

# Double and single- $\beta$ decays and nuclear structures in the mapped IBM

Kosuke Nomura

*Hokkaido University*

Osaka, June 2024

# Contents

- $\beta\beta$  decays, and the Interacting Boson Model
- Low-lying structures of even-even and odd-odd nuclei
- $2\nu\beta\beta$  decays
- Single- $\beta$  decay of neutron-rich N~60 nuclei

# Study of $\beta\beta$ decay

- Nature of neutrinos, test of fundamental symmetries, ... neutrino-less  $\beta\beta$  decay
- Experiments: GERDA, NEMO, KamLAND ...
- Predicted nuclear matrix elements (NMEs) differ by a factor 2-3

Avignone et al., RMP (2008)

Agostini et al., RMP (2023)

Engel, Menendez, RPP (2017), etc.

# Two-neutrino ( $2\nu$ ) $\beta\beta$ decay

- wealth of experimental data available
- needs calculation of the intermediate states (non-closure approx.)

Theoretical descriptions without the closure approx. by

- **Interacting Boson Model** Yoshida-Iachello, PTEP (2013)
- QRPA Suhonen-Civitarese, Phys. Rep.(1998)  
Pirinen-Suhonen PRC (2015)  
Simkovic, Smetana, Vogel, PRC (2018), etc.
- NSM Caurier, Nowacki, Poves, IJMPE (2007)  
Yoshinaga et al., PTEP (2018)  
Coraggio et al., PRC (2019), etc.

# This work

- Consistent description of nuclear structure,  $\beta$ , and  $\beta\beta$  decays
- Framework: EDF-mapped IBM
- No closure approximation: calculations for odd-odd nuclei

K. Nomura, Phys. Rev. C 105 (2022) 044301

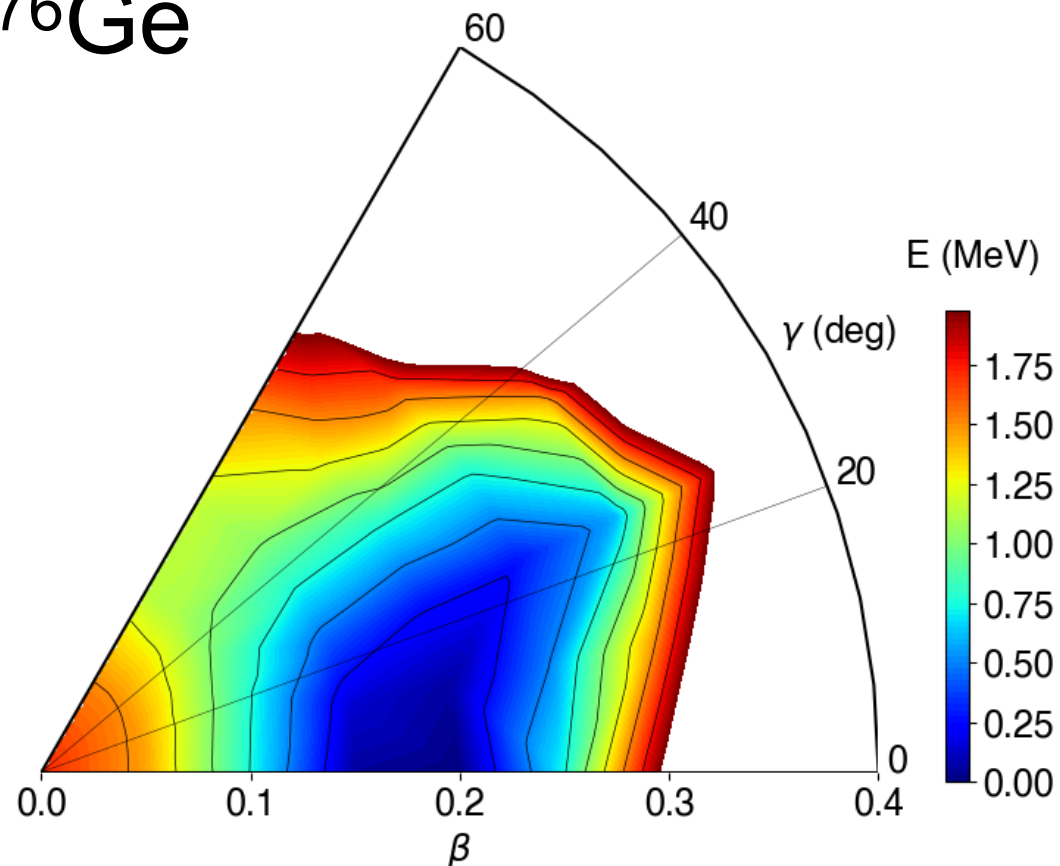
K. Nomura, arXiv: 2406.02986

K. Nomura, Phys. Rev. C 109 (2024) 034319

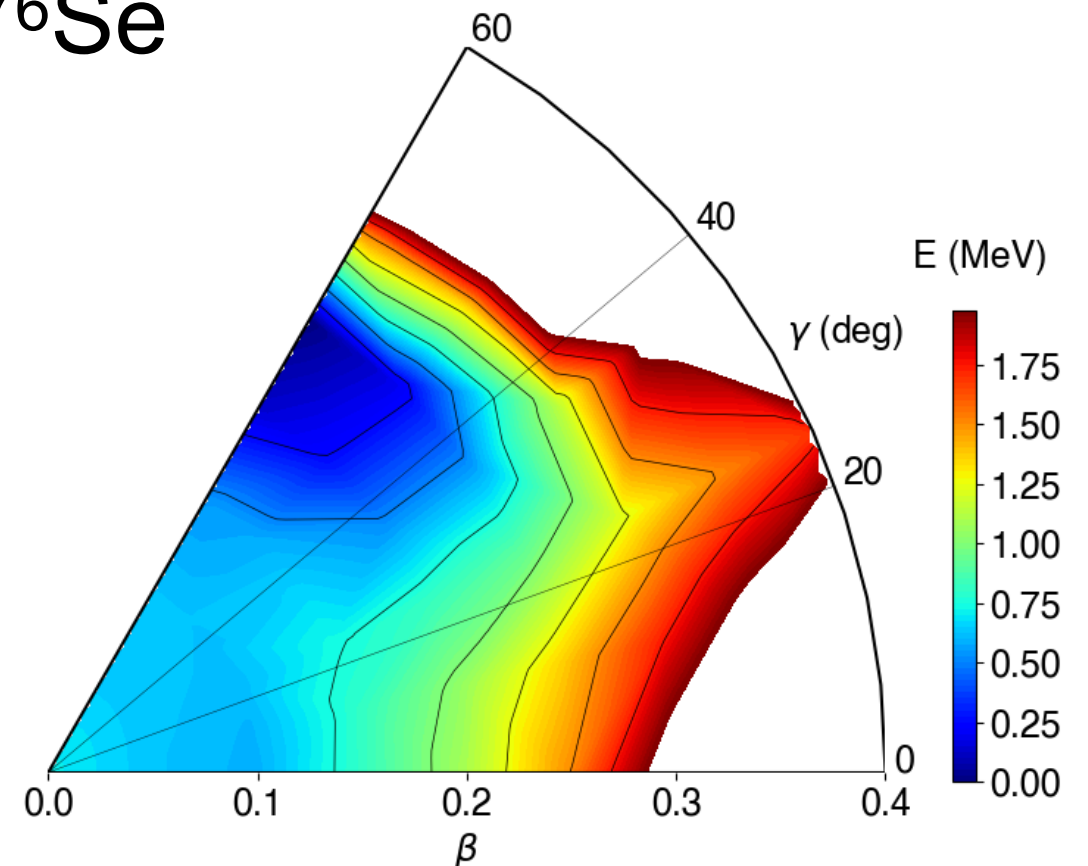
M. Homma, and K. Nomura, arXiv: 2404.14624, to appear in PRC

# Potential energy surfaces

$^{76}\text{Ge}$



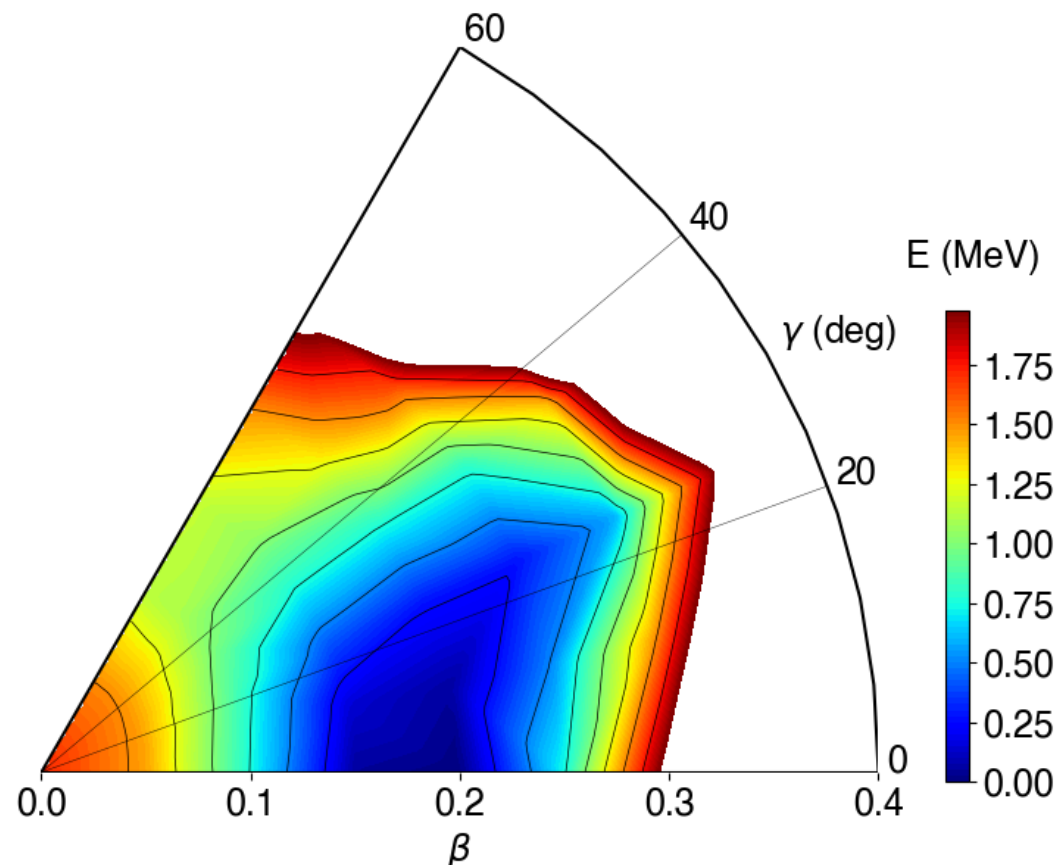
$^{76}\text{Se}$



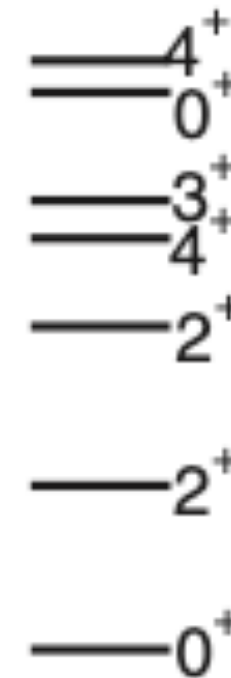
... calculated with the relativistic Hartree-Bogoliubov method with DD-PC1 energy density functional (EDF) and separable pairing

# Computing energy spectra

**Intrinsic frame**



**lab. frame**



Beyond-mean-field treatments

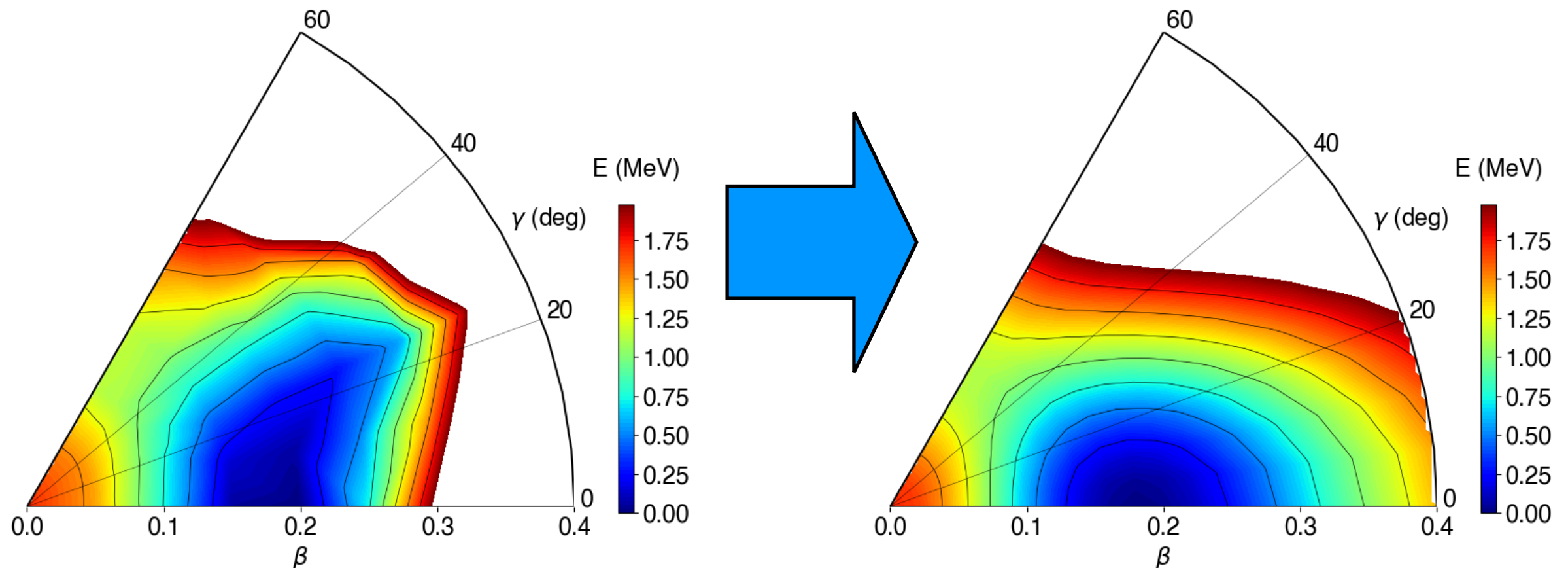
- Symmetry projections, GCM
- Collective Hamiltonian
- Interacting Boson Model

**Observables:**  
Excitation spectra, EM  
properties,  $\beta$ ,  $\beta\beta$  decay?

# Mean-field to IBM

Fermionic

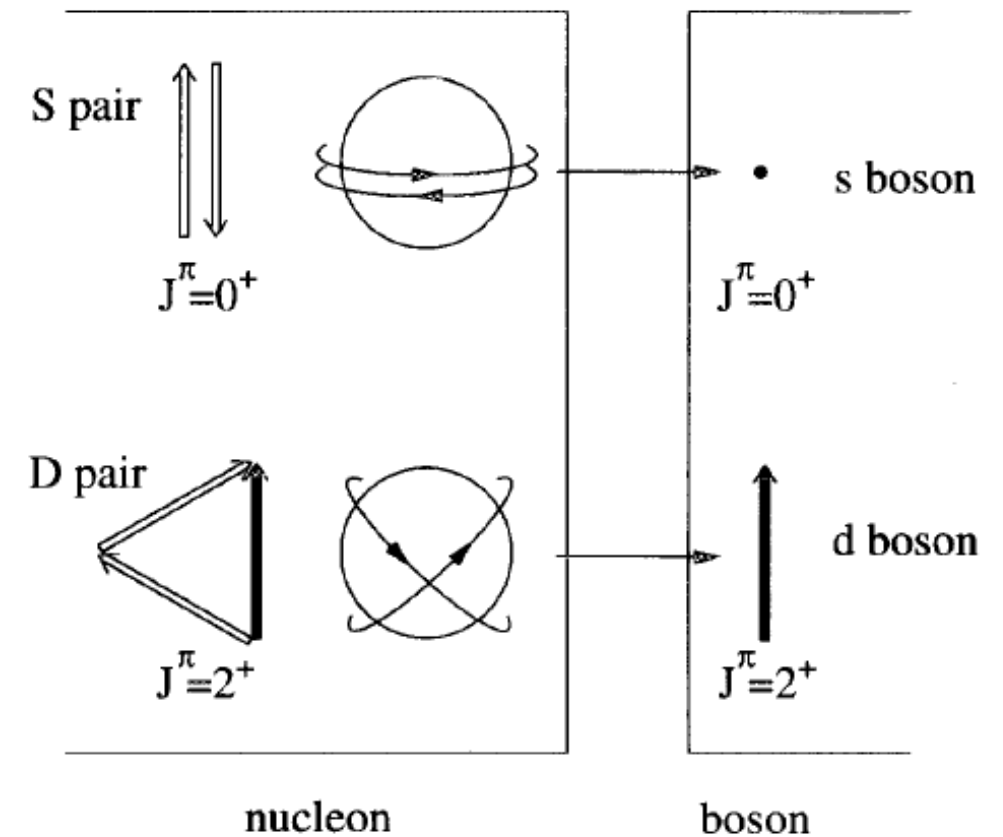
Bosonic



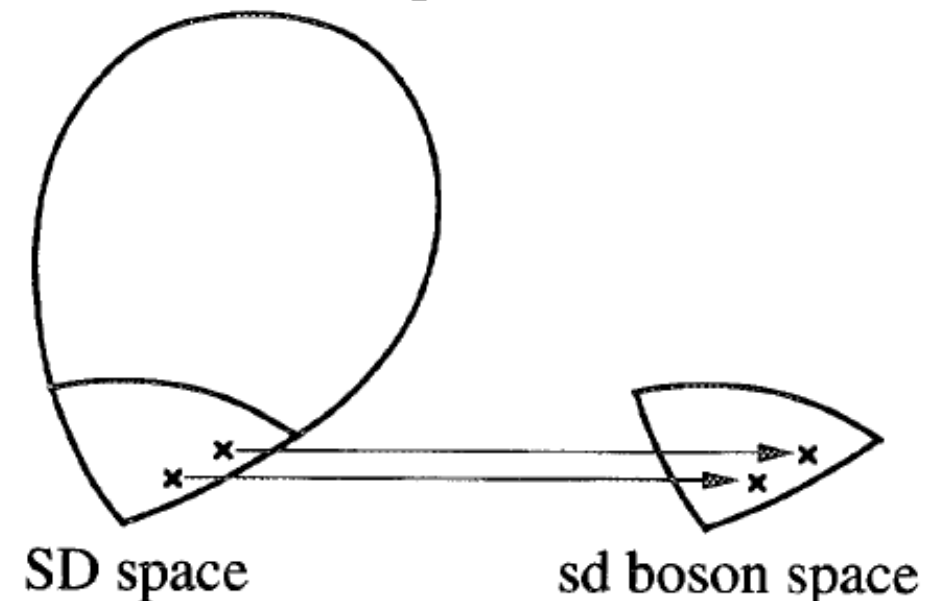
- SCMF energy surface is mapped onto that of the IBM
- Diagonalization of the mapped Hamiltonian yields energy spectra

# Interacting Boson Model (IBM)

- Collective  $J=0^+$  and  $2^+$  pairs of valence nucleons  $\mapsto$  s, d bosons
- exactly solvable if the Hamiltonian has the dynamical symmetries  $U(5)$ ,  $SU(3)$ , and  $O(6)$
- Microscopic derivations from nucleonic degrees of freedom



full shell-model space



Arima, Iachello (1975)  
 Otsuka, Arima, Iachello (1979)  
 Mizusaki, Otsuka (1997)  
 Nomura, Shimizu, Otsuka (2008)

# Neutron-proton IBM (IBM-2)

- building blocks:  $(s_\nu, s_\pi)$  and  $(d_\nu, d_\pi)$  bosons

- Hamiltonian:

$$\hat{H}_{\text{IBM}} = \epsilon_d (\hat{n}_{d_\nu} + \hat{n}_{d_\pi}) + \kappa \hat{Q}_\nu \cdot \hat{Q}_\pi + \kappa' \hat{L} \cdot \hat{L}$$

pairing-like  
(spherical driving)

quadrupole-quadrupole  
(deformation driving)

rotational term

$$\hat{Q}_\rho = s_\rho^\dagger \tilde{d}_\rho + d_\rho^\dagger s_\rho + \chi_\rho (d_\rho^\dagger \times \tilde{d}_\rho)^{(2)}$$

$$\hat{L} = \sqrt{10} [(d_\nu^\dagger \times \tilde{d}_\nu)^{(1)} + (d_\pi^\dagger \times \tilde{d}_\pi)^{(1)}]$$

... with 5 parameters

# Geometry of the IBM

Energy surface:

$$E_{\text{IBM}}(\beta, \gamma) = \langle \phi | \hat{H}_{\text{IBM}} | \phi \rangle$$

... with boson coherent state

$$| \phi \rangle \propto \prod_{\rho=\nu, \pi} \left[ s_{\rho}^{\dagger} + \beta \cos \gamma d_{\rho,0}^{\dagger} + \frac{1}{\sqrt{2}} \beta \sin \gamma \left( d_{\rho,+2}^{\dagger} + d_{\rho,-2}^{\dagger} \right) \right]^{N_{\rho}} | 0 \rangle$$

Ginocchio-Kirson (1980)

IBM Hamiltonian is determined by

$$E_{\text{SCMF}}(\beta, \gamma) \approx E_{\text{IBM}}(\beta, \gamma)$$

KN et al. PRL101 (2008) 142501

# Interacting Boson-Fermion-Fermion Model

$$\hat{H}_{\text{IBFFM}} = \hat{H}_{\text{IBM}} + \hat{H}_{\text{F}} + \hat{V}_{\text{BF}} + \hat{V}_{\nu\pi}$$

Single-fermion Hamiltonian

$$\hat{H}_{\text{F}} = \sum_{j_{\rho}} \epsilon_{j_{\rho}} \hat{n}_{j_{\rho}}$$

Boson-fermion interactions

$$\hat{V}_{\text{BF}} = \Gamma_0 \hat{V}_{\text{dyn}} + \Lambda_0 \hat{V}_{\text{exc}} + A_0 \hat{V}_{\text{mon}}$$

dynamical  
(direct) term

exchange  
term

monopole  
term

neutron-proton interaction

$$\begin{aligned} \hat{V}_{\nu\pi} = & 4\pi [v_{\text{d}} + v_{\text{ssd}} \boldsymbol{\sigma}_{\nu} \cdot \boldsymbol{\sigma}_{\pi}] \delta(\mathbf{r}) \delta(\mathbf{r}_{\nu} - r_0) \delta(\mathbf{r}_{\pi} - r_0) \\ & - \frac{1}{\sqrt{3}} v_{\text{ss}} \boldsymbol{\sigma}_{\nu} \cdot \boldsymbol{\sigma}_{\pi} + v_{\text{t}} \left[ \frac{3(\boldsymbol{\sigma}_{\nu} \cdot \mathbf{r})(\boldsymbol{\sigma}_{\pi} \cdot \mathbf{r})}{r^2} - \boldsymbol{\sigma}_{\nu} \cdot \boldsymbol{\sigma}_{\pi} \right] \end{aligned}$$

Iachello, Van Isacker, "The interacting boson-fermion model" (1991)

# Boson-fermion interactions

$$\hat{V}_{\text{dyn}}^{\rho} = \sum_{j_{\rho} j'_{\rho}} \gamma_{j_{\rho} j'_{\rho}} (a_{j_{\rho}}^{\dagger} \times \tilde{a}_{j'_{\rho}})^{(2)} \cdot \hat{Q}_{\rho'},$$

$$\hat{V}_{\text{exc}}^{\rho} = - \left( s_{\rho'}^{\dagger} \times \tilde{d}_{\rho'} \right)^{(2)} \cdot \sum_{j_{\rho} j'_{\rho} j''_{\rho}} \sqrt{\frac{10}{N_{\rho}(2j_{\rho} + 1)}} \beta_{j_{\rho} j'_{\rho}} \beta_{j''_{\rho} j_{\rho}} : \left[ (d_{\rho}^{\dagger} \times \tilde{a}_{j''_{\rho}})^{(j_{\rho})} \times (a_{j'_{\rho}}^{\dagger} \times \tilde{s}_{\rho})^{(j'_{\rho})} \right]^{(2)} : + (\text{H.c.}) ,$$

$$\hat{V}_{\text{mon}}^{\rho} = \hat{n}_{d_{\rho}} \hat{n}_{j_{\rho}} ,$$

with (u,v)-dependent factors:

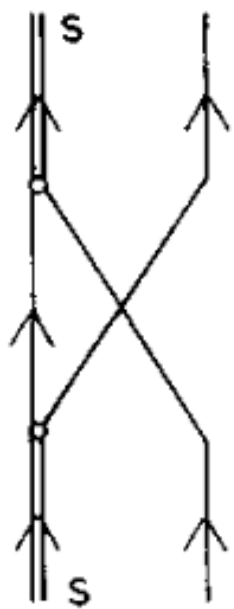
$$\gamma_{j_{\rho} j'_{\rho}} = (u_{j_{\rho}} u_{j'_{\rho}} - v_{j_{\rho}} v_{j'_{\rho}}) Q_{j_{\rho} j'_{\rho}}$$

$$\beta_{j_{\rho} j'_{\rho}} = (u_{j_{\rho}} v_{j'_{\rho}} + v_{j_{\rho}} u_{j'_{\rho}}) Q_{j_{\rho} j'_{\rho}}$$

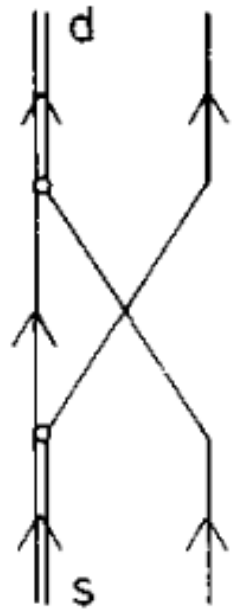
... derived within the generalized seniority

e.g., Schoten, PPNP (1985)

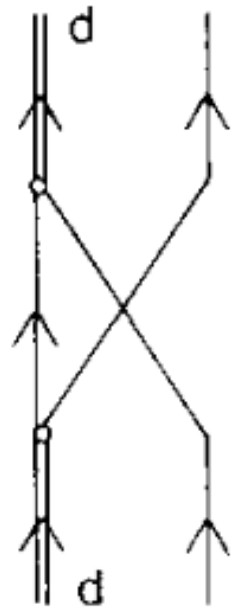
# Boson-fermion interactions



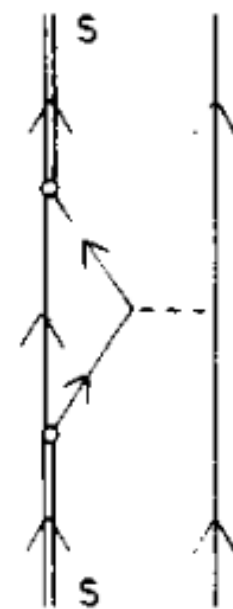
(a)



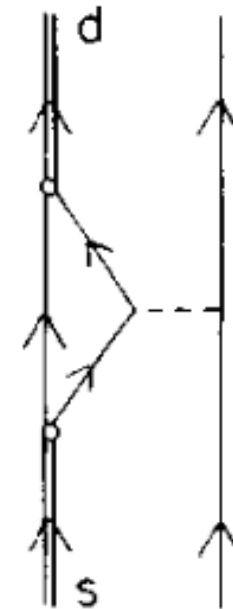
(b)



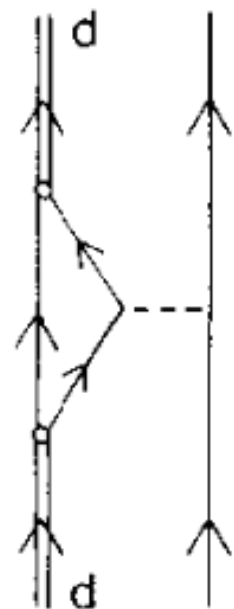
(c)



(d)



(e)



(f)

exchange terms

direct terms

# Building the IBFFM Hamiltonian

$$\hat{H}_{\text{IBFFM}} = \hat{H}_{\text{IBM}} + \hat{H}_{\text{F}} + \hat{V}_{\text{BF}} + \hat{V}_{\nu\pi}$$

mapping the PES

spherical s.p. e. ( $\epsilon_j$ )

occupation probabilities  $v_j^2$   
at spherical configuration

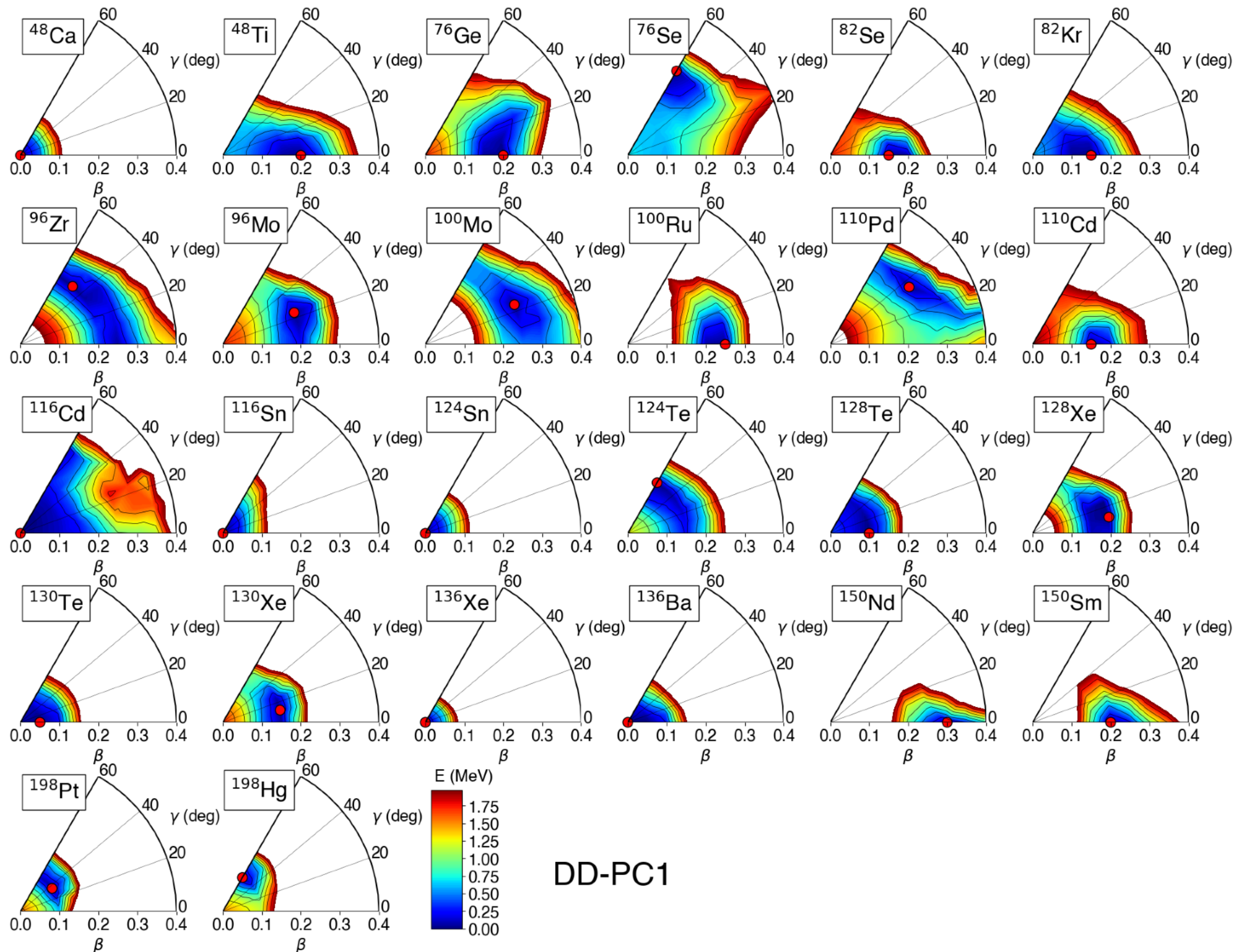
Microscopic  
input from EDF

- ... 3 strength parameters for  $\hat{V}_{\text{BF}}$  fitted for each nucleus (odd-N, odd-Z, and parity)
- ... 4 strength parameters for  $\hat{V}_{\nu\pi}$  fitted for each odd-odd nucleus

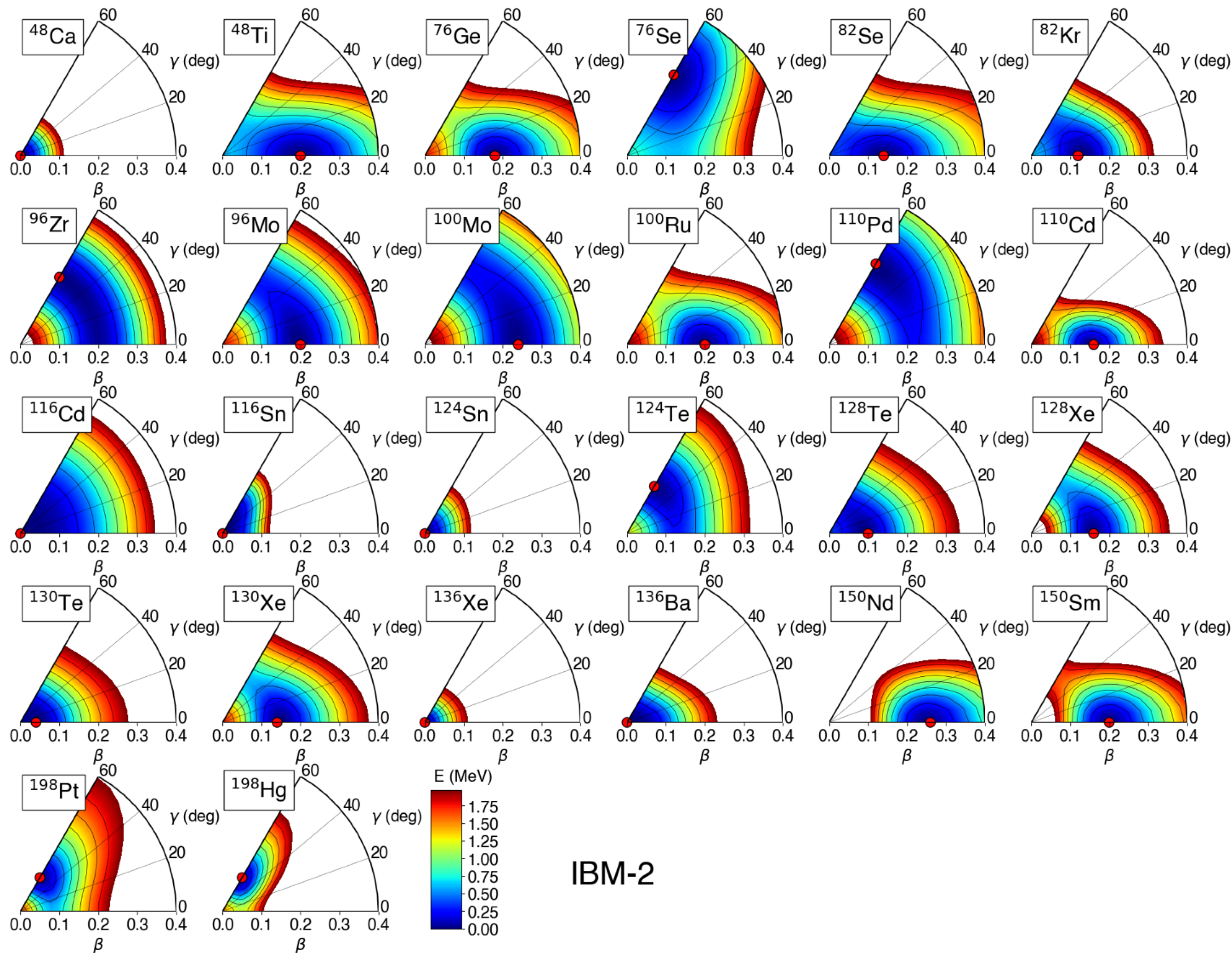
- IBFM: KN et al. PRC93 (2016) 054305
- IBFFM-2: KN et al., PRC99 (2019) 034308

Low-lying structure

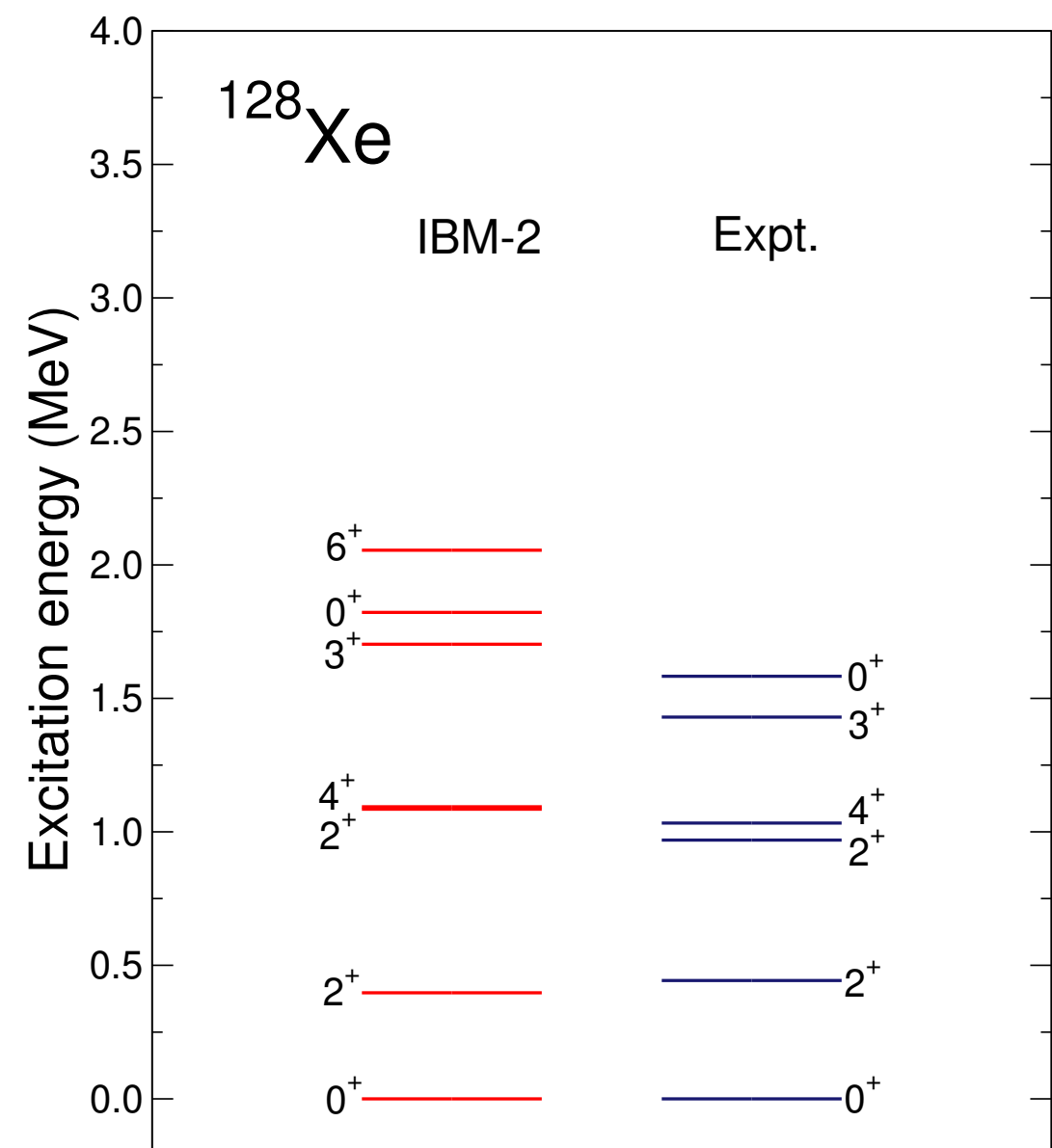
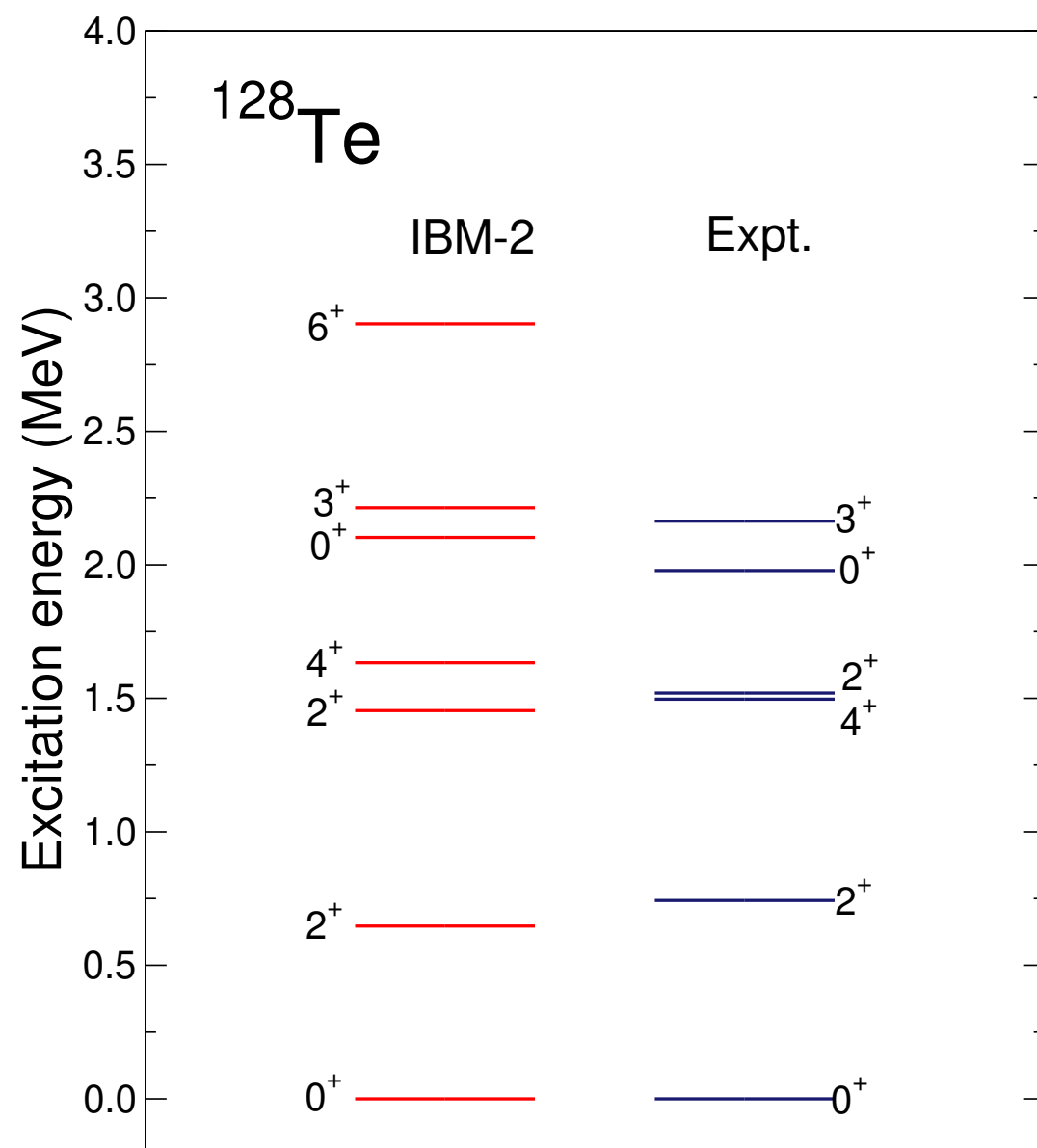
# EDF PESs for even-even nuclei



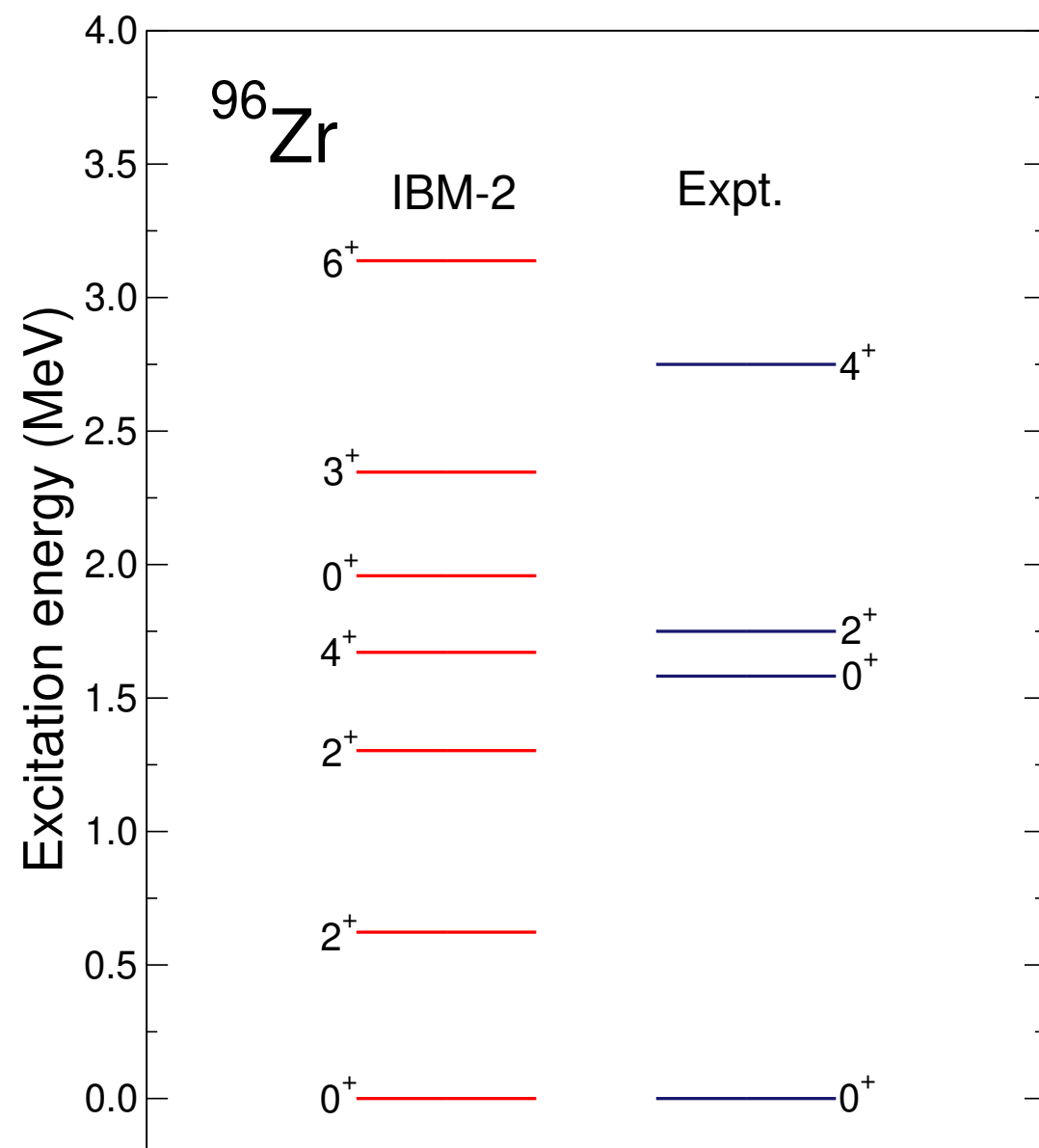
# IBM PESs



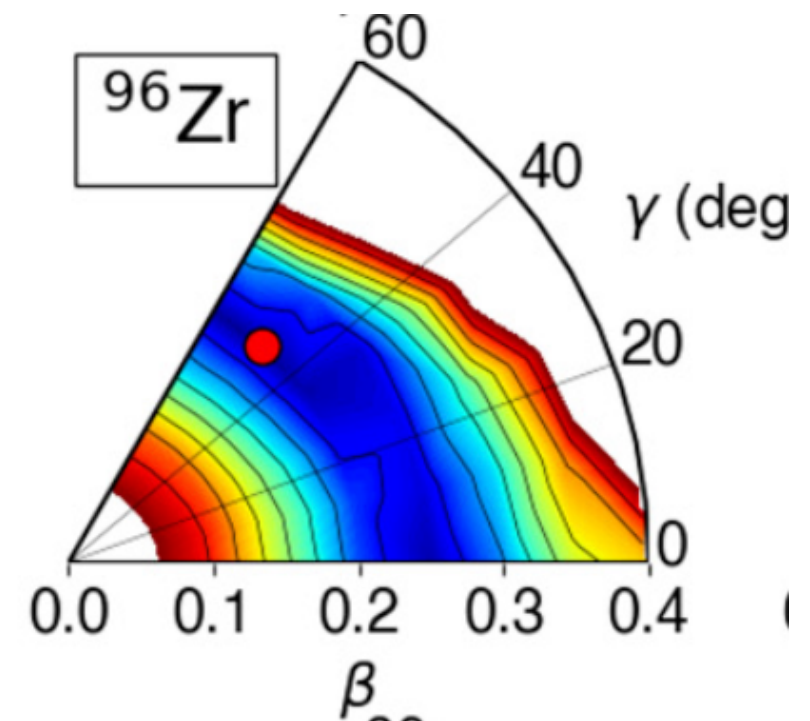
# $^{128}\text{Te}$ and $^{128}\text{Xe}$ spectra



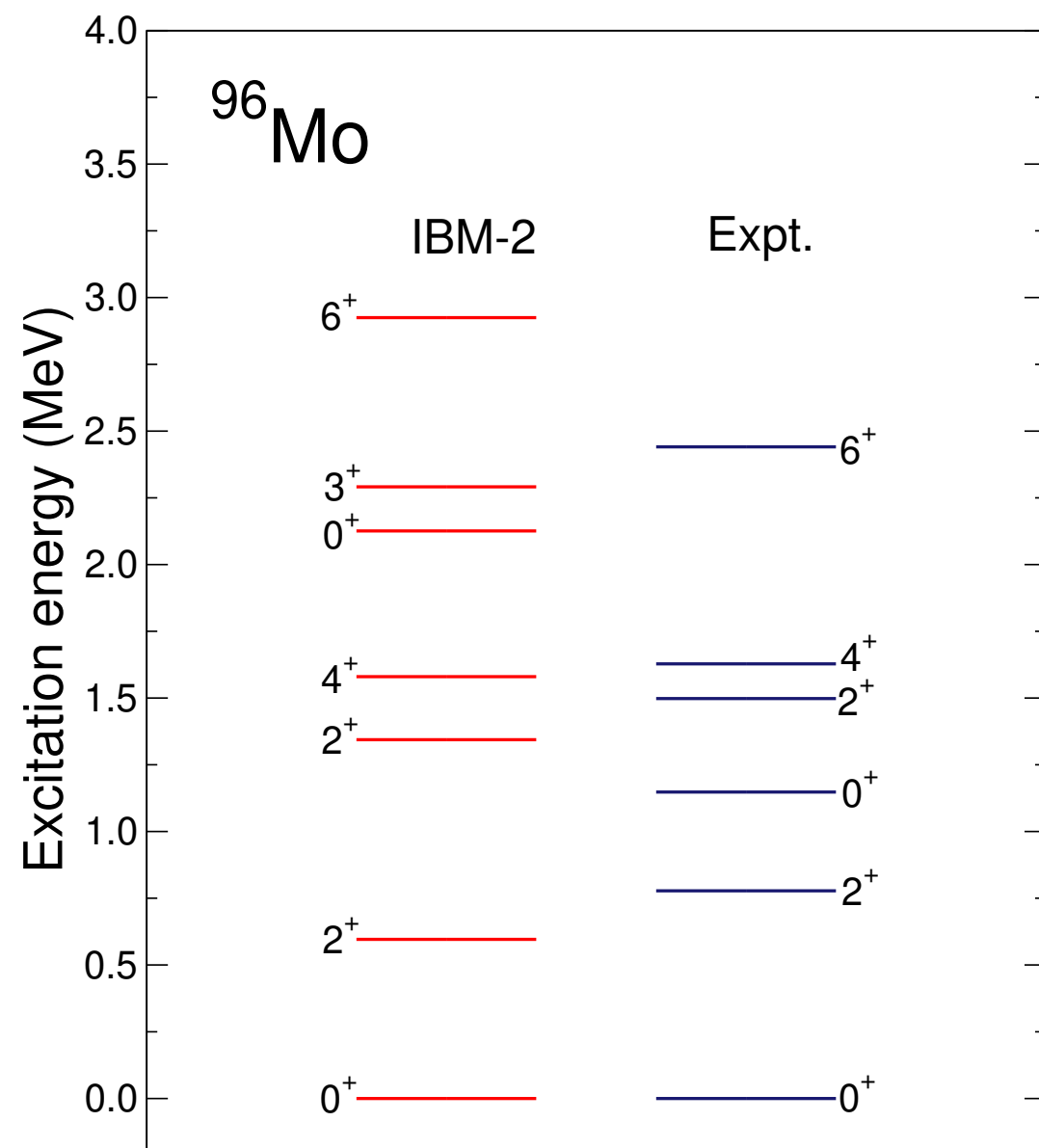
# $^{96}\text{Zr}$ spectrum



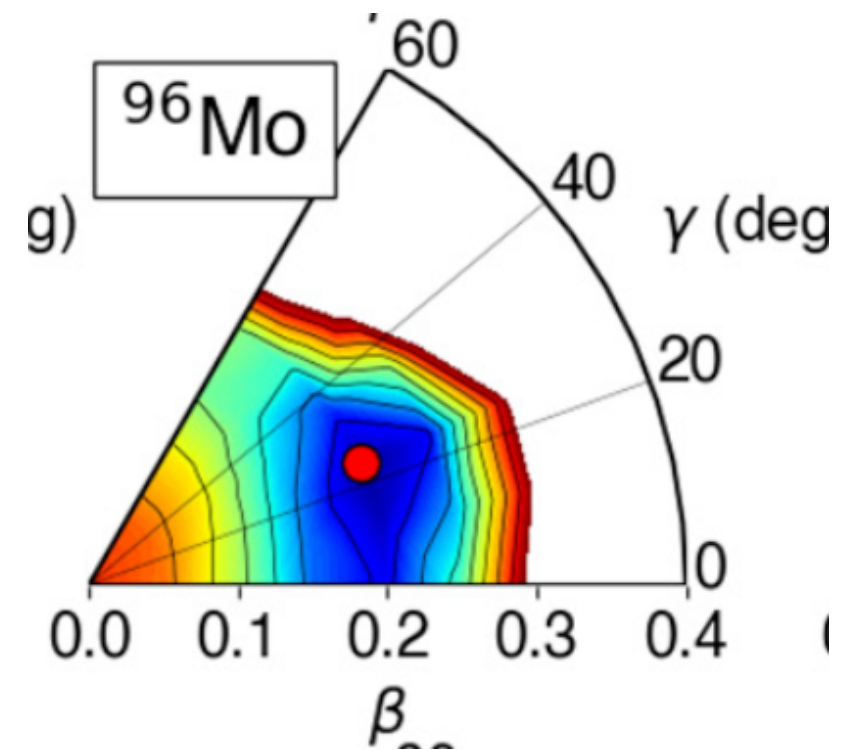
... unable to reproduce N=56 sub-shell effects



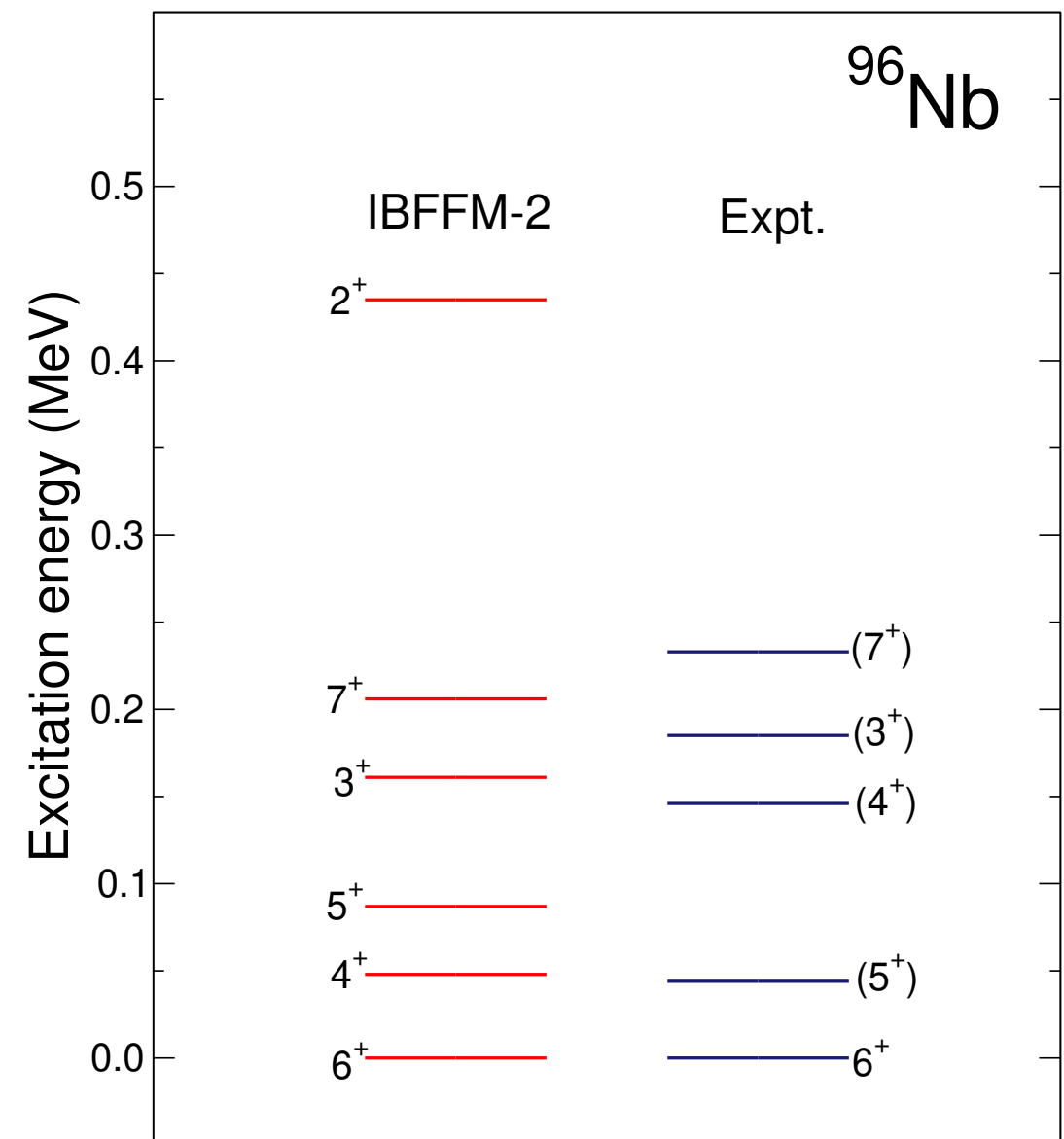
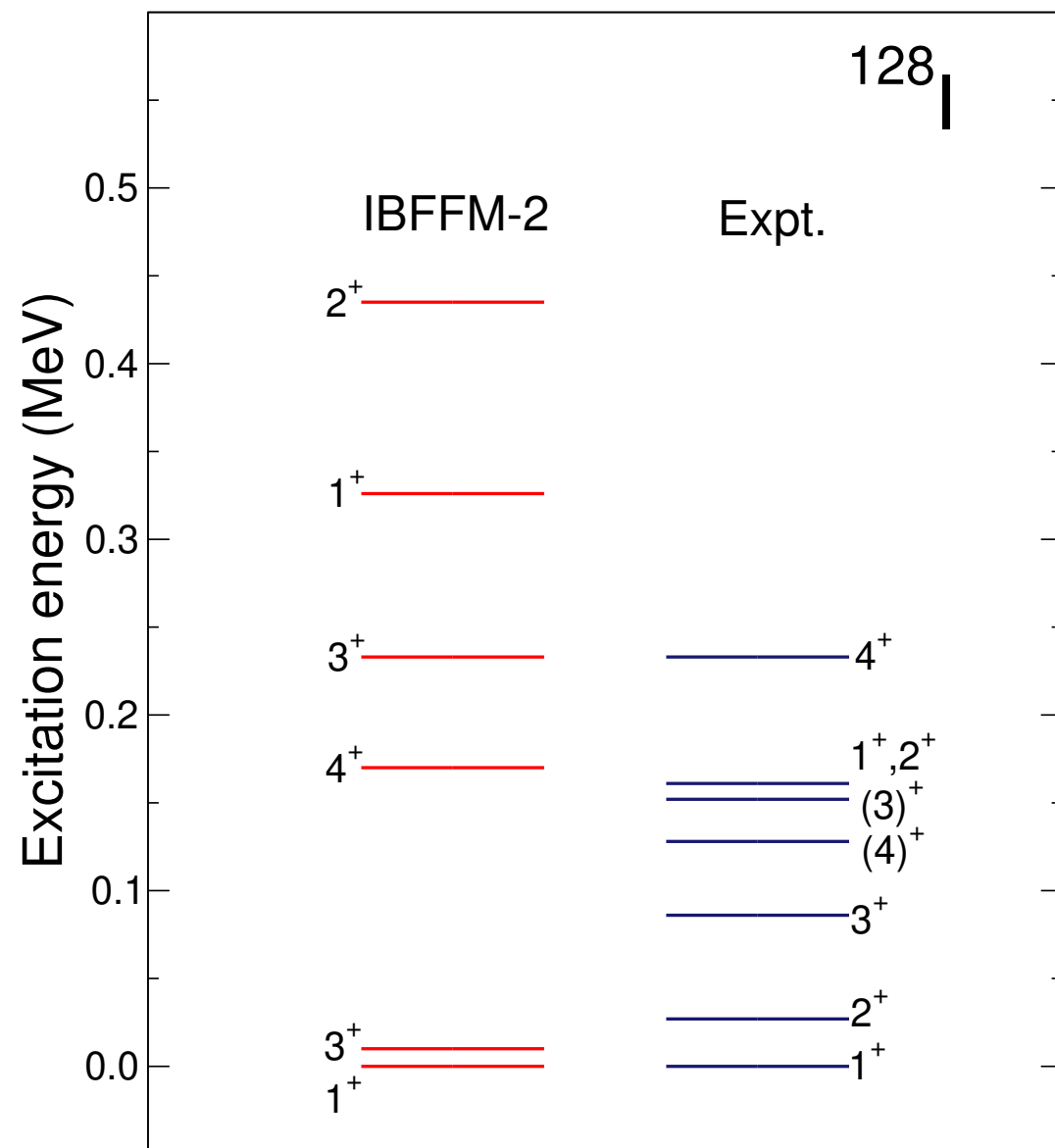
# $^{96}\text{Mo}$ spectrum



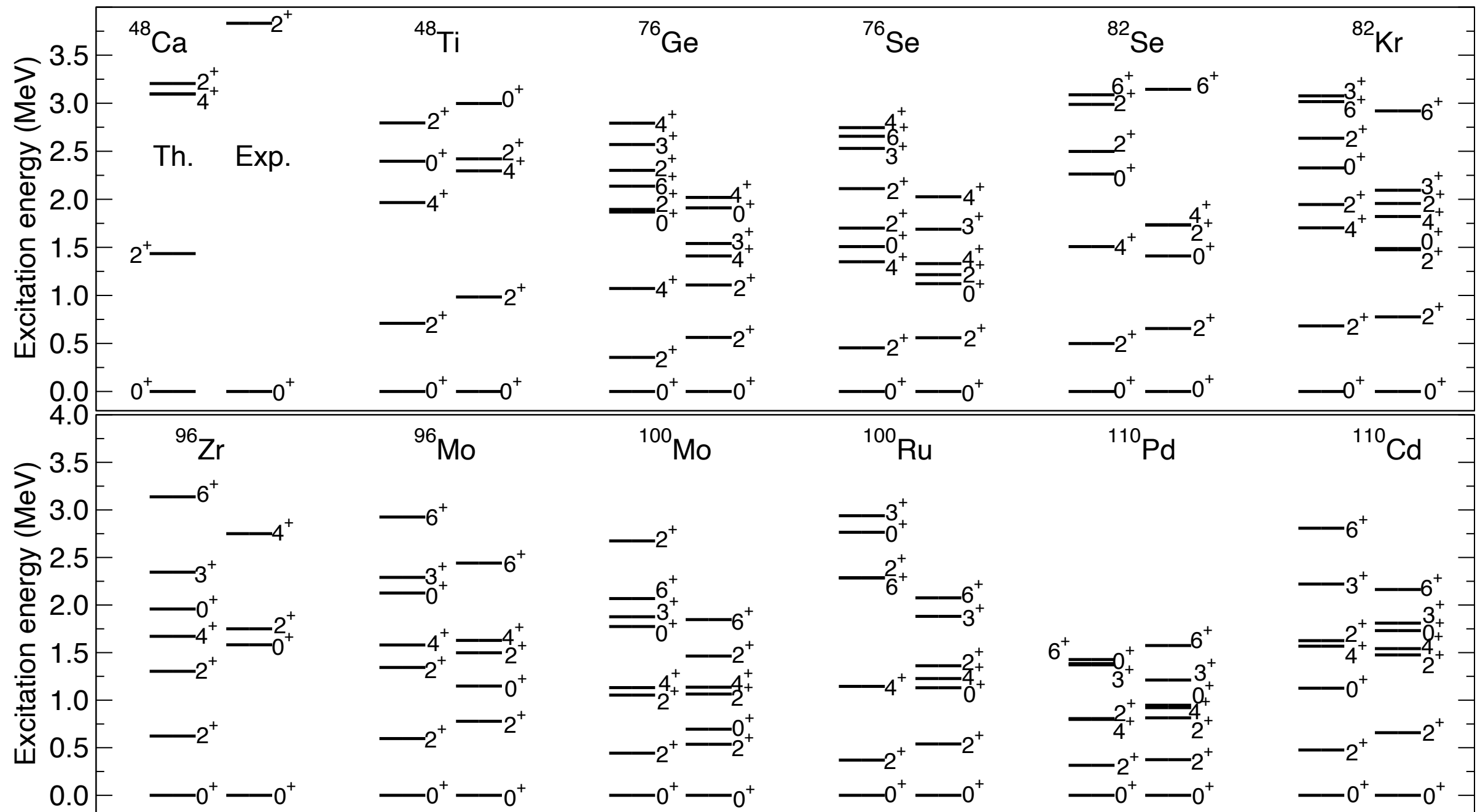
... overestimates the  $0_2^+$  level: a common problem of the mapped IBM



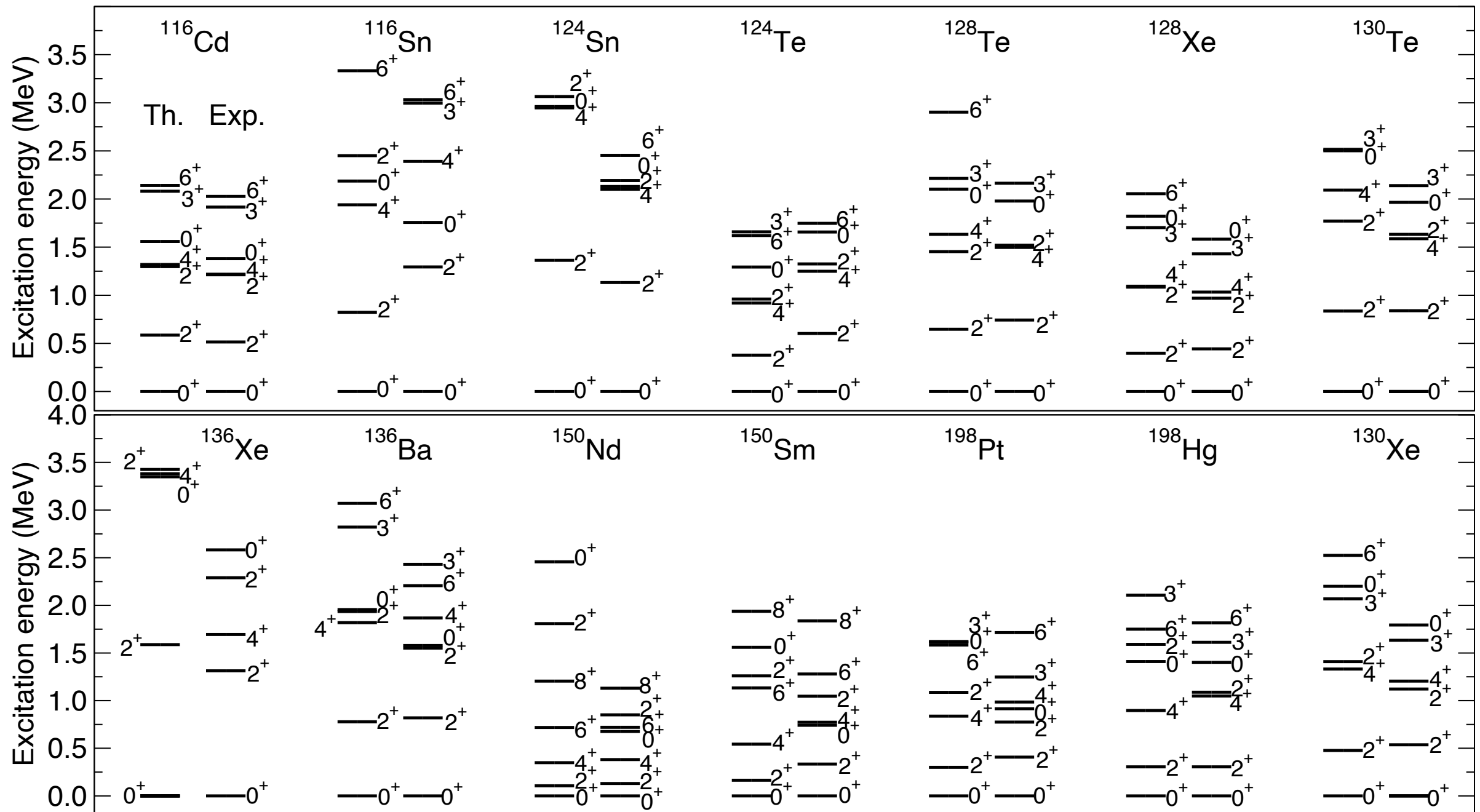
# Energy spectra of odd-odd nuclei



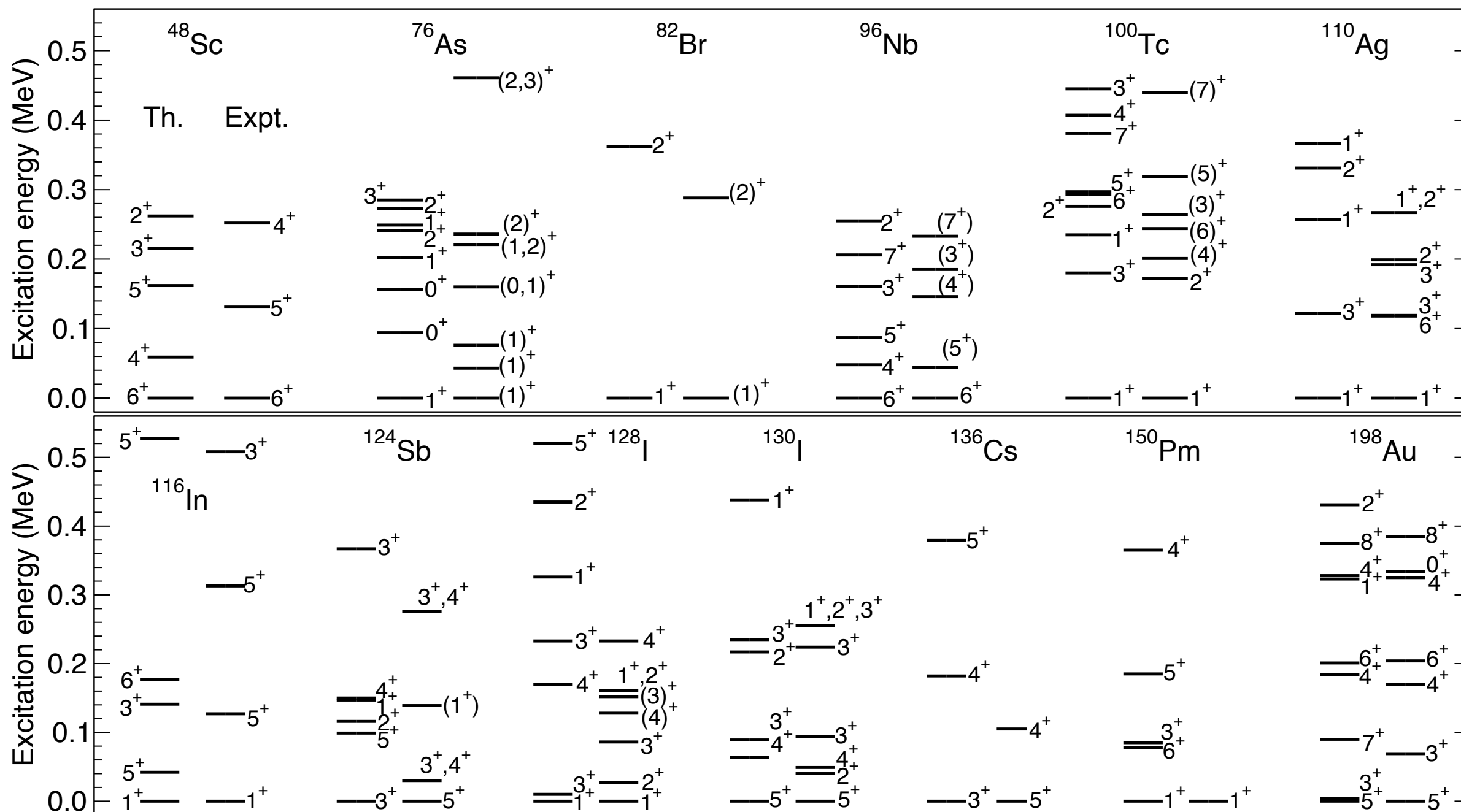
# Energy spectra of even-even nuclei



# Energy spectra of even-even nuclei



# Energy spectra of odd-odd nuclei



$2\nu\beta\beta$  decay

# Calculation of NME

$$M_{2\nu} = g_A^2 \cdot m_e c^2 \left[ M_{2\nu}^{\text{GT}} - \left( \frac{g_V}{g_A} \right)^2 M_{2\nu}^{\text{F}} \right],$$

$$M_{2\nu}^{\text{GT}} = \sum_N \frac{\langle 0_F^+ \| t^+ \sigma \| 1_N^+ \rangle \langle 1_N^+ \| t^+ \sigma \| 0_{1,I}^+ \rangle}{E_N - E_I + \frac{1}{2}(Q_{\beta\beta} + 2m_e c^2)},$$

$$M_{2\nu}^{\text{F}} = \sum_N \frac{\langle 0_F^+ \| t^+ \| 0_N^+ \rangle \langle 0_N^+ \| t^+ \| 0_{1,I}^+ \rangle}{E_N - E_I + \frac{1}{2}(Q_{\beta\beta} + 2m_e c^2)},$$

GT and Fermi  
operators

$$t^\pm \mapsto \hat{T}^{\text{F}} = \sum_{j_\nu j_\pi} \eta_{j_\nu j_\pi}^{\text{F}} (\hat{P}_{j_\nu} \times \hat{P}_{j_\pi})^{(0)},$$

$$t^\pm \sigma \mapsto \hat{T}^{\text{GT}} = \sum_{j_\nu j_\pi} \eta_{j_\nu j_\pi}^{\text{GT}} (\hat{P}_{j_\nu} \times \hat{P}_{j_\pi})^{(1)},$$

with  $\hat{P}_j$  given as one of

$$A_{j_\rho m_\rho}^\dagger = \zeta_{j_\rho} a_{j_\rho m_\rho}^\dagger + \sum_{j'_\rho} \zeta_{j_\rho j'_\rho} s_\rho^\dagger (\tilde{d}_\rho \times a_{j'_\rho}^\dagger)_{m_\rho}^{(j_\rho)}$$

$$B_{j_\rho m_\rho}^\dagger = \theta_{j_\rho} s_\rho^\dagger \tilde{a}_{j_\rho m_\rho} + \sum_{j'_\rho} \theta_{j_\rho j'_\rho} (d_\rho^\dagger \times \tilde{a}_{j'_\rho})_{m_\rho}^{(j_\rho)}$$

and their H.C.

# (u,v)-dependent forms

Dellagiacoma-Iachello (1989), etc.

## Coefficients

$$\zeta_{j_\rho} = u_{j_\rho} \frac{1}{K'_{j_\rho}},$$

$$\zeta_{j_\rho j'_\rho} = -v_{j_\rho} \beta_{j'_\rho j_\rho} \sqrt{\frac{10}{N_\rho(2j_\rho + 1)}} \frac{1}{KK'_{j_\rho}},$$

$$\theta_{j_\rho} = \frac{v_{j_\rho}}{\sqrt{N_\rho}} \frac{1}{K''_{j_\rho}},$$

$$\theta_{j_\rho j'_\rho} = u_{j_\rho} \beta_{j'_\rho j_\rho} \sqrt{\frac{10}{2j_\rho + 1}} \frac{1}{KK''_{j_\rho}}.$$

with factors

$$K = \left( \sum_{j_\rho j'_\rho} \beta_{j'_\rho j_\rho}^2 \right)^{1/2},$$

$$K'_{j_\rho} = \left[ 1 + 2 \left( \frac{v_{j_\rho}}{u_{j_\rho}} \right)^2 \frac{\langle (\hat{n}_{s_\rho} + 1) \hat{n}_{d_\rho} \rangle_{0_1^+}}{N_\rho(2j_\rho + 1)} \frac{\sum_{j'_\rho} \beta_{j'_\rho j_\rho}^2}{K^2} \right]^{1/2},$$

$$K''_{j_\rho} = \left[ \frac{\langle \hat{n}_{s_\rho} \rangle_{0_1^+}}{N_\rho} + 2 \left( \frac{u_{j_\rho}}{v_{j_\rho}} \right)^2 \frac{\langle \hat{n}_{d_\rho} \rangle_{0_1^+}}{2j_\rho + 1} \frac{\sum_{j'_\rho} \beta_{j'_\rho j_\rho}^2}{K^2} \right]^{1/2},$$

... (u,v) amplitudes provided by the DFT

# $Q_{\beta\beta}$ values

$$Q_{\beta\beta} = 2(m_n - m_p - m_e)c^2 + E_{\text{gs}}^I - E_{\text{gs}}^F$$

- $Q_{\beta\beta,\text{th}}$  : calculated by using the IBM eigenenergy:  $E_{\text{gs}} = E_{\text{IBM}}(0_1^+) + E_0$
- $Q_{\beta\beta,\text{ex}}$  : experimental value

| Nucleus           | $Q_{\beta\beta,\text{th}}$ (MeV) | $Q_{\beta\beta,\text{ex}}$ (MeV) |
|-------------------|----------------------------------|----------------------------------|
| $^{48}\text{Ca}$  | 1.8479                           | 4.2681                           |
| $^{76}\text{Ge}$  | 0.8831                           | 2.0391                           |
| $^{82}\text{Se}$  | 1.6356                           | 2.9979                           |
| $^{96}\text{Zr}$  | 4.1285                           | 3.3560                           |
| $^{100}\text{Mo}$ | 2.8338                           | 3.0344                           |
| $^{110}\text{Pd}$ | 2.9081                           | 2.0171                           |
| $^{116}\text{Cd}$ | 6.1166                           | 2.8135                           |
| $^{124}\text{Sn}$ | -0.3795                          | 2.2927                           |
| $^{128}\text{Te}$ | -0.1784                          | 0.8680                           |
| $^{130}\text{Te}$ | 1.4466                           | 2.5290                           |
| $^{136}\text{Xe}$ | 0.0989                           | 2.4579                           |
| $^{150}\text{Nd}$ | 3.3123                           | 3.3714                           |
| $^{198}\text{Pt}$ | 1.2895                           | 1.0503                           |

... 1~2 MeV difference

# GT and Fermi matrix elements

| Decay                                         | $0_1^+$                    |                            |                            |                            | $0_2^+$                    |                            |                            |                            |
|-----------------------------------------------|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|
|                                               | $M_{2\nu}^{\text{GT}}$     |                            | $M_{2\nu}^{\text{F}}$      |                            | $M_{2\nu}^{\text{GT}}$     |                            | $M_{2\nu}^{\text{F}}$      |                            |
|                                               | $Q_{\beta\beta,\text{th}}$ | $Q_{\beta\beta,\text{ex}}$ | $Q_{\beta\beta,\text{th}}$ | $Q_{\beta\beta,\text{ex}}$ | $Q_{\beta\beta,\text{th}}$ | $Q_{\beta\beta,\text{ex}}$ | $Q_{\beta\beta,\text{th}}$ | $Q_{\beta\beta,\text{ex}}$ |
| $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$   | 0.060                      | 0.042                      | 0.024                      | 0.016                      | 0.325                      | 0.066                      | -0.142                     | -0.075                     |
| $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$   | 0.040                      | 0.034                      | -0.007                     | -0.007                     | 0.097                      | 0.078                      | -0.085                     | -0.069                     |
| $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$   | -0.060                     | -0.045                     | 0.017                      | 0.015                      | 0.124                      | 0.070                      | -0.081                     | -0.064                     |
| $^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$   | 0.139                      | 0.154                      | -0.001                     | -0.001                     | 0.053                      | 0.063                      | -0.000                     | -0.000                     |
| $^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$ | 0.513                      | 0.483                      | -0.000                     | -0.000                     | -0.007                     | -0.020                     | 0.000                      | 0.000                      |
| $^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$ | 0.071                      | 0.080                      | 0.000                      | 0.000                      | -0.052                     | -0.062                     | 0.000                      | 0.000                      |
| $^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$ | 0.148                      | 0.275                      | 0.001                      | 0.001                      | 0.032                      | -0.037                     | -0.001                     | -0.001                     |
| $^{124}\text{Sn} \rightarrow ^{124}\text{Te}$ | 0.123                      | 0.074                      | -0.054                     | -0.045                     | 0.345                      | -0.066                     | 0.016                      | 0.012                      |
| $^{128}\text{Te} \rightarrow ^{128}\text{Xe}$ | -0.139                     | -0.102                     | 0.006                      | 0.005                      | 0.108                      | 0.032                      | 0.002                      | 0.002                      |
| $^{130}\text{Te} \rightarrow ^{130}\text{Xe}$ | -0.041                     | -0.037                     | 0.025                      | 0.022                      | 0.043                      | 0.037                      | -0.019                     | -0.017                     |
| $^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$ | -0.173                     | -0.102                     | 0.028                      | 0.028                      | -2.807                     | 0.010                      | 0.009                      | 0.001                      |
| $^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$ | -0.375                     | -0.369                     | 0.000                      | 0.000                      | -0.414                     | -0.390                     | -0.000                     | -0.000                     |
| $^{198}\text{Pt} \rightarrow ^{198}\text{Hg}$ | -0.016                     | -0.016                     | 0.001                      | 0.001                      | -0.008                     | -0.010                     | -0.000                     | -0.000                     |

Results are similar between  $Q_{\beta\beta,\text{th}}$  and  $Q_{\beta\beta,\text{ex}}$  for the  $0_1^+ \rightarrow 0_1^+$  decay, and significantly different for the  $0_1^+ \rightarrow 0_2^+$  decay

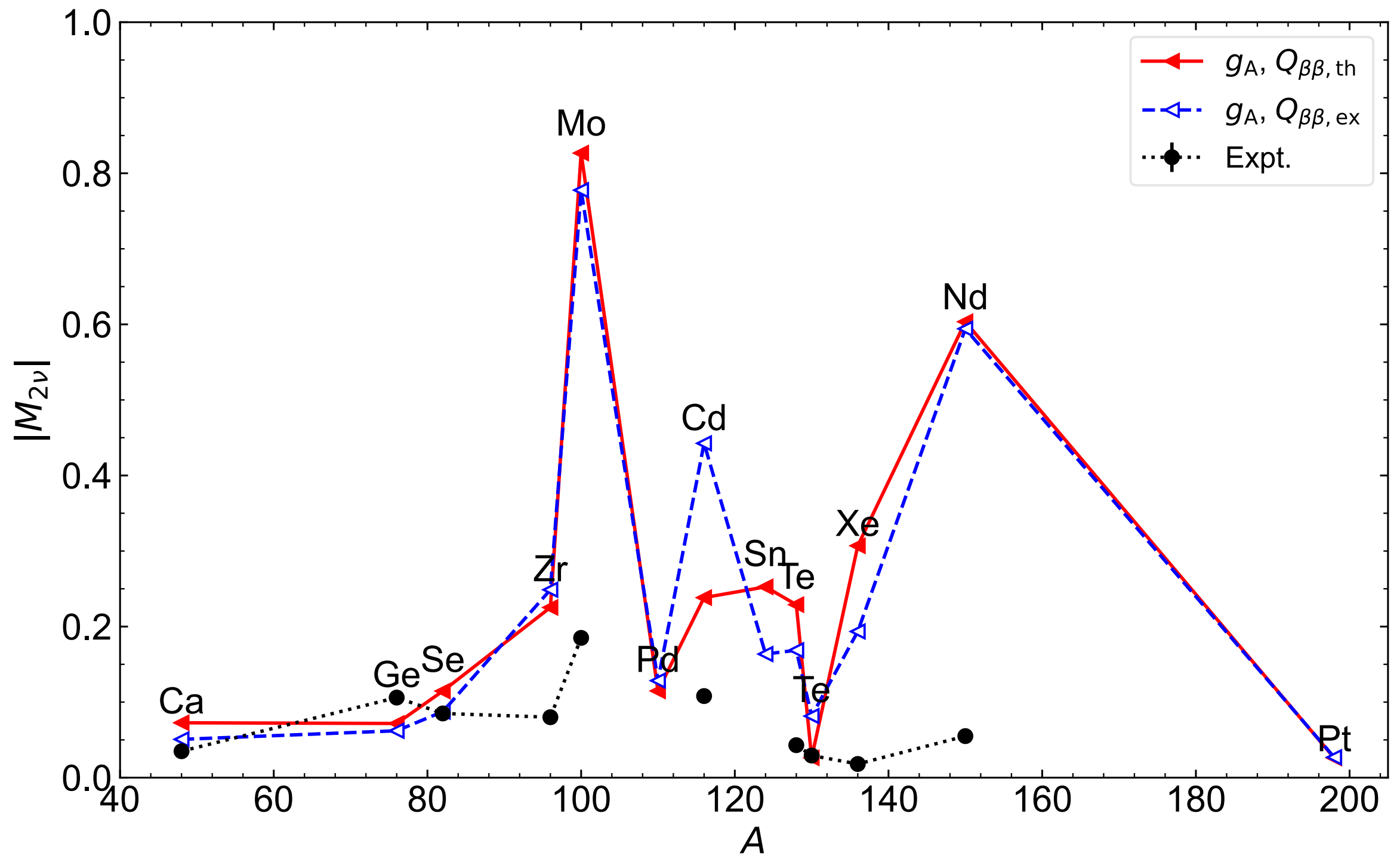
# Isospin symmetry breaking

$$\chi_F(0^+) = M_{2\nu}^F / M_{2\nu}^{\text{GT}}$$

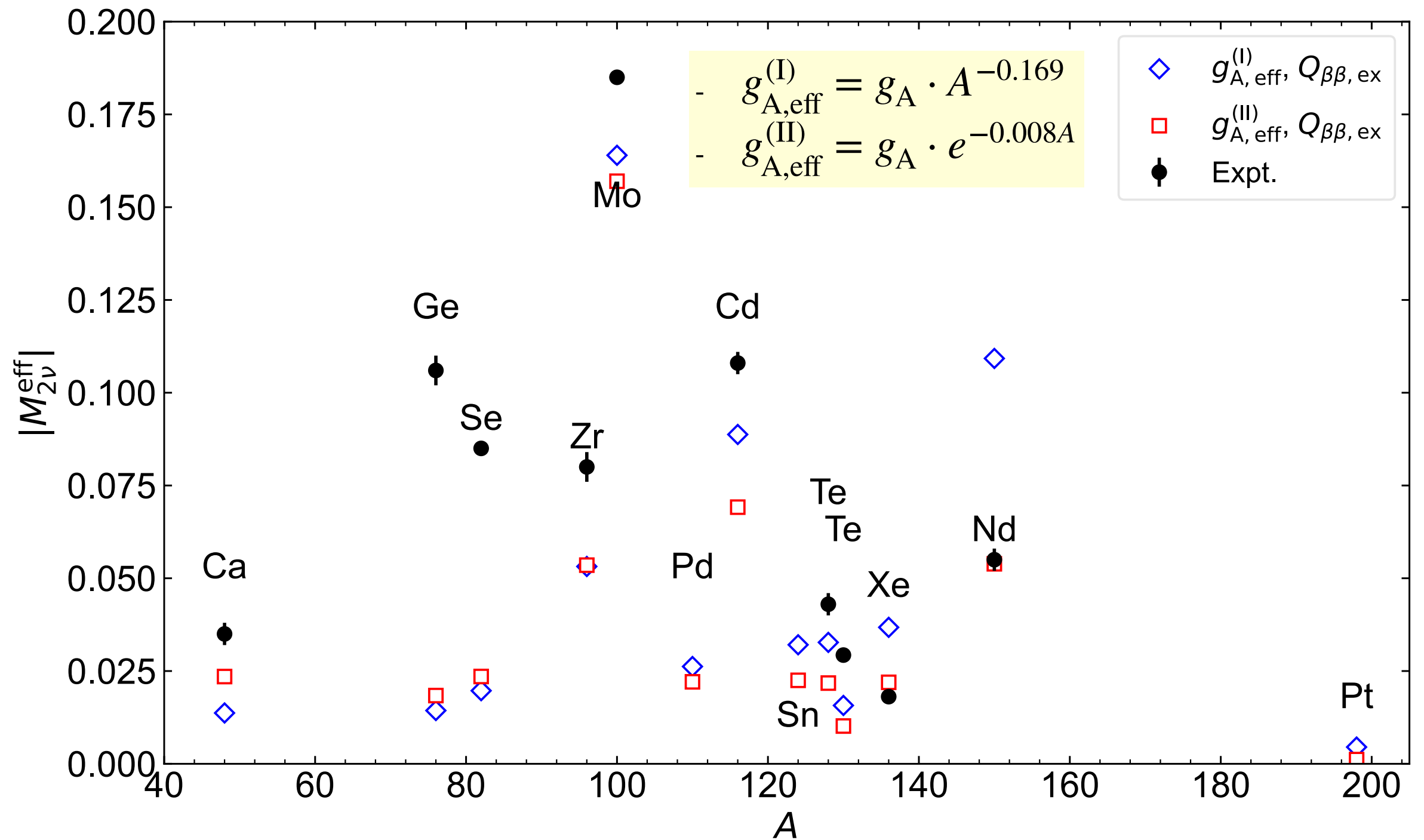
... appears when the protons and neutrons are in the same shells

| Nucleus           | $Q_{\beta\beta,\text{th}}$ |                 |
|-------------------|----------------------------|-----------------|
|                   | $\chi_F(0_1^+)$            | $\chi_F(0_2^+)$ |
| <sup>48</sup> Ca  | 0.401                      | −0.435          |
| <sup>76</sup> Ge  | −0.179                     | −0.874          |
| <sup>82</sup> Se  | −0.287                     | −0.650          |
| <sup>96</sup> Zr  | −0.006                     | −0.003          |
| <sup>100</sup> Mo | −0.000                     | −0.005          |
| <sup>110</sup> Pd | 0.000                      | −0.000          |
| <sup>116</sup> Cd | 0.004                      | −0.021          |
| <sup>124</sup> Sn | −0.443                     | 0.046           |
| <sup>128</sup> Te | −0.040                     | 0.016           |
| <sup>130</sup> Te | −0.104                     | −0.093          |
| <sup>136</sup> Xe | −0.163                     | −0.003          |
| <sup>150</sup> Nd | −0.001                     | 0.000           |
| <sup>198</sup> Pt | −0.060                     | 0.036           |

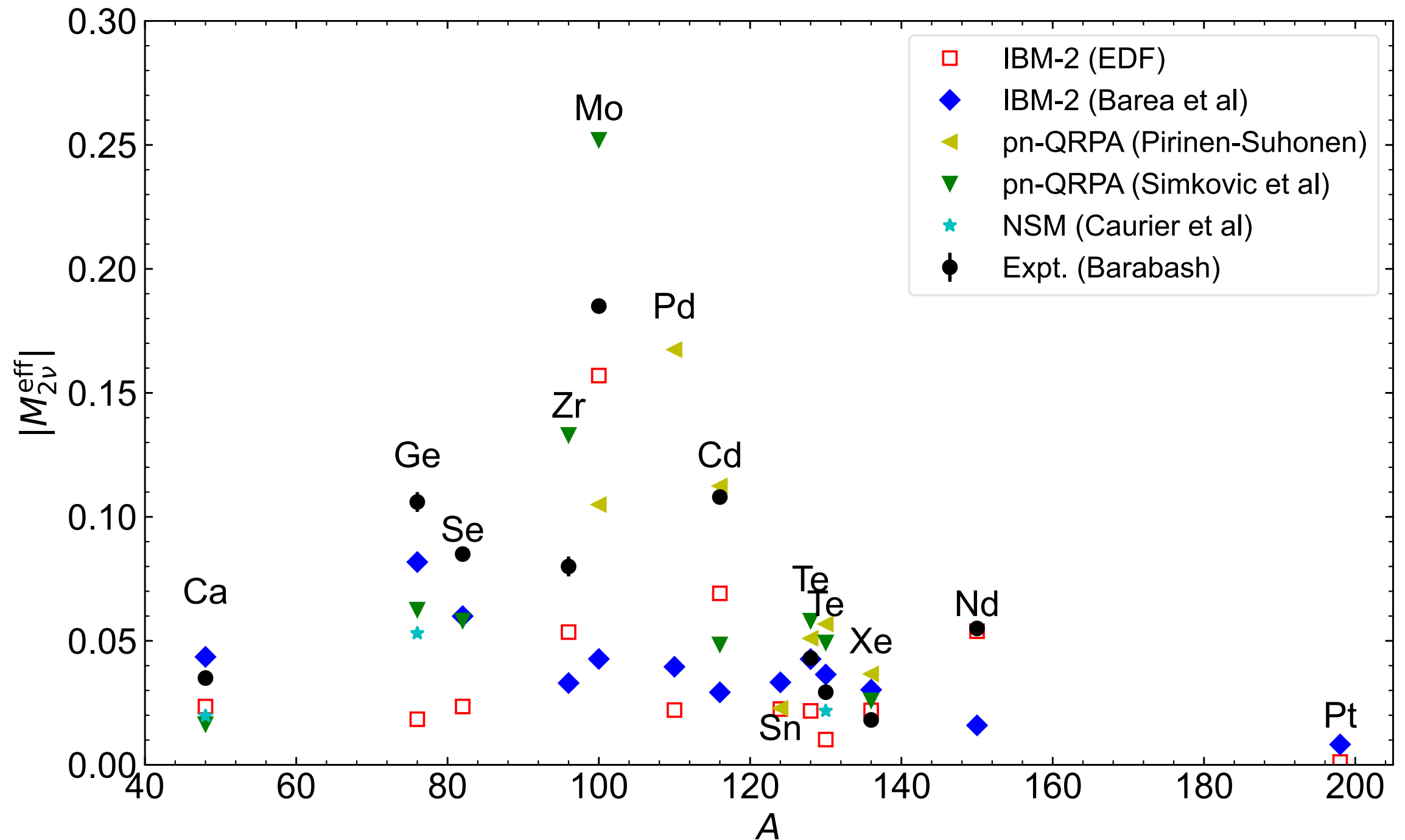
# NMEs



# Effective NMEs



# Comparison with other predictions



# Half-lives

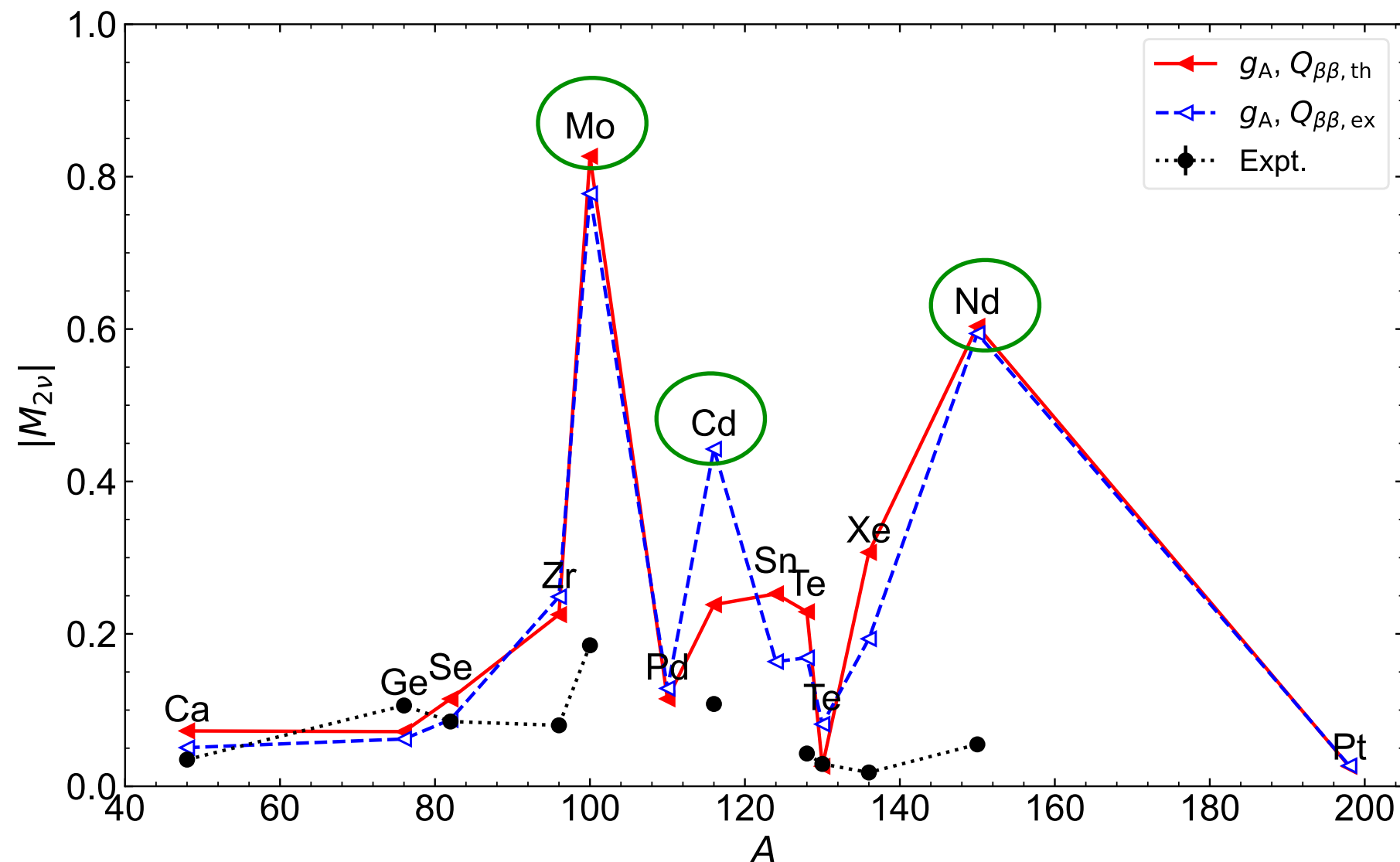
| Decay                                                | $\tau_{1/2}^{(2\nu)}$ (yr), with $Q_{\beta\beta,ex}$ |                       |                       |                                         |
|------------------------------------------------------|------------------------------------------------------|-----------------------|-----------------------|-----------------------------------------|
|                                                      | $g_A$                                                | $g_{A,eff}^{(I)}$     | $g_{A,eff}^{(II)}$    | Expt. [12]                              |
| $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$          | $2.50 \times 10^{19}$                                | $3.43 \times 10^{20}$ | $1.16 \times 10^{20}$ | $5.3_{-0.8}^{+1.2} \times 10^{19}$      |
| $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$          | $5.39 \times 10^{21}$                                | $1.01 \times 10^{23}$ | $6.14 \times 10^{22}$ | $(1.88 \pm 0.08) \times 10^{21}$        |
| $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$          | $8.20 \times 10^{19}$                                | $1.61 \times 10^{21}$ | $1.13 \times 10^{21}$ | $(0.87_{-0.01}^{+0.02}) \times 10^{20}$ |
| $^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$          | $2.37 \times 10^{18}$                                | $5.19 \times 10^{19}$ | $5.12 \times 10^{19}$ | $(2.3 \pm 0.2) \times 10^{19}$          |
| $^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$        | $5.00 \times 10^{17}$                                | $1.12 \times 10^{19}$ | $1.23 \times 10^{19}$ | $(7.06_{-0.13}^{+0.15}) \times 10^{18}$ |
| $^{100}\text{Mo} \rightarrow ^{100}\text{Ru}(0_2^+)$ | $1.59 \times 10^{22}$                                | $3.57 \times 10^{23}$ | $3.90 \times 10^{23}$ | $6.7_{-0.4}^{+0.5} \times 10^{20}$      |
| $^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$        | $4.40 \times 10^{20}$                                | $1.06 \times 10^{22}$ | $1.49 \times 10^{22}$ |                                         |
| $^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$        | $1.85 \times 10^{18}$                                | $4.59 \times 10^{19}$ | $7.56 \times 10^{19}$ | $(2.69 \pm 0.09) \times 10^{19}$        |
| $^{124}\text{Sn} \rightarrow ^{124}\text{Te}$        | $6.76 \times 10^{19}$                                | $1.76 \times 10^{21}$ | $3.57 \times 10^{21}$ |                                         |
| $^{128}\text{Te} \rightarrow ^{128}\text{Xe}$        | $1.31 \times 10^{23}$                                | $3.48 \times 10^{24}$ | $7.86 \times 10^{24}$ | $(2.25 \pm 0.09) \times 10^{24}$        |
| $^{130}\text{Te} \rightarrow ^{130}\text{Xe}$        | $9.85 \times 10^{19}$                                | $2.65 \times 10^{21}$ | $6.31 \times 10^{21}$ | $(7.91 \pm 0.21) \times 10^{20}$        |
| $^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$        | $1.86 \times 10^{19}$                                | $5.16 \times 10^{20}$ | $1.45 \times 10^{21}$ | $(2.18 \pm 0.05) \times 10^{21}$        |
| $^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$        | $7.78 \times 10^{16}$                                | $2.30 \times 10^{18}$ | $9.45 \times 10^{18}$ | $(9.34 \pm 0.65) \times 10^{18}$        |
| $^{150}\text{Nd} \rightarrow ^{150}\text{Sm}(0_2^+)$ | $5.84 \times 10^{17}$                                | $1.73 \times 10^{19}$ | $7.10 \times 10^{19}$ | $1.2_{-0.2}^{+0.3} \times 10^{20}$      |
| $^{198}\text{Pt} \rightarrow ^{198}\text{Hg}$        | $8.95 \times 10^{22}$                                | $3.20 \times 10^{24}$ | $5.09 \times 10^{25}$ |                                         |

$$[\tau_{1/2}^{(2\nu)}]^{-1} = G_{2\nu} |M_{2\nu}|^2$$

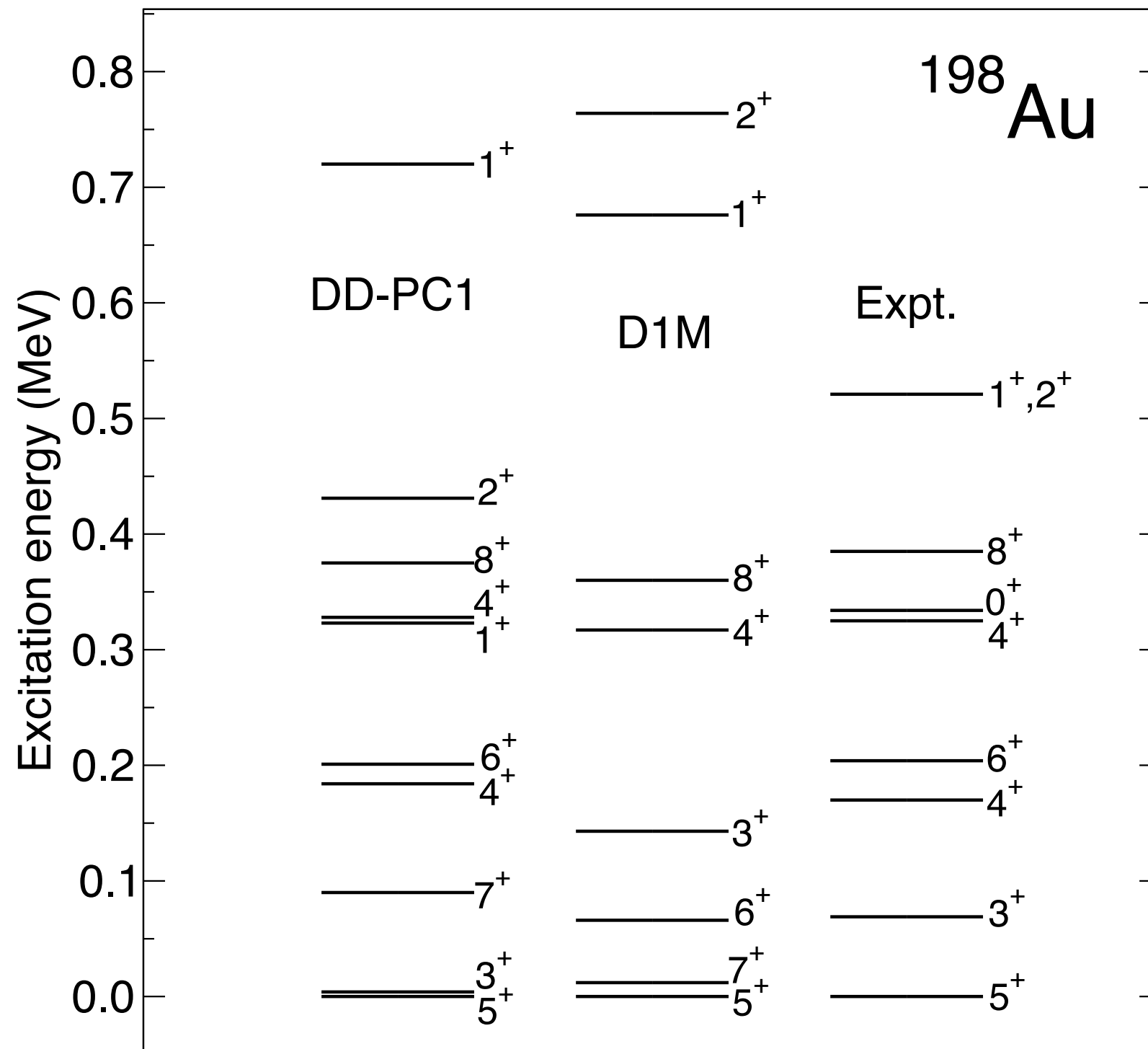
Phase-space factor: Kotila-Iachello (2012)  
Experiment: Barabash, Universe (2020)

# Source of uncertainties

- SCMF: choice of the EDF, pairing properties, ... etc.
- IBM/IBFFM: Hamiltonian, model space, ... etc.



# Sensitivity to the EDFs



... from the mapped IBM using the Gogny-D1M EDF.

# Sensitivity to the EDFs

| EDF    | $0_1^+$                |                       |              | $0_2^+$                |                       |              |
|--------|------------------------|-----------------------|--------------|------------------------|-----------------------|--------------|
|        | $M_{2\nu}^{\text{GT}}$ | $M_{2\nu}^{\text{F}}$ | $ M_{2\nu} $ | $M_{2\nu}^{\text{GT}}$ | $M_{2\nu}^{\text{F}}$ | $ M_{2\nu} $ |
| DD-PC1 | -0.016                 | 0.001                 | 0.026        | -0.008                 | -0.000                | 0.012        |
| D1M    | -0.074                 | 0.000                 | 0.120        | -0.034                 | -0.000                | 0.054        |

larger  $|M_{2\nu}|$  with Gogny EDF

# Pairing interaction in the SCMF

... separable pairing force of finite range

Tian, Ma, Ring (2009)

$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}'_1, \mathbf{r}'_2) = -V \delta(\mathbf{R} - \mathbf{R}') P(\mathbf{r}) P(\mathbf{r}') \frac{1}{2} (1 - P^\sigma),$$

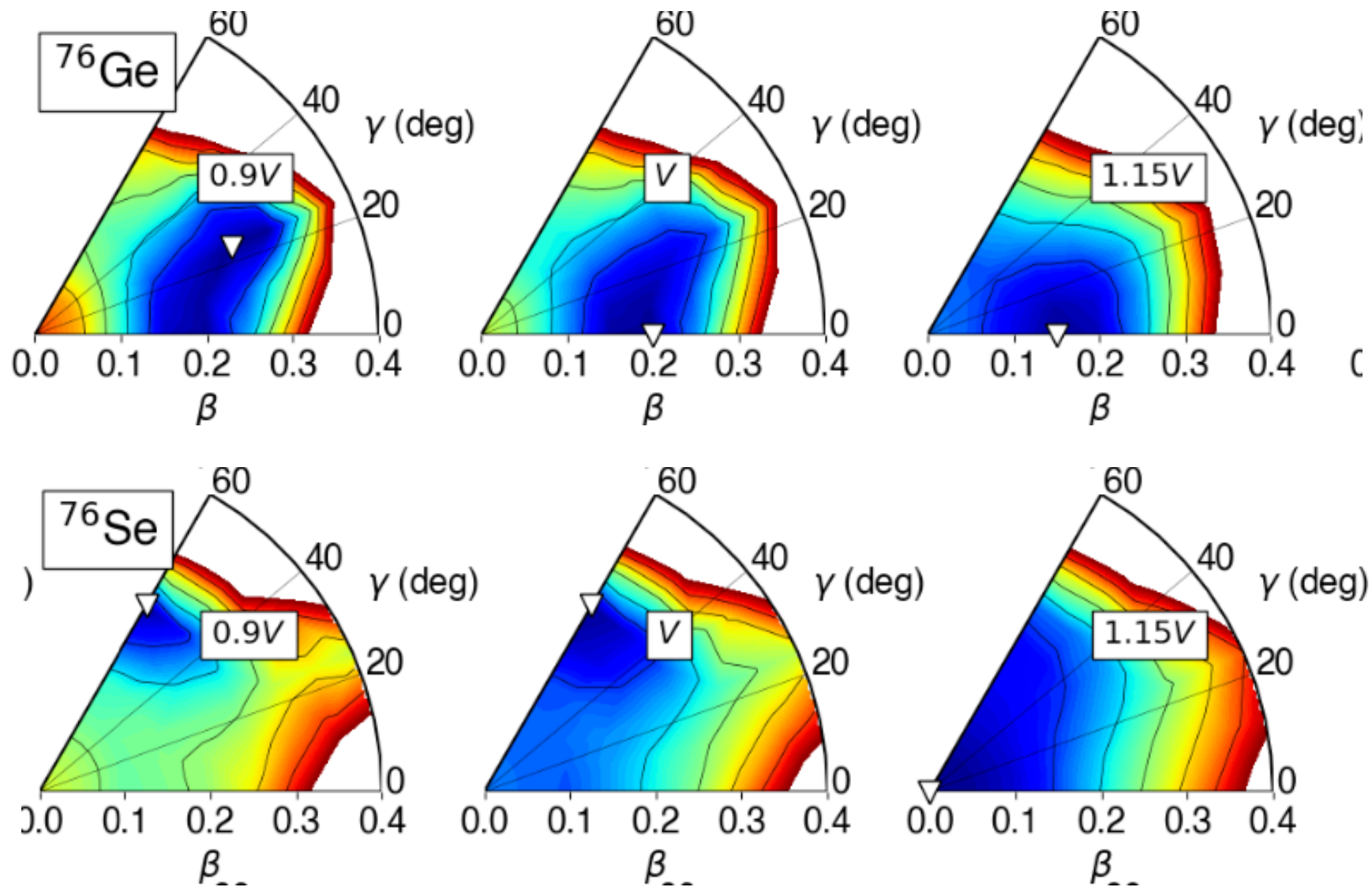
$$P(\mathbf{r}) = \frac{1}{(4\pi a^2)^{3/2}} e^{-\mathbf{r}^2/4a^2}$$

... strength  $V=728 \text{ MeV fm}^3$  fit to Gogny D1S pairing gap

compare results with the pairing strength

- 10 % reduced
- default
- 15 % enhanced

# Sensitivity to the pairing strength



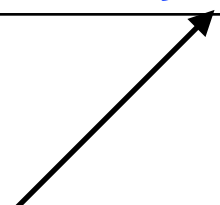
**increased pairing strength favors less pronounced deformation**

# Influence on IBM parameters

IBM-2  
Hamiltonian:

$$\hat{H}_{\text{IBM}} = \epsilon_d (\hat{n}_{d_\nu} + \hat{n}_{d_\pi}) + \kappa \hat{Q}_\nu \cdot \hat{Q}_\pi + \kappa' \hat{L} \cdot \hat{L}$$

pairing-like  
(spherical driving)

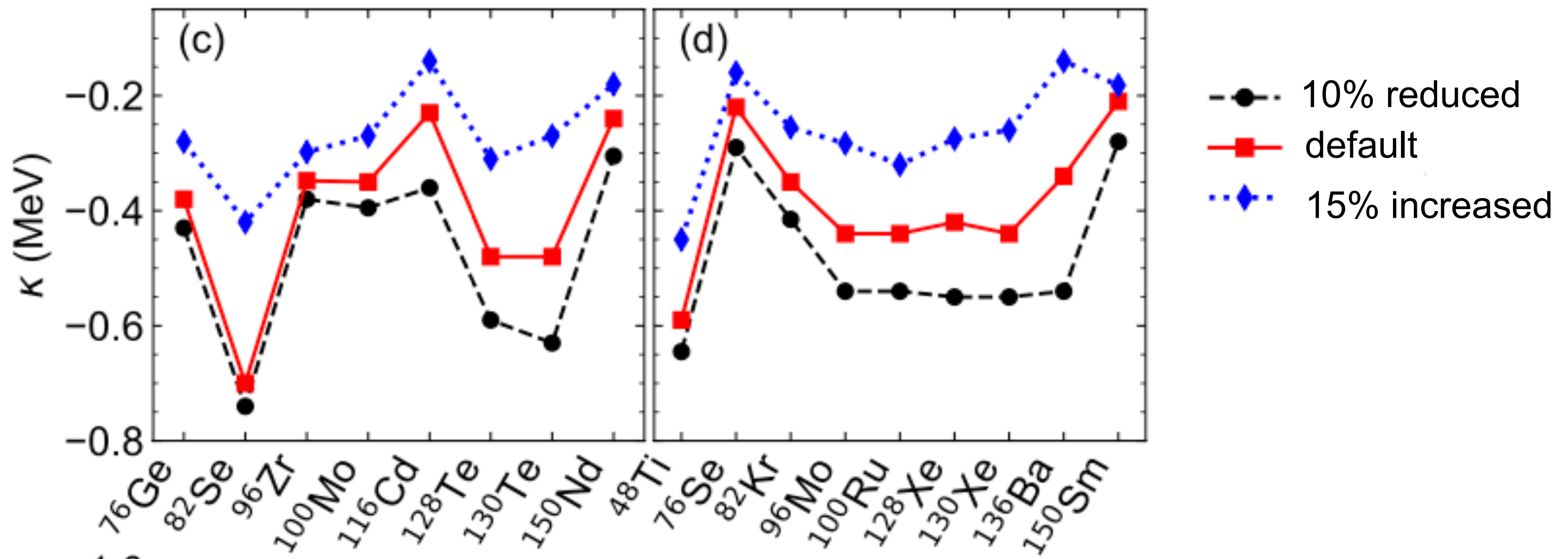


quadrupole-quadrupole  
(deformation driving)



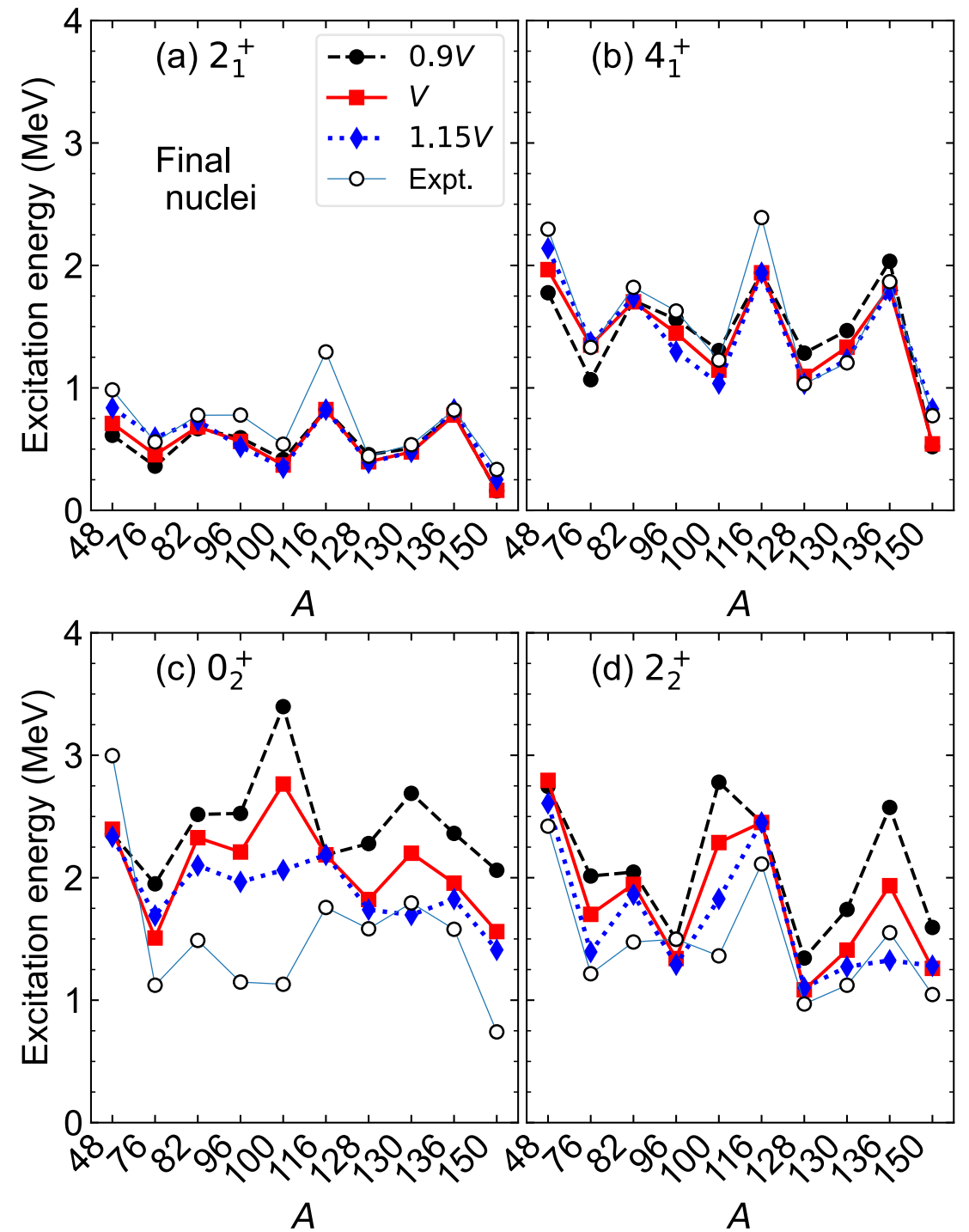
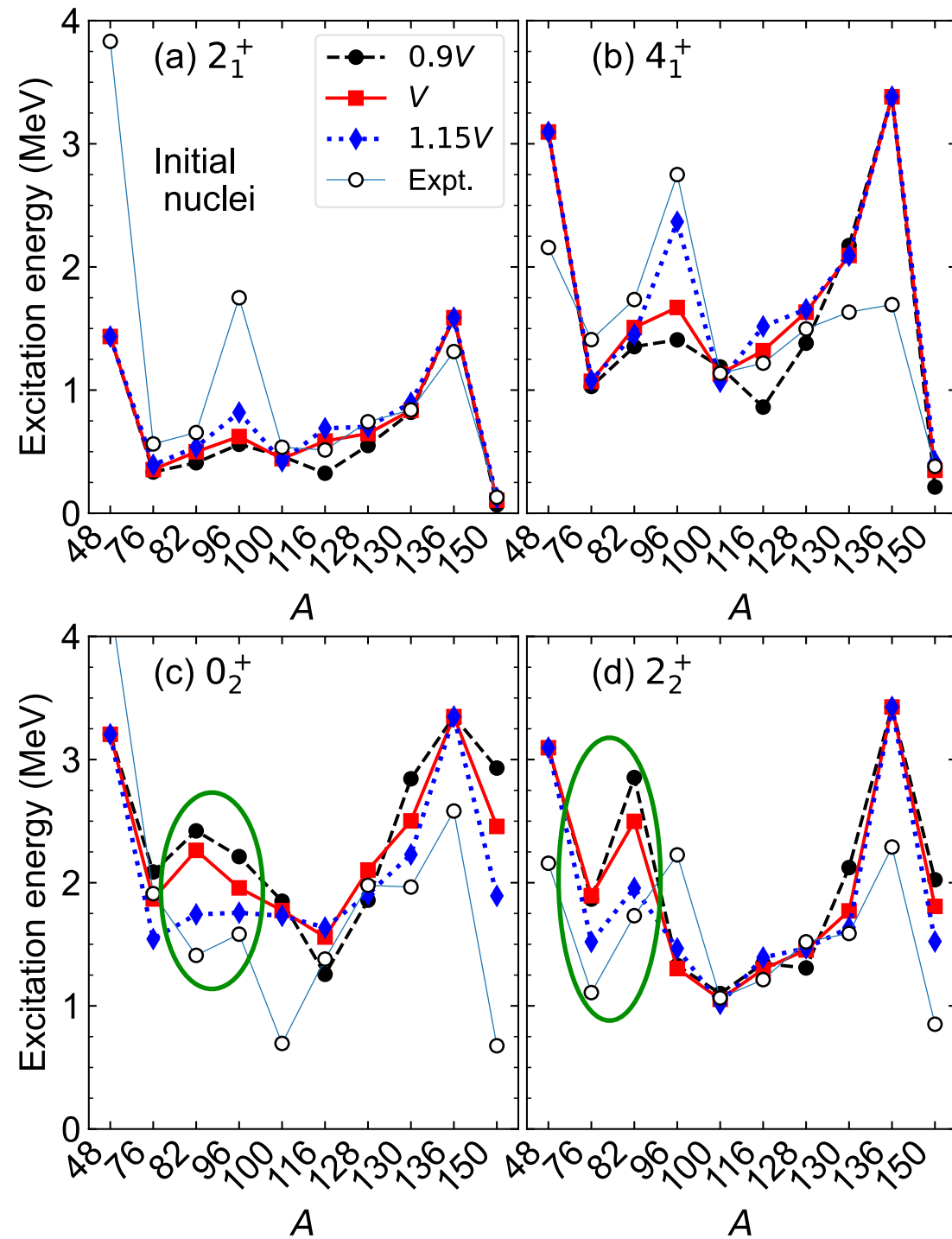
... disagreement with the  $0_2^+$  states is due to a too large QQ strength,  
as the SCMF PES exhibits a too pronounced deformation

# Derived QQ strength



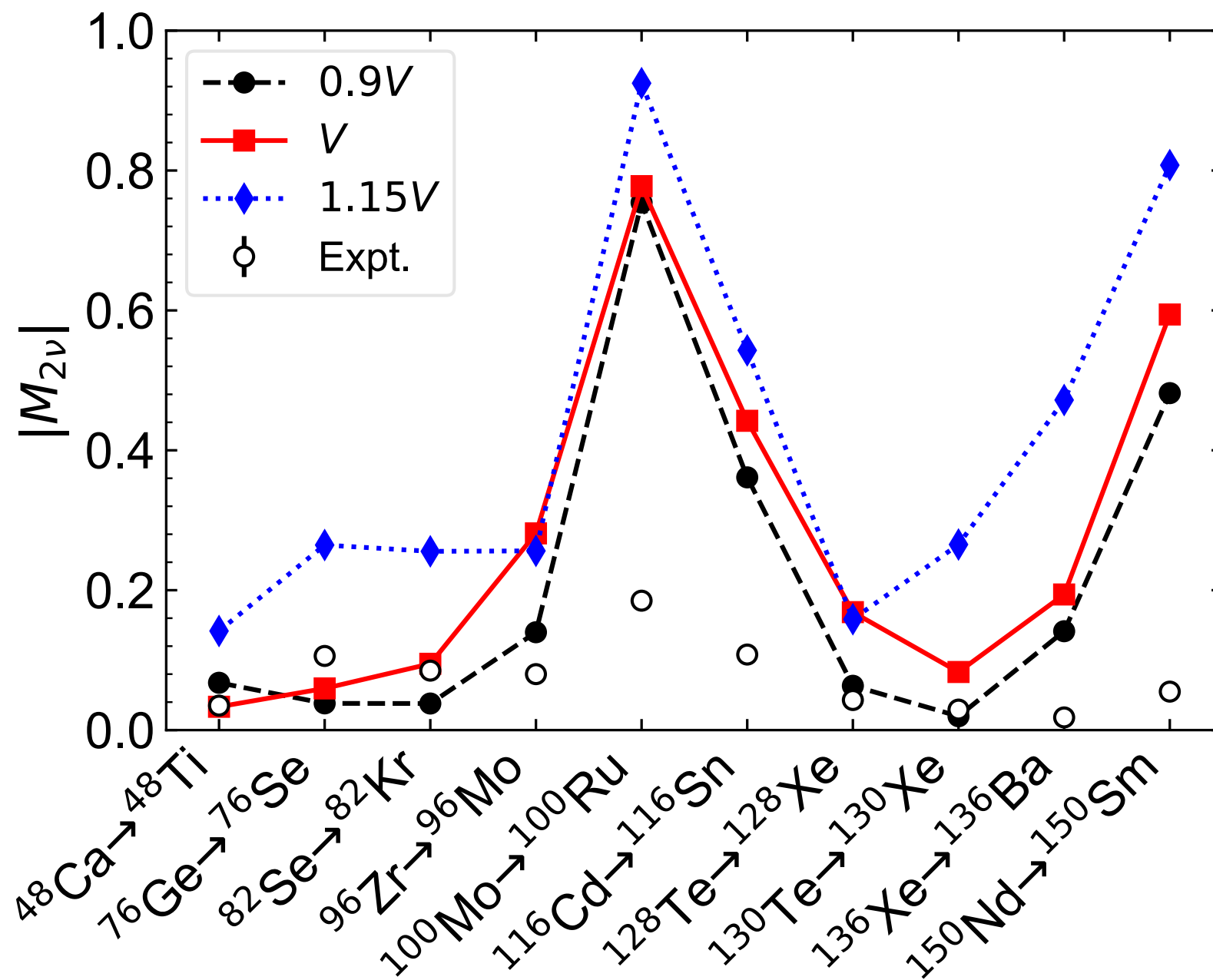
... QQ strength reduced with the enhanced pairing

# Impacts on energy spectra

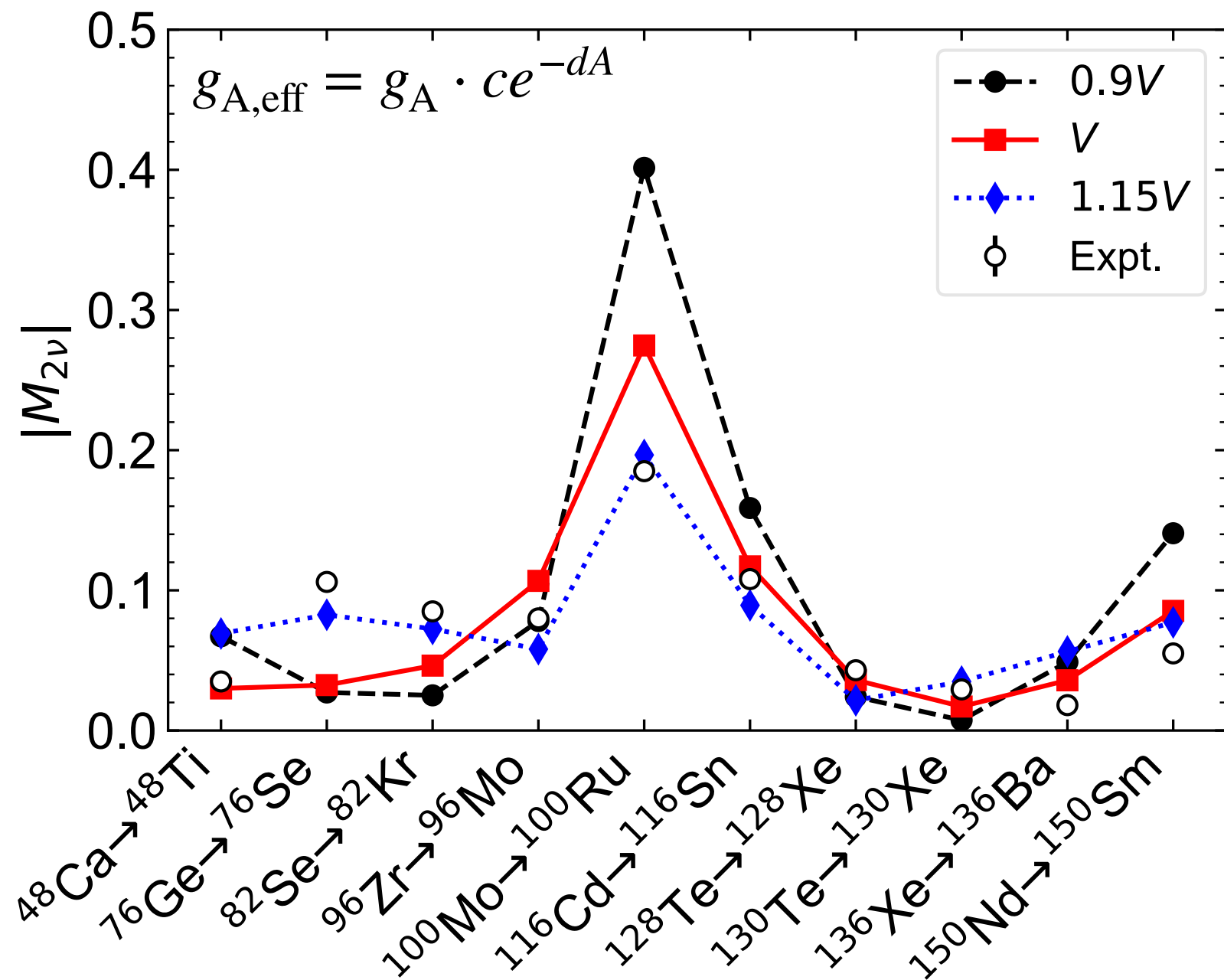


$0_2^+$  and  $2_2^+$  energy levels are lowered with the increased (+15%) pairing

# Unquenched NMEs

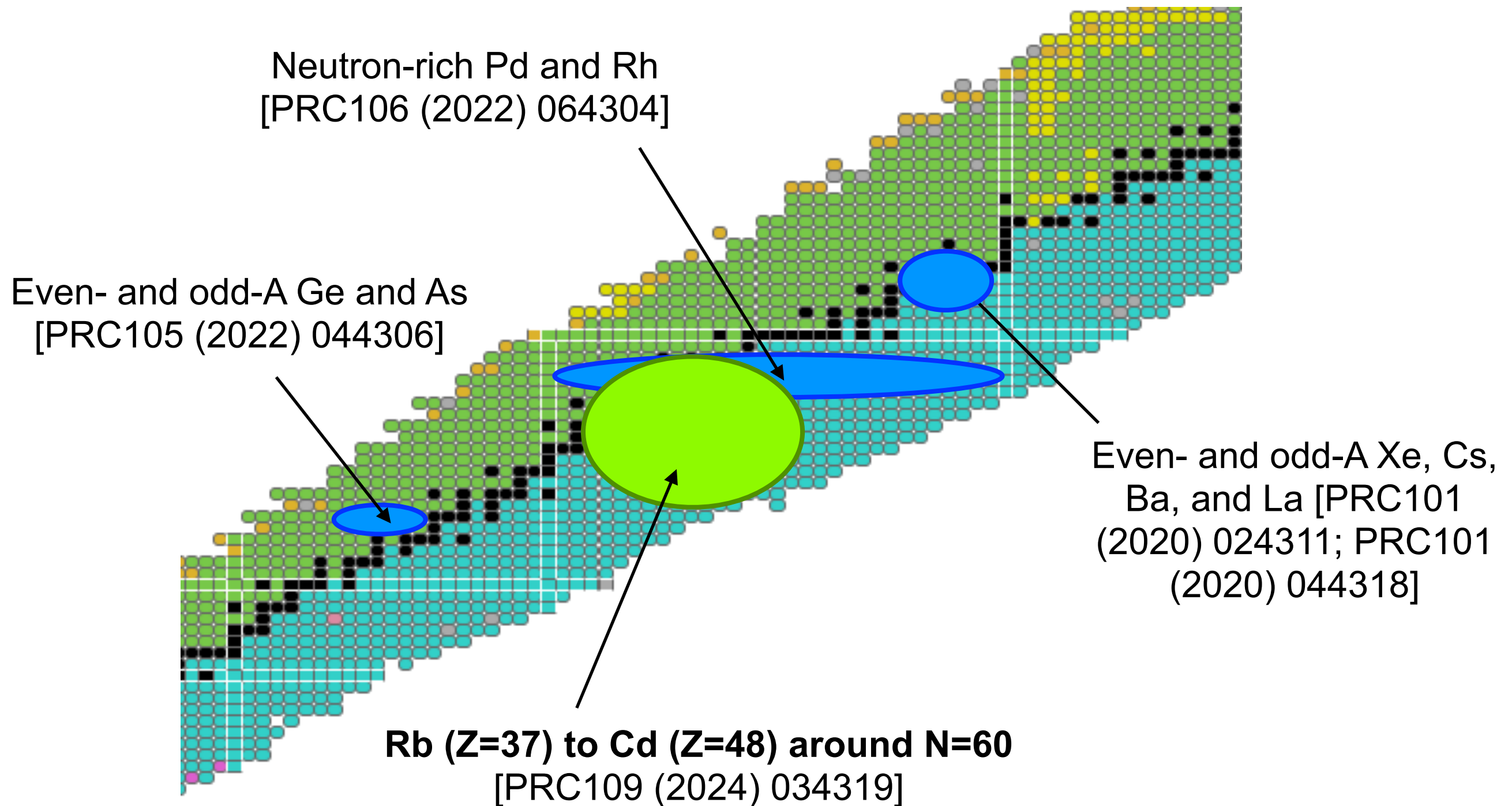


# Effective NMEs

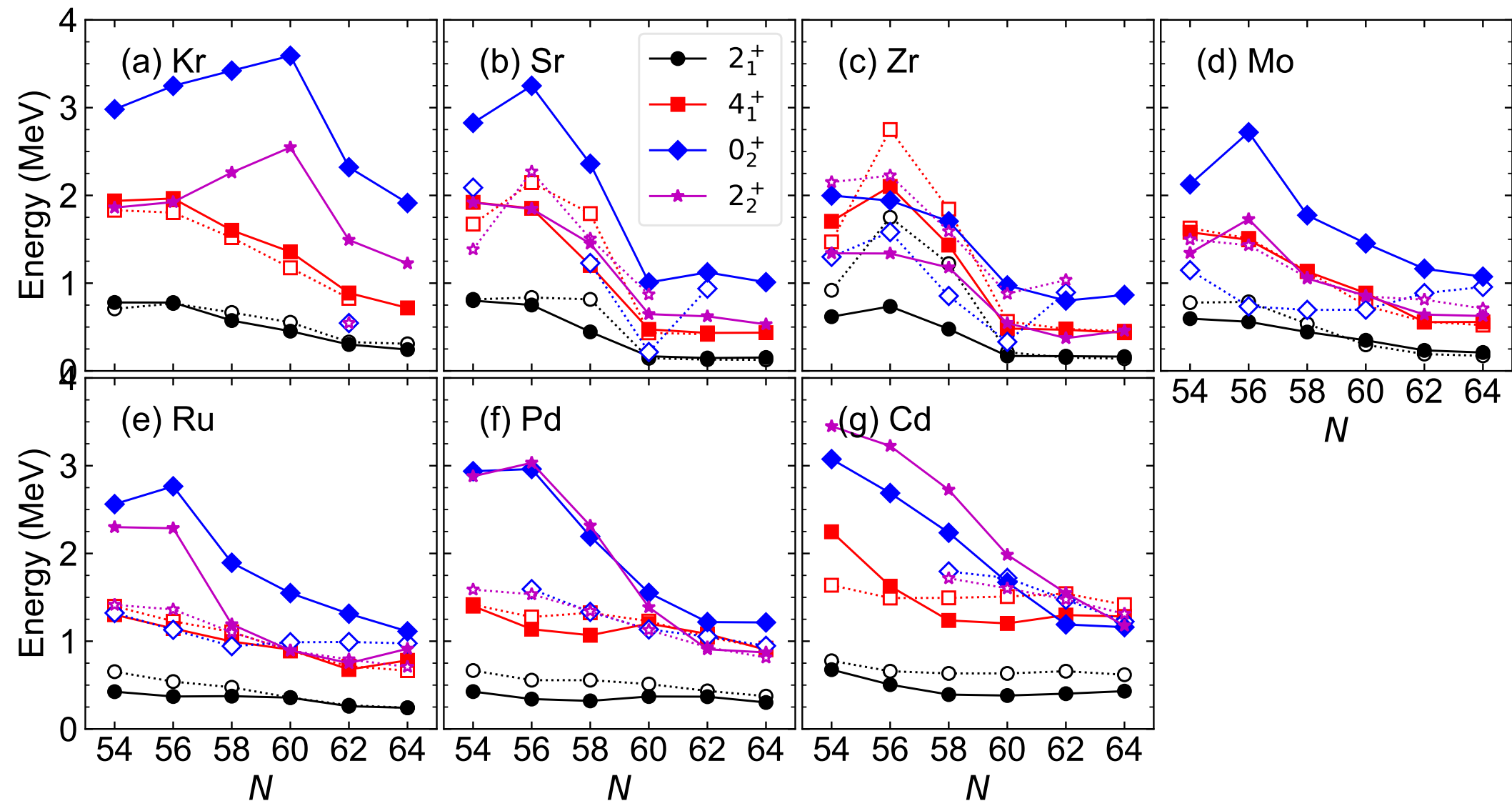


Single  $\beta$  decay

# Mapped IBM applied to $\beta$ decay

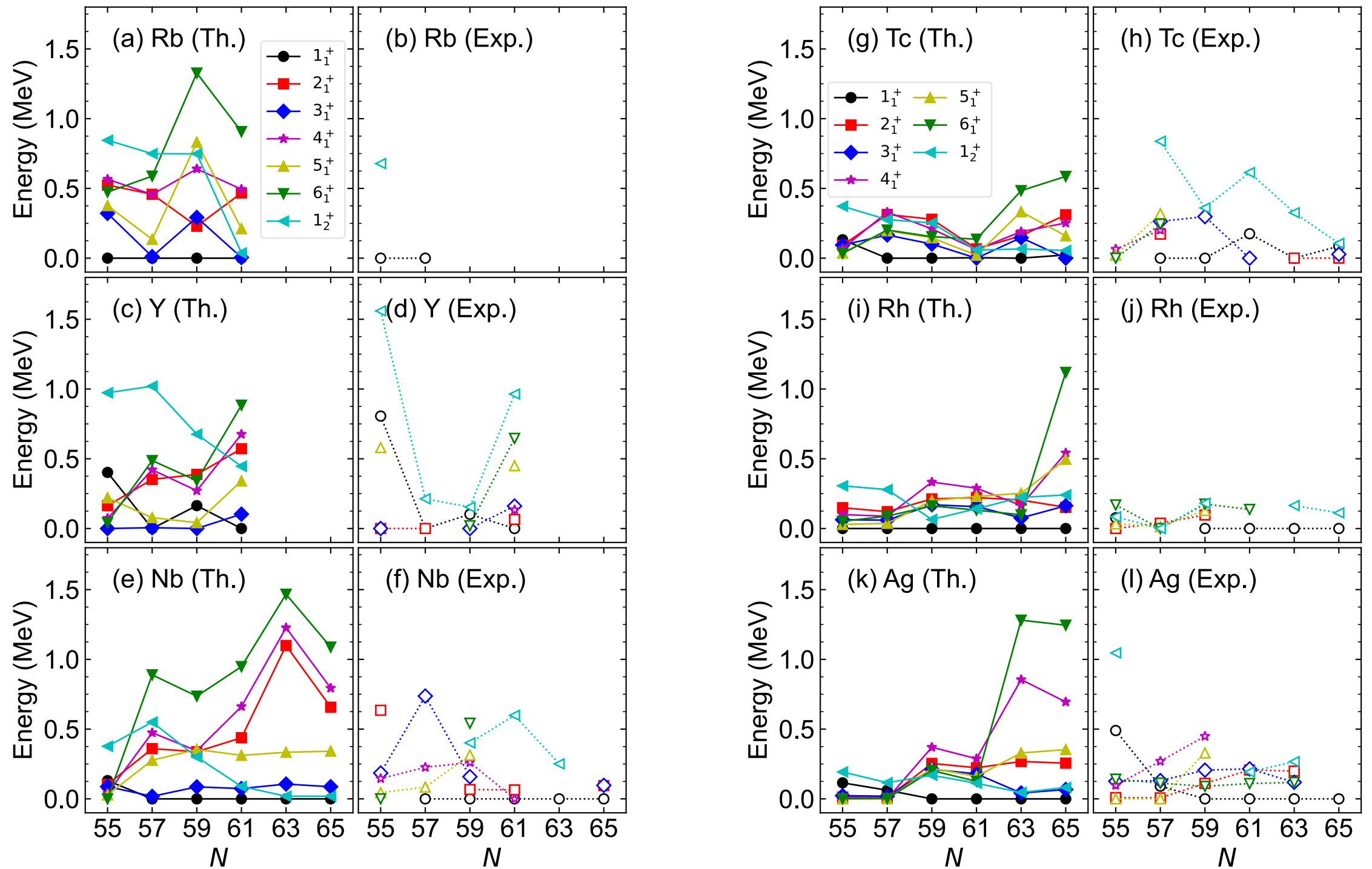


# Energy spectra of even-even nuclei

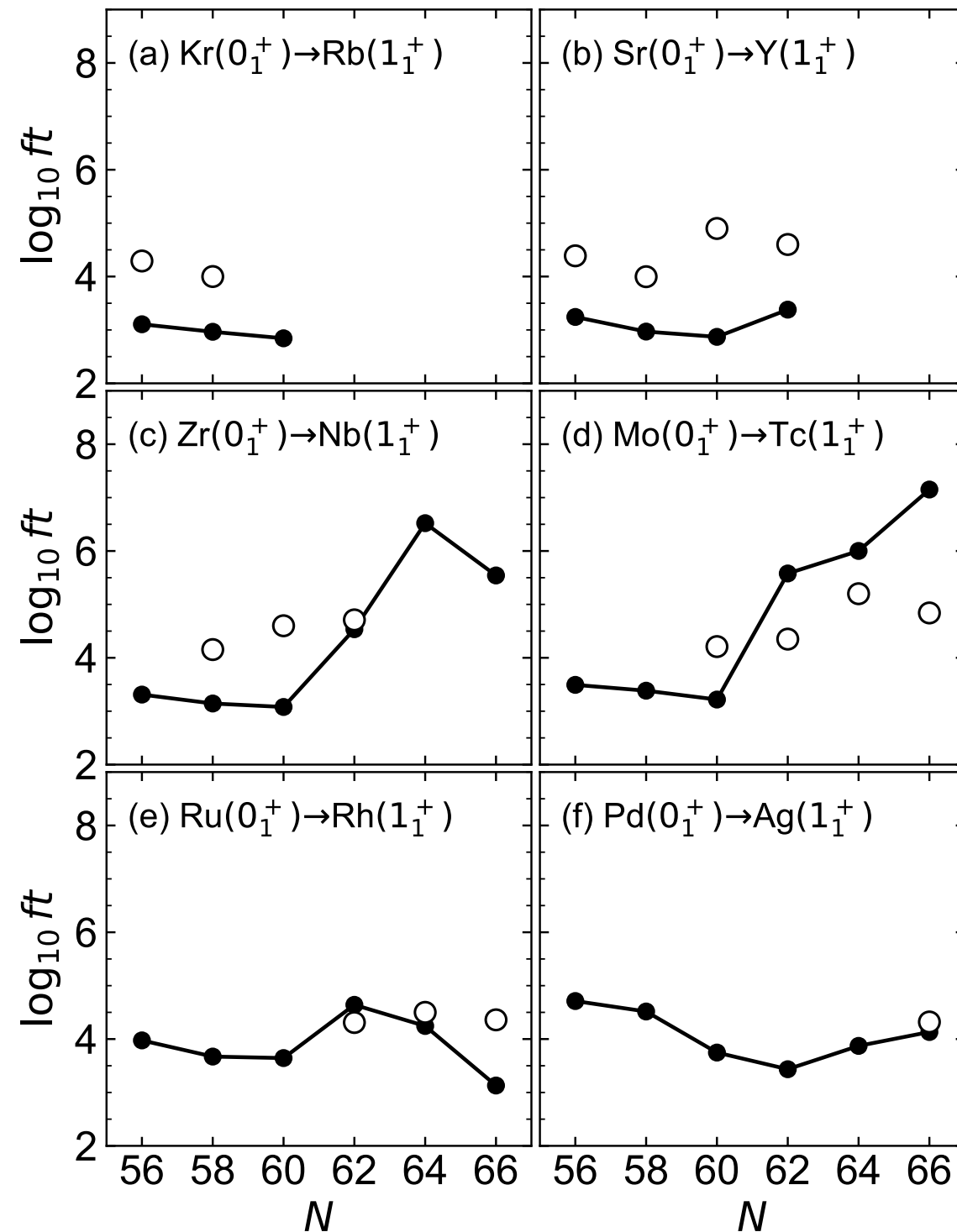


mapped IBM results based on the DD-PC1 EDF

# Energy spectra of odd-odd nuclei

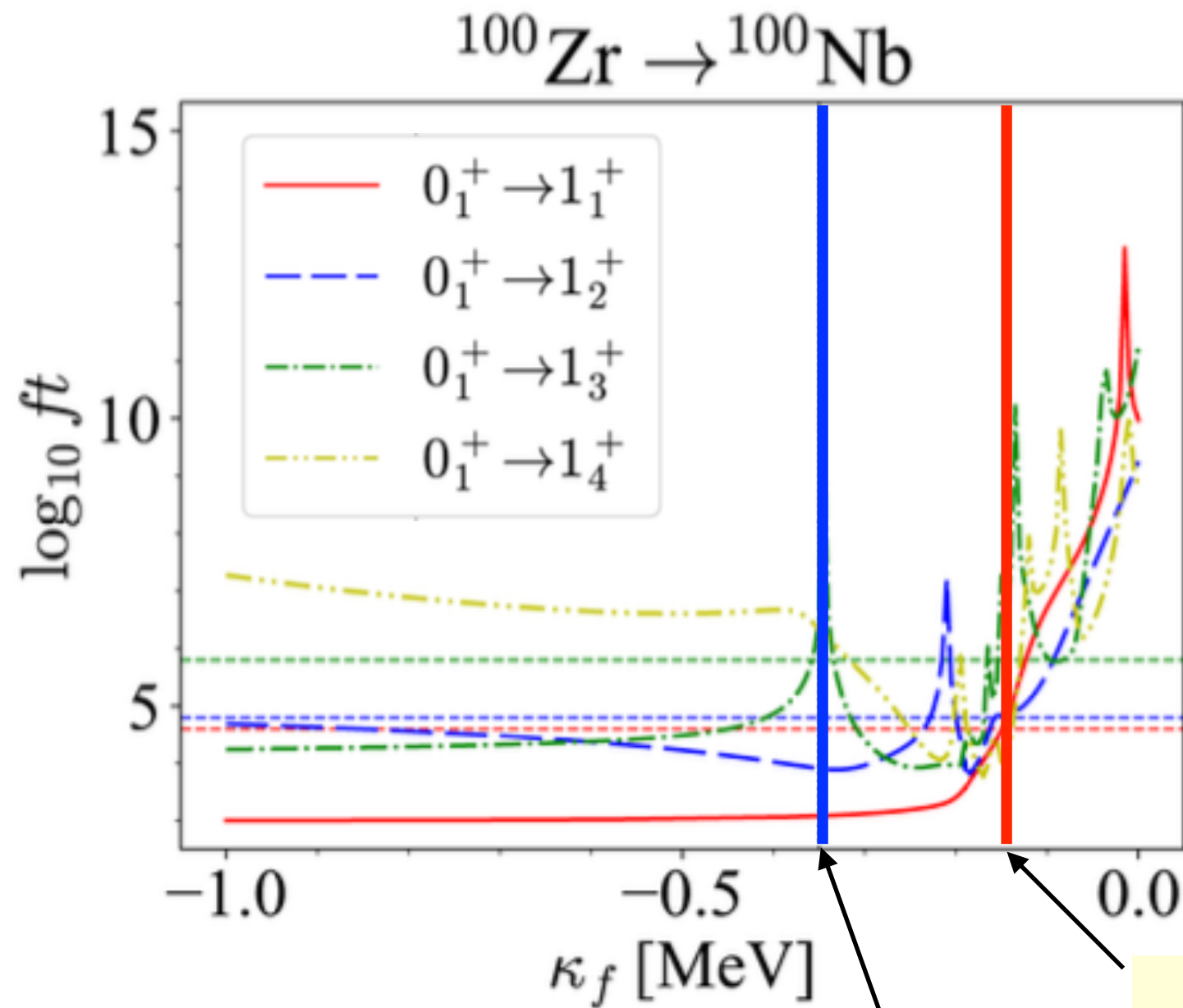


# $\beta^-$ decay $\log(ft)$



Significant change of  $\log(ft)$  around  $N=60$ , reflecting a shape phase transition

# $\log(ft)$ as a constraint to IBM-2



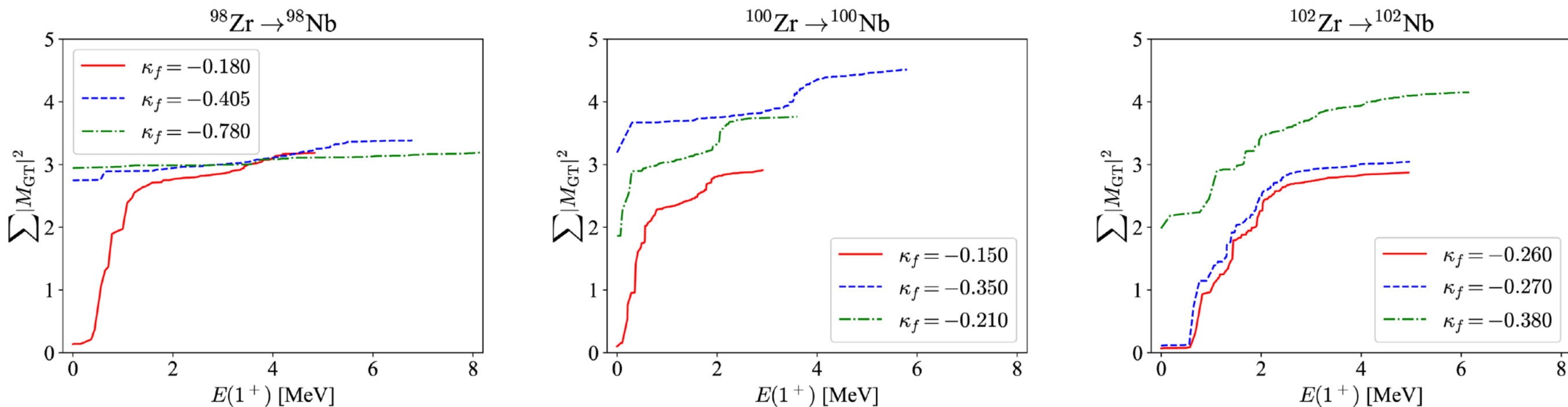
... QQ strength  $\kappa_f$  for the IBFFM-2 should be reduced by  $\sim 50\%$

derived value from the mapping

value required to reproduce experimental  $\log(ft)$

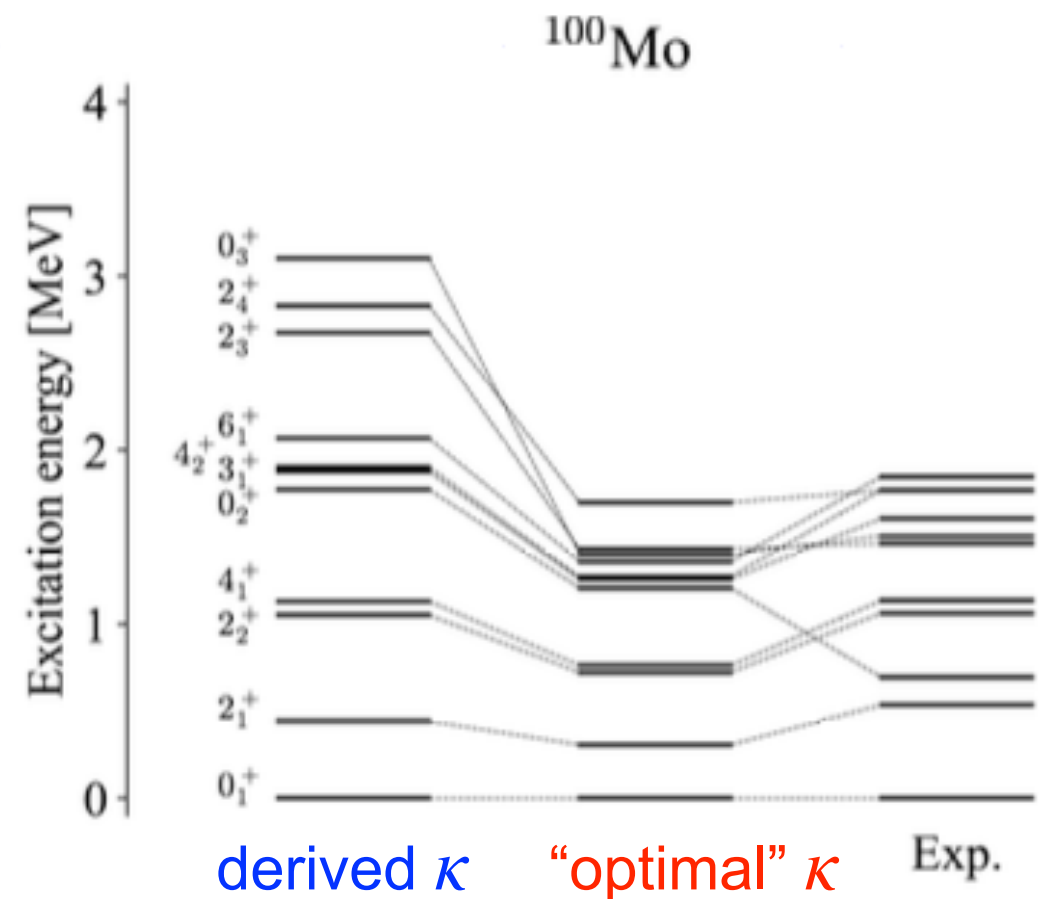
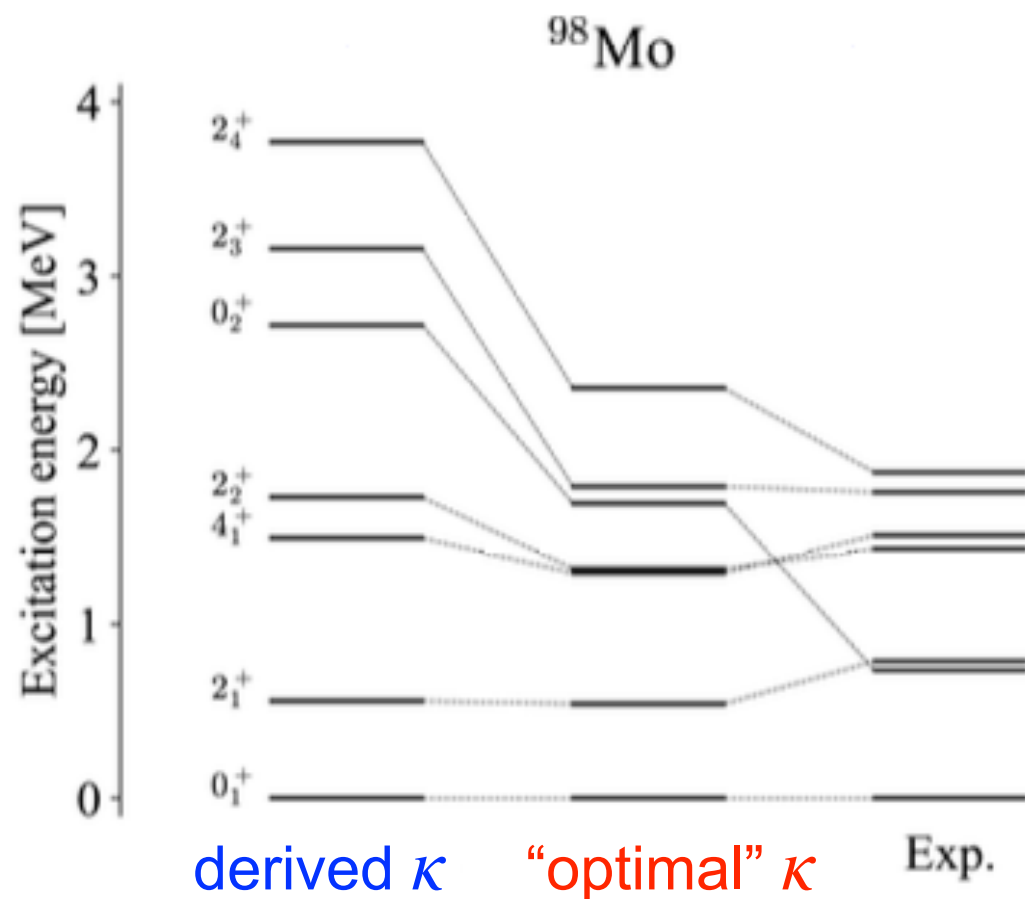
# GT running sums

— “optimal” QQ strength



... reduced QQ strength giving smaller GT matrix element for neutron-rich Zr

# Comparisons of spectra



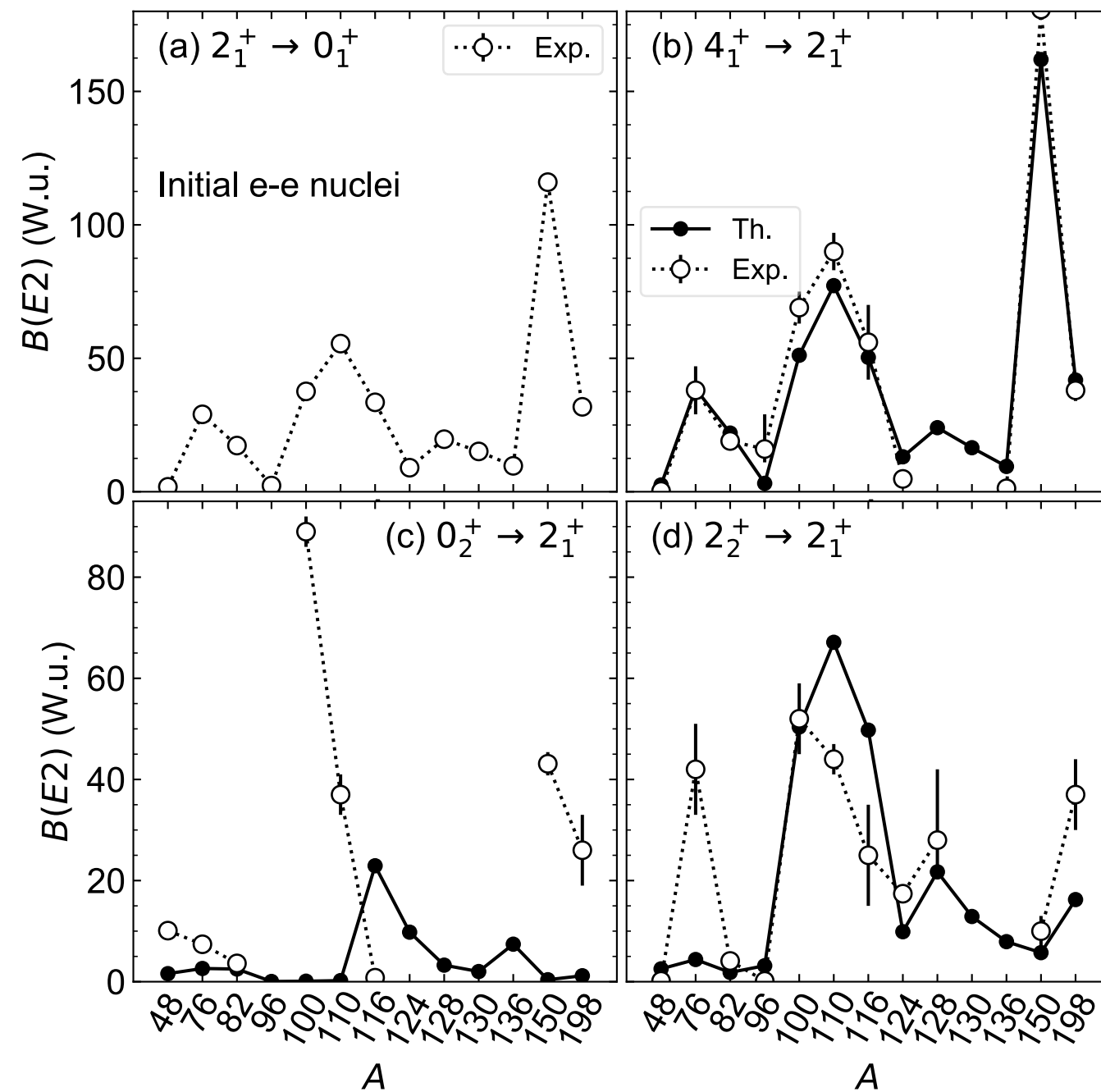
improvement over the mapped IBM-2 results for  $^A\text{Mo}$  ( $= ^A\text{Nb} + p - n$ )

# Summary

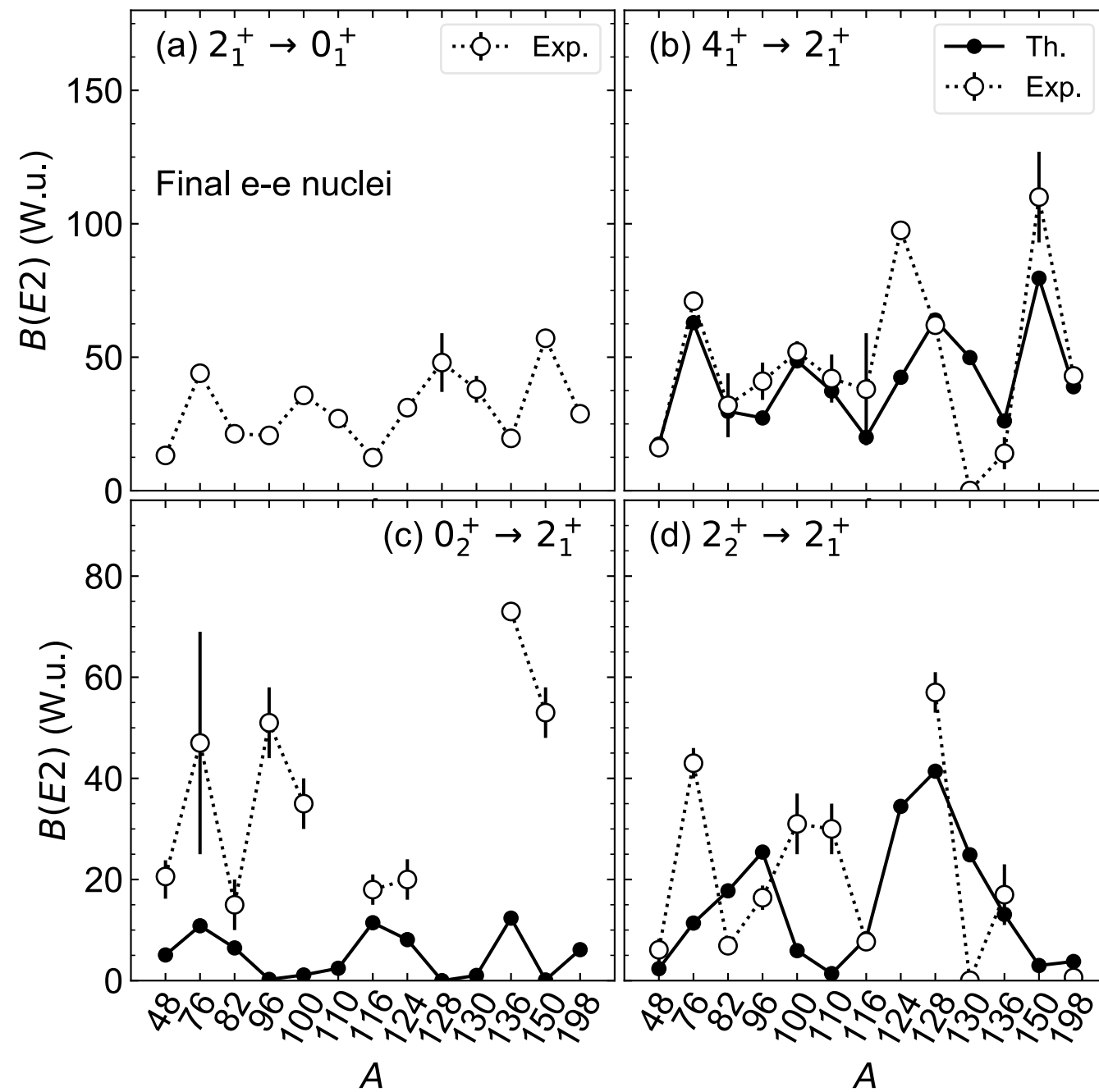
- Consistent description of low-lying states and  $2\nu\beta\beta$  NME
- Further improvements by assessing model deficiency, coupling to higher-order deformations, shape coexistence...?
- Extension to the  $0\nu$  mode is in progress

Thank you

# B(E2)s of parent nuclei



# B(E2)s of daughter nuclei



# NMEs

| Decay                                                | $\mathcal{Q}_{\beta\beta,\text{th}}$ |                           |                            | $\mathcal{Q}_{\beta\beta,\text{ex}}$ |                           |                            | $ M_{2\nu}^{\text{eff}} $ [12] |
|------------------------------------------------------|--------------------------------------|---------------------------|----------------------------|--------------------------------------|---------------------------|----------------------------|--------------------------------|
|                                                      | $ M_{2\nu} $                         | $ M_{2\nu}^{(\text{I})} $ | $ M_{2\nu}^{(\text{II})} $ | $ M_{2\nu} $                         | $ M_{2\nu}^{(\text{I})} $ | $ M_{2\nu}^{(\text{II})} $ |                                |
| $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$          | 0.073                                | 0.020                     | 0.034                      | 0.051                                | 0.014                     | 0.024                      | $0.035 \pm 0.003$              |
| $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$          | 0.072                                | 0.017                     | 0.021                      | 0.062                                | 0.014                     | 0.018                      | $0.106 \pm 0.004$              |
| $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$          | 0.115                                | 0.026                     | 0.031                      | 0.087                                | 0.020                     | 0.024                      | $0.085 \pm 0.001$              |
| $^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$          | 0.225                                | 0.048                     | 0.048                      | 0.249                                | 0.053                     | 0.054                      | $0.088 \pm 0.004$              |
| $^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$        | 0.827                                | 0.174                     | 0.167                      | 0.778                                | 0.164                     | 0.157                      | $0.185 \pm 0.002$              |
| $^{100}\text{Mo} \rightarrow ^{100}\text{Ru}(0_2^+)$ | 0.011                                | 0.002                     | 0.002                      | 0.032                                | 0.007                     | 0.007                      | $0.151 \pm 0.004$              |
| $^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$        | 0.115                                | 0.023                     | 0.020                      | 0.128                                | 0.026                     | 0.022                      |                                |
| $^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$        | 0.238                                | 0.048                     | 0.037                      | 0.443                                | 0.089                     | 0.069                      | $0.108 \pm 0.003$              |
| $^{124}\text{Sn} \rightarrow ^{124}\text{Te}$        | 0.253                                | 0.050                     | 0.035                      | 0.164                                | 0.032                     | 0.022                      |                                |
| $^{128}\text{Te} \rightarrow ^{128}\text{Xe}$        | 0.229                                | 0.044                     | 0.030                      | 0.169                                | 0.033                     | 0.022                      | $0.043 \pm 0.003$              |
| $^{130}\text{Te} \rightarrow ^{130}\text{Xe}$        | 0.091                                | 0.017                     | 0.011                      | 0.081                                | 0.016                     | 0.010                      | $0.0293 \pm 0.0009$            |
| $^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$        | 0.307                                | 0.058                     | 0.035                      | 0.194                                | 0.037                     | 0.022                      | $0.0181 \pm 0.0006$            |
| $^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$        | 0.604                                | 0.111                     | 0.055                      | 0.594                                | 0.109                     | 0.054                      | $0.055 \pm 0.003$              |
| $^{150}\text{Nd} \rightarrow ^{150}\text{Sm}(0_2^+)$ | 0.666                                | 0.122                     | 0.060                      | 0.629                                | 0.116                     | 0.057                      | $0.044 \pm 0.005$              |
| $^{198}\text{Pt} \rightarrow ^{198}\text{Hg}$        | 0.026                                | 0.004                     | 0.001                      | 0.027                                | 0.005                     | 0.001                      |                                |

data: Barabash, Universe (2020)