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Double beta decay: Standard mass mechanism and beyond

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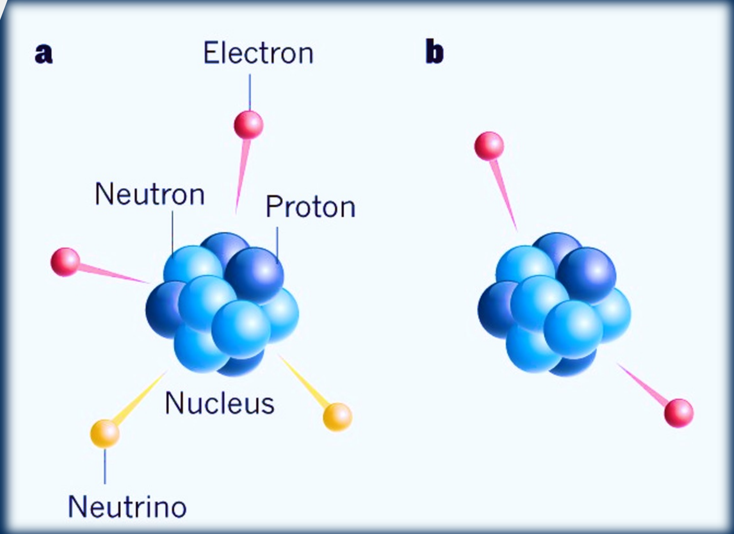
OUTLINE

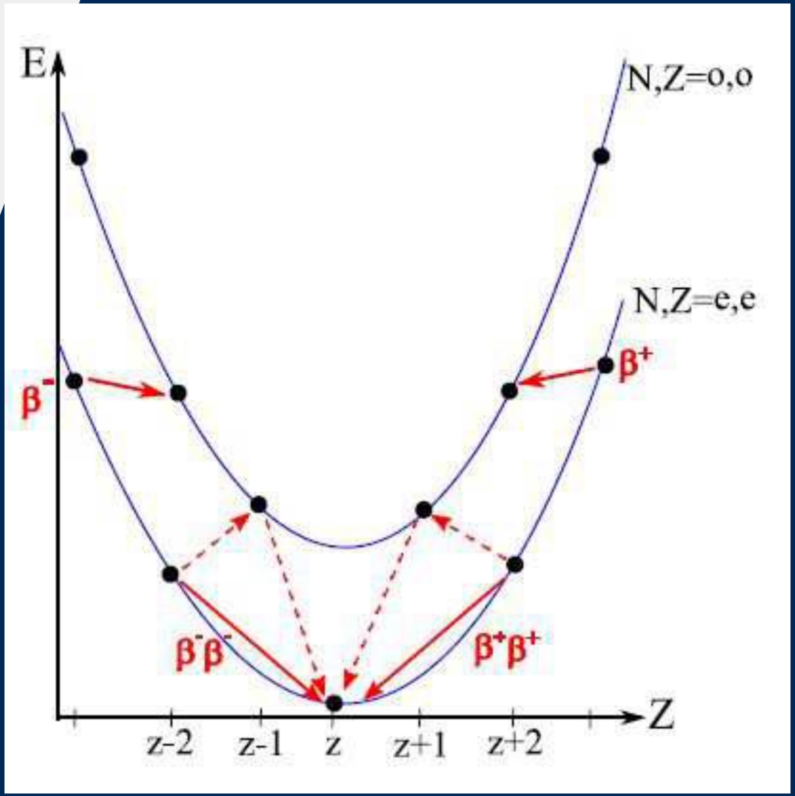
- Motivation/History
- Theoretical ingredients
- Standard mass mechanism
- General description
- Sterile neutrinos
- Left-Right models
- Susy models
- Majoron emitting DBD
- Conclusions



$\beta\beta$-decay: History

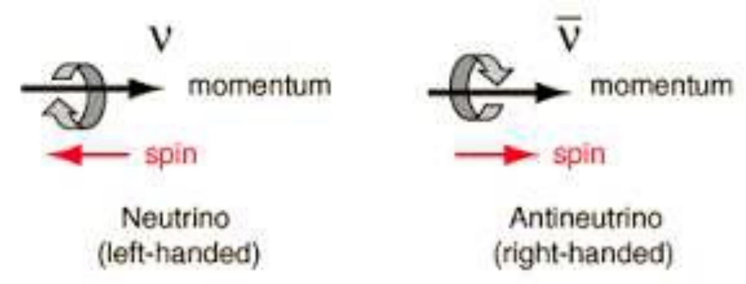
- 1930's: The idea of double beta decay was suggested by Eugene Paul Wigner as a second-order weak transition between isobars differing by two units in atomic number

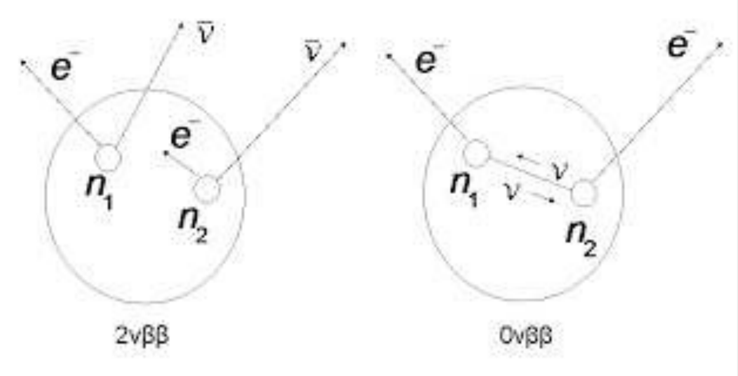
- 1935: Assuming the emission of two electrons and two neutrinos Maria Goeppert-Mayer made the first theoretical estimate of the extremely low rates for this process $\tau^{1/2} > 10^{20}$yr



$\beta\beta$-decay: **History**

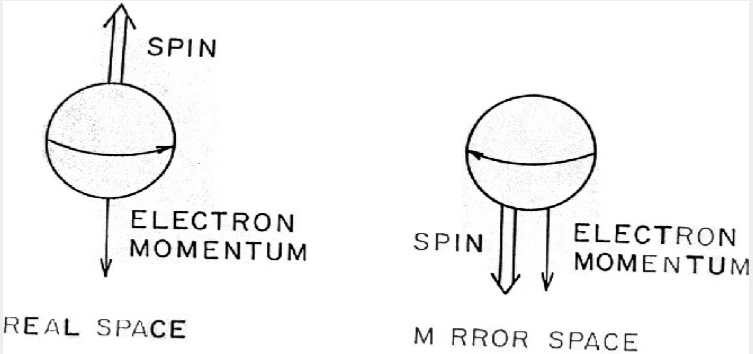
- 1937: Ettore Majorana demonstrated that all results of beta decay theory remain unchanged if neutrino is its own antiparticle (Majorana particle)
- 1939: Wendell H. Furry proposed that if neutrinos are Majorana particles, then double beta decay could proceed without the emission of any neutrinos ($0\nu\beta\beta$)
- In 1940's the predicted half-lives were of the order of 10^{15-16} years, and $0\nu\beta\beta$ was thought to be more likely to occur than $2\nu\beta\beta$





$\beta\beta$-decay: **History**

- 1948: Edward L. Fireman tried to measure the $\beta\beta$-decay half-life of ^{124}Sn with Geiger counter, without success ($\tau_{1/2}^{2\nu\beta\beta} > 3 \times 10^{15}$yr)
- 1950: $\beta\beta$-decay half-life of 1.4×10^{21}yr for ^{130}Te was measured by geochemical methods
- 1956: Parity violation in weak interactions was established and it became clear that $2\nu\beta\beta$-decay would be much more likely to occur than $0\nu\beta\beta$-decay



$\beta\beta$-decay: **History**

- 1987: First observation of $2\nu\beta\beta$-decay in laboratory by Elliott, Hahn, and Moe: $\tau_{1/2}^{2\nu\beta\beta}$ (^{82}Se)=$1.1^{+0.8}_{-0.3}$ x 10^{20}yr

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Direct Evidence for Two-Neutrino Double-Beta Decay in ^{82}Se

S. R. Elliott, A. A. Hahn, and M. K. Moe
Department of Physics, University of California, Irvine, Irvine, California 92717
(Received 31 August 1987)

The two-neutrino mode of double-beta decay in ^{82}Se has been observed in a time-projection chamber at a half-life of $(1.1^{+0.8}_{-0.3})\times 10^{20}$ yr (68% confidence level). This result from direct counting confirms the earlier geochemical measurements and helps provide a standard by which to test the double-beta-decay matrix elements of nuclear theory. It is the rarest natural decay process ever observed directly in the laboratory.

- Since then, $2\nu\beta\beta$-decay has been observed in laboratory in 12 different nuclei in several different experiments, with half-lives of $10^{18–22}$yr
- 2001: A sub-group of the Heidelberg-Moscow experiment claimed first evidence for $0\nu\beta\beta$-decay. This remains unconfirmed.

And now…

- $0\nu\beta\beta$-decay remains unobserved and continues to intrigue both theorists and experimentalists
- It has unique potential for neutrino physics, beyond Standard Model physics, and the understanding of matter-antimatter asymmetry of the universe.
- It remains the most sensitive probe to test lepton number and to answer the following open questions:
  - What is the absolute neutrino mass scale?
  - Are neutrinos Dirac or Majorana particles?
  - How many neutrino species are there?
- Current experiments are reporting lower half-life limits up to the order of 10^{26} yr

Theoretical ingredients

- The half-life triggered by a single mechanism may be generically expressed

$$T_{1/2}^{-1} = |\epsilon_I|^2 G_I |\mathcal{M}_I|^2$$

- **G** is the phase space factor which varies depending on the decaying nucleus, Q-value of the decay, as well as the, mechanism of the decay
- **M** is the nuclear matrix element which is calculated using chosen theoretical model. The model gives the wave functions of the initial and final states, and they are connected by proper transition operator, that varies depending on the mechanisms of the decay
- The coupling constant ε parametrizes the underlying particle physics dynamics and contains the physics beyond standard model
  - Different for different mechanisms: exchange of light or heavy neutrino, emission of Majoron(s), contribution of sterile neutrino(s)…

Calculation of phase space factor G

- The key ingredient for the evaluation of phase space factors are the electron wave functions $e_s(\epsilon,\mathbf{r}) = e_s^{S_{1/2}}(\epsilon,\mathbf{r}) + e_s^{P_{1/2}}(\epsilon,\mathbf{r}) + e_s^{P_{3/2}}(\epsilon,\mathbf{r}) + \cdots$

$$e_s^{S_{1/2}}(\epsilon,\mathbf{r}) = \begin{pmatrix} g_{-1}(\epsilon,r)\chi_s \\ f_1(\epsilon,r)(\hat{\mathbf{p}}\cdot\vec{\sigma})\chi_s \end{pmatrix},$$

$$e_s^{P_{1/2}}(\epsilon,\mathbf{r}) = \begin{pmatrix} ig_1(\epsilon,r)(\hat{\mathbf{r}}\cdot\vec{\sigma})(\hat{\mathbf{p}}\cdot\vec{\sigma})\chi_s \\ -if_{-1}(\epsilon,r)(\hat{\mathbf{r}}\cdot\vec{\sigma})\chi_s \end{pmatrix},$$

$$e_s^{P_{3/2}}(\epsilon,\mathbf{r}) = \begin{pmatrix} ig_{-2}(\epsilon,r)[3(\hat{\mathbf{r}}\cdot\hat{\mathbf{p}}) - (\hat{\mathbf{r}}\cdot\vec{\sigma})(\hat{\mathbf{p}}\cdot\vec{\sigma})]\chi_s \\ if_2(\epsilon,r)[3(\hat{\mathbf{r}}\cdot\hat{\mathbf{p}})(\hat{\mathbf{p}}\cdot\vec{\sigma}) - (\hat{\mathbf{r}}\cdot\vec{\sigma})]\chi_s \end{pmatrix}$$

- To simulate realistic situation, we take radial functions that satisfy Dirac equation

$$\frac{dg_\kappa(\epsilon,r)}{dr} = -\frac{\kappa}{r}g_\kappa(\epsilon,r) + \frac{\epsilon - V + m_e c^2}{c\hbar}f_\kappa(\epsilon,r)$$

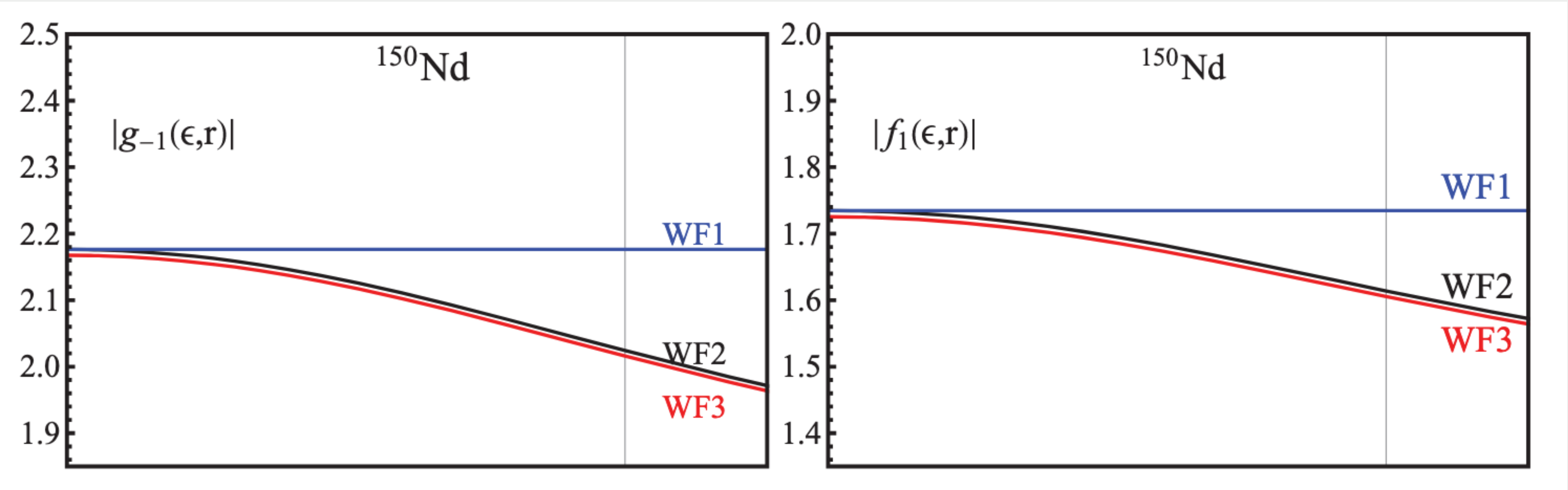
$$\frac{df_\kappa(\epsilon,r)}{dr} = -\frac{\epsilon - V - m_e c^2}{c\hbar}g_\kappa(\epsilon,r) + \frac{\kappa}{r}f_\kappa(\epsilon,r)$$

- And potential that takes into account the finite nuclear size and the electron screening

$$V(r) \equiv \varphi(r) \times \begin{bmatrix} -\frac{Z_d(\alpha\hbar c)}{r}, & r \geqslant R, \\ -Z_d(\alpha\hbar c)\left(\frac{3-(r/R)^2}{2R}\right), & r < R. \end{bmatrix}$$

Calculation of phase space factor G

- Example: $Z_d = 62, \epsilon = 2.0$ MeV, and R = 6.38 fm (vertical line)



WF1 = leading finite-size Coulomb

WF2 = exact finite-size Coulomb

WF3 = exact finite-size Coulomb with electron screening

Calculation of phase space factor G

- The combinations f_{jk} of energy dependent electron wavefunctions are integrated over the available kinetic energy to obtain phase space factors

$$G_{jk}^{(i)} = \frac{2}{\ln 2} \int f_{jk}^{(i)} w_{0\nu} \mathrm{d}E_1 \ , \quad i = 0, 1$$

- From these one also obtains:
  - Single electron spectrum: $\frac{dW}{d\epsilon_1} = \mathcal{N} \frac{dG^{(0)}}{d\epsilon_1}$
  - Energy dependent angular correlation: $\alpha(\epsilon_1) = \frac{dG^{(1)}/d\epsilon_1}{dG^{(0)}/d\epsilon_1}$

Can be compared with experimental data!

Calculation of nuclear matrix element M

- The double beta decay nuclear matrix elements are calculated by connecting the initial and final state wavefunctions with proper transition operator depending on the mechanism and mode of the decay:

$$\mathcal{M}_K \equiv \langle \mathcal{O}_F^+ | \mathcal{H}_K | \mathcal{O}_I^+ \rangle$$

- Three types of NMEs: Fermi, Gamow-Teller and tensor
- Six types of form factors:

$$\tilde{h}_{XX}(q^2) = \frac{1}{(1+q^2/m_V^2)^4}$$

$$\tilde{h}_{PP}(q^2) = \frac{1}{(1+q^2/m_A^2)^4}\frac{1}{(1+q^2/m_\pi^2)^2}$$

$$\tilde{h}_{AA}(q^2) = \frac{1}{(1+q^2/m_A^2)^4}$$

$$\tilde{h}_{AP}(q^2) = \frac{1}{(1+q^2/m_A^2)^4}\frac{1}{1+q^2/m_\pi^2}$$

$$\tilde{h}_{AX}(q^2) = \frac{1}{(1+q^2/m_V^2)^2}\frac{1}{(1+q^2/m_A^2)^2}$$

$$\tilde{h}_{XP}(q^2) = \frac{1}{(1+q^2/m_V^2)^2}\frac{1}{(1+q^2/m_A^2)^2}\frac{1}{1+q^2/m_\pi^2}$$

Calculation of nuclear matrix element M

- The wavefunctions are calculated in nuclear models, such as the quasiparticle random phase approximation, **QRPA**, the nuclear shell model, **NSM**, energy density functional theory, **EDF** and the microscopic interacting boson model, **IBM-2**
- Recent ab initio calculations: multi-reference version of the similarity renormalization group **IMSRG** and coupled-cluster **CC** theory.

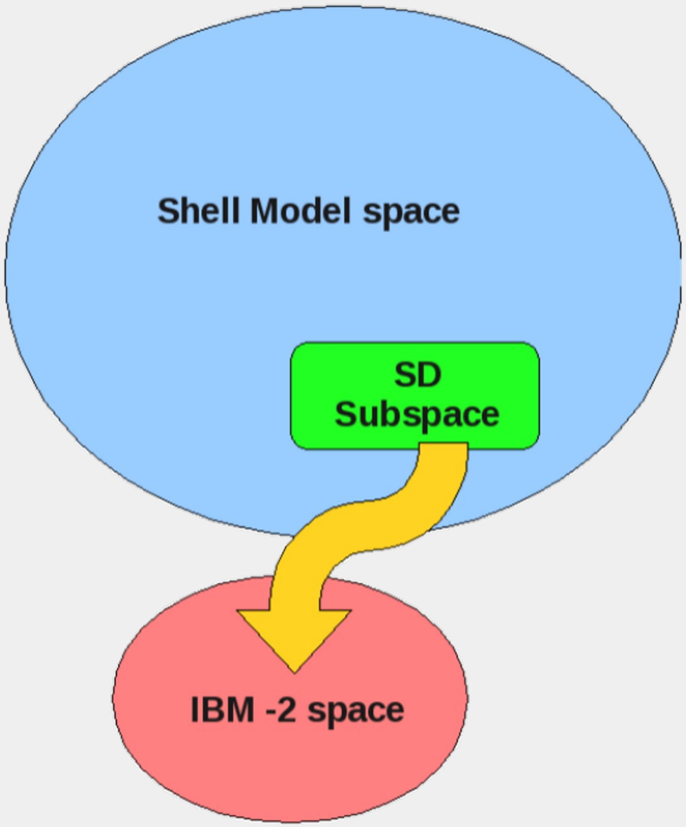
- The fact that $0\nu\beta\beta$ -decay is a unique process, and there is no direct probe which connects the initial and final states other than the process itself makes the prediction challenging for theoretical models.
- The reliability of the used wavefunctions, and eventually $M^{(0\nu)}$, must be then tested using other available relevant data.

Nuclear model: IBM-2

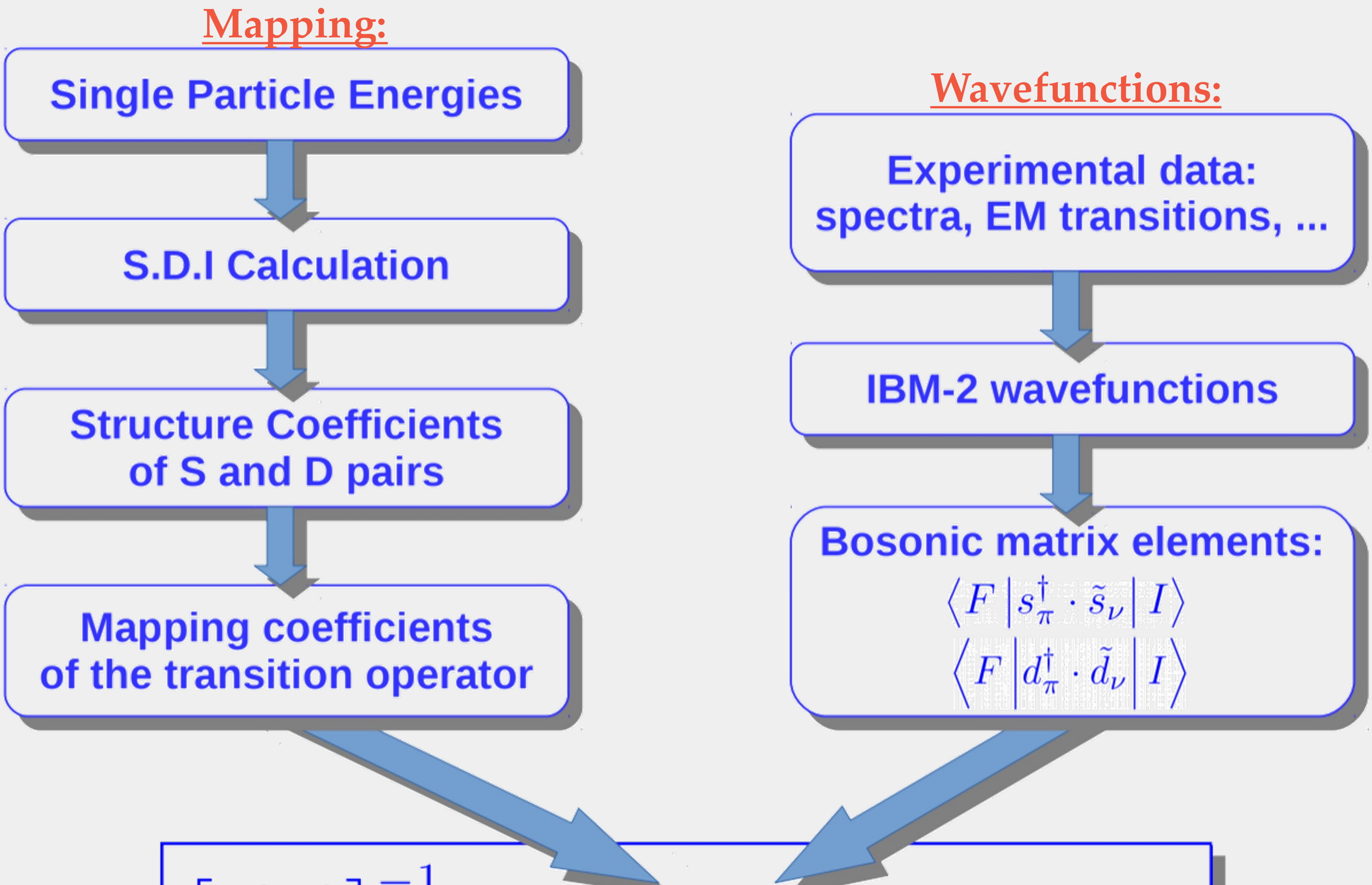
- Why IBM-2?
  - The interacting boson model has been one of the most successful models in reproducing collective features of the low-lying levels of medium, as well as heavy nuclei
  - Extensive studies of several isotopic chains available in literature
  - Can be used in any nucleus and thus all nuclei of interest can be calculated within the same model making it easier to recognize model dependent uncertainties

Nuclear model: IBM-2

- In IBM-2 the very large shell model space is truncated to states built from pairs of nucleons with J = 0 and 2
- These pairs are assumed to be collective and are taken as bosons
- The Hamiltonian is constructed phenomenologically and two- and four valence-nucleon states are generated by a schematic interaction
- The fermion operators are mapped onto a boson space and the matrix elements of the mapped operators are then evaluated with realistic wavefunctions



Nuclear model: IBM-2

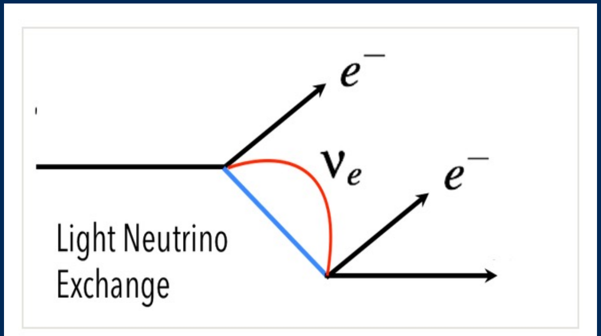


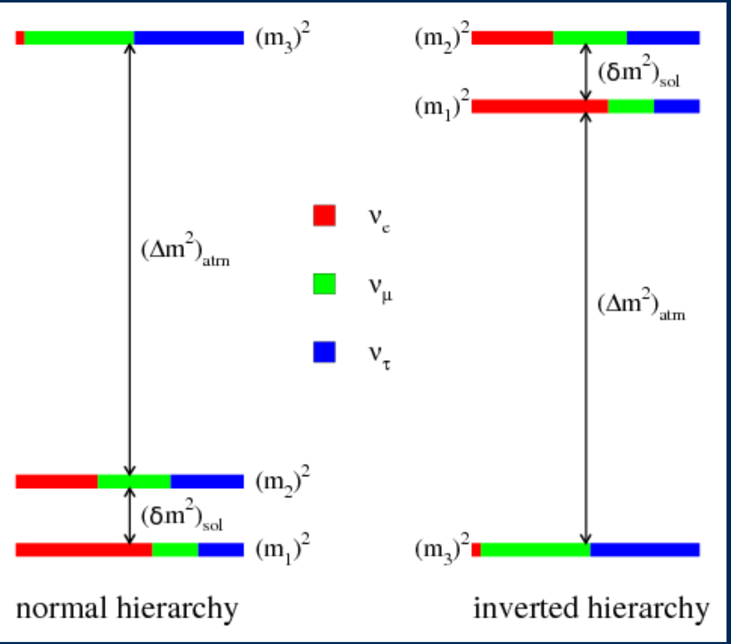
$$\left[\tau_{1/2}^{(0\nu)}\right]^{-1} \simeq G_{0\nu} \left|M_{0\nu}\right|^2 \left|f(m_i, U_{ei})\right|^2$$

Standard mass mechanism

- After the discovery of neutrino oscillations, attention has been mostly focused on the mass mechanism of $0\nu\beta\beta$-decay, wherein the three species of neutrinos have masses m_i and couplings to the electron neutrino U_{ei}

- Obtained information on mass differences and their mixing leaves two possibilities: Normal and inverted hierarchy





Standard mass mechanism

- The effective light neutrino mass is constrained by atmospheric, solar, reactor and accelerator neutrino oscillation experiments

$$\langle m_\nu \rangle = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\varphi_2} + s_{13}^2 m_3 e^{i\varphi_3} \right|$$

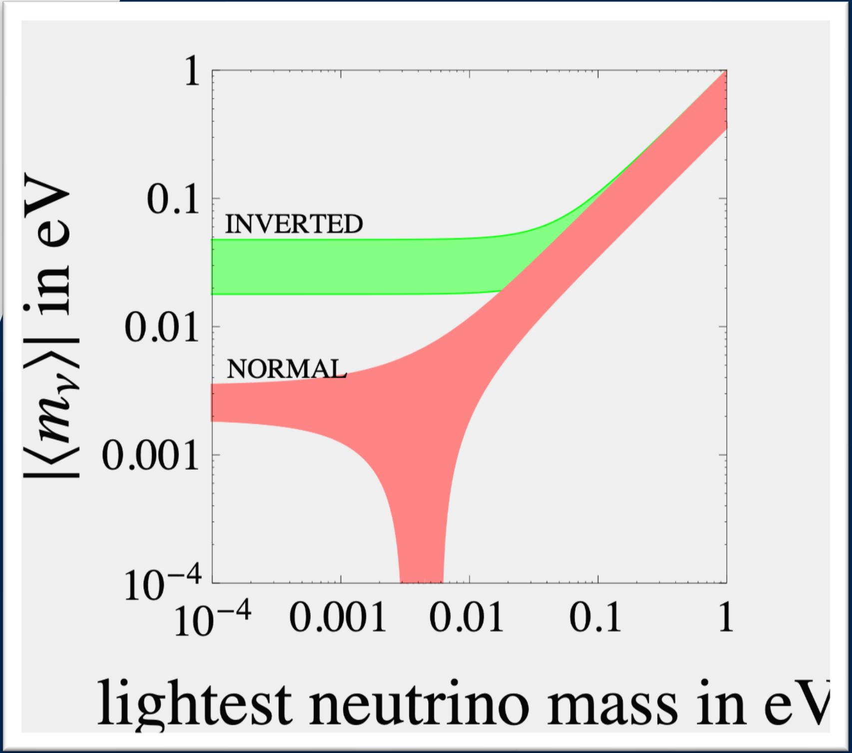
$$c_{ij} = \cos\vartheta_{ij}, \quad s_{ij} = \sin\vartheta_{ij}, \quad \varphi_{2,3} = [0, 2\pi],$$

Normal:

$$m_1 = m_{\min},$$
$$m_2 = \sqrt{m_{\min}^2 + \Delta m_{\rm SOL}^2},$$
$$m_3 = \sqrt{m_{\min}^2 + \Delta m_{\rm ATM}^2 + \Delta m_{\rm SOL}^2/2}$$

Inverted:

$$m_3 = m_{\min},$$
$$m_1 = \sqrt{m_{\min}^2 + \Delta m_{\rm ATM}^2 - \Delta m_{\rm SOL}^2/2}$$
$$m_2 = \sqrt{m_{\min}^2 + \Delta m_{\rm ATM}^2 + \Delta m_{\rm SOL}^2/2}$$



Standard mass mechanism

- In this case, the inverse decay rate is given by

$$\left[\tau_{1/2}^{0\nu}(0^+ \to 0^+)\right]^{-1} = G_{0\nu}\left|M_{0\nu}\right|^2 \left|\frac{\langle m_\nu\rangle}{m_e}\right|^2$$

- The physics beyond standard model is proportional to average light neutrino mass

$$\langle m_\nu\rangle = \sum_{k=light} (U_{ek})^2 m_k$$

Standard mass mechanism

- Phase space factors: $G_{jk}^{(i)} = \frac{2}{\ln 2}\int_{m_ec^2}^{Q_{\beta\beta}+m_ec^2} f_{11}^{(i)} w_{0\nu} d\epsilon_1$

$$f_{11}^{(0)} = |f^{-1-1}|^2 + |f_{11}|^2 + |f^{-1}{}_1|^2 + |f_1{}^{-1}|^2$$

$$f_{11}^{(1)} = -2\mathrm{Re}[f^{-1-1}f_{11}^{*} + f^{-1}{}_1 f_1{}^{-1*}]$$

$$\begin{aligned} f^{-1-1} &= g_{-1}(\epsilon_1)g_{-1}(\epsilon_2), \\ f_{11} &= f_1(\epsilon_1)f_1(\epsilon_2), \\ f^{-1}{}_1 &= g_{-1}(\epsilon_1)f_1(\epsilon_2), \\ f_1{}^{-1} &= f_1(\epsilon_1)g_{-1}(\epsilon_2). \end{aligned}$$

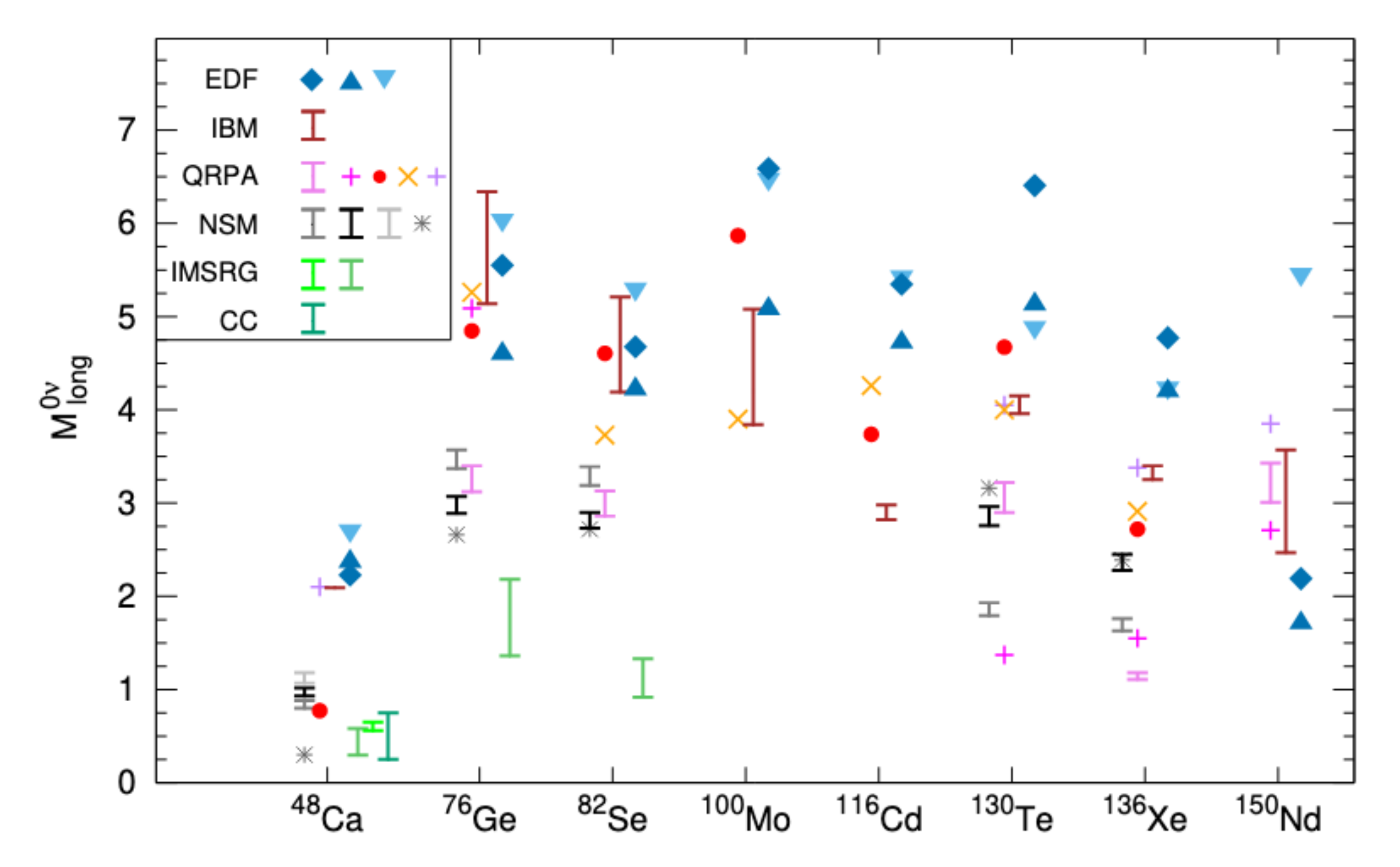
- Neutrino potential: $v_1(q) = \frac{2}{\pi}\frac{1}{q\left(q+\tilde{A}\right)}$ ← Long-range

- Nuclear matrix element: $\mathcal{M}_\nu = g_A^2\left[\left(\frac{g_V}{g_A}\right)^2\mathcal{M}_F - \mathcal{M}_{\mathrm{GT}} + \mathcal{M}_T\right]$

$$\mathcal{M}_{\mathrm{GT}} = \mathcal{M}_{\mathrm{GT}}^{AA} - \frac{g_P}{6g_A}\mathcal{M}_{\mathrm{GT}}^{\prime AP} + \frac{(g_V+g_W)^2}{6g_A^2}\mathcal{M}_{\mathrm{GT}}^{\prime WW} + \frac{g_P^2}{48g_A^2}\mathcal{M}_{\mathrm{GT}}^{\prime\prime PP}$$

$$\mathcal{M}_T = \frac{g_P}{6g_A}\mathcal{M}_T^{\prime AP} + \frac{(g_V+g_W)^2}{12g_A^2}\mathcal{M}_T^{\prime WW} - \frac{g_P^2}{48g_A^2}\mathcal{M}_T^{\prime\prime PP}$$

Standard mass mechanism



Summary of current situation:
M. Agostini *et al.*
arXiv:2202.01787
(2022)

- IMSRG and CC smaller than NSM NMEs, EDF theory the largest, and IBM and QRPA somewhere in between
- Still large differences between the different models but some of them can be explained by model dependent quenching of g_A
- NME + PSFs: can be compared with experimental half-life limits

Experimental aspects: $\tau_{1/2}$

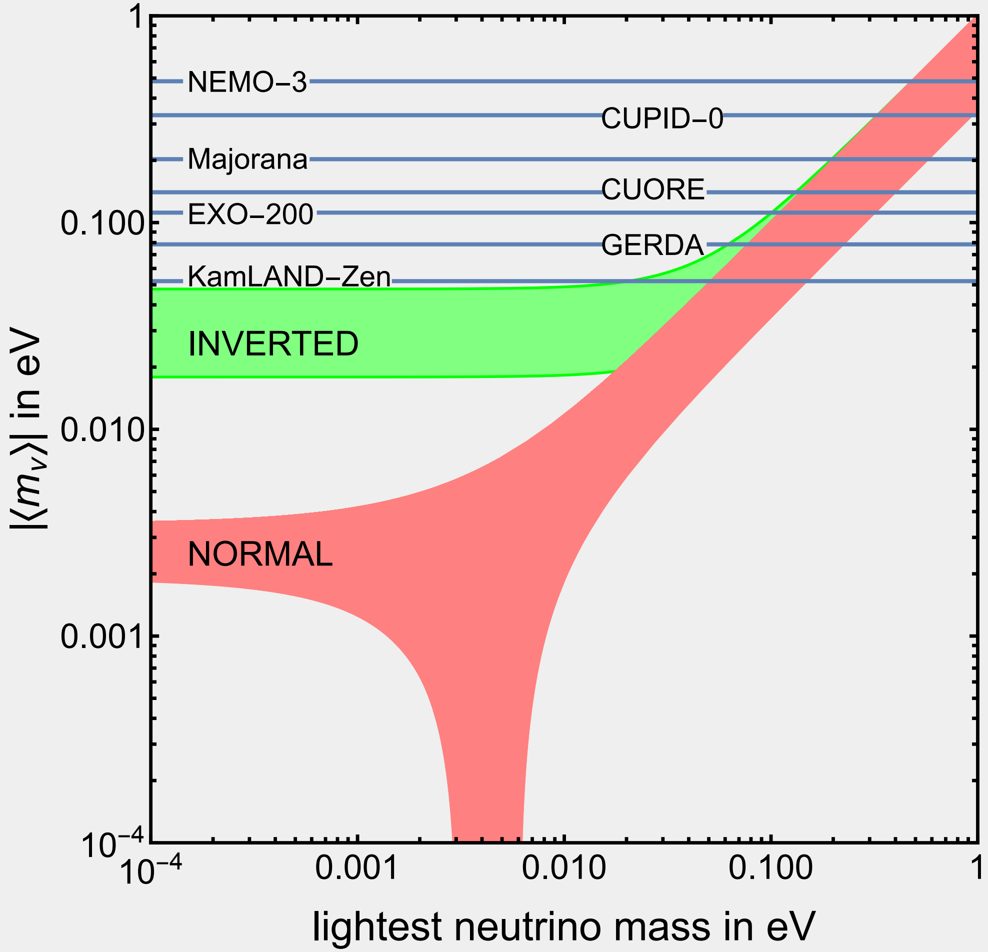
Current lower half-life limits coming from different experiments:

Experiment	nucleus	$\tau_{1/2}$	$\langle m_\nu \rangle$
Majorana	^{76}Ge	> 2.7 x 10^{25}yr	< 0.21 eV
GERDA	^{76}Ge	> 1.8 x 10^{26}yr	< 0.079 eV
CUPID-0	^{82}Se	> 3.5 x 10^{24}yr	< 0.33 eV
NEMO-3	^{100}Mo	> 1.1 x 10^{24}yr	< 0.44 eV
CUORE	^{130}Te	> 2.2 x 10^{25}yr	< 0.14 eV
EXO-200	^{136}Xe	> 5.0 x 10^{25}yr	< 0.12 eV
KamLAND-Zen	^{136}Xe	> 2.3 x 10^{26}yr	< 0.052 eV

$$\tau_{1/2} \Rightarrow \langle m_\nu \rangle < \frac{m_e}{\sqrt{\tau_{1/2}^{exp} G_{0\nu} g_A^2} |M^{(0\nu)}|}$$

^{76}Ge: Majorana: Alvis S I *et al.* 2019 PRC 100 025501; GERDA: M. Agostini *et al.* 2020 PRL 125 252502
^{82}Se: CUPID-0: Azzolini O *et al*. 2019 PRL 123 032501
^{100}Mo: NEMO-3: Arnold R *et al*. 2015 PRD 92 072011
^{130}Te: CUORE: D.Q Adams *et al.* Nature 2022 604 53–58
^{136}Xe: EXO-200: G. Anton *et al*. 2019 , PRL 123 161802; KamLAND-Zen: S. Abe *et al*. 2022 arXiv:2203.02139

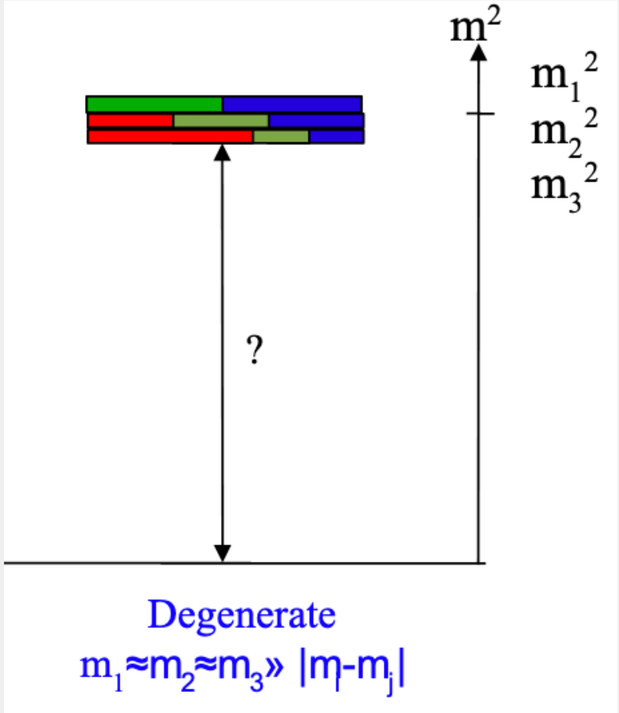
Theory+Experiments: Limits on $\langle m_\nu \rangle$



NEMO-3: Arnold R et al. 2015 PRD 92 072011; CUPID-0: Azzolini O et al. 2019 PRL 123 032501; Majorana: Alvis S I et al. 2019 PRC 100 025501; CUORE: D.Q Adams et al. Nature 2022 604 53–58; EXO-200: G. Anton et al.2019 , PRL 123 161802; , GERDA: M. Agostini et al. 2020 PRL 125 252502; KamLAND-Zen: S. Abe et al. 2022 arXiv:2203.02139

Mass mechanism: Heavy neutrinos

- It might also be that there are heavy neutrinos, $\langle m_{\nu heavy}\rangle$ >>1GeV
- Neutrino potential: $$v(q) = \frac{2}{\pi}\frac{1}{m_e m_p}$$ Short-range
- Half-life: $$[\tau_{1/2}^{0\nu_h}]^{-1} = G_{0\nu}^{(0)}|M_{0\nu_h}|^2|\eta|^2,$$ $$\eta \equiv m_p\langle m_{\nu_h}^{-1}\rangle = \sum_{k=\text{heavy}} (U_{ek_h})^2 \frac{m_p}{m_{k_h}}.$$



- Effective heavy neutrino mass is not constrained by experiments, and only model dependent limits of $\langle m_{\nu heavy}\rangle$ can be set
- If both light and heavy neutrino exchange contribute, the half-lives are given by

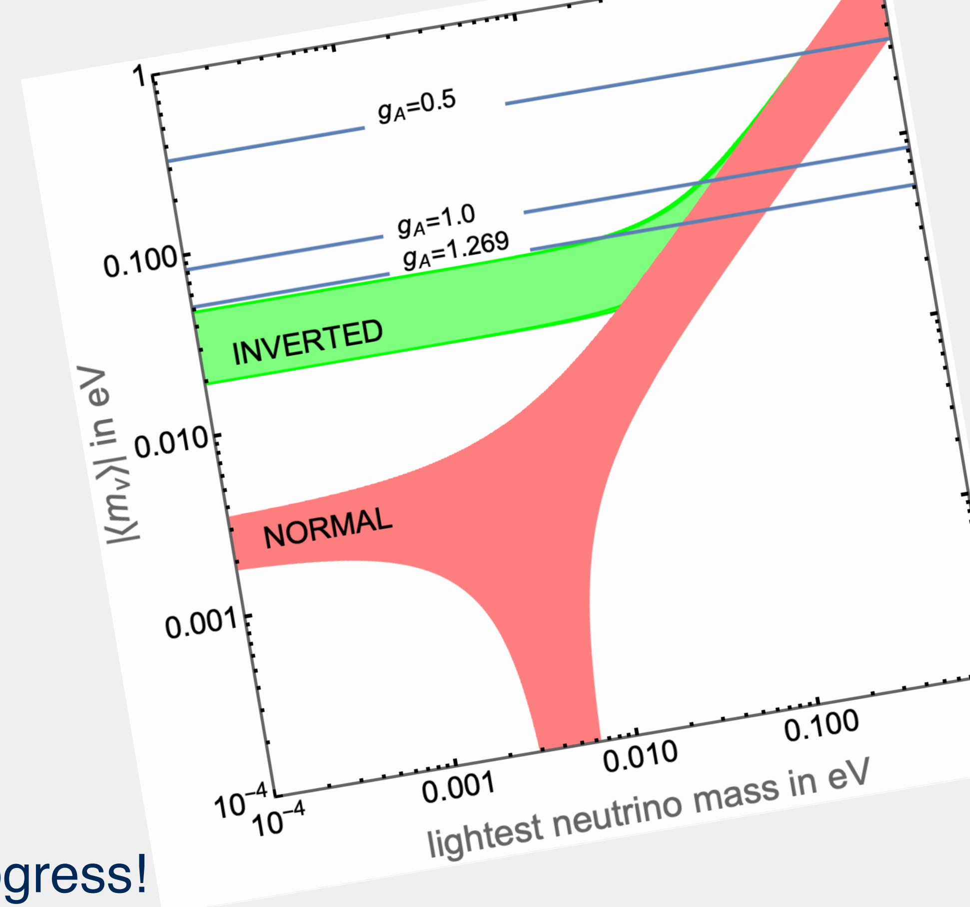
$$[\tau_{1/2}^{0\nu}]^{-1} = G_{0\nu}^{(0)}\left|M_{0\nu}\frac{\langle m_\nu\rangle}{m_e} + M_{0\nu_h}\eta\right|^2$$

Cause of worry: Quenching of g_A

- It is well known from single beta decay and electron capture that g_A is renormalized in models of nuclei. Two reasons for this are:
  - The omission of non-nucleonic degrees of freedom
  - The limited model space in which the calculations done
- The former effect is not expected to be present in $0\nu\beta\beta$-decay
  - The average neutrino momentum is ~100 MeV, while in $2\nu\beta\beta$-decay is of the order of 1–2 MeV
- The latter effect instead appears both in $0\nu\beta\beta$ and $2\nu\beta\beta$-decays
- $2\nu\beta\beta$ may be used to get an idea of the model dependent quenching, but
  - In $2\nu\beta\beta$ only the 1^+ (GT) multipole contributes. In $0\nu\beta\beta$ all multipoles 1^+, 2^-, ...; 0^+, 1^-, ... contribute. Some of which could be even unquenched
- This is a critical issue: g_A enters the equations to the power of 4!

Quenching of g_A

- Three suggested scenarios are:
  - Free value: 1.269
  - Quark value: 1
  - Even stronger quenching: $g_{A,\mathrm{eff}} < 1$



- Effective value of g_A is a work in progress!
  - Studies using effective field theory (EFT) to estimate the effect of non-nucleonic degrees of freedom (two-body currents)
  - Measures of both single and double charge exchange reaction intensities
  - Comparison of the shapes of the calculated and measured spectra of forbidden non-unique beta decays

General description

- The underlying interaction does not have to be as simple as standard neutrino mass mechanism
- General Lagrangian can be written in terms of effective couplings ε corresponding to the point like vertices at the Fermi scale:

$$\mathcal{L}_{0\nu\beta\beta} = \mathcal{L}_{\mathrm{LR}} + \mathcal{L}_{\mathrm{SR}}$$

- Long range mechanisms include standard light neutrino exchange, light sterile neutrinos, left-right models, SUSY models…
- Short range mechanisms include heavy neutrino exchange, heavy sterile neutrinos…
- Mechanisms where additional light states are either mediating the decay or are emitted in it (e.g., Majorons) are also possible

Short-range mechanisms

- Short-range transitions with no mediating particle lighter than ≈100 MeV.

- For short-range mechanisms neutrino potential is defined as

$$v(q) = \frac{2}{\pi}\frac{1}{m_e m_p},$$

- In this case $0\nu\beta\beta$-decay probes lepton number violating physics around the TeV scale
- The most prominent scenario where this kind of operator is generated is through the inclusion of heavy sterile neutrinos [Simkovic et al. PRC60 (1999) 055502]

Short-range mechanisms

- Altogether 18 nuclear matrix elements to be calculated for each double beta decay candidate nucleus
  - 8 Compound NMEs
- 4 x 2 PSFs
- In this case the full inverse $0\nu\beta\beta$-decay half-life is then given by

Standard light neutrino exchange (LR)

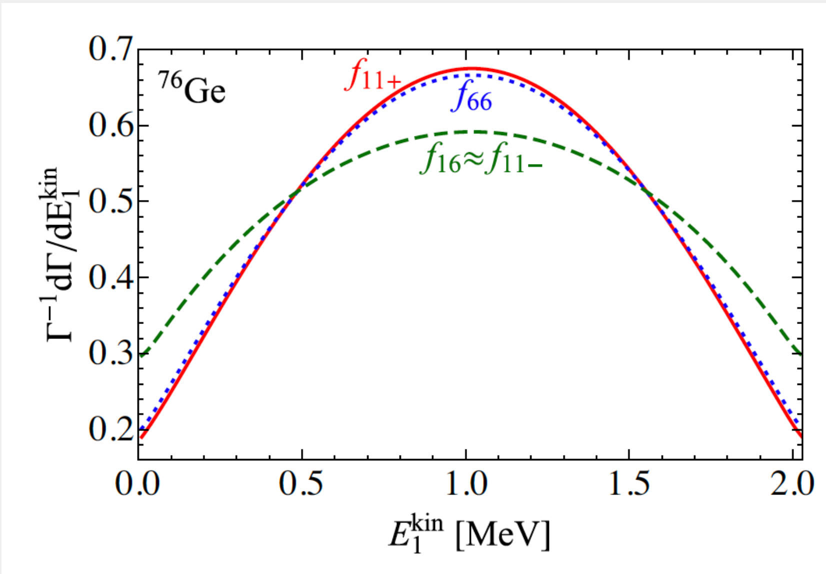
$$T_{1/2}^{-1} = G_{11+}^{(0)} \left| \sum_{I=1}^{3} \epsilon_I^L \mathcal{M}_I + \epsilon_\nu \mathcal{M}_\nu \right|^2 + G_{11+}^{(0)} \left| \sum_{I=1}^{3} \epsilon_I^R \mathcal{M}_I \right|^2 + G_{66}^{(0)} \left| \sum_{I=4}^{5} \epsilon_I \mathcal{M}_I \right|^2$$

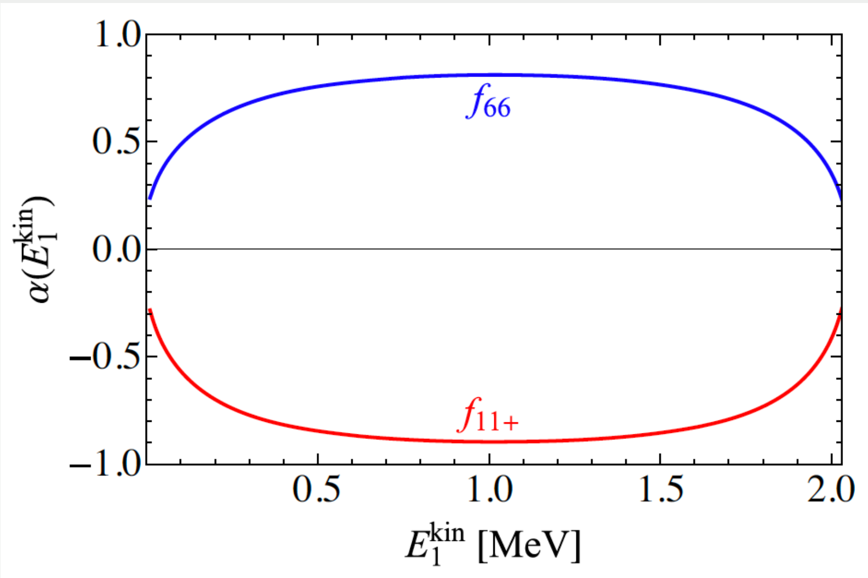
$$+ G_{11-}^{(0)} \times 2\mathrm{Re}\left[\left(\sum_{I=1}^{3} \epsilon_I^L \mathcal{M}_I + \epsilon_\nu \mathcal{M}_\nu \right) \left(\sum_{I=1}^{3} \epsilon_I^R \mathcal{M}_I \right)^* \right]$$

$$+ G_{16}^{(0)} \times 2\mathrm{Re}\left[\left(\sum_{I=1}^{3} \epsilon_I^L \mathcal{M}_I - \sum_{I=1}^{3} \epsilon_I^R \mathcal{M}_I + \epsilon_\nu \mathcal{M}_\nu \right) \left(\sum_{I=4}^{5} \epsilon_I \mathcal{M}_I \right)^* \right].$$

Short-range mechanisms

- Example of normalized single electron spectra and angular correlation





- The two electrons preferably share the available kinetic energy equally
- Only a small difference between the f_{11} and f_{66} case, with the latter having a slightly flatter profile, that is unlikely to be distinguishable experimentally
- The term f_{16}, corresponding to an interference between mechanisms 1, 2, 3 and 4, 5 has a significantly flatter profile
- In view of the opposite sign for 11 and 66 measurements of the angular correlation would allow a discrimination of the two types of mechanisms

Long-range mechanisms

- Long-range transitions proceed through the exchange of ν_{light}
- 3 different neutrino potentials corresponding to 3 terms of

$$\frac{1}{\omega}\left[\frac{\omega\gamma^0 - \mathbf{k}\cdot\gamma + m_i}{\omega + A_1} - \frac{\omega\gamma^0 + \mathbf{k}\cdot\gamma - m_i}{\omega + A_2}\right]$$

  - m_i terms $$v_1(q) = \frac{2}{\pi}\frac{1}{q(q+\tilde{A})}$$

  - ω terms $$v_3(q) = \frac{2}{\pi}\frac{1}{(q+\tilde{A})^2}$$

  - **k** terms $$v_4(q) = \frac{2}{\pi}\frac{q+2\tilde{A}}{q(q+\tilde{A})^2}$$

Long-range mechanisms

- Altogether there are 18 x 3 NMEs to be calculated for each double beta decay candidate nucleus
- In addition, for rank-1 tensors under rotation several new terms appear
- 14 x 2 PSFs
- Examples:
  - Light sterile neutrinos
    - 1 x 2 PSFs
    - 1 compound NME
  - LR models
    - 11 x 2 PSFs
    - 14 compound NMEs
  - SUSY models
    - 10 x 2 PSFs
    - 12 compound NMEs

Sterile neutrinos

- Neutrinos with no standard model interaction
- If there are sterile neutrinos, the equation for half-life is different…

$$[\tau_{1/2}^{0\nu}]^{-1} = G_{0\nu} \left| \sum_N (U_{eN})^2 M_{0\nu}(m_N) \frac{m_N}{m_e} \right|^2$$

- Nuclear matrix element:

$$M^{(0\nu)}(m_N) = M_{GT}^{(0\nu)}(m_N) - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)}(m_N) + M_T^{(0\nu)}(m_N)$$

function containing physics beyond standard model:

$$f = m_N / m_e$$

and neutrino potential all depend on sterile neutrino mass

$$fv(p) = \frac{m_N}{m_e} \frac{2}{\pi} \frac{1}{\sqrt{p^2 + m_N^2}\left(\sqrt{p^2 + m_N^2} + \tilde{A}\right)}$$

Sterile neutrinos

- Several types of sterile neutrinos have been suggested: Light sterile neutrinos: $m_N \sim 1$eV or at keV mass range, heavy sterile neutrinos: $m_N \gg 100$ MeV
- Limits

$$m_N \to 0 \quad : \quad fv = \frac{m_N}{m_e}\frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}$$

$$m_N \to \infty \quad : \quad fv = \frac{m_N}{m_e}\frac{2}{\pi}\frac{1}{m_N^2} = \frac{m_p}{m_N}\left(\frac{2}{\pi}\frac{1}{m_e m_p}\right)$$

Known neutrinos — Unknown light sterile ν (eV, keV)

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}\left|\left[\frac{1}{m_e}\sum_{k=1}^{3}U_{ek}^2 m_k + \frac{1}{m_e}\sum_i U_{ei}^2 m_i + \frac{1}{m_e}\sum_j U_{ej}^2\right]M^{(0\nu)} + \left[m_p\sum_N U_{eN}^2\frac{m_N}{\langle p^2\rangle + m_N^2} + m_p\sum_{k_h=1}^{3}U_{ek_h}^2\frac{1}{m_{k_h}}\right]M^{(0\nu_h)}\right|$$

Unknown heavy sterile ν — Unknown heavy neutrinos

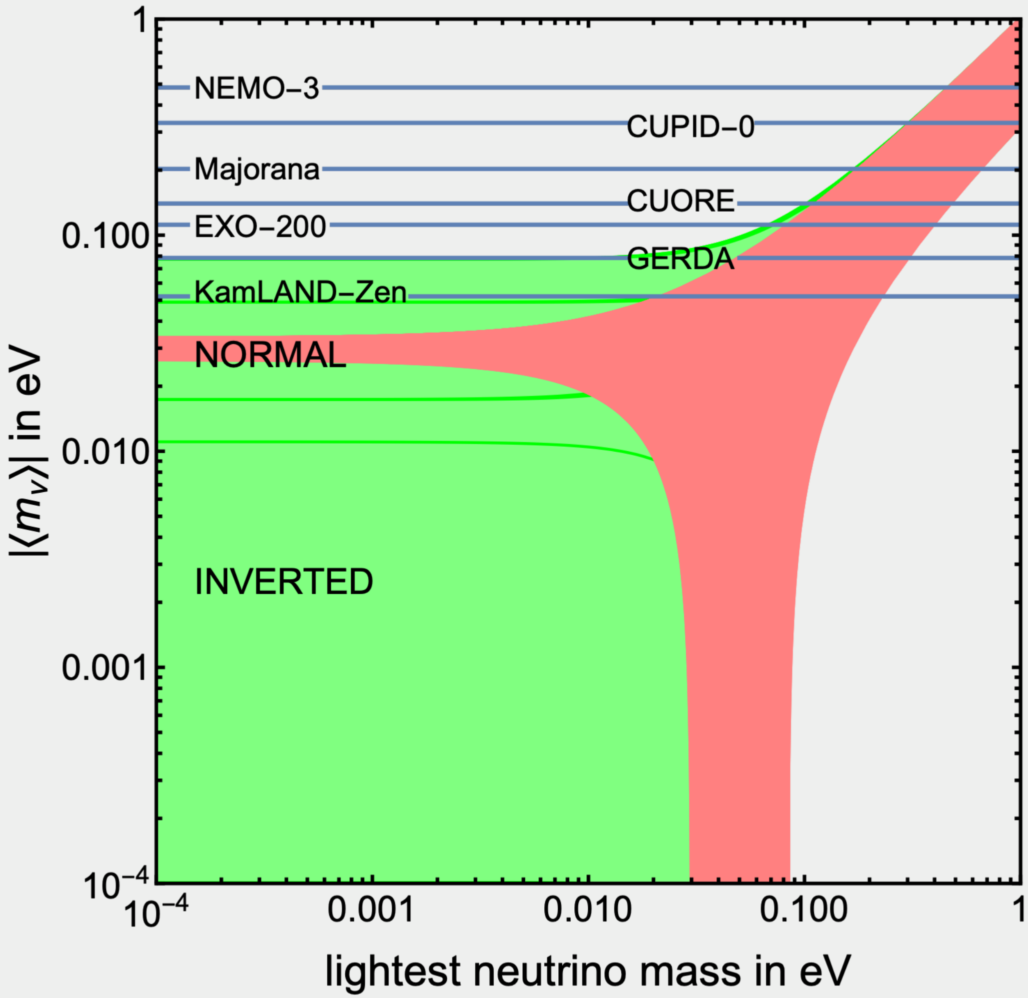
PRD 92 (2015) 093001

Sterile neutrinos

- Example: Light 4th neutrino: mass m_4=1 eV and $|U_{e4}|^2$=0.03

$$\langle m_{N,light} \rangle = \sum_{k=1}^{3} U_{ek}^2 m_k + U_{e4}^2 e^{i\alpha_4} m_4$$

- The effect: adds to the effective mass a contribution of 30 meV
- Makes the spread of the allowed values larger
- Improves the possibility to detect it in the next generation experiments



Sterile neutrinos

- Example: Heavy sterile neurino

$$T_{1/2}^{-1} = G_{11+}^{(0)} \left| \frac{m_{\beta\beta}}{m_e} \mathcal{M}_\nu + \epsilon_3^{LLL} \mathcal{M}_3^{LL} \right|^2$$

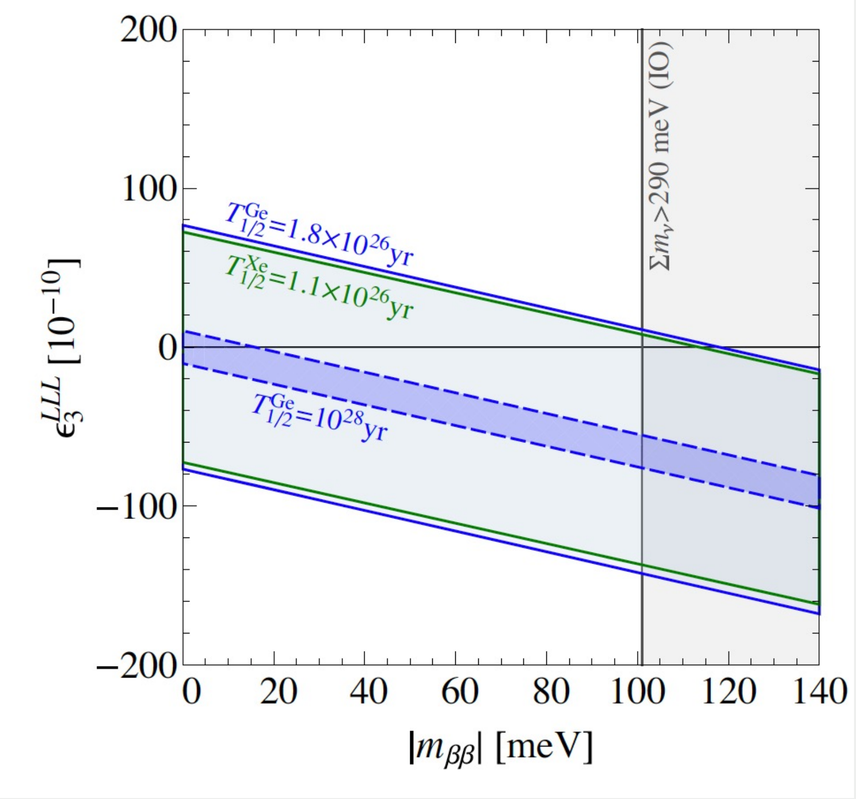
$$\epsilon_3^{LLL} = \sum_{i=1}^{n_N} V_{eN_i}^2 \frac{m_p}{m_{N_i}}, \quad (m_{N_i} \gg 100 \text{ MeV})$$

- The light shaded areas: allowed given the current limits from 0νββ decays searches in ^{76}Ge and ^{136}Xe
- Defining effective operator scale

$$1/(\Lambda_3^{LLL})^5 = G_F^2 \cos^2\theta_C \epsilon_3^{LLL}/(2m_p)$$

gives $|\Lambda_3^{LLL}| \gtrsim 4.5$ TeV

- The dark shaded area: sensitivity from future searches at $T_{1/2}(^{76}\text{Ge}) = 10^{28}$yr



Left-right models

- Non-zero contributions

$$\langle m_\nu \rangle = \sum_i m_i U_{ei}^2$$

$$\langle \lambda \rangle = \lambda \sum_i U_{ei} V_{ei} \equiv \bar{\epsilon}_{V+A}^{V+A}$$

$$\langle \eta \rangle = \eta \sum_i U_{ei} V_{ei} \equiv \bar{\epsilon}_{V-A}^{V+A}$$

- The differential rate for these models is obtained by combining matrix elements

$$\mathcal{M}_3 = g_V^2 \mathcal{M}_F \;\; (- \;\; - \;\; +) \; g_A^2 \mathcal{M}_{GT}^{AA} \;\; (+ \;\; + \;\; -) \; \frac{g_A g_{P'}}{6} \left(\mathcal{M}'^{AP'}_{GT} + \mathcal{M}'^{AP'}_{T} \right) + \frac{(g_V + g_W)^2}{12} \left(-2\mathcal{M}'^{WW}_{GT} + \mathcal{M}'^{WW}_{T} \right) \;\; (- \;\; - \;\; +) \; \frac{g_{P'}^2}{48} \left(\mathcal{M}''^{P'P'}_{GT} + \mathcal{M}''^{P'P'}_{T} \right)$$

with appropriate phase space factors

Left-right models

- Half-life:

$$\left[\tau_{1/2}^{0\nu}\left(0^+ \rightarrow 0^+\right)\right]^{-1} = C_{mm}^{(i)}\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2 + C_{\lambda\lambda}^{(i)}\langle\lambda\rangle^2 + C_{\eta\eta}^{(i)}\langle\eta\rangle^2 + 2C_{m\lambda}^{(i)}\frac{\langle m_\nu\rangle}{m_e}\langle\lambda\rangle + 2C_{m\eta}^{(i)}\frac{\langle m_\nu\rangle}{m_e}\langle\eta\rangle + 2C_{\lambda\eta}^{(i)}\langle\lambda\rangle\langle\eta\rangle$$

$$\begin{aligned}
C_{mm}^{(i)} &= G_{11}^{(i)}\left|\mathcal{M}_1^{LL}\right|^2 \\
C_{\lambda\lambda}^{(i)} &= G_{33}^{(i)}\left|\mathcal{M}_3^{RR}\right|^2 + G_{44}^{(i)}\left|\mathcal{M}_4'^{RR}\right|^2 + 2G_{34}^{(i)}\mathcal{M}_3^{RR}\mathcal{M}_4'^{RR} \\
C_{\eta\eta}^{(i)} &= G_{33}^{(i)}\left|\mathcal{M}_3^{RL}\right|^2 + G_{44}^{(i)}\left|\mathcal{M}_4'^{RL}\right|^2 + 2G_{34}^{(i)}\mathcal{M}_3^{RL}\mathcal{M}_4'^{RL} \\
&\quad + G_{55}^{(i)}\left|\mathcal{M}_5^{LR}\right|^2 + G_{66}^{(i)}\left|\mathcal{M}_6^{LR}\right|^2 + 2G_{56}^{(i)}\mathcal{M}_5^{LR}\mathcal{M}_6^{LR} \\
C_{m\lambda}^{(i)} &= G_{13}^{(i)}\mathcal{M}_1^{LL}\mathcal{M}_3^{RR} + G_{14}^{(i)}\mathcal{M}_1^{LL}\mathcal{M}_4'^{RR} \\
C_{m\eta}^{(i)} &= G_{13}^{(i)}\mathcal{M}_1^{LL}\mathcal{M}_3^{RL} + G_{14}^{(i)}\mathcal{M}_1^{LL}\mathcal{M}_4'^{RL} + G_{15}^{(i)}\mathcal{M}_1^{LL}\mathcal{M}_5^{LR} + G_{16}^{(i)}\mathcal{M}_1^{LL}\mathcal{M}_6^{LR} \\
C_{\lambda\eta}^{(i)} &= G_{33}^{(i)}\mathcal{M}_3^{RR}\mathcal{M}_3^{RL} + G_{44}^{(i)}\mathcal{M}_4'^{RR}\mathcal{M}_4'^{RL} + G_{34}^{(i)}\left(\mathcal{M}_3^{RR}\mathcal{M}_4'^{RL} + \mathcal{M}_3^{RL}\mathcal{M}_4'^{RR}\right)
\end{aligned}$$

Left-right models

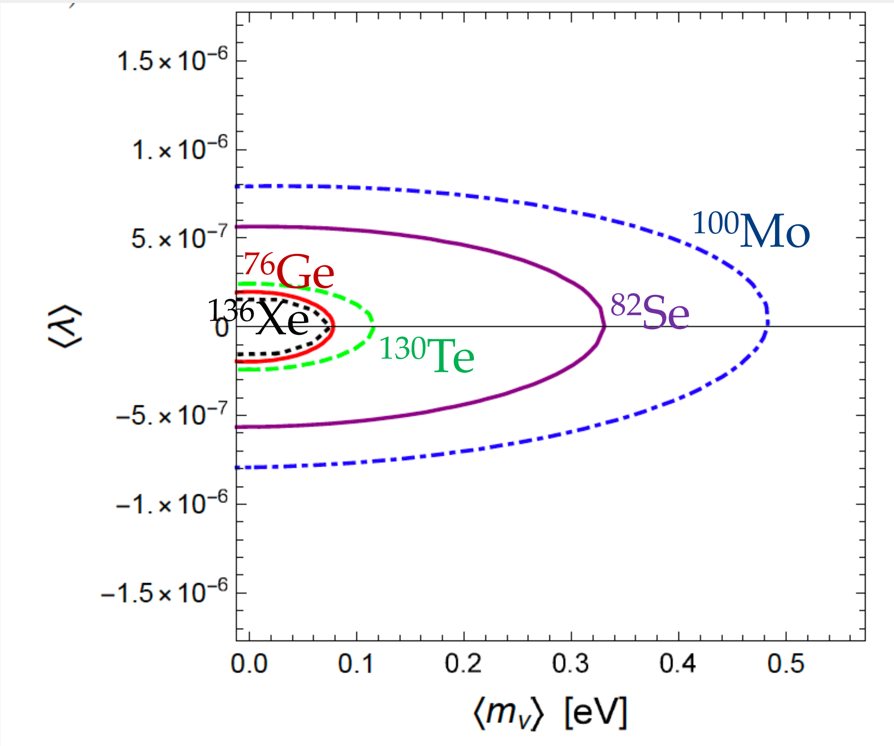
- Limits from experimental half-life limits

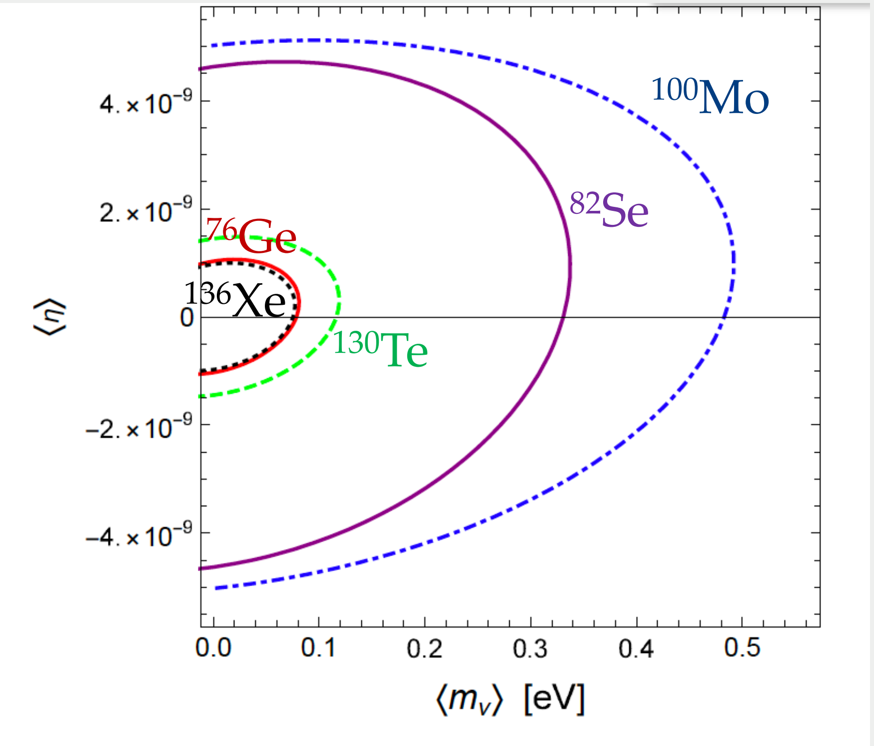
	$T_{1/2}^{\exp}$ [y]		$\frac{\langle m_\nu \rangle}{m_e}$	$\langle \lambda \rangle$	$\langle \eta \rangle$
^{76}Ge	1.8×10^{26}	[2]	1.5×10^{-7}	2.0×10^{-7}	1.0×10^{-9}
^{82}Se	3.5×10^{24}	[7]	6.5×10^{-7}	5.7×10^{-7}	4.6×10^{-9}
^{100}Mo	1.1×10^{24}	[5]	9.4×10^{-7}	7.9×10^{-7}	5.0×10^{-9}
^{130}Te	3.2×10^{25}	[1]	2.3×10^{-7}	2.4×10^{-7}	1.4×10^{-9}
^{136}Xe	1.1×10^{26}	[4]	1.5×10^{-7}	1.6×10^{-7}	1.0×10^{-9}

[1] CUORE, PRL 124, 1022501 (2020); [2] GERDA PRL 125, 252502 (2020); [4] KamLAND-Zen PRL 117, 082503 (2016); [5] NEMO-3 PRD 92, 072011 (2015); [7] CUPID-0 PRL 123, 032501 (2019)

Left-right models

- Limits on the combination





SUSY models

- The half-life depends on four non-zero parameters

$$\langle m_\nu \rangle = \sum_i m_i U_{ei}^2$$

$$\bar{\epsilon}_{S+P}^{S+P} = \theta \sum_i U_{ei} V'_{ei} \equiv \langle \theta \rangle$$

$$\bar{\epsilon}_{S+P}^{S-P} = \tau \sum_i U_{ei} V'_{ei} \equiv \langle \tau \rangle$$

$$\bar{\epsilon}_{T+T_5}^{T+T_5} = \varphi \sum_i U_{ei} V''_{ei} \equiv \langle \varphi \rangle$$

- The differential rate for these models is obtained by combining the matrix elements

$$\mathcal{M}_4 = (- \ - \ + +)\, i g_A g_{T_1} \mathcal{M}_{GT}^{AT_1} \ \ (+ \ + \ - -)\, i \frac{g_{P'} g_{T_1}}{12} \left(\mathcal{M}_{GT}^{\prime P'T_1} + \mathcal{M}_{T}^{\prime P'T_1} \right)$$

$$\mathcal{M}_5 = g_S g_V \mathcal{M}_F \ \ (+ \ + \ - -)\, \frac{g_A g_P}{12} \left(\tilde{\mathcal{M}}_{GT}^{AP} + \tilde{\mathcal{M}}_{T}^{AP} \right) \\ (- \ - \ + +)\, \frac{g_P g_{P'}}{24} \left(\mathcal{M}_{GT}^{\prime q_0 PP'} + \mathcal{M}_{T}^{\prime q_0 PP'} \right)$$

with appropriate phase space factors

SUSY models

$$\left[\tau_{1/2}^{0\nu}\left(0^{+}\to 0^{+}\right)\right]^{-1} = C_{mm}^{(i)}\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2 + C_{\theta\theta}^{(i)}\langle\theta\rangle^2 + C_{\tau\tau}^{(i)}\langle\tau\rangle^2 + C_{\varphi\varphi}^{(i)}\langle\varphi\rangle^2$$
$$+2C_{\theta\tau}^{(i)}\langle\theta\rangle\langle\tau\rangle + 2C_{\theta\varphi}^{(i)}\langle\theta\rangle\langle\varphi\rangle + 2C_{\tau\varphi}^{(i)}\langle\tau\rangle\langle\varphi\rangle$$
$$+2C_{m\theta}^{(i)}\frac{\langle m_\nu\rangle}{m_e}\langle\theta\rangle + 2C_{m\tau}^{(i)}\frac{\langle m_\nu\rangle}{m_e}\langle\tau\rangle + 2C_{m\varphi}^{(i)}\frac{\langle m_\nu\rangle}{m_e}\langle\varphi\rangle$$

$$\begin{aligned}
C_{mm}^{(i)} &= G_{11}^{(i)}\left|\mathcal{M}_{3,1}^{LL}\right|^2 \\
C_{\theta\theta}^{(i)} &= G_{33}^{\prime(i)}\left|\mathcal{M}_{5,3}^{RR}\right|^2 + G_{44}^{\prime(i)}\left|\mathcal{M}_{5,4}^{\prime RR}\right|^2 + 2G_{34}^{\prime(i)}\,\mathrm{Re}[\mathcal{M}_{5,3}^{RR}\mathcal{M}_{5,4}^{\prime RR*}] \\
C_{\tau\tau}^{(i)} &= G_{33}^{\prime(i)}\left|\mathcal{M}_{5,3}^{LR}\right|^2 + G_{44}^{\prime(i)}\left|\mathcal{M}_{5,4}^{\prime LR}\right|^2 + 2G_{34}^{\prime(i)}\,\mathrm{Re}[\mathcal{M}_{5,3}^{LR}\mathcal{M}_{5,4}^{\prime LR*}] \\
C_{\varphi\varphi}^{(i)} &= G_{33}^{\prime(i)}\left|\mathcal{M}_{4,3}^{RR}\right|^2 + G_{44}^{\prime(i)}\left|\mathcal{M}_{4,4}^{\prime RR}\right|^2 + 2G_{34}^{\prime(i)}\,\mathrm{Re}[\mathcal{M}_{4,3}^{RR}\mathcal{M}_{4,4}^{\prime RR}] \\
C_{\theta\tau}^{(i)} &= G_{33}^{\prime(i)}\,\mathrm{Re}[\mathcal{M}_{5,3}^{RR}\mathcal{M}_{5,3}^{LR*}] + G_{44}^{\prime(i)}\,\mathrm{Re}[\mathcal{M}_{5,4}^{\prime RR}\mathcal{M}_{5,4}^{\prime LR*}] \\
&\quad + G_{34}^{\prime(i)}\left(\mathrm{Re}[\mathcal{M}_{5,3}^{RR}\mathcal{M}_{5,4}^{\prime LR*}] + \mathrm{Re}[\mathcal{M}_{5,3}^{LR}\mathcal{M}_{5,4}^{\prime RR*}]\right) \\
C_{\theta\varphi}^{(i)} &= G_{33}^{\prime(i)}\,\mathrm{Re}[\mathcal{M}_{5,3}^{RR}\mathcal{M}_{4,3}^{RR*}] + G_{44}^{\prime(i)}\,\mathrm{Re}[\mathcal{M}_{5,4}^{\prime RR}\mathcal{M}_{4,4}^{\prime RR*}] \\
&\quad + G_{34}^{\prime(i)}\left(\mathrm{Re}[\mathcal{M}_{5,3}^{RR}\mathcal{M}_{4,4}^{\prime RR*}] + \mathrm{Re}[\mathcal{M}_{4,3}^{RR}\mathcal{M}_{5,4}^{\prime RR*}]\right) \\
C_{\tau\varphi}^{(i)} &= G_{33}^{\prime(i)}\,\mathrm{Re}[\mathcal{M}_{5,3}^{LR}\mathcal{M}_{4,3}^{RR*}] + G_{44}^{\prime(i)}\,\mathrm{Re}[\mathcal{M}_{5,4}^{\prime LR}\mathcal{M}_{4,4}^{\prime RR*}] \\
&\quad + G_{34}^{\prime(i)}\left(\mathrm{Re}[\mathcal{M}_{5,3}^{LR}\mathcal{M}_{4,4}^{\prime RR*}] + \mathrm{Re}[\mathcal{M}_{4,3}^{RR}\mathcal{M}_{5,4}^{\prime LR*}]\right) \\
C_{m\theta}^{(i)} &= G_{13}^{\prime\prime(i)}\,\mathrm{Re}[\mathcal{M}_{3,1}^{LL}\mathcal{M}_{5,3}^{RR*}] + G_{14}^{\prime\prime(i)}\,\mathrm{Re}[\mathcal{M}_{3,1}^{LL}\mathcal{M}_{5,4}^{\prime RR*}] \\
C_{m\tau}^{(i)} &= G_{13}^{\prime\prime(i)}\,\mathrm{Re}[\mathcal{M}_{3,1}^{LL}\mathcal{M}_{5,3}^{LR*}] + G_{14}^{\prime\prime(i)}\,\mathrm{Re}[\mathcal{M}_{3,1}^{LL}\mathcal{M}_{5,4}^{\prime LR*}] \\
C_{m\varphi}^{(i)} &= G_{13}^{\prime\prime(i)}\,\mathrm{Re}[\mathcal{M}_{3,1}^{LL}\mathcal{M}_{4,3}^{RR*}] + G_{14}^{\prime\prime(i)}\,\mathrm{Re}[\mathcal{M}_{3,1}^{LL}\mathcal{M}_{4,4}^{\prime RR*}]
\end{aligned}$$

SUSY models

- Limits from experimental half-life limits

	$T_{1/2}^{\mathrm{exp}}$ [y]		$\frac{\langle m_\nu \rangle}{m_e}$	$\langle \theta \rangle$	$\langle \tau \rangle$	$\langle \varphi \rangle$
^{76}Ge	1.8×10^{26}	[2]	1.5×10^{-7}	2.9×10^{-7}	1.2×10^{-7}	3.3×10^{-8}
^{82}Se	3.5×10^{24}	[7]	6.5×10^{-7}	1.0×10^{-6}	4.4×10^{-7}	1.2×10^{-7}
^{100}Mo	1.1×10^{24}	[5]	9.4×10^{-7}	2.2×10^{-6}	4.1×10^{-7}	2.0×10^{-7}
^{130}Te	3.2×10^{25}	[1]	2.3×10^{-7}	2.2×10^{-7}	1.0×10^{-7}	3.5×10^{-8}
^{136}Xe	1.1×10^{26}	[4]	1.5×10^{-7}	1.4×10^{-7}	6.6×10^{-8}	2.3×10^{-8}

[1] CUORE, PRL 124, 1022501 (2020); [2] GERDA PRL 125, 252502 (2020);
[4] KamLAND-Zen PRL 117, 082503 (2016); [5] NEMO-3 PRD 92, 072011 (2015);
[7] CUPID-0 PRL 123, 032501 (2019)

Majoron emitting $0\nu\beta\beta$

- Requires the emission of one or two additional massless bosons, Majorons $\Longrightarrow$ similarities with $2\nu\beta\beta$
- There are many different models with different spectral index n and different number of emitted majorons
-

Model	Decay mode	NG boson	L	n	NME
IB	$0\nu\beta\beta\chi_0$	No	0	1	M_1
IC	$0\nu\beta\beta\chi_0$	Yes	0	1	M_1
ID	$0\nu\beta\beta\chi_0\chi_0$	No	0	3	M_3
IE	$0\nu\beta\beta\chi_0\chi_0$	Yes	0	3	M_3
IIB	$0\nu\beta\beta\chi_0$	No	−2	1	M_1
IIC	$0\nu\beta\beta\chi_0$	Yes	−2	3	M_2
IID	$0\nu\beta\beta\chi_0\chi_0$	No	−1	3	M_3
IIE	$0\nu\beta\beta\chi_0\chi_0$	Yes	−1	7	M_3
IIF	$0\nu\beta\beta\chi_0$	Gauge boson	−2	3	M_2
"Bulk"	$0\nu\beta\beta\chi_0$	Bulk field	0	2	

- Experimental limits on $\tau_{1/2}$ give information about the Majoron-neutrino coupling constant

$$\left[\tau_{1/2}^{0\nu M}\right]^{-1} = G^{(0)}_{m\chi_0 n}\left|\langle g_{\chi^M_{ee}}\rangle\right|^{2m}\left|M^{(m,n)}_{0\nu M}\right|^2$$

Majoron emitting $0\nu\beta\beta$

- Different types on nuclear matrix elements:

$$M_1 = g_A^2 \mathcal{M}_1 = g_A^2 \left[-\left(\frac{g_V^2}{g_A^2}\right) \mathcal{M}_F + \mathcal{M}_{GT} - \mathcal{M}_T \right],$$

$$M_2 = g_A^2 \mathcal{M}_2 = g_A^2 \left[\left(\frac{g_V}{g_A}\right) \frac{f_W}{3} \mathcal{M}_{GTR} - \left(\frac{g_V}{g_A}\right) \frac{f_W}{6} \mathcal{M}_{TR} \right],$$

$$M_3 = g_A^2 \mathcal{M}_3 = g_A^2 \left[-\left(\frac{g_V^2}{g_A^2}\right) \mathcal{M}_{F\omega^2} + \mathcal{M}_{GT\omega^2} - \mathcal{M}_{T\omega^2} \right],$$

- M_1

$$\mathcal{M}_F = \langle f \| v_m \| i \rangle,$$
$$\mathcal{M}_{GT} = \langle f \| v_m \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \| i \rangle,$$
$$\mathcal{M}_T = \langle f \| v_m S_{12} \| i \rangle,$$

$$v_m = \frac{2}{\pi} \frac{1}{q(q+\tilde{A})}$$

Same as for ν_{light}

- M_2

$$\mathcal{M}_{GTR} = \langle f \| v_R \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \| i \rangle,$$
$$\mathcal{M}_{TR} = \langle f \| v_R S_{12} \| i \rangle,$$

$$v_R = \frac{2}{\pi} \frac{1}{R m_p} \frac{q + \frac{\tilde{A}}{2}}{q(q+\tilde{A})^2}$$

- M_3

$$\mathcal{M}_{F\omega^2} = \langle f \| v_{\omega^2} \| i \rangle,$$
$$\mathcal{M}_{GT\omega^2} = \langle f \| v_{\omega^2} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \| i \rangle,$$
$$\mathcal{M}_{T\omega^2} = \langle f \| v_{\omega^2} S_{12} \| i \rangle,$$

$$v_{\omega^2} = \frac{2}{\pi} m_e^2 \frac{q^2 + \frac{9}{8} q\tilde{A} + \frac{3}{8}\tilde{A}^2}{q^3 (q+\tilde{A})^3}$$

Majoron emitting $0\nu\beta\beta$

- Differential decay rate $dW_{m\chi_0 n} = \left(a^{(0)} + a^{(1)}\cos\theta_{12}\right) w_{m\chi_0 n} d\epsilon_1 d\epsilon_2 d(\cos\theta_{12})$

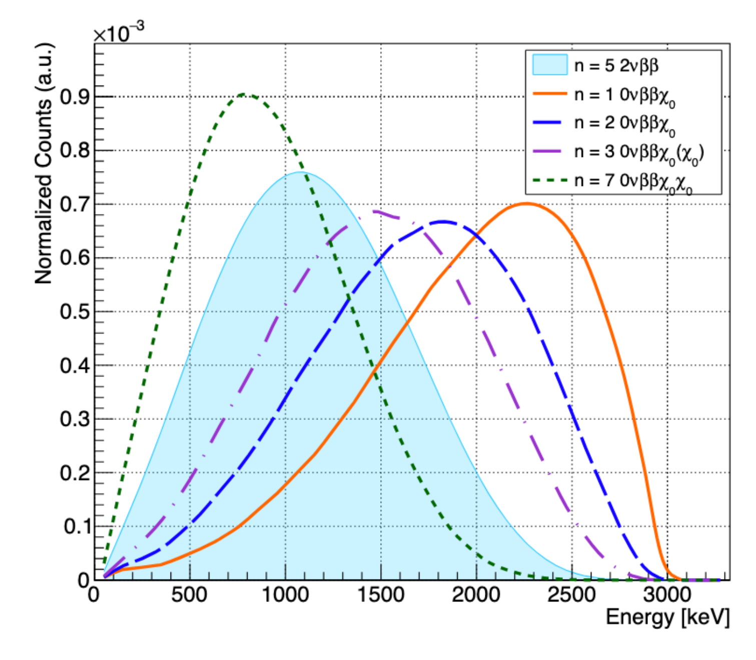
$$w_{1\chi_0 1} = \frac{g_A^4 (G\cos\theta_C)^4}{64\pi^7\hbar}\left(\frac{\hbar c}{2R}\right)^2 q(p_1c)(p_2c)\epsilon_1\epsilon_2$$

$$w_{1\chi_0 3} = \frac{g_A^4 (G\cos\theta_C)^4}{64\pi^7\hbar} q^3(p_1c)(p_2c)\epsilon_1\epsilon_2$$

$$w_{2\chi_0 3} = \frac{g_A^4 (G\cos\theta_C)^4}{3072\pi^9\hbar}(m_ec^2)^{-2}\left(\frac{\hbar c}{2R}\right)^2 q^3(p_1c)(p_2c)\epsilon_1\epsilon_2$$

$$w_{2\chi_0 7} = \frac{g_A^4 (G\cos\theta_C)^4}{53760\pi^9\hbar}(m_ec^2)^{-6}\left(\frac{\hbar c}{2R}\right)^2 q^7(p_1c)(p_2c)\epsilon_1\epsilon_2$$

- Summed electron spectrum: shape makes the different Majoron emitting modes experimentally recognizable



Majoron emitting $0\nu\beta\beta$

/ Limits on majoron-neutrino coupling constant for different models *PRC 103 (2021)* 044302

Decay mode	Spectral index	Model type	$\mathcal{M}$	$G^{(0)}_{m\chi_0 n}[10^{-18}$ yr]	$\tau_{1/2}$ [yr]	$\lvert\langle g_{\chi^M_{ee}}\rangle\rvert$
^{76}Ge [Gerda, EPJ+ 130(2015) 139]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	6.64	44.2	$>4.2\times10^{23}$	$<3.5\times10^{-5}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0026	0.22	$>0.8\times10^{23}$	<1.7
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.381	0.073	$>0.8\times10^{23}$	$<0.34\times10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0026	0.420	$>0.3\times10^{23}$	<1.9
$0\nu\beta\beta\chi_0$	2	Bulk			$>1.8\times10^{23}$	
^{130}Te [CUORE, PLB557 (2003) 167]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	4.40	413	$>2.2\times10^{21}$	$<2.4\times10^{-4}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0013	3.21	$>0.9\times10^{21}$	<3.8
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.199	1.51	$>2.2\times10^{21}$	$<0.87\times10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0013	14.4	$>0.9\times10^{21}$	<2.6
$0\nu\beta\beta\chi_0$	2	Bulk			$>2.2\times10^{21}$	
^{130}Te [NEMO3, PRL107 (2011) 062504]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	4.40	413	$>1.6\times10^{22}$	$<8.8\times10^{-5}$
^{136}Xe [EXO-200, PRD90 (2014) 092004]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	3.60	409	$>1.2\times10^{24}$	$<1.3\times10^{-5}$
$0\nu\beta\chi_0\chi_0$	3	ID,IE,IID	0.0011	3.05	$>2.7\times10^{22}$	<1.8
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.160	1.47	$>2.7\times10^{22}$	$<0.31\times10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0011	12.5	$>6.1\times10^{21}$	<1.8
$0\nu\beta\beta\chi_0$	2	Bulk			$>2.5\times10^{23}$	
^{136}Xe [KamLAND-Zen, PRC86 (2012) 021601(R)]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	3.60	409	$>2.6\times10^{24}$	$<8.5\times10^{-6}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0011	3.05	$>4.5\times10^{24}$	<0.49
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.160	1.47	$>4.5\times10^{24}$	$<0.24\times10^{-2}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0011	12.5	$>1.1\times10^{22}$	<1.6
$0\nu\beta\beta\chi_0$	2	Bulk			$>1.0\times10^{24}$	

Majoron emitting $0\nu\beta\beta$

/ Limits on majoron-neutrino coupling constant for different models *PRC 103 (2021) 044302*

Decay mode	Spectral index	Model type	$\mathcal{M}$	$G^{(0)}_{m\chi_0 n}[10^{-18}\ \text{yr}]$	$\tau_{1/2}$ [yr]	$\vert\langle g_{\chi^M_{ee}}\rangle\vert$
^{76}Ge [32]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	6.64	44.2	$>4.2\times10^{23}$	$<3.5\times10^{-5}$
$0\nu\beta\beta\chi_0\chi_0$				0.22	$>0.8\times10^{23}$	<1.7
$0\nu\beta\beta\chi_0$		IIC,IIF		0.073	$>0.8\times10^{23}$	$<0.34\times10^{-1}$
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB		409	$>1.2\times10^{24}$	$<1.3\times10^{-5}$
$0\nu\beta\chi_0\chi_0$	3	ID,IE,IID		3.05		<1.8
$0\nu\beta\beta\chi_0$	3	IIC,IIF		1.47		
$0\nu\beta\beta\chi_0\chi_0$		IIE		12.5		
$0\nu\beta\beta\chi_0$		Bulk				
^{136}Xe [30]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	3.60	409		
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0011	3.05		
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.160	1.47		
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0011	12.5		
$0\nu\beta\beta\chi_0$	2	Bulk				

Model type	$\mathcal{M}$	$G^{(0)}_{m\chi_0 n}[10^{-18}\ \text{yr}]$	$\tau_{1/2}$ [yr]	$\vert\langle g_{\chi^M_{ee}}\rangle\vert$
IB,IC,IIB	6.64	44.2	$>4.2\times10^{23}$	$<3.5\times10^{-5}$
ID,IE,IID	0.0026	0.22	$>0.8\times10^{23}$	<1.7
IIC,IIF	0.381	0.073	$>0.8\times10^{23}$	$<0.34\times10^{-1}$
IIE	0.0026	0.420	$>0.3\times10^{23}$	<1.9

LARGE

LARGE

SMALL

SMALL

MUCH WEAKER LIMITS COMPARED TO m=1, n=1 models

Majoron emitting $0\nu\beta\beta$

- Fresh limits for ^{82}Se on majoron-neutrino coupling constant [CUPID-0, (2022) arXiv:2209.09490v1]

Decay	n	$t_{1/2}$ 90%C.I. (yr)	$\lvert\langle g_{\chi_0}\rangle\rvert$
$0\nu\beta\beta\chi_0$	1	$>1.2\times10^{23}$	$<(1.8–4.4)\times10^{-5}$
$0\nu\beta\beta\chi_0$	2	$>3.8\times10^{22}$	–
$0\nu\beta\beta\chi_0$	3	$>1.4\times10^{22}$	<0.020
$0\nu\beta\beta\chi_0\chi_0$	3	$>1.4\times10^{22}$	<1.2
$0\nu\beta\beta\chi_0\chi_0$	7	$>2.2\times10^{21}$	<1.1

Conclusions

- Even though many milestones in the research of double beta decay has been achieved, $0\nu\beta\beta$ remains yet to be observed and fully understood.
- Observation of $0\nu\beta\beta$ would clarify many fundamental aspects of neutrino physics
  - lepton number non-conservation
  - neutrino nature: whether the neutrino is a Dirac or a Majorana particle
  - absolute neutrino mass scale
  - the type of neutrino mass ordering (normal or inverted)

- We do not yet know what is the mechanisms of $0\nu\beta\beta$-decay. Number of different mechanisms can trigger $0\nu\beta\beta$–decay and several mechanisms may contribute with different relative phases.

Joint effort of theory and experiments (i) to interpret the data, (ii) to identify new good candidates to study

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THANK YOU!

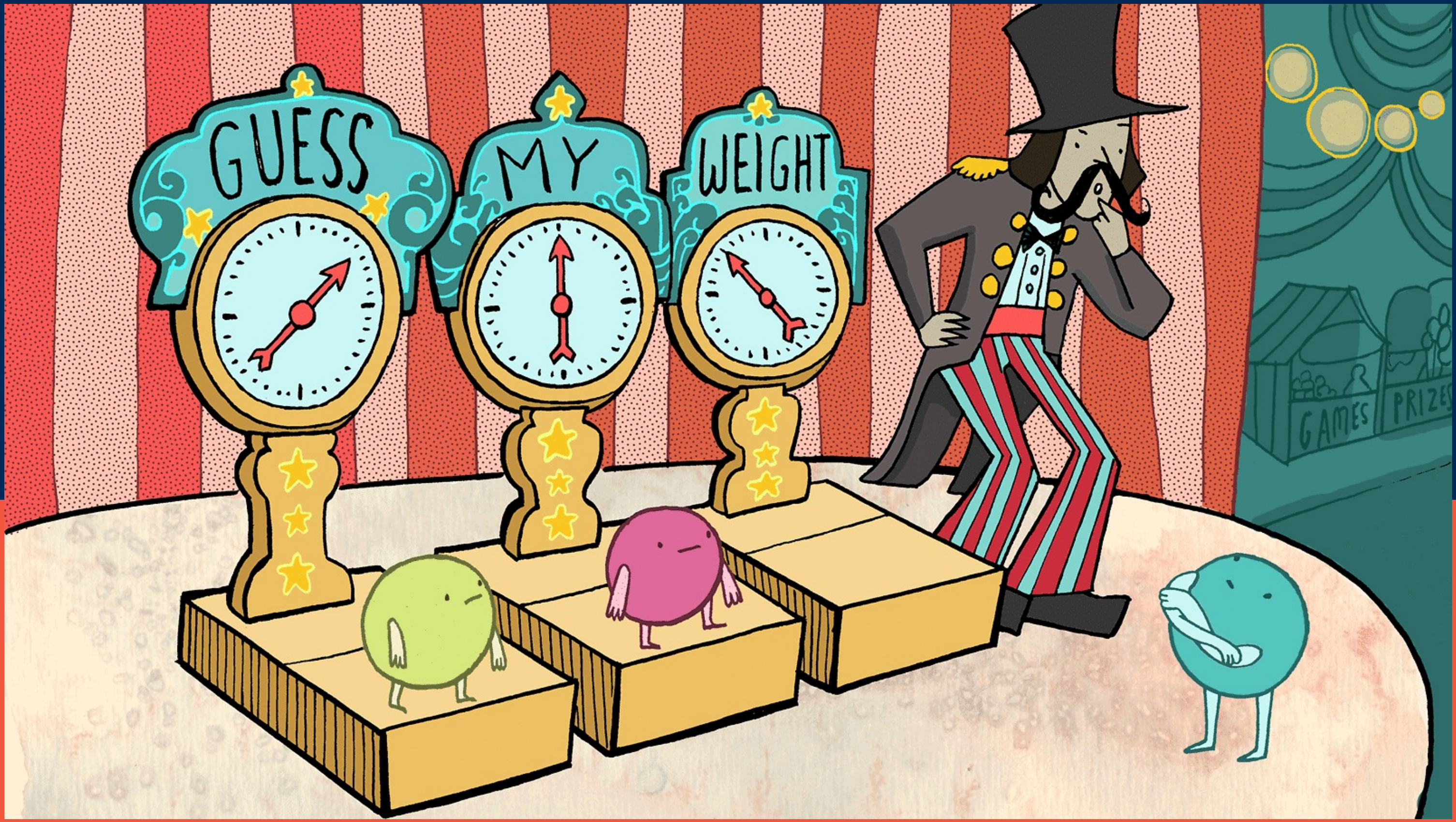

Illustration by Sandbox Studio, Chicago with Corinne Mucha