal{O}(10^{-8})$

(just below the current experimental upper limit)

Summary

- Mu-to-$\overline{\mathrm{Mu}}$ transition
  - ✓ rare process with $\Delta L_\mu = -\Delta L_e = 2$
  - ✓ good probe for the leptonic structure of the new physics model
  - ✓ future experiments are planned in Japan & China
  - ✓ We investigate how large impacts Mu-to-$\overline{\mathrm{Mu}}$ gives for many models.

    T. Fukuyama, Y. Mimura, & Y. Uesaka, PRD**105**, 015026 (2022).

- e.g. Zee-Babu model
  - ✓ one of radiative neutrino models (two loop)
  - ✓ Mu-to-$\overline{\mathrm{Mu}}$ rate can be the same as the current limit with reproducing neutrino masses & satisfying other LFV constraints.
  - ✓ It is interesting to cross-check τ rare decay with Mu-to-$\overline{\mathrm{Mu}}$.