Exploring the potential γγ-decay to constrain 0νββ-decay NMEs

NEWS Colloquium

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Donostia International Physics Center

Brief historic introduction to DBD

1935 — Maria Goeppert Mayer idea of $2\nu\beta\beta$

1937 — Etore Majorana new representation of neutral fermions $\chi = \bar{\chi}$; G.Racah applied massive neutrinos

1939 — Wendell H. Furry idea of $0\nu\beta\beta$

1950 — First **indirect** observation of 2ν-decay (geochemical) detection ^{130}Te Inghram and Reynolds $T^{2\nu}_{1/2} = 1.4 \times 10^{21} y$

1957 — Prediction of ν-osc. B. Pontecorvo

1970 — First counter experiment (scintillator+stream chamber) ^{48}Ca Bardin, Gollon, Ullman,Wu $T^{0\nu(2\nu)}_{1/2} > 2(3.6) \times 10^{21} y$

1982 — Schechter-Valle Theorem $0\nu\beta\beta \implies \nu$-Majorana mass

1985 — Theoretical analysis 2β Doi, Kotani, Takasugi

1987 — First **direct** observation of 2ν-decay **TPC** (S.R.Elliot et al.) ^{82}Se $T^{2\nu}_{1/2} = 1.1 \times 10^{20} y$

1989 — First counter high sensitivity experiment (Caldwell et al.) ^{76}Ge $T^{0\nu}_{1/2} > 1.2 \times 10^{24} y$

90's — ELEGANT-V ^{100}Mo,^{116}Cd (H. Ejiri et al.); NEMO-2 ^{100}Mo,^{116}Cd,^{82}Se,^{96}Zr R. (Arnold et al.); IGEX ^{76}Ge (C.E. Aalseth et al.); Heidelberg-Moscow ^{76}Ge (H.V.Klapdor-Kleingrothaus et al.)

1998 — ν**-oscillation discovery** (SK & SNO) revived interest $0\nu2\beta$ searches

21st century-today

$T^{2\nu}_{1/2} \sim 10^{18} - 10^{21} y$

$T^{0\nu}_{1/2} > 10^{26} y$

NEMO-3 (2003-2011)
CUORICINO (2003-2008)
CUORE (0:2013-2015;2017-…)
SuperNEMO (dem. Under construction)
GERDA(2011-2020) } LEGEND (2022-…)
MAJORANA dem(2015-2021) } LEGEND (2022-…)
KamLand-ZEN(400:2011-2015;800:2019-…)
EXO(200:2011-2014;n:2022-…)
NEXT(NEW:2017-2021;100:under construction)

… AND MORE!

INCOMPLETE!

For more detailed discussion see A. S. Barabash, Phys. Atom. Nuclei **74**, 603–613 (2011)

Motivation and the role of Particle Physics

The discovery that neutrino have mass through neutrino oscillations raised a fundamental question

ORIGIN of ν-mass

Intrinsically related with the nature (Dirac or Majorana) of neutrinos

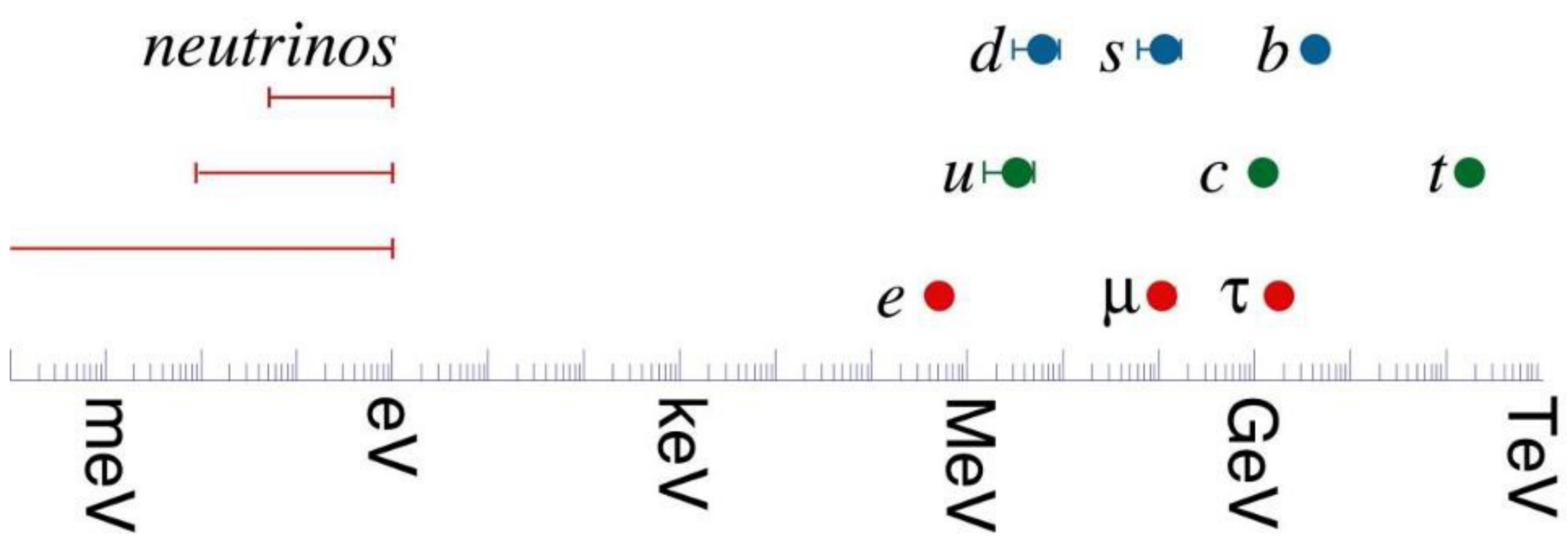
ν-masses at least 10^5 smaller other fermion masses → different mass-generation mechanism?

"Majorana mass" term can produce light neutrinos without fine-tuned coupling to the Higgs

Majorana nature predicted by models that explain small mass by **Lepton Number symmetry Violation** (LNV)

ν-oscillation conserve L, the most feasible process to observe LNV induced by light Majorana ν is $0\nu\beta\beta$

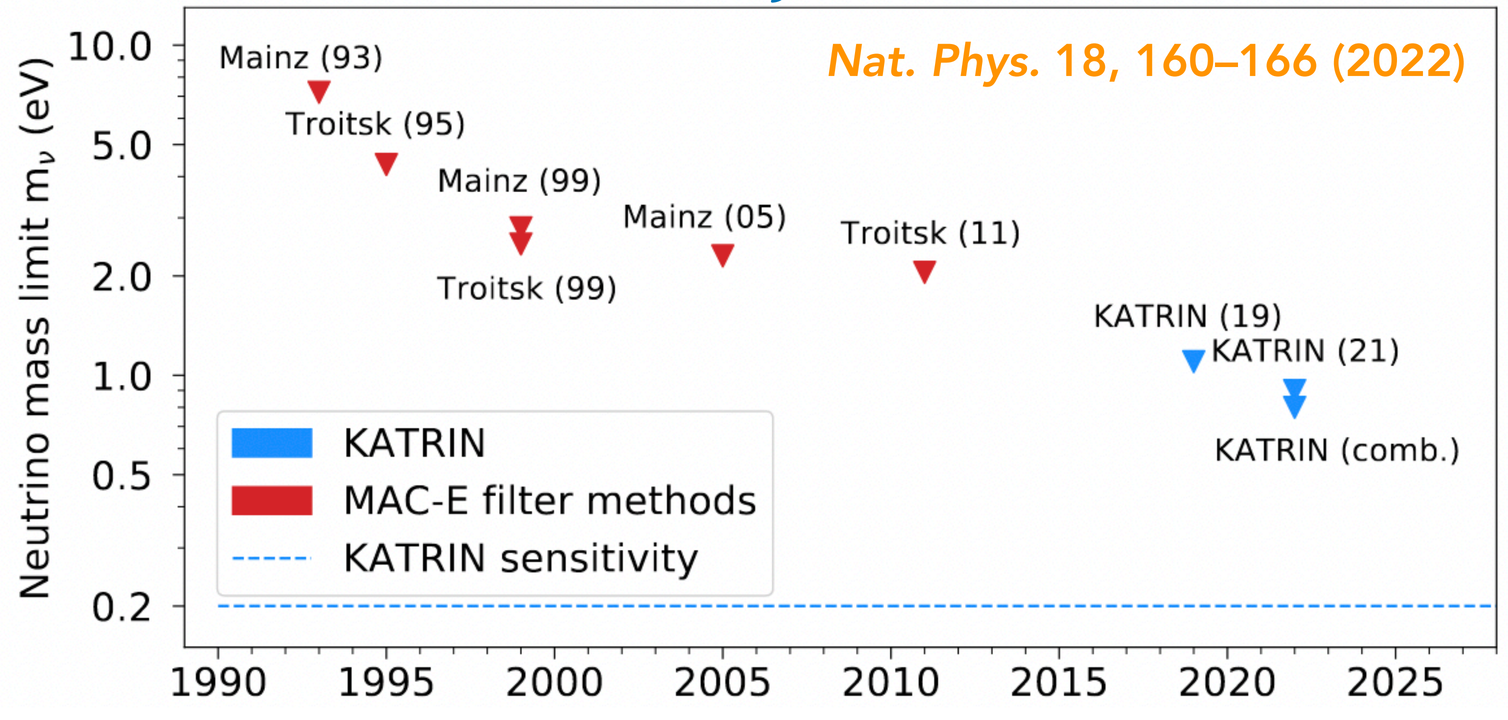
Fermion masses



Why they are so small?

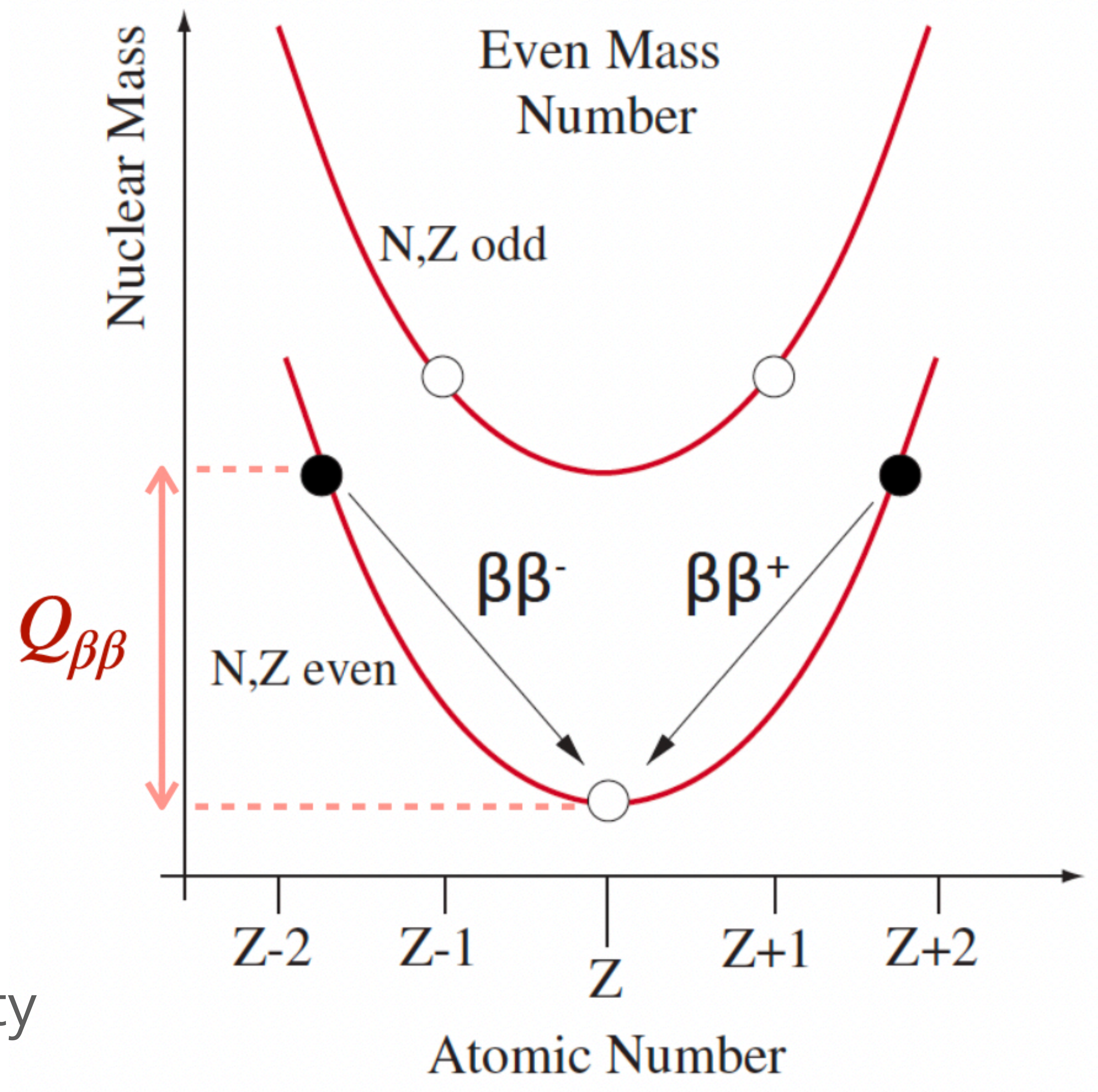
… also absolute ν-mass remains a pressing open question, oscillation experiments can probe Δm^2_{ij}

KATRIN $m_\nu < 0.8\ \mathrm{eV/c^2}$ at 90 % CL

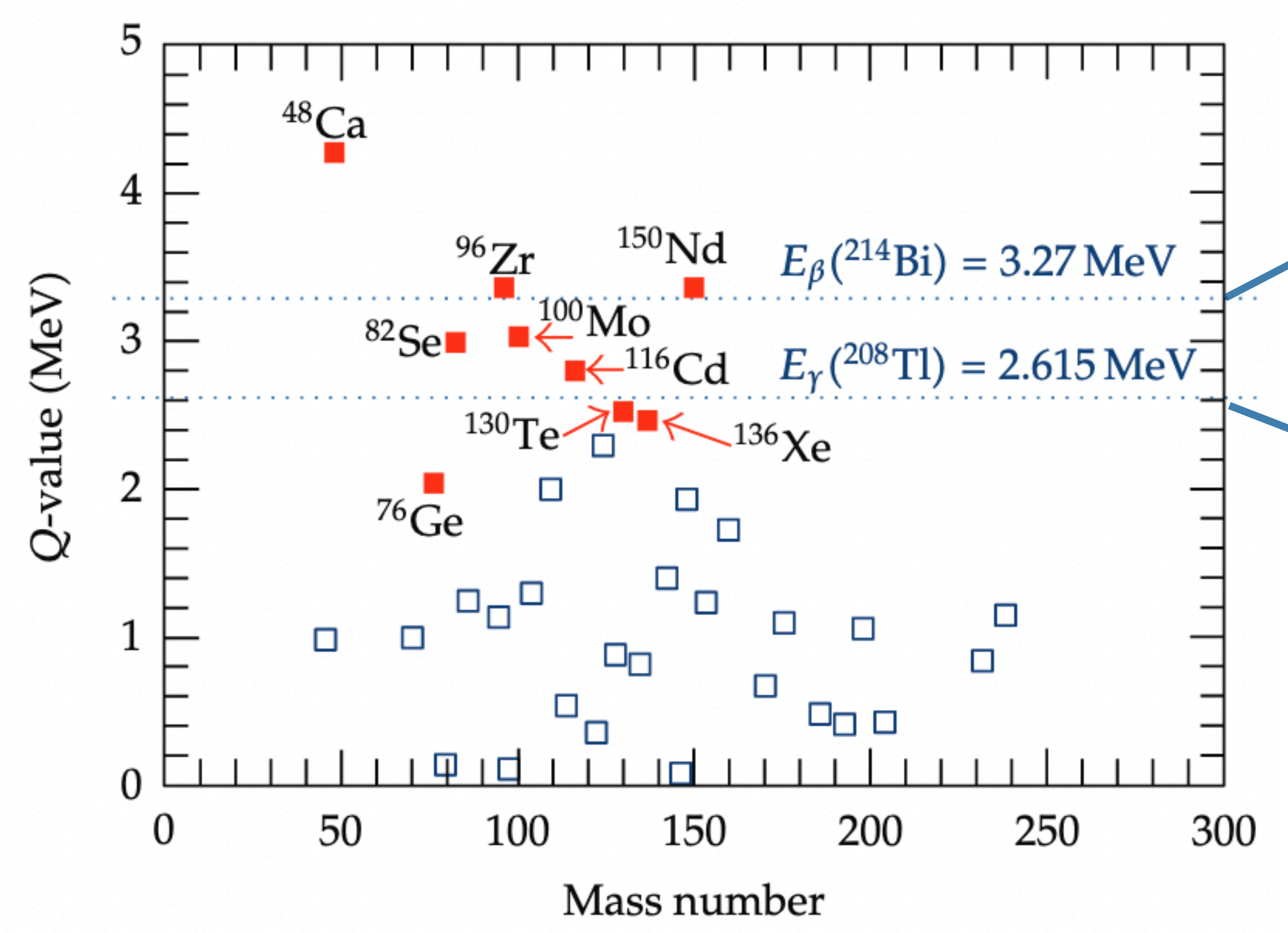


Motivation and the role of Nuclear Physics

Candidate isotopes are **even-even** nuclei which due to nuclear pairing force are lighter than the odd-odd (A,Z-1) nucleus (single beta decay kinematically forbidden)



Possible for 35 nuclei, but **only 9 of interest in DBD searches** (Q-value, isotopic abundance and enrichment ease, compatibility with a good detection technique)

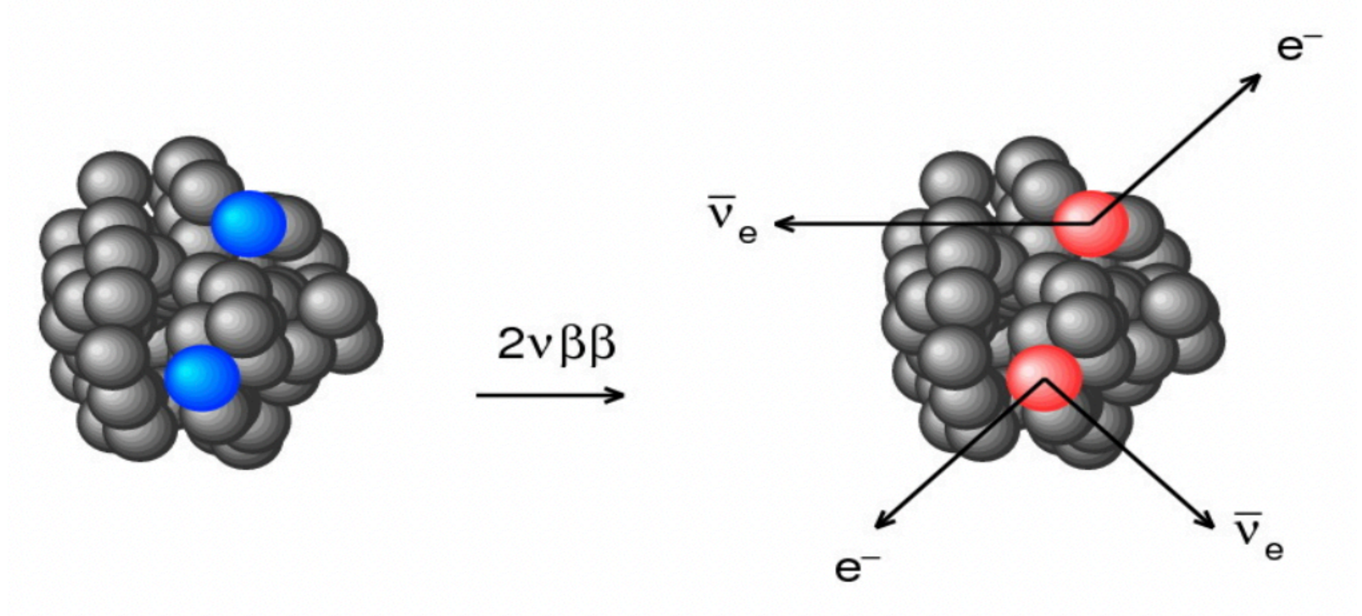


End-point of ^{222}Rn induced radioactivity

End-point of natural γ-radioactivity

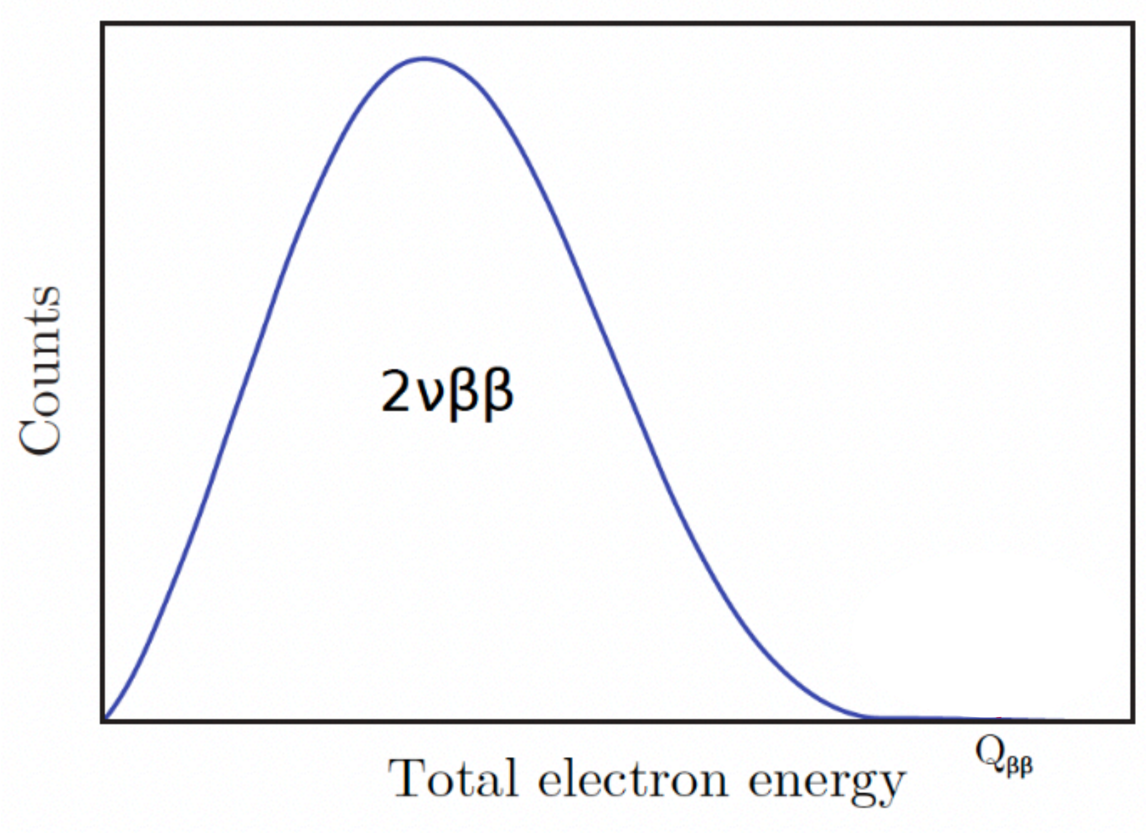
Motivation and the role of Nuclear Physics

Standard $2\nu\beta\beta$-decay



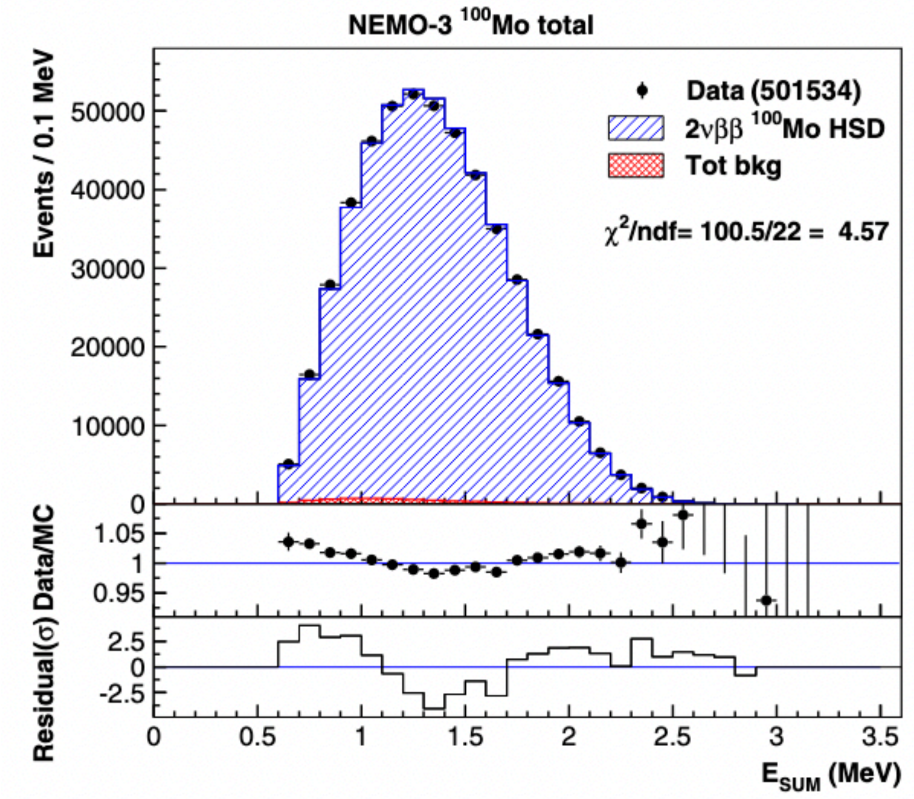
(A, Z) $\qquad$ $(A, Z+2) + 2e^- + 2\bar{\nu}_e$

Deteted signal (energy outgoing e-s)



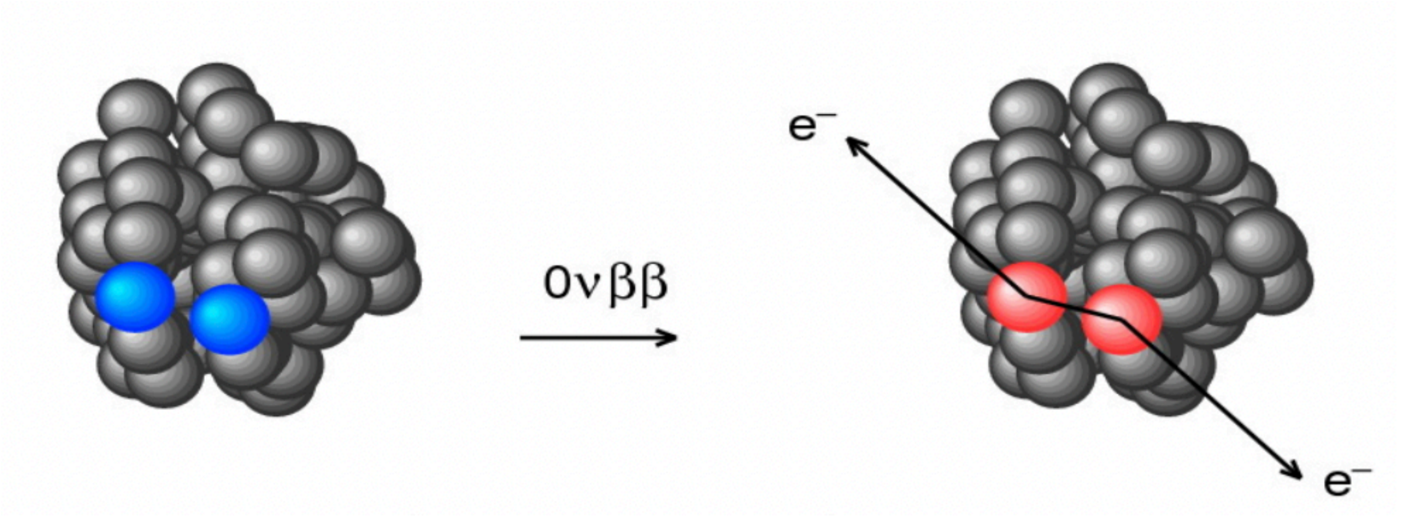
- Allowed by SM-Letpon number conserving
- Observed in 14 isotopes
- Half-lives $10^{18} - 10^{21}y$

Example in ^{100}Mo in Arnold, R *et al.* Eur. Phys. J. C 79, 440 (2019)



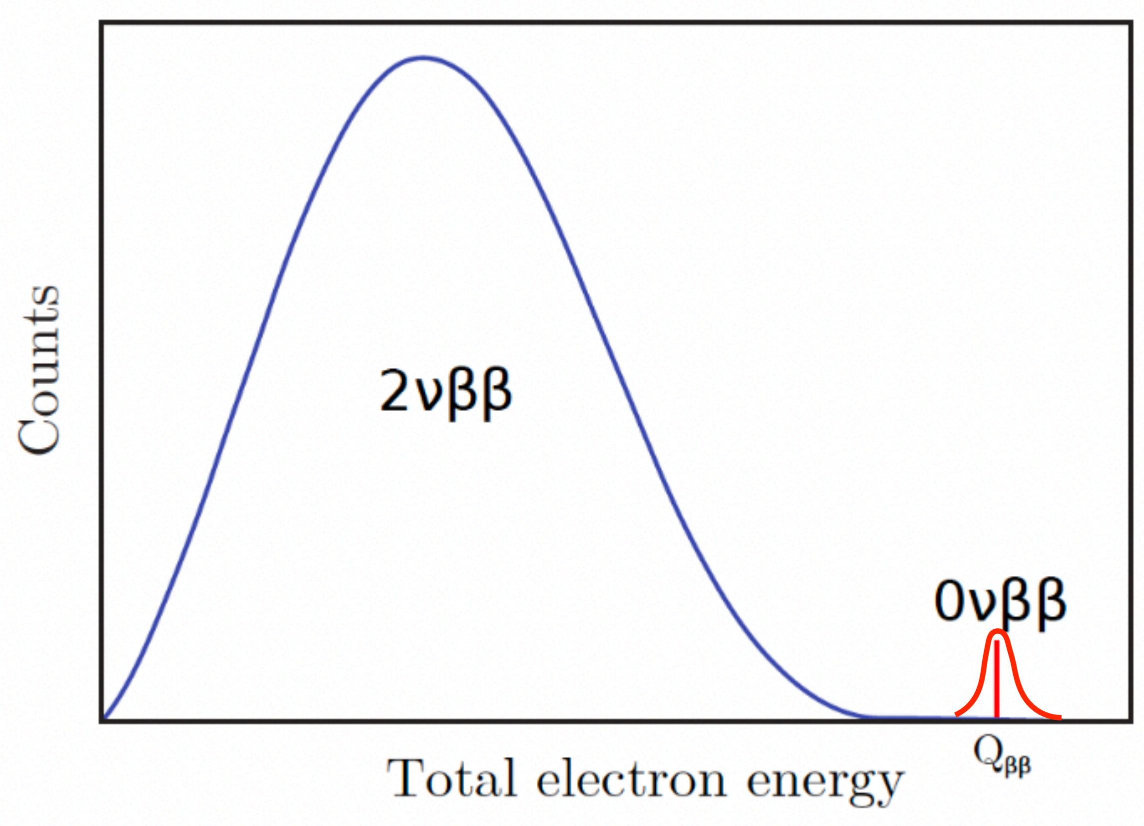
Motivation and the role of Nuclear Physics

BSM $0\nu\beta\beta$-decay



(A, Z) $\quad$ $(A, Z+2) + 2e^-$

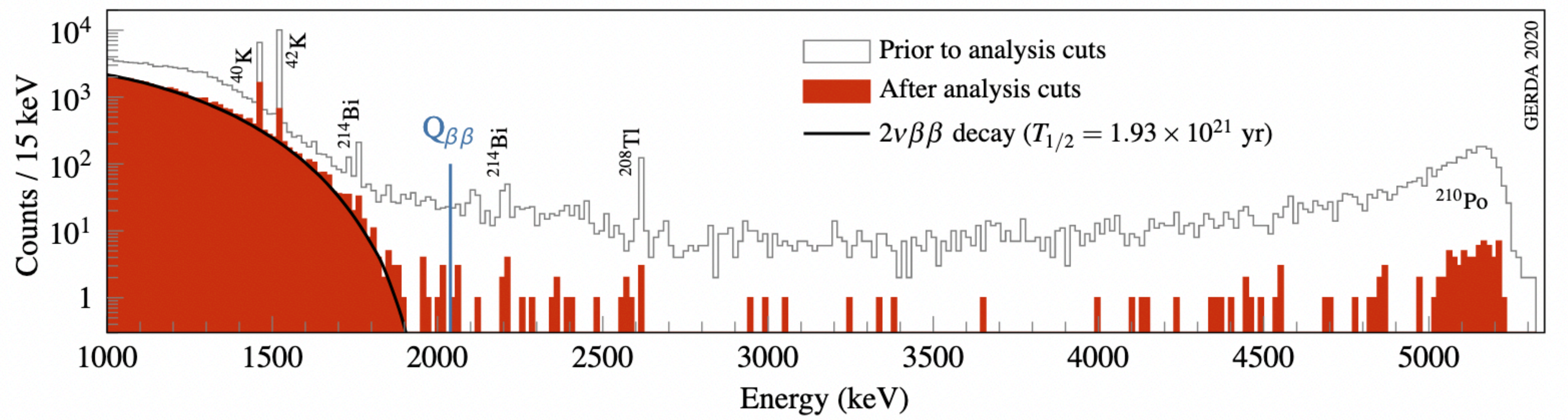
Deteted signal (energy outgoing e-s)



Enlarged by detector energy resolution

- Not allowed within SM-Letpon number violating
- Immediately demonstrate Majorana nature of ν
- Current limits $> 10^{26}y \sim 10^{15}$ age of the universe!

Example in ^{76}Ge Phys. Rev. Lett 125, 252502 (2020)



Motivation and the role of Nuclear Physics

Generically

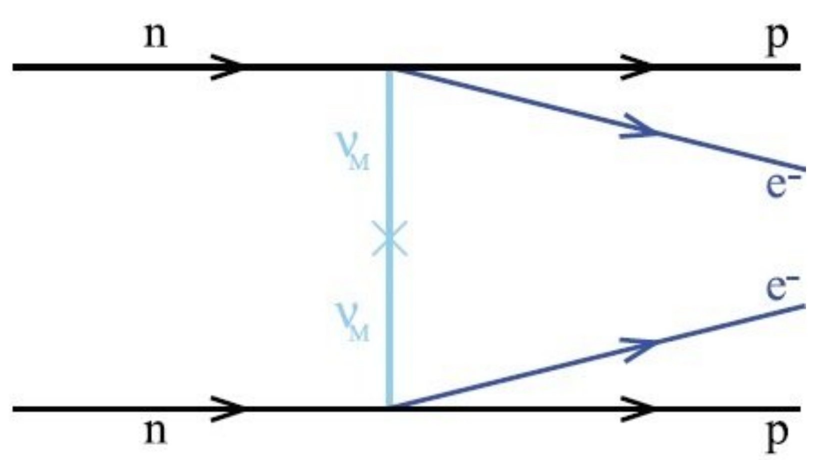
$$[T_{1/2}^{0\nu}]^{-1} = g_A^4 G_{0\nu}(Q,Z)\,|M^{0\nu}|^2 \phi^2$$

Phase Space Factor (PSF)

represent the distortion of the electron plane waves in the Coulomb field of the nucleus

New Physics parameter

particle physics parameter that we would like to extract from experiment



Light neutrino exchange

$$\phi \equiv |<m_{\beta\beta}>| = |\sum_{i=1}^{3} m_i |U_{ei}|^2 e^{i\alpha_i}|$$

effective mass parameter

$g_A^2 M^{0\nu}$ **Nuclear Matrix Element** (NME) a theoretical input, brings the largest uncertainties in DBD half-lifes, essential to obtain predictions if a positive signal is observed

$$M^{0\nu} = M_F^{0\nu} + M_{GT}^{0\nu} + M_T^{0\nu}$$

$H_K(r_{12},\bar{E})$ neutrino potentials

$\bar{E}$ average nuclear excitation energy

F ≡ vector $\tau_1^- \tau_2^- H_F(r_{12},\bar{E})$

GT≡ axial-vector $\tau_1^- \tau_2^- \sigma_1 \cdot \sigma_2 H_{GT}(r_{12},\bar{E})$

T≡ tensor $\tau_1^- \tau_2^- \{3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2\} H_T(r_{12},\bar{E})$

NMEs also needed to project physics reach of experiments from expected sensitivities

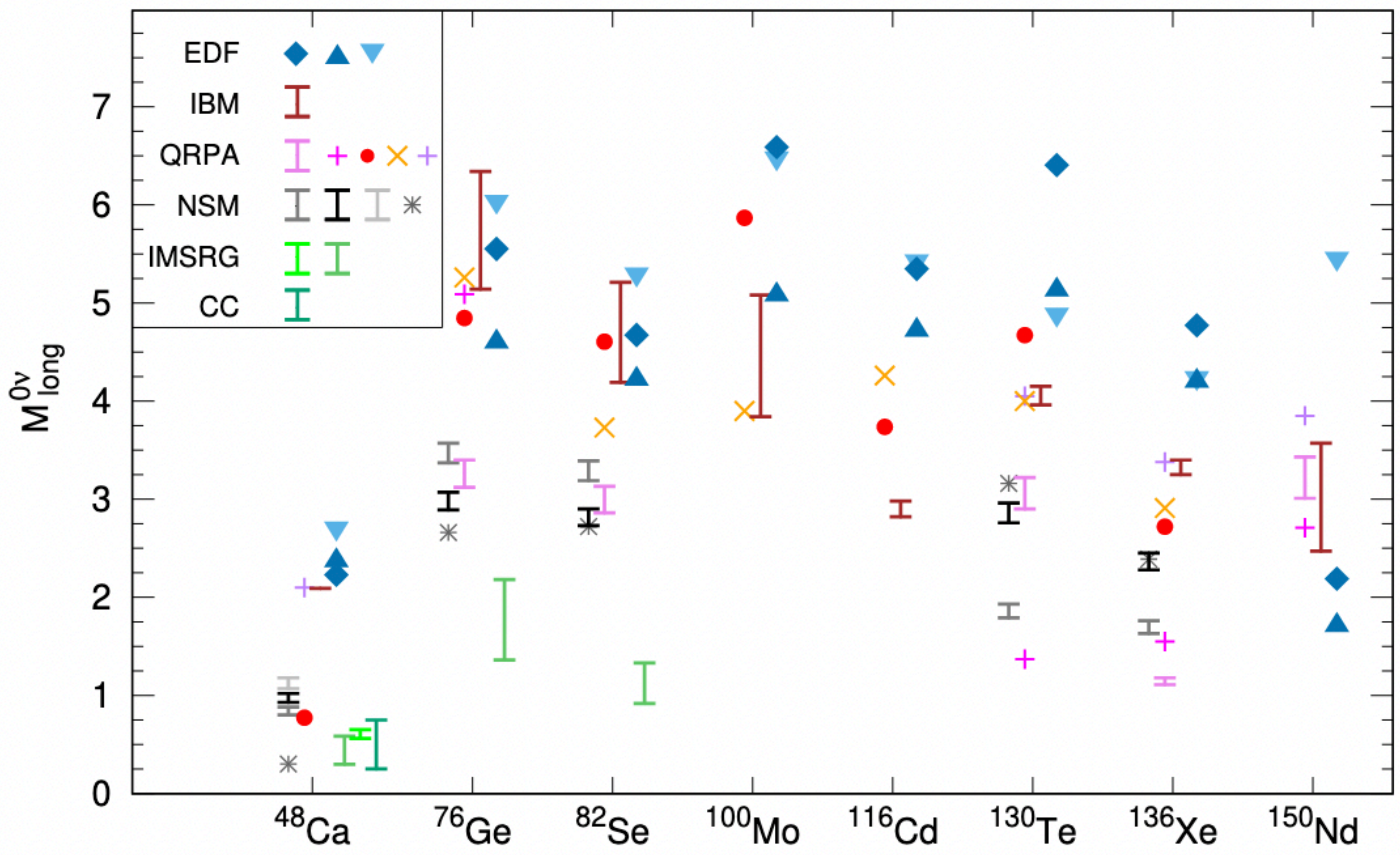
Nuclear models predictions of NMEs

Calculations need to give a good description of the nuclear structure initial and final nuclei and $0\nu\beta\beta$ transition operator (hard problem)

Variation of the NME in a factor 3 shows the uncertainties introduced by the approximate solutions of the many-body problem

Phenomenological character of most calculations prevent reliable uncertainty estimation

Approaches like NCSM, QMC, couple-cluster and IMSRG, are being developed, not yet applicable to heavy nuclei



2202.01787[hep-ex]

Experimental search for $0\nu\beta\beta$

Next generation of $0\nu\beta\beta$ experiments aim to cover $m_{\beta\beta} \gtrsim 10$ meV

Technical issue with the $\beta\beta$ isotope mass for n-ton detectors

Number of $0\nu\beta\beta$ events (background-free)

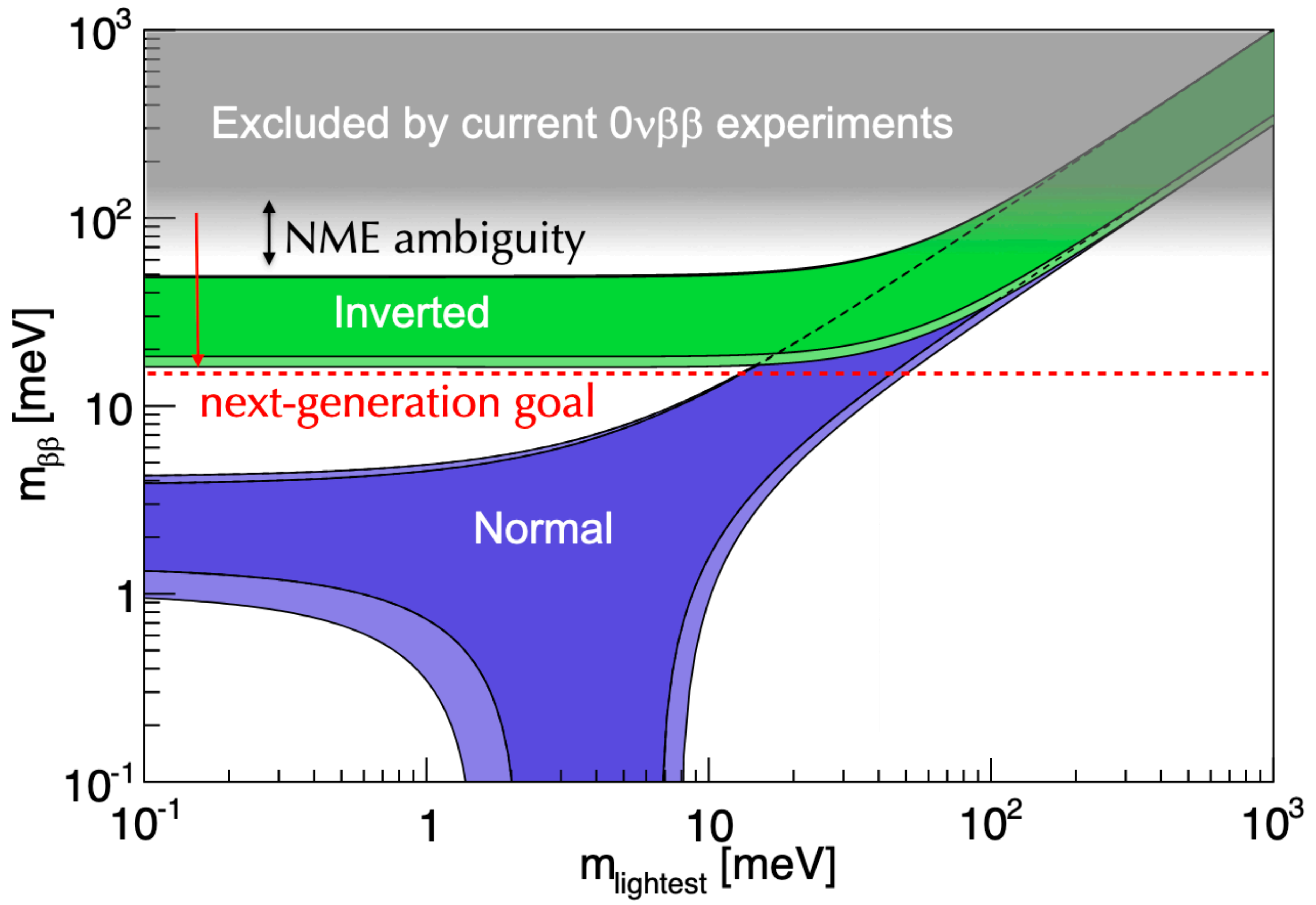
$\beta\beta$ isotope mass

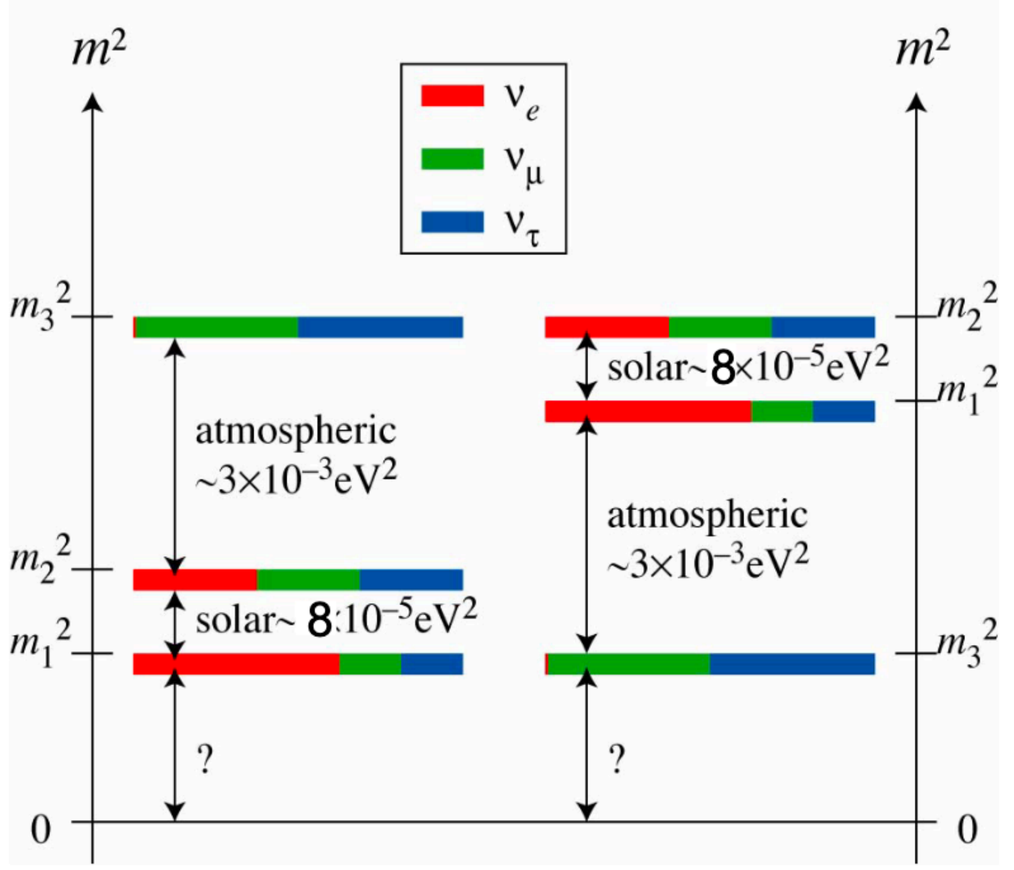
data taking time

$$N = \eta\varepsilon \frac{M \cdot t}{T^{0\nu}_{1/2}} \Rightarrow M = \left(\frac{N}{\eta\varepsilon t G_{0\nu} m^2_{\beta\beta}} \right) \frac{1}{|M_{0\nu}|^2}$$

isotope dependent number

detection efficiency





Normal $\Delta m_{31}^2 > 0$

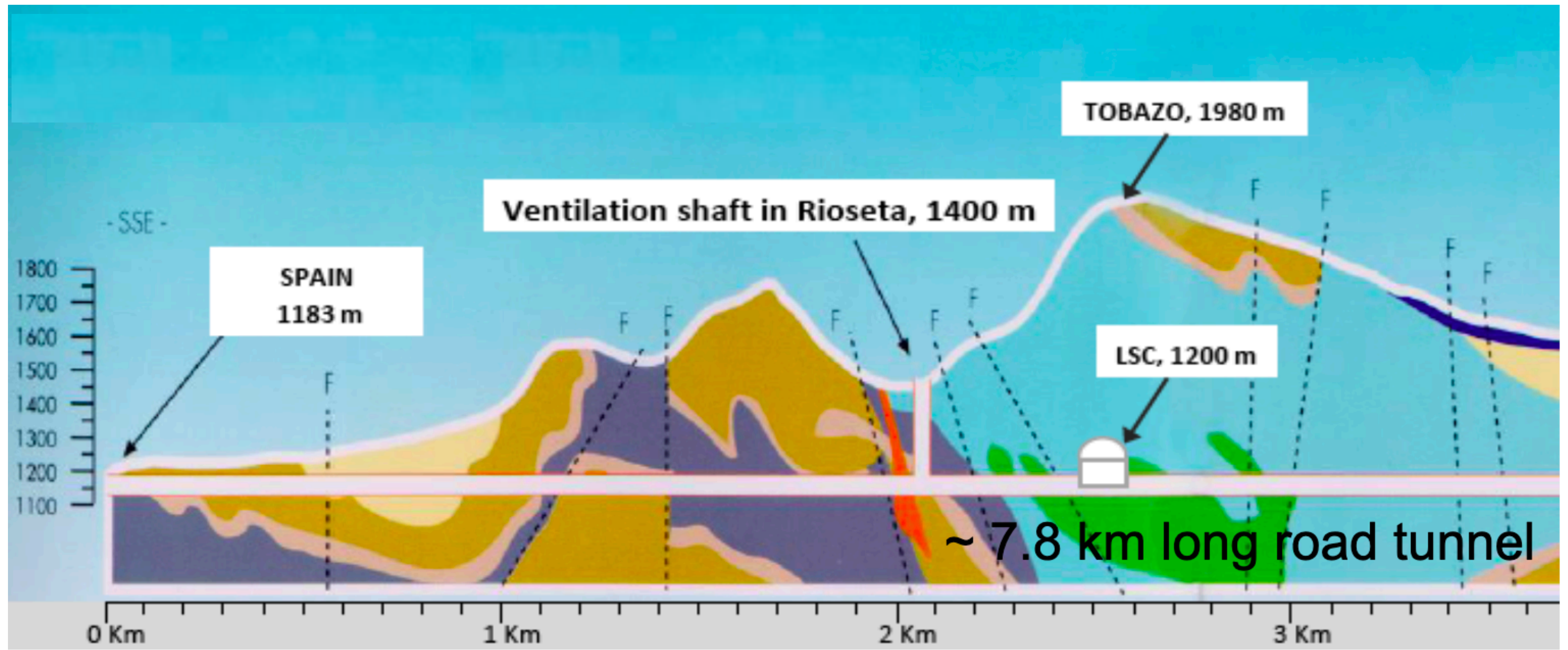
Inverted $\Delta m_{31}^2 < 0$

Reduce NME uncertainty to plan target mass (M) of experiments needed to reach $m_{\beta\beta}$ goal

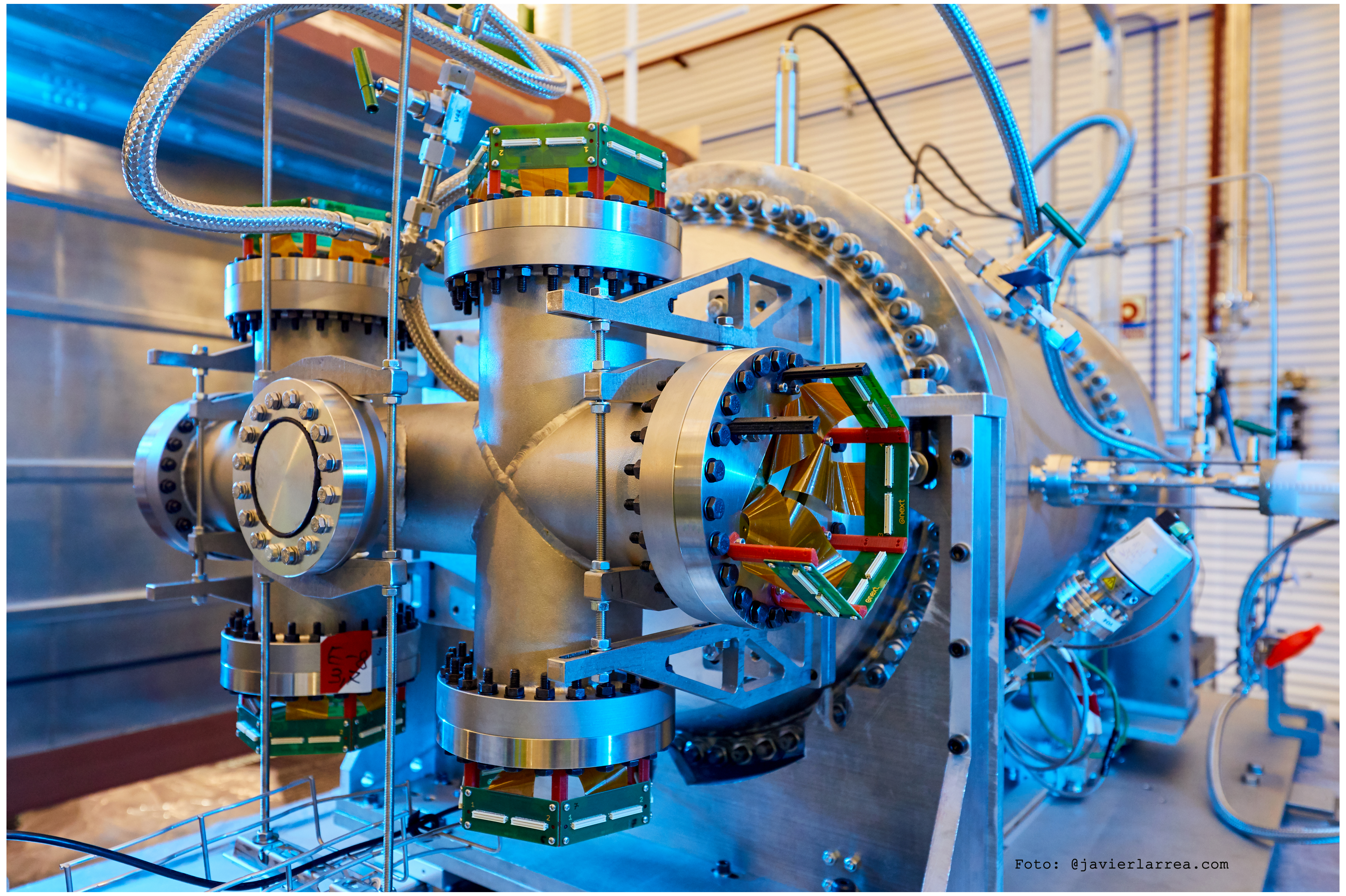
It is complex the choice of the isotope and experimental technique: good energy resolution, detection efficiency, natural isotopic abundance & enrichment ease, background in ROI,…

Experimental search for $0\nu\beta\beta$

Neutrino Experiment with a Xenon high pressure TPC (**NEXT**) located in Canfranc's Underground Laboratory



Exploit the Electroluminescence effect, good calorimetry and tracking capabilities



^{136}Xe is a good candidate for $0\nu\beta\beta$ searches:

- Relatively high $Q_{\beta\beta} = 2458keV$ less affected by radioactive background events
- Natural isotopic abundance ~ 8.86% and $^{136}\mathrm{Xe}$ easily enriched
- Provides scintillation and ionisation signal, source ≡ detector

Experimental search for $0\nu\beta\beta$

Very good energy resolution at $Q_{\beta\beta}$ (~0.9% FWHM) **NEXT-White** 4.3kg@10b with enriched (~91%) ^{136}Xe

JINST 13 (2018) 10, P10020; JHEP 10 (2019) 230

Powerful topological discrimination in gaseous Xe (signal vs bkg rejection factor 27 for 57% signal efficiency at 1.6MeV)

JHEP 10 (2019) 052; JHEP 01 (2021) 189; JHEP 07 (2021) 146

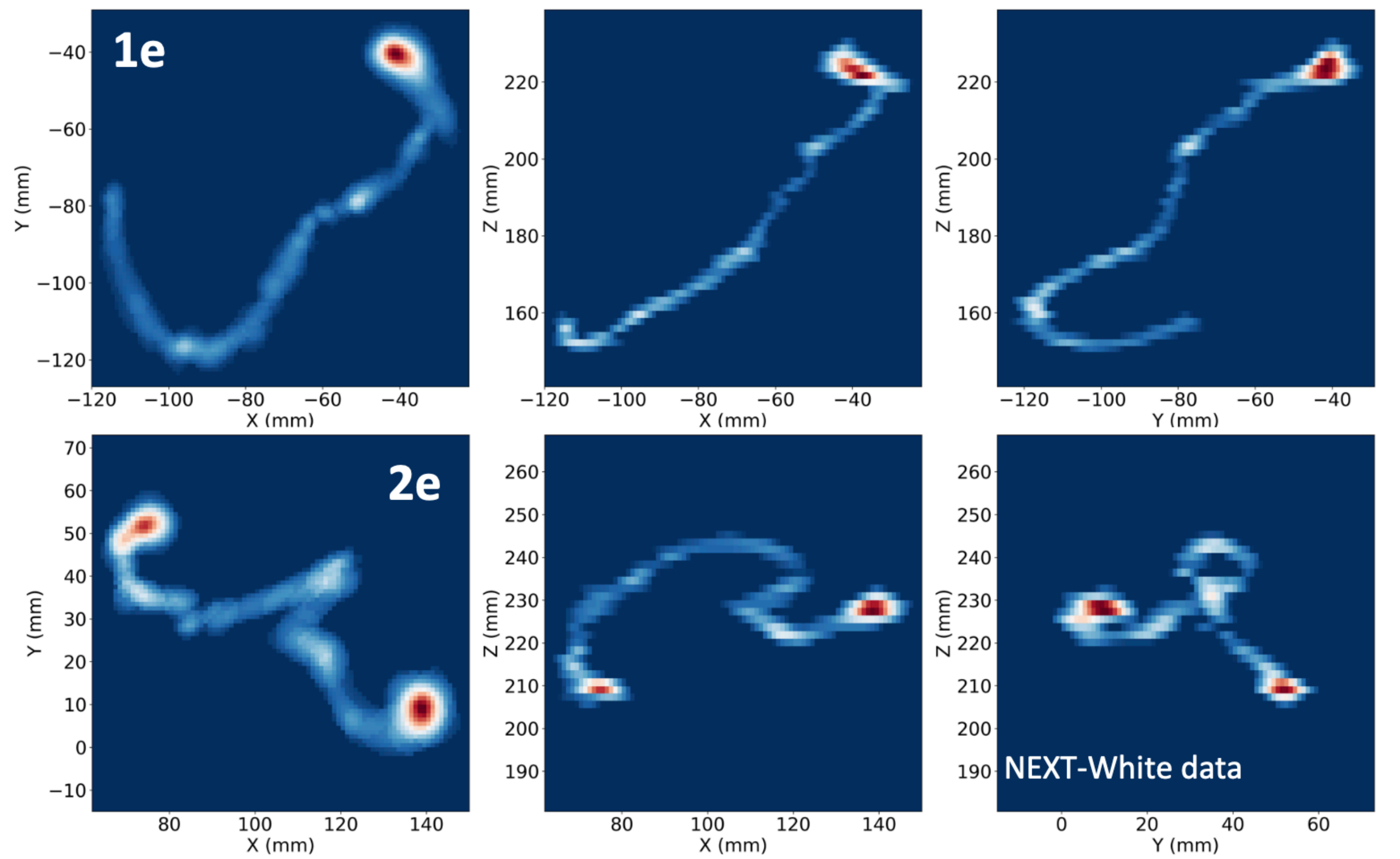
Validation of background model and measurement of $2\nu\beta\beta$ half-life (fiducial mass **3.5kg**)

JHEP 10 (2018) 112; JHEP 10 (2019) 051; Phys.Rev.C 105 (2022) 5, 055501

$$T^{2\nu}_{1/2} = 2.34^{+0.80}_{0.46}(\mathrm{stat})^{+0.30}_{-0.17}(\mathrm{sys}) \times 10^{21}\mathrm{y}$$

$$T^{0\nu}_{1/2} > 0.6 - 1.3 \times 10^{24}\mathrm{y}$$ (at 90%CL)

PRELIMINARY



NEXT-100 goals: sensitivity comparable to current generation detectors, demonstration of nearly background-free conditions $b \sim 1\mathrm{count}/(\mathrm{ROI} \cdot \mathrm{y})$ at 100kg scale, technology demonstrator for ton scale

JHEP 05 (2016) 159

Experimental search for $0\nu\beta\beta$

Future plans ton-scale detector

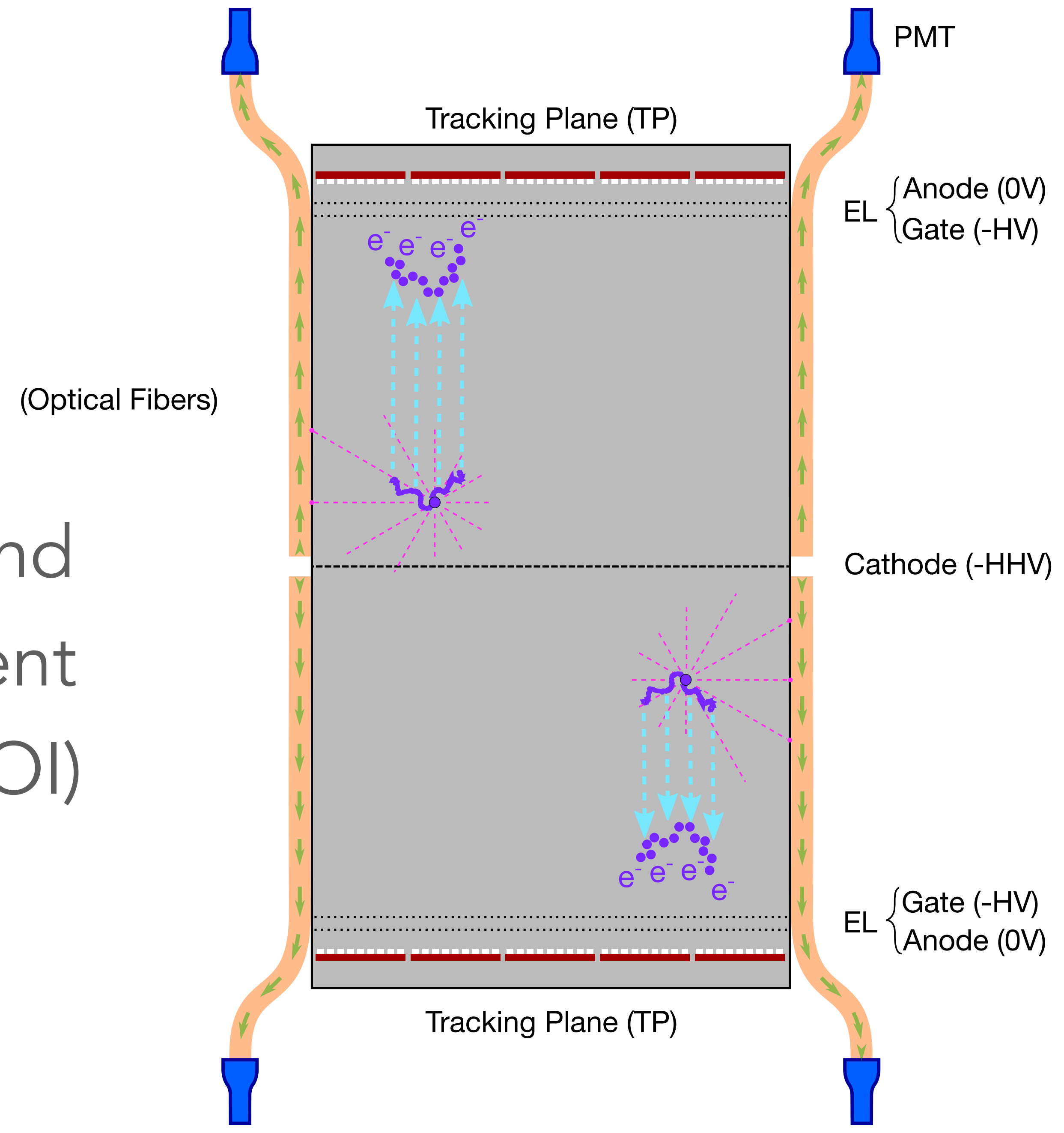
"High-Definition" (**NEXT-HD**)

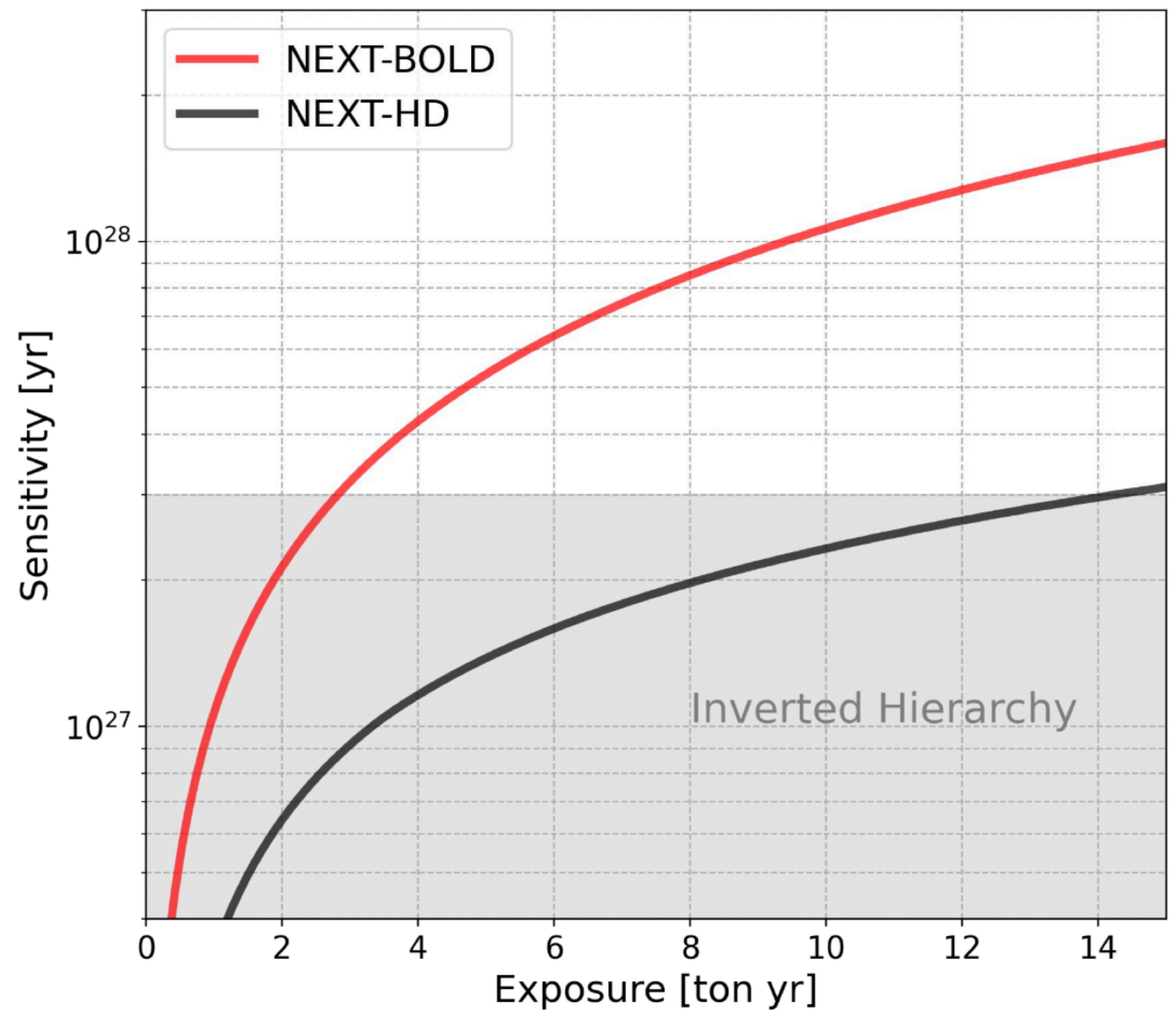
Symmetric TPC with central cathode

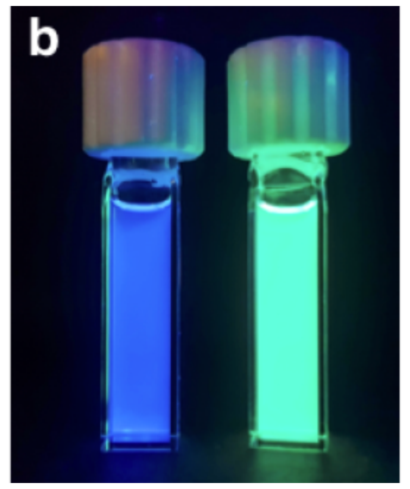
Replace PMTs plane by SiPMs to reduce background

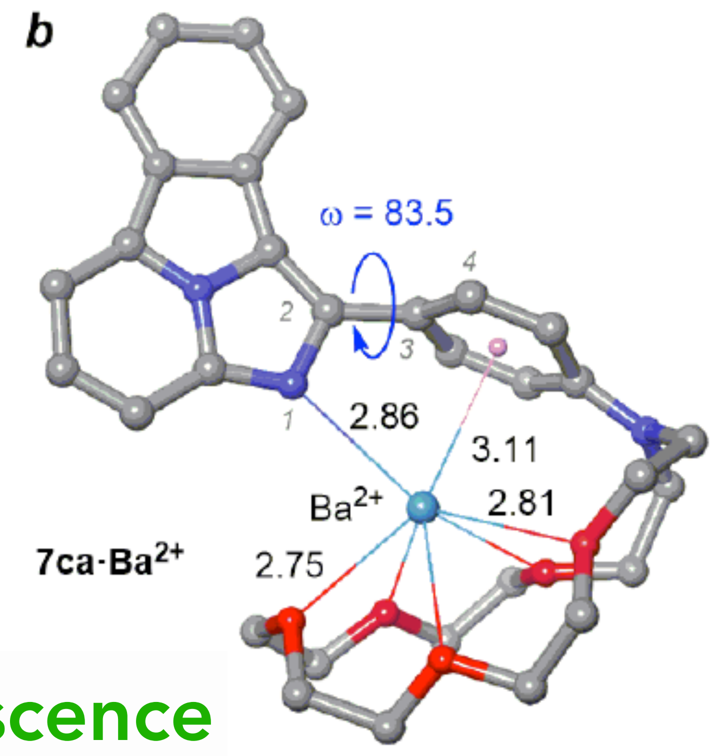
Optical fibers around barrel for energy measurement

Estimated background 0.09-0.27 counts/(ton·yr·ROI)

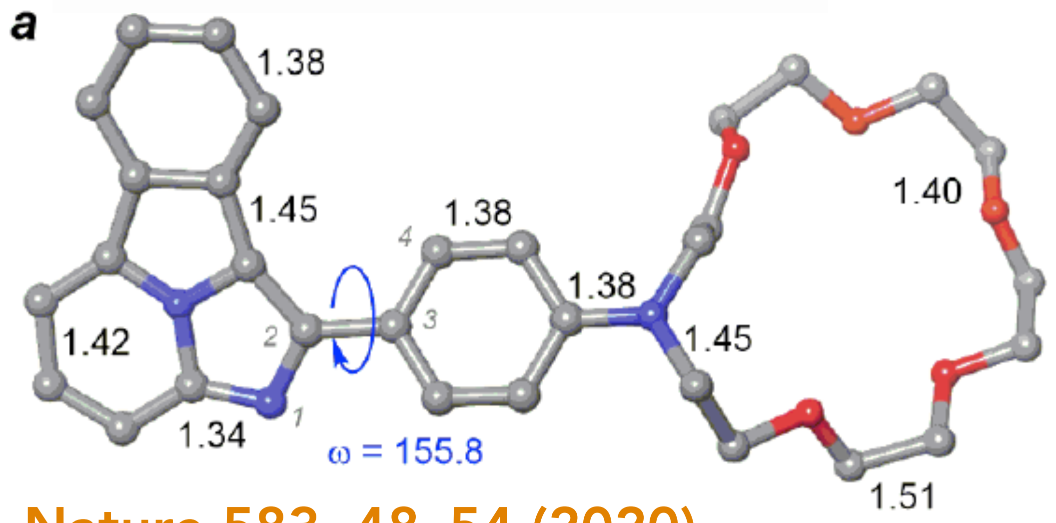








Bi-color fluorescence



Nature 583, 48–54 (2020)

"**Barium iOn Light Detection**"(**NEXT-BOLD**)

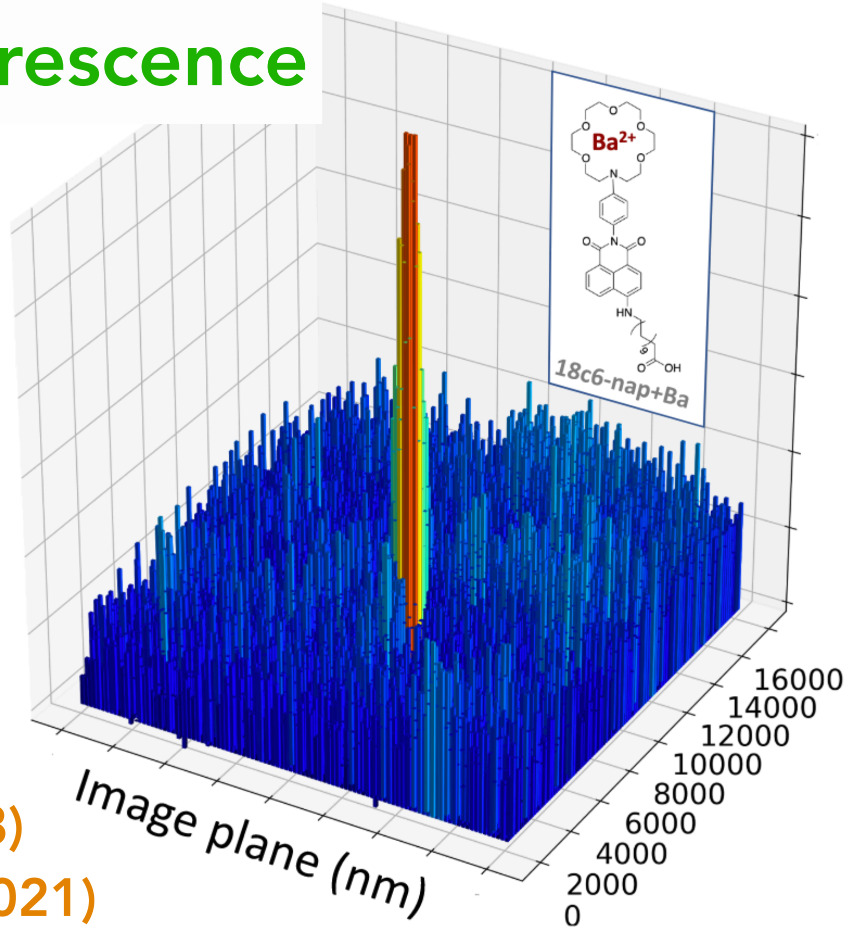
Ba-tagging based on **Single Molecule Fluorescence imaging**

J. Phys Conf. Series 650,012002(2015)

FMIs (Fluorescent Monocolor Indicators) **"Turn ON"** approach

FBIs (Fluorescent Bi-color Indicators) **"Bi-color"** approach

On-Off fluorescence



Phys. Rev. Lett. 120,132504 (2018)
ACS Sens. 2021, 6, 1, 192–202 (2021)

Other approaches to NMEs problem

Other approaches

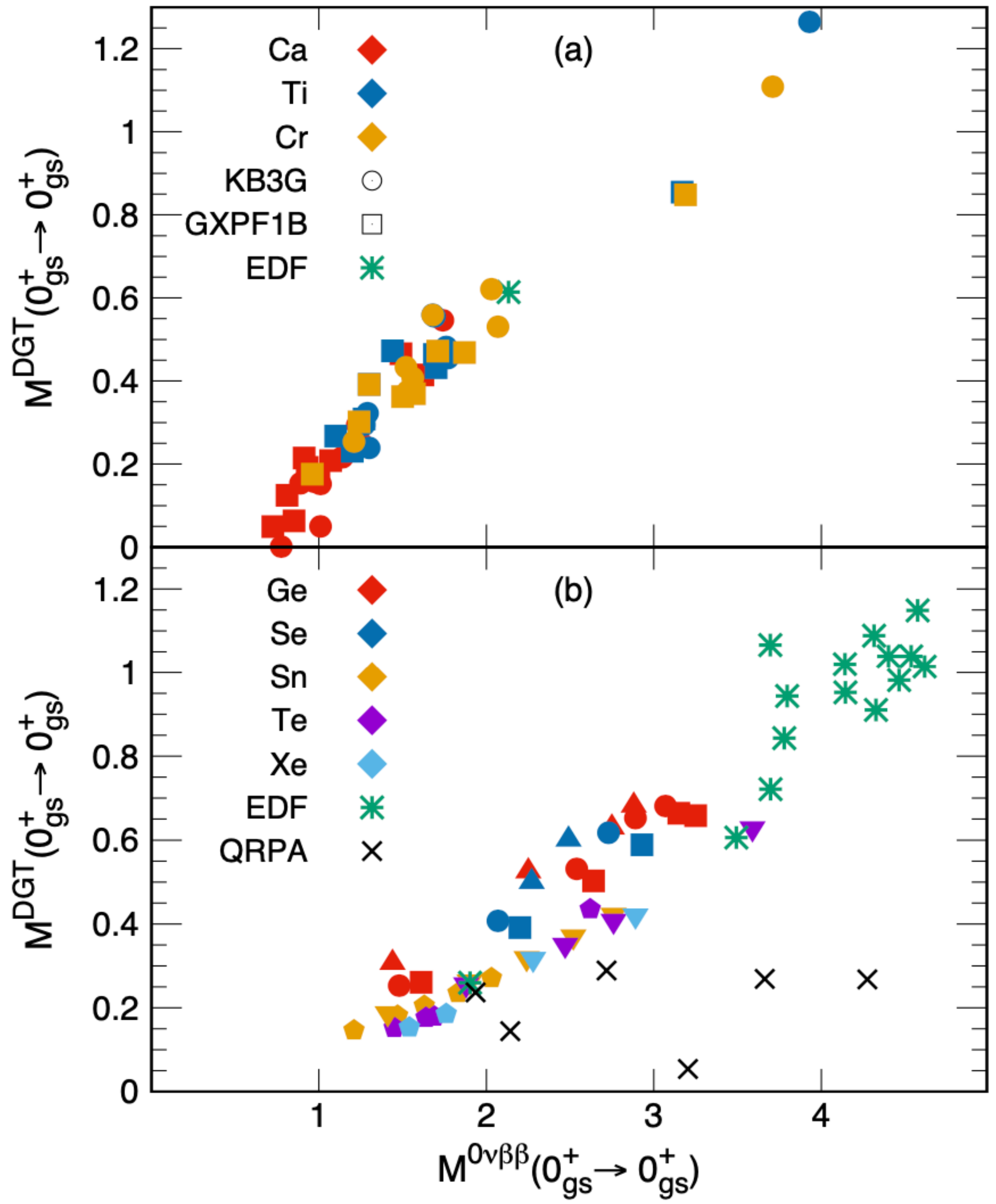
search of **observables** that are **linked** to $0\nu\beta\beta$ even not mediated by same interaction

Double charge-exchange reactions (DCE), isobaric 2nd order nuclear transitions where $|i\rangle$ and $|f\rangle$ are the same as in $0\nu\beta\beta$, **transition operators similar**

Few years ago Shimizu et al. (SM) and recently Lotta (pnQRPA) and [Yao22] (IMSRG) found very good linear correlation between **DGT transition** to the ground state of the final nucleus and the $0\nu\beta\beta$

$$M^{DGT} := B^{1/2}(DGT^-; 0; 0^+_{gs,i} \to 0^+_{gs,f}) = |\langle 0^+_{gs,f} || \sum_{j,k} [\sigma_j \tau_j^- \times \sigma_k \tau_k^-]^0 || 0^+_{gs,i} \rangle|$$

Key: look for more accessible experiments and try to measure this transition giving valuable information for $0\nu\beta\beta$ NME



N. Shimizu, J. Menéndez and K. Yako Phys. Rev. Lett. 120, 142502 (2018)

Other approaches to NMEs problem

Measure M^{DGT} represents a challenge (small fraction of the total strength, tiny σ), but it could be more "accesible" than $0\nu\beta\beta$

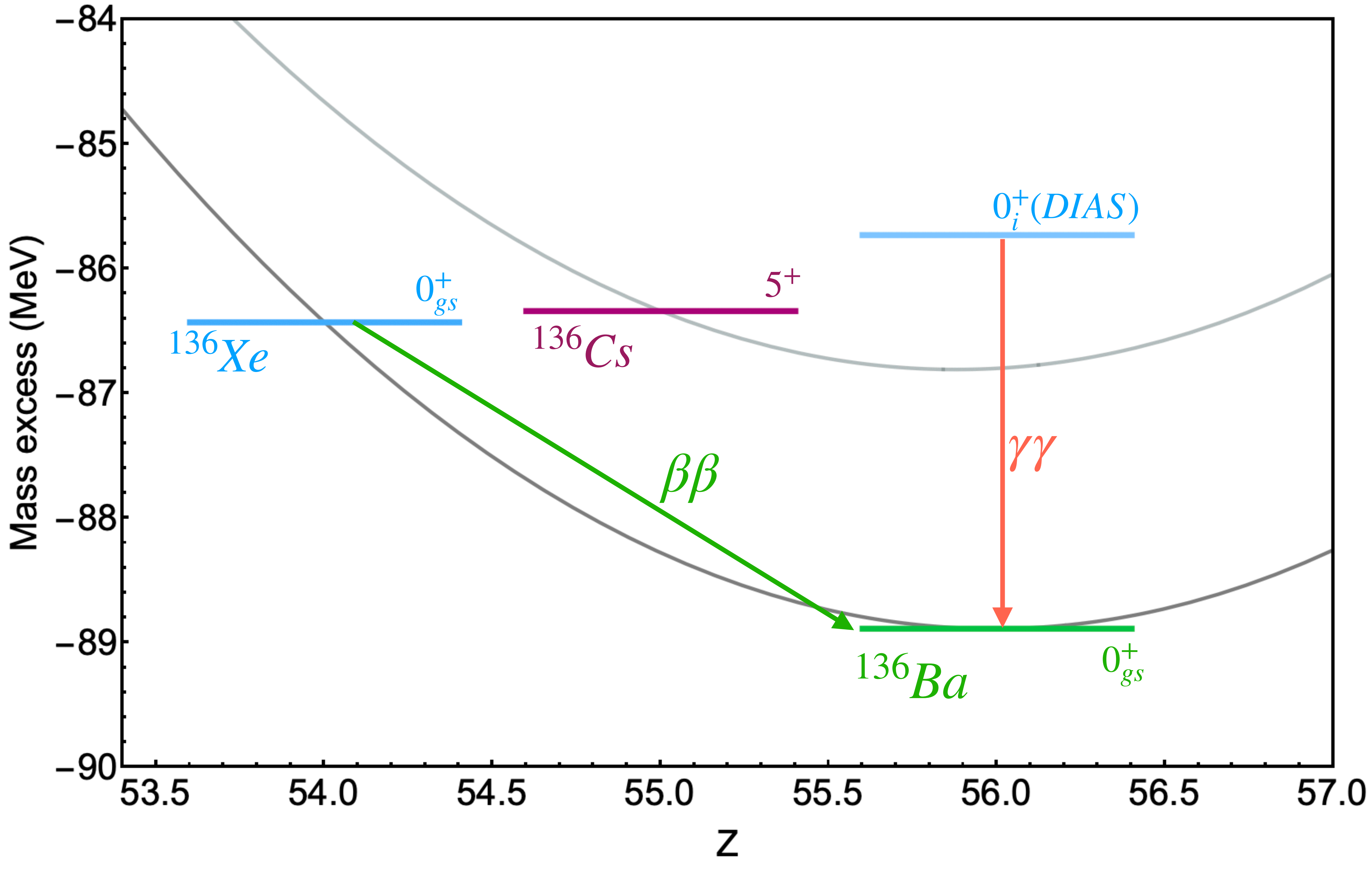
Experiments at **RCNP** [^{56}Fe(^{11}B,^{11}Li)^{56}Ni] **RIBF RIKEN** [^{12}C(^{18}O,^{18}Ne)^{12}Be] and **INFN-LNS [NUMEN** project, ^{40}Ca(^{18}O,^{18}Ne)^{40}Ar] and others aim to measure DGT via HIDCE reactions among other observables that could constrain $M^{0\nu}$

What about second-order EM transitions?

Double magnetic dipole (M1M1) $\gamma\gamma$-transition operator similar to **DGT** (same isovector $\sigma\tau$ term)

$$|0_i^+\rangle_{\gamma\gamma} \equiv |0_i^+\rangle_{\beta\beta}(DIAS) = \frac{T_- T_-}{N_f}|0_i^+\rangle_{\beta\beta}$$

$$|0_f^+\rangle_{\gamma\gamma} \equiv |0_f^+\rangle_{\beta\beta}$$



Isospin symmetry holds very well in nuclei, nuclear structure aspects DIAS to GS $\gamma\gamma$ and $0\nu\beta\beta$ be very similar

Double gamma decay transition amplitude

$$\boxed{N_i(P_i) \longrightarrow N_f(P_f) + \gamma_\lambda(k) + \gamma_{\lambda'}(k')}$$

$$\hat{H} = \hat{H}_0 + \hat{H}_\gamma + \underbrace{\int d^4x \hat{J}_\mu(x) A^\mu(x) + \frac{1}{2}\int d^4x d^4y \hat{B}_{\mu\nu}(x,y) A^\mu(x) A^\nu(y)}_{\hat{H}_I}$$

[Fri75]

Nucleus

$N_\alpha(P_\alpha) \longrightarrow |\alpha \mathbf{P}_\alpha\rangle$

$\hat{H}_0 |\alpha \mathbf{P}_\alpha\rangle = E_\alpha |\alpha \mathbf{P}_\alpha\rangle$

Photon

$\gamma_\lambda(k) \longrightarrow \varepsilon^\mu(k, \lambda)$

Nuclear current operator

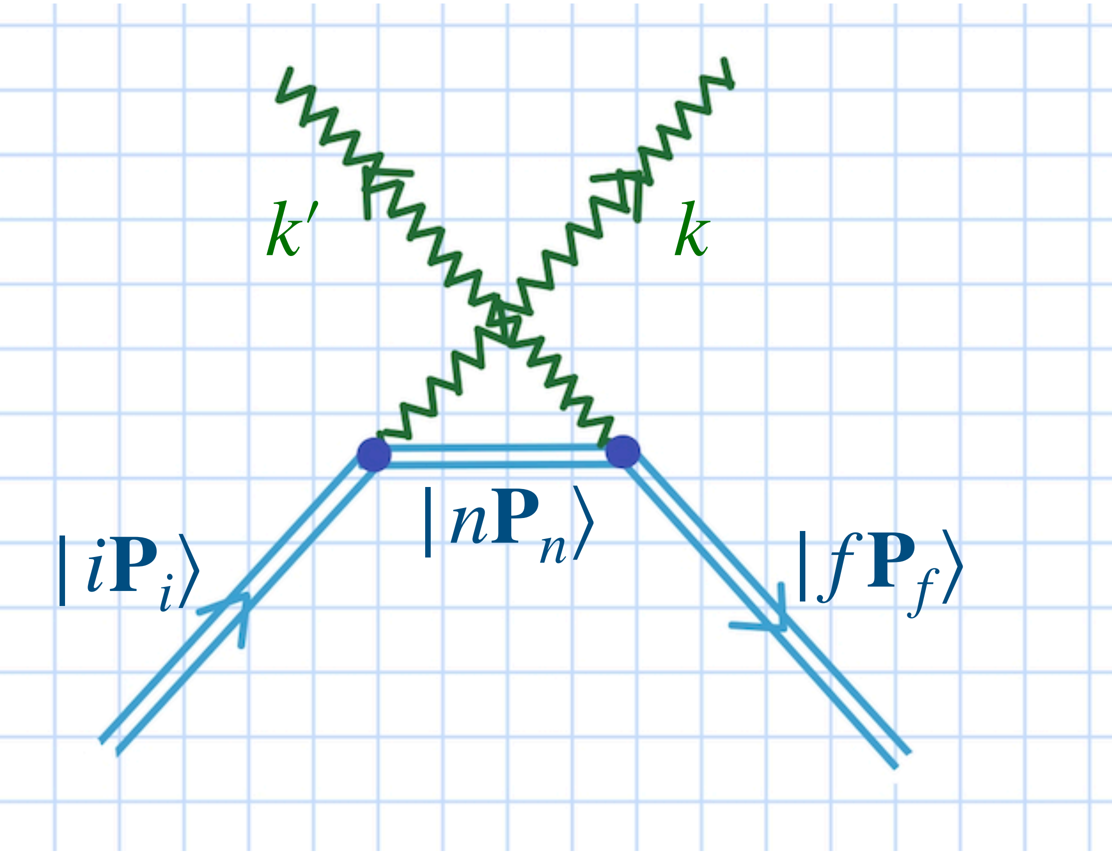
Seagull operator (SG)

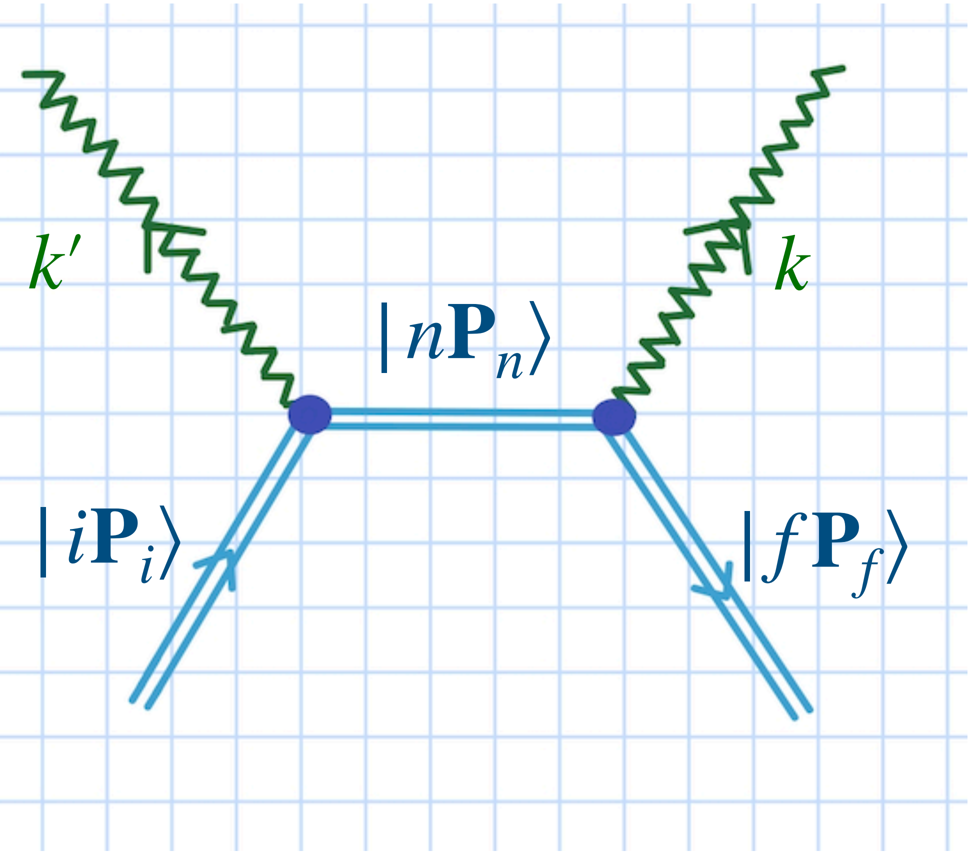
$$\mathcal{S}^{(2)} = -\frac{1}{2}\int d^4x d^4y T[\hat{J}_\mu(x)\hat{J}_\mu(y)] T[A^\mu(x) A^\mu(y)]$$

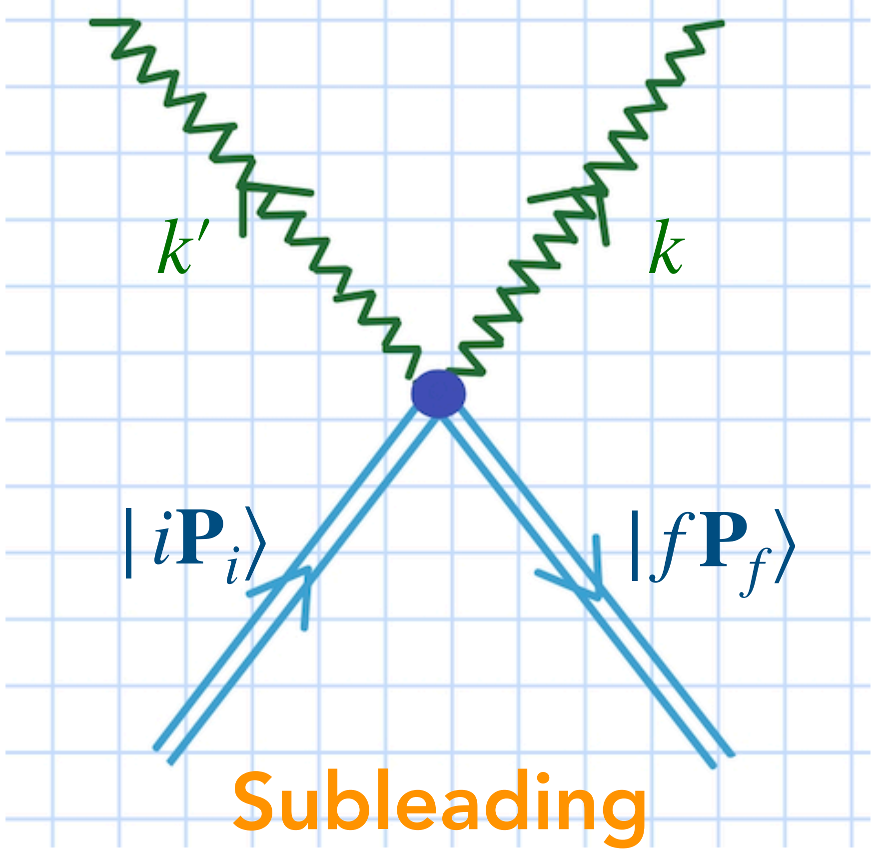
$$\mathcal{S}^{(1)} = -\frac{i}{2}\int d^4x d^4y T[\hat{B}_{\mu\nu}(x,y) A^\mu(x) A^\nu(y)]$$

$$\mathcal{M}_{fi} = (2\pi)\delta(k_0 + k_0' + E_f - E_i) T_{fi}$$

$$\mathcal{M}_{fi}^{sg} = (2\pi)\delta(k_0 + k_0' + E_f - E_i) T_{fi}^{sg}$$







Double gamma decay transition amplitude

$$T_{fi} = \varepsilon^*_{\mu\lambda}(\mathbf{k})\varepsilon^*_{\nu\lambda'}(\mathbf{k}') \sum_n \int d^3\mathbf{x}d^3\mathbf{y}e^{-i\mathbf{k}\cdot\mathbf{x}}e^{-i\mathbf{k}'\cdot\mathbf{y}} \left(\frac{\langle f\mathbf{P}_f|\hat{J}_\mu(\mathbf{x})|n\mathbf{P}_n\rangle\langle n\mathbf{P}_n|\hat{J}_\nu(\mathbf{y})|i\mathbf{P}_i\rangle}{E_i - k'_0 - E_n + i\epsilon} + \frac{\langle f\mathbf{P}_f|\hat{J}_\nu(\mathbf{y})|n\mathbf{P}_n\rangle\langle n\mathbf{P}_n|\hat{J}_\mu(\mathbf{x})|i\mathbf{P}_i\rangle}{E_i - k_0 - E_n + i\epsilon} \right)$$

Multipolar expansion of the photon field $A^\mu(k,\lambda;x)$

$$\varepsilon^{\mu*}(k,\lambda)e^{-i\mathbf{k}\cdot\mathbf{x}} = -(2\pi)^{\frac{1}{2}} \sum_{\substack{L,M \\ S=0,1}} \sqrt{2L+1}\lambda^S(-1)^{L+M-1+\Delta_{\mu 0}}\widetilde{A}^\mu_{L,-M}(S,k_0,\mathbf{x})D^{L*}_{M\lambda}(R)$$

$S = 0$ for E multipoles

$S = 1$ for M multipoles

[Kr77] use **LANDAU gauge**, it simplifies the result in the **long wave approximation** $R(\sim fm) \ll \lambda(\sim 10^2 fm)$ (good approximation in the range of energies $\sim MeV$ we work with)

$$\widetilde{A}^0_{L,M}(E,k_0,\mathbf{x}) = \alpha(L)k_0^L|\mathbf{x}|^L Y_{LM}(\hat{\mathbf{x}})$$

$$\widetilde{\mathbf{A}}_{L,M}(E,k_0,\mathbf{x}) \Rightarrow 0$$

$$\widetilde{A}^0_{L,M}(M,k_0,\mathbf{x}) \Rightarrow 0$$

$$\widetilde{\mathbf{A}}_{L,M}(M,k_0,\mathbf{x}) = \alpha'(L)k_0^L(\mathbf{x}\times\nabla)Y_{LM}(\hat{\mathbf{x}})$$

Double gamma decay resonant transition amplitude

Transition amplitude is proportional to

$$P_J(S'L'SL, k_0, k_0') = (2\pi)(-1)^{J_f+J_i}\hat{L}\hat{L}'\sum_{n,J_n}\left[\begin{Bmatrix} L & L' & J \\ J_i & J_f & J_n \end{Bmatrix}\frac{\langle J_f||\mathcal{O}(SL,k_0)||J_n\rangle\langle J_n||\mathcal{O}(S'L',k_0')||J_i\rangle}{E_n - E_i + k_0'} + \right.$$

$$\left. +(-1)^{J-L-L'}\begin{Bmatrix} L' & L & J \\ J_i & J_f & J_n \end{Bmatrix}\frac{\langle I_f||\mathcal{O}(S'L',k_0')||J_n\rangle\langle J_n||\mathcal{O}(SL,k_0)||J_i\rangle}{E_n - E_i + k_0}\right]$$

Generalized polarizability

The transition operators are

$$\mathcal{O}_M(SL, k_0) = \int d^3\mathbf{x}(-1)^{\Delta_{\mu 0}}J_\mu(\mathbf{x})\widetilde{A}^{\mu}_{LM}(S, k_0, \mathbf{x})$$

non null components

$\widetilde{A}^0_{L,M}(E, k_0, \mathbf{x})$ $\quad$ $\widetilde{\mathbf{A}}_{L,M}(M, k_0, \mathbf{x})$

Nucleus as a collection of non-relativistic point nucleons with charge and magnetic moments

Finally

$$\mathcal{O}_M(EL, k_0) = k_0^L\alpha(L)\sum_{i=1}^{A} e(i)r_i^L Y_{LM}(\Omega_i)$$

$$\mathcal{O}_M(ML, k_0) = i\alpha(L)\frac{ek_0^L}{2m}\left[\sum_{i=1}^{A}\left(\frac{2}{L+1}g_l^{(i)}\mathbf{L}_i + g_s^{(i)}\mathbf{S}_i\right)\cdot\nabla_i\left(r_i^L Y_{LM}(\Omega_i)\right)\right]$$

Double gamma decay $0_i^+ \longrightarrow 0_f^+$ selection rules

We are interested in calculate NME between $J_i^P = 0'^+$ and $J_f^P = 0^+$

Parity conservation $\pi_i \pi_f = \pi_{\gamma\gamma} = (-1)^{L+S+L'+S'}$

Angular momentum conservation $J_i = J_f = 0 \Rightarrow J = 0,\, L = L'$

$S = S'$

2ML,2EL for the dominant amplitude

2ML for the contact amplitude

$P_0 \sim (k_0 k_0')^L$ which means that statistically photons have the same energy most of the transitions

$$E_n - E_i + k_0 \approx E_n - E_f - \frac{1}{2}(E_i - E_f)$$

In order to avoid dependence on photon energy we study $k_0 = k_0'$

Double gamma decay NMEs occupation number representation

Essentially what we have is **one-body reduced matrix elements** and **products of two of them**

reduced ME of rank λ operator

$$\langle \xi_f J_f || T_{\lambda\mu} || \xi_i J_i \rangle = \hat{\lambda}^{-1} \sum_{a,b} \underbrace{\langle a || \mathbf{T}_\lambda || b \rangle}_{} \underbrace{\langle \xi_f J_f || [c^\dagger_\alpha \tilde{c}_\beta]_\lambda || \xi_i J_i \rangle}_{=: \rho^{(L)}_{fi,\alpha\beta}}$$

$c^\dagger_\alpha, c_\alpha$ creation, annihilation op's

$\phi_\alpha(\mathbf{x}) = \langle \mathbf{x} | c^\dagger_\alpha | 0 \rangle$

Reduced single-particle matrix element **encodes information of the properties of the one-body op $T_{\lambda\mu}$** (analytical)

Reduced one-body transition density **characterises the many nucleon properties of $|\xi_i J_i\rangle, |\xi_f J_f\rangle$** (numerical)

$$P_0^{res}(MLML, k_0, k_0') = \sum_{n,L} \eta(L, k_0, k_0') \frac{Tr(ML\rho^{(L)}_{fn})Tr(ML\rho^{(L)}_{ni})}{E_n - E_f + \frac{1}{2}(E_i - E_f)}$$

Double gamma decay NMEs

$\gamma\gamma$-M1M1 NME

$$M^{\gamma\gamma}(M1M1) = \sum_{1_n} \sum_{\substack{a,b\\c,d}} (a \parallel \mathbf{M1} \parallel b)\,(c \parallel \mathbf{M1} \parallel d)\,\frac{\langle 0_f^+ \parallel [c_\alpha^\dagger \tilde{c}_\beta]_1 \parallel 1_n^+\rangle\langle 1_n^+ \parallel [c_\gamma^\dagger \tilde{c}_\delta]_1 \parallel 0_i^+\rangle}{E_n - E_f - \frac{1}{2}(E_i - E_f)}$$

$$\mathbf{M}1 = \mu_N \sqrt{\frac{3}{4\pi}} \sum_{i=1}^{A} g_l^{(i)} \mathbf{l}_i + g_s^{(i)} \sigma_i$$

We want to study

$|0_f^+\rangle \equiv |0_{gs}^+(\beta\beta_{daugther})\rangle$

$T = T_z^d \equiv T_f$

$|0_i^+\rangle \equiv |0_1^+(DIAS)\rangle = \frac{T\_T\_}{N_f}|0_{gs}^+(\beta\beta_{parent})\rangle$

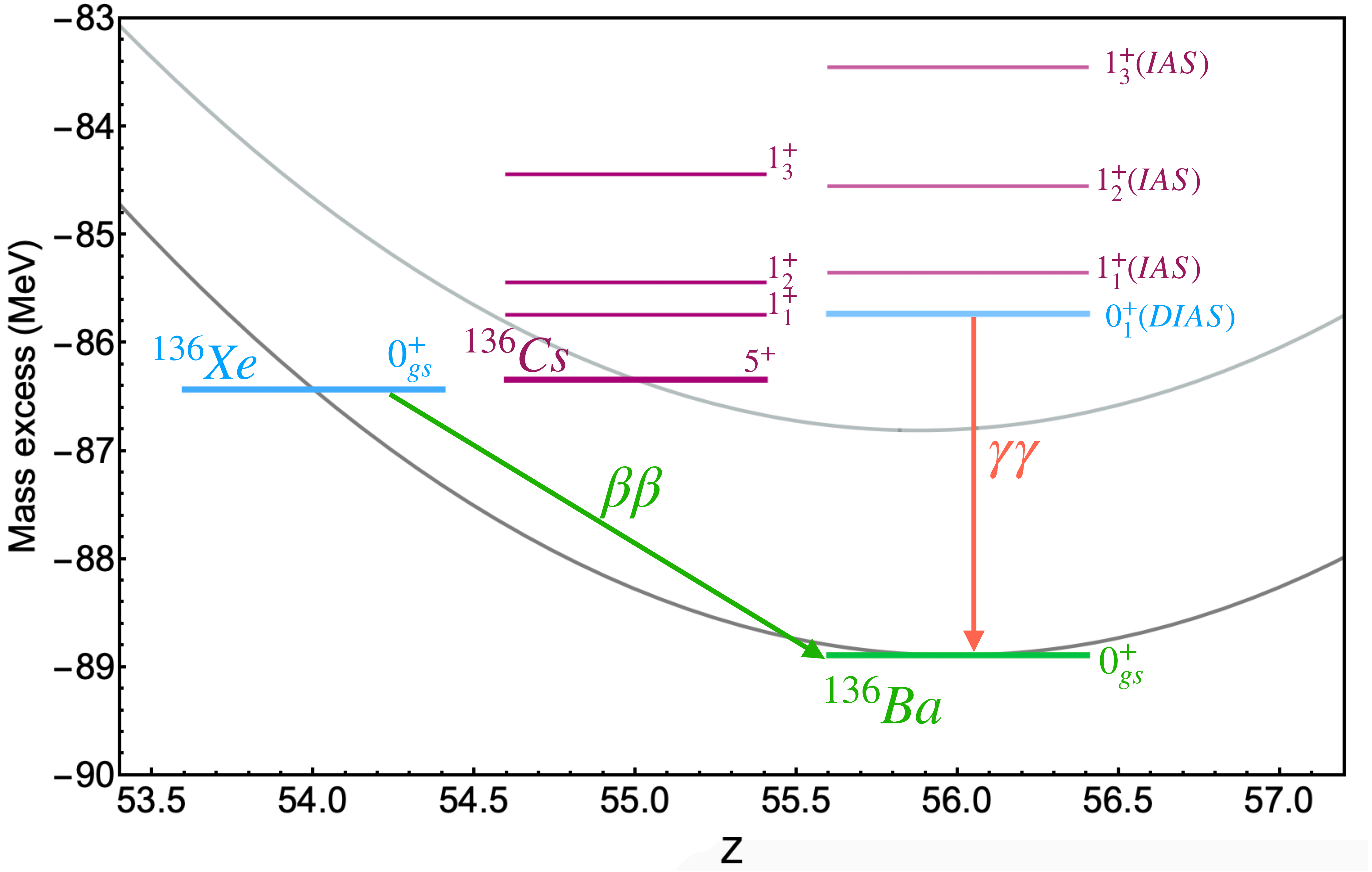
$T_i = T_f + 2,\ T_z = T_z^d$

M1 $\Delta T = 0, 1$

only connected by **isovector** transitions

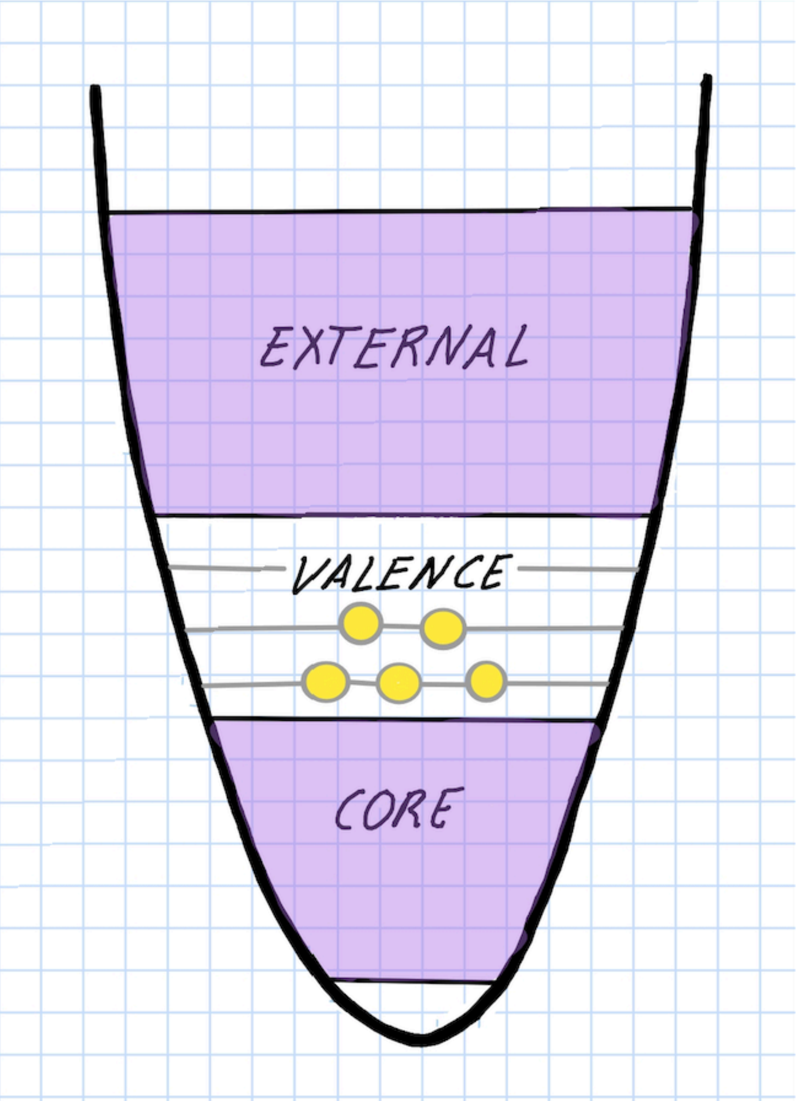
$|1_n^+\rangle \equiv |1_n^+(IAS)\rangle$

$T_n = T_f + 1,\ T_z = T_z^d$



Nuclear Shell Model

Microscopic treatment of the nucleus requires to solve $H\Psi_i = E_i\Psi_i$



Nuclear shell model $H_{eff}|\overline{\Psi}_i\rangle = E_i|\overline{\Psi}_i\rangle$

$|\overline{\Psi}_i\rangle$ configuration (valence) space wf

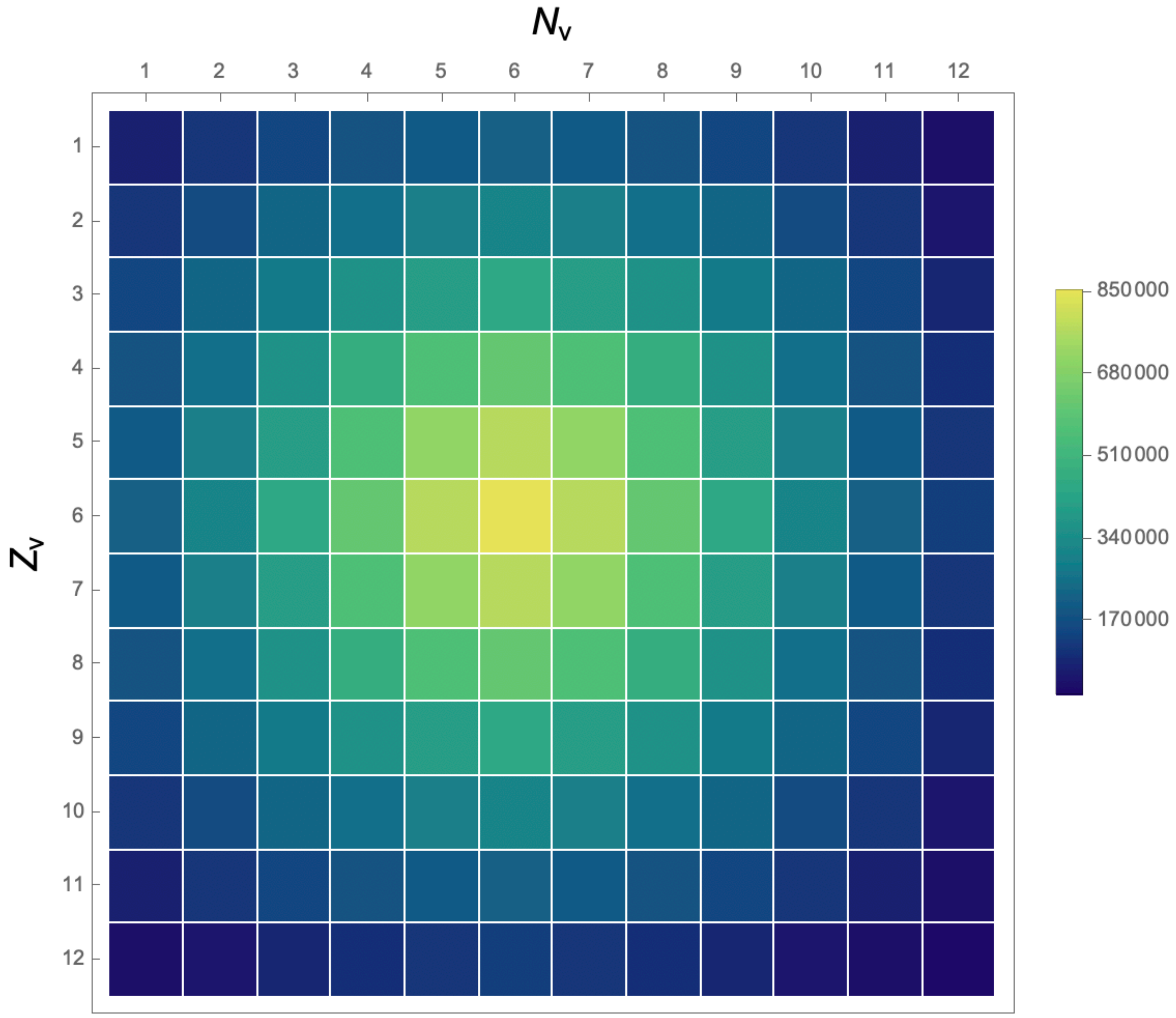
$$|\overline{\Psi}_i\rangle = \sum_{k=1}^{d} a_{ij}|\Phi_j\rangle$$ a_{ij} obtained from diagonalising H_{eff}

$$|\Phi_\alpha\rangle = \prod_{\alpha_i=(n_i,l_i,j_i,m_i,m_{t_i})}^{A} c^\dagger_{\alpha_i}|0\rangle$$ SDs

ANTOINE (Caurier and Nowacki,1999)

m-scheme shell code, SDs with definite M_J and M_T but not **J** and **T**

Matrix size maximal but sparse and easy to compute → **Diagonalization** using **LANCZOS** method



number of SDs in the sd shell

Nuclear interactions and configuration spaces

Several approaches to obtain H_{eff}, we use **phenomenological** interactions where TBMEs and SPEs have been fitted to reproduce experimental low-energy spectra

Valence space

pf-shell [^{40}Ca CORE] $0f_{7/2}1p_{3/2}1p_{1/2}0f_{5/2}$

$\gamma\gamma$ decay in $^{46-58}Ti$ $^{50-60}Cr$ $^{54-60}Fe$

$0\nu\beta\beta$ of ^{48}Ca

pfg-shell [^{56}Ni CORE] $1p_{3/2}1p_{1/2}0f_{5/2}0g_{9/2}$

$\gamma\gamma$ decay in $^{72-78}Zn$ $^{74-80}Ge$ $^{76-82}Se$ $^{82-84}Kr$

$0\nu\beta\beta$ in ^{76}Ge

sdgh-shell [^{100}Sn CORE] $0g_{7/2}1d_{5/2}1d_{3/2}1s_{1/2}0h_{11/2}$

$\gamma\gamma$ decay in $^{124-132}Te$ $^{130-134}Xe$ $^{134-136}Ba$

$0\nu\beta\beta$ of ^{136}Xe

H_{eff}

KB3G A. Poves et al., Nucl. Phys. A 649, 157(2001)
GXPF1B M. Honma et al., RIKEN Accelerator. Progress Report 41,32(2008)

GCN2850 A. Gniady, E. Caurier, and F. Nowacki
JUN45 M. Honma et al., Phys. Rev. C80,064323 (2009)
JJ4BB B.A. Brown and A.F. Lisetskiy

GCN5082, A. Gniady, E. Caurier, and F. Nowacki
QX Chong Qi and Z.X. Xu,Phys. Rev. C 86, 044323 (2012)

Nuclear shell model analysis of M1M1

Steps followed to obtain $M^{\gamma\gamma}(M1M1)$

$$M^{\gamma\gamma}(M1M1) = \sum_{1_n} \sum_{\substack{a,b \\ c,d}} \left(a \parallel \mathbf{M1} \parallel b\right) \left(c \parallel \mathbf{M1} \parallel d\right) \frac{\langle 0_f^+ \parallel [c_\alpha^\dagger \tilde{c}_\beta]_1 \parallel 1_n^+(IAS)\rangle\langle 1_n^+(IAS) \parallel [c_\gamma^\dagger \tilde{c}_\delta]_1 \parallel 0_i^+(DIAS)\rangle}{E_n - E_f + \frac{1}{2}(E_i - E_f)}$$

1) Obtain the ground state of $|0_{gs}^+(\beta\beta_{emiter})\rangle := |0_i^+\rangle$ and E_i

2) Rotate twice in isospin space to obtain $|0_i^{\prime +}\rangle \longrightarrow |0_i^{\prime +}\rangle := |0_i^+(DIAS)\rangle = \dfrac{T\_T\_}{(\langle 0_i^+ | T_+^2 T_-^2 | 0_i^+\rangle)^{1/2}} |0_i^+\rangle$

3) Obtain the ground state of $|0_{gs}^+(\gamma\gamma_{emiter})\rangle := |0_f^+\rangle$ and E_f

4) Obtain the intermediate states $|1_n^+(IAS)\rangle$ and E_n

Using **Lanczos Strength Function**

initial vector for Lanczos algorithm, $|v_1\rangle$

good $J^P = 1^+$ and $T_n = T_f + 1, T_z = T_z^d$

$$|v_1\rangle = \frac{M1_{IV}|0_f^+\rangle}{\left(\langle 0_f^+ | M1_{IV}^\dagger M1_{IV} | 0_f^+\rangle\right)^{1/2}}$$

Good approximation for eigenstates in a wide range of E's

5) Finally, we obtain transition density matrixes for each 1_n^+

$$\rho_{ni} = \langle 1_n^+(IAS) \parallel [c_\gamma^\dagger \tilde{c}_\delta]_1 \parallel 0_i^+(DIAS)\rangle$$
$$\rho_{fn} = \langle 0_f^+ \parallel [c_\alpha^\dagger \tilde{c}_\beta]_1 \parallel 1_n^+(IAS)\rangle$$

Nuclear shell model analysis of M1M1

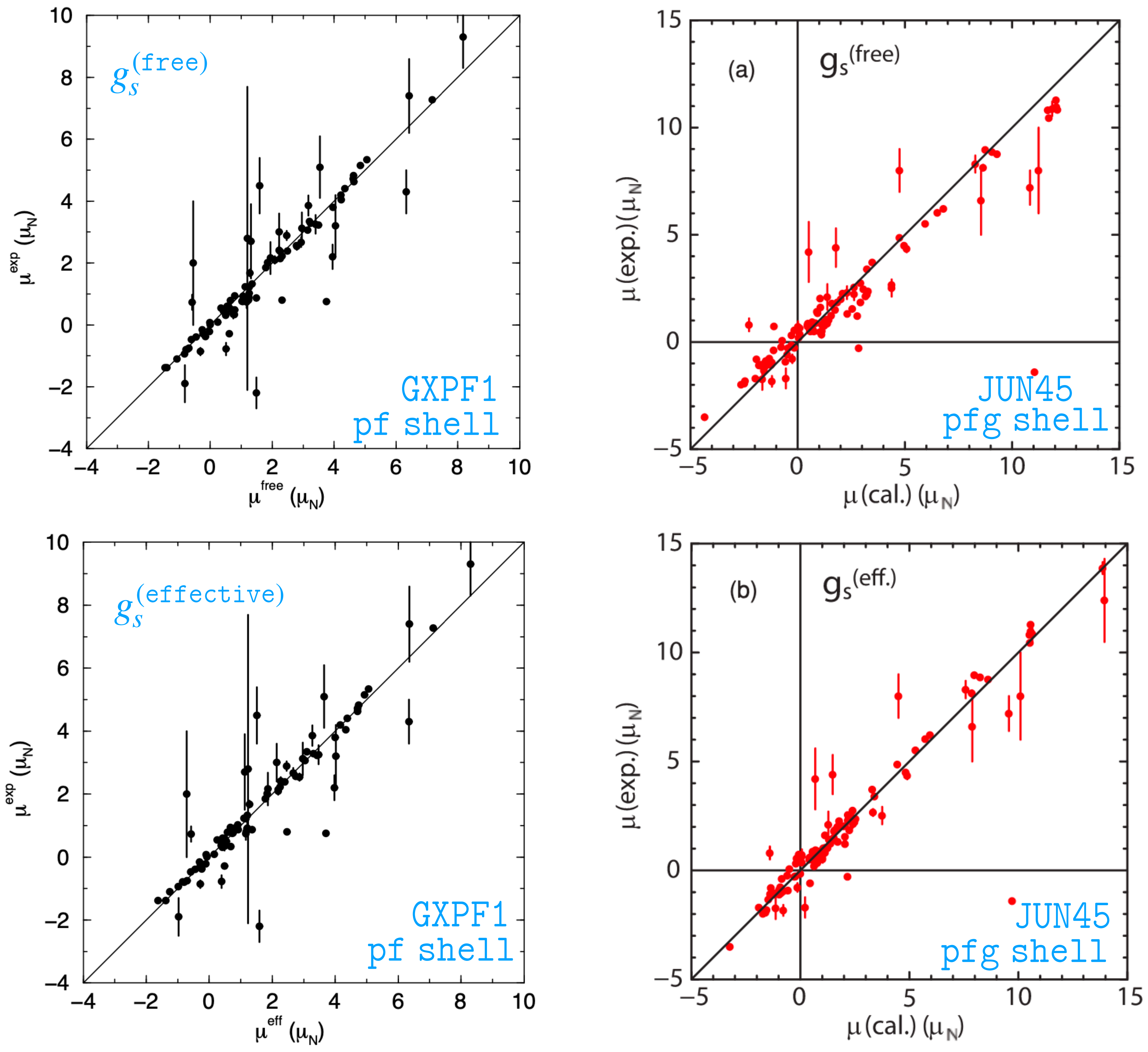
The results have been obtained using the ***bare*** (or free-nucleon) values for the ***g factors***

$$\mu = g_s \mathbf{s} + g_l \mathbf{l}$$

$g_s^{\nu} = -3.826$ $g_s^{\pi} = 5.586$ $g_l^{\nu} = 0$ $g_l^{\pi} = 1$ (free g-factors)

The agreement between μ^{free} and μ^{exp} seems good for the *sd*, *pf* shells, but may demands a correction factor $q_s = 0.7$ for the *pfg* shell

Our first approach to the problem was to use free values, but the results vary slightly when we take effective values for g_s



M. Honma *et al.*, Phys.Rev. C 69, 034335 (2004)

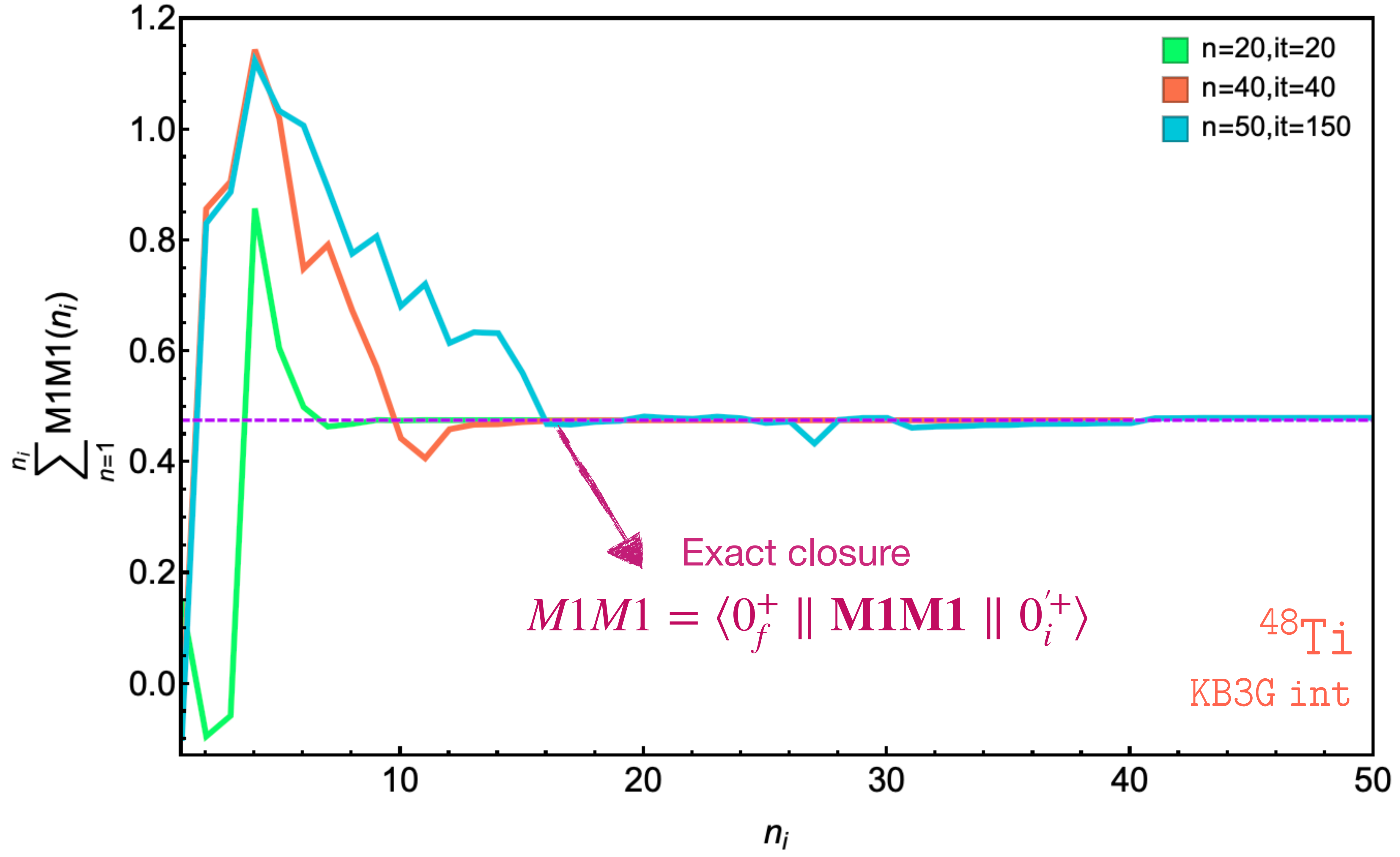
M. Honma *et al.*, Phys.Rev. C 80, 064323 (2009)

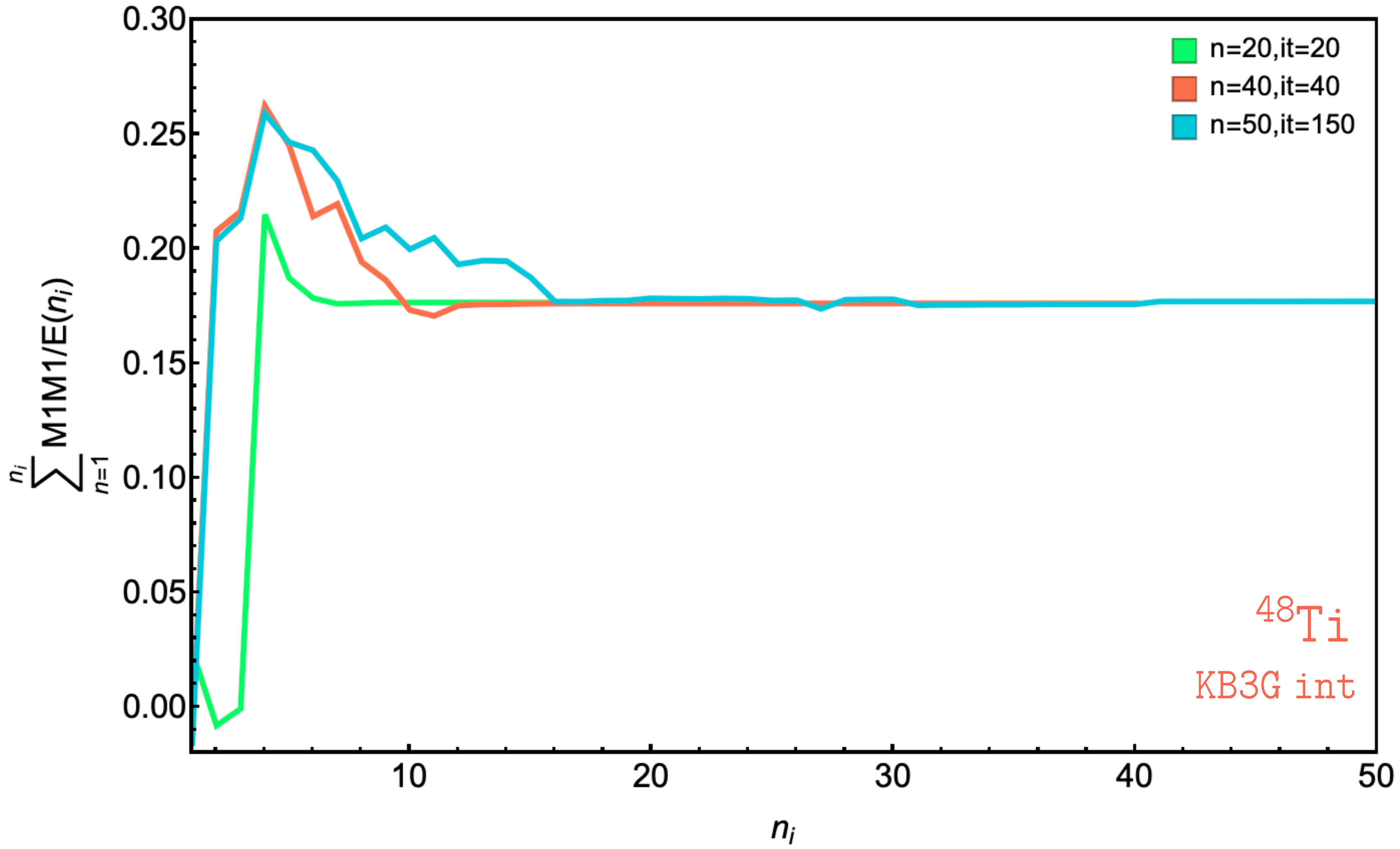
Results: convergence

Evolution of NMEs with the number of intermediate states and Lanczos SF iterations

$$M1M1 = \sum_{1_n^+} \langle 0_{gs}^+ \parallel \mathbf{M1} \parallel 1_n^+(IAS)\rangle\langle 1_n^+(IAS) \parallel \mathbf{M1} \parallel 0_1^+(DIAS)\rangle$$

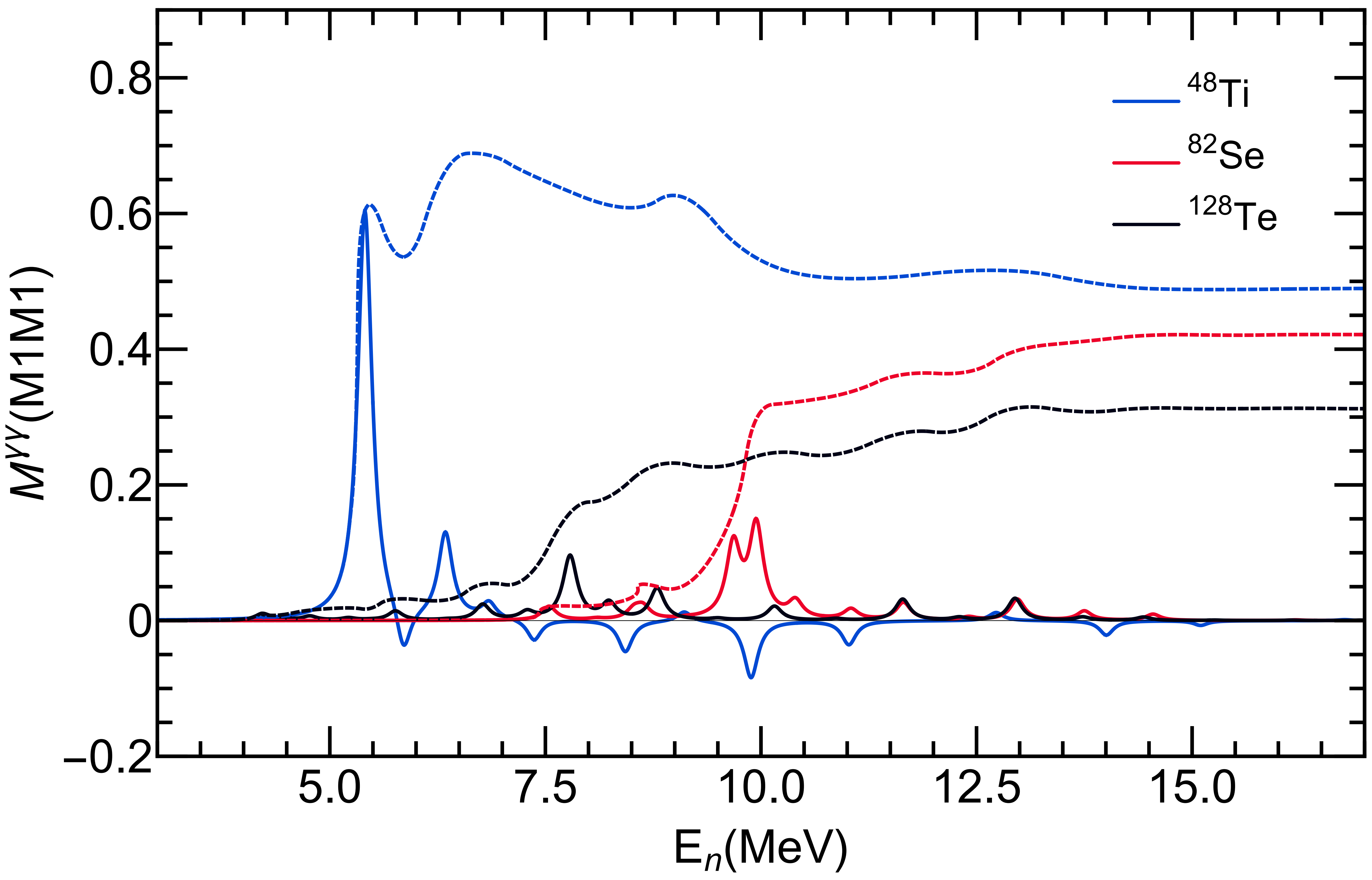
$$M^{\gamma\gamma}(M1M1) = \sum_{1_n^+} \frac{\langle 0_{gs}^+ \parallel \mathbf{M1} \parallel 1_n^+(IAS)\rangle\langle 1_n^+(IAS) \parallel \mathbf{M1} \parallel 0_1^+(DIAS)\rangle}{E_n - E_f + \frac{1}{2}(E_i - E_f)}$$





The value of the exact closure NME has been used as a **criteria** of good convergence (errors $\sim 1\,\%$)

Results: convergence

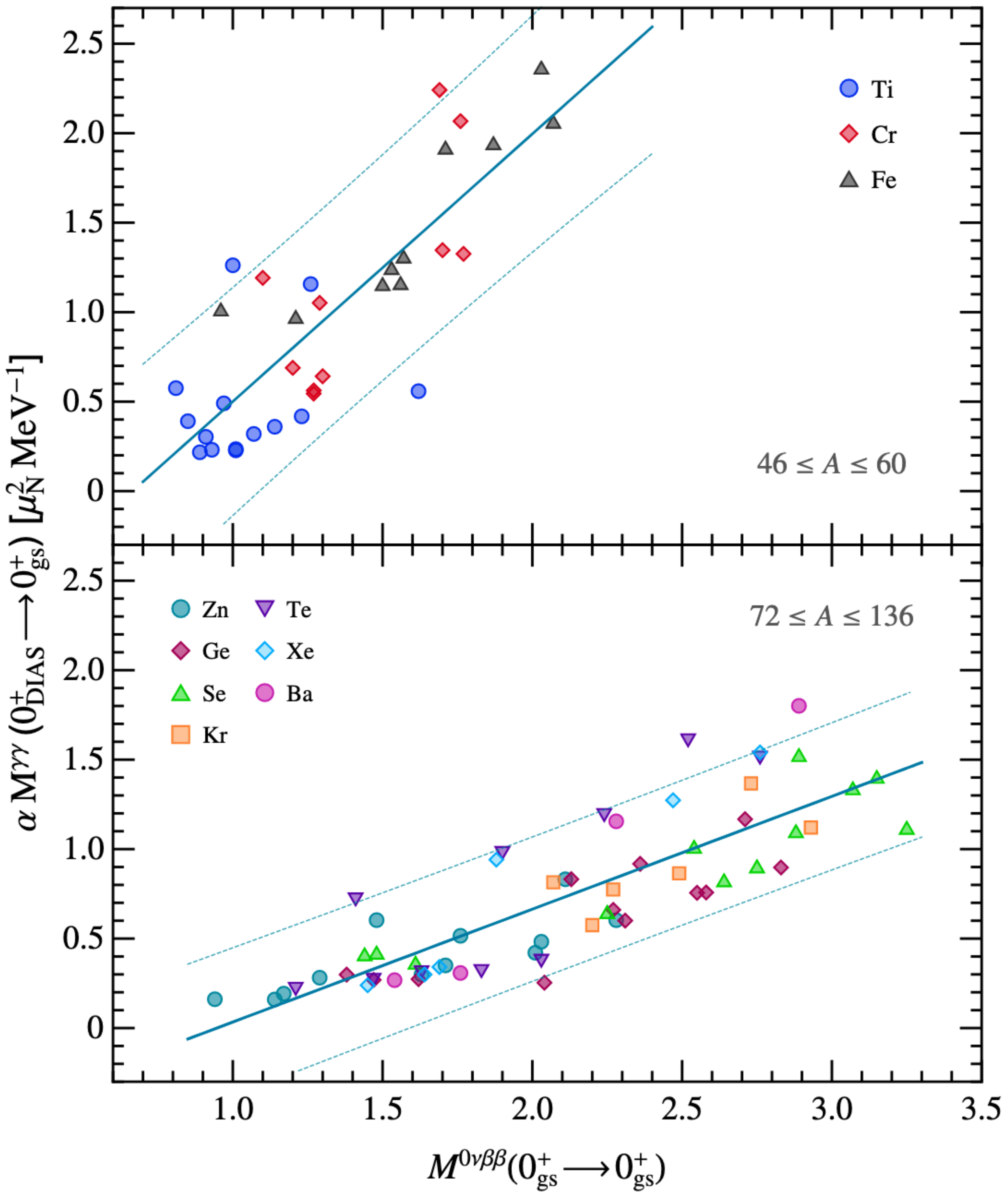


$$M^{\gamma\gamma}(M1M1) = \sum_{1_n^+} \frac{\langle 0^+_{gs} \parallel \mathbf{M1} \parallel 1^+_n \rangle \langle 1^+_n \parallel \mathbf{M1} \parallel 0_1^{1+}(DIAS) \rangle}{E_n - E_f + \frac{1}{2}(E_i - E_f)}$$

Evolution of $M^{\gamma\gamma}(M1M1)$ with the energy of the intermediate state (solid lines) and its accumulated value (dashed lines)

Few states contribute to the total ME

Results: $0\nu\beta\beta$-$\gamma\gamma$ correlation



A separate analysis give a very *good correlation coefficients*

$$\rho_{\rm pf} = 0.80 \quad \rho_{\rm pfg+gds} = 0.84$$

α come from apply Wigner-Eckart Th. to isospin space, both op's isospin tensor of same rank but different isospin projections

$$\frac{M^{0\nu\beta\beta}}{M^{\gamma\gamma}(M1M1)} = \alpha(T_f)\ \frac{\bar{M}^{0\nu\beta\beta}}{\bar{M}^{\gamma\gamma}(M1M1)}$$

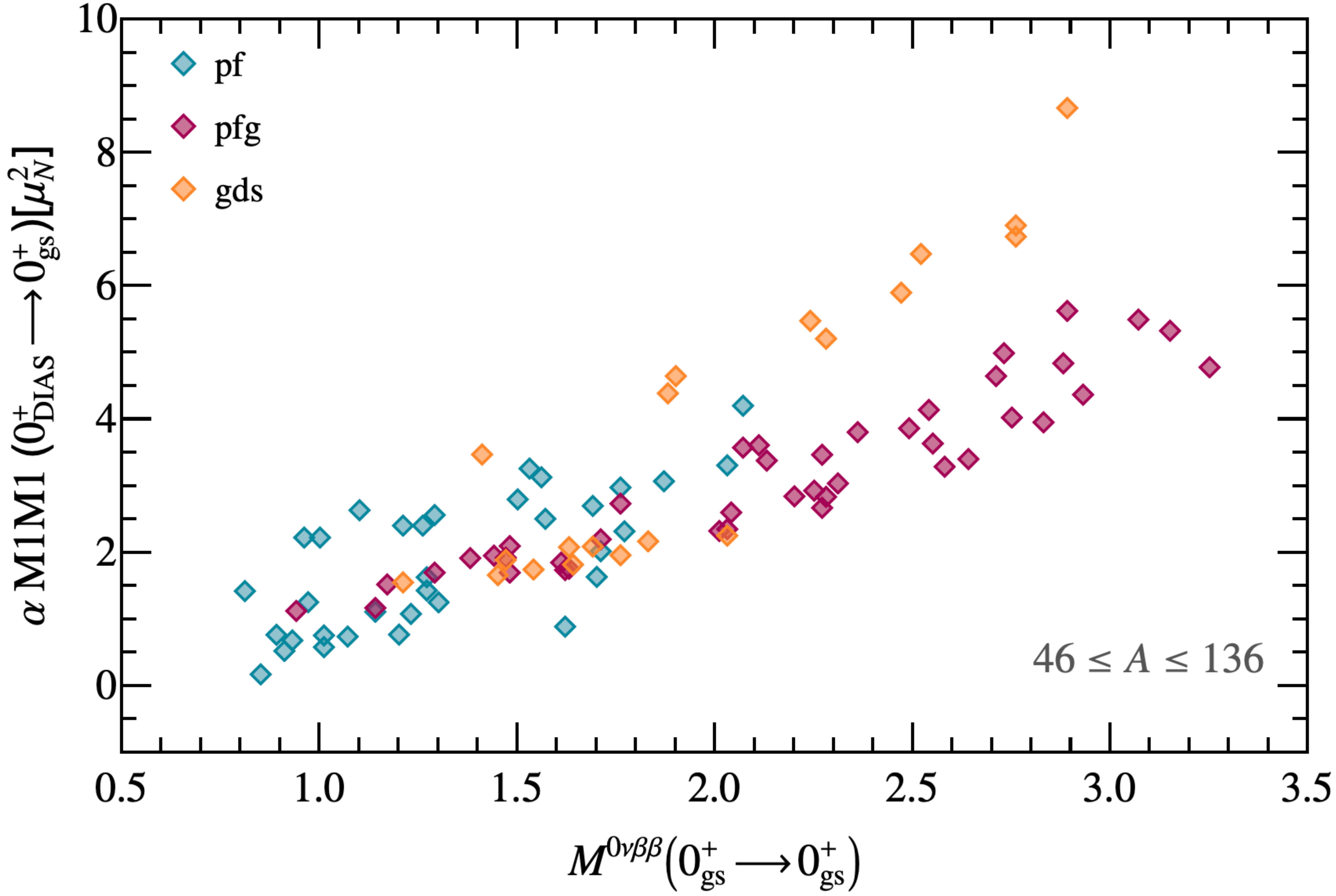
$\bar{M}$ (reduced in isospin space)

$$\alpha(T_f) = \frac{1}{2}\left[(2+T_f)(3+2T_f)\right]^{1/2}$$

Two different slopes

Behind is the energy denominator dominant states

Results: Double Magnetic Dipole NMEs



Behind this different slope it is the **energy denominator**

$$M^{\gamma\gamma}(M1M1) = \sum_{1_n} \frac{\langle 0_f^+ \| \mathbf{M1} \| 1_n^+ \rangle \langle 1_n^+ \| \mathbf{M1} \| 0_i'^+ \rangle}{E_n - E_f + \frac{1}{2}(E_i - E_f)}$$

Analysing only

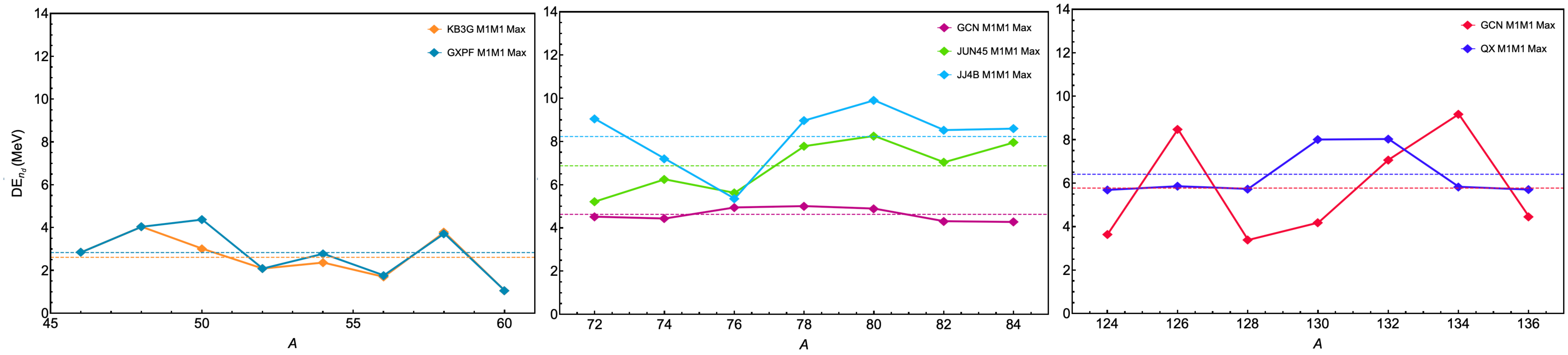
$$M1M1 = \sum_{1_n} \langle 0_f^+ \| \mathbf{M1} \| 1_n^+ \rangle \langle 1_n^+ \| \mathbf{M1} \| 0_i'^+ \rangle$$

All nuclei studied lie in the same correlation

Results: Dominant energy denominator

We had a few states that dominated the M1M1 op

DE_{n_d} represents the energy denominator for the dominant state (maximum M1M1matrix element)



Qualitatively, making the mean for the DE_{n_d} both in pf and pfg+gds

$$\frac{\overline{DE}_{n_d}(pfg+gds)}{\overline{DE}_{n_d}(pf)} = 2.32$$

Doing a separate linear fit to pf and pfg+gds nuclei, the ratio between slopes gives

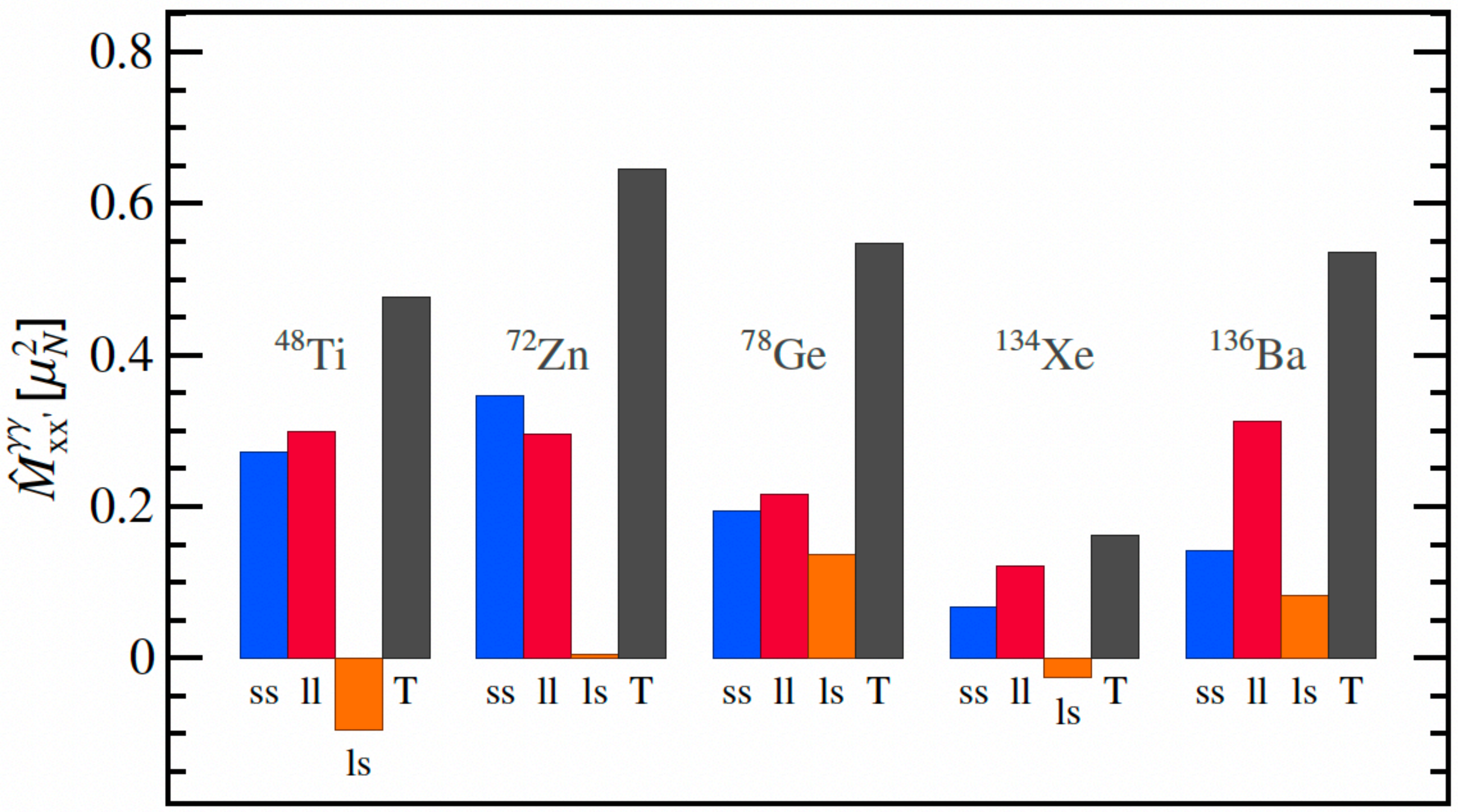
$$\frac{m_{pfg+gds}}{m_{pf}} = 2.31$$

Results: Spin Orbital and Interference contributions

Additional insight on the $\gamma\gamma$- $0\nu\beta\beta$ correlation by decomposing $\gamma\gamma$-M1M1 into spin, orbital and interference parts

Energy denominator plays a minor role we focus on M1M1(SS,LL,2LS)

$\hat{M}^{\gamma\gamma} = \hat{M}^{\gamma\gamma}_{\rm ss} + \hat{M}^{\gamma\gamma}_{\rm ll} + \hat{M}^{\gamma\gamma}_{\rm ls}$ and $\hat{M}^{\gamma\gamma}_{\rm ss} \propto \hat{M}^{DGT}$



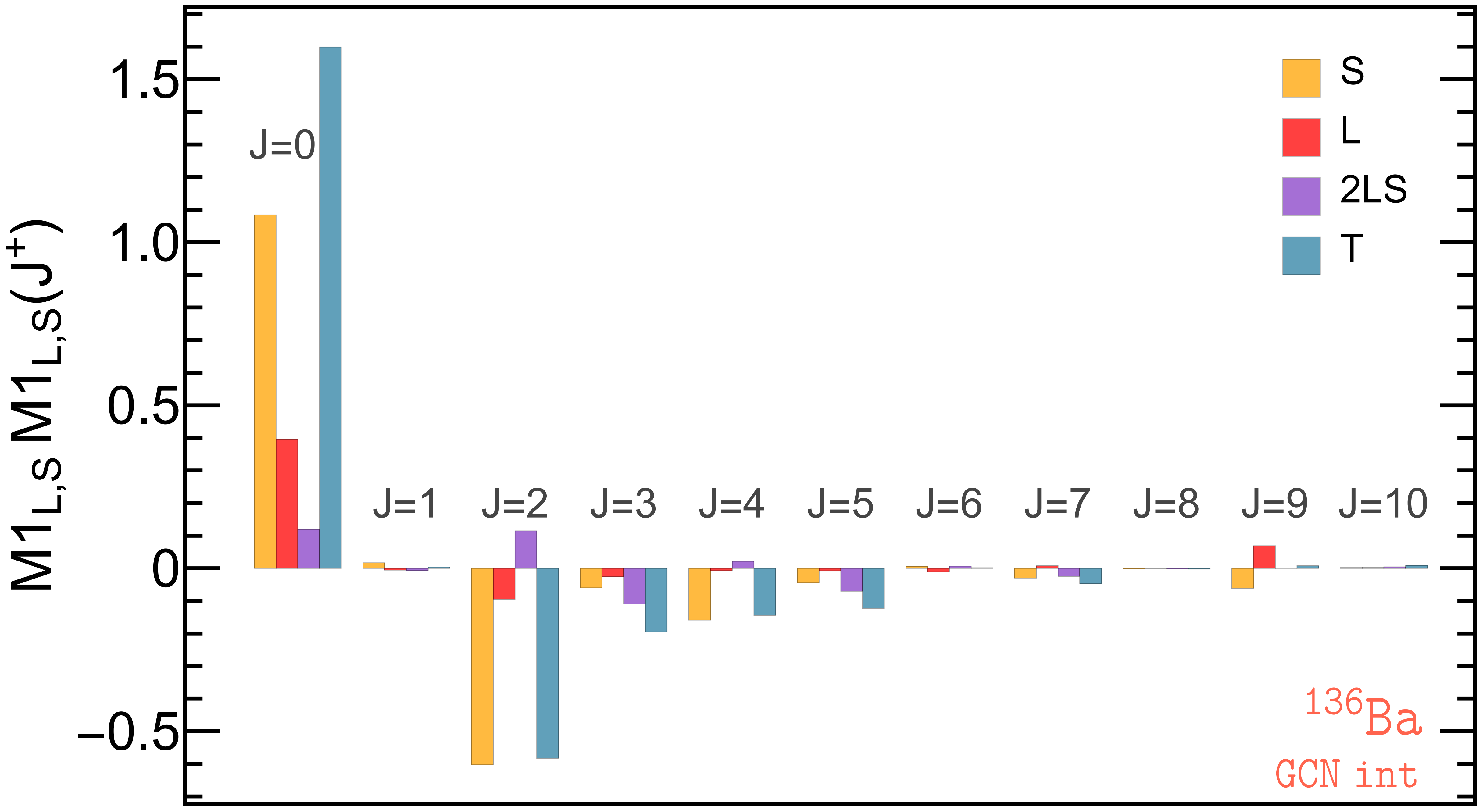
The orbital part represents a relevant contribution but is generally of the same order and sign than the spin

For Xe, Ba is L is greater but still lies in the correlation, the correlation with $0\nu\beta\beta$ is not limited to op's driven by nuclear spin

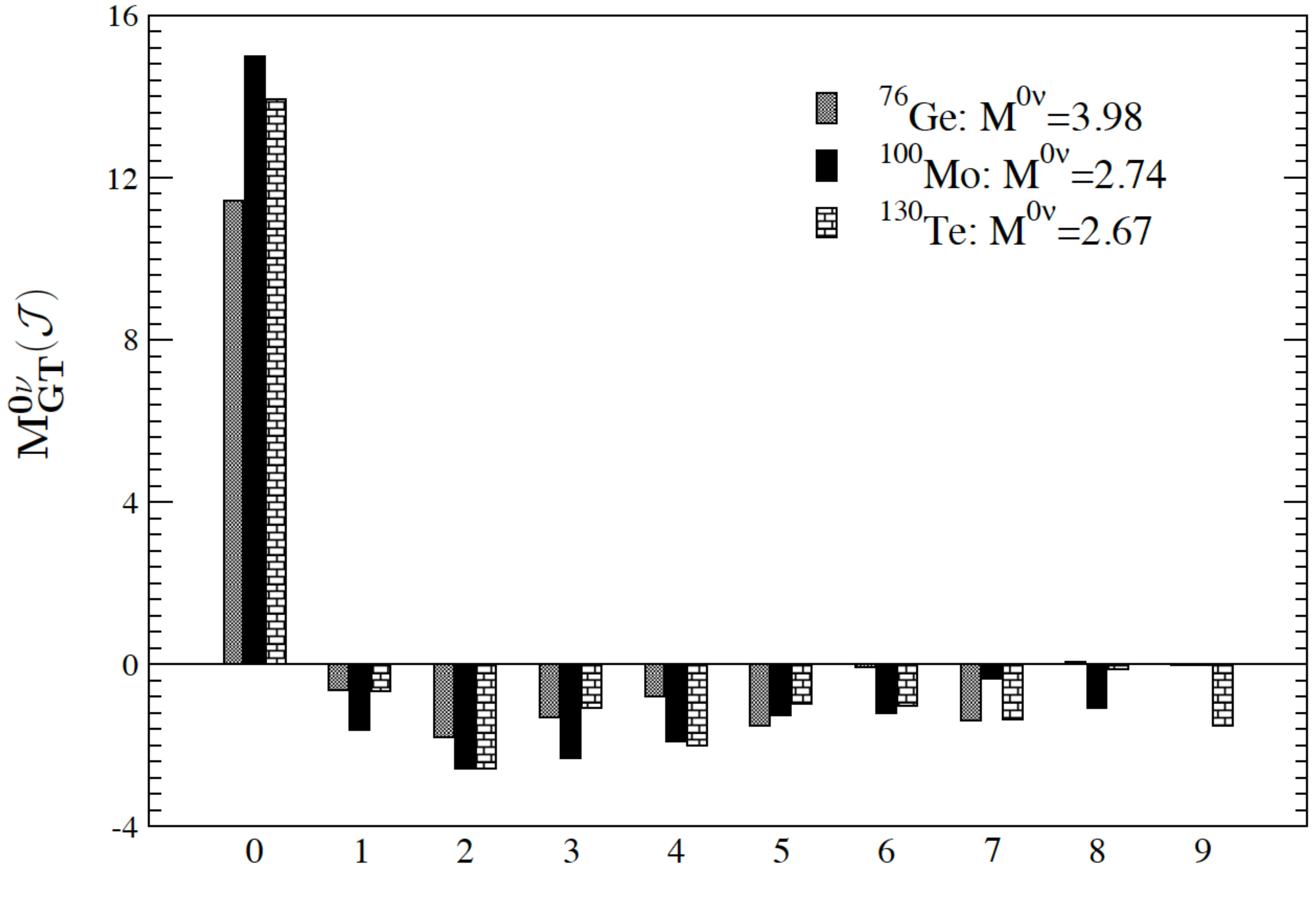
This behaviour is systematic in other nuclei that we have analysed

Results: J^P NME decomposition

For $M^{0\nu\beta\beta}$ is revealing the J^P decomposition, nn and pp coupled to J^P



$$M1_{L,S}M1_{L,S} = \langle 0_f^+ \parallel \mathbf{M1}_{L,S}\mathbf{M1}_{L,S} \parallel 0_1^+(DIAS)\rangle$$

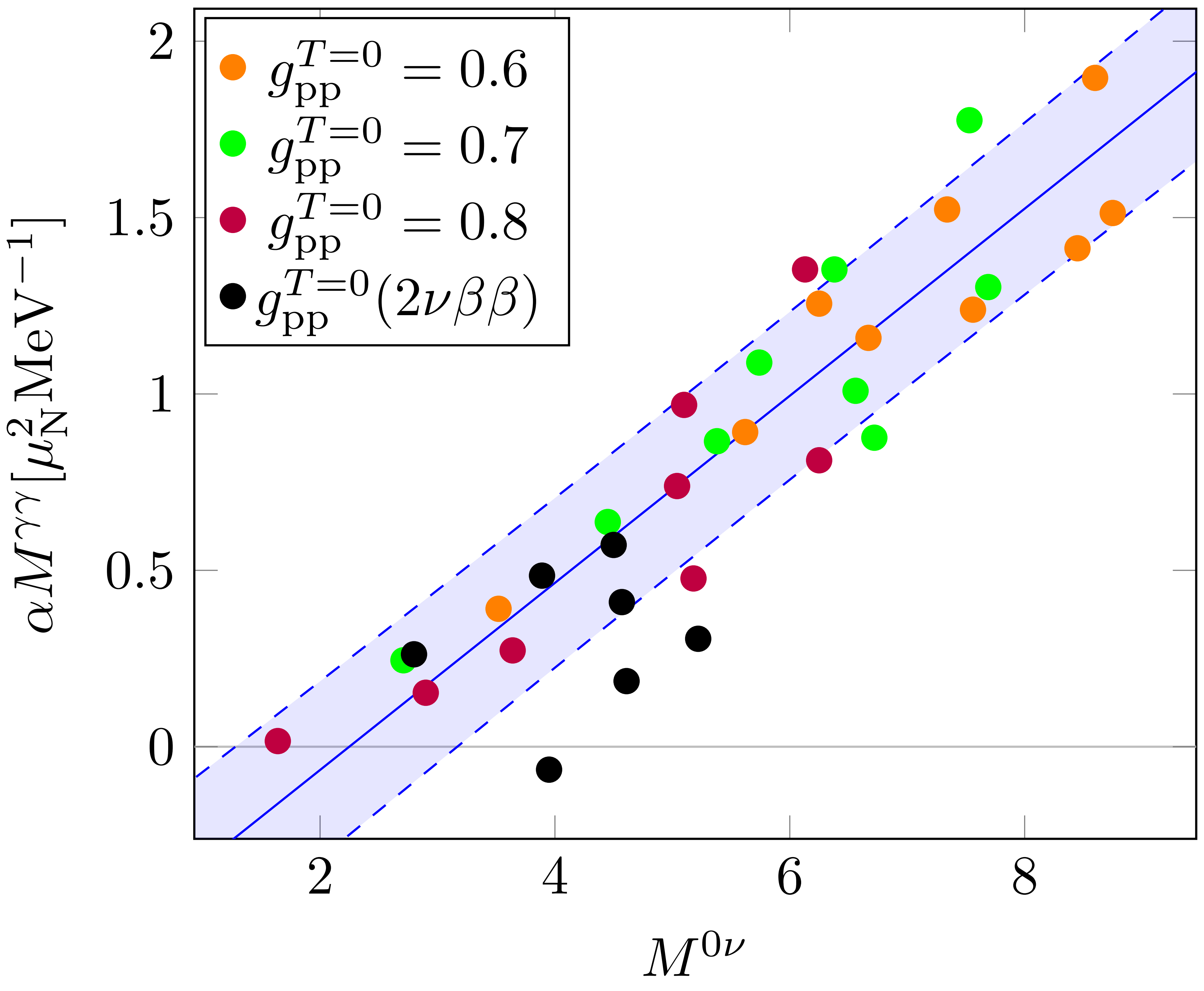


[Simk08]

The spin component is the dominant contribution for J=0 pairs as in $M^{0\nu}$

But also it is where the strongest cancellation is observed

Results: $0\nu\beta\beta$-$\gamma\gamma$ correlation in QRPA



Lotta Jokiniemi has found also a very good correlation $R^2 = 0.80$ in spherical pnQRPA

with different values of particle-particle parameter $g_{\mathrm{pp}}^{\mathrm{T=0}}$ for A=76,82,116,128,130 and 136

Since isospin is not a good quantum number in QRPA, not able to describe DIAS

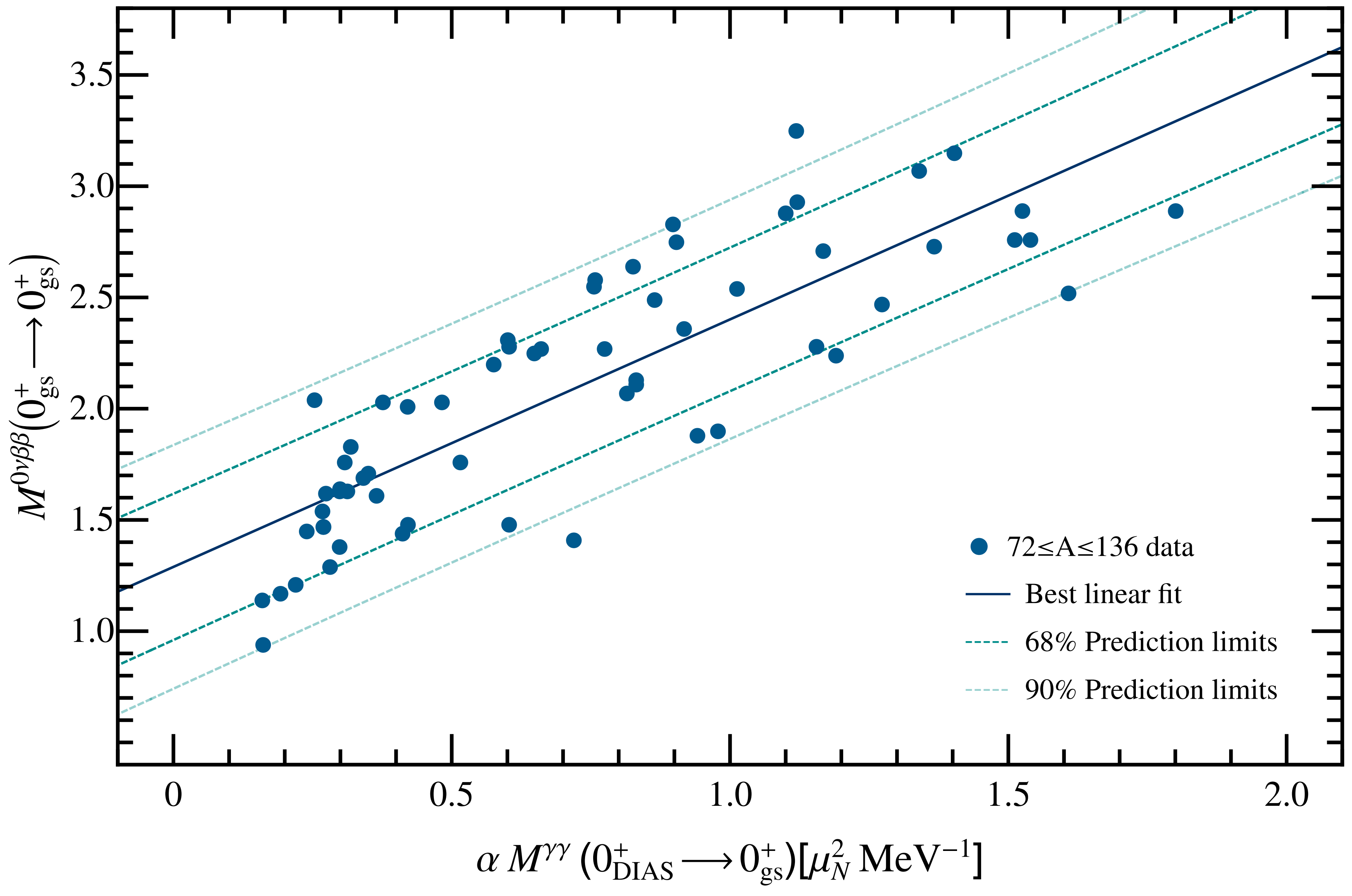
M1M1 decay calculated as charge-changing transitions between the different isotopes (between the initial/final even-even nucleus and the intermediate odd-odd nucleus of the double-beta-decay triplet)

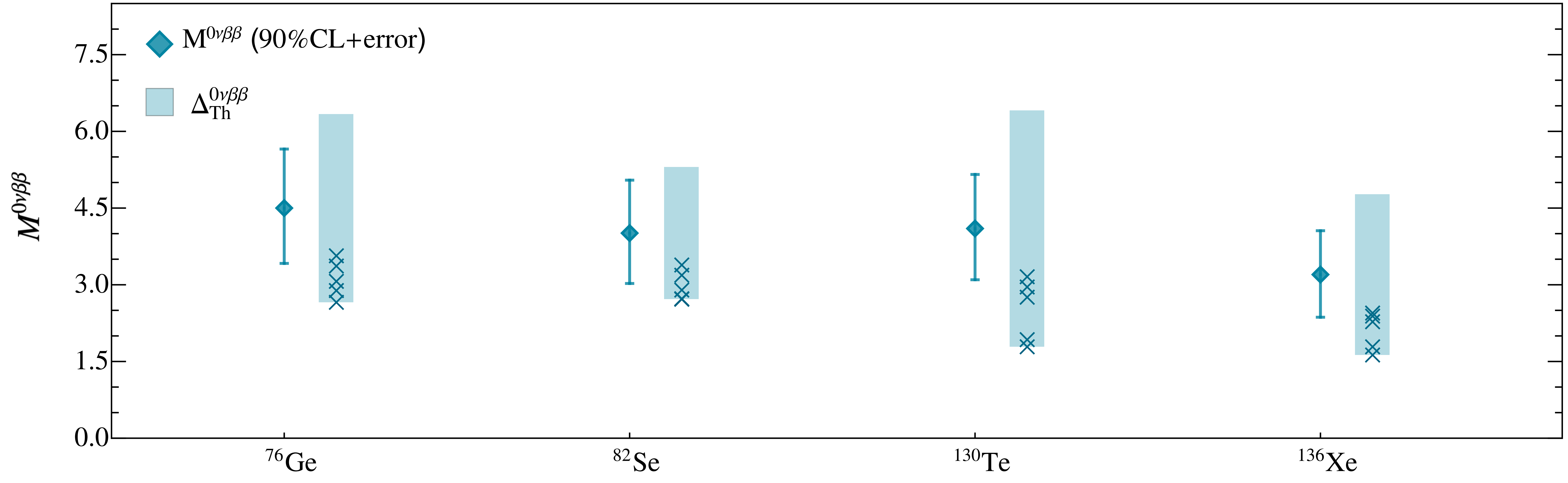
Results: potential of measuring $2\gamma(M1M1)$

Using linear regression equation $M^{0\nu} = a + bM^{\gamma\gamma}$ and prediction bands we want to predict $0\nu\beta\beta$ NMEs from a hypothetical measurement of $\gamma\gamma(M1M1)$ from DIAS to GS

Give the uncertainty in each of these single estimated NMEs, have an estimate of what we would expect from single measure of $0\nu\beta\beta$

Assume $\gamma\gamma(M1M1)$ measurement can be achieve with $\pm 15\,\%$ uncertainty in the BR (as in [Wal15, Söd20])



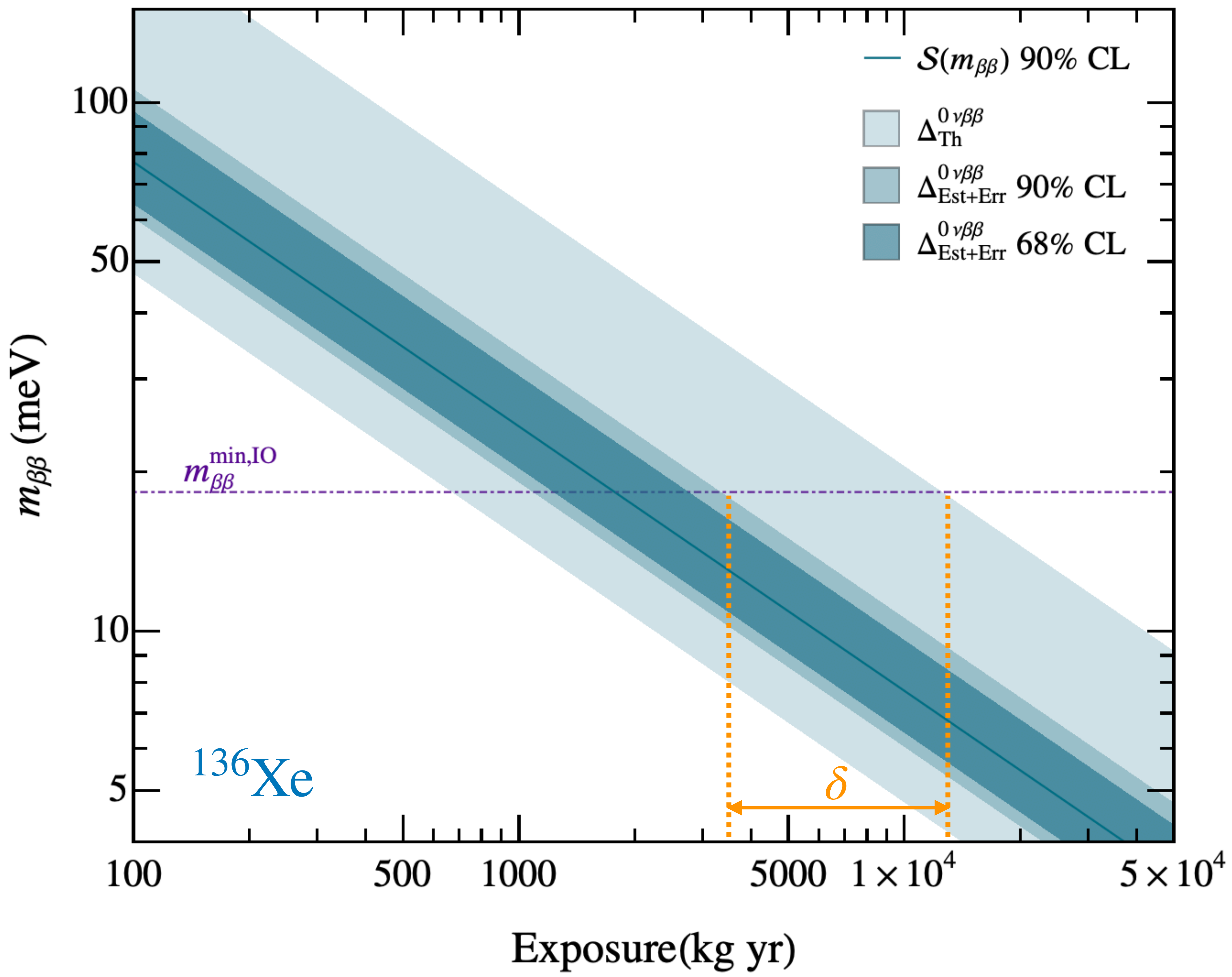


"×" ShM NMEs literature

Shaded bands combined calculated NME (SM,EDF,IBM)

2202.01787 [hep-ex]

Results: potential of measuring $2\gamma(M1M1)$



Sensitivity to $m_{\beta\beta}$ $\quad \mathcal{S}(m_{\beta\beta}) = \mathcal{K}\sqrt{\dfrac{\bar{N}}{Mt}}$

$\mathcal{K}$ bb isotope dependent

$\bar{N}$ average upper limit on the number of events expected under no-signal hypothesis

Minimum effective mass parameter from PDG

$$m^{\mathrm{min,IO}}_{\beta\beta} = 18.4 \pm 1.3eV$$

Quantitative reduction δ in the exposure to completely cover the inverted ordering region $\sim 9000\mathrm{kg}\cdot\mathrm{yr}$

Outlook

This correlation suggest a new avenue to reduce $0\nu\beta\beta$ NMEs uncertainties if $2\gamma(M1M1)$ DIAS to GS can be measured

Relation between em decays from IAS and ew decays has been analysed and measured previously [Eji68,Fuj11,Eji19]

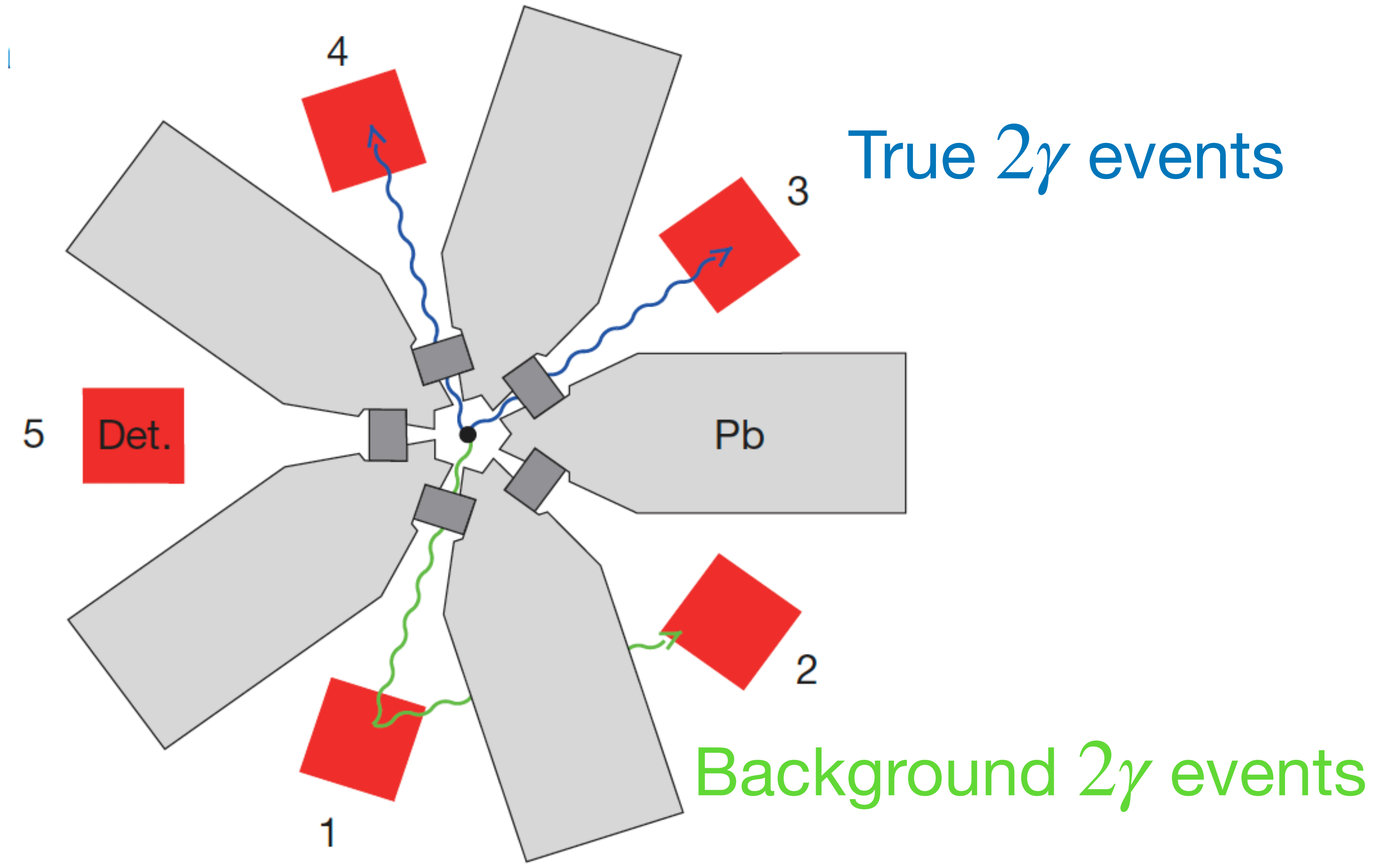
Measurements of second order *em* decays are difficult but they have been done (^{16}O,^{40}Ca,^{90}Zr in $0^+_2 \longrightarrow 0^+_1$), and recently the competitive $2\gamma/\gamma$ decay $11^+/2 \longrightarrow 3^+/2$ in ^{137}Cs has been observed [Wal15, Söd20]

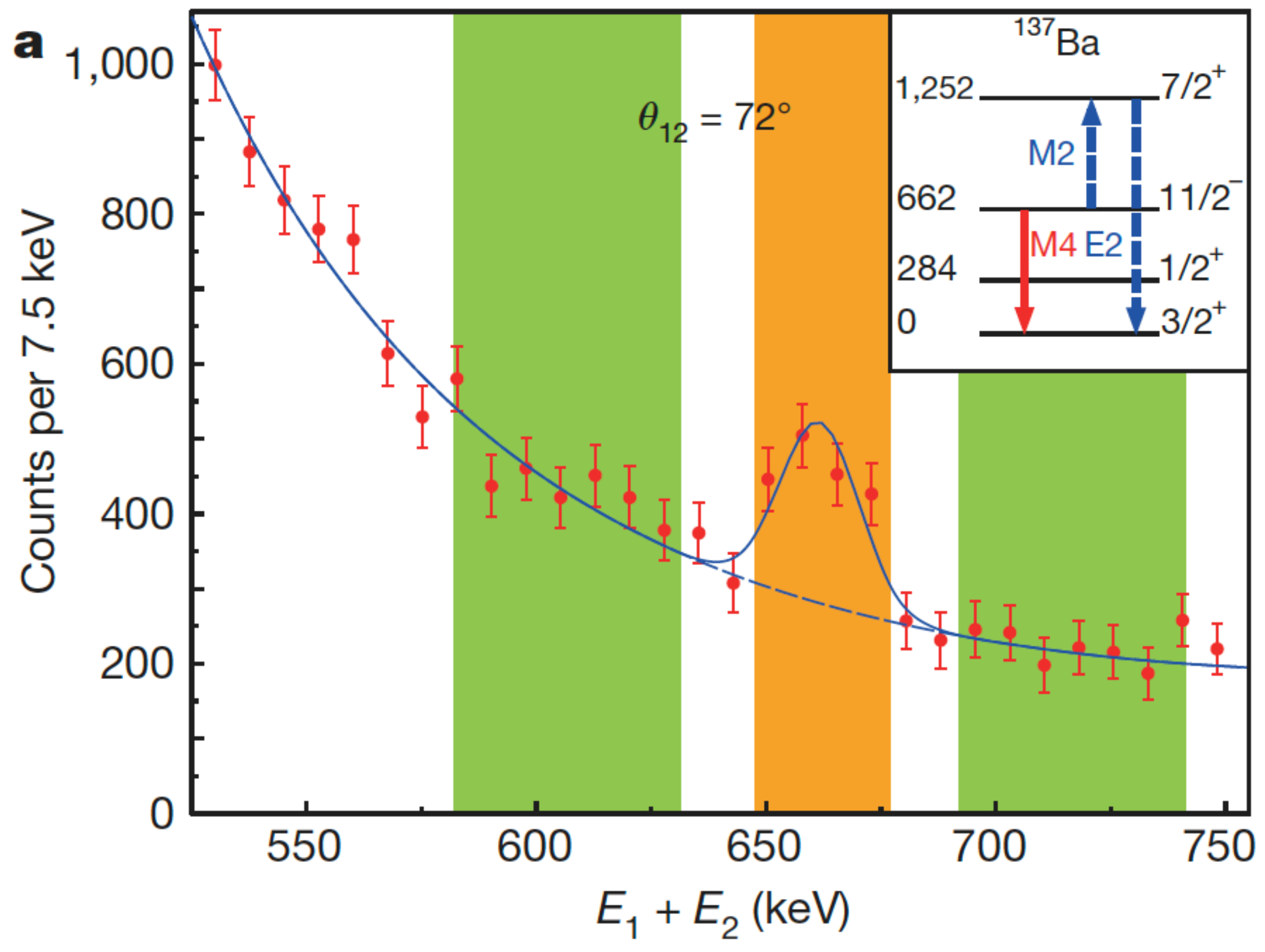
$\Gamma_{\gamma\gamma}/\Gamma \simeq 10^{-4}$ $\quad$ $0^+_2 \longrightarrow 0^+_1$

$\Gamma_{\gamma\gamma}/\Gamma_\gamma \simeq 10^{-6}$ $\quad$ $11^+/2 \longrightarrow 3^+/2$

$\Gamma_{\gamma\gamma}/\Gamma \simeq 10^{-8} - 10^{-9}$ $\quad$ $0^+(DIAS) \longrightarrow 0^+_{gs}$

0(DIAS) lies above $S_{p,n}$, can decay via p,n emission but is isospin forbidden





[Wal15]

Sumary & Outlook

A good correlation between $0\nu\beta\beta$ and $2\gamma(M1M1)$ for a large number of nuclei in different model spaces and different effective nuclear interactions

Nuclei in the ***pf*** model space (A~50) follow a correlation with different slope than nuclei in ***pfg*** (A~80) and ***sdgh*** (A~130) model spaces; behind energy denominator of the dominant intermediate states

This correlation is present although the dominant contribution of $2\gamma(M1M1)$ is not always the spin part, the orbital part is of the same order and sign than the spin part

The correlation is present in a different nuclear many-body method pnQRPA

Future work try to understand different correlations observed by many-body methods (SM,QRPA)

Study $2\gamma(M1M1)$ NMEs using ab-initio many body methods (VS-IMSRG) and evaluate the effect of two-nucleon current contribution to M1M1


Thank you!

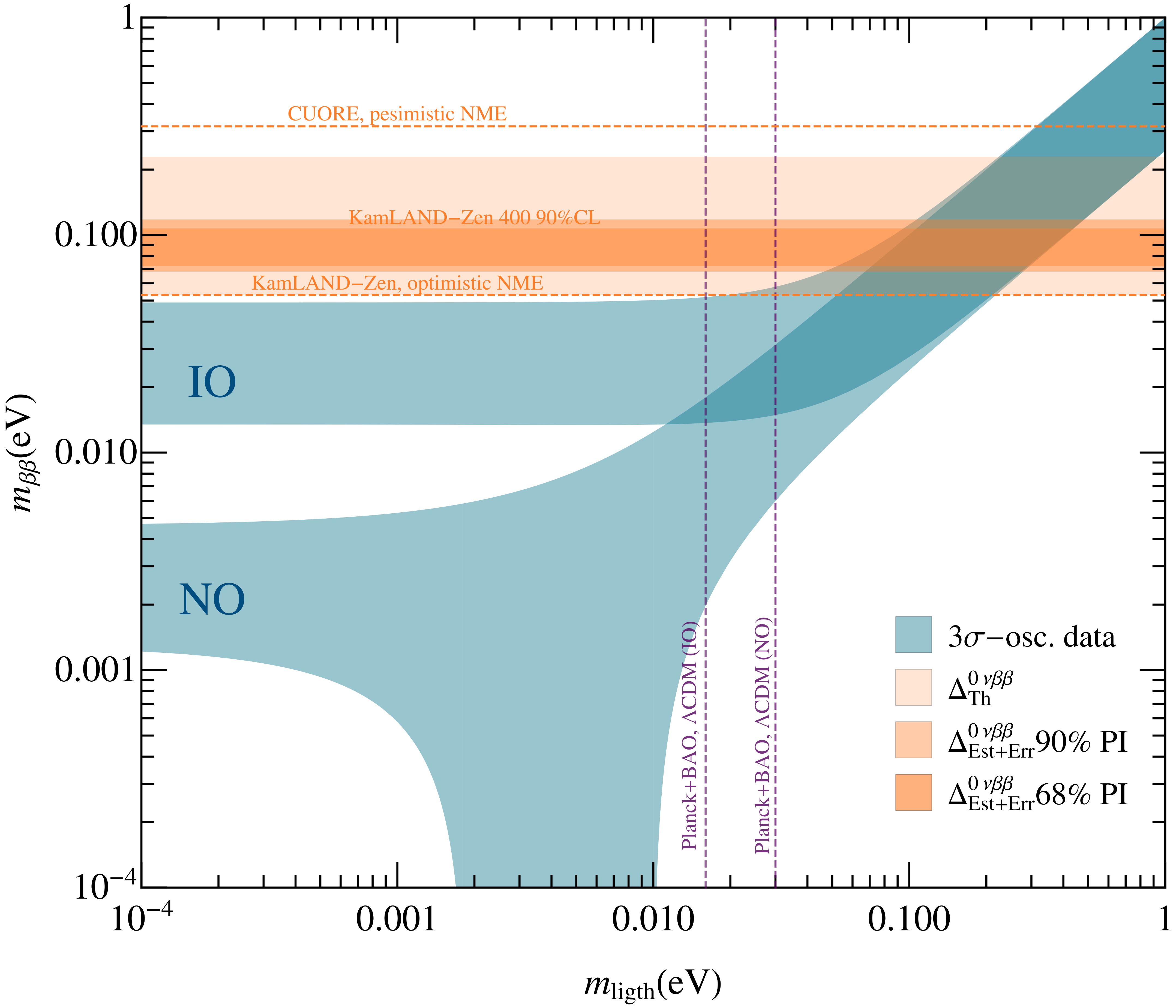
Results: potential of measuring $2\gamma(M1M1)$

Horizontal dashed lines show 90%CL current upper limits from $0\nu\beta\beta$ searches

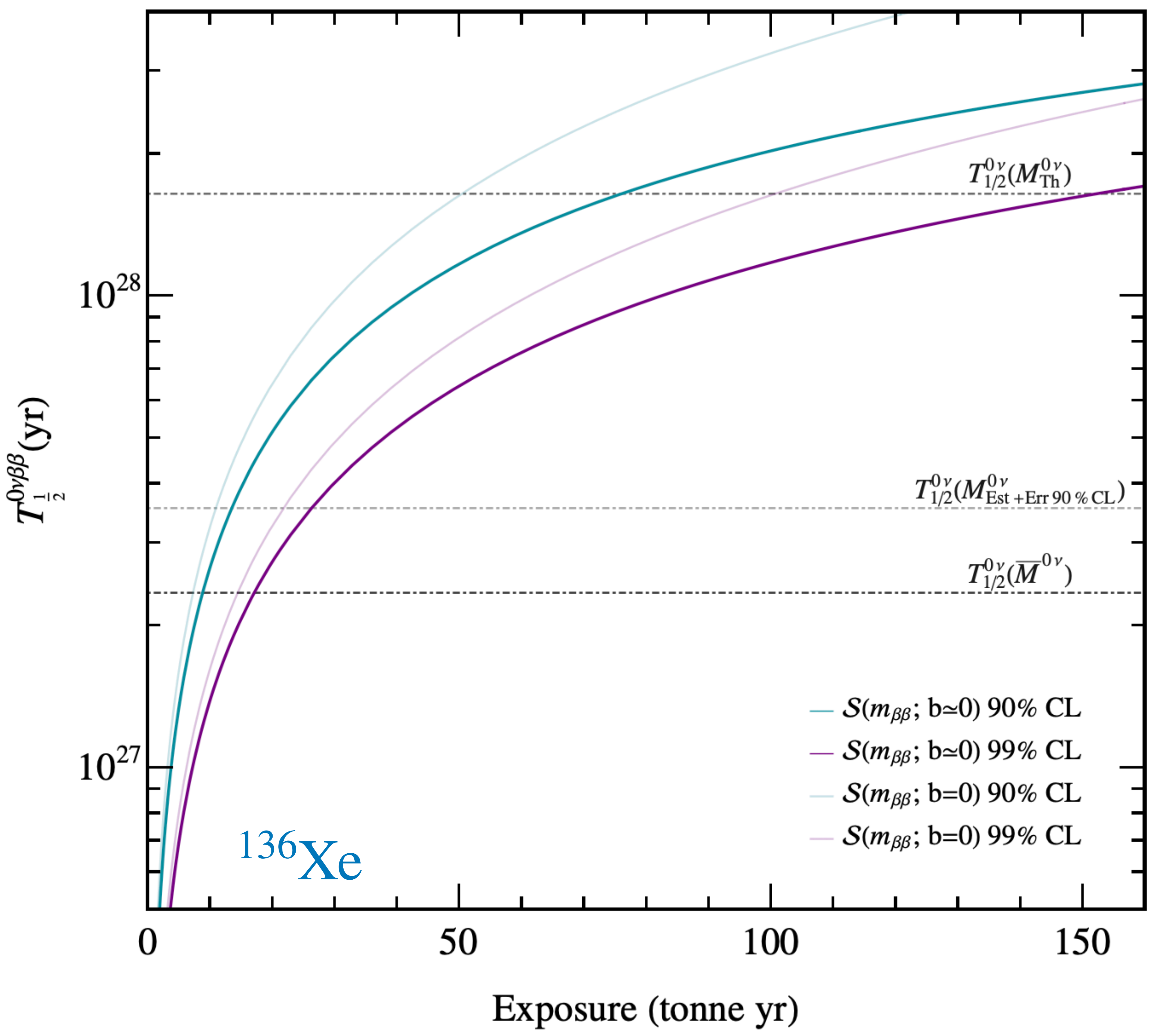
Tightest and loosest limits among those reported in the literature, most stringent from KamLAND-Zen (with largest NMEs) and less stringent from CUORE (with lowest NMEs)

Orange lightest shaded area: uncertainty current calculations

Orange darker shaded areas: uncertainty estimated from correlation at 68(90)%CL



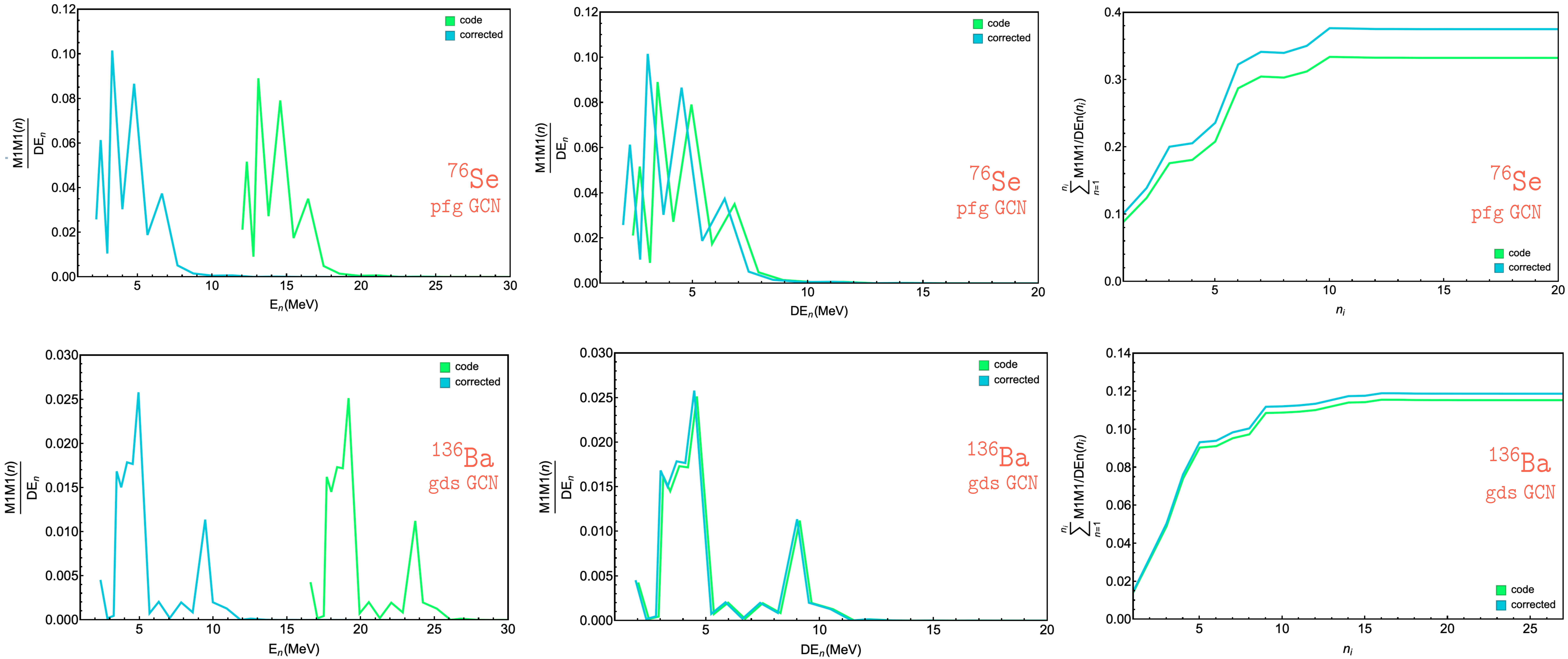
Results: potential of measuring $2\gamma(M1M1)$



In terms of the half-life for a **low-background** (dark colors) and an **ideal**(light colors) experiment

A reduction of $\sim 63(125)\mathrm{kg}\cdot\mathrm{yr}$ to the sensitivity to the half-life at 90(99)%CL

Energy correction



Double gamma decay contact amplitude

The contact amplitude is proportional to

$$P_J^{sg}(S'L'SL, k_0, k_0') = (2\pi)(-1)^J \frac{\hat{L}\hat{L}'}{\hat{J}} \langle J_f || \int d^3\mathbf{x} d^3\mathbf{y} (-1)^{\Delta_{\mu 0}+\Delta_{\nu 0}} \hat{B}_{\mu\nu}(\mathbf{x},\mathbf{y}) \left[\widetilde{A}^{\mu}_{L'} \times \widetilde{A}^{\nu}_{L} \right]_J || J_i \rangle$$

Point-like nucleons, absence of exchange currents

$$B_{\mu\nu}(\mathbf{x},\mathbf{y}) = \frac{\delta_{\mu\nu}}{m} \sum_{i=1}^{A} e^2(i) \delta^{(3)}(\mathbf{x}-\mathbf{x}_i)(\mathbf{x},\mathbf{y}) \delta^{(3)}(\mathbf{y}-\mathbf{x}_i) \longrightarrow P_J^{sg}(ML'ML, k_0, k_0') = O(k_0, k_0', L, L', J) \langle J_f || \underbrace{\sum_{i=1}^{A} \frac{e^2(i)}{m} r_i^{L+L'} Y_J(\Omega_i)}_{\displaystyle =: O^{SG}_{LL'J}} || J_i \rangle$$

$$\delta_{00} = \delta_{0k} = 0$$

$B_{ij}(\mathbf{x},\mathbf{y}) \rightarrow$ Only magnetic multipoles contribute

$$\widetilde{\mathbf{A}}_{L,M}(E, k_0, \mathbf{x}) = 0 \qquad \widetilde{\mathbf{A}}_{L,M}(M, k_0, \mathbf{x}) \neq 0$$

Nuclear shell model analysis of M1M1 contact term

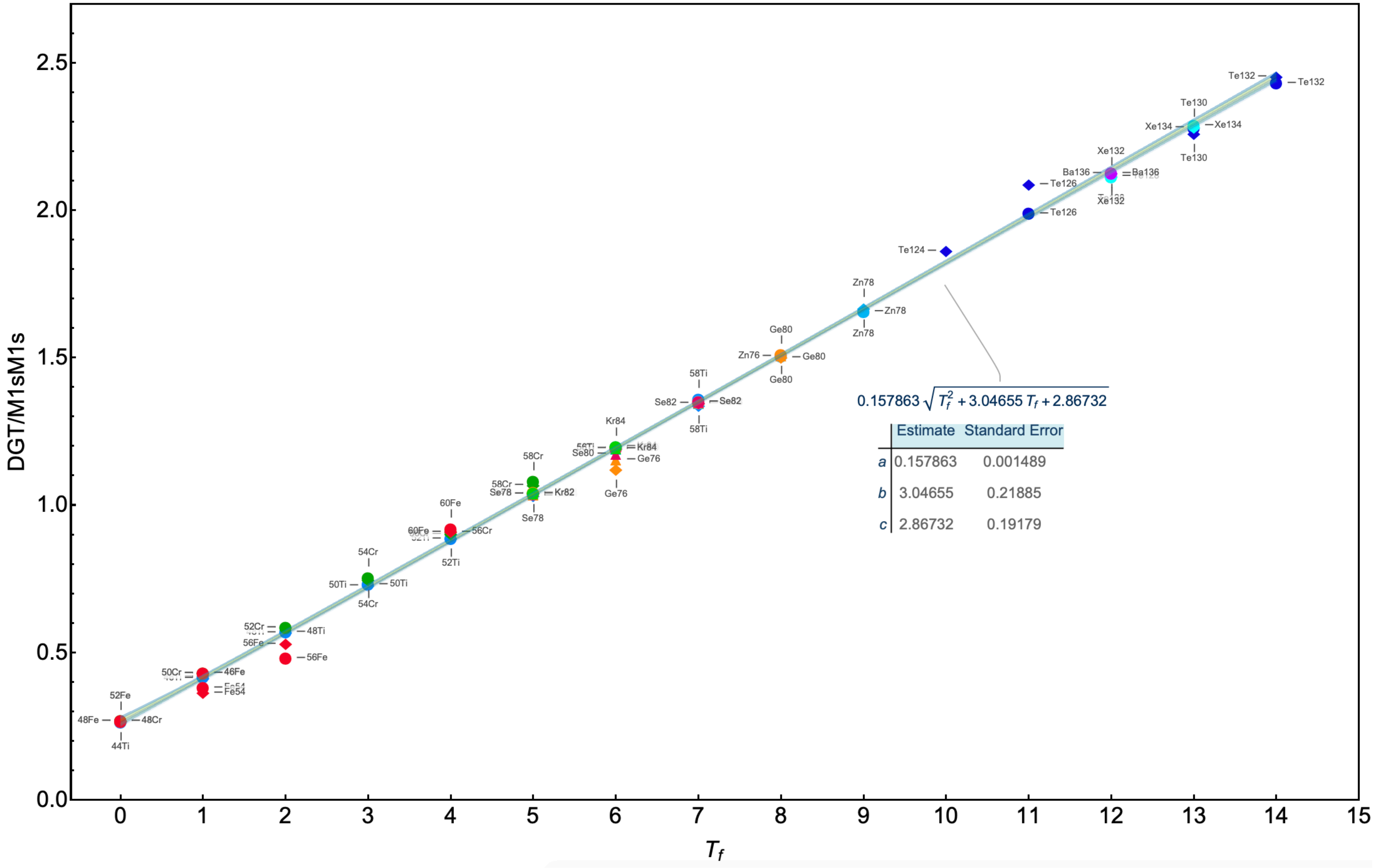
Steps followed to obtain $M^{\gamma\gamma}_{sg}(M1M1)$

$$M^{\gamma\gamma}_{sg}(2M1) = \sum_{a,b} (a \parallel \mathbf{O}^{\mathbf{sg}}_{\mathbf{1}} \parallel b)\langle 0^+_f \parallel [c^\dagger_\alpha \tilde{c}_\beta]_0 \parallel 0^+_i(DIAS)\rangle$$

The steps would be the same but without obtaining intermediate estates

However the isospin tensor structure makes not possible to connect $|0^+_i(DIAS)\rangle$ and $|0^+_f\rangle$ $\longrightarrow$ $M^{\gamma\gamma}_{sg}(M1M1) \ll 1$

DGT and M1sM1s



We are comparing two second order reduced (in J) nuclear matrix elements

$$\frac{M^{0\nu\beta\beta}}{M^{\gamma\gamma}_{res}(M1M1)} = \alpha(T_f)\,\frac{\bar{M}^{0\nu\beta\beta}}{\bar{M}^{\gamma\gamma}_{res}(M1M1)}$$

$$0^+_{gs} \longrightarrow 0^+_{gs} \qquad \alpha(T_f) = \frac{1}{2}\left[(2+T_f)(3+2T_f)\right]^{1/2}$$

For the pure spin part of M1M1, we have

$$\frac{M^{DGT}}{M^{\gamma\gamma}_{res}(M1_sM1_s)} = \alpha(T_f)\frac{\bar{M}^{DGT}}{\bar{M}^{\gamma\gamma}_{res}(M1_sM1_s)}$$

$$= \frac{4\pi\sqrt{6}}{9g^2_{s,IV}}\left[T_f^2 + \frac{7}{2}T_f + 3\right]^{1/2}$$

$$\frac{4\pi\sqrt{6}}{9g^2_{s,IV}} = 0.1544$$

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