Nuclear Matrix Elements for Neutrinoless Double Beta Decays and Spin Dipole Giant Resonances.

Hiro Ejiri
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1. **Neutrinos studies by neutrinoless double beta decays (DBDs), and nuclear matrix elements (NMEs).**
2. **Giant isospin spin (τ σ) resonances and τ σ responses.**
3. **Experimental spin dipole (SD) single-beta responses and quenching of GT and SD single beta NMEs.**
4. **GT and SD strengths and DBD NMEs.**
5. **Impact on DBD exps. and discussions on DBD NMEs.**
6. **Concluding remarks**

1. H. Ejiri, J. Suhonen and K. Zuber, Phys. Rep. 797, 1 (2019).
2. H. Ejiri, Universe 6, 225 (2020); Frontiers in Physics 9, 650421 (1921).
3. L. Jokiniemi, H. Ejiri, D. Frekers, and J. Suhonen, P. R. C 98, 24608 (2018).
4. H. Ejiri, L. Jokiniemi and J. Suhonen, Phys. Rev. C. Lett, 105 L022501

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Letter

Nuclear matrix elements for neutrinoless $\beta\beta$ decays and spin-dipole giant resonances

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Nuclear matrix element (NME) for neutrinoless $\beta\beta$ decay (DBD) is required for studying neutrino physics beyond the standard model by using DBD. Experimental information on nuclear excitation and decay associated with DBD is crucial for theoretical calculations of the DBD-NME. The spin-dipole (SD) NME for DBD via the intermediate SD state is one of the major components of the DBD-NME. The experimental SD giant-resonance energy and the SD strength in the intermediate nucleus are shown for the first time to be closely related to the DBD-NME and are used for studying the spin-isospin correlation and the quenching of the axial-vector coupling, which are involved in the NME. So they are used to help the theoretical model calculation of the DBD-NME. Impact of the SD giant resonance and the SD strength on the DBD study is discussed.

I. H. Ejiri
Experimental aspects and impact on DBD experiments.

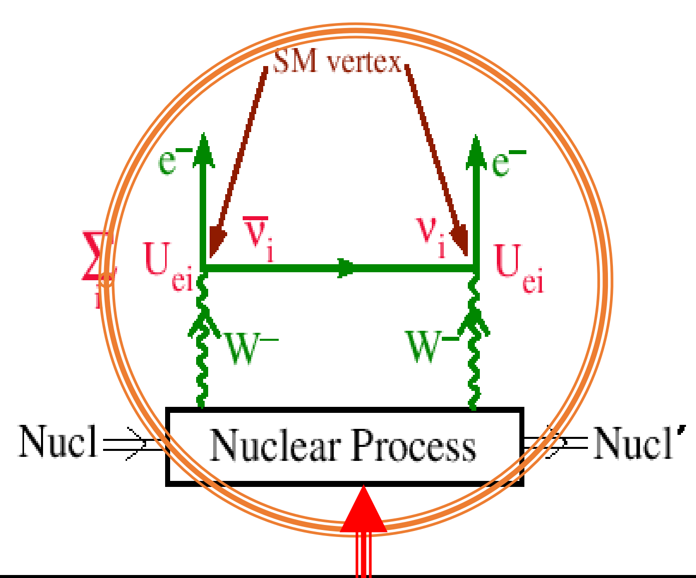
II. L. Jokiniemi
Theoretical aspects on pnQRPA calculations for NMEs

1. Neutrinos studies by neutrinoless double beta decays and nuclear matrix elements (NMEs).

Neutrino-less ββ decays DBDs

$A = B + \beta + \beta$

Lepton number $\Delta L=2$ beyond SM.



Part. ν physics

Majorana ν, m_ν CP

Transition rate : Exp.

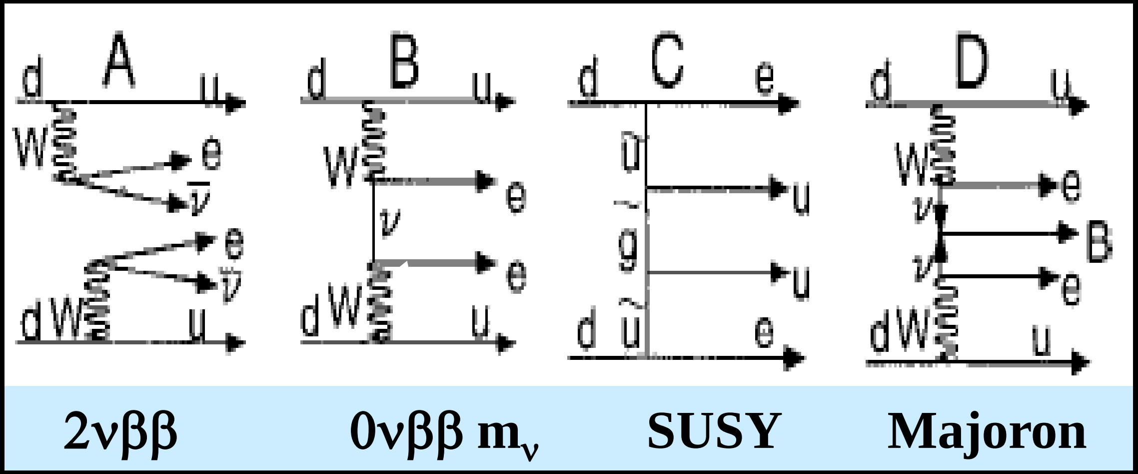
$$T^{0\nu} = G^{0\nu} [M^{0\nu} \; m_\nu]^2$$

FEMT(fm)
Nucl. micro-lab.
to enhance
. ν-exchange

Atomic physics
Phase space
E and Z
dependences

Nuclear physics
Matrix elements
(NME) pp,ph,
τ σ correlation,
Nuclear medium.

A. Neutrinoless double beta decays $0\nu\beta\beta$



* A realistic and sensitive probe for new physics beyond the standard electro-weak model (SM).

* Majorana nature of ν ,

* Mass scale of ν, mass hierarchy, Majorana phase .

* Right-handed current*, and others beyond the SM

Exp. observable = product of $M^{0\nu}$ x m_ν , need NME

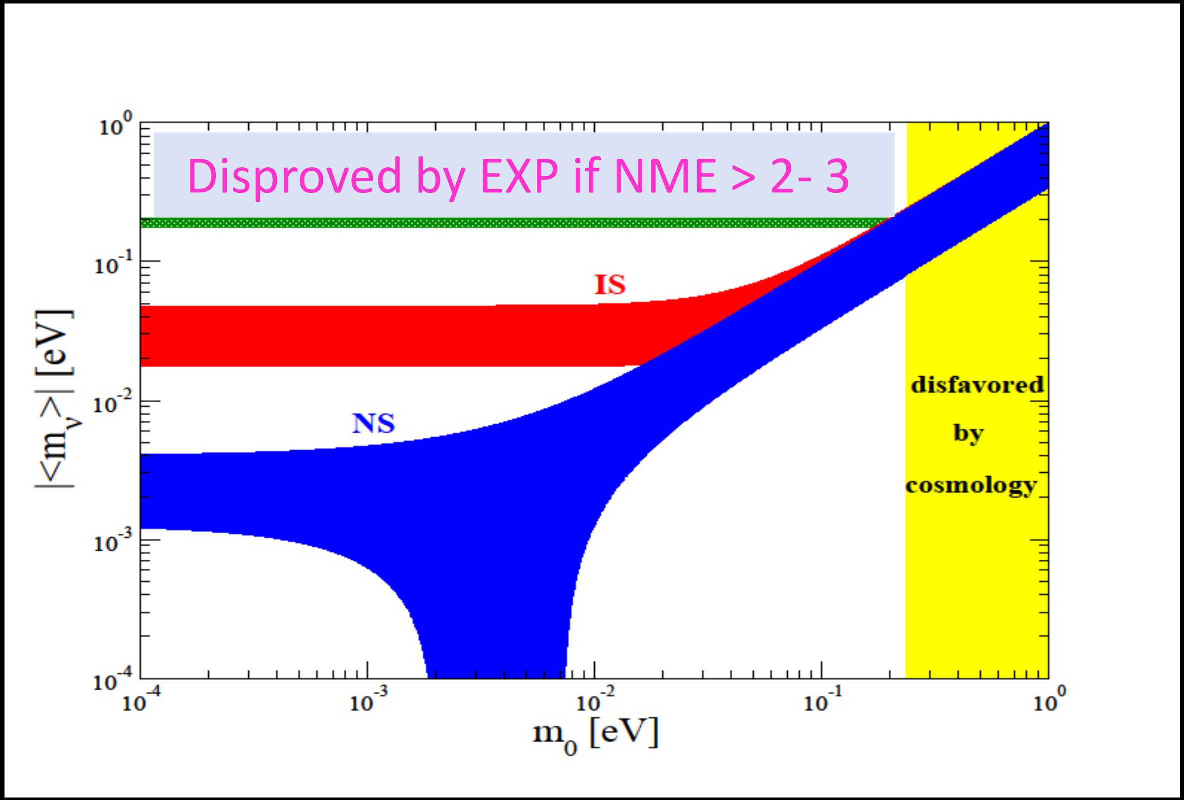
NME = nuclear detector sensitivity (response) to m_ν

* RHC: Fukuyama Iwata NEWS

Light ν mass process

$< m_\nu > = |\Sigma\, U_i^2\ \exp(i\,\phi_i)\, m_i| \qquad \phi_2=\alpha_2-\alpha_1, \qquad \phi_3 = -\alpha_2-2\delta$

are given by using U_i Δm_S, Δm_A measured by ν oscillations.



NME within a factor 2 and 30% to get IS/NS and the phase,

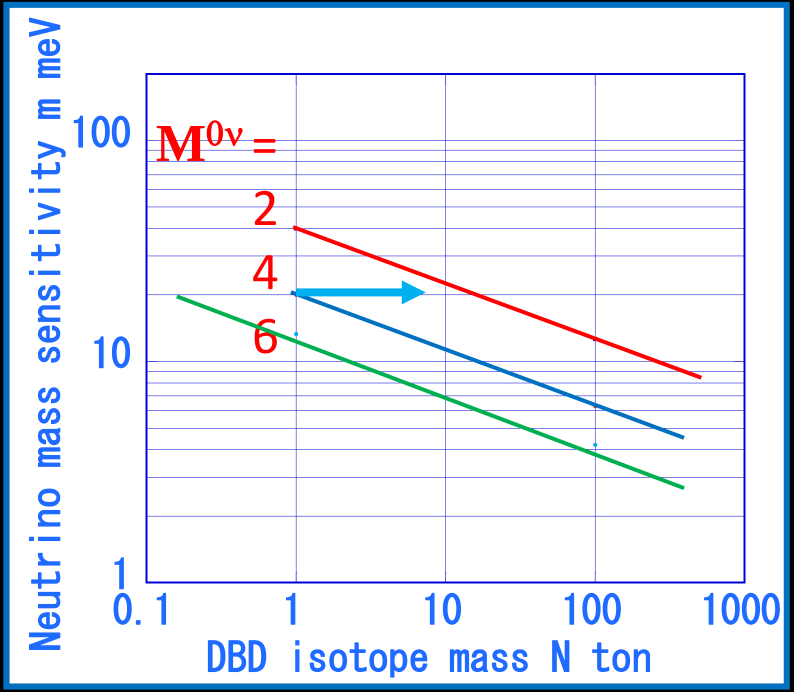
J. Vergados, H. Ejiri, F. Simkovic, Rep. Prog. Phys. 75 (2012) 106301.

Experimental sensitivity= m_ν : mass to be detected

NT= Isotope ton and year B= BG/ton year

$m_\nu = 2\ m_0\ [B/NT]^{1/4}$ with detector efficiency ε=0.5

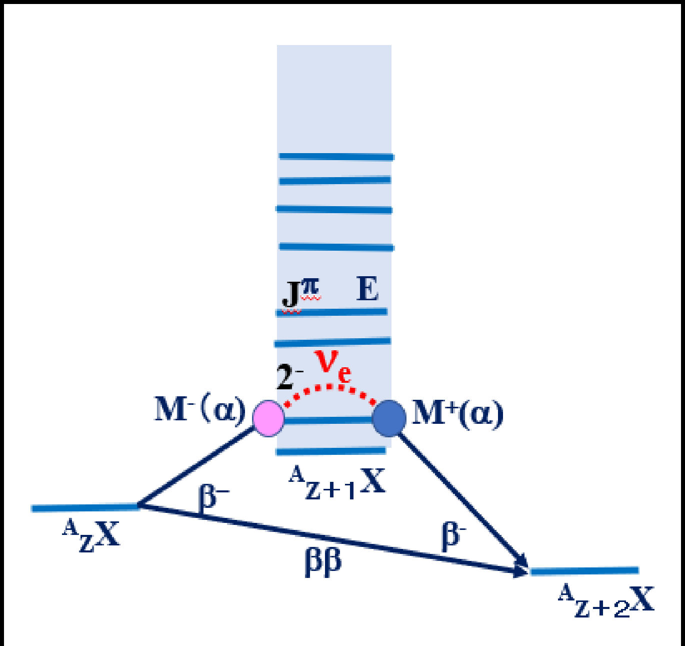
m_0 =40 meV /$M^{0\nu}$ for Se, Mo, Cd Te, Xe m_0 =80 meV /$M^{0\nu}$ for Ge



B=1/ton year, 20 meV sensitivity is achieved by
NT=0.2-16 ton-year experiments in cases if $M^{0\nu}$ = 6– 2 .

DBD 0νββ NME

$$M^{0\nu} = \left(\frac{g_A^{\text{eff}}}{g_A}\right)^2 \left[M_{GT}^{0\nu} + \left(g_V/g_A^{\text{eff}}\right)^2 M_F^{0\nu} + M_T^{0\nu}\right],$$



Quenching due to effects not in model

=1 for free 2n

Model NMEs

$$M_{GT}^{0\nu} = \sum_k \langle t_\pm \boldsymbol{\sigma} h_{GT}(r_{12}, E_k) t_\pm \boldsymbol{\sigma} \rangle$$

$$M_F^{0\nu} = \sum_k \langle t_\pm h_F(r_{12}, E_k) t_\pm \rangle ,$$

$$M_T^{0\nu} = \sum_k \langle t_\pm h_T(r_{12}, E_k) S_{12} t_\pm \rangle ,$$

$H(r_{12}) \sim 1/r_{12}$ ν potential for ν-exchange,

$M^{0\nu} = \Sigma_J M(J)$ J= Multipole sum

$M(J) = \Sigma_k M_k(J)$, Sum over all intermediate state k.

Key elements : 1. Spin (σ) isospin (τ) correlation

2. Dipole SD (L=1) to match the ν momentum

Nuclear $\tau\sigma$ symmetry, $\tau\sigma$ GR , $\tau\sigma$ polarization

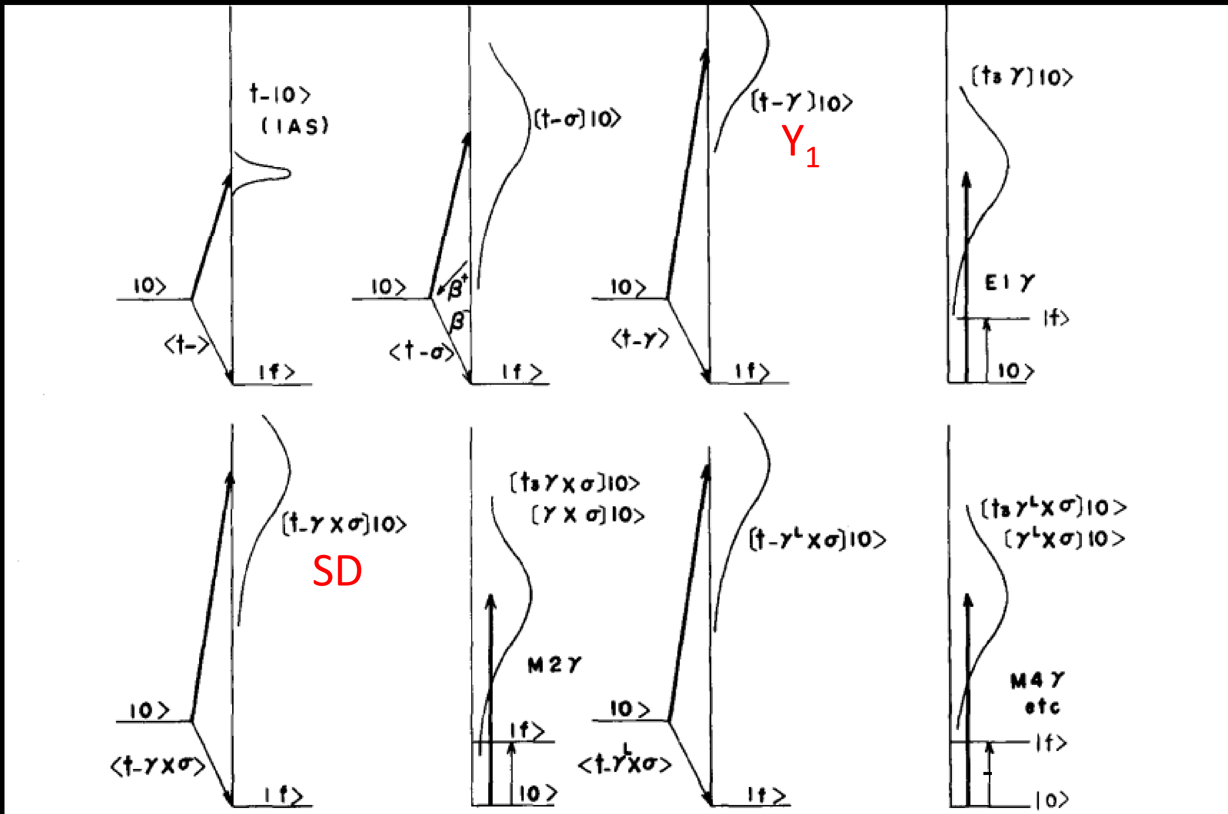
1. T= β,γ,CER operators : vector T= τY_l, Axial-vector T=$\tau\sigma\, Y_l$

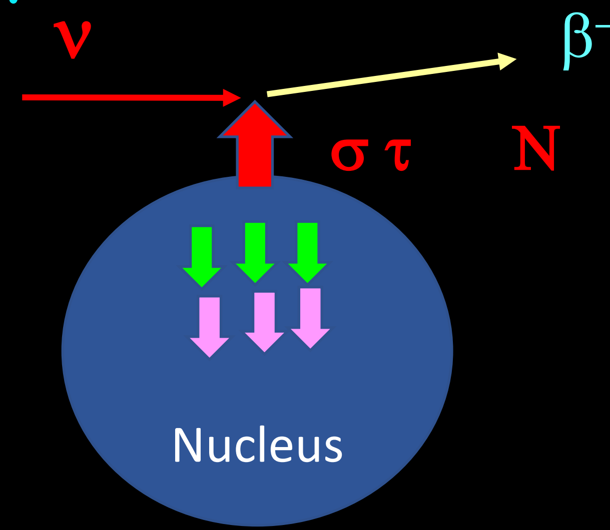
2. [H, T] ~ $E_G T$
   T|i> ; T GR, giant resonance: most T strengths, and little <f|T|i>
   T phonon = Coherent sum of all (N) ph excitations
   GR NME=M_{GR}= $N^{1/2}$ Ms , E_{GR}=Es+χN

   T=τ T|i> = IAS No τ Fermi strength
   T=$\tau\sigma$, T|i> = GT GR , little (~ 10^{-1}) GT strength to low states
   T=$\tau\sigma$rY, T|i> = SD GR , little 2^- strength to low states

3. T isospin and spin isospin polarization
   $|f\rangle = |f\rangle_0 - \varepsilon|GR\rangle$
   $M \sim M_0\,[1 - \varepsilon M_{GR}/M_0]$
   $= k^{eff} M_0$ $k^{eff} = 1/[1+\chi]$ $\chi=\tau/\tau\sigma$ susceptivility
   $\varepsilon \sim 0.07$ admixture of GR M_{GR}= 6 makes k^{eff} =0.6 as exps.

Spin isospin giant resonances and spin isospin core polarization in β–γ and CERs



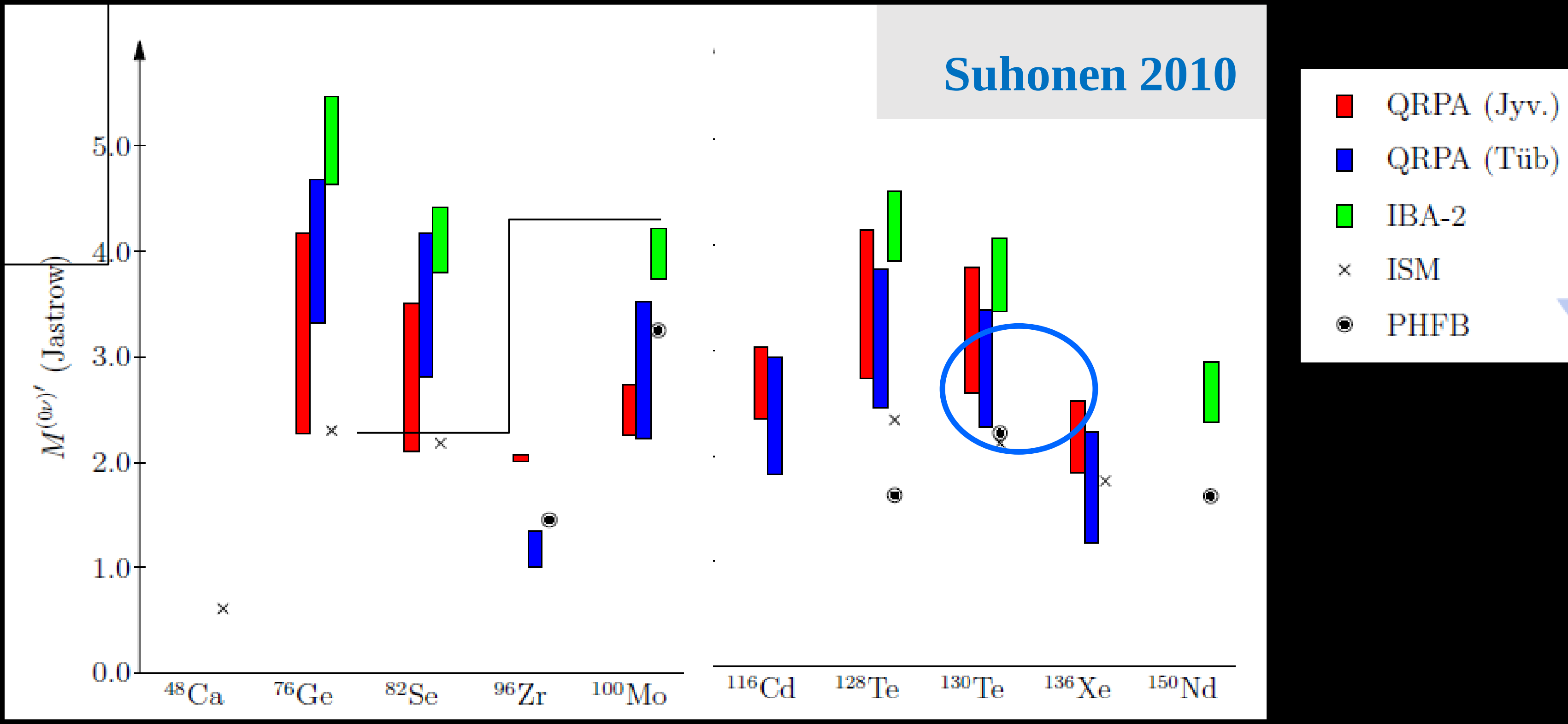


Nucleons and quark τσ polarizations reduce nucleon σ τ for a nucleon at surface

Nucleon τ σ giant resonances at 10-30 MeV region,
Quark τ σ Δ-isobar nucleon-hole at 250 MV region.

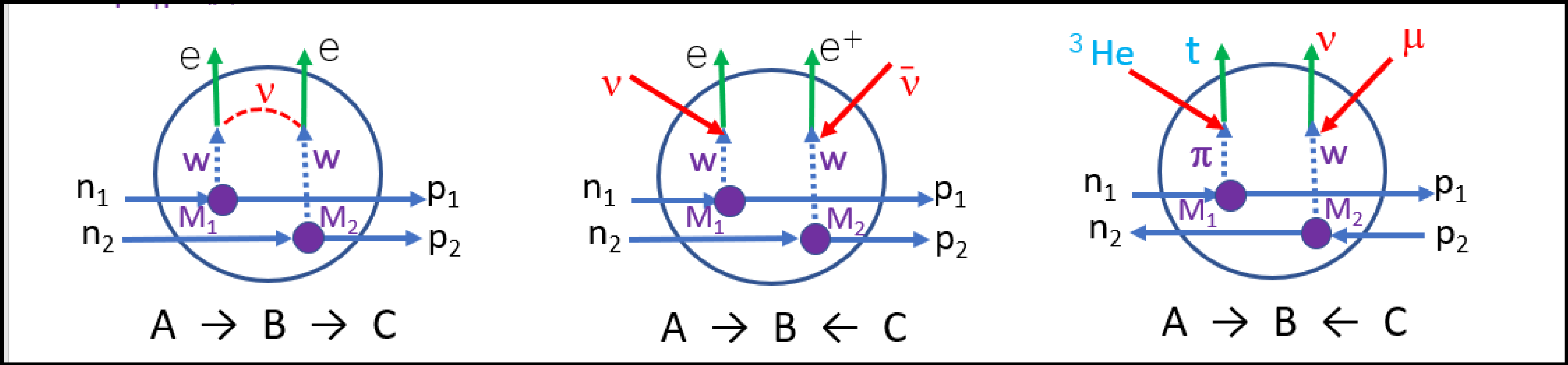
Ejiiri , J. Fujuta Ikeda Phys. Rev. 176 1968
H. Ejiri, J.I. Fujita Phys. Rep. 38 1978

NMEs are very sensitive to nuclear models and parameters



Experimental inputs are crucial, NEXT NEWS

Double β decay, single β&ν and CERs



DBD M_1 , M_2 via neutrino potential by single β, ν, μ. CER NMEs

$M(\alpha, \beta^{\pm}) = (g_A^{eff})^{\pm}\, M(\text{QRPA } \alpha\, \beta^{\pm})$ α=GT, SD. SQ, ・ ・

(g_A^{eff}) for renormalization effects due to non-nucleonic and nuclear medium effects which are not in pnQRPA.

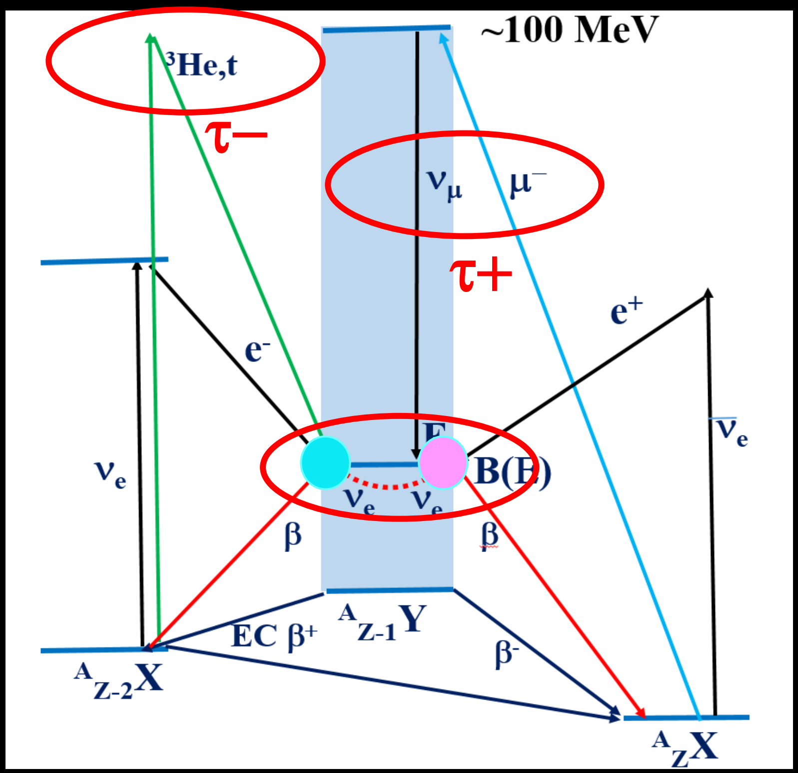
$(g_A^{eff})^{-} \sim (g_A^{eff})^{+}$ for β^{-} , β^{+} and $(g_A^{eff})^2$ for ββ

$M(\alpha, \beta\beta) = (g_A^{eff})^2\, M(\text{QRPA } \beta\beta)$

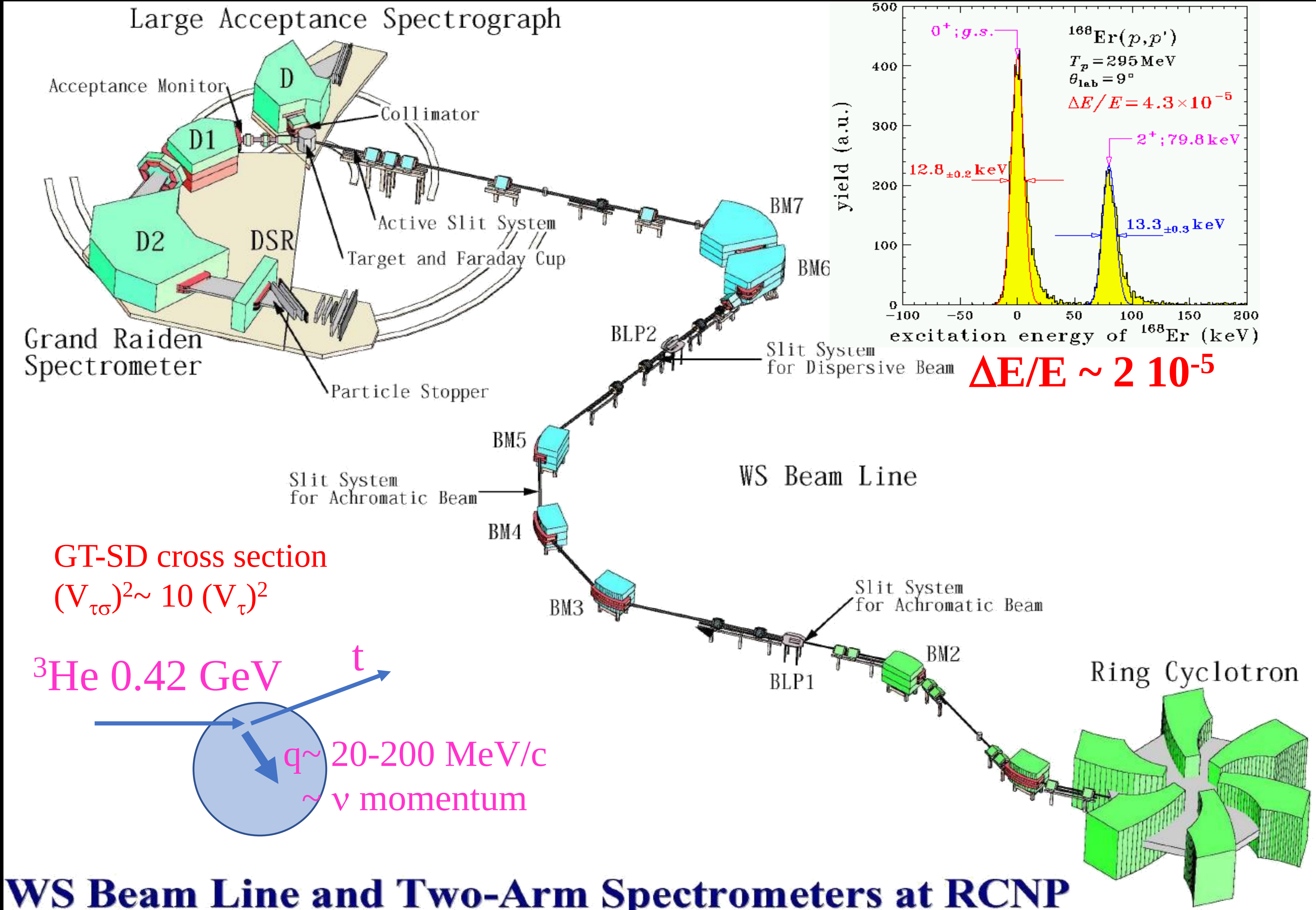
2. Experimental spin isospin (F,GT, SD) strengths

Nuclear and lepton CERs provide τ , τσ , Y_L
NMEs associated with the τ , τσ , Y_L DBD NMEs, & help DBD NME model calculations and evaluation of g_A.

Nuclear & μ CERs E=1-30 MeV P ~ 60-120 MeV
similar as DBD with r~2 fm ν-exchange.



High E resolution (^{3}He,t) CERs at RCNP Osaka



$\Delta E/E \sim 2\ 10^{-5}$

GT-SD cross section

$(V_{\tau\sigma})^2 \sim 10\ (V_\tau)^2$

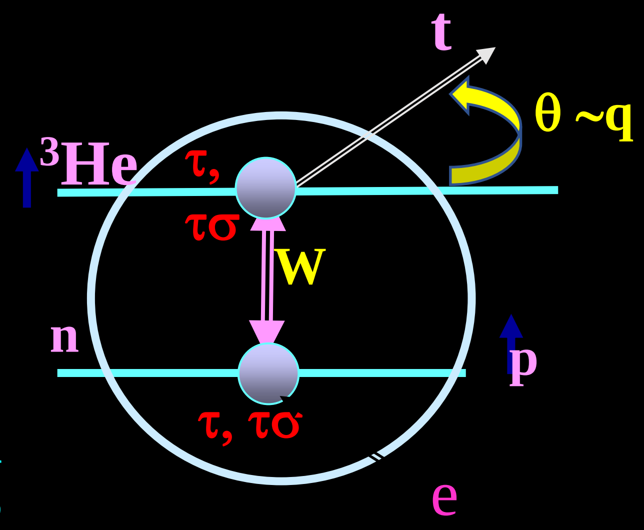
^{3}He 0.42 GeV

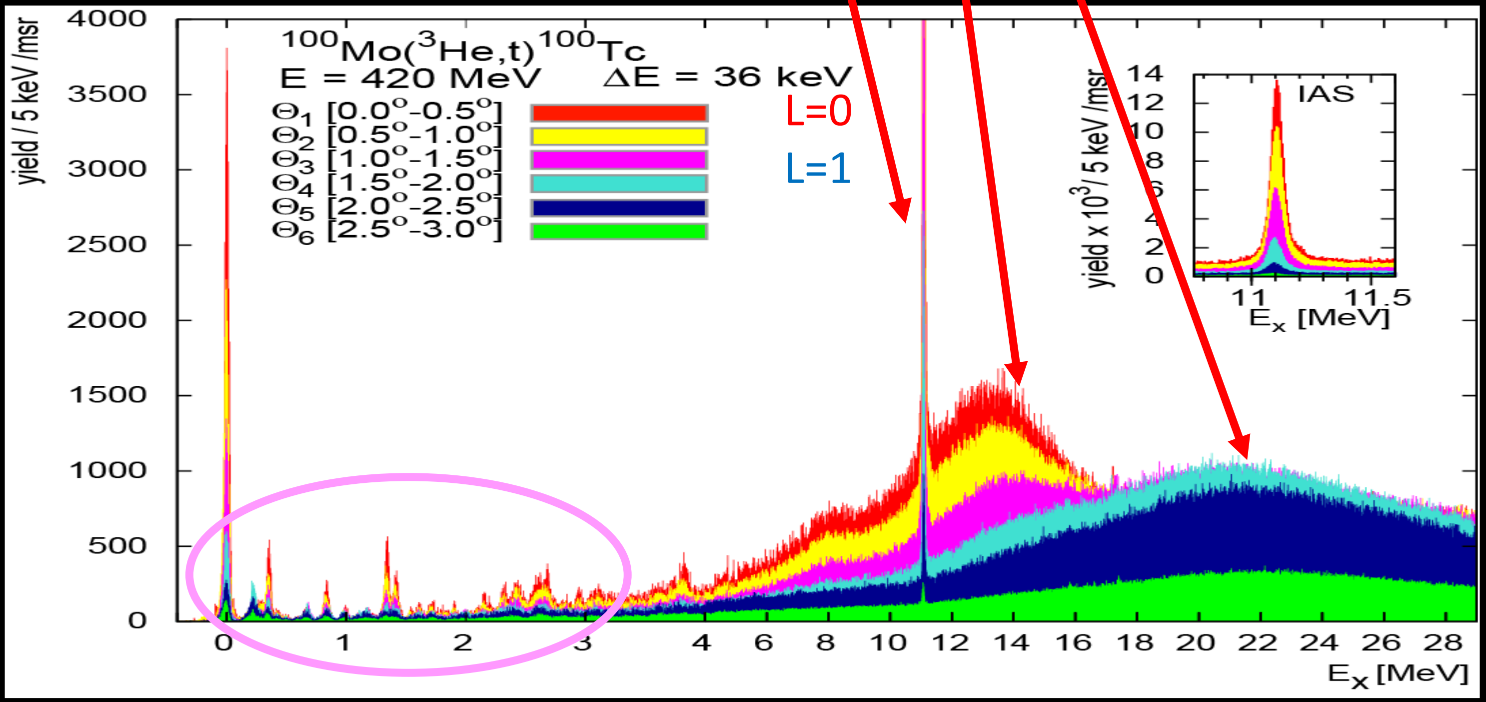
q~ 20-200 MeV/c

~ ν momentum

CERs at RCNP
Most strengths
GRs (Giant resonances)

Fermi No at low states, all in F-GR: IAS
GT   A few % at low states, 50% GT-GR
SD   A few % at low states, main SD-GR



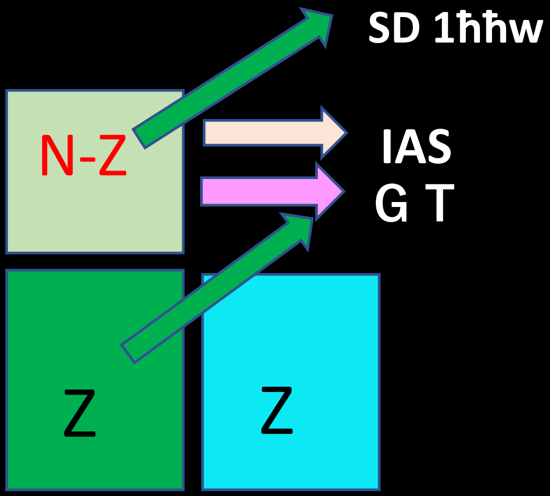


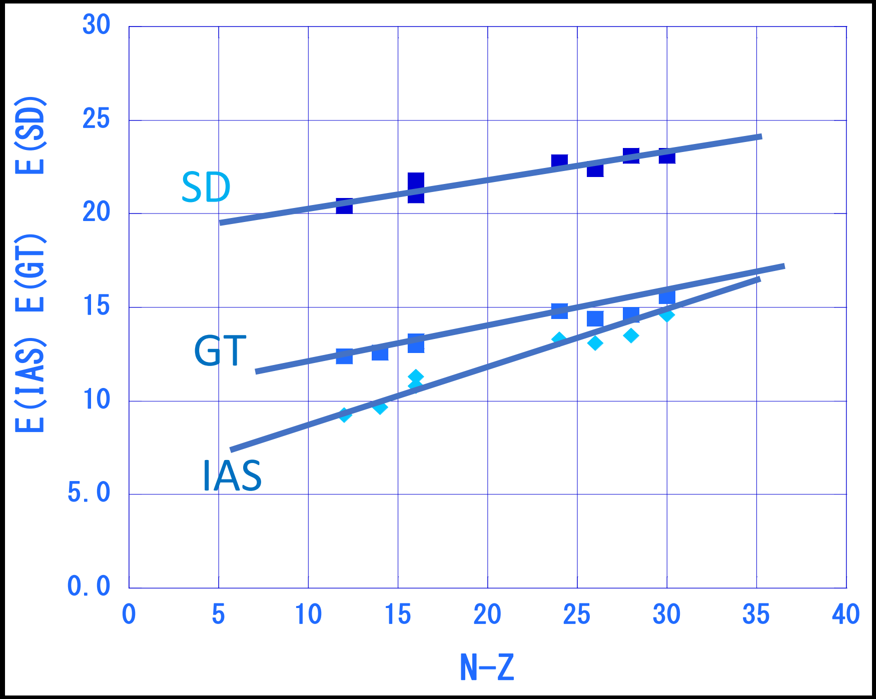
Ejiri, Suhonen, Zuber PR 797 1 2019

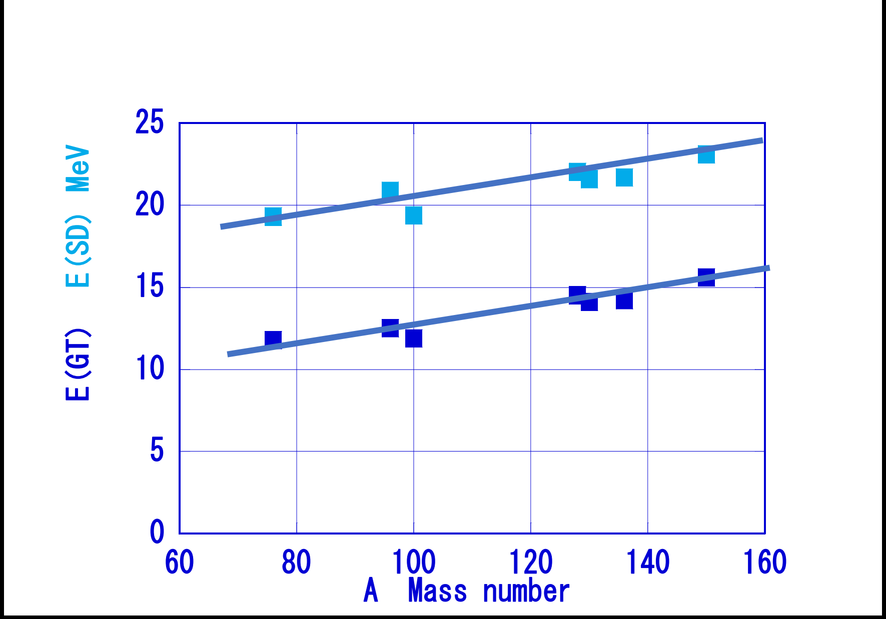
IAS, GT and SD GRs

E_G (IAS) = 5+0.3(N-Z)
E_G (GT) = 0.2 (N-Z)+9=0.06A + 6.5
E_G (SD) = 0.2 (N-Z)+16.5=0.06A+ 14

GT and SD same A dependence
E(SD)~E(GT)+0.9 ħω L=I excitation







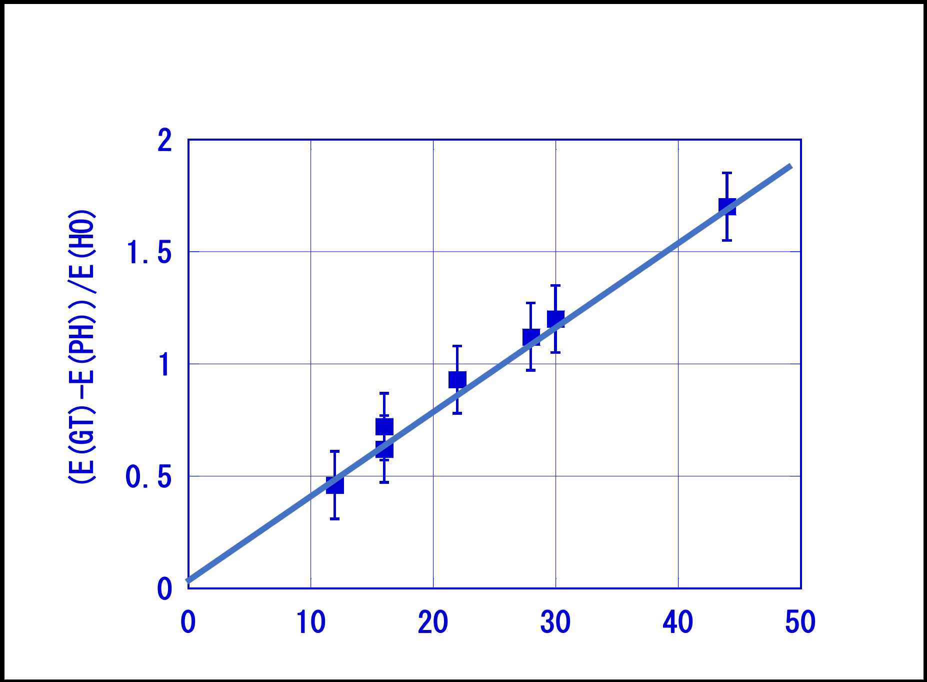
E_G GR –Energies increase smoothly as N-Z and A, reflecting nuclear core property

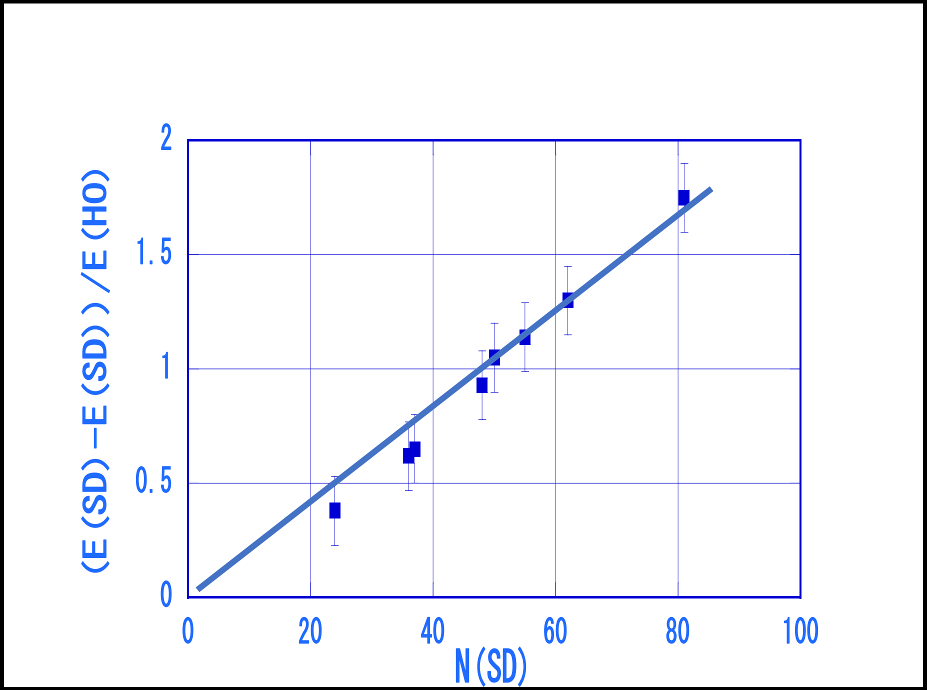
$E(GT) - E(PH) = 0.04\hbar\omega \times (N\text{-}Z)$

$\hbar\omega = 14A^{-1/3}\text{MeV}$

$E(SD) - E(PH) = 0.02\,\hbar\omega \times N(SD)$

$\hbar\omega = 41A^{-1/3}\text{MeV}$





N-Z

$2T_Z = N\text{-}Z$

Summed strengths of GRs and low-QP states

B_S(IAS) = N-Z,

B_S(GT) =3 (N-Z)
Nucleon sum*

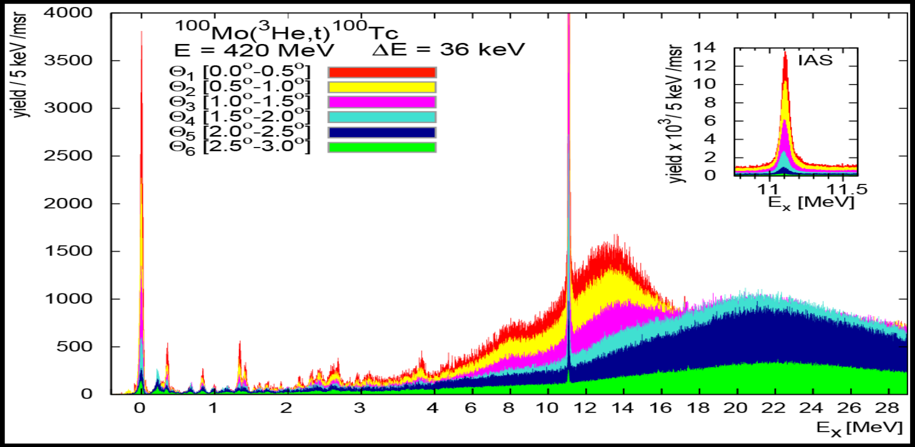
B_{GR}(GT)~ B_A(GT) = 0.55

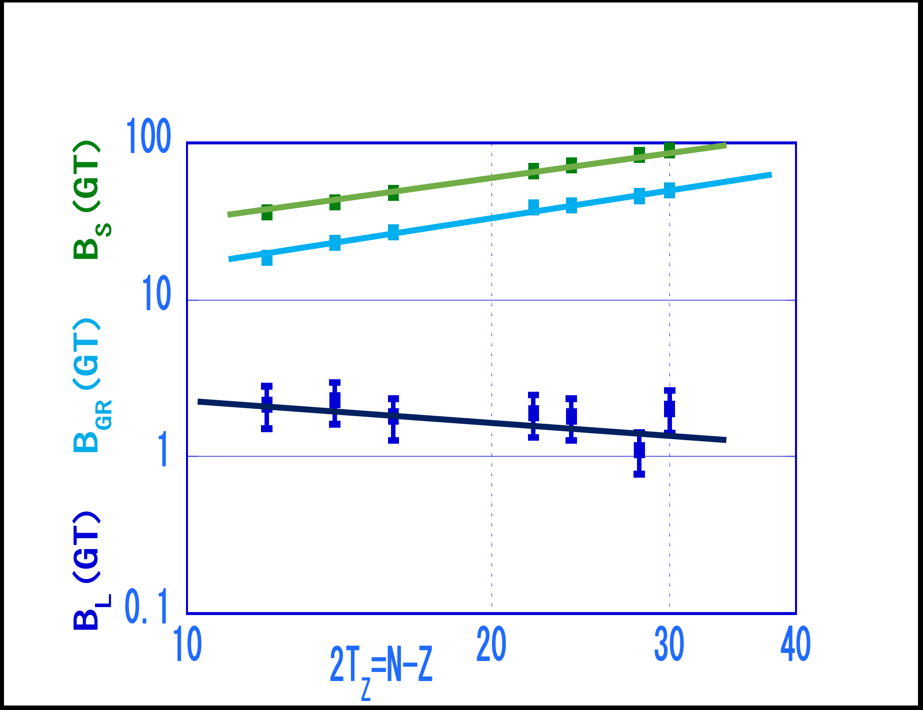
B_L(GT) for E= 0-6MeV ~0.2- 0.1
not increase as N-Z

B_{GR}(SD)~ B_A(GT)

B_L(SD for E= 0-10 MeV ~ 0.1
not increase as N-Z

* Ikeda Fujita Fujii Sum -rule

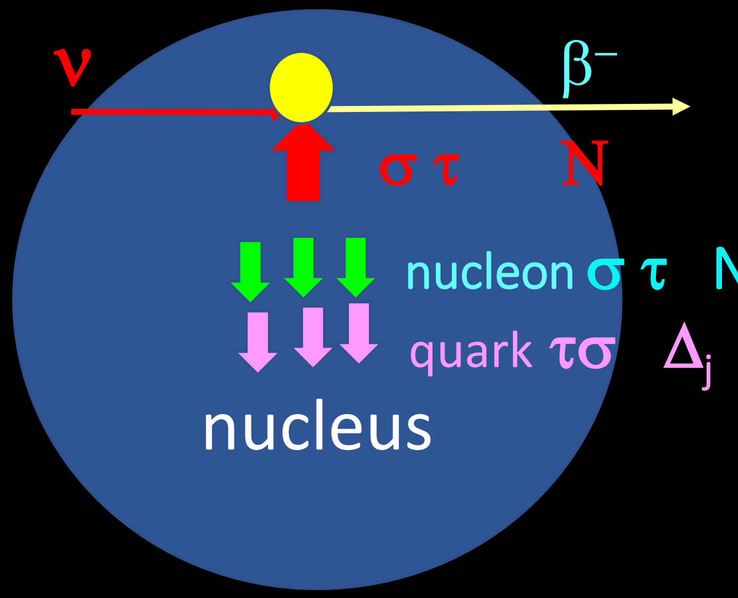




3. Quenching of axial vector weak coupling

E. Spin isospin correlations/polarizations
Nucleonic $\tau\sigma$ $N^{-1}N$ GR , non-nucleonic $N^{-1}\Delta$ GR

Nuclear medium
$\tau\sigma$ polarization



$|I\rangle = |SP\rangle - \varepsilon\,|GRn\rangle - \delta\,|GR\,\Delta\rangle$

$M^{\beta} \sim k^{eff}\, M_0 \quad M_0 = QP$

$k^{eff}\ (\tau\sigma) \sim 1/(1+\chi_{\tau\sigma}) = 0.3$

$\chi_{\tau\sigma}$: susceptibility ~ 2

due to nuclear and isobar polarizations.

Nuclear $\tau\sigma$ polarization effects

Ejiri Fujita 1968-1978

Isobar polarization effects for GT
Bohr Mottelson
PL B 10 '81 10 Isobar

Nuclear τσ susceptibility due τσ polarization interaction.

$$H^{P}_{\alpha} = \chi_{\alpha} T_{\alpha} \cdot T_{\alpha}.$$

H. Ejiri NP 166 594 1970,
H. Ejiri and J.I. Fujita PR 34 1978.

T=τσ repulsive interaction gives rise to the $\tau\sigma Y_L$ mode giant resonances (phonon) at the high E, and reduce the τσ NMEs , as the attractive Y_2 interactions E2 phonons at the low E and enhance E_2NMEs ($e^{eff} > e$).

Nuclear τσ correlations, which are not in QP, are incorporated by g^{eff} and κ

$$\langle f \| T_{\alpha} \| i \rangle \approx \frac{g^{eff}}{g} \langle J_1 \| T_{\alpha} \| i_1 \rangle_p$$

$$\frac{g^{eff}}{g} = \frac{1}{1+\kappa} = \frac{1}{1+\kappa^{-}+\kappa^{+}}$$

$$\kappa^{-}_{Ii} = \frac{h_{Ii}}{E(Ii) - E_1} V_i^2 U_I^2, \qquad \kappa^{+}_{Ii} = \frac{h_{Ii}}{E(Ii) + E_1} V_I^2 U_i^2.$$

$$h_{Ii} = \chi_{\alpha} G^2_{Ii} / [(2i+1)(2I+1)].$$

$E(GR) \sim E(QP) + N\chi G^2 \quad \kappa \sim N\chi G^2/E(Ij)$

$N\chi G^2$: the total τσ strength of the nuclear core increase as A and N-Z.

Likewise, quark (isobar) τσ correlations, which are not in the model, are incorporated by g^{eff} and κ.

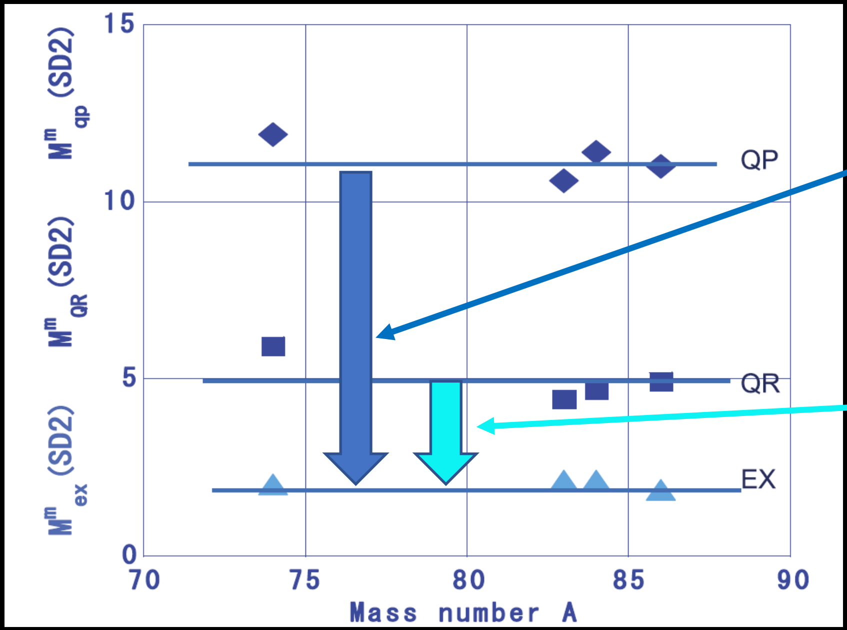
Renormalization (quenching) of spin dipole $(\tau\sigma Y_1)_2$ NMEs

Geometrical mean of beta + and beta – NMEs to avoid effects of QP occupation and vacancy coefficients

$$M^{\mathrm{m}}(\mathrm{SD2}) = \left[M^{+}(\mathrm{SD2})M^{-}(\mathrm{SD2})\right]^{1/2},$$

$$M^{\mathrm{m}}_{\mathrm{qp}}(\mathrm{SD2}) = M_{\mathrm{sp}}(\mathrm{SD2})\left(V_p U_n V'_n U'_p\right)^{1/2},$$

0.43



QP only nuclear paring interactions, but no $\tau\sigma$ correlations. Uniformly reduced by $\tau\sigma$ correlations

QRPA : nuclear pp and ph interactions. Uniformly reduced by non-nucleonic and nuclear medium effects, which are not in QRPA

H. Ejiri N. Soucouti, J. Suhonen PL B 729 2014 .

M4 gamma transitions

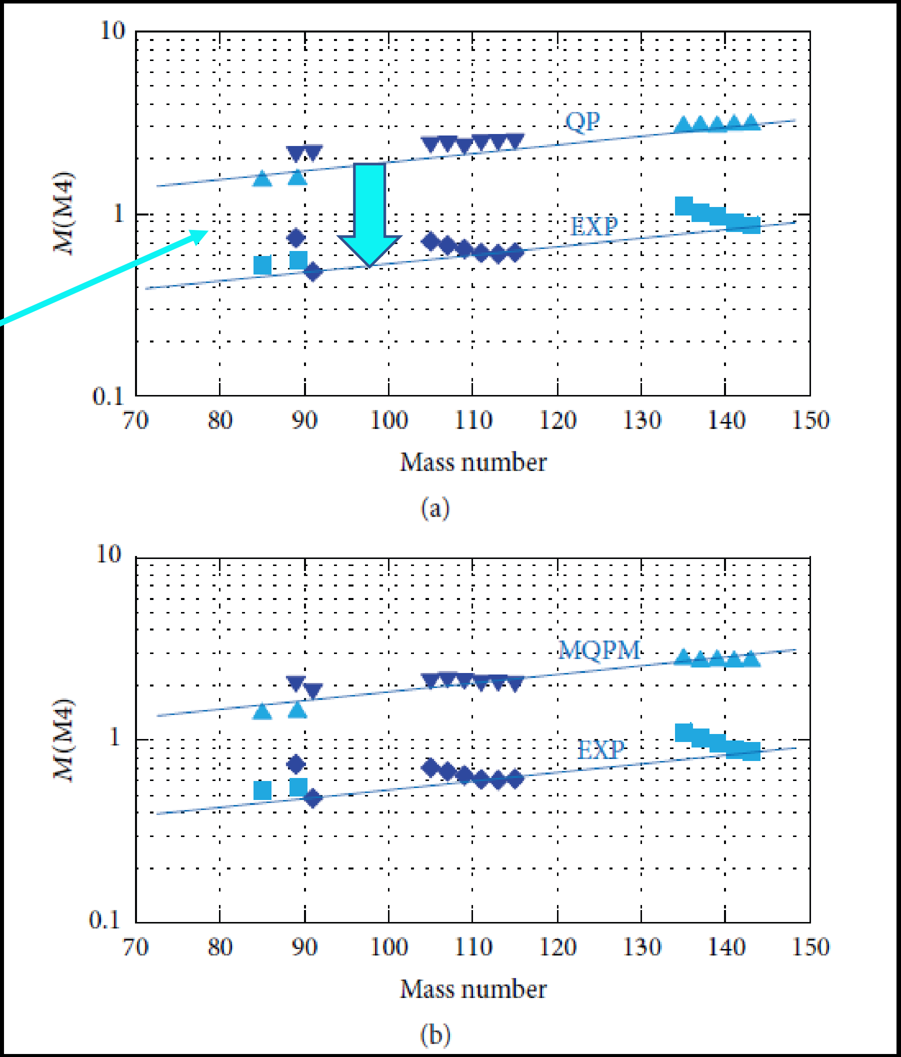
Mainly isovector

$[\tau\sigma r^3 Y_3]_4$

$M_{EXP} \sim k\ M_{QP}$

K=0.29

M increase as A $\sim r^3$



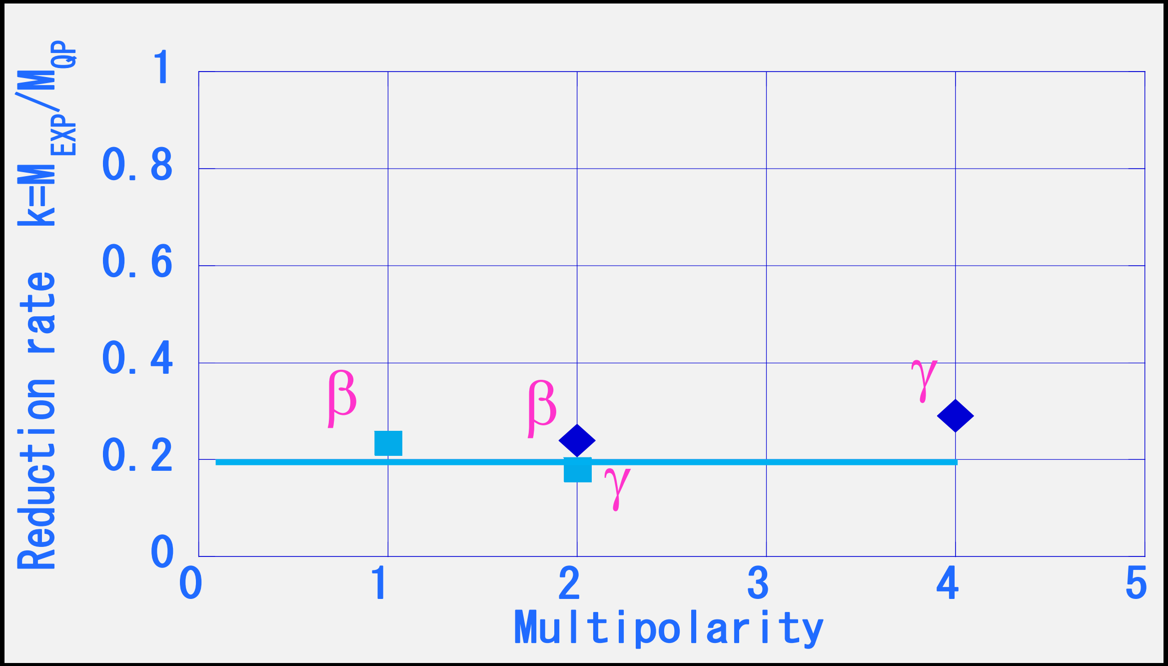
MQPPM=

Microscopic QP phonon model

L. Jokiniemi J. Suhonen H. Ejiri AHEP2016 ID8417598

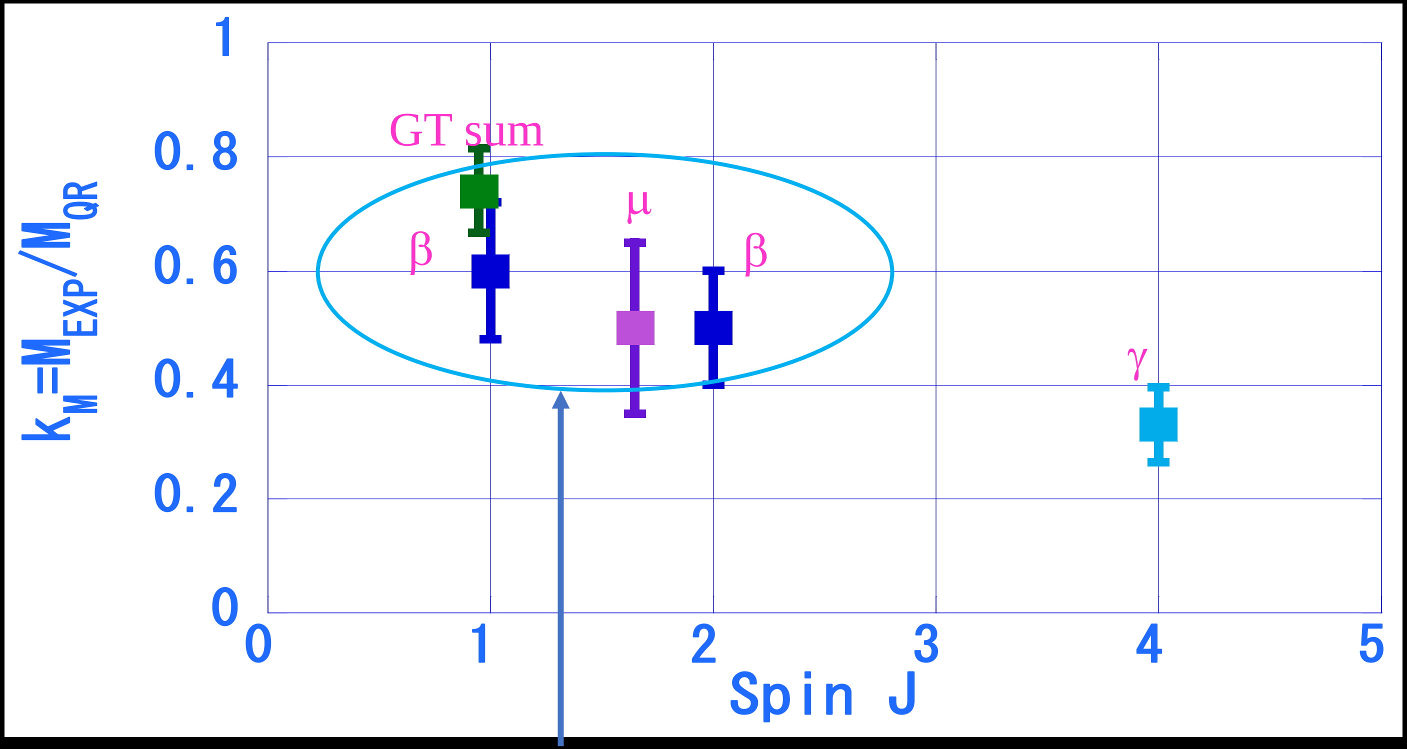
Renormalization of axial vector β & γ in low p

Spin isospin (στ) repulsive interactions push up most strengths into the τ στ GRs (IAS, GT, SD), thus στ weak /EM couplings are renormalized much with respect to the QP(quasi-particle NMEs) without the τσ correlation .



$\kappa_{\tau\sigma} = M_{EXP}/M_{QP} = 0.2\text{-}0.3$ with respect to QP, due to the nucleonic and non-nucleonic στ correlations which are not in QP model.

Renormalization/reductions of axial vector β & γ coupling/NME with respect to QRPA



$g_A^{eff}(\beta\beta)/g_A \sim 0.55\text{-}0.75$

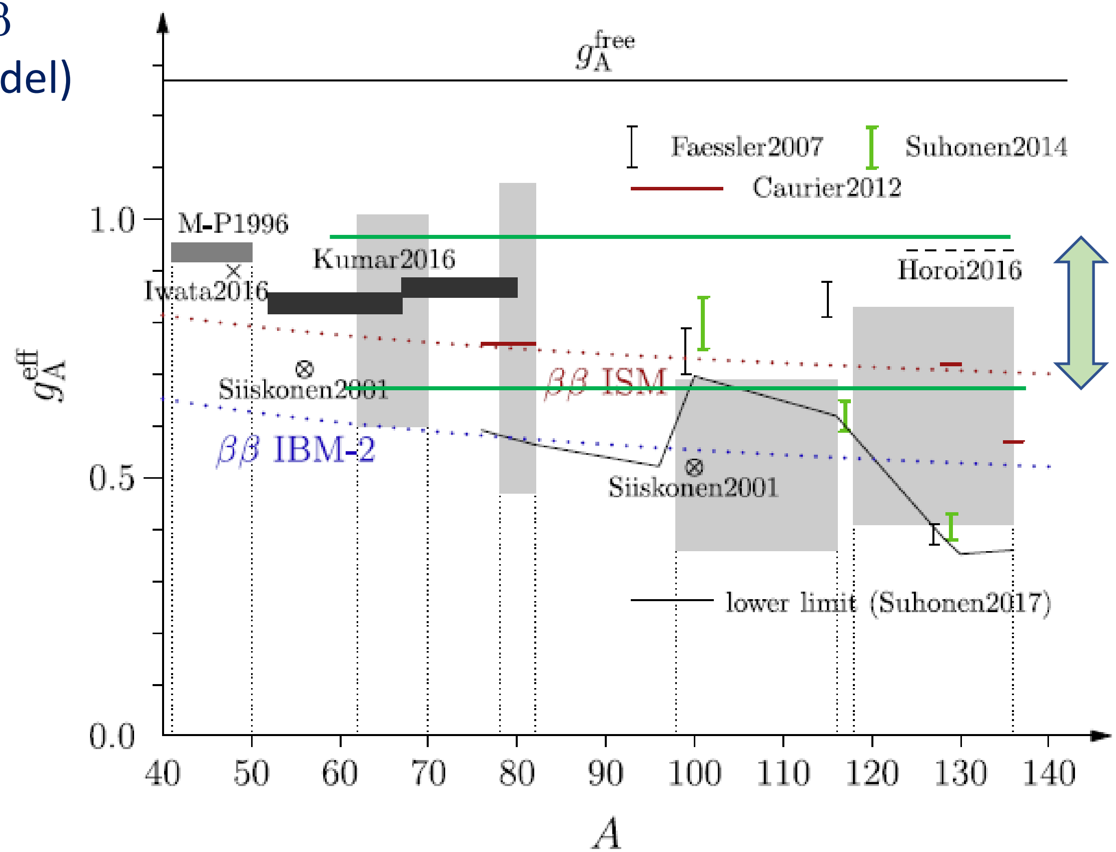
H, Ejiri J. Suhonen J. Phys. G. 42 2015

H. Ejiri N. Soucouti, J. Suhonen PL B 729 2014 .

L. Jokiniemi J. Suhonen H. Ejiri AHEP2016 ID8417598

L. Jokiniemi J. Suhonen. H. Ejiri and I. Hashim PL B 794 143 (2019)

g^{eff} from $2\nu\beta\beta$
M(EXP)/M(Model)



Present
0.74x1.27
0.55x1.27

Fig. 29. Effective values of g_A in different theoretical β and $2\nu\beta\beta$ analyses for the nuclear mass range $A = 41 - 136$. The quoted references are *Suhonen2017* [216], *Caurier2012* [233], *Faessler2007* [242], *Suhonen2014* [243] and *Horoi2016* [235]. These studies are contrasted with the ISM β-decay studies of *M-P1996* [229], *Iwata2016* [230], *Kumar2016* [231] and *Siiskonen2001* [228]. For more information see the text and Table 3 in Section 3.1.2 and the text in Section 3.1.3.

. Ejiri H, Suhonen J and Zuber Z 2019 Phys. Rep. 797 1

4. Comparisons with pnQRPA NMEs

Model NME: pnQRPA, which consider well $\tau - \sigma$ correlations and SD giant resonances..

$$R^{0\nu} = \ln 2 \, g_A^4 G^{0\nu} [m_{\beta\beta} |M^{0\nu}|]^2,$$

g_A=1.27 in unit of g_V for free nucleon weak coupling.

$$M^{0\nu} = \left(\frac{g_A^{\text{eff}}}{g_A}\right)^2 [M_{\text{GT}}^{0\nu} + M_T^{0\nu}] - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu}, \qquad (2)$$

g_A^{eff} for non-nuleonic , nuclear medium effects that are not in QRPA.

pn QRPA. NMEs

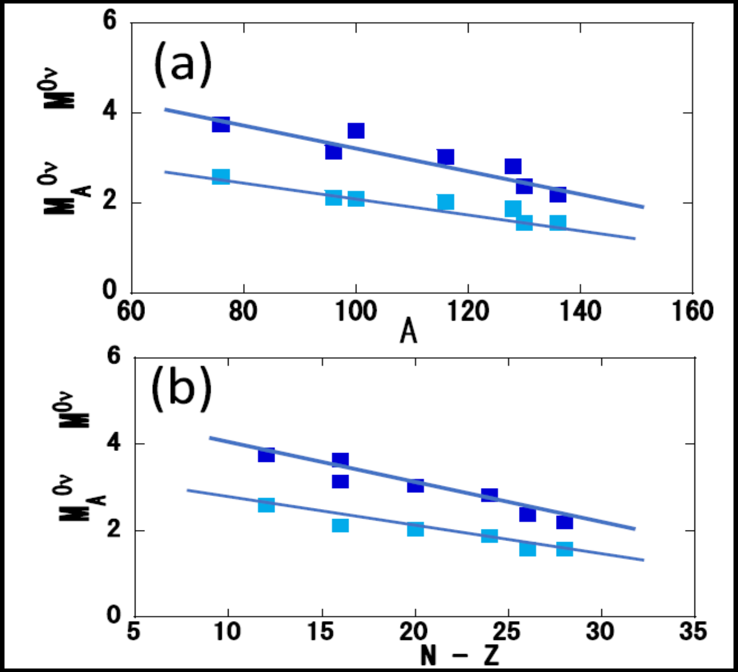
g_V^{eff} put g_V free nucleon

Nucleons in nucleus are different from nucleons in a free space, and also different from nucleons in a model space, i.e. different meson -isobar clouds and nuclear medium. Accordingly, the mass, the charge, the weak, strong and EM couplings are renormalized in models unless all effects are properly taken into accounts.

Next talk by Lotta Jokiniemi on the pnQRPA used

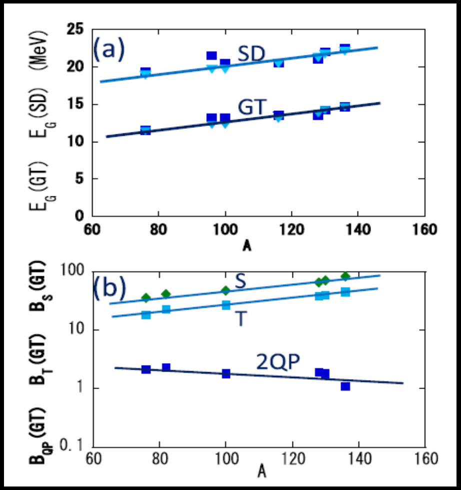
$$M^{0\nu} = \left(\frac{g_A^{eff}}{g_A}\right)^2 \left[M_{GT}^{0\nu} + \left(g_V/g_A^{eff}\right)^2 M_F^{0\nu} + M_T^{0\nu}\right],$$

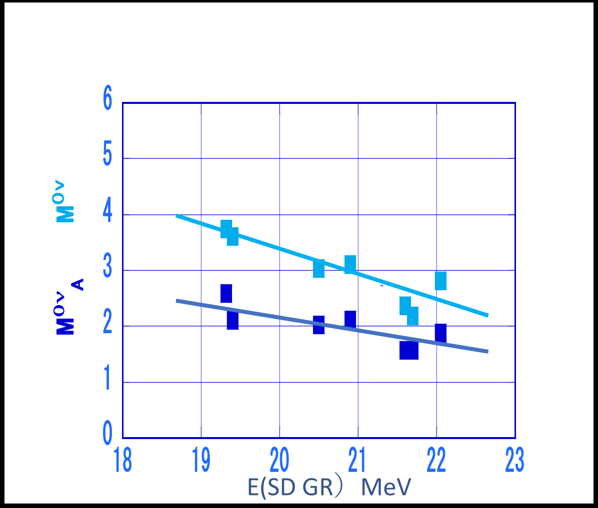
1. Use $g_A^{eff}/g_A = 0.75$:
   Sum of GT strength /sum rule .
   $M^{0\nu}$ and $M^{0\nu}_A$ decreases as A and N-Z, in contrast to F, GT, SD GR energies and GR strengths which increase as A and N-Z.

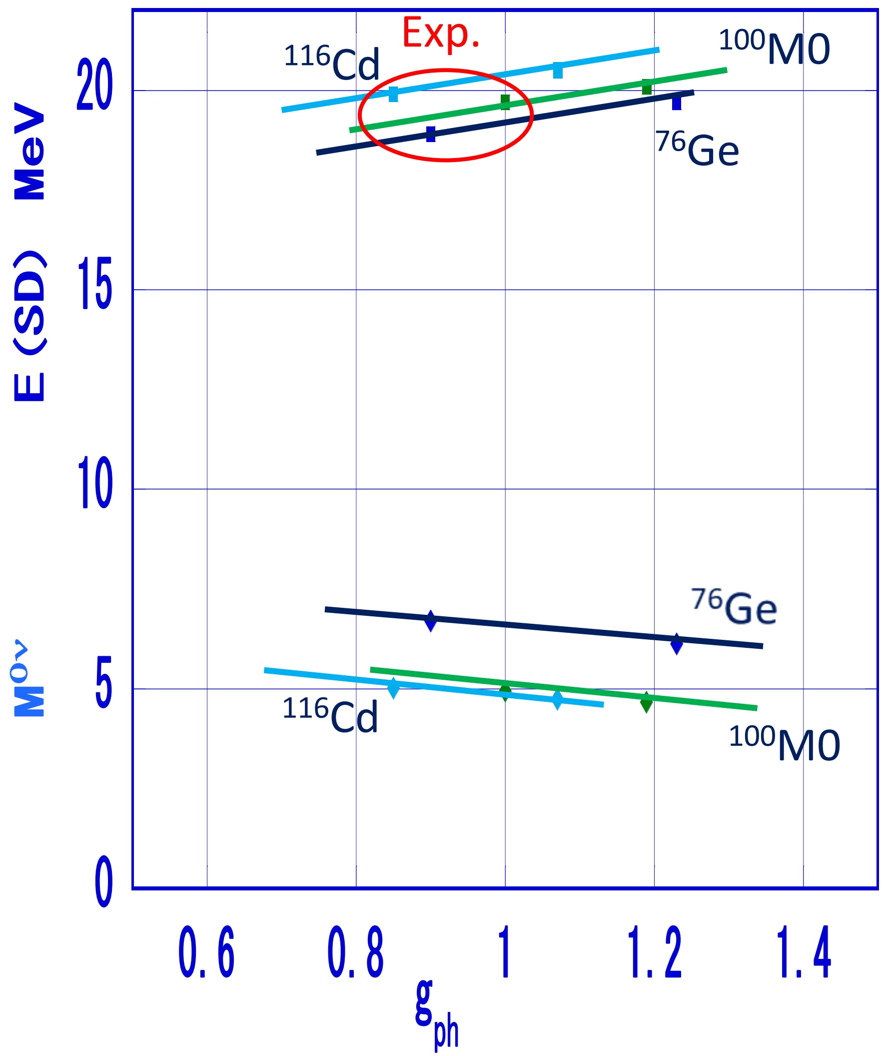


2. The Axial-vector NMEs(GT T) are 60 %, and the F-NME.~40%.

3. The model NMEs smoothly decrease as A and N-Z, reflecting the nuclear core effects. They are less sensitive to the valence nucleon configurations.

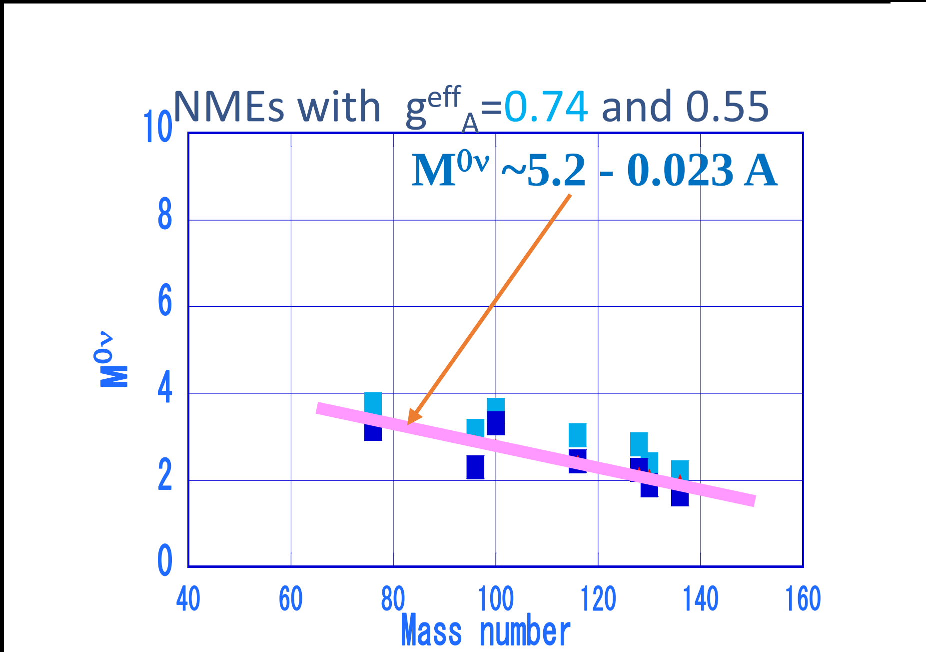


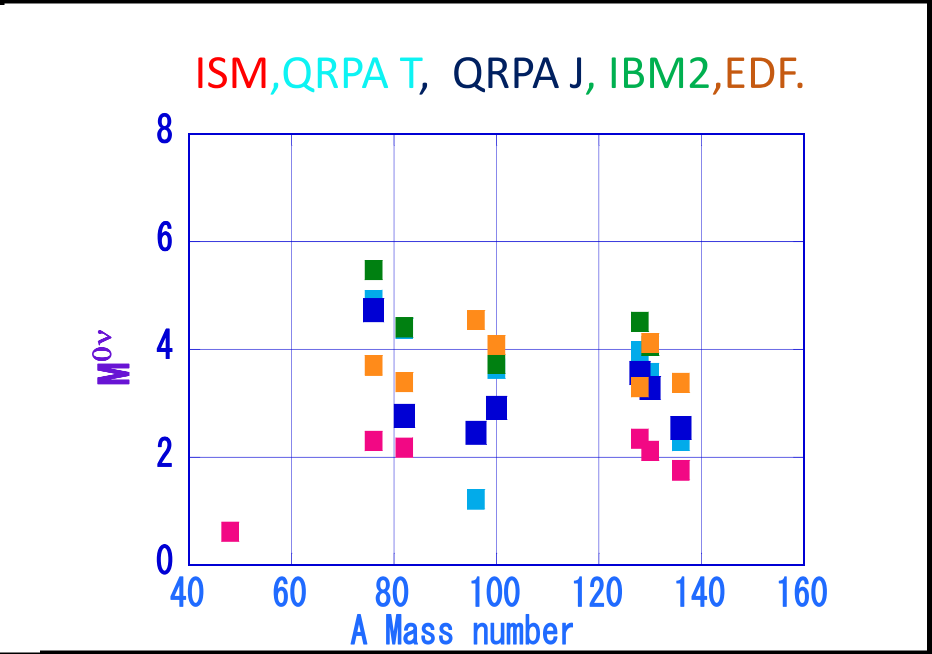




1. L. Jokiniemi, H. Ejiri, D. Frekers, and J. Suhonen, P. R. C 98, 24608 (2018).

$M^{0\nu}$ (pnQRPA) with exp. g_A^{eff}/g_V =0.65 ± 0.1 from GT and SD exps , g_{ph} from SD GR exps and g_{pp} from 2νββ exps.





$M^{0\nu}$~ pnQRPA with =0.65 ± 0.1

$M^{0\nu}$ =3-2 ~ 5.2 - 0.023 A ± 10%

$(0.55/0.74)^2$ =0.55, while NMEs ratio=0.8

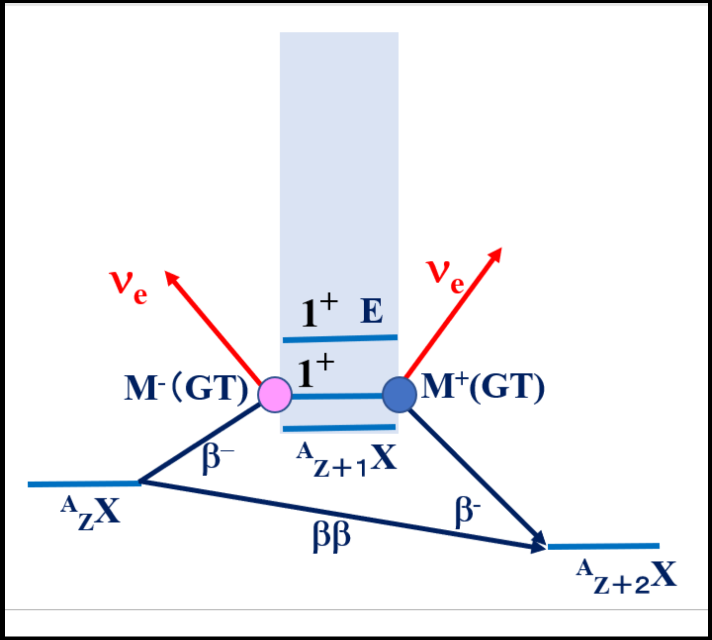
$M^{0\nu}$~ depends on models (g_A^{eff}/g_V=1)

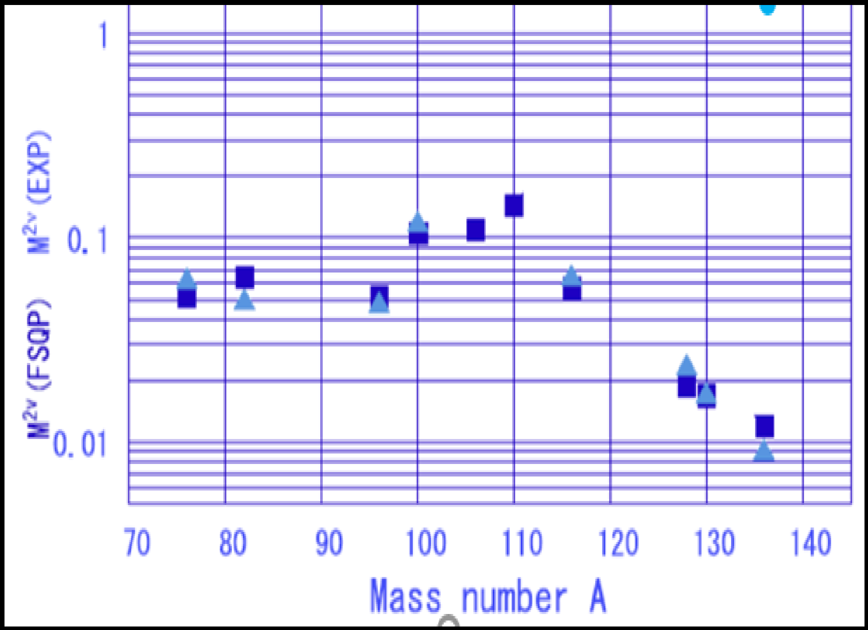
ROPP 2014 Vergados Ejiri Simkovic

Contrast to DBD 2νββ NME, which are sensitive to

QP-GT states in very low (0-2 MeV) region.

$$M^{2\nu} = \left(\frac{g_A^{eff}}{g_A}\right)^2 \sum_i \left[\frac{M_i(\beta^-)M_i(\beta^+)}{\Delta_i}\right],$$





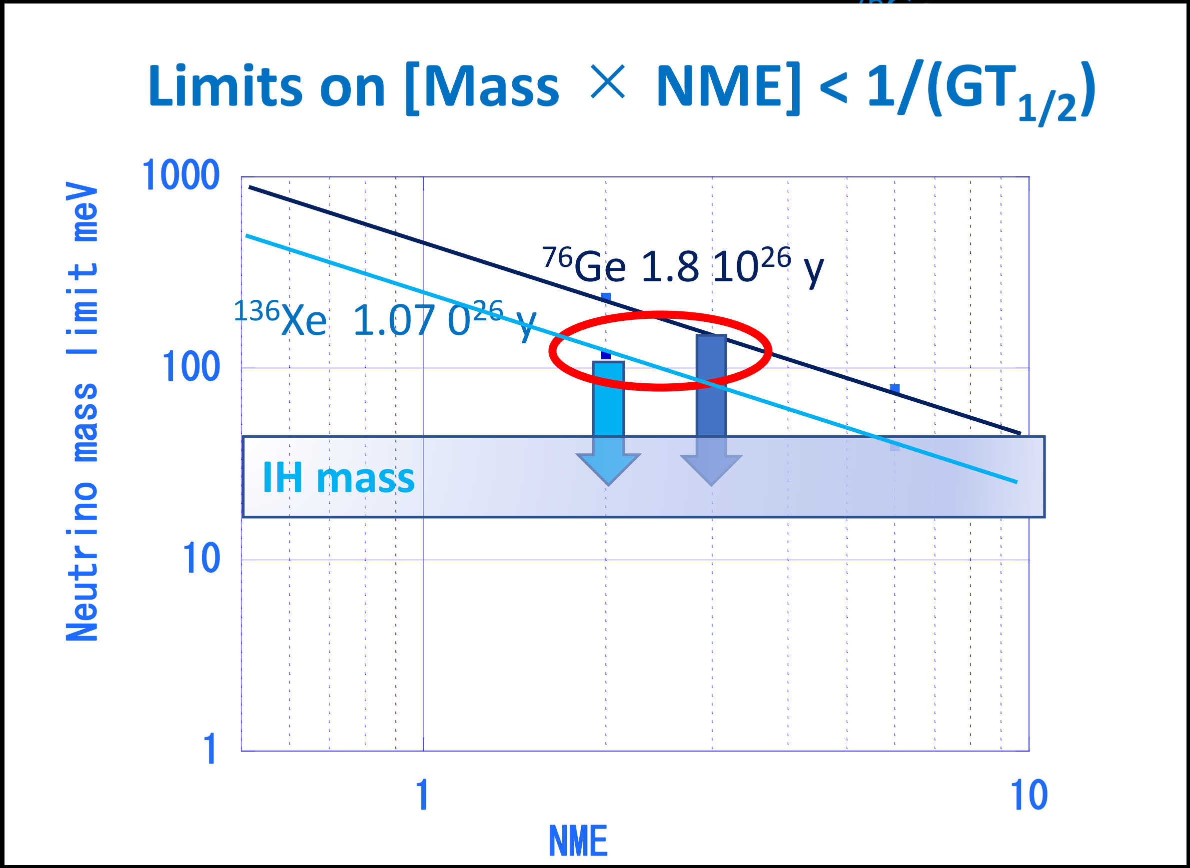
Triangles: Exp. Squares: Ejiri H, J. Phys. 2017 44 115201 , JSPS 2009 78 074201

Depend much on QP-GT states at the nuclear Fermi-surface of individual nucleus, thus NMEs change by an order of magnitude.

5. Impact on DBD exps. Discussions on NMEs

1. DBD -Exps to search for the IH neutrino mass,

NT (isotopes year) wih **NMEs ~ 2-3** is one order more than that with NMEs~4-6.

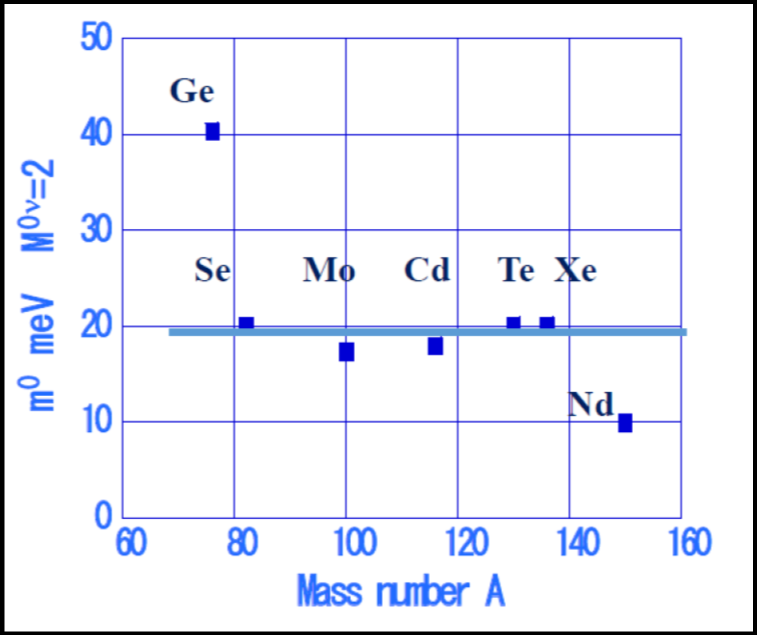


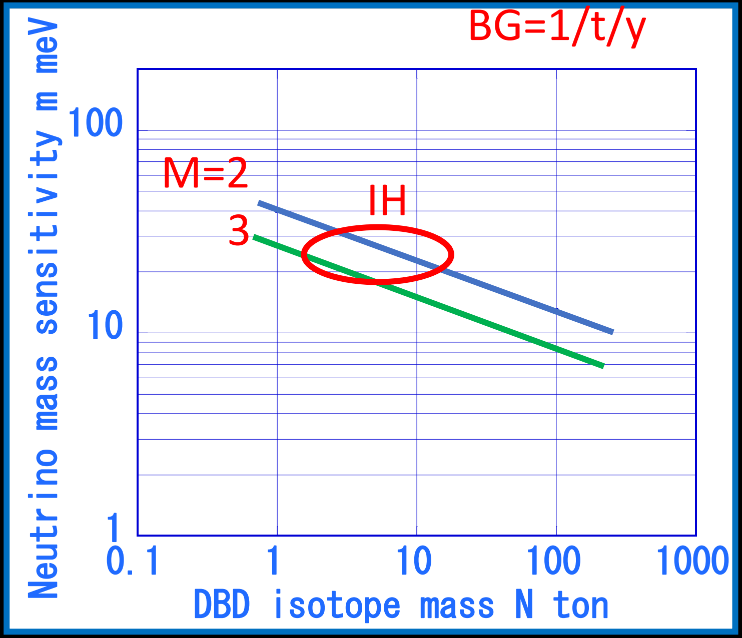
Current limits (GERDA PRL 123 16182 and KamLAND PRL 117 10903) are 2-1 10^{26} for Ge and Xe. To reach IH mass, a factor ~ 10 in ν-mass and >10^4 in NT/B

2. DBD detector sensitivity m_ν :mass to be detected.

NT= Isotope ton year B= BG/ton year

$m_\nu = 2\ m_0\ [B/NT]^{1/4}$ $m_0 \sim 40$ meV $/M^{0\nu}$ with ε=0.5

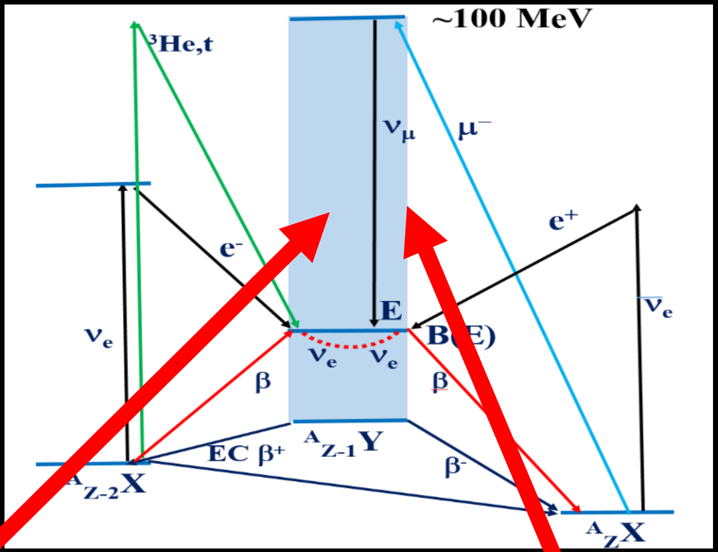


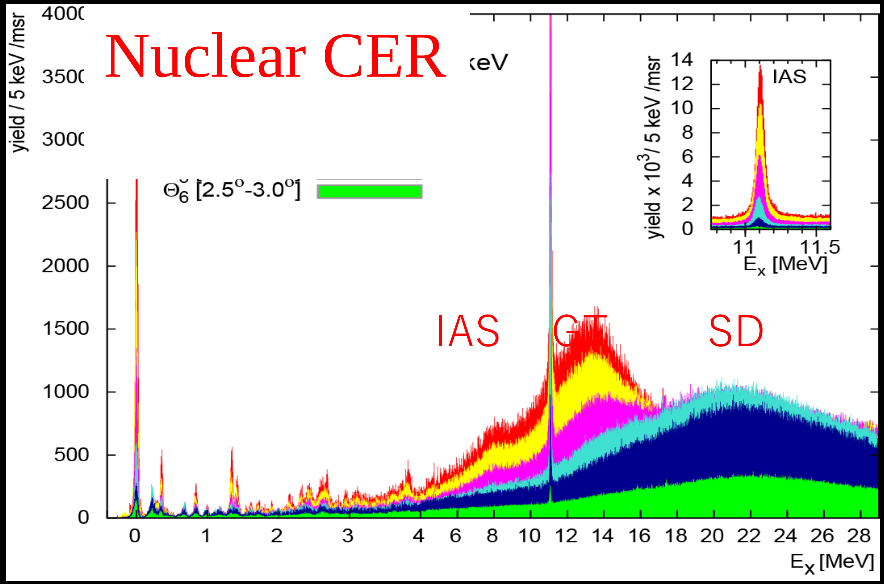


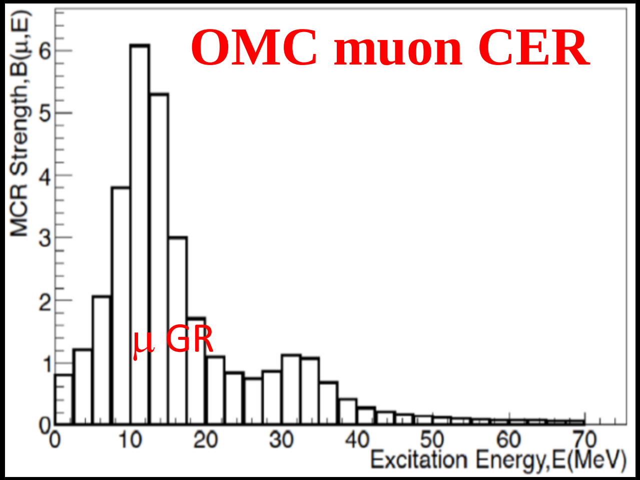
M=2~3 smooth function of A, depends little on individual nuclei, and m_0 is around constant for Se-Xe. Then isotopes to be studied should be selected by experimental requirements such as enriched ton scale isotopes N and low-BG B

3. DBD Models.

DBD model |i> and |f> are such that have realistic $\tau - \tau\sigma$ correlations and/or effective weak coupling to reproduce the quenched and enhanced $\tau - \tau\sigma$ at low-states and giant resonances in intermediate nucleus .





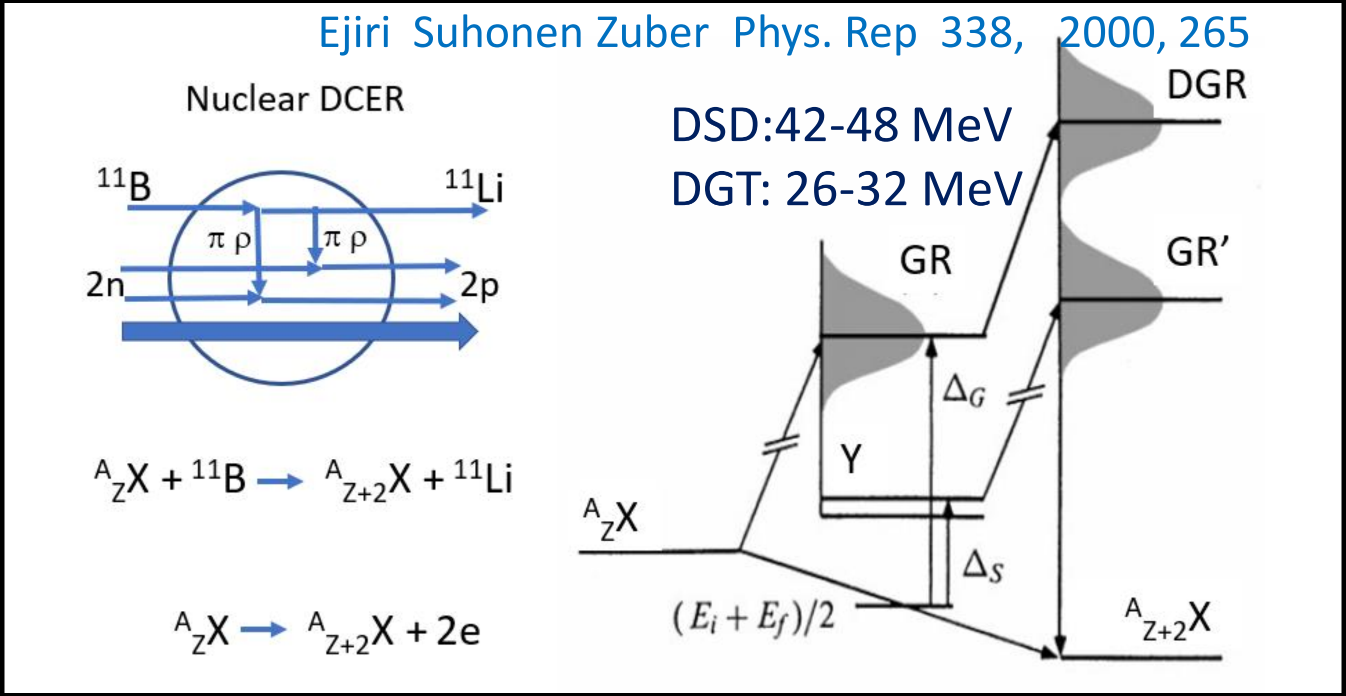


Hashim Ejiri et al. PR C 97 2018

4. Double charge exchange reactions (DCERs)

Mainly double GRs (GT, SD).

Little strengths at low-states of the DBD interest



NEWS: Cappuzzello, Agodi, Menendez, Lenske

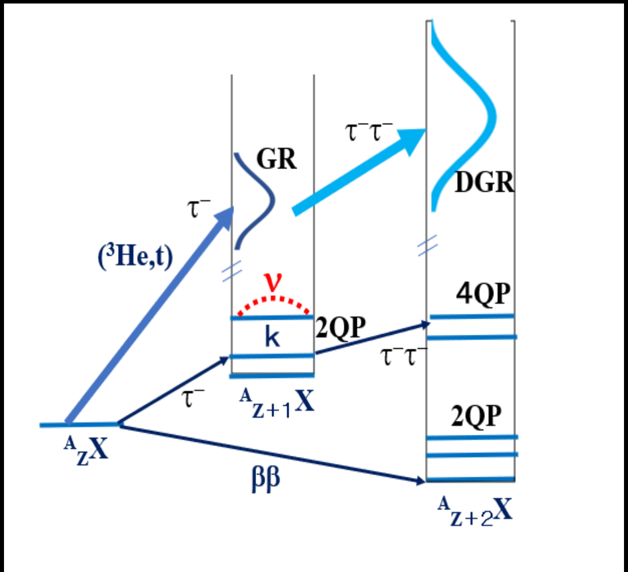
F. Cappuzzello et al Eur. Phys. J. A 51 2015 145. NEUMEN

C. Agodi et al., NEWS , Catania HI CER Project

N. Shimizu, J. Menendez, K. Yako Phys. Rev. Lett. 120 142502 2018

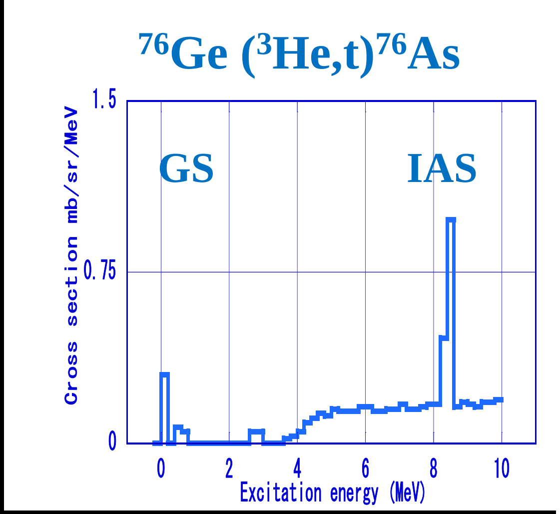
H. Lenske et al, Universe 7() 98 2021.

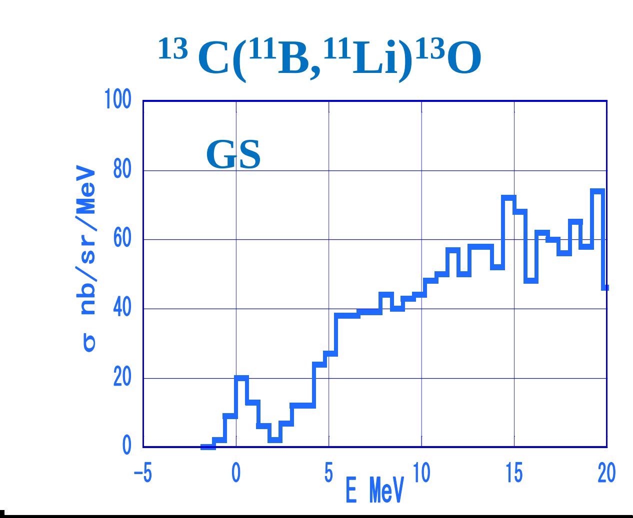
Double Charge Exchange Reaction

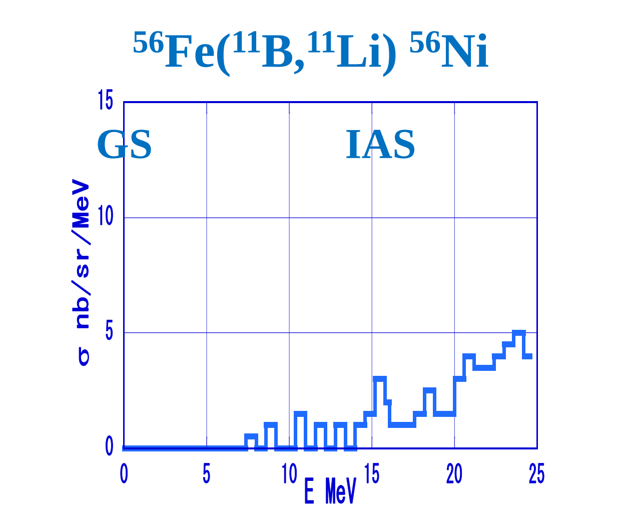


RCNP $^{56}Fe(^{11}B,^{11}Li)\,^{56}Ni$ at E=0.88 GeV.

1. $(V_{\tau\sigma}/V_\tau)^2$ ~ 3.4 enhance τσ GT SD excitation
2. Q value = - 50 MeV, p-transfer 100 MEV/c same as DBD, and L=1 enhances SD







SCER $^{76}Ge\,(^3He,t)^{76}As$ at p=70 MeV/c SD strength 0.1 of QP with $k_{\tau\sigma}$ ~ 0.3 .

$^{13}C(^{11}B,^{11}Li)^{13}O$ excites well the ground state and other low states

DCER $^{56}Fe(^{11}B,^{11}Li)\,^{56}Ni$ excites little low-QP GT-SD states with $(k_{\tau\sigma})^2$~ 0.3

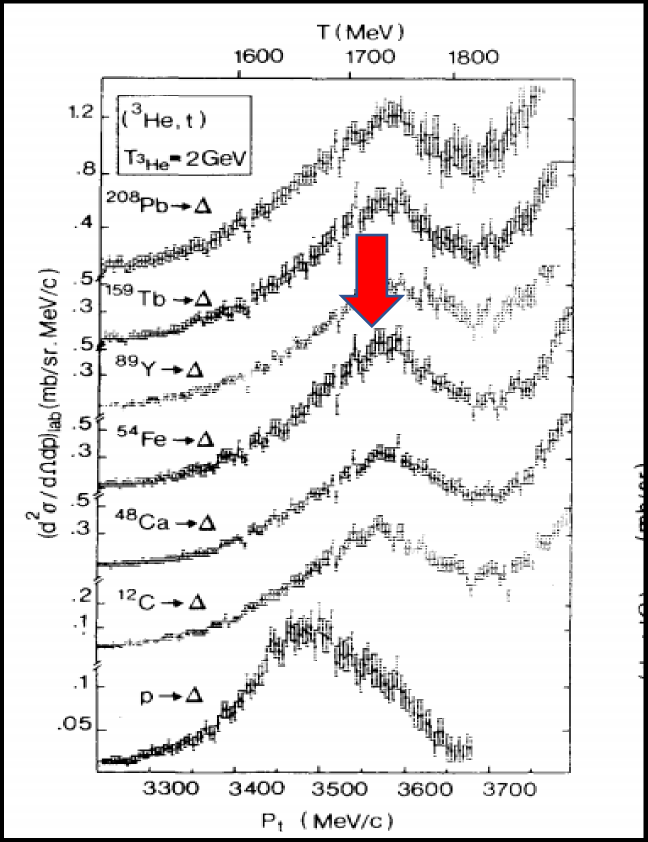
5. Quark στ)flip GR=Delta Δ and quenching of στ -g_A

Bohr Mottelson PL B 100 1981
Rho NP A 231 1974
H. Ejiri PRC 26 '82 2628

$$|I\rangle \sim |QP\rangle - \varepsilon\,|GR\,N\rangle - \delta\,|GR\,\Delta\rangle$$

$$M \sim k^{eff} M_0 \quad k^{eff}(\Delta) \sim 1/[1+\chi_\Delta]$$

$$\chi_\Delta \sim 0.4, \quad k^{eff}(\Delta) \sim 0.7$$



Kirchuk et al., Phys. Scripta 59 1999

$$V = g'_{NN} C\delta^3(\boldsymbol{r}_{12})\sigma_1\cdot\sigma_2\tau_1\cdot\tau_2 + g'_{\Delta N}\frac{f_\pi N\Delta}{f_\pi NN} C\delta^3(\boldsymbol{r}_{12})\boldsymbol{S}_1\cdot\sigma_2\boldsymbol{T}_1\cdot\tau_2$$

(^{3}He,t) with E=2 GeV
150 MeV/c SQ 3^+ S0=4^-
Quark τσ excit to Δ
D. Contard et al. PL B 168

$g_{\Delta N}'/g_{NN}'$=0.6 B(GT) quench 0.5
g^{eff}_A/g_A=0.7 at A=209

Delta Δ quenching effect

$k = g_A^{eff}/g_A = (1+\chi_\Delta)^{-1}$

χ_Δ = $k\, h_\Delta A$ since all nucleons are involved in the Δ excitation.

* Assume $k_\Delta = g_A^{eff}/g_A \sim 0.74$ from GT total strength/sum without Δ.

*A dependence of h_Δ

1. E(GR)-E(ph) =0.013 ħ ω 3(N–Z)

   $h_N = 0.013\, \hbar\omega = \kappa A^{-1/3}$

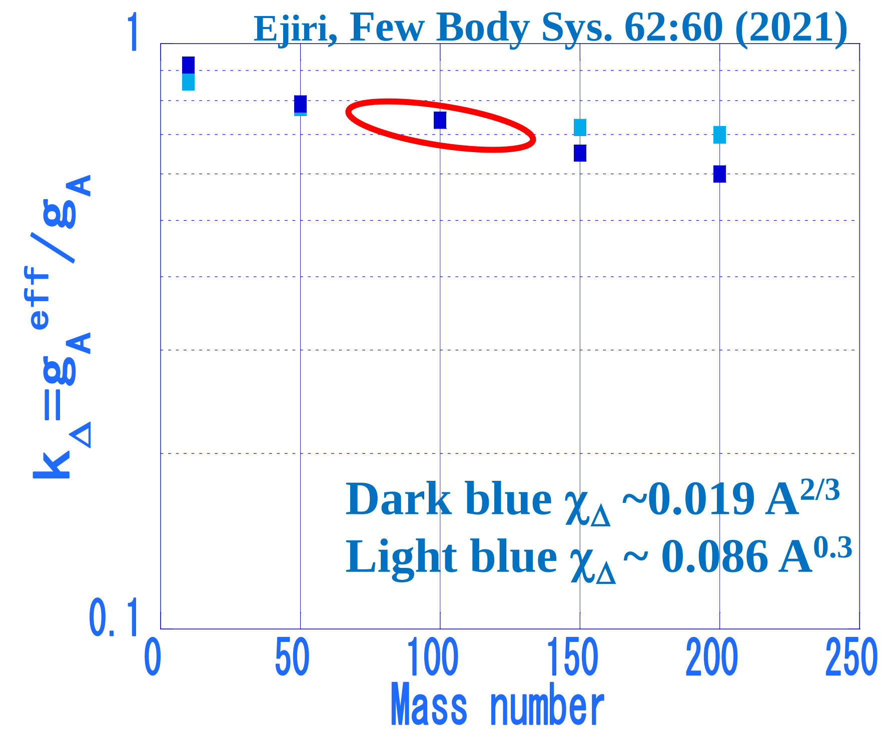
   $\chi_\Delta = 0.019\, A^{2/3}$

2. Quench of B(GT) at A=50-150

   QRPA Homma et al

   $h_N = 2.6\, A^{-0.7}$

   $\chi_\Delta \sim 0.086\, A^{0.3}$



Δ reduces τσ NMEs by 0.65-0.65

The effect of 5-10 % can be seen

even at A~15-10 where accurate NMEs are available from shell models.

6. Concluding remarks

1. SD strengths in intermediate nuclei are mainly in the high E SD GR. The GT and SD GR energies and their strengths increase smoothly with A and N-Z, reflecting $\sigma\tau$ correlations in the nuclear core.

2. The summed GT strength over the GR region is quenched by (g_A^{eff}/g_A)~0.75 with respect to the nucleon-based sum-rule and the pnQRPA sum, GT and SD NMEs for low-lying 2QP states are reduced with respect to the pnQRPA by (g_A^{eff}/g_A)~0.5-0.6
Then, one may use for the pnQRPA (g_A^{eff}/g_A)~0.65 $\pm$ 0.1 to incorporate non-nucleonic and nuclear medium effects.

3. The pnQRPA $M^{0\nu}$ values are much smaller than the QP model NMEs and decreases as A , N-Z, reflecting the negative effect of the $\sigma\tau$ core polarization. The DBD NMEs for A=76-136 are around $M^{0\nu}$~5.2 – 0.023 A = 3-2 for A=7-136.

4. $M^{0\nu}$ values are small and depend little on individual nuclei. Then one may select DBD isotopes from experimental requirements as ton-scale enriched isotopes with large phase space, low-backgrounds and good E resolution.

5. Experimental CERs, OMC, and DCERs and theoretical calculations of the NMEs including Δ are encouraged.

Thanks for your attention.