Nuclear Matrix Elements for Neutrinoless $\beta\beta$ Decays and Spin-Dipole Giant Resonances - Theory Aspects

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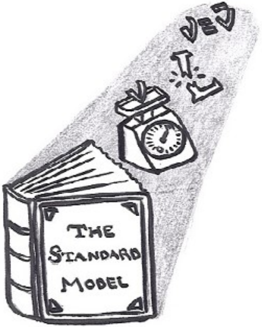
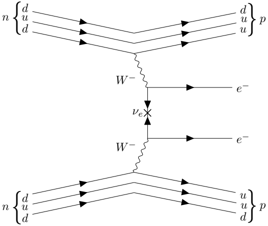
Motivation

- Current knowledge on particles and interactions between them is based on the Standard Model (SM) of particle physics
- According to the SM, neutrinos are extremely weakly interacting, massless fermions
- Yet we know from solar neutrino experiments that neutrinos must have a non-zero mass
  - But what is the absolute mass scale?
  - What else is there beyond the SM?
- *Observing neutrinoless double-beta decay would provide answers!*



Neutrinoless Double-Beta ($0\nu\beta\beta$) Decay



$$^{A}_{Z}\mathrm{X}_{N} \rightarrow ^{A}_{Z+2}\mathrm{Y}_{N-2} + 2e^{-}$$

- Requires that the neutrino is its own antiparticle
- Violates the lepton-number conservation law by two units
- $\frac{1}{t^{0\nu}_{1/2}} \propto \left|\frac{m_{\beta\beta}}{m_e}\right|^2, \quad m_{\beta\beta} = \sum_i^{\mathrm{light}} U_{ei}^2 m_i \rightarrow$ Neutrino masses!
- Has not (yet) been measured!

Half-life of $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_{\rm A}^4 G_{0\nu} |M^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

[1] Agostini *et al.*, arXiv:2202.01787 (2022)

Half-life of $0\nu\beta\beta$ Decay

What would be measured

$$\boxed{\frac{1}{t_{1/2}^{0\nu}} = g_{\mathrm{A}}^4 G_{0\nu} |M^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2}$$

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- Axial-vector coupling ($g_{\mathrm{A}}^{\mathrm{free}} \approx 1.27$)

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- Axial-vector coupling ($g_A^{\rm free} \approx 1.27$)
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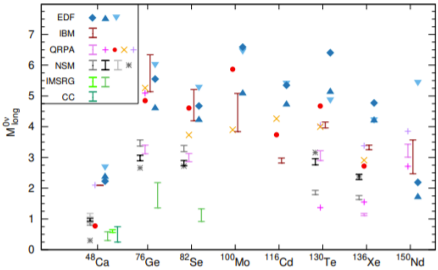


Matrix elements of $0\nu\beta\beta$ decays [1]

[1] Agostini *et al.*, arXiv:2202.01787 (2022)

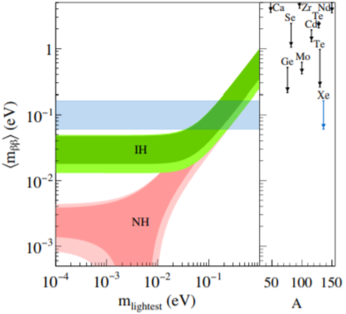
Current Status of $0\nu\beta\beta$-Decay Experiments

$$\frac{1}{t_{1/2}^{0\nu}} = g_{\mathrm{A}}^4 G_{0\nu} |M^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

- Large-scale experiments: CUORE(Italy), GERDA(Italy), CUPID(Italy), MAJORANA(US), EXO-200(US), KamLAND-Zen(Japan), ...
- Currently, most stringent half-life limit $t_{1/2}^{0\nu}(^{76}\mathrm{Ge}) \geq 1.8 \times 10^{26}\ y$ [2]

NH: $m_1 < m_2 < m_3$

IH: $m_3 < m_1 < m_2$



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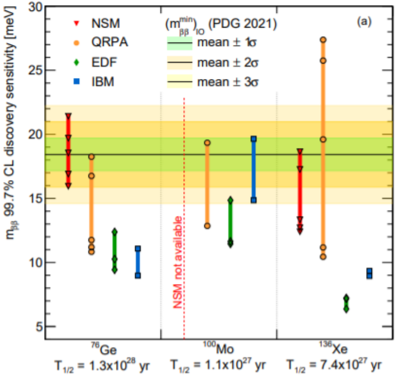
[Engel and Menéndez, Rep. Prog. Phys. **80***,046301 (2017)]*

[2] GERDA collab., *Phys. Rev. Lett.* **125**, 252502 (2020)

Next Generation Experiments

GOAL

Reaching the inverted-hierarchy region of neutrino masses



[Agostini et al., Phys. Rev. C **104**, *L042501 (2021)]*

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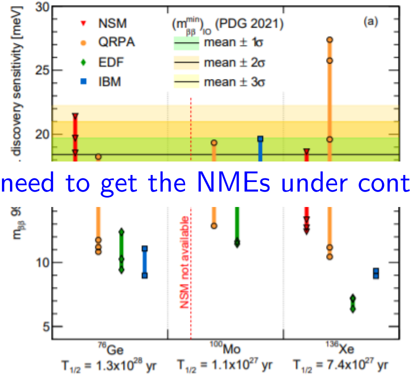
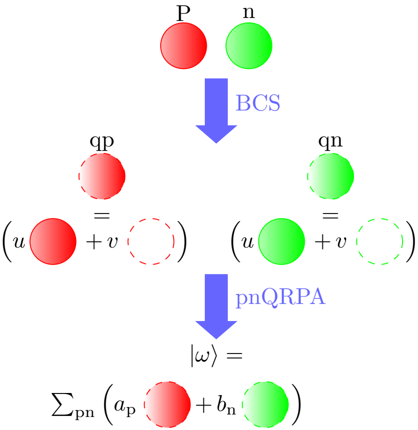


We need to get the NMEs under control!

[Agostini et al., Phys. Rev. C **104**, *L042501 (2021)]*

Proton-Neutron Quasiparticle Random-Phase Approximation (pnQRPA)

- Start from (large) **Woods-Saxon** single-particle bases
  - Adjust, if needed
- **BCS**: Particles → quasiparticles
- **pnQRPA**: proton-neutron two-quasiparticle excitations → excitations in odd-odd nuclei
  - + Large model spaces, high excitation energies
  - – Missing correlations



$0\nu\beta\beta$-Decay NME in pnQRPA

- Total NME:

$$M^{0\nu} = \left(\frac{g_{\rm A}^{\rm eff}}{g_{\rm A}}\right)^2 \left[M_{\rm GT}^{0\nu} + M_{\rm T}^{0\nu}\right] - \left(\frac{g_{\rm V}}{g_{\rm A}}\right)^2 M_{\rm F}^{0\nu}$$

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- $M^{0\nu}_K$, $K = \mathrm{GT}, \mathrm{F}, \mathrm{T}$, computed without closure approximation:

$$M^{0\nu}_K = \sum_{J^\pi, k_1, k_2, \mathcal{J}} \sum_{p,p',n,n'} (-1)^{j_n + j_{p'} + J + \mathcal{J}} \sqrt{2\mathcal{J}+1} \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{Bmatrix}$$
$$\times (0^+_f || [c^\dagger_{p'} \tilde{c}_{n'}]_J || J^\pi_{k_1}) \langle J^\pi_{k_1} | J^\pi_{k_2} \rangle (J^\pi_{k_2} || [c^\dagger_p \tilde{c}_n]_J || 0^+_i)$$
$$\times (pp' : \mathcal{J} || \mathcal{O}_K || nn' : \mathcal{J})$$

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- $M_K^{0\nu}$, $K = {\rm GT, F, T}$, computed without closure approximation:

$$M_K^{0\nu} = \sum_{J^\pi, k_1, k_2, \mathcal{J}} \sum_{p,p',n,n'} (-1)^{j_n + j_{p'} + J + \mathcal{J}} \sqrt{2\mathcal{J}+1} \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{Bmatrix}$$

$$\ldots J^\pi_{k_1}) \langle J^\pi_{k_1} | J^\pi_{k_2} \rangle (J^\pi_{k_2} || [c_p^\dagger \tilde{c}_n]_J || 0_i^+)$$

$$\ldots n' : \mathcal{J})$$

Sum over intermediate states

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- $M_K^{0\nu}$, $K = \mathrm{GT}, \mathrm{F}, \mathrm{T}$, computed without closure approximation:

$$M_K^{0\nu} = \sum_{J^\pi, k_1, k_2, \mathcal{J}} \sum_{p,p',n,n'} \cdots \mathcal{J}+1 \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{Bmatrix}$$
$$\times (0_f^+ || [c_{p'}^\dagger \tilde{c}_{n'}]_J || J_{k_1}^\pi) \langle J_{k_1}^\pi | J_{k_2}^\pi \rangle (J_{k_2}^\pi || [c_p^\dagger \tilde{c}_n]_J || 0_i^+)$$
$$\times (pp' : \mathcal{J} || \mathcal{O}_K || nn' : \mathcal{J})$$

One-body transition densities

$0\nu\beta\beta$-Decay NME in pnQRPA

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- $M_K^{0\nu}$, $K = \mathrm{GT}, \mathrm{F}, \mathrm{T}$, computed without closure approximation:

Overlap

$$M_K^{0\nu} = \sum_{J^\pi, k_1, k_2, \mathcal{J}} \sum_{p,p',n,n'} (-1)^{j_n + j_{p'} + J + \mathcal{J}} \sqrt{2\mathcal{J}+1} \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{Bmatrix}$$
$$\times \left(0_f^+ || [c_{p'}^\dagger \tilde{c}_{n'}]_J || J_{k_1}^\pi\right) \langle J_{k_1}^\pi | J_{k_2}^\pi \rangle \left(J_{k_2}^\pi || [c_p^\dagger \tilde{c}_n]_J || 0_i^+\right)$$
$$\times \left(pp' : \mathcal{J} || \mathcal{O}_K || nn' : \mathcal{J}\right)$$

$0\nu\beta\beta$-Decay NME in pnQRPA

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- $M_K^{0\nu}$, $K = \rm GT, F, T$, computed without closure approximation:

Two-body matrix element

$$M_K^{0\nu} \ldots 1)^{j_n+j_{p'}+J+\mathcal{J}} \sqrt{2\mathcal{J}+1} \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{Bmatrix}$$

$$\times \left(0_f^+ \middle|\middle| [c_{p'}^\dagger \tilde{c}_{n'}]_J \middle|\middle| J_{k_1}^\pi\right) \left\langle J_{k_1}^\pi \middle| J_{k_2}^\pi \right\rangle \left(J_{k_2}^\pi \middle|\middle| [c_p^\dagger \tilde{c}_n]_J \middle|\middle| 0_i^+\right)$$

$$\times (pp' : \mathcal{J} || \mathcal{O}_K || nn' : \mathcal{J})$$

- Operators:

$$\mathcal{O}_{GT} = h_{\rm GT}(r, E_k)[f_{\rm SRC}(r)]^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

$$\mathcal{O}_F = h_{\rm F}(r, E_k)[f_{\rm SRC}(r)]^2$$

$$\mathcal{O}_T = h_{\rm T}(r, E_k)[f_{\rm SRC}(r)]^2 S_{12}^{\rm T}$$

Spherical pnQRPA and Adjustable Parameters

- Excitations $|J_k^\pi M\rangle = \sum_{pn}(X_{pn}^{J_k^\pi}[a_p^\dagger a_n^\dagger]_{JM} - Y_{pn}^{J_k^\pi}[a_p^\dagger a_n^\dagger]_{JM}^\dagger)|\mathrm{QRPA}\rangle$

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- ...obtained from pnQRPA equation:

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X^\omega \\ Y^\omega \end{pmatrix} = E_\omega \begin{pmatrix} X^\omega \\ Y^\omega \end{pmatrix} ,$$

$$\begin{aligned} A_{pn,p'n'}(J) =& (E_p + E_n)\delta_{pp'}\delta_{nn'} \\ &+ (u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'}) \times g_{\mathrm{pp}}\langle pn; J|V|p'n'; J\rangle \\ &+ (u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'}) \times g_{\mathrm{ph}}\langle pn^{-1}; J|V'|p'n'^{-1}; J\rangle , \\ B_{pn,p'n'}(J) =& -(u_p u_n v_{p'} v_{n'} + v_p v_n u_{p'} u_{n'}) \times g_{\mathrm{pp}}\langle pn; J|V|p'n'; J\rangle \\ &+ (u_p v_n v_{p'} u_{n'} + v_p u_n u_{p'} v_{n'}) \times g_{\mathrm{ph}}\langle pn^{-1}; J|V'|p'n'^{-1}; J\rangle \end{aligned}$$

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solved from BCS equations

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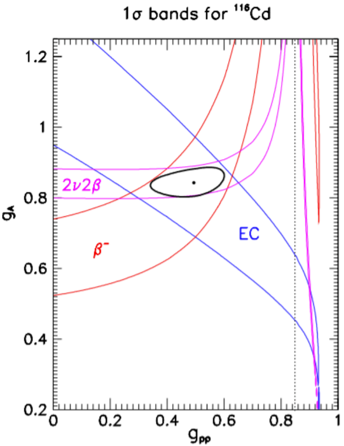
adjustable parameters

$g_{\rm pp}$-Problem of pnQRPA

$$[t_{1/2}^{2\nu}]^{-1} = g_{\rm A}^4 G_{2\nu} \, |M^{2\nu}|^2$$

$$\log ft_{{\rm EC}/\beta} = \log_{10}(3\kappa/(g_{\rm A}^2 |M_{{\rm EC}/\beta}|^2))$$

- It is hard to simultaneously reproduce experimental $2\nu\beta\beta$, EC and β^- data
  - Small values of $g_{\rm pp}$ AND quenched $g_{\rm A} \ll 1.27$ needed
- Usually, $g_{\rm pp}$ adjusted to observed $2\nu\beta\beta$ decays with $g_{\rm A} = 1.27$ or $g_{\rm A}^{\rm eff} < g_{\rm A}$



[Faessler et al., J. Phys. G: Nucl. Part. Phys. **35**, *075104 (2008)]*

Partial Isospin Restoration Scheme

$$g_{\rm pp}\langle pn;J|V|p'n';J\rangle \rightarrow \; g_{\rm pp}^{T=0}\langle pn;J,T=0|V|p'n';J,T=0\rangle + g_{\rm pp}^{T=1}\langle pn;J,T=1|V|p'n';J,T=1\rangle$$

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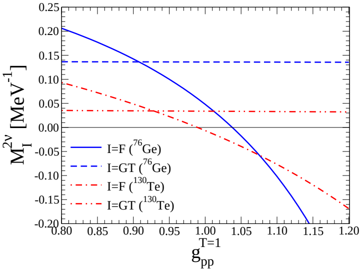

[Šimkovic, Rodin, Faessler, Vogel, Phys. Rev. C **87***, 045501, (2013)]*

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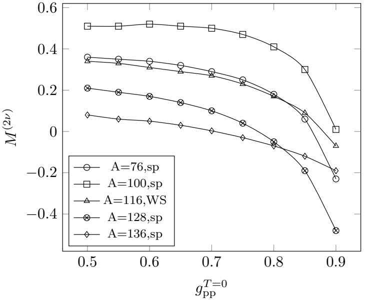

[LJ, Ejiri, Frekers, Suhonen, Phys. Rev. C **98***, 024608, (2018)]*

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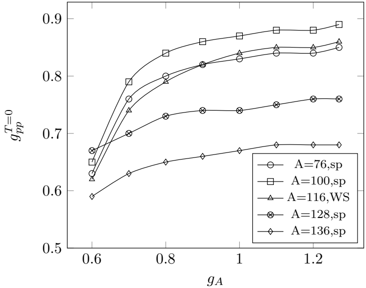
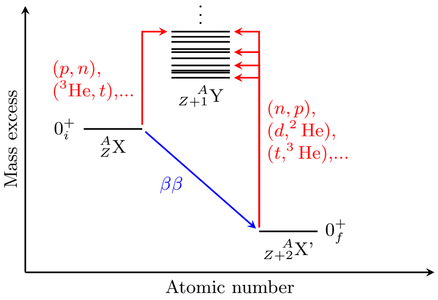
- Note: $g_{\rm pp}^{T=0}$ is $g_{\rm A}$-dependent



[LJ, Ejiri, Frekers, Suhonen, Phys. Rev. C **98***, 024608, (2018)]*

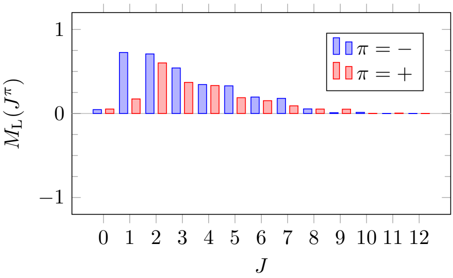
Charge-Exchange Reactions as Probes of $0\nu\beta\beta$ Decay

- Virtual states of $\beta\beta$ decay (weak interaction) can be probed by charge-exchange reactions (strong interaction)
- Gamow-Teller (GT) type transitions to 1^+ states probed by partial-wave $L = 0$ reactions
- Higher J^π states can be probed by higher-L reactions



Adjusting g_{ph}

- g_{ph} moves giant resonances
- Normally, g_{ph} adjusted in the 1^+ channel to Gamow-Teller giant resonance (GTGR)
  - + Lot of experimental data and empirical models available
  - − 2^- states normally more important in $0\nu\beta\beta$ decays
- How about adjusting to isovector spin-dipole (IVSD), $L = 1$, $J^\pi = 2^-$ data, instead?



Multipole decomposition of $M^{0\nu}$ for ^{76}Ge

Adjusting $g_{\rm ph}$ to IVSD $J^\pi = 2^-$ Data

IVSD $J^\pi = 2^-$ strength function:

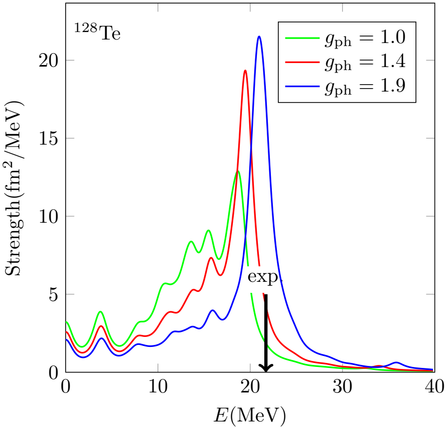
$$S^-_{1,2}(\mathrm{f}) = |(2^-_f||ir[\mathbf{Y}_1\boldsymbol{\sigma}]_2 t^-||0^+_i)|^2$$

- $g_{\rm ph}$ adjusted to empirical location of $J^\pi = 2^-$ IVSD giant resonance

$$E(\mathrm{SD}) \approx 16.5 + 0.4\,T_Z\ \mathrm{MeV},$$

where $T_z = (N - Z)/2$

- Total strength insensitive to $g_{\rm ph}$



Parameters for $0\nu\beta\beta$-Decay NMEs

- Use different g_A quenchings:
  - $g_A^{eff}/g_A = 0.74$ (typical shell-model quenching of GT strength [3])
  - $g_A^{eff}/g_A = 0.55$ (average QRPA quenching of GT and SD strengths [4])
- Adjust $g_{ph}(1^+)$ to GT and $g_{ph}(2^-)$ to SD giant resonances
  - Normally, g_{ph}(GTGR) used for all multipoles
- Adjust g_{pp} according to partial isospin-restoration scheme with the chosen g_A^{eff}

[3] Balasi, Langanke, Martínez-Pinedo, *Prog. Part. Nucl. Phys.* **85**, 33, (2015)
[4] Ejiri, *Front. Phys.* **7**, 30 (2019)

Results: Particle-Hole Parameters

- g_{ph} typically increases by $\sim 20-25\%$

Nucleus	$g_{\mathrm{ph}}(1^+)$	$g_{\mathrm{ph}}(2^-)$
^{76}Ge	1.03 ± 0.13	1.2 ± 0.3
^{96}Zr	0.84 ± 0.09	0.8 ± 0.2
^{100}Mo	1.19 ± 0.08	1.0 ± 0.2
^{116}Cd	0.85 ± 0.13	1.07 ± 0.09
^{128}Te	1.40 ± 0.09	1.9 ± 0.2
^{130}Te	1.36 ± 0.09	1.9 ± 0.2
^{136}Ge	1.18 ± 0.08	0.9 ± 0.2

[LJ, Ejiri, Frekers, Suhonen, Phys. Rev. C **98**, *024608 (2018)]*

Results: Particle-Hole Parameters

- $g_{\rm ph}$ typically increases by $\sim 20-25\%$
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[LJ, Ejiri, Frekers, Suhonen, Phys. Rev. C **98***, 024608 (2018)]*

Results: Particle-Hole Parameters

- g_{ph} typically increases by $\sim 20-25\%$
  - Decreases $M^{0\nu}$ by $\approx 5-15\%$
- With a couple of exceptions

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[LJ, Ejiri, Frekers, Suhonen, Phys. Rev. C **98**, *024608 (2018)]*

Results: $0\nu\beta\beta$-Decay NMEs

$$M^{0\nu} = \left(\frac{g_{\rm A}^{\rm eff}}{g_{\rm A}}\right)^2 \left[M_{\rm GT}^{0\nu} + M_{\rm T}^{0\nu}\right] - \left(\frac{g_{\rm V}}{g_{\rm A}}\right)^2 M_{\rm F}^{0\nu}$$

- Ratio of NMEs calculated with different $g_{\rm A}^{\rm eff}$ values larger than $(0.55/0.74)^2 \approx 0.62$
  - This is due to $g_{\rm pp}^{T=0}$ fixed to $t_{1/2}^{2\nu}$ with $g_{\rm A}^{\rm eff}$
  - And the Fermi NME
- With an average value $g_{\rm A}^{\rm eff}/g_{\rm A} \approx 0.65 \pm 0.1$ we get

$$M^{0\nu} \approx 5.2 - 0.023A \,,$$
$$M^{0\nu} \approx 4.2 - 0.08(N - Z)$$

Nucleus	$M^{0\nu}$	
	$g_{\rm A}^{\rm eff} = 0.74$	$g_{\rm A}^{\rm eff} = 0.55$
^{76}Ge	3.75	3.18
^{96}Zr	3.14	2.31
^{100}Mo	3.62	3.32
^{116}Cd	3.03	2.44
^{128}Te	2.82	2.24
^{130}Te	2.37	1.89
^{136}Ge	2.19	1.68

[Ejiri, LJ, Suhonen, Phys. Rev. C **105**, *L022501 (2022)]*

$$M^{0\nu} = \left(\frac{g_A^{\rm eff}}{g_A}\right)^2 \left[M_{\rm GT}^{0\nu} + M_{\rm T}^{0\nu}\right] - \left(\frac{g_V}{g_A}\right)^2 M_{\rm F}^{0\nu}$$

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Ratio ≈ 0.7

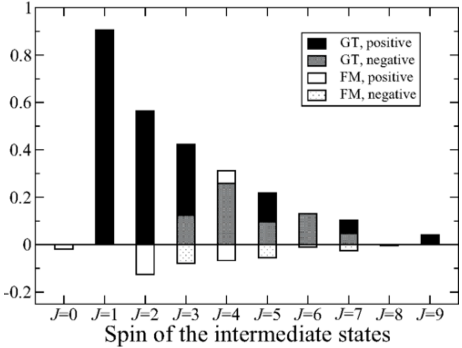
[Ejiri, LJ, Suhonen, *Phys. Rev. C* **105**, L022501 (2022)]

Summary

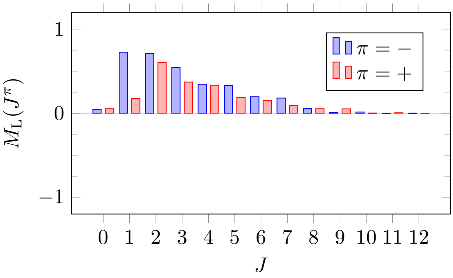
- Observing $0\nu\beta\beta$ decay would shed light on neutrino properties and physics beyond the standard model
- Reliable nuclear matrix elements crucial for $0\nu\beta\beta$ studies
- Data on spin-dipole charge-exchange reactions can help constrain the NMEs in the pnQRPA framework
- A simple relation between $M^{0\nu}$ and A has been found

Thank you!

J^{π} Decomposition of $M^{0\nu}$ of ^{76}Ge



NSM [5]



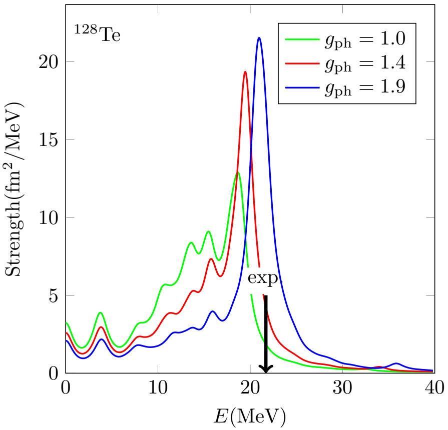
pnQRPA

[5]Sen'kov, Horoi, *Phys. Rev. C* **90**, 051301(R) (2014)

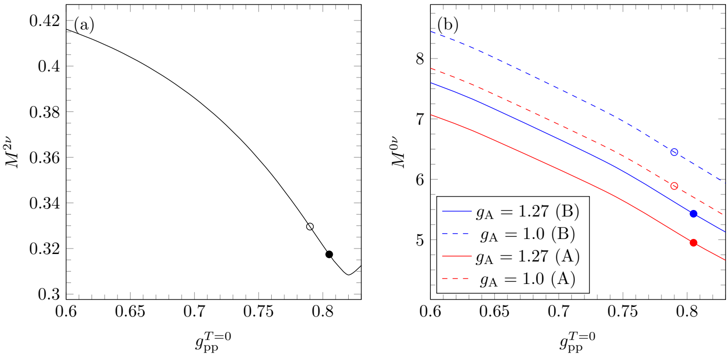
Total IVSD $J^{\pi} = 2^{-}$ Strengths

Total strengths with $g_{\mathrm{A}}^{\mathrm{eff}} = 1.0$:

- $g_{\mathrm{ph}} = 1.0$: $S^{-}_{\mathrm{tot.}} = 129.28\ \mathrm{fm}^2$
- $g_{\mathrm{ph}} = 1.4$: $S^{-}_{\mathrm{tot.}} = 128.32\ \mathrm{fm}^2$
- $g_{\mathrm{ph}} = 1.9$: $S^{-}_{\mathrm{tot.}} = 127.41\ \mathrm{fm}^2$



g_{pp}-Dependence of the NMEs



[Nesterenko et al., EPJA ***98****, 44 (2022)]*