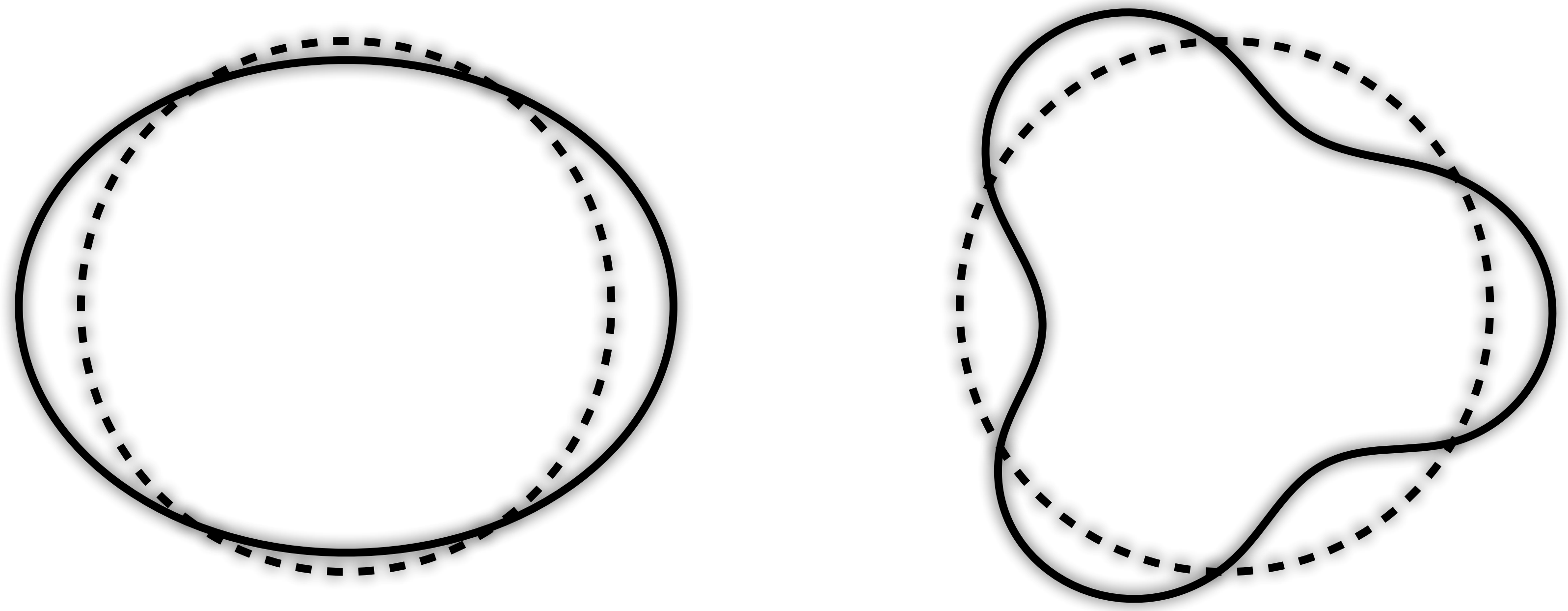
NEWS2310, Nov. 2, 2023

Spin and isospin responses in nuclei: roles of deformation and neutron excess

Kenichi Yoshida, RCNP

Giant resonances: collective mode of surface vibration

classical and intuitive picture



L=2: Giant Quadrupole Resonance (GQR)
L=3: High Energy Octupole Resonance (HEOR)

strongly excited by a one-body operator, exhaust a sum-rule value

$$\hat{O} = \sum_{\sigma\sigma'} \sum_{\tau\tau'} \int \vec{r} r^L Y_L(\hat{r}) \hat{\psi}^\dagger(\vec{r}\sigma\tau) \langle\sigma \begin{Bmatrix} 1 \\ \vec{\sigma} \end{Bmatrix} \sigma'\rangle\langle\tau \begin{Bmatrix} 1 \\ \vec{\tau} \end{Bmatrix} \tau'\rangle \hat{\psi}(\vec{r}\sigma'\tau')$$

space — spin — isospin

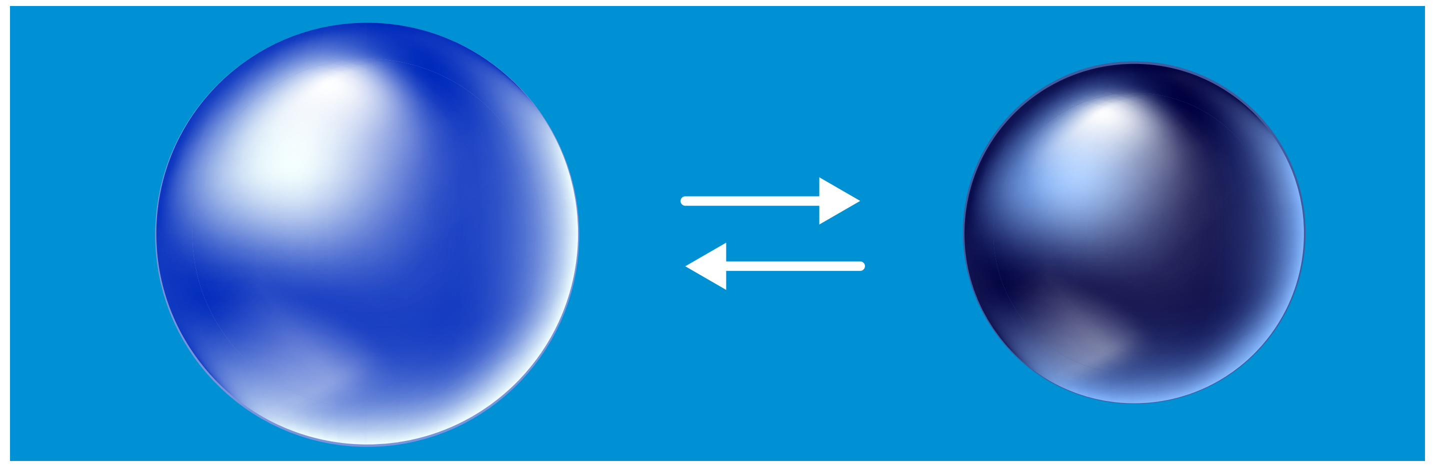
rich variety of modes depending on ΔL, ΔS, ΔT, and ΔN
affected by many-body correlations (deformation and superfluidity)

Giant Monopole Resonance (GMR)

$$\hat{O} = \sum_{\sigma\tau} \int d\vec{r} r^2 \psi^\dagger(\vec{r}\sigma\tau)\psi(\vec{r}\sigma\tau)$$

volume change ⟷ incompressibility of nuclear matter

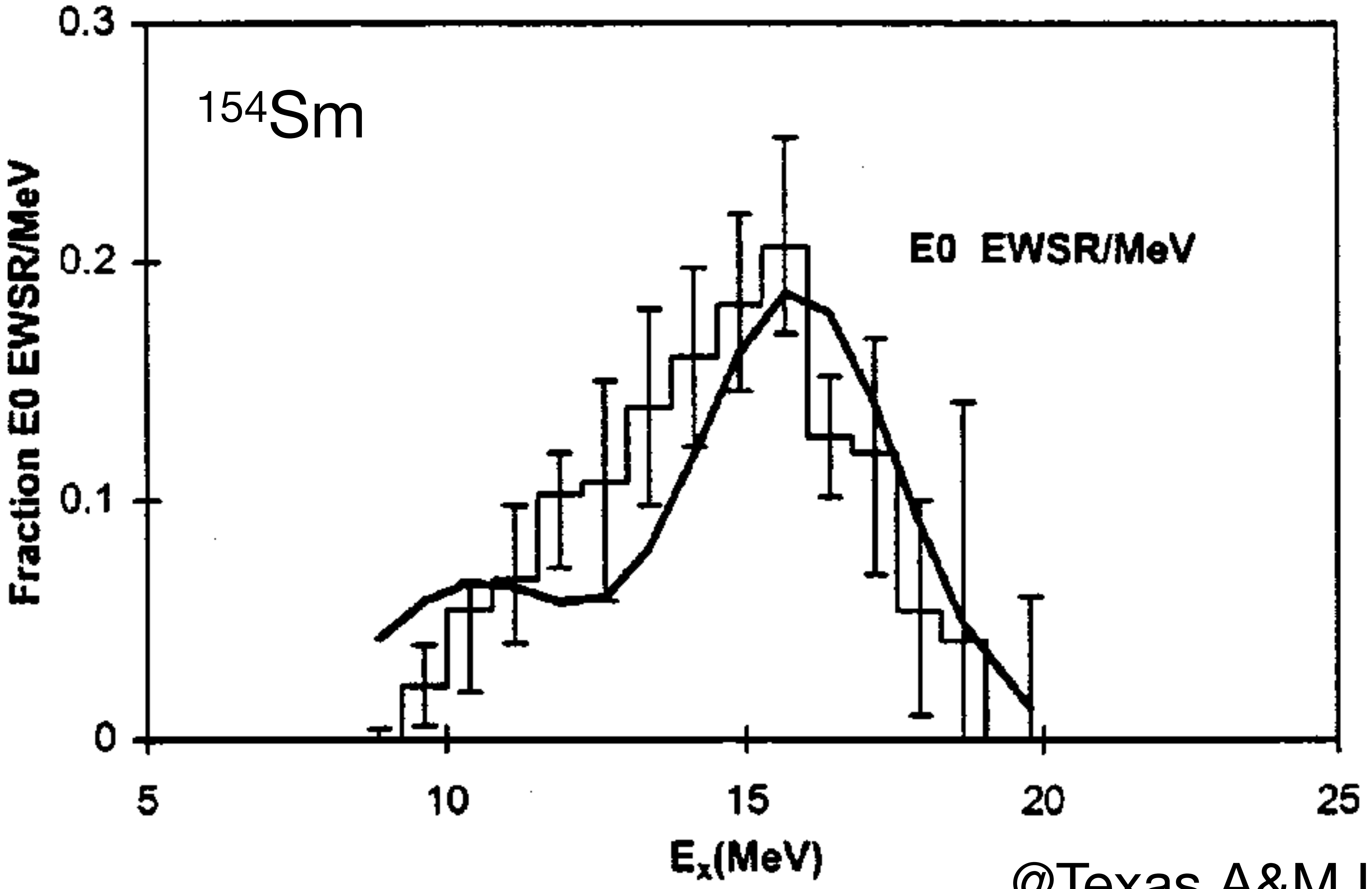
Blaizot ('80)



deformation splitting?

Garg+ ('80)

Youngblood+ ('99)

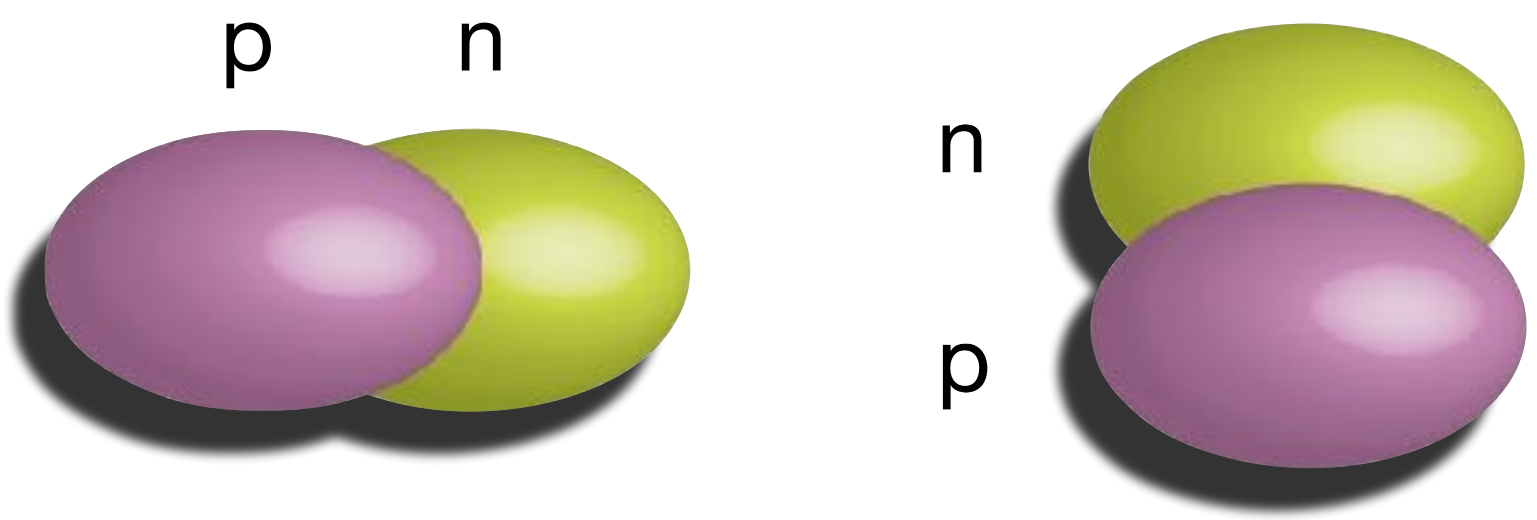


@Texas A&M Univ.

Deformation splitting?

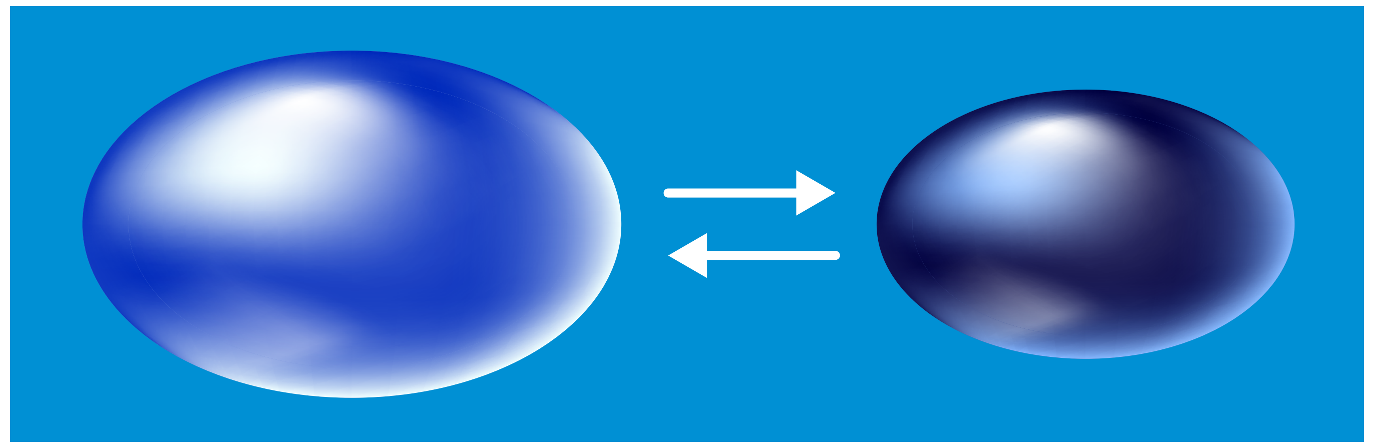
IVGDR

$$\omega_{\parallel} \propto \frac{1}{R_{\parallel}} \qquad \omega_{\perp} \propto \frac{1}{R_{\perp}}$$



K=0 K=1

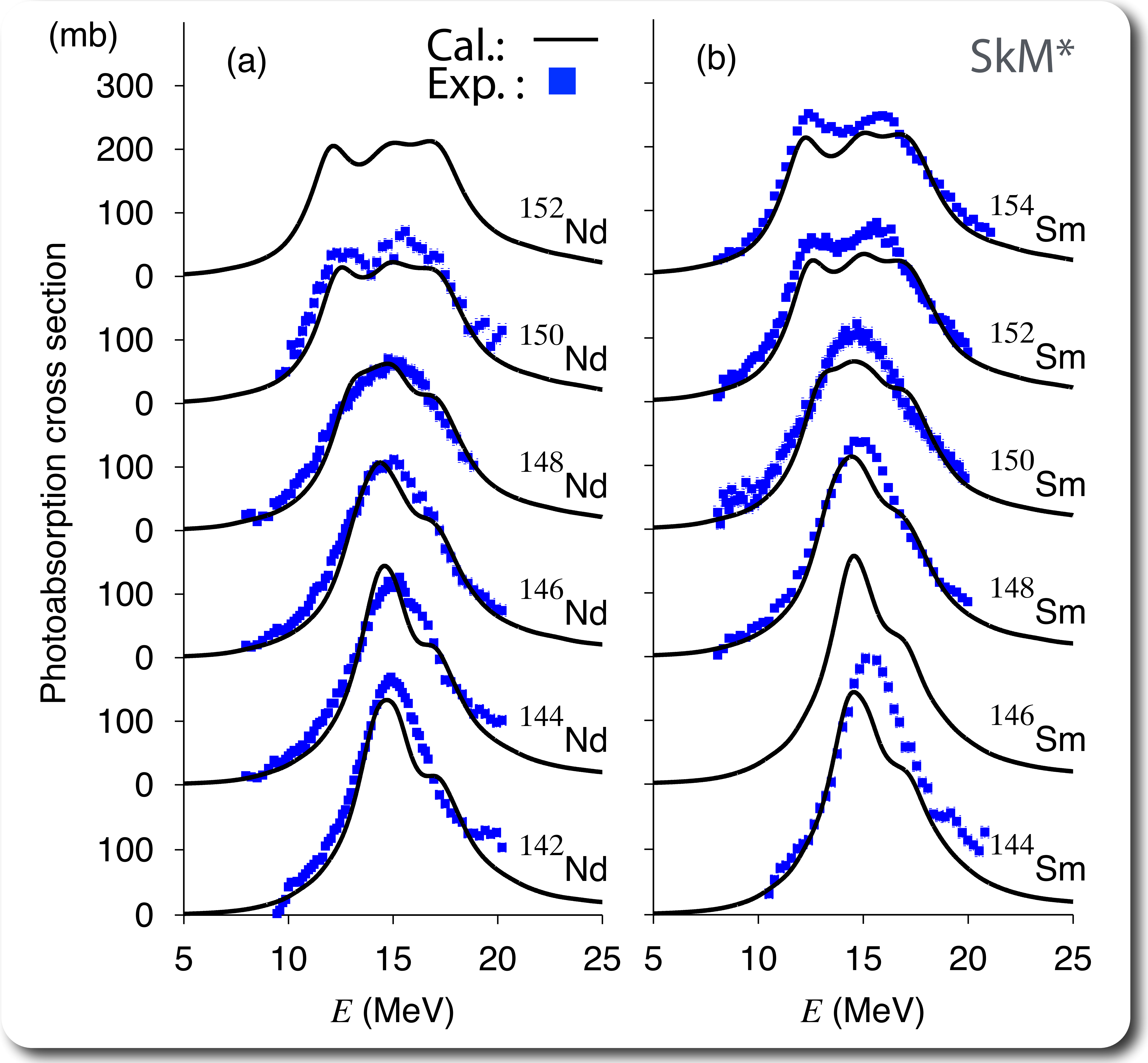
GMR



no angle dependence contrary to GDR

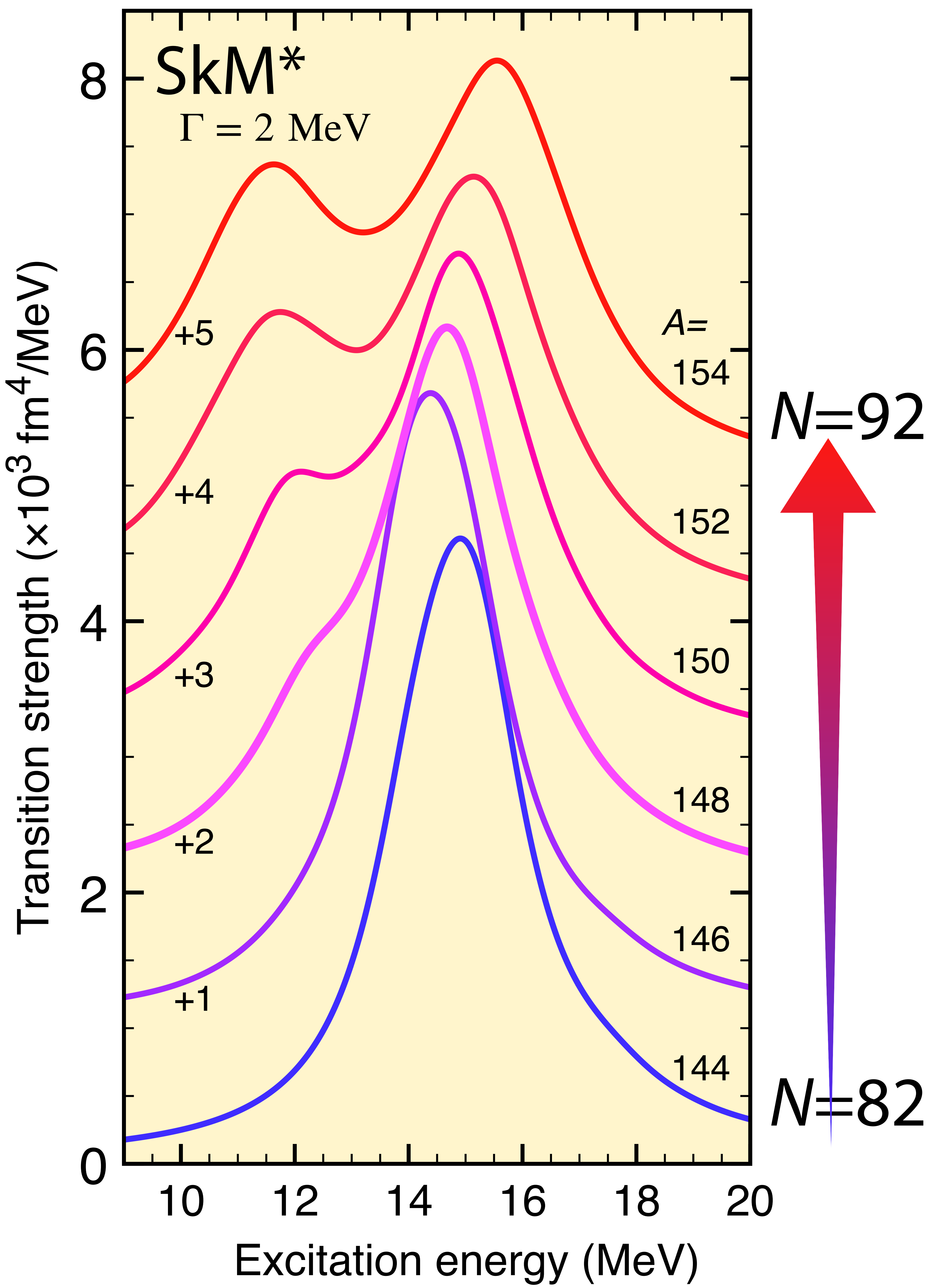
$Y_0(\hat{r})$ $Y_{1K}(\hat{r})$

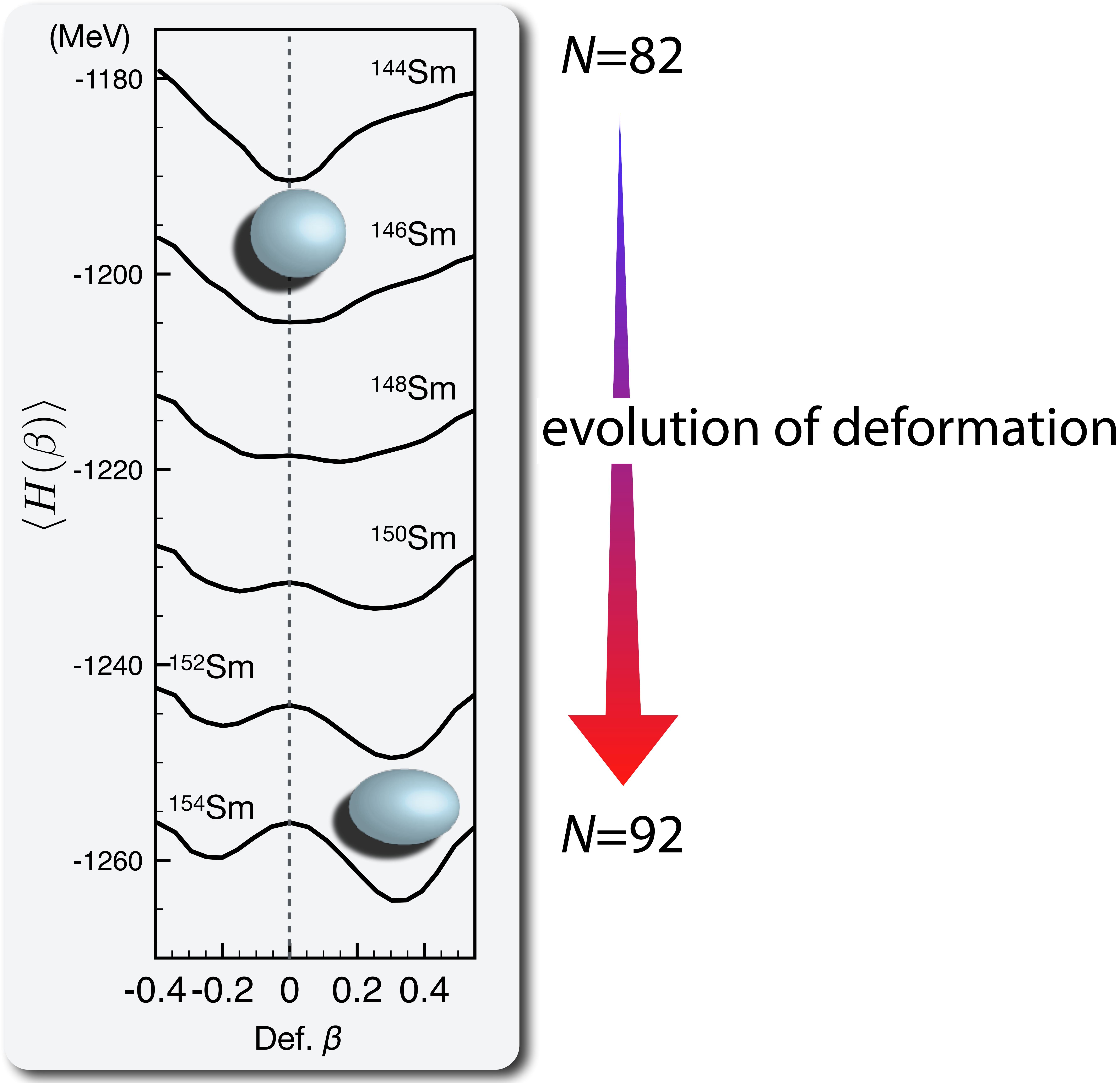
Yoshida–Nakatsukasa ('11)



GMR in the Sm isotopes

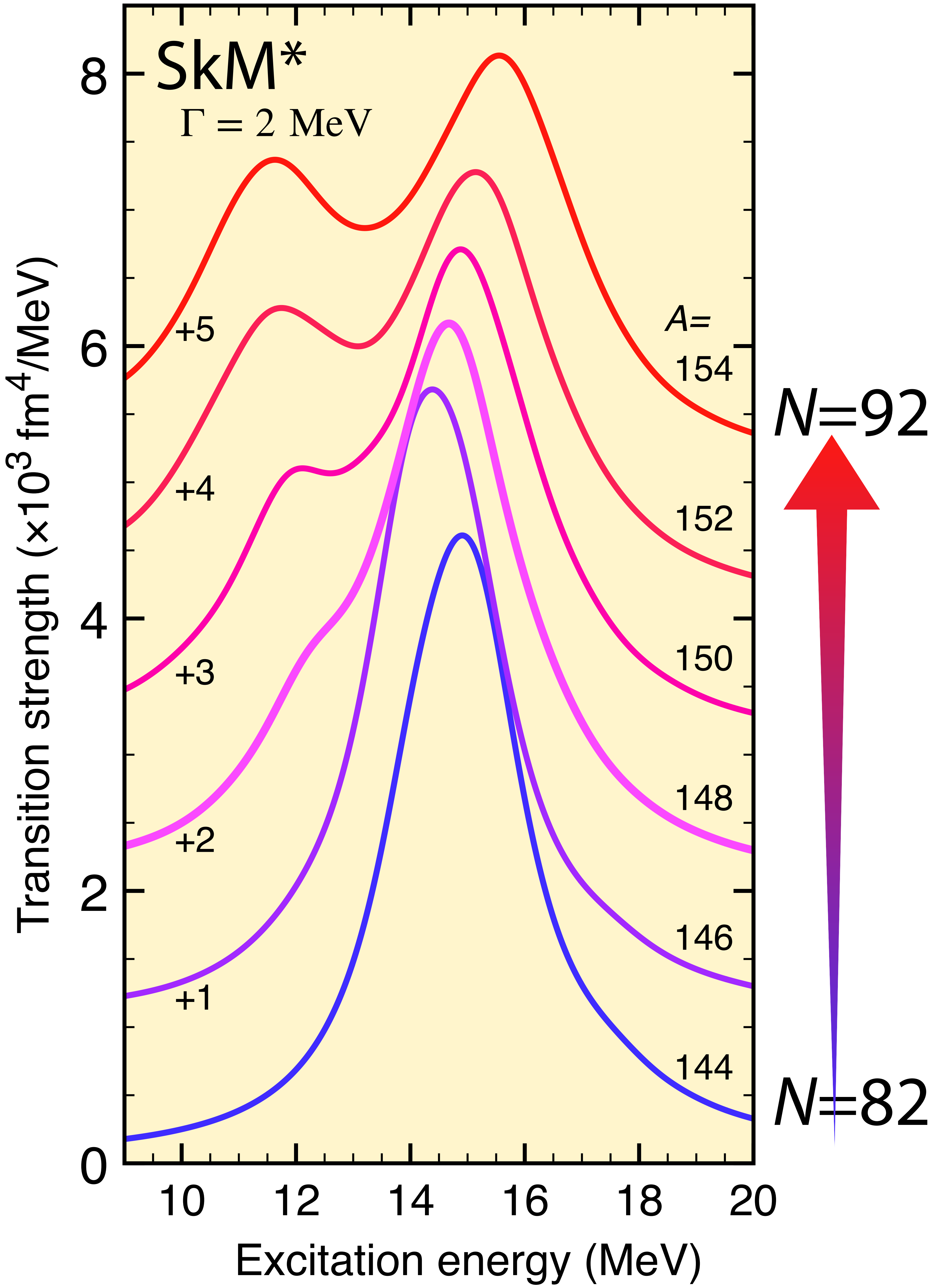
Yoshida–Nakatsukasa ('13)

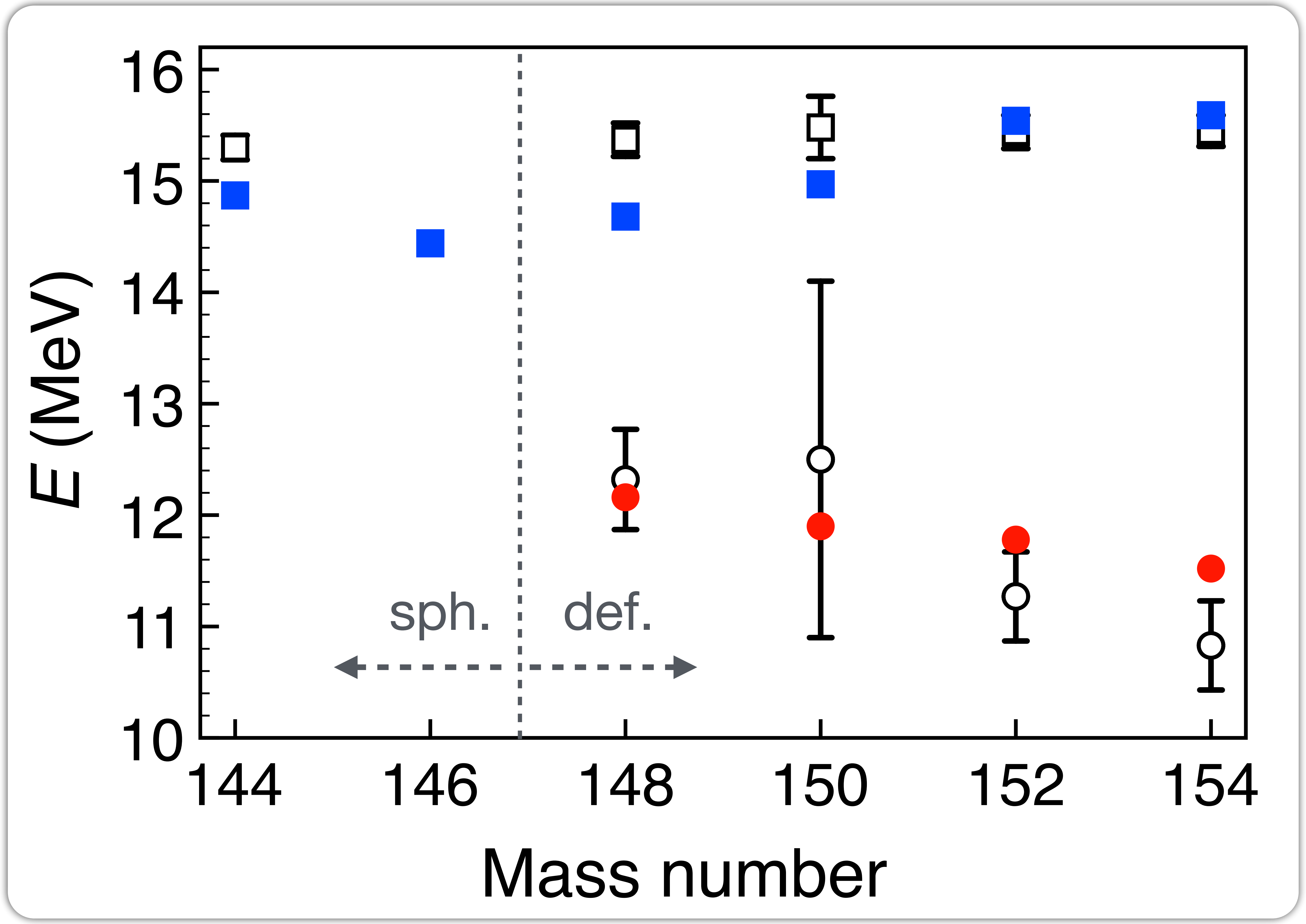




GMR in the Sm isotopes

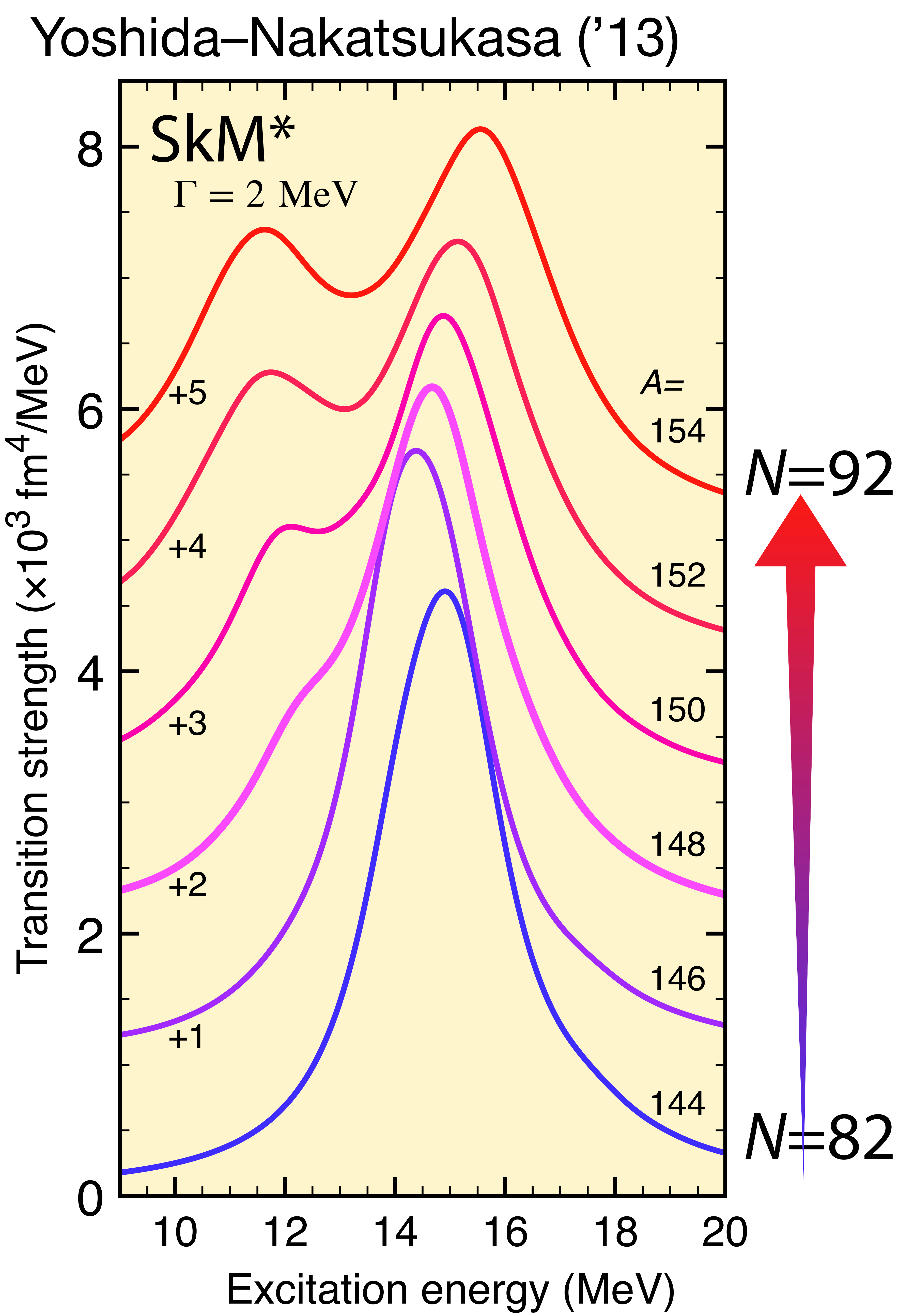
Yoshida–Nakatsukasa ('13)

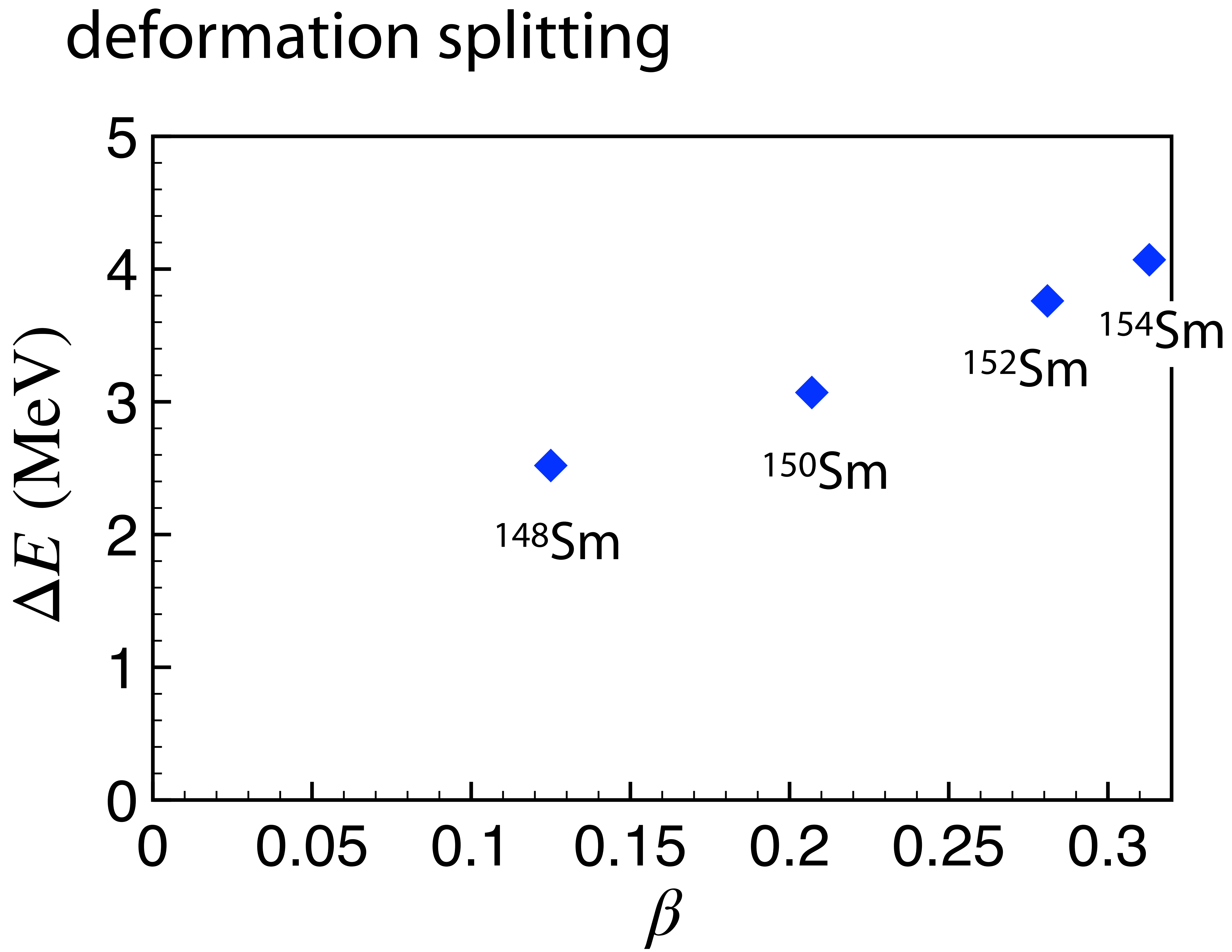




Exp.: Itoh+ ('03)

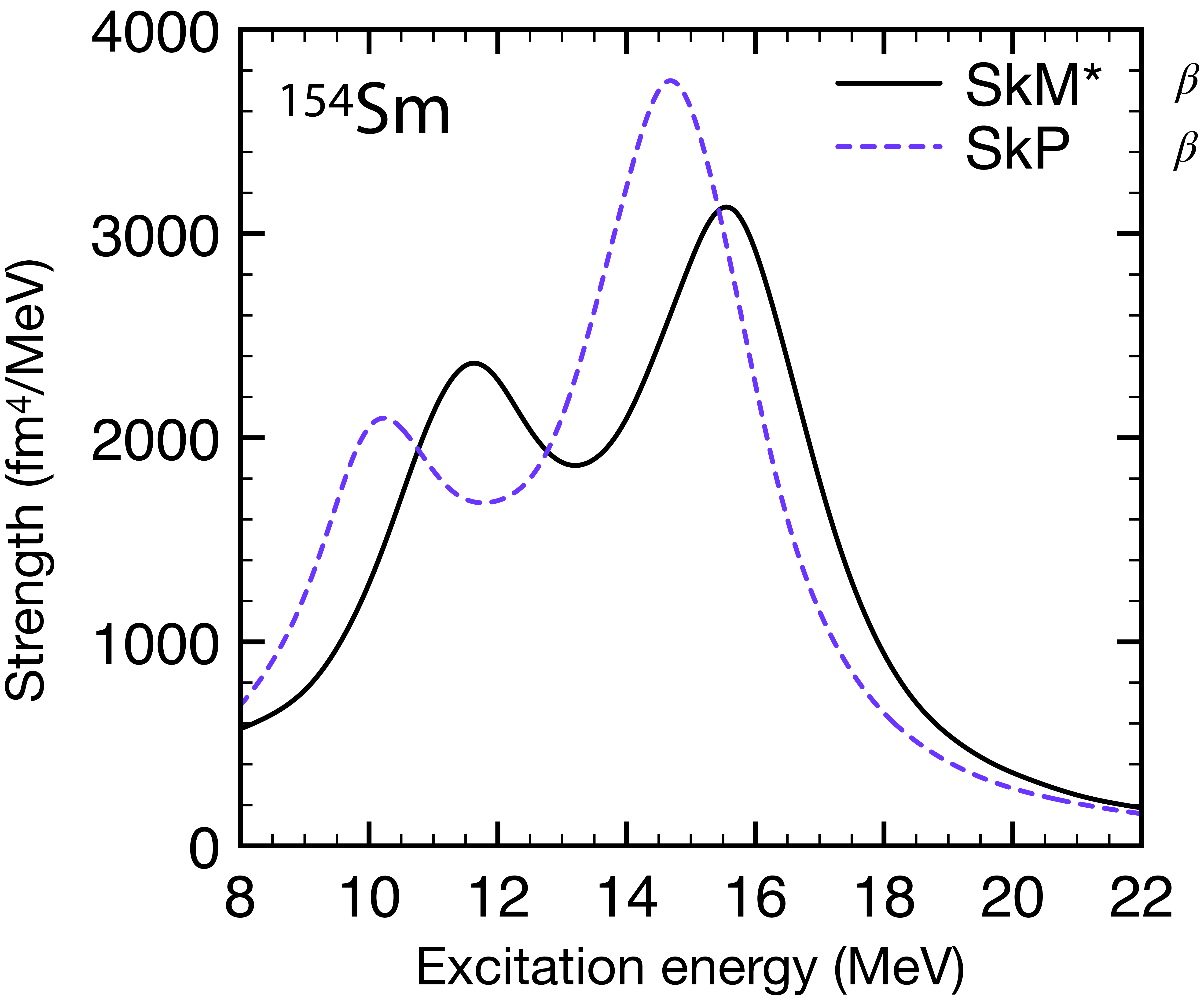
GMR in the Sm isotopes





GMR in the Sm isotopes

Yoshida–Nakatsukasa ('13)



	K_0 [MeV]	Ratio of EWS higher/lower
$\beta = 0.31$	217	1.9
$\beta = 0.29$	201	3.2

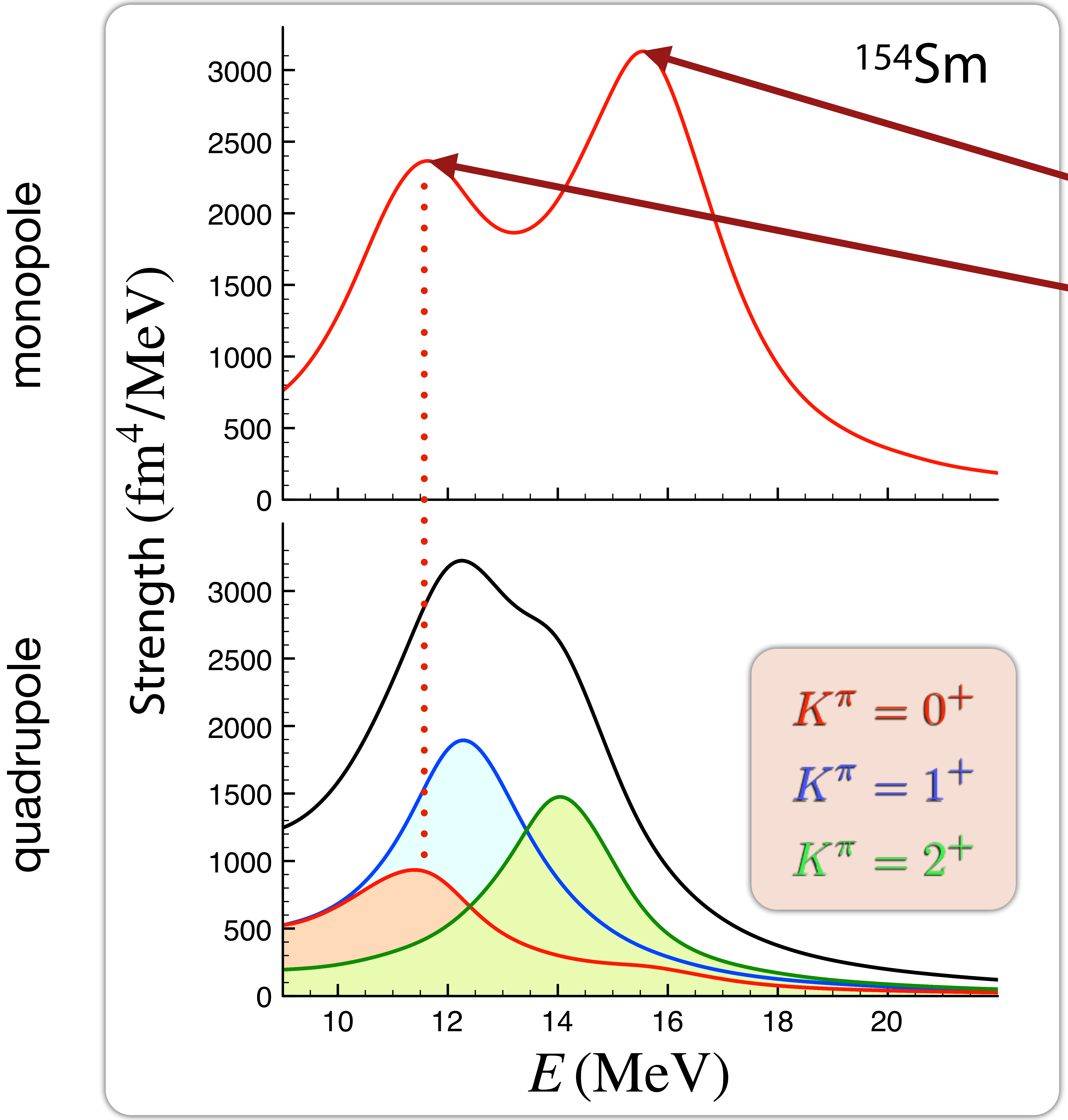
larger strengths in the lower peak in a strongly-deformed nucleus

stronger coupling between GMR and GQR as deformation increases

splitting energy

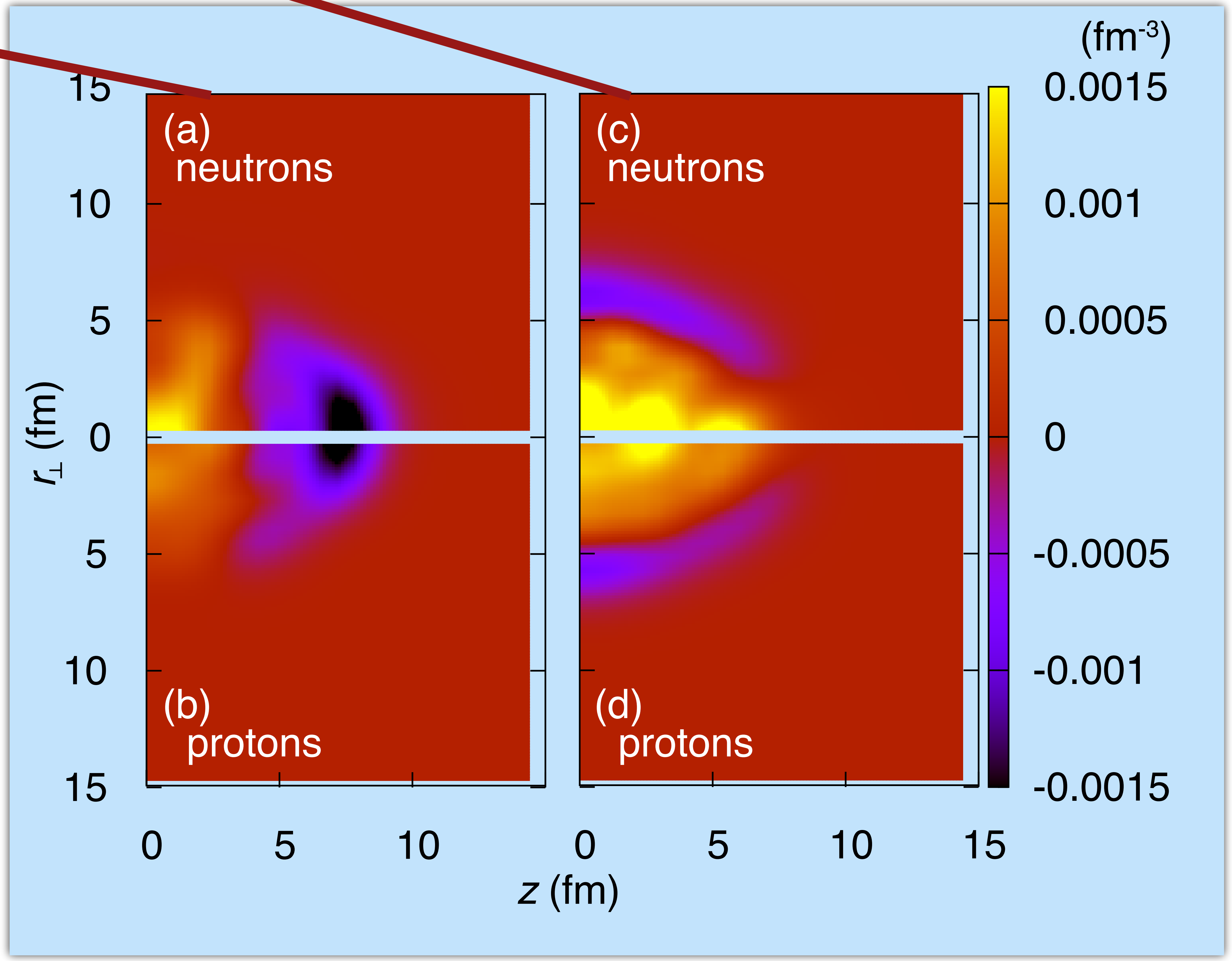
ratio of strengths

Coupling between GMR and GQR

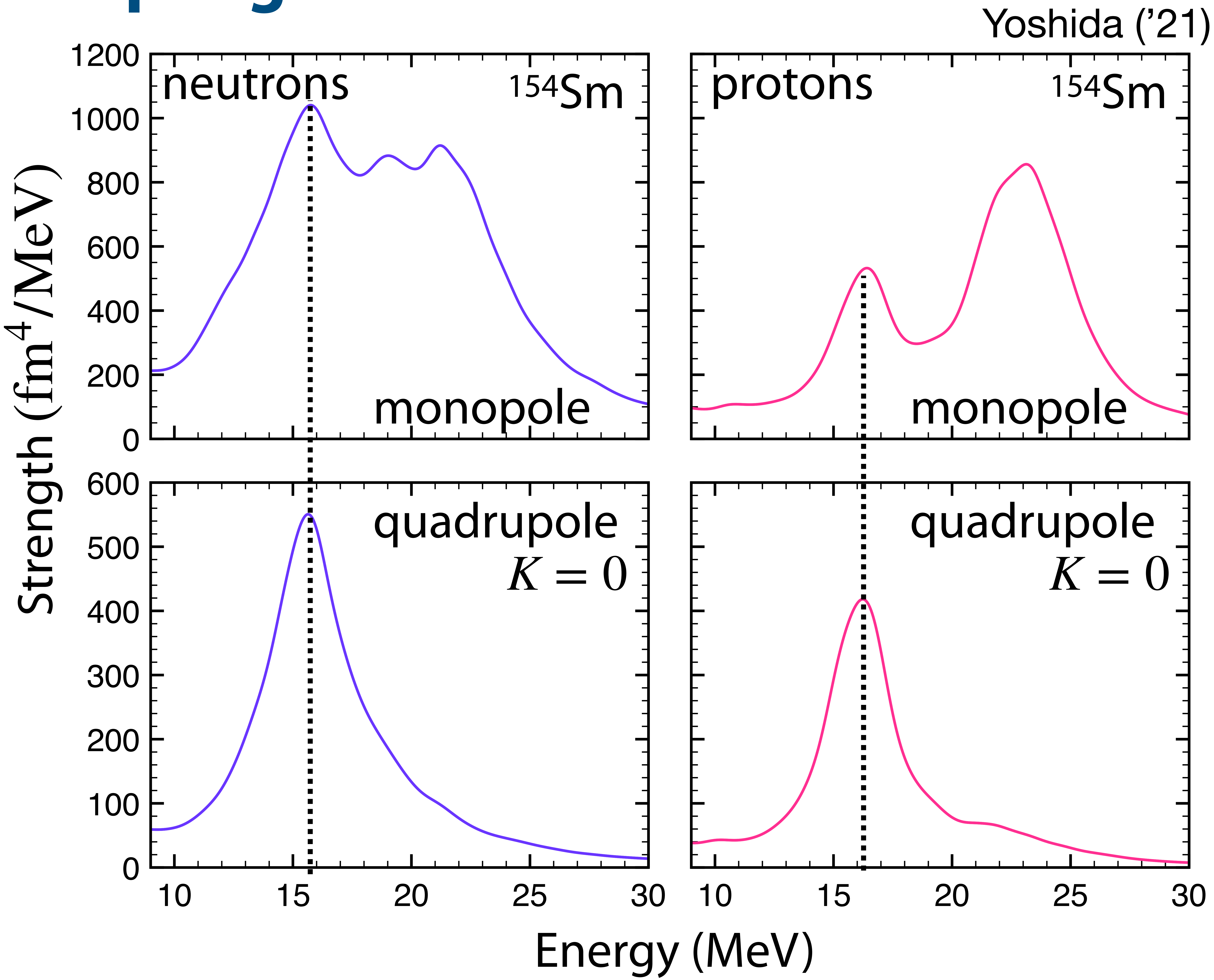


transition density

$$\delta\rho_\lambda(\vec{r}) = \langle \Psi_\lambda \;\; \psi^\dagger(\vec{r})\psi(\vec{r}) \;\; \Psi_0 \rangle$$



Coupling at the static level



Unperturbed strengths

w/o the RPA (dynamic) correlations

deformation-induced coupling

static effect

Peaks

monopole and K=0 quadrupole

coincide in energy

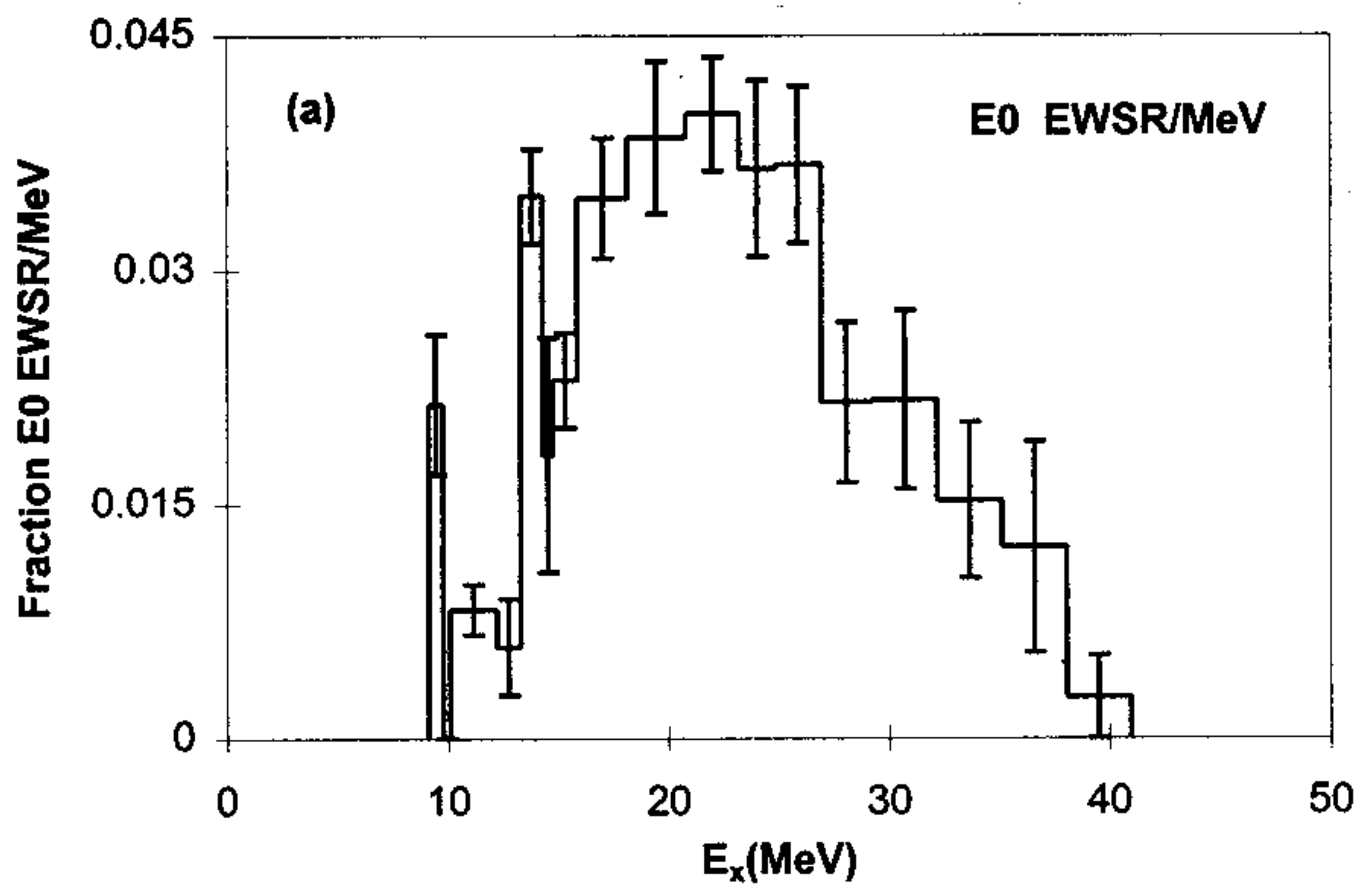
residual interactions

Coexistence persists

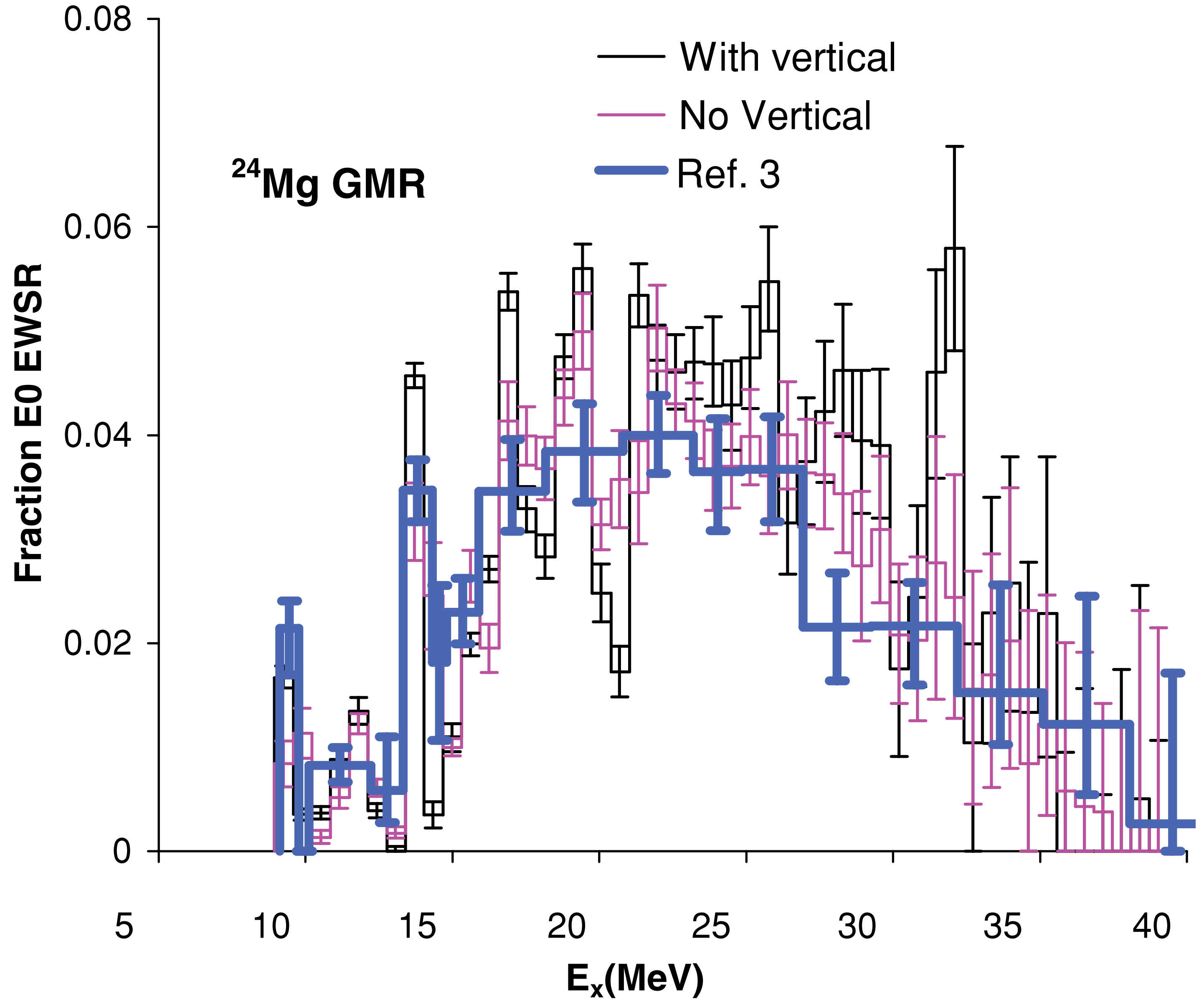
Deformation effect on GMR in light nuclei: universality

^{24}Mg @ TAMU

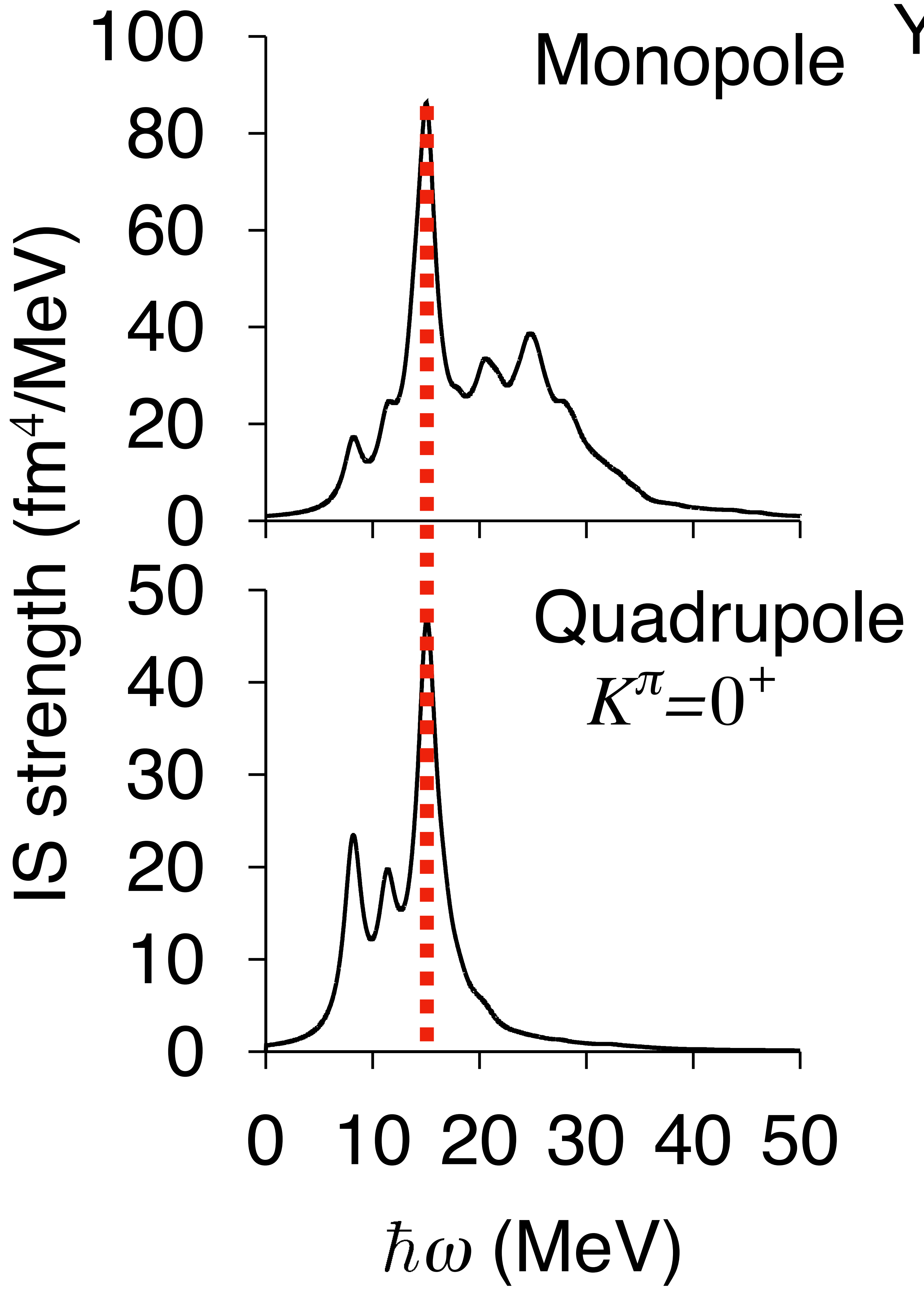
Youngblood+('99)



Youngblood+('09)



Yoshida('10)



occurrence of the "lower-energy (~15 MeV)" peak due to coupling to the *K*=0 of GQR

Deformation splitting in a light nucleus

Physics Letters B 748 (2015) 343–346

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Splitting of ISGMR strength in the light-mass nucleus ^{24}Mg due to ground-state deformation

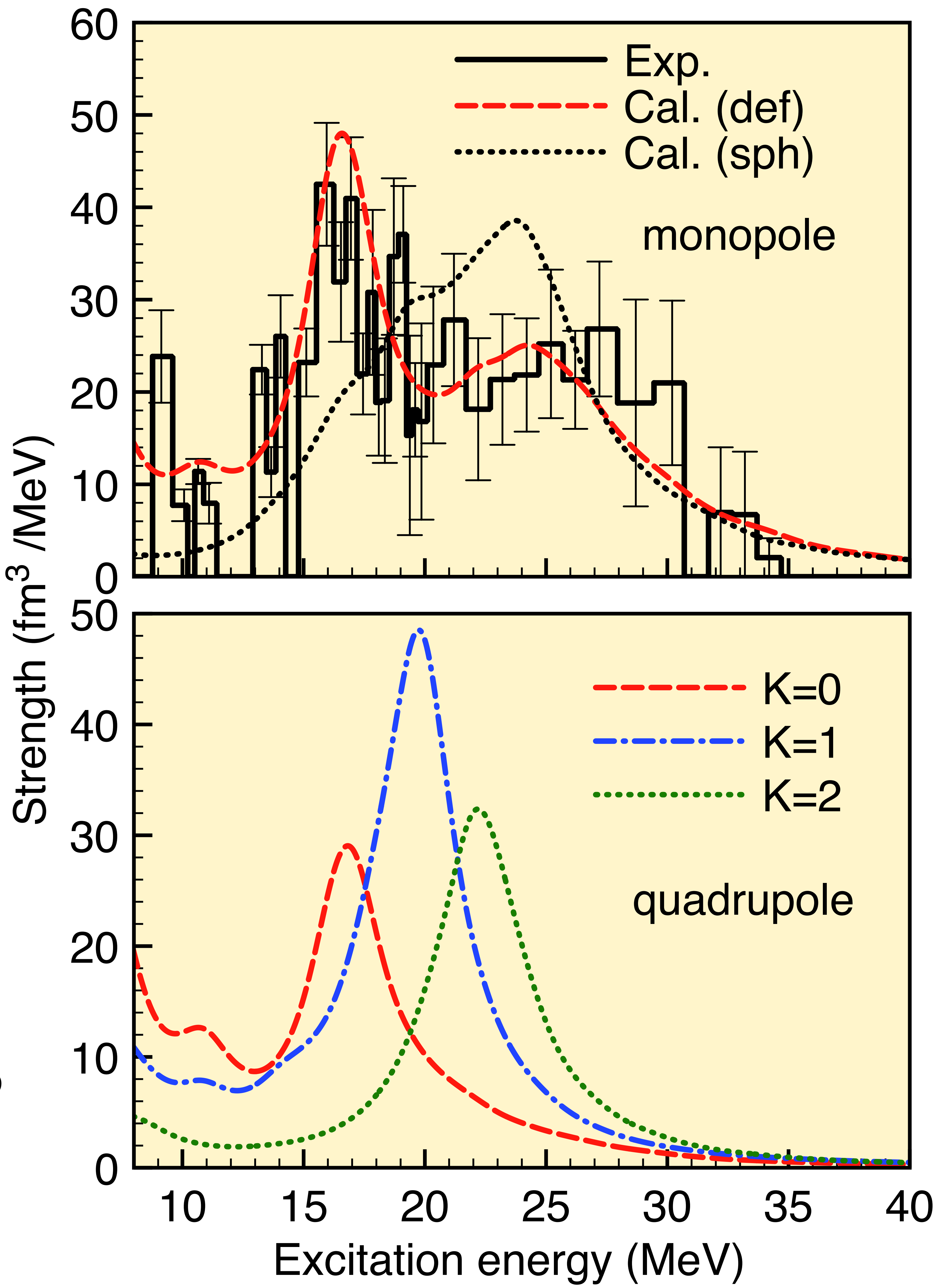
Y.K. Gupta [a,1], U. Garg [a], J.T. Matta [a], D. Patel [a], T. Peach [a], J. Hoffman [a,2], K. Yoshida [b,c], M. Itoh [d,3], M. Fujiwara [d], K. Hara [d], H. Hashimoto [d], K. Nakanishi [d], M. Yosoi [d], H. Sakaguchi [e], S. Terashima [e], S. Kishi [e], T. Murakami [e], M. Uchida [e,4], Y. Yasuda [e], H. Akimune [f], T. Kawabata [g,5], M.N. Harakeh [h]

First observation of the splitting of GMR strengths in a light system

universal feature in deformed nuclei

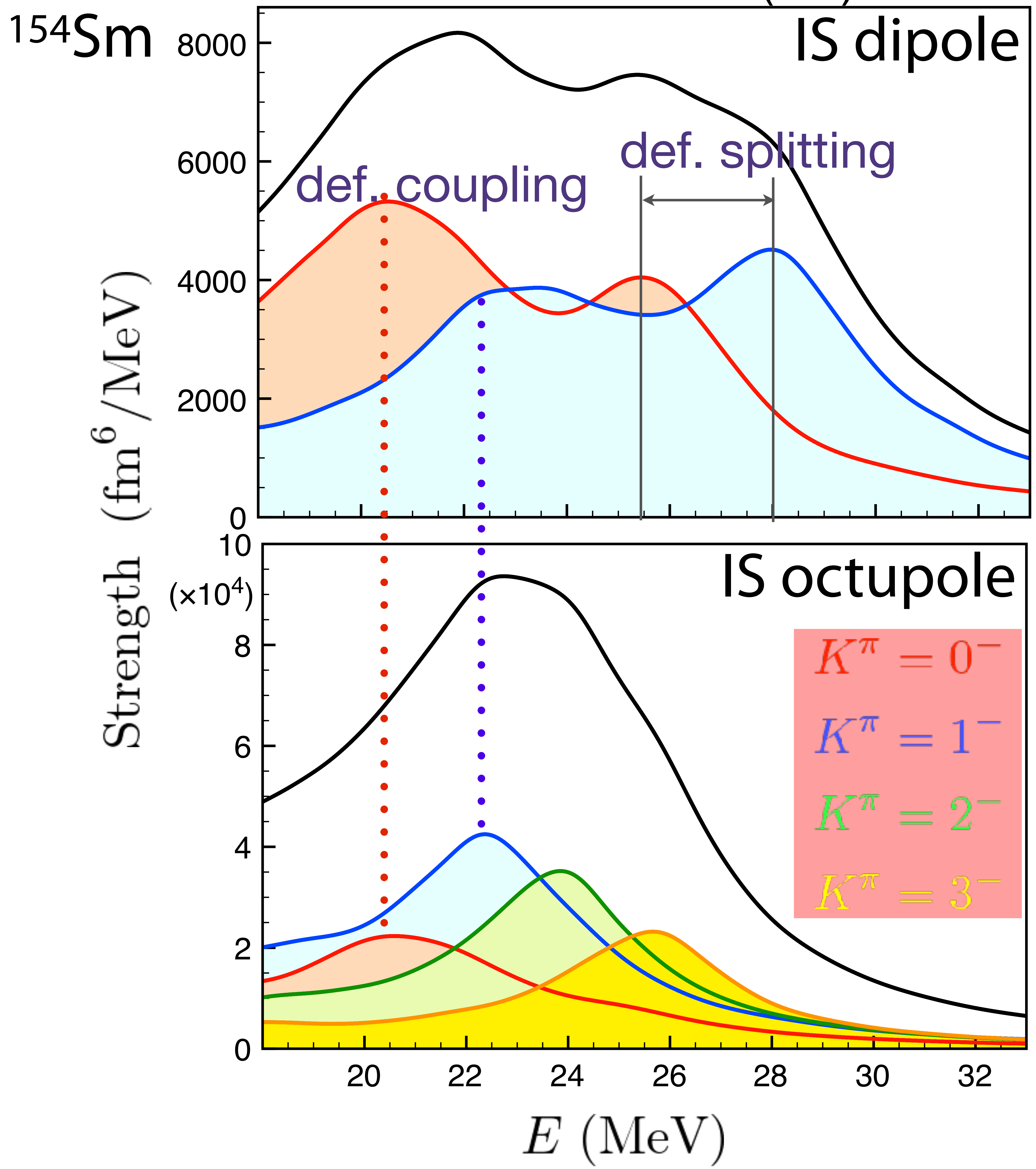
background-free high-resolution experiment @RCNP

parameter-free nuclear DFT calculation



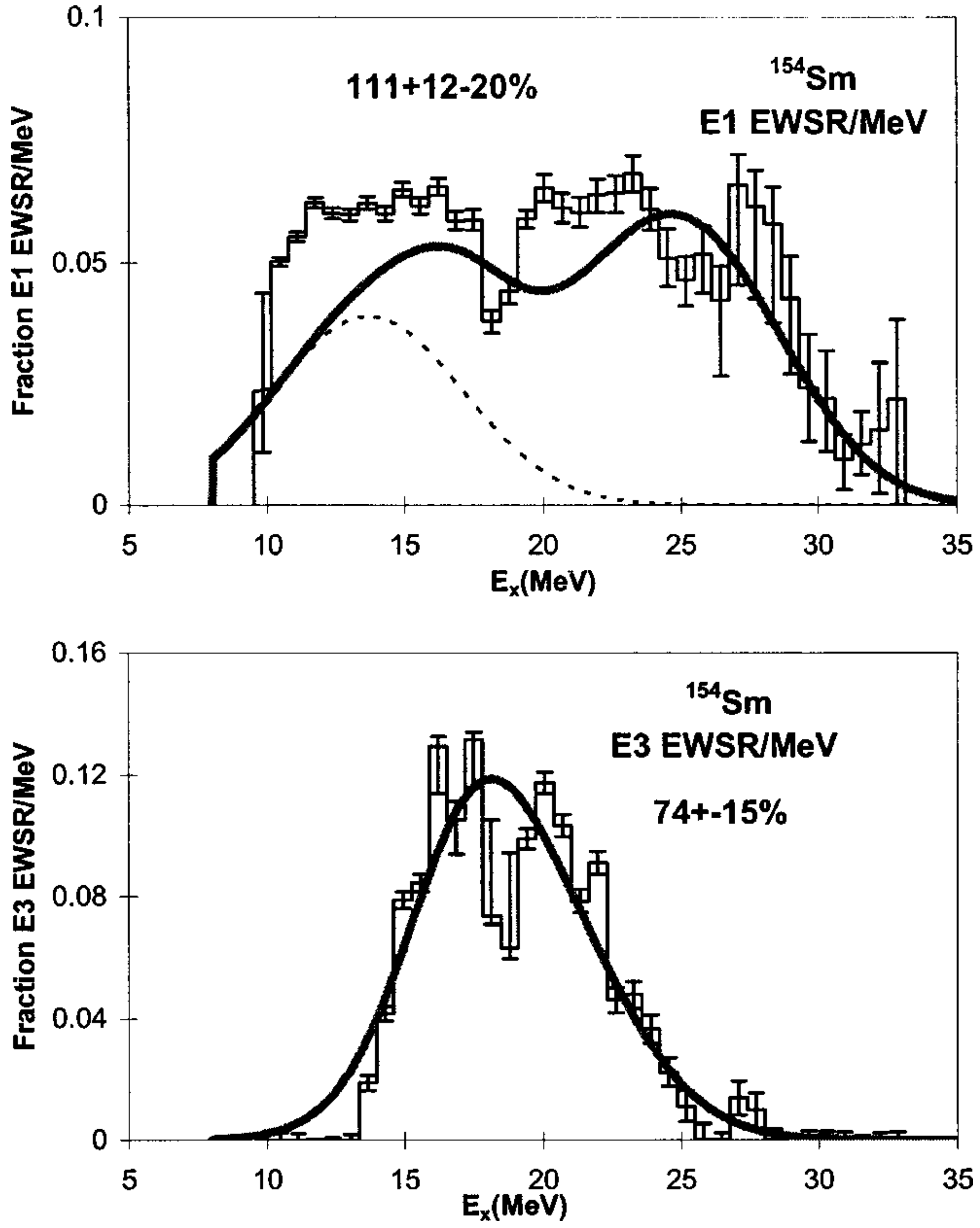
Coupling between $\Delta L = 2$

Yoshida–Nakatsukasa ('13)

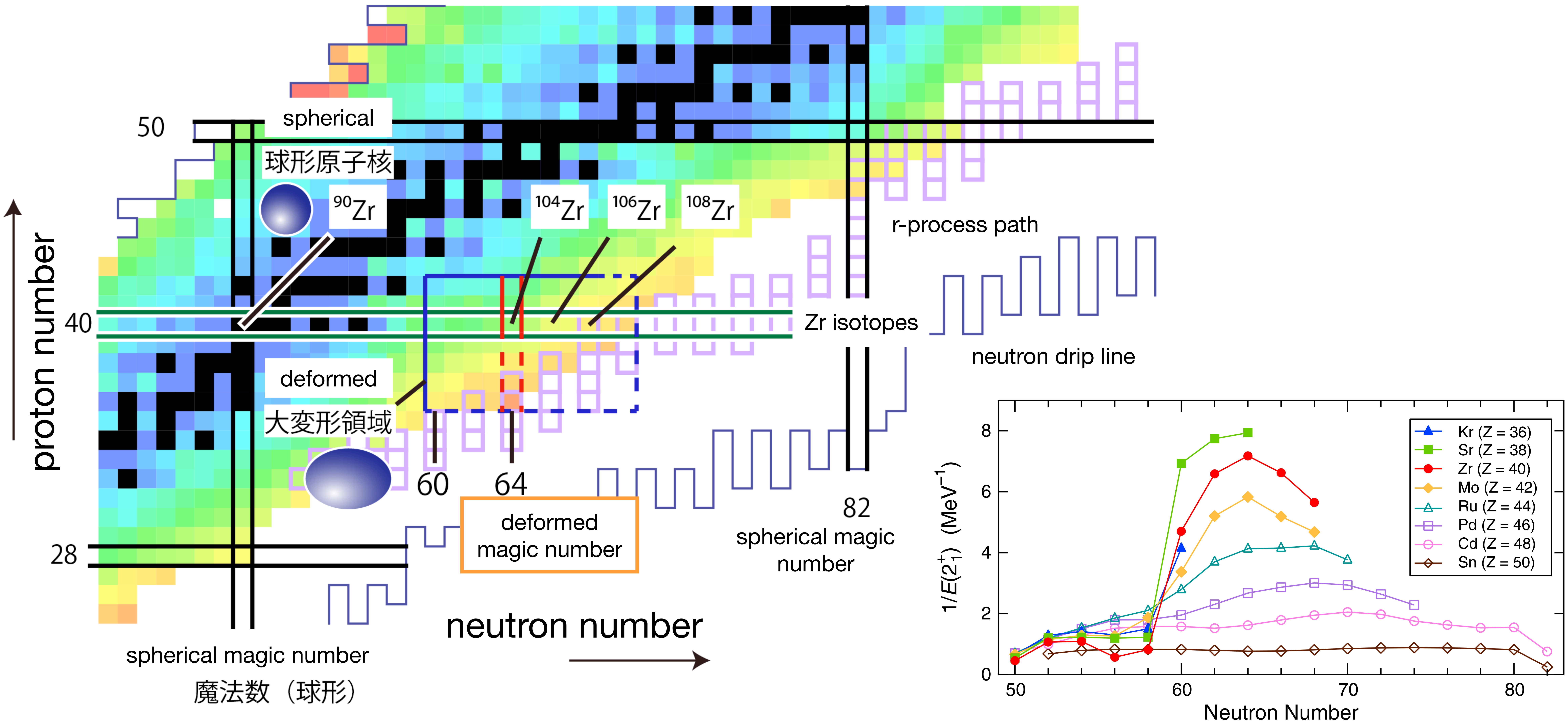


leading to a large width of the ISGDR

Youngblood+ ('04)

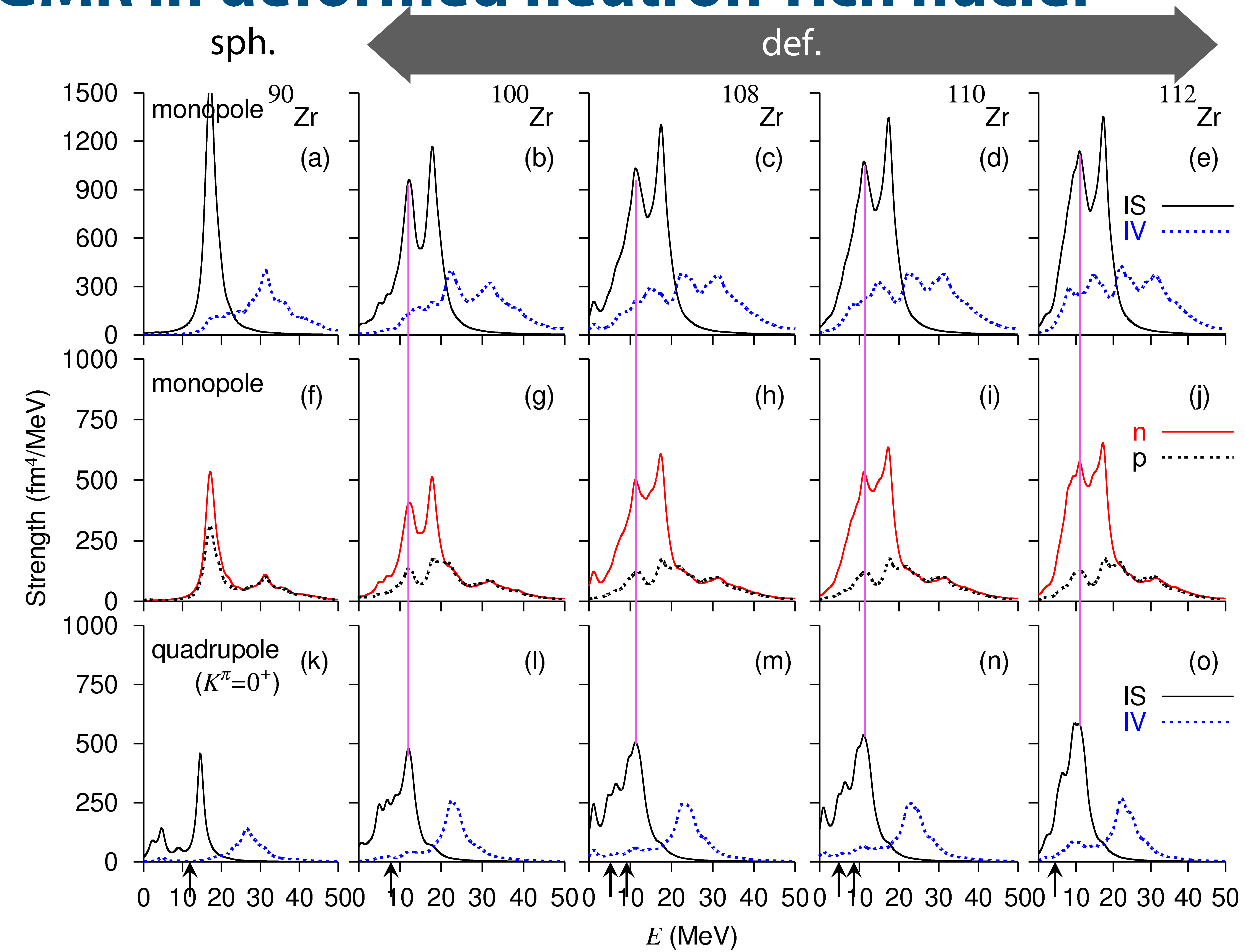


Large deformation of neutron-rich Zr isotopes



Sumikama+ ('11)
Figure taken from RIDAI-RIKEN press release

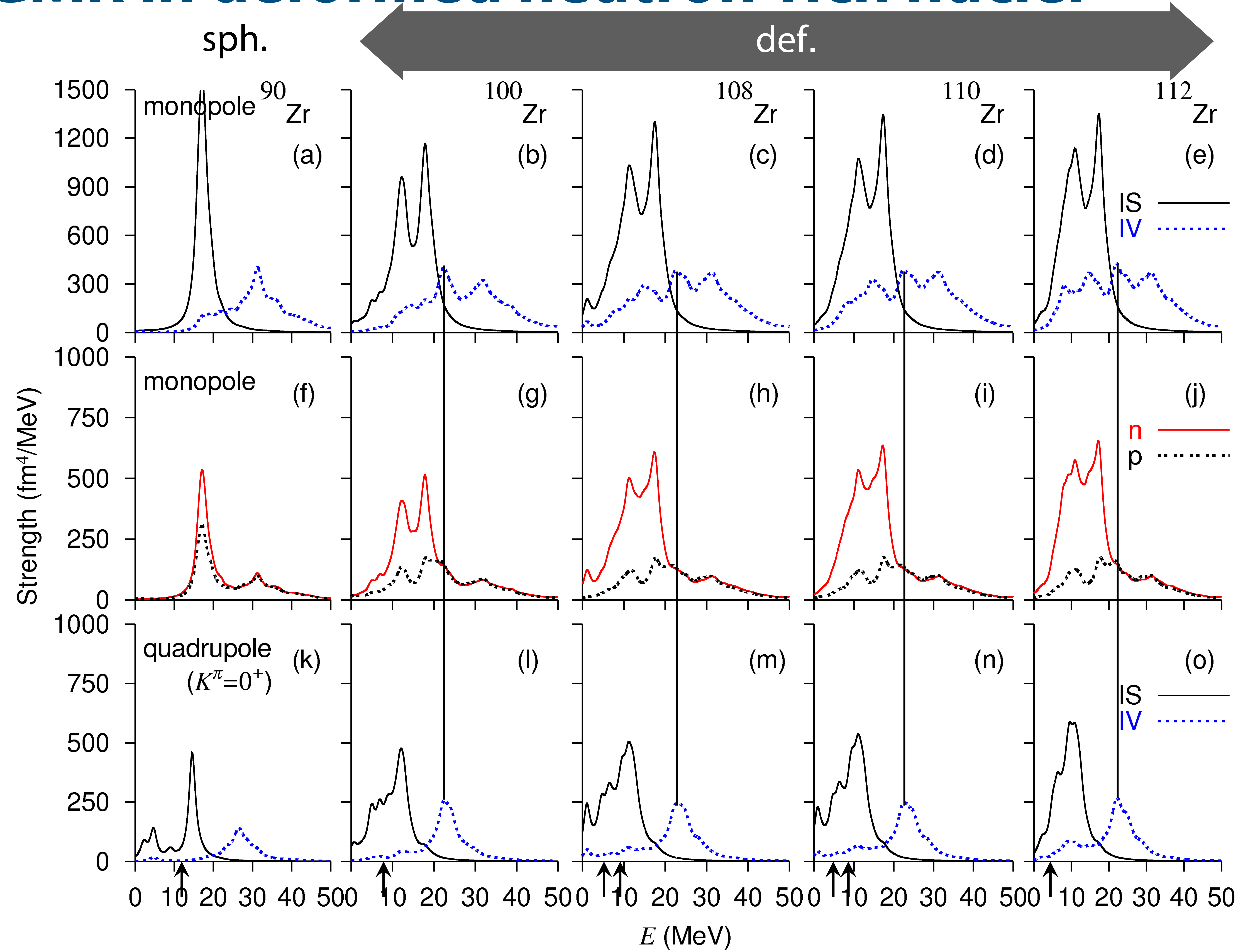
GMR in deformed neutron-rich nuclei



SkM* Yoshida('10)

$\Gamma = 2$ MeV

GMR in deformed neutron-rich nuclei



SkM* Yoshida('10)

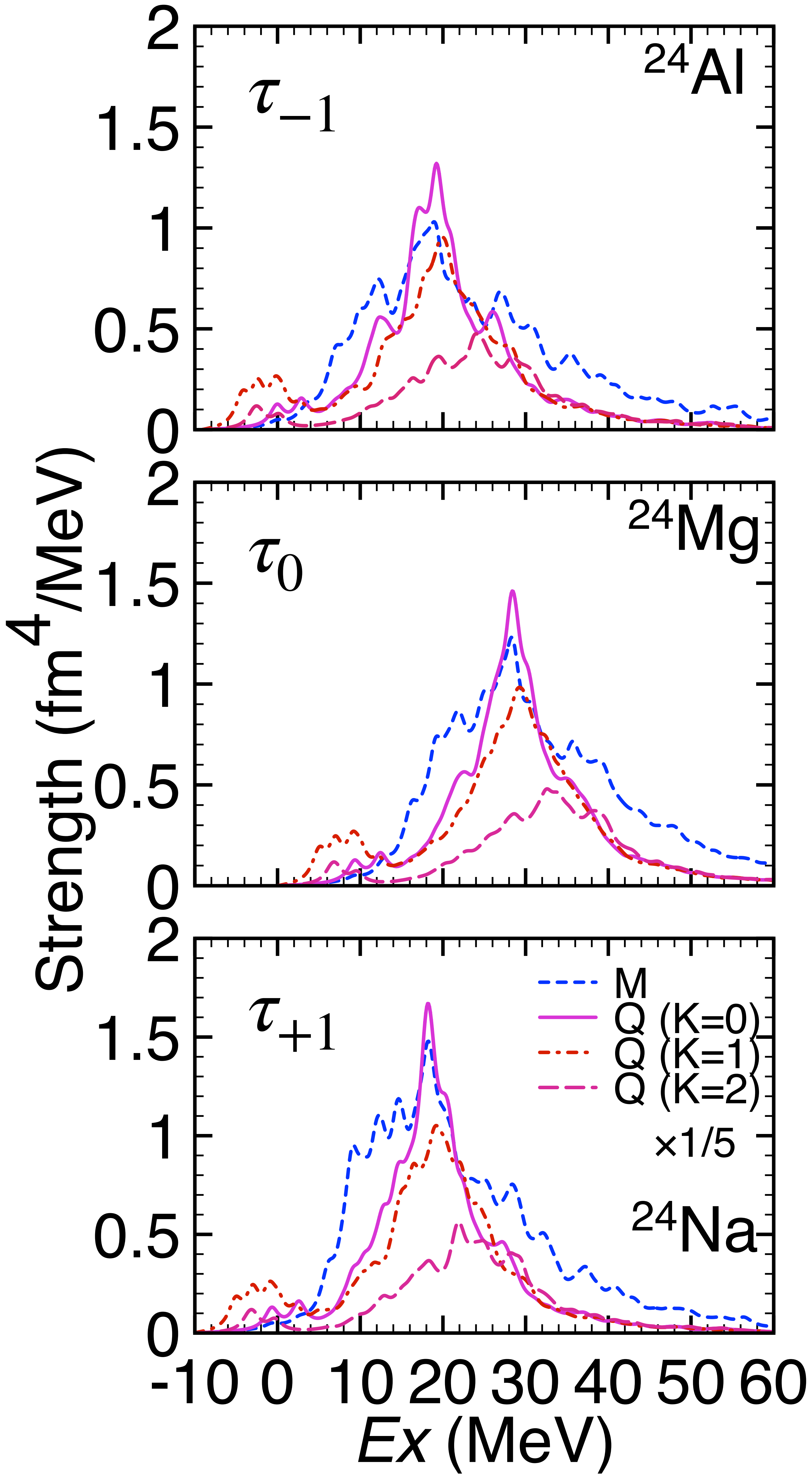
$\Gamma = 2$ MeV

IV strengths in low energy excitation of neutrons

deformation splitting in IVGMR

IV GMR and GQR in $N = Z$ deformed nuclei

Yoshida('21)



$$O = \int d\vec{r} f(\vec{r}) \psi^\dagger(\vec{r}\tau) \langle \tau \quad \tau_\mu \quad \tau' \rangle \psi(\vec{r}\tau')$$

$f = r^2$ monopole

$f = r^2 Y_{2K}$ quadrupole

The shape of distributions are similar for $\mu = 0, \pm 1$

degeneracy of isotriplet states

NEWSR: $S_- - S_+ \propto N\langle r^4 \rangle_\nu - Z\langle r^4 \rangle_\pi$

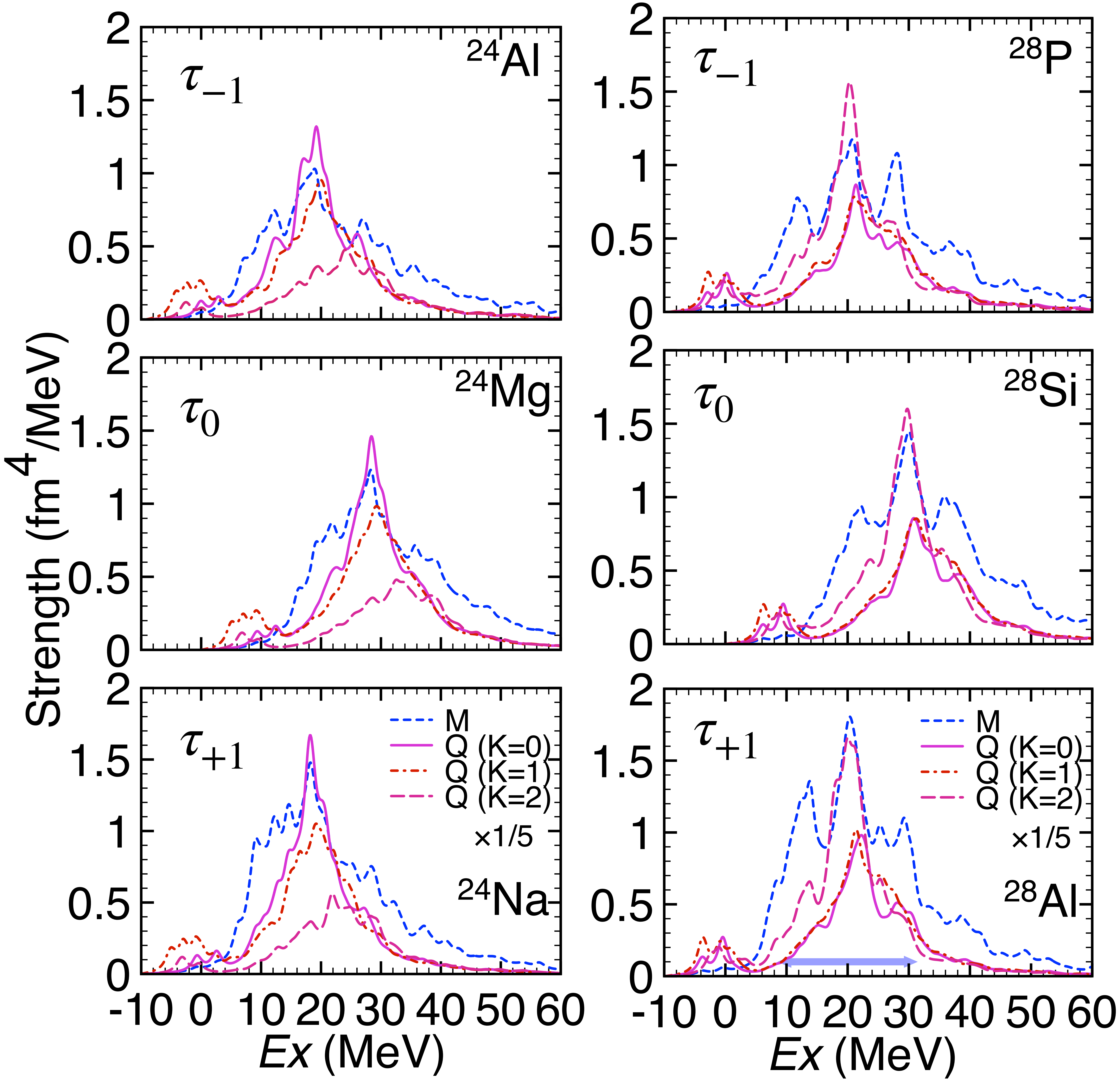
A simple RPA analysis assuming a single mode by BM

$$\frac{1}{2}(S_- + S_+) = S_0 \left[1 + \mathcal{O}\left(\frac{N-Z}{A} \right) \right]$$

The present self-consistent cal. agrees well with this estimation.

The K-splitting for IVGQR is similar to that for ISGQR.

IV GMR and GQR in $N = Z$ deformed nuclei



24Mg: prolate deformation: $\beta_2 = 0.39$

$$E(K=0) < E(K=1) < E(K=2)$$

24Si: oblate deformation: $\beta_2 = -0.22$

$$E(K=2) < E(K=1) \simeq E(K=0)$$

universal feature:

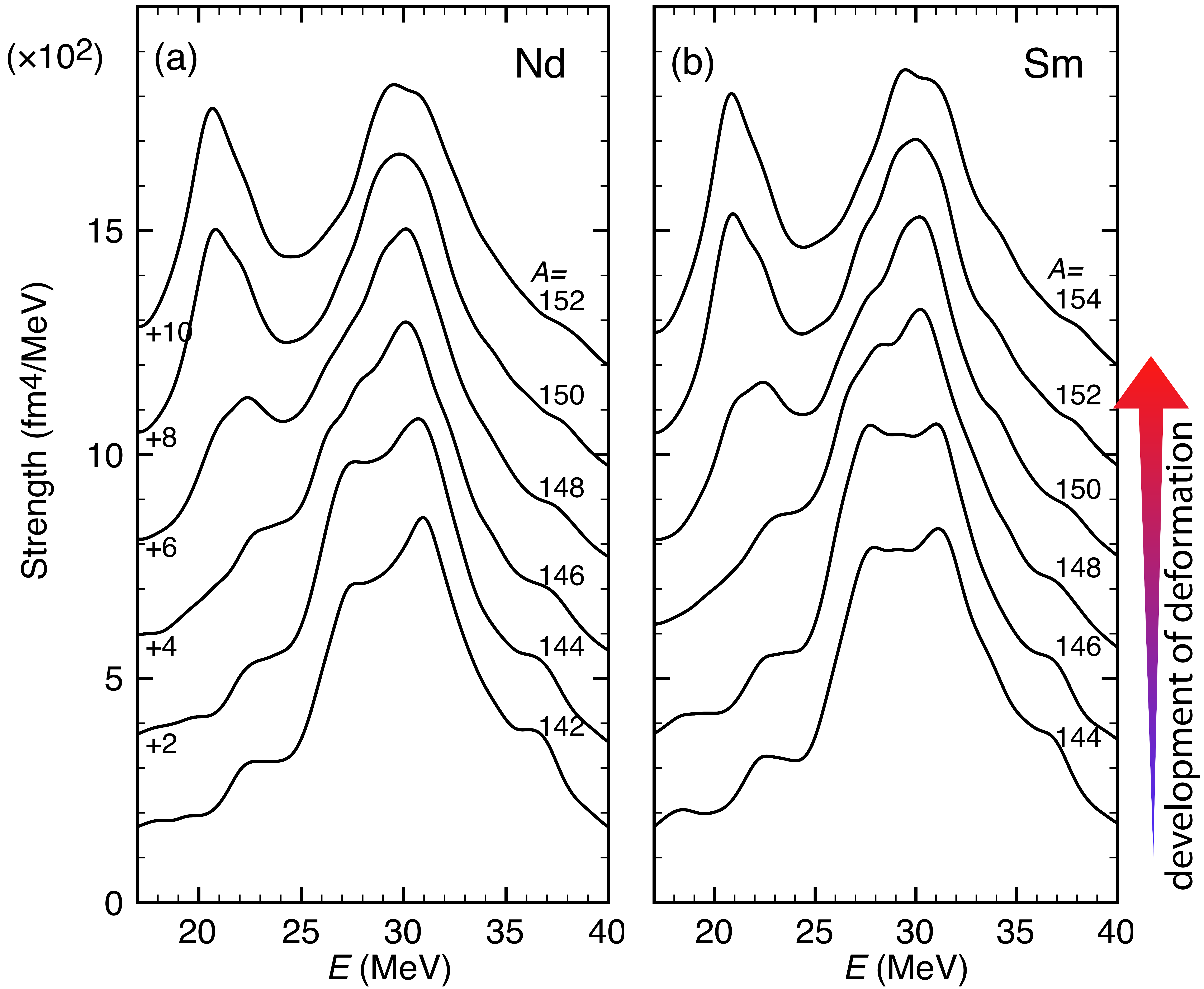
coupling of IVGMR and K=0 component of IVGQR

$^{28}\mathrm{Si}(^{10}\mathrm{Be}, ^{10}\mathrm{B}*)$

Scott, Zegers+, PRL118 (2017)

Isovector (IV)-GMR in deformed nuclei

Yoshida–Nakatsukasa ('13)



$$O = \int d\vec{r} r^2 \psi^\dagger(\vec{r}\tau)\langle\tau \;\; \tau_0 \;\; \tau'\rangle\psi(\vec{r}\tau')$$

emergence of deformation "splitting"

$\Delta E \sim 10$ MeV @ ^{154}Sm

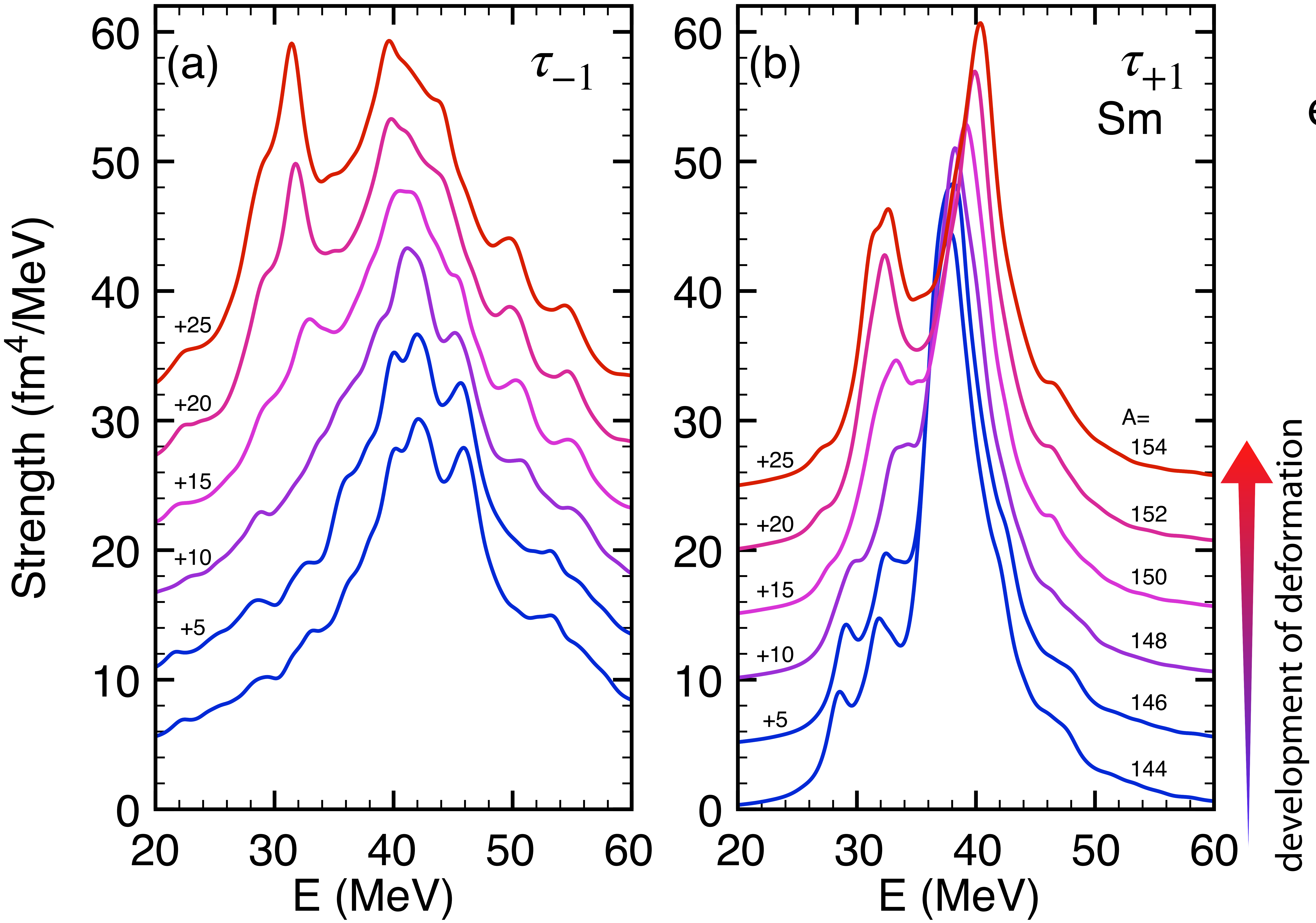
$\sim 2 \times \Delta E$(ISGMR)

due to the coupling to the $K = 0$ of IV-GQR

Isovector (IV)-GMR in deformed nuclei

Yoshida ('21)

$$O = \int d\vec{r} r^2 Y_0(\hat{r}) \psi^\dagger(\vec{r}\tau) \langle \tau \; \tau_{\pm 1} \; \tau' \rangle \psi(\vec{r}\tau')$$

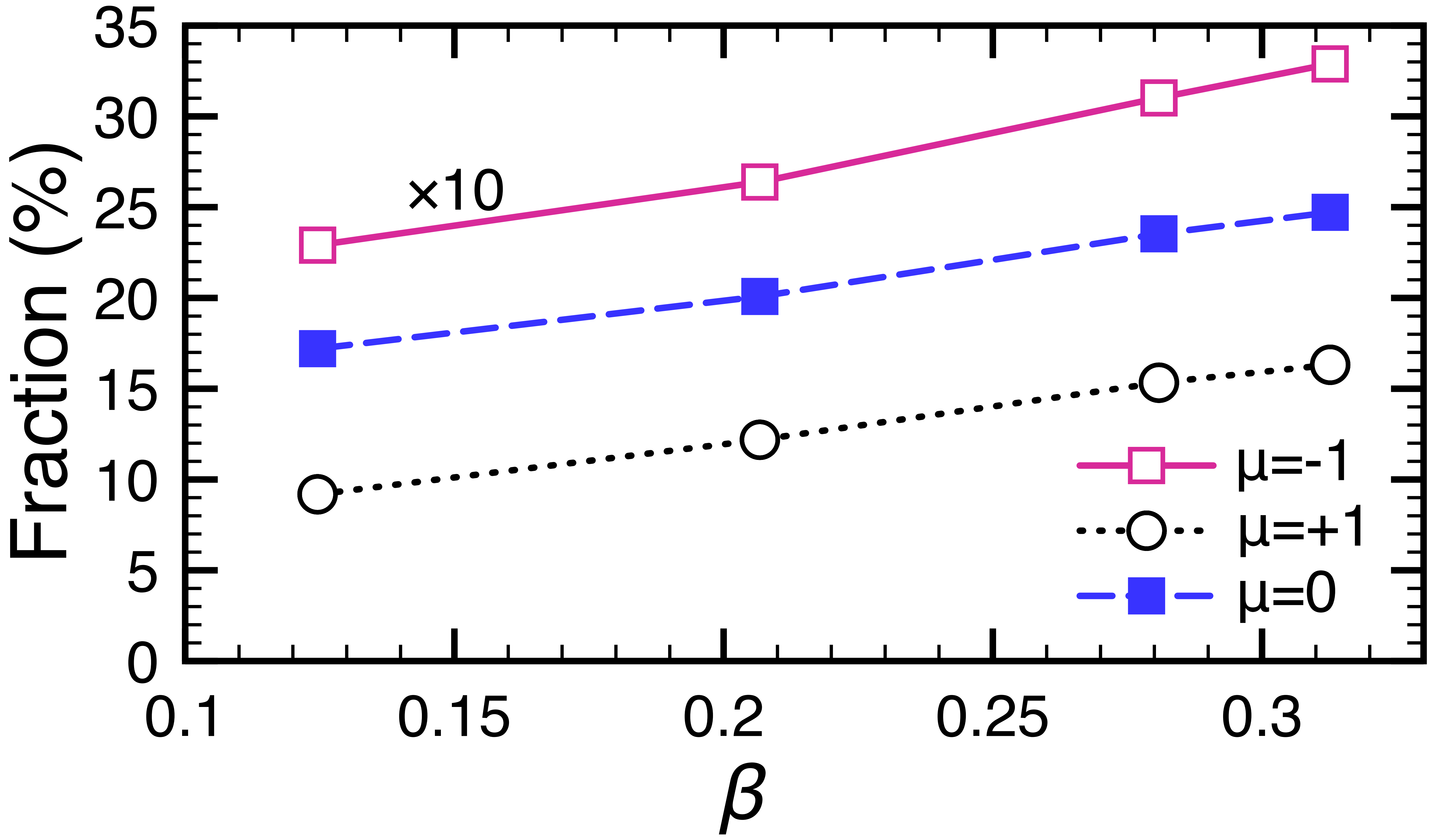


emergence of
deformation "splitting"
$\Delta E \sim 10$ MeV @ ^{154}Sm

universal in IV excitations
$\mu_\tau = -1, 0, +1$

Isovector (IV)-GMR in deformed nuclei

coupling of IVGMR and K=0 component of IVGQR in the strengths



* For $\mu = -1$, most of the strengths concentrate on the IAS.

IV dipole excitation: *K* and ΔT_z splittings

Isospin symmetry

degeneracy for μ for N=Z nuclei
w/o the Coulomb int.

broken by the Coulomb int.

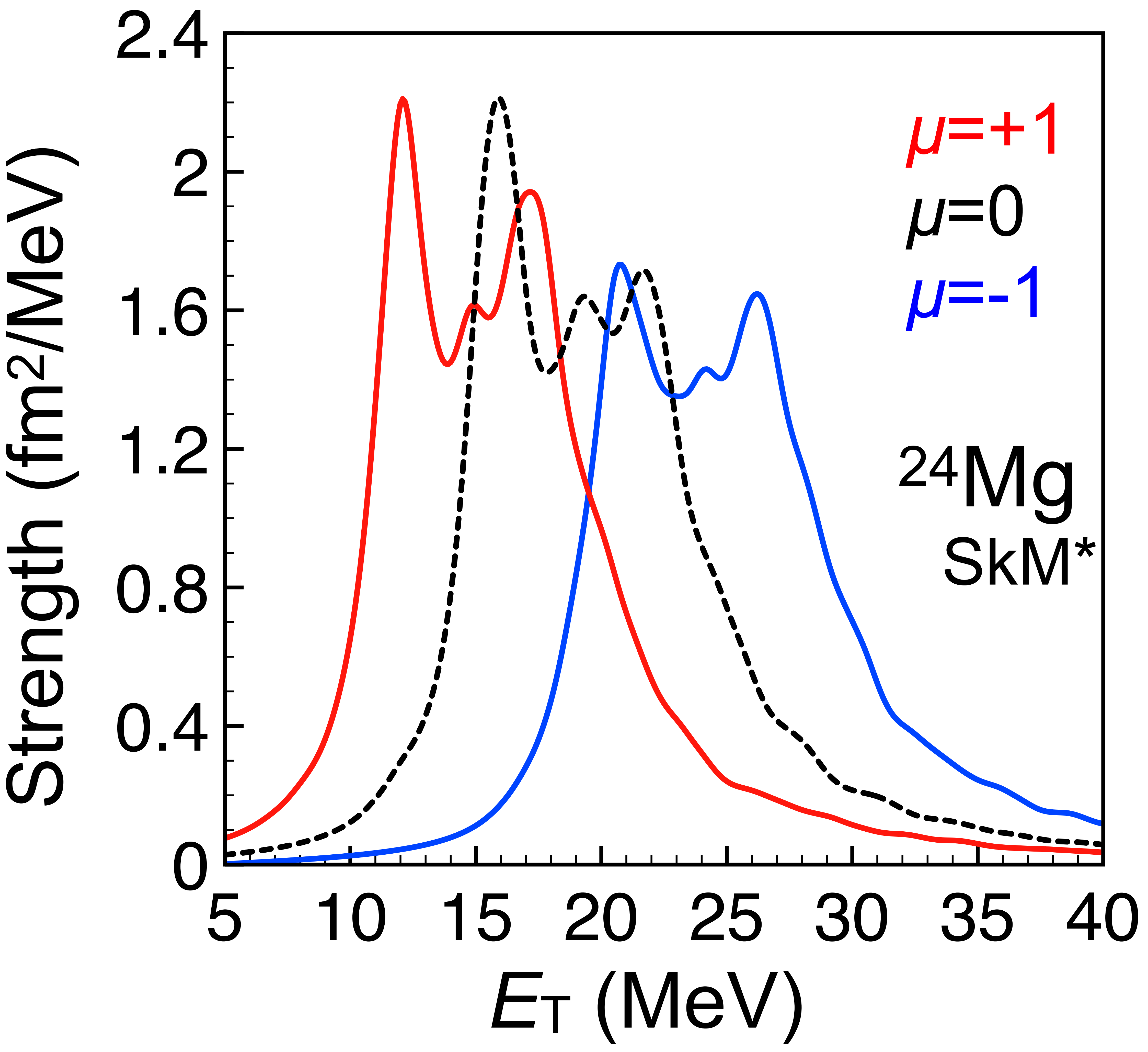
$\langle r^2\rangle_\nu$=9.0 fm^2
$\langle r^2\rangle_\pi$=9.2 fm^2

$$S_- - S_+ \propto N\langle r^2\rangle_\nu - Z\langle r^2\rangle_\pi$$

Shape deformation effect

K-splitting

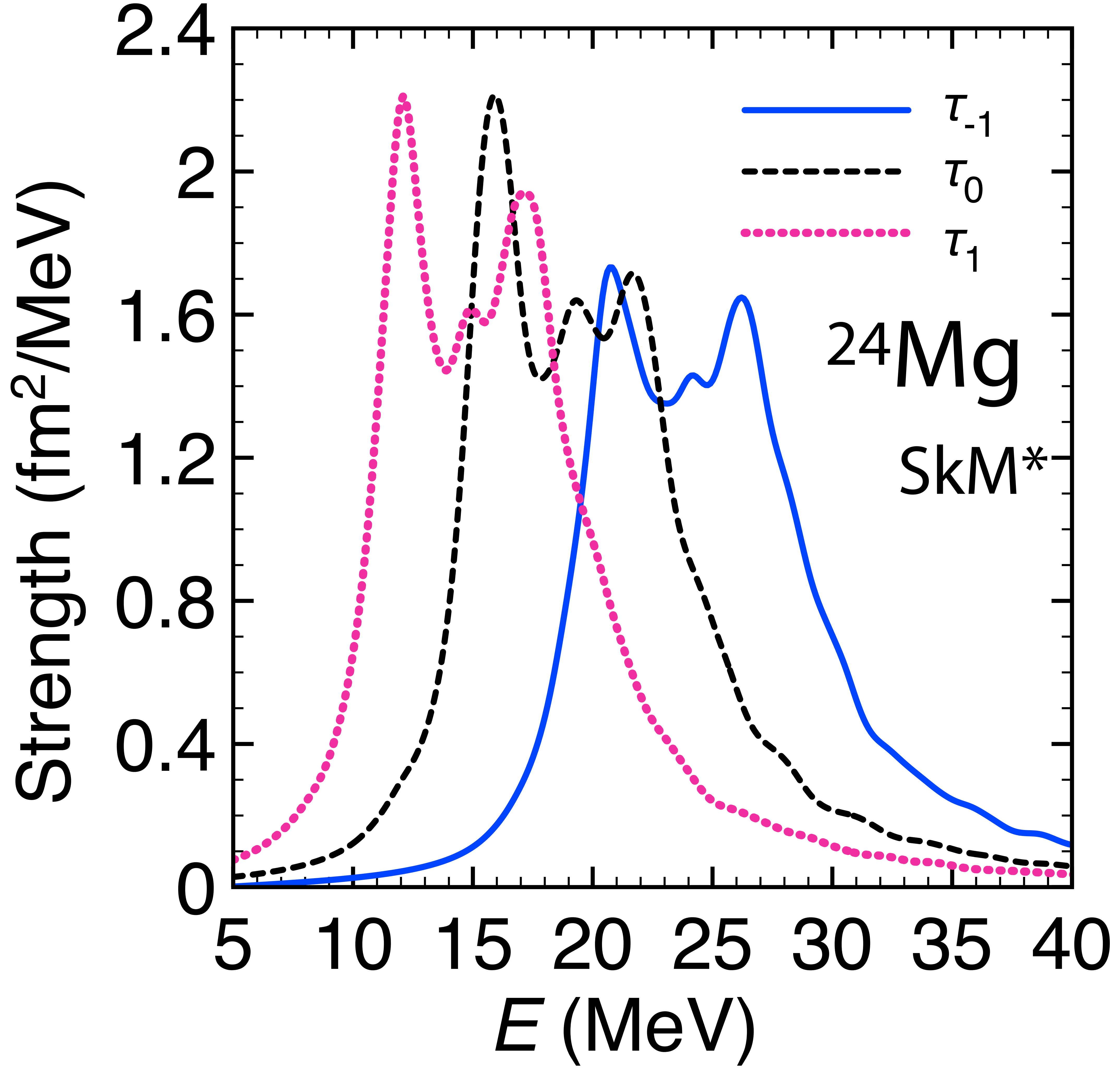
Yoshida('20)



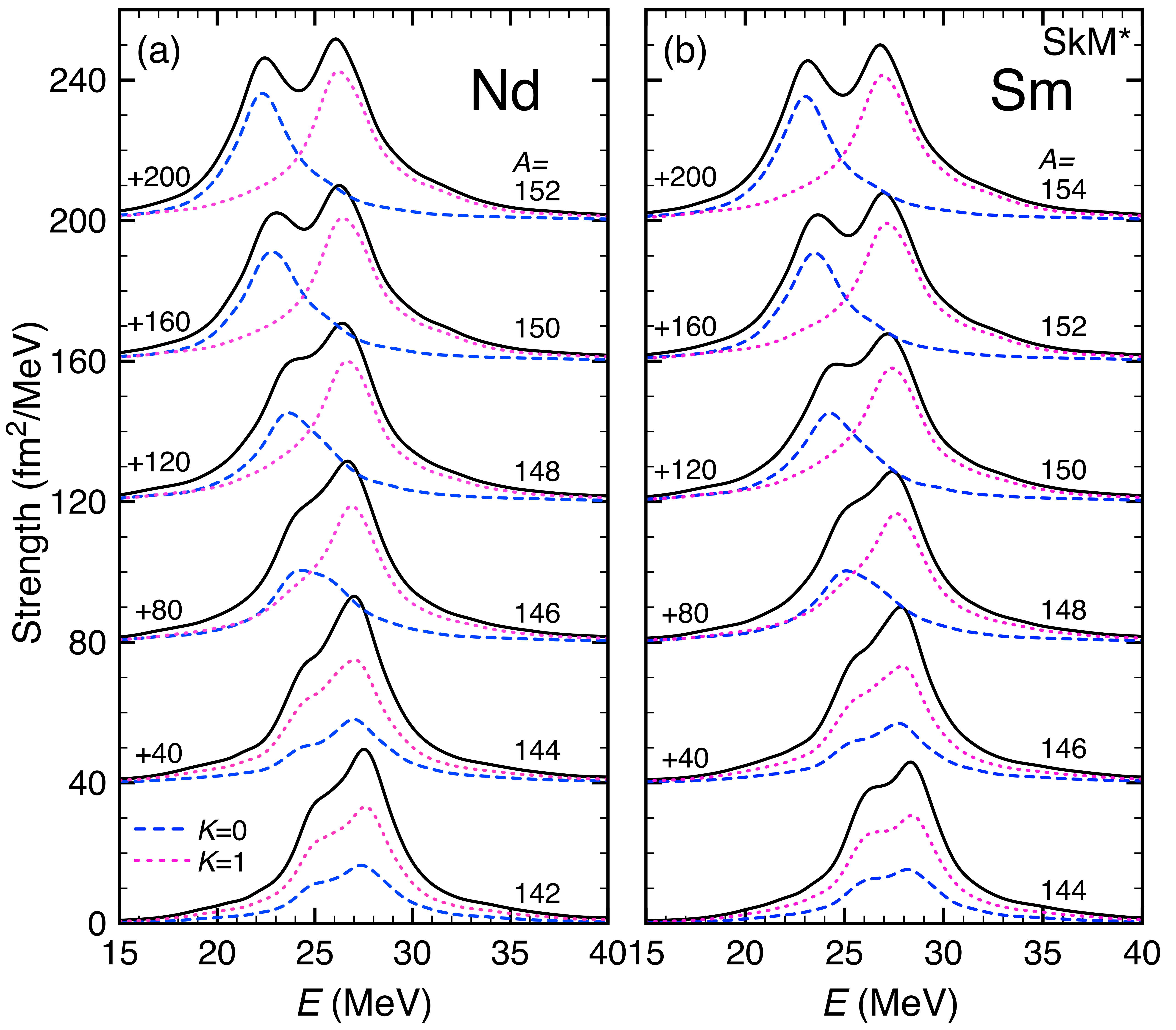
Deformation effects in IV excitations for $\tau_{\pm 1}$

Yoshida('20)

K-splitting: general feature

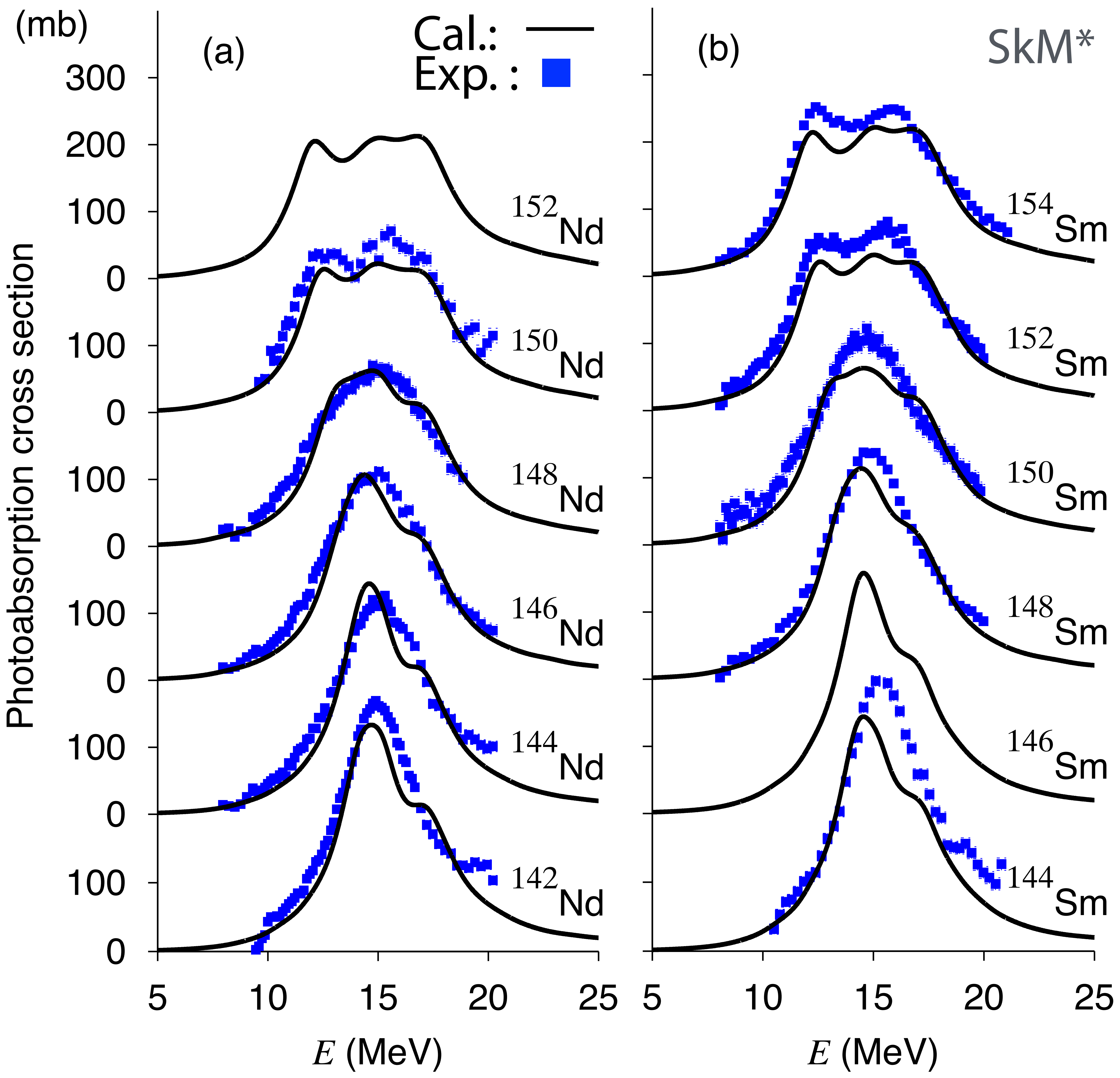


IV dipole for τ_{-1}

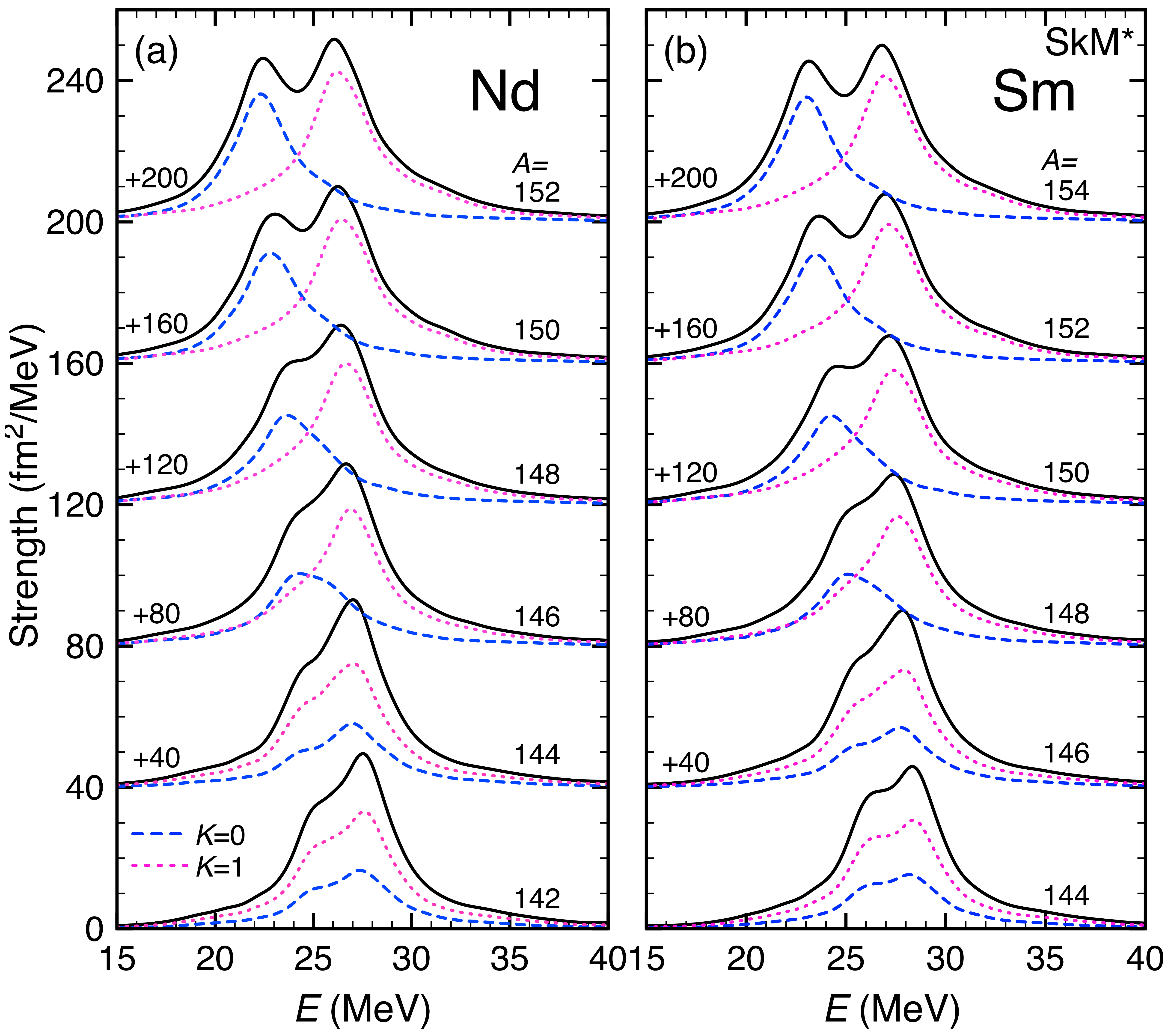


Deformation effects in IV excitations for $\tau_{\pm 1}$

Yoshida('20)

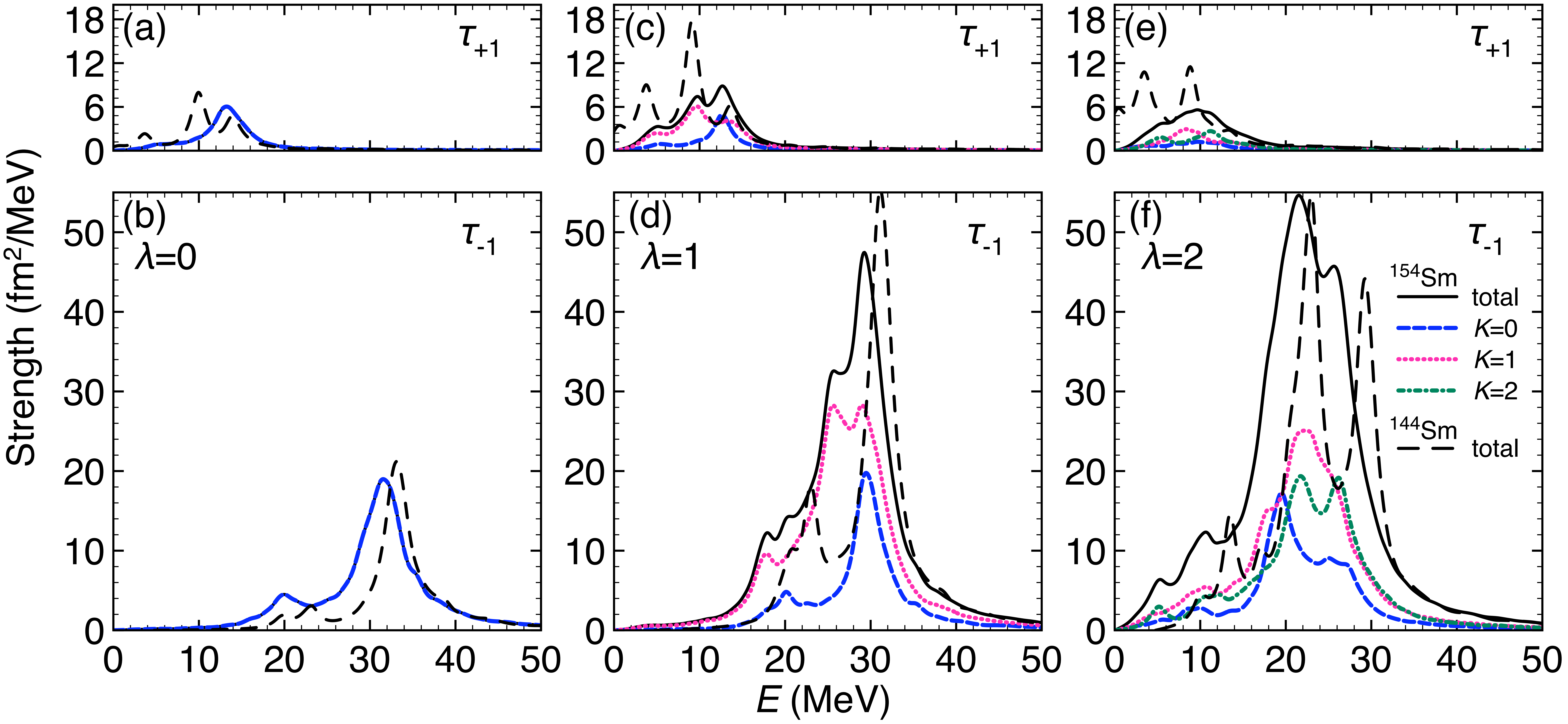


IV dipole for τ_{-1}



Deformation effects in spin-dipole excitations

$$r[Y_1 \otimes \sigma]^{\lambda}\tau_{\pm 1}$$



Deformation effects in spin-dipole excitations

Sum rule:

$$\int dE[S^{-}_{\lambda K}(E) - S^{+}_{\lambda K}(E)]$$

$$= \frac{1}{2\pi}[N\langle r^2\rangle_N - Z\langle r^2\rangle_Z]$$

to rst order in deformation

for $\lambda = 1$

$$= \begin{cases} 2\frac{3}{8\pi}[N\langle r_\perp^2\rangle_N - Z\langle r_\perp^2\rangle_Z] & K = 0 \\ 4\frac{3}{16\pi}[N\langle r_\perp^2 + 2z^2\rangle_N - Z\langle r_\perp^2 + 2z^2\rangle_Z] & K = 1 \end{cases}$$

$K = 0$	$-\frac{1}{6\pi}[N\langle r^2\rangle_N - Z\langle r^2\rangle_Z]\delta$	↘
$K = 1$	$+\frac{1}{6\pi}[N\langle r^2\rangle_N - Z\langle r^2\rangle_Z]\delta$	↗

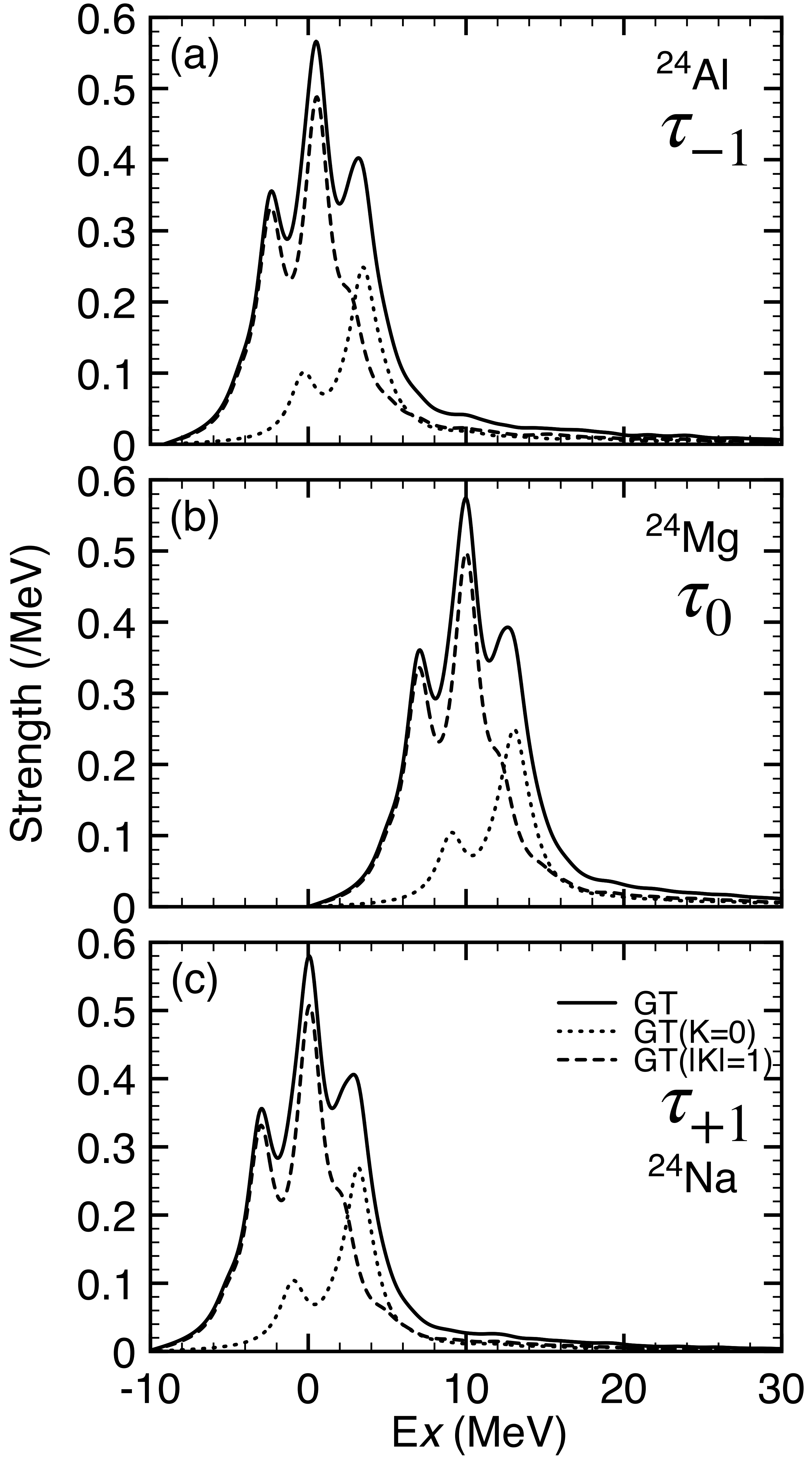
for $\lambda = 2$

$$= \begin{cases} 2\frac{1}{8\pi}[N\langle r_\perp^2 + 4z^2\rangle_N - Z\langle r_\perp^2 + 4z^2\rangle_Z] & K = 0 \\ 4\frac{3}{16\pi}[N\langle r_\perp^2 + 2z^2\rangle_N - Z\langle r_\perp^2 + 2z^2\rangle_Z] & K = 1 \\ 4\frac{3}{8\pi}[N\langle r_\perp^2\rangle_N - Z\langle r_\perp^2\rangle_Z] & K = 2 \end{cases}$$

$K = 0$	$+\frac{1}{6\pi}[N\langle r^2\rangle_N - Z\langle r^2\rangle_Z]\delta$	↗
$K = 1$	$+\frac{1}{6\pi}[N\langle r^2\rangle_N - Z\langle r^2\rangle_Z]\delta$	↗
$K = 2$	$-\frac{2}{6\pi}[N\langle r^2\rangle_N - Z\langle r^2\rangle_Z]\delta$	↘

Gamow–Teller and spin M1: *K* and ΔT_z splittings

Yoshida('21)



$$O = \int d\vec{r}\psi^\dagger(\vec{r}\tau)\langle\tau\ \tau_\mu\ \tau'\rangle\psi(\vec{r}\tau')$$

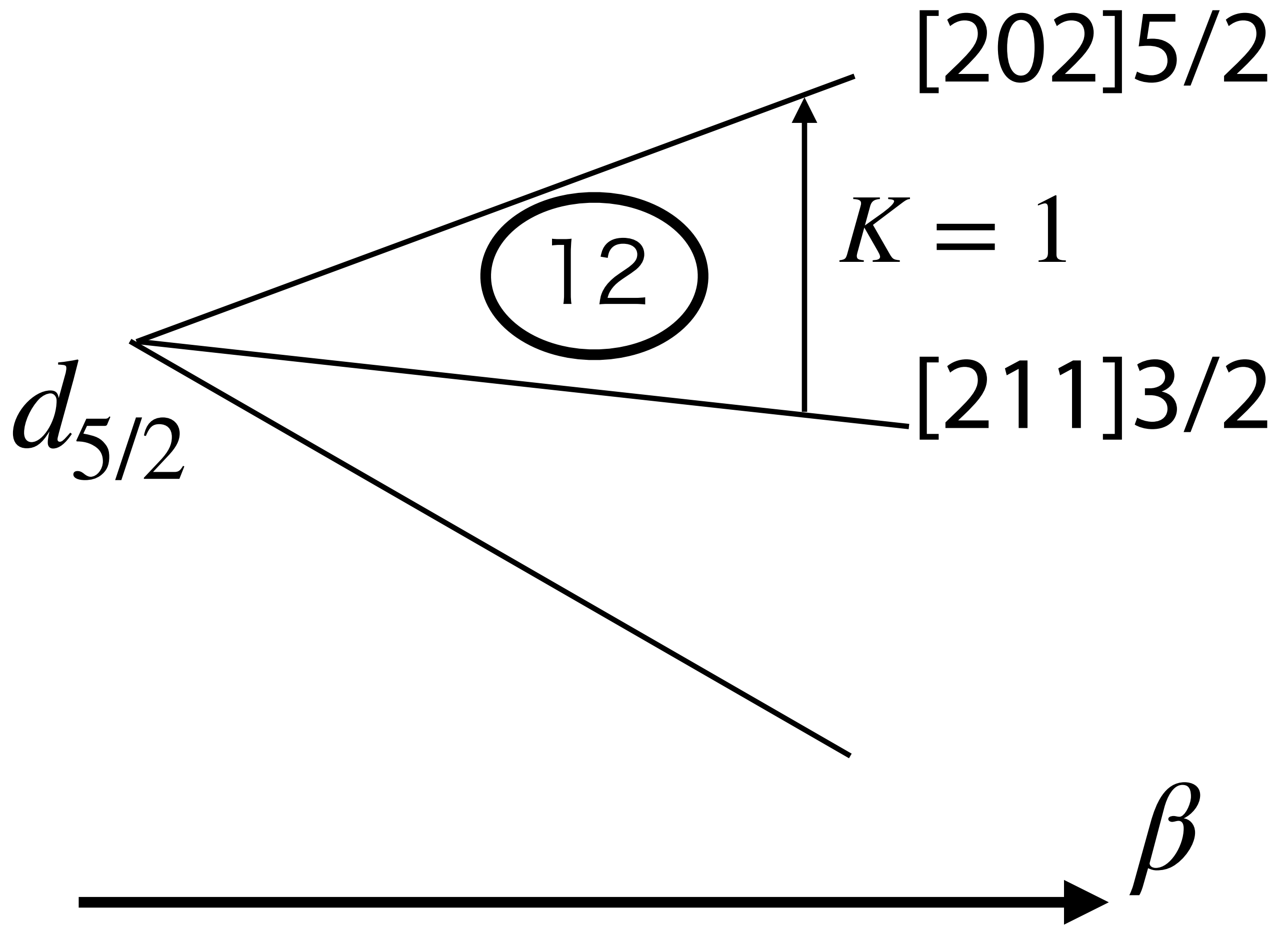
The shape of distributions are similar for $\mu = 0, \pm 1$

insensitive to the spatial structure of the single-particle wfs

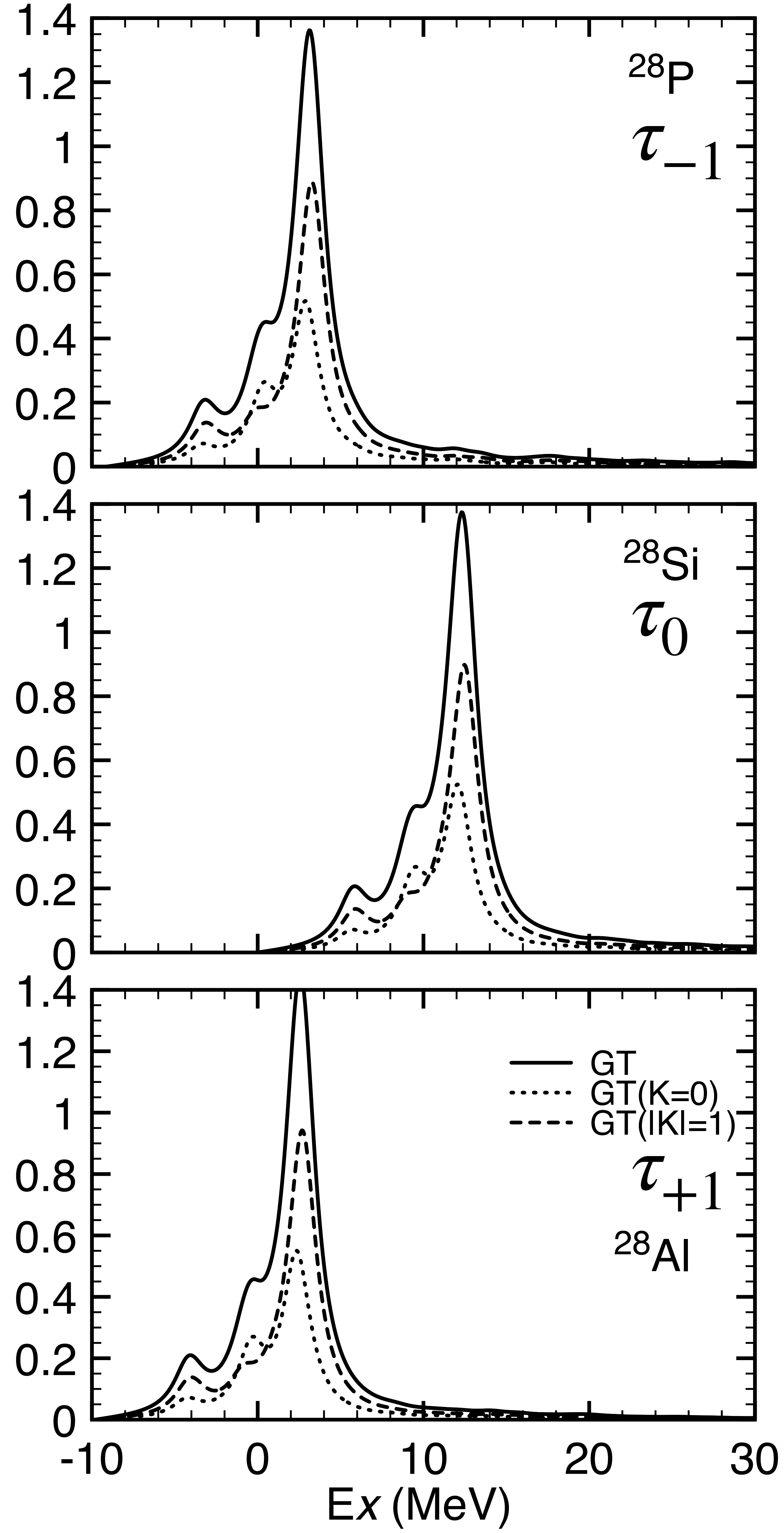
The *K* splitting due to deformation

is not the collective effect but the shell effect.

K=1 appears lower in energy:

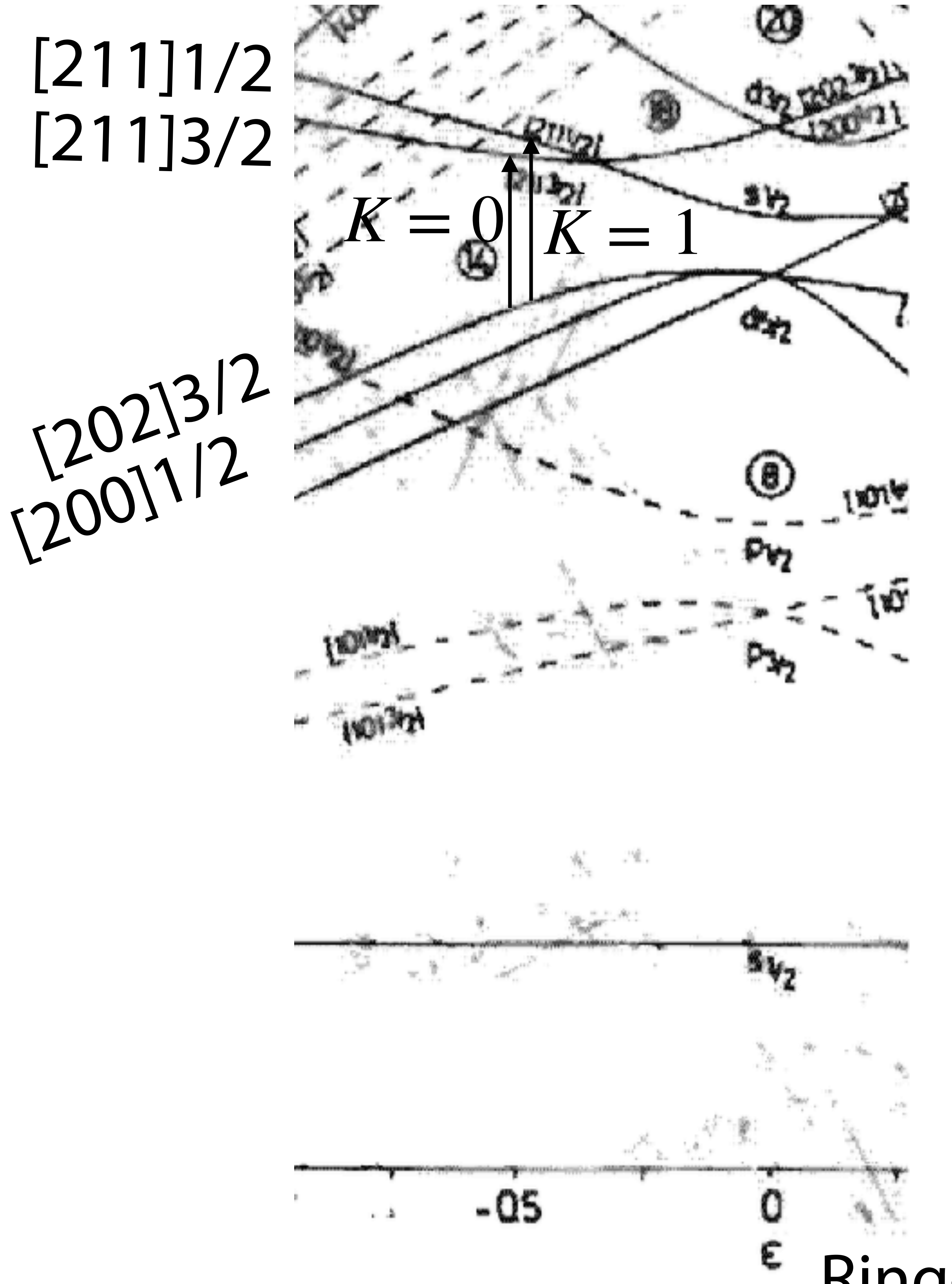


Gamow–Teller and spin M1: *K* and ΔT_z splittings



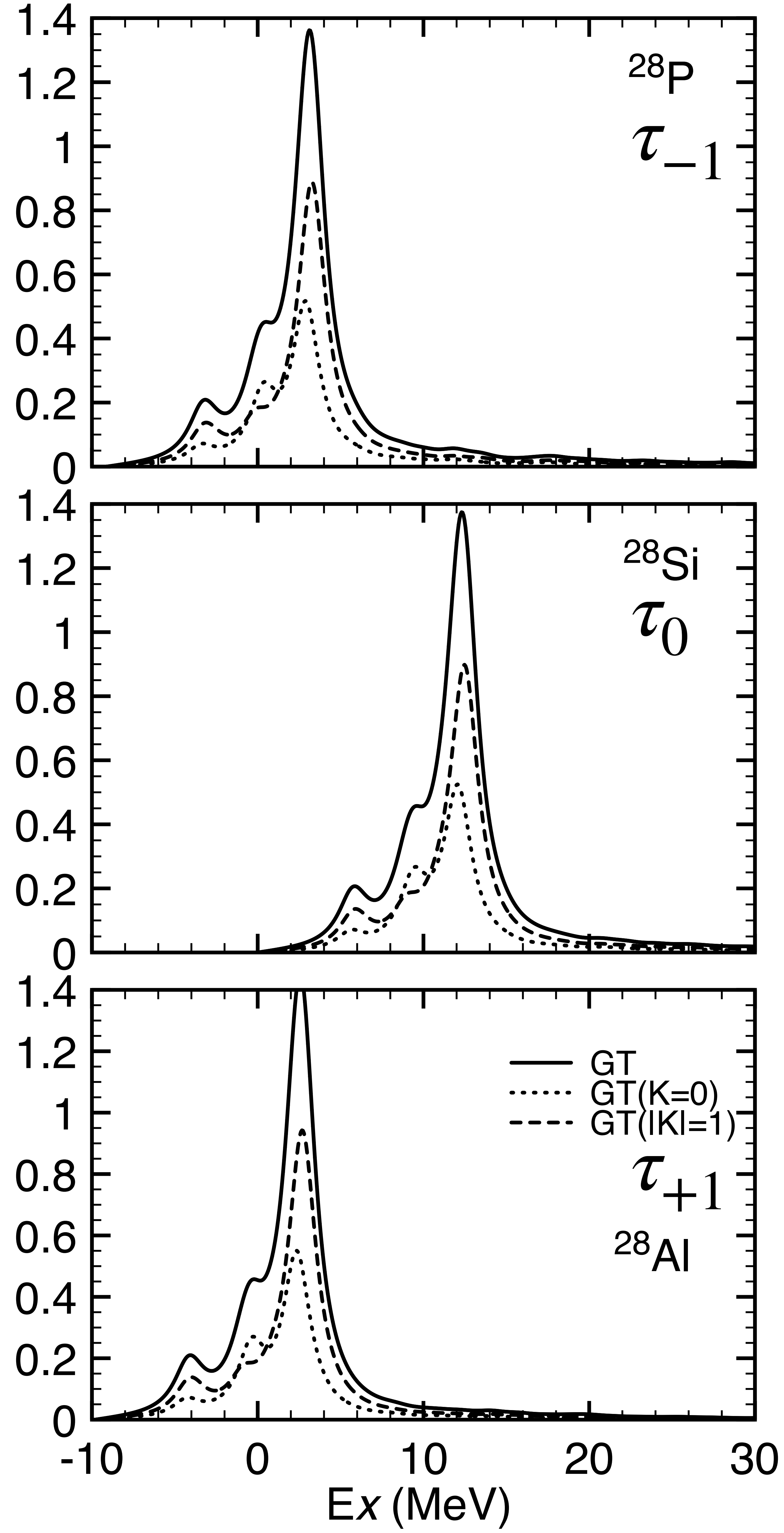
The shape of distributions are similar for $\mu = 0, \pm 1$

The *K* splitting is weak



Ring–Schuck

Gamow–Teller and spin M1: *K* and ΔT_z splittings



The shape of distributions are similar for $\mu = 0, \pm 1$

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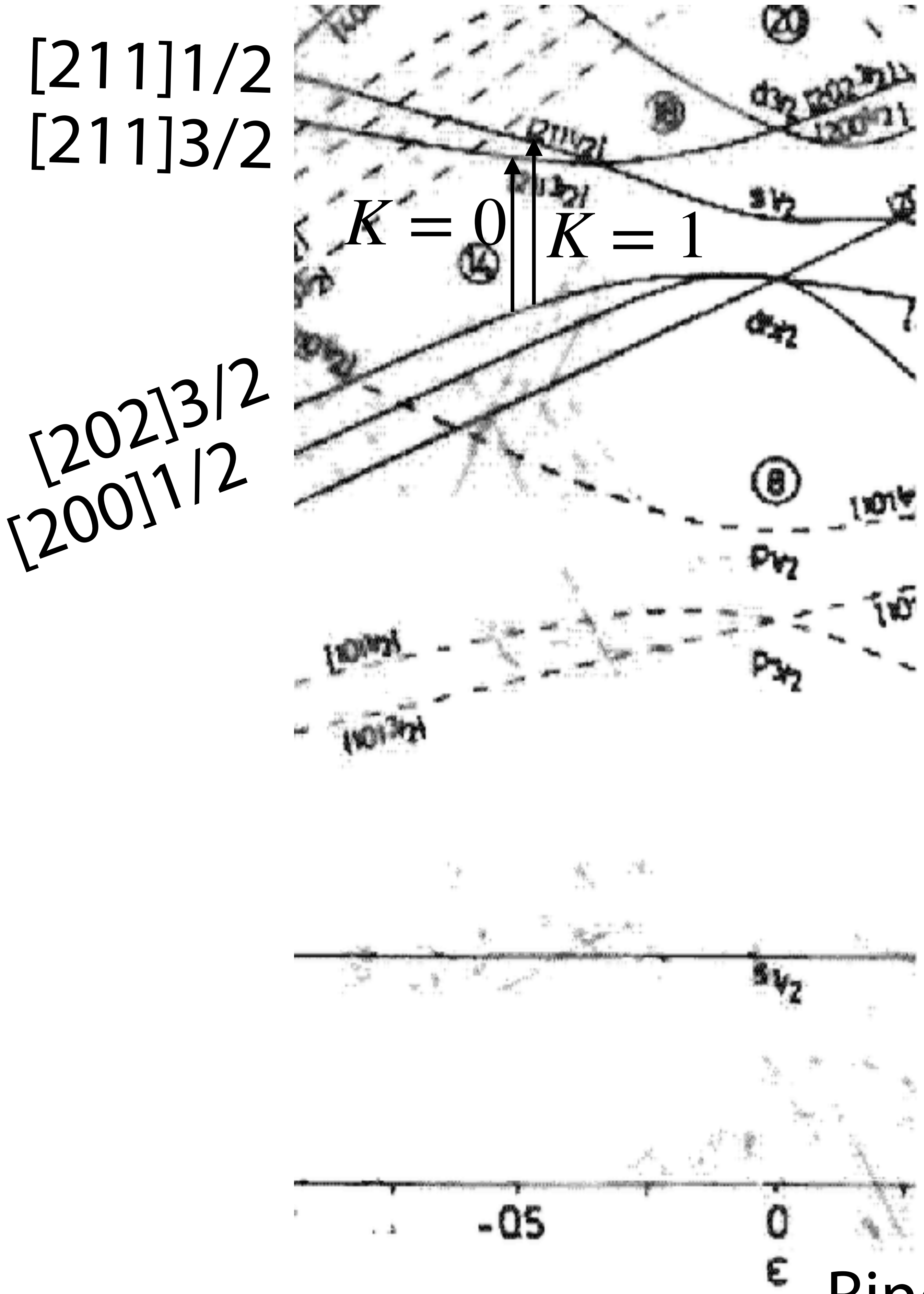


Ring–Schuck

GT/spin-M1: $0\hbar\omega$ excitation

sensitive to details of the shell structure

Evolution of collectivity of GT/spin-M1 excitations

Sum-rule-based approach for the systematic investigation

Isovector magnetic susceptibility

Yoshida ('21)

$$\chi_z = 2\mu^2 \sum_i \frac{\langle i \;\; S_z T_z \;\; 0\rangle^{\;2}}{\omega_i}$$

$$S_z = \mu \int d\vec{r} \sum_{ss'} \psi^\dagger(\vec{r}s)\psi(\vec{r}s')\langle s \;\; \frac{\sigma_z}{2} \;\; s'\rangle$$

IV spin M1

$$\chi_\perp = \mu^2 \left[\sum_i \frac{\langle i \;\; S_z T_- \;\; 0\rangle^{\;2}}{\omega_i} + \sum_i \frac{\langle i \;\; S_z T_+ \;\; 0\rangle^{\;2}}{\omega_i} \right]$$

Gamow–Teller

$$S_z T_\pm = \mu \int d\vec{r} \sum_{ss'} \sum_{tt'} \psi^\dagger(\vec{r}st)\psi(\vec{r}s't')\langle s \;\; \frac{\sigma_z}{2} \;\; s'\rangle\langle t \;\; \tau_\pm \;\; t'\rangle$$

Response function and static susceptibility

$$R_F(\omega) = \sum_n \left[\frac{\langle n \;\; F^\dagger \;\; 0\rangle^{\;2}}{\omega - (E_n - E_0) + i\epsilon} - \frac{\langle n \;\; F \;\; 0\rangle^{\;2}}{\omega + (E_n - E_0) + i\epsilon} \right]$$

IV spin M1

$$F = F^\dagger = S_z = \int d\vec{r} \sum_{ss'} \psi^\dagger(\vec{r}s)\psi(\vec{r}s')\langle s \;\; \frac{\sigma_z}{2} \;\; s'\rangle$$

$$\chi_z = -R_{S_z}(0) = 2\mu^2 \sum_i \frac{\langle i \;\; S_z T_z \;\; 0\rangle^{\;2}}{\omega_i}$$

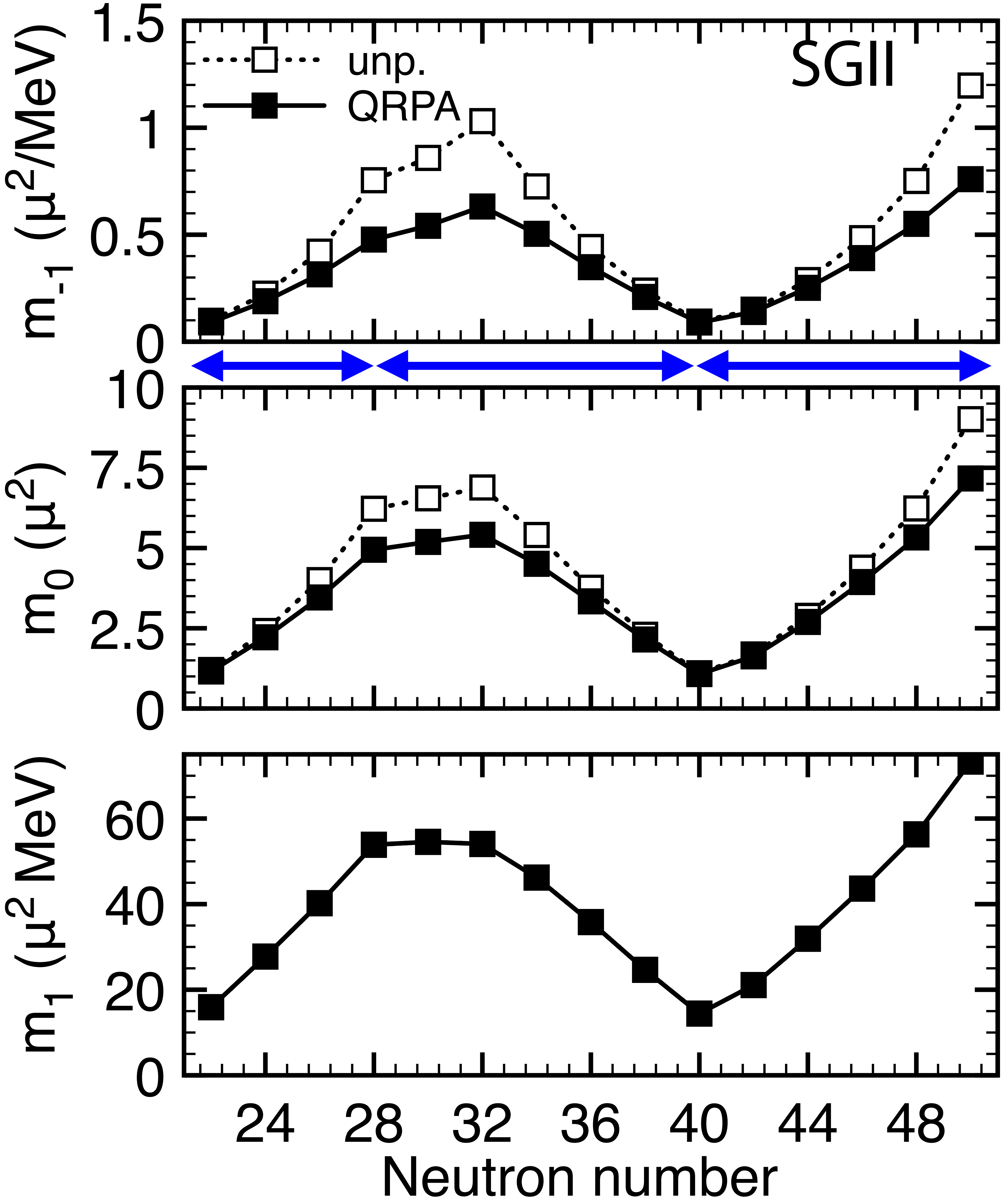
Gamow–Teller

$$F = S_z T_- = \mu \int d\vec{r} \sum_{ss'} \sum_{tt'} \psi^\dagger(\vec{r}st)\psi(\vec{r}s't')\langle s \;\; \frac{\sigma_z}{2} \;\; s'\rangle\langle t \;\; \tau_{-1} \;\; t'\rangle$$

$$F^\dagger = S_z T_+ = \mu \int d\vec{r} \sum_{ss'} \sum_{tt'} \psi^\dagger(\vec{r}st)\psi(\vec{r}s't')\langle s \;\; \frac{\sigma_z}{2} \;\; s'\rangle\langle t \;\; \tau_{+1} \;\; t'\rangle$$

$$\chi_\perp = -R_{S_z T_-}(0) = \mu^2 \left[\sum_i \frac{\langle i \;\; S_z T_- \;\; 0\rangle^{\;2}}{\omega_i} + \sum_i \frac{\langle i \;\; S_z T_+ \;\; 0\rangle^{\;2}}{\omega_i} \right]$$

Systematics: isovector spin-M1 in Ca isotopes



moments:

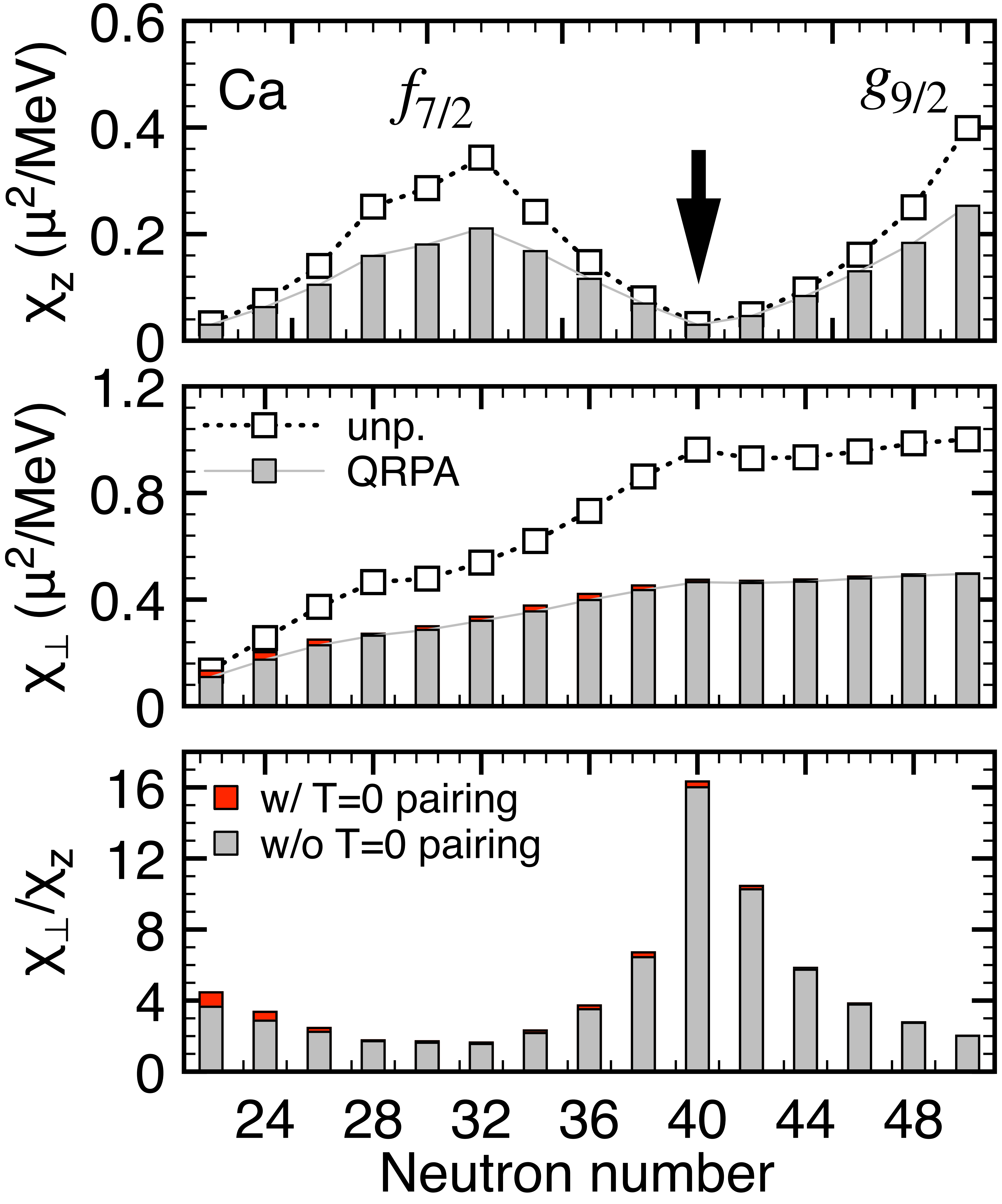
isotopic evolution of collectivity of spin-M1

shell effect

effect of the RPA correlations clearly seen in m_{-1}

$$\chi_z = \frac{m_{-1}}{3}$$

Magnetic property: isovector spin susceptibility in Ca isotopes



stronger collectivity: $j_>$ orbital is occupied

(sub)-shell effect at *N*=40

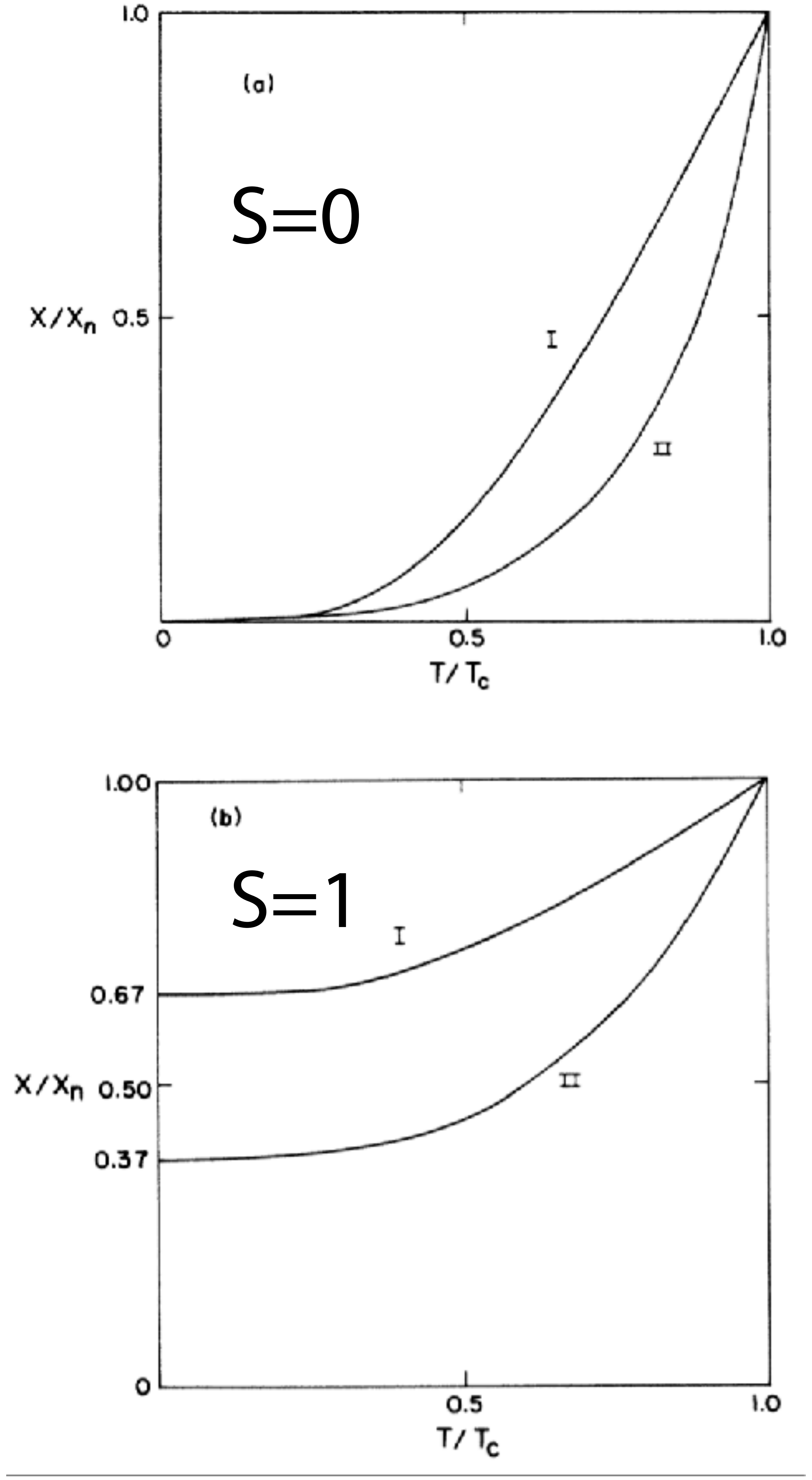
stronger collectivity in neutron-rich nuclei

"saturation" due to the appearance of $-1\hbar\omega_0$ excitations in very neutron-rich nuclei

shell effect in χ_z dominates the isotopic dep.

role of S=1 pn-pair int. around (sub)-shell closures

Magnetic susceptibility as evidence of spin-triplet pairing



A. J. Legett, PRL14 ,536 (1965)

$$\chi(T)=\frac{\chi_0 f_{\text{eff}}(T)}{1+f_{\text{eff}}(T)Z_0/4},$$

Yosida function: $f_{\text{eff}}(T)$

Z_0 :Landau parameter

$$G_0=\frac{Z_0}{4}$$

$$\chi = 2\mu^2 \sum_i \frac{\langle i\ S_z\ 0\rangle^2}{\omega_i}$$

$$S_z = \int d\vec{r} \sum_{\sigma\sigma'} \psi^\dagger(\vec{r}\sigma)\psi(\vec{r}\sigma')\langle\sigma\ \frac{\sigma_z}{2}\ \sigma'\rangle$$

“isoscalar” magnetic susceptibility

Isovector magnetic susceptibility for the spin-triplet pairing

$$\chi_z = \frac{1 + \frac{1}{3}F_0}{1 + G_0'}\chi_0$$

$$\chi_\perp = \frac{1 + \frac{1}{3}F_0}{1 + G_0' + V_{S=1}}\chi_0$$

repulsive spin-spin int. attractive int.

$$\frac{\chi_\perp}{\chi_z} = \frac{1 + G_0'}{1 + G_0' + V_{S=1}} > 1$$

enhancement due to $V_{S=1}$?

In the actual calculations (based on the mean- eld aprrox.),
it is not so obvious to see the effect of the S=1 pairing

Summary

DFT approach for nuclear responses: revealing various properties of nuclear structure

- nuclear deformation effect

  well establish for IVGDR, and well known for ISGMR

  coupling between the monopole and the K=0 quadrupole

  universal for IVGDR with charge exchange

  as well as for IVGMR

  cannot be seen in GT/spin-M1

  complicated in spin-dipole excitations

- spin susceptibility

  the systematic trend in the collectivity of the spin excitation, spin fluctuation $\delta\langle\vec{s}\rangle$

  spin-triplet pairing: needs more work

References

K. Yoshida, PRC82(2010)034324

K. Yoshida, T. Nakatsukasa, PRC83(2011)021304R

K. Yoshida, T. Nakatsukasa, PRC88(2013)034309

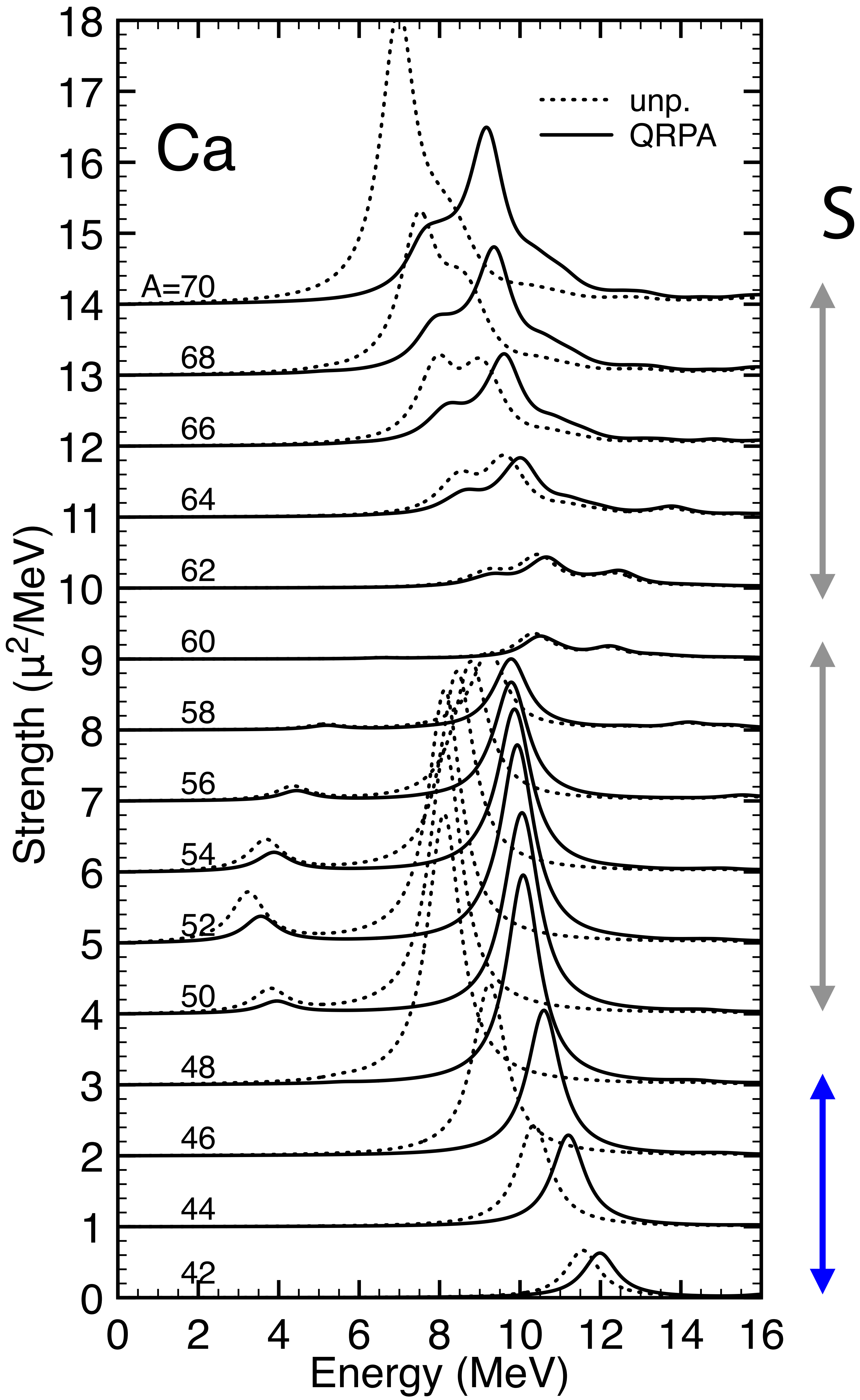
K. Yoshida, PRC102(2020)054336

K. Yoshida, PRC104(2021)014309

K. Yoshida, PRC104(2021)044309

Backups

Neutral channel: isovector spin-M1



$$\vec{F}_z = \int d\vec{r} \sum_{\sigma\sigma'} \psi^\dagger(\vec{r}\sigma\tau)\psi(\vec{r}\sigma'\tau')\langle\sigma \ \vec{\sigma} \ \sigma'\rangle\langle\tau \ \tau_z \ \tau'\rangle$$

SGII + T=1 pairing

^{42}Ca–^{48}Ca

generated dominantly by $\mathrm{n}1f_{7/2} \to \mathrm{n}1f_{5/2}$

gradual increase in the occupation of $\mathrm{n}1f_{7/2}$

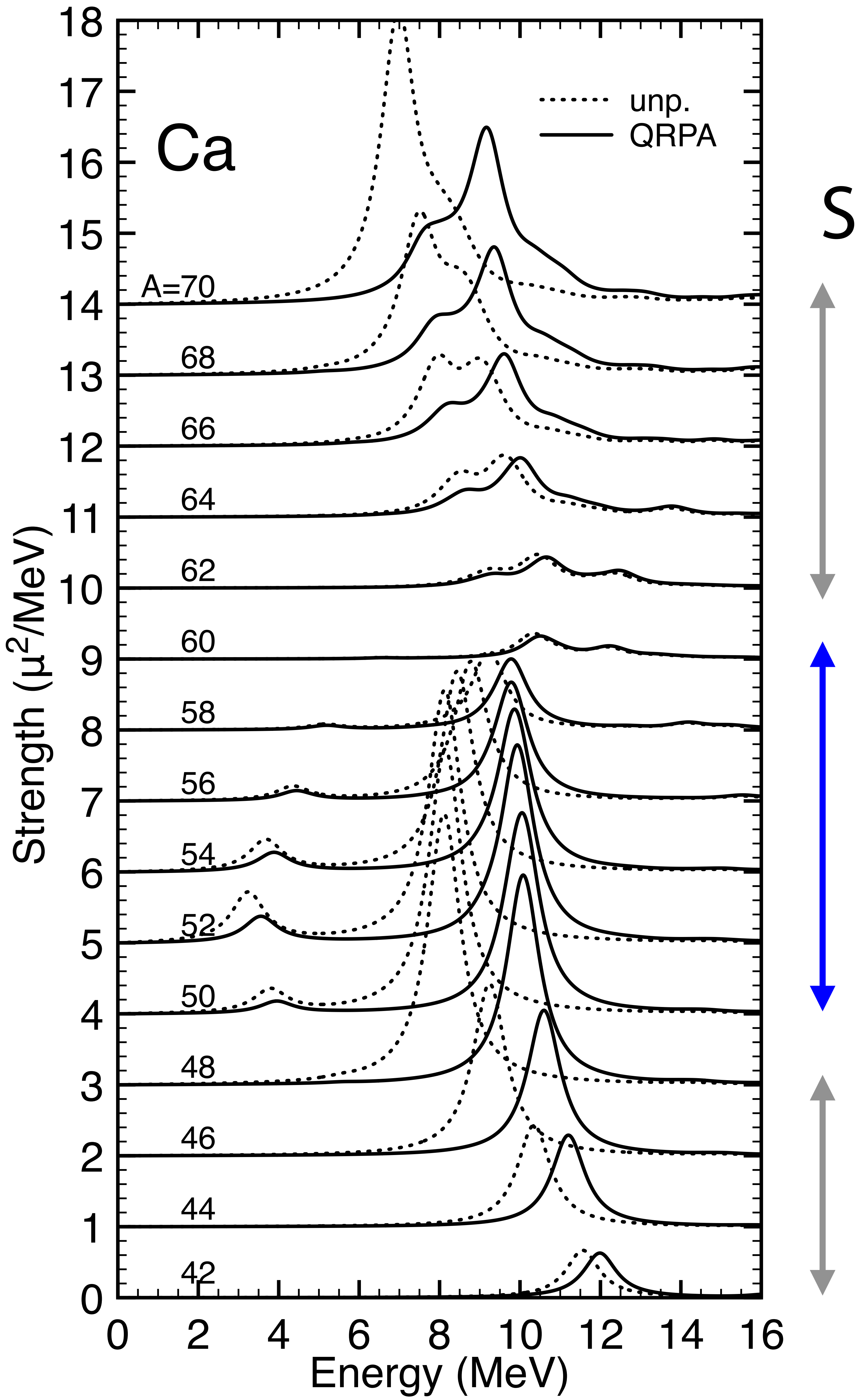
neutrons pairing

stronger effect of the RPA correlations

diagonal matrix element: $\propto u_{f_{5/2}}^2 v_{f_{7/2}}^2$

repulsive spin–isospin int.

Neutral channel: isovector spin-M1



SGII + T=1 pairing

^{50}Ca–^{60}Ca

appearance of low-lying state: generated by $\mathrm{n}1p_{3/2} \rightarrow \mathrm{n}1p_{1/2}$

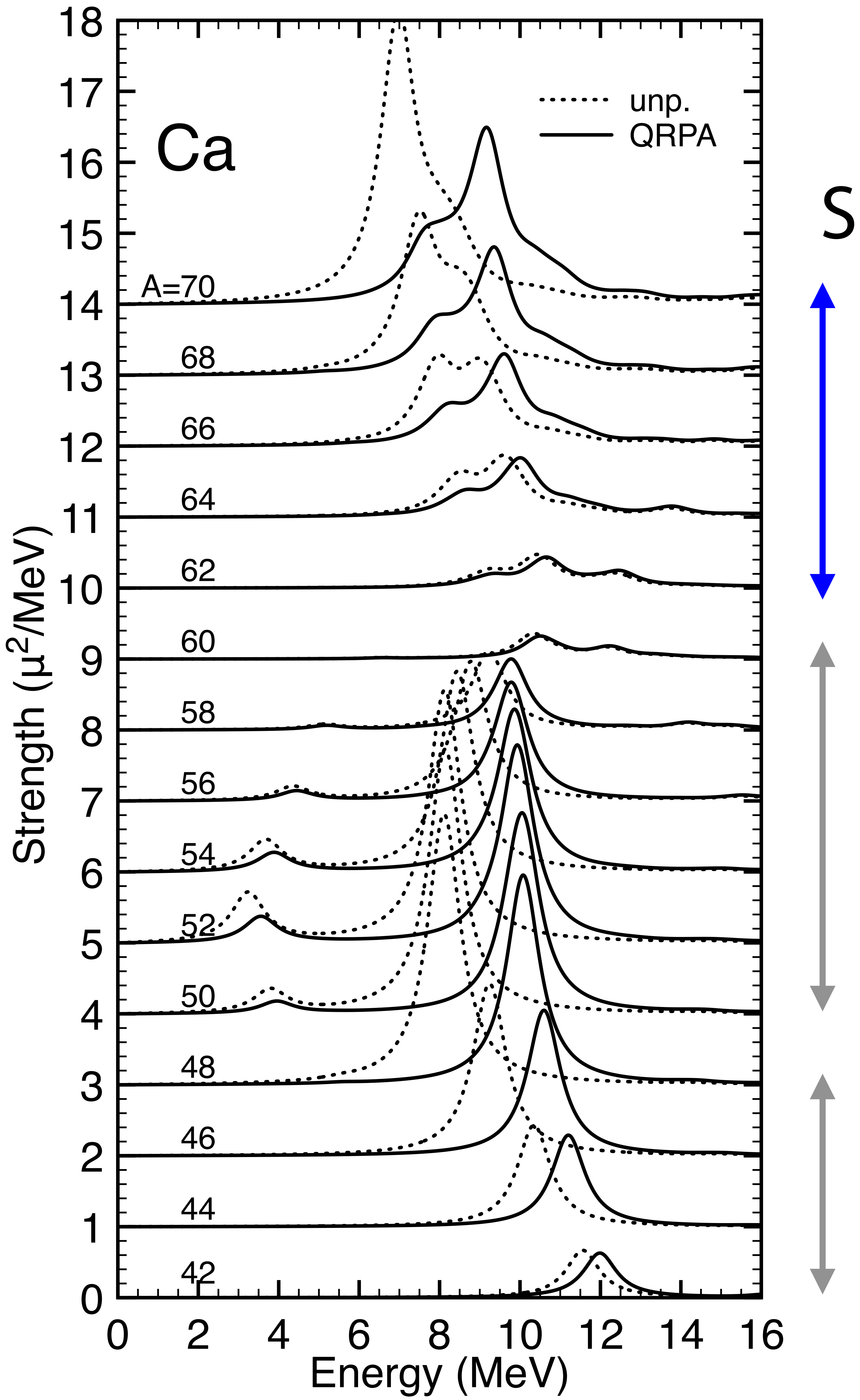
gradual increase in the occupation of $\mathrm{n}1f_{5/2}$

weaker effect of the RPA correlations

diagonal matrix element: $\propto (1 - v^2_{f_{5/2}})v^2_{f_{7/2}}$

separation energy of $f_{5/2}$ is 1.4 MeV

Neutral channel: isovector spin-M1



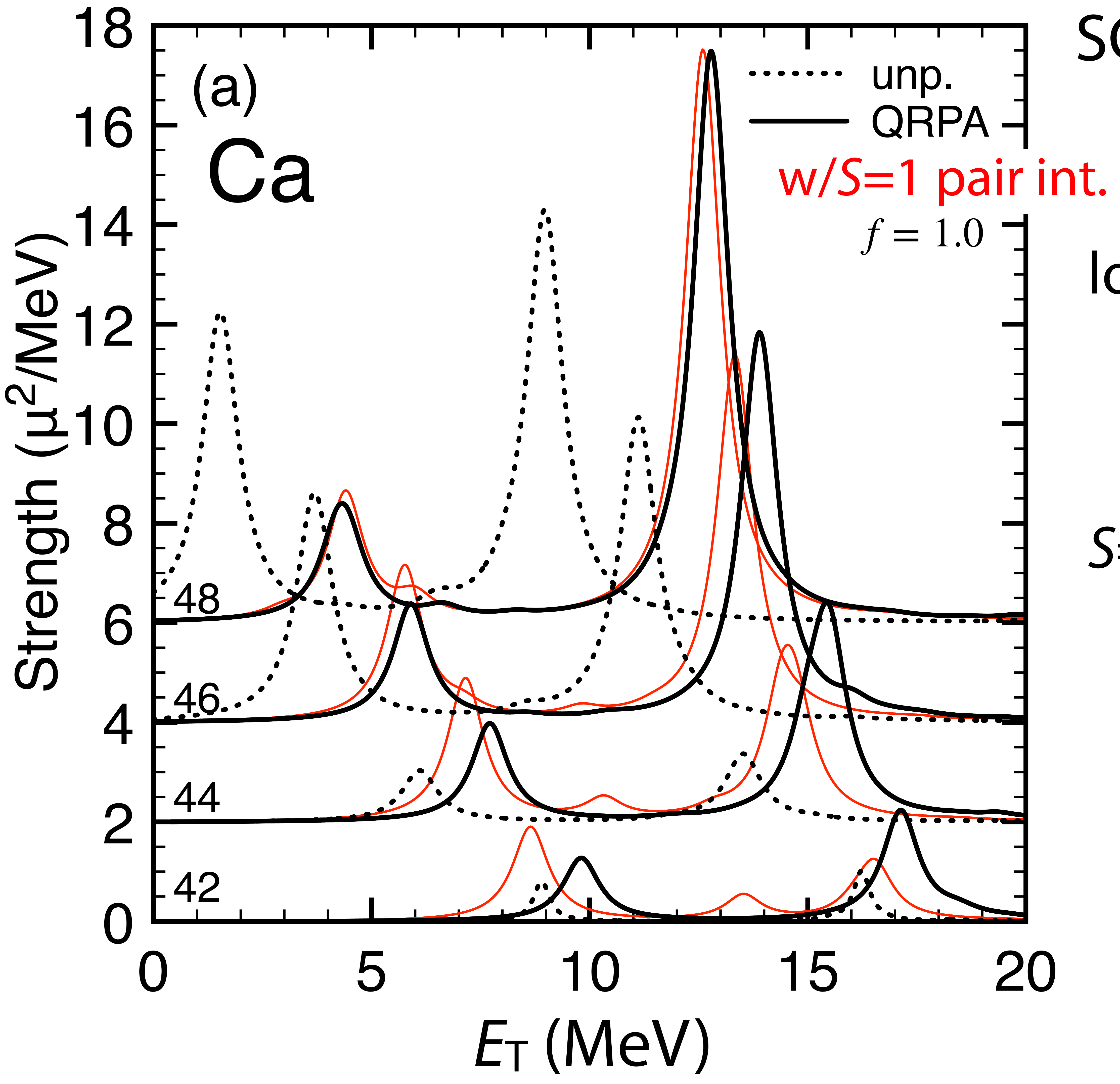
SGII + T=1 pairing

^{62}Ca$-^{70}$Ca

M1 resonance generated dominantly by $n1g_{9/2} \to n1g_{7/2}$ unbound

2qp excitations in the continuum

Charge-exchange channel: Gamow–Teller



SGII + T=1 pairing

low-lying state generated mainly by $n1f_{7/2} \rightarrow p1f_{7/2}$

c.f.: $(n1f_{7/2})^2$ is prohibited in spin M1 excitations

S=1 pn-pair int. increases the collectivity

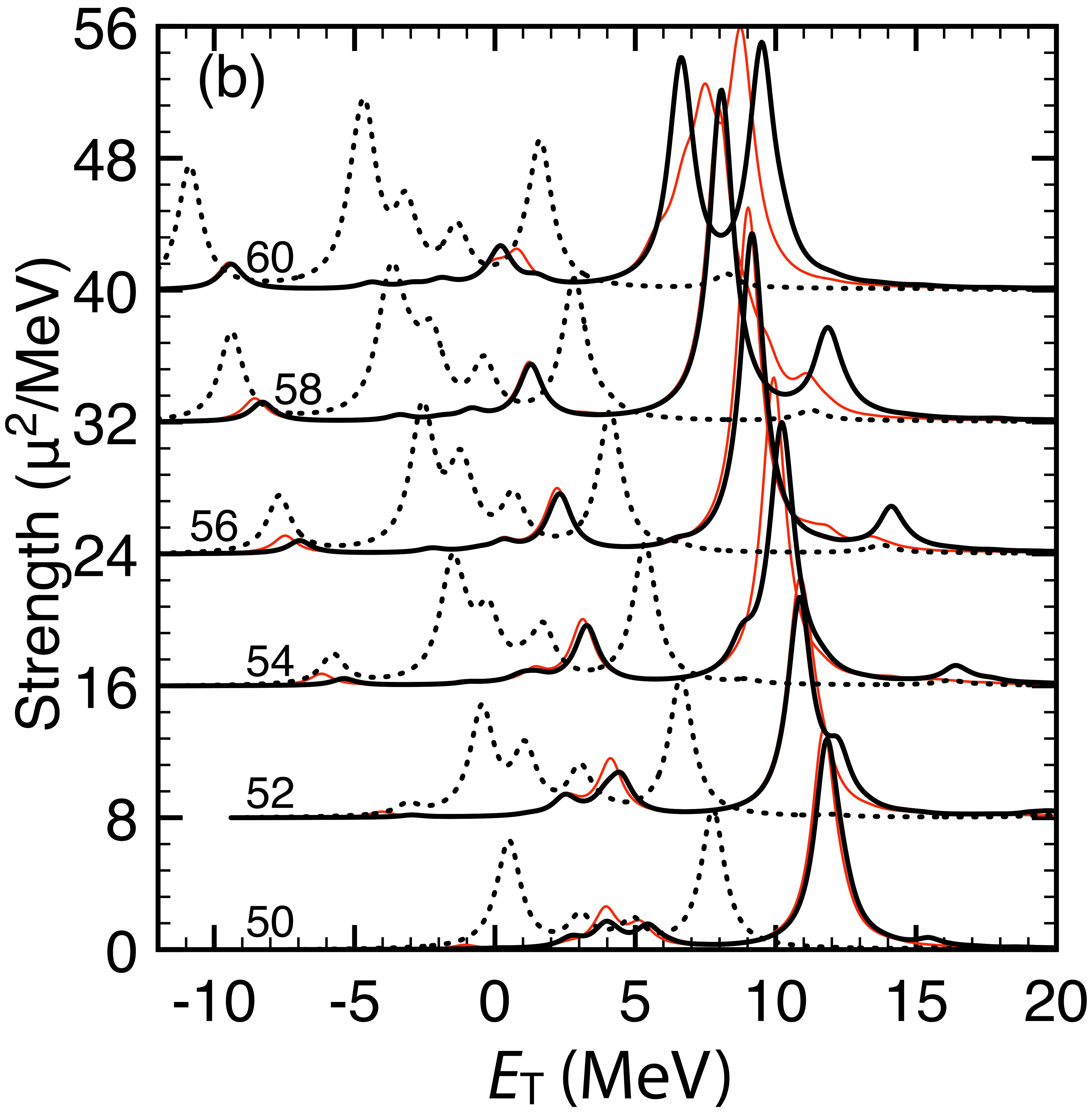
$n1f_{7/2} \rightarrow p1f_{7/2}$

$n1f_{7/2} \rightarrow p1f_{5/2}$

particle–particle excitations

large matrix elements of pairing

Charge-exchange channel: Gamow–Teller



low-lying states: $n2p_{3/2} \rightarrow p2p_{3/2}$

in $N \geq 34$: $n1f_{5/2} \rightarrow p1f_{7/2}$

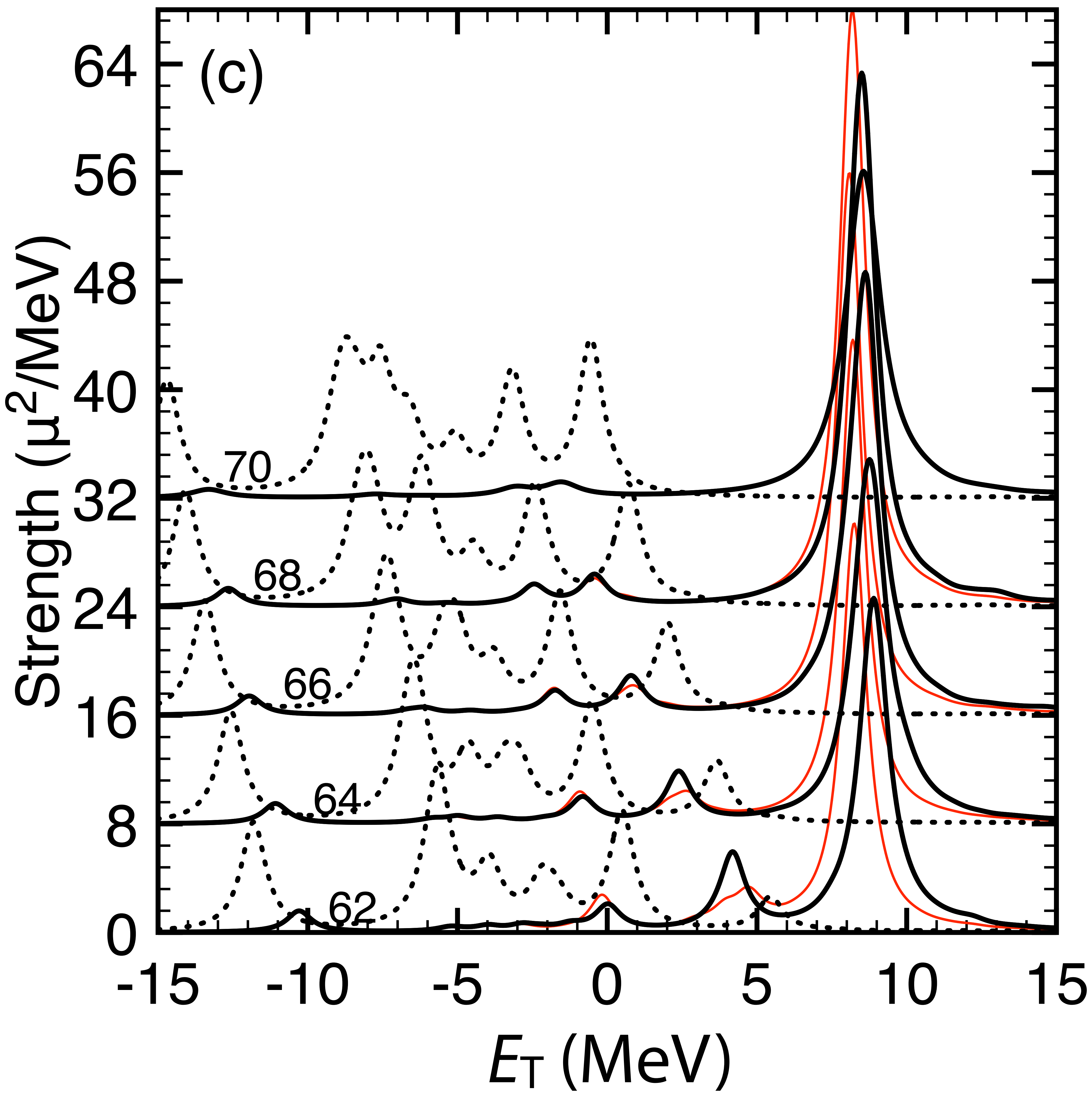
S=1 pn-pair int. slightly affects the state

high-lying states around N=40: $n1g_{9/2} \rightarrow p1p_{9/2}$

particle–particle excitations

large matrix elements of pairing

Charge-exchange channel: Gamow–Teller



high-lying states around N=40: $\mathrm{n}1g_{9/2} \rightarrow \mathrm{p}1g_{9/2}$

particle–particle excitations

large matrix elements of pairing

most of the strengths are concentrated in GR

many p–h/2qp excitations available

strong collectivity in neutron-rich nuclei