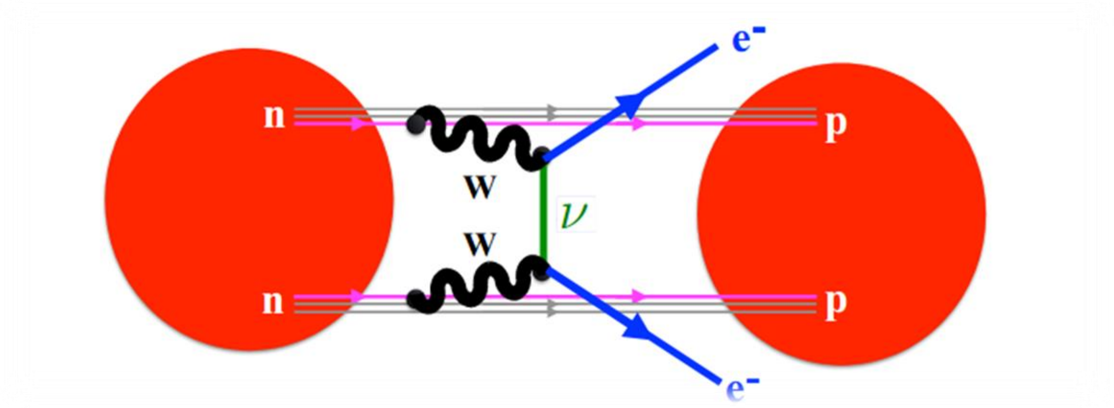
Neutrinos Electro-Weak interactions and Symmetries (NEWS) colloquium on

# Study of neutrinoless double beta decay of $^{136}$Xe using nuclear shell model


**Dr. Shahariar Sarkar**

Postdoc Fellow at Department of Physics, Indian Institute of Technology Roorkee, India



**In Collaboration with**

**Dr. Yoritaka Iwata**

Osaka University of Economics and Law, Osaka, Japan


NEWS2407, RNCP, Osaka University, July 30, 2024

# Outline

Neutrinoless double beta ($0\nu\beta\beta$) decay and its importance in neutrino physics

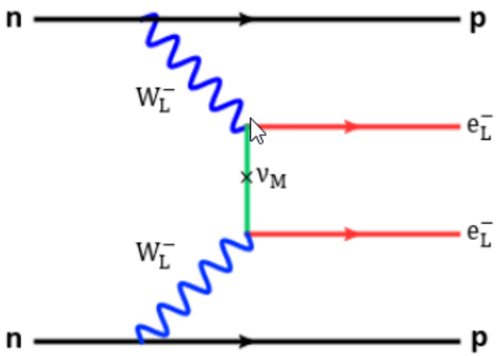
↓

Method of Nuclear Shell Model to Study $0\nu\beta\beta$ Decay and Nuclear Matrix Elements

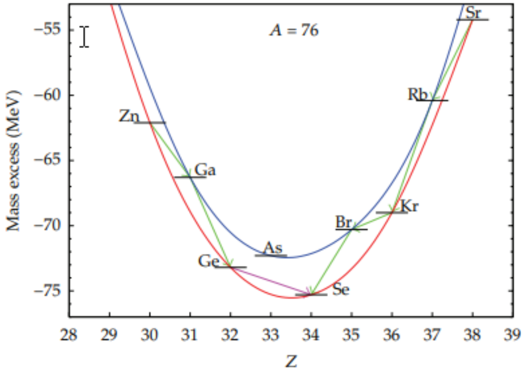
↓

Results for Nuclear Matrix Elements Mostly of $^{136}Xe$ Using Nuclear Shell Model in Both Closure and Nonclosure Approximations

# Neutrinoless Double Beta Decay (0νββ)




Feynman diagram for light neutrino exchange 0νββ

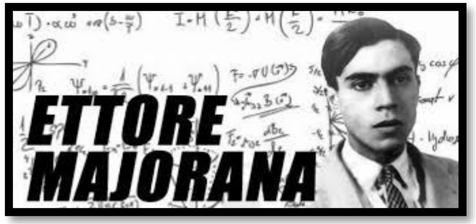



Representation of the energies of the A  76 isobars

## If this process is observed…

- Neutrinos must be their own antiparticle (Majorana particle), which has important implications in BSM physics theories.
- Absolute Majorana Neutrino Mass Can be Calculated Which Can Help to Establish Mass Hierarchy of Neutrinos.
- Lepton number violation ($\Delta L = 2$) will be observed.

## History




1937: **Ettore Majorana** predicted a that neutrino can be its own antiparticle

1939: **Wolfgang Furry** first predicted Neutrinoless double beta decay based on Majorana neutrino

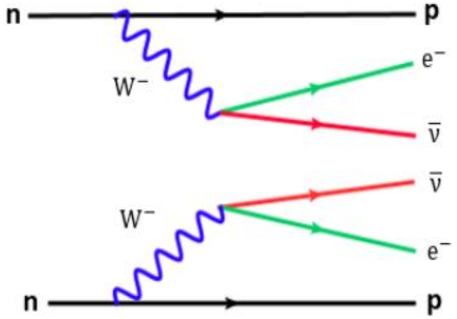
## Possible Decaying Isotopes

$^{48}$Ca, $^{76}$Ge, $^{82}$Se, $^{96}$Zr, $^{100}$Mo, $^{116}$Cd, $^{124}$Sn , $^{130}$Te ,$^{136}$Xe, $^{150}$Nd

## Rarity of the process:

Half life can be in the range $> 10^{26}$ Years

## But….0νββ is still unobserved even after 80 years.




Feynman diagram for two-neutrino double beta decay

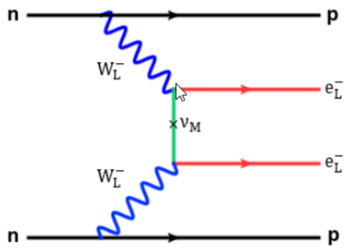
## Two-Neutrino Double Beta decay

Observed in the experiment

# Half-life of Neutrinoless Double Beta Decay 0νββ: light-neutrino exchange

**Half-life of 0νββ**

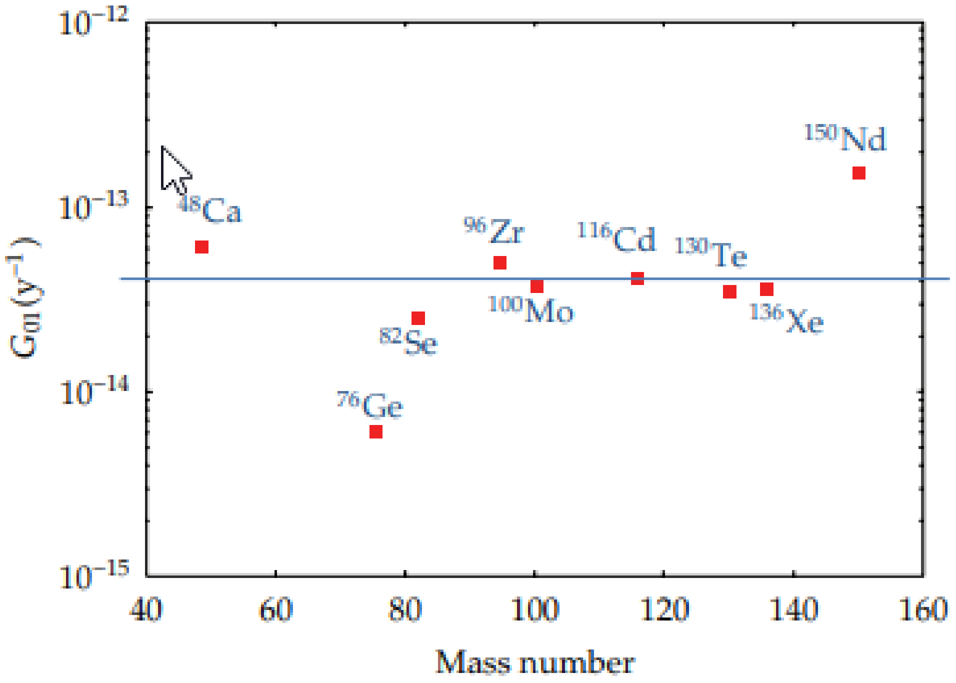
$$\frac{1}{T^{0\nu}_{\frac{1}{2}}} = G^{0\nu}(Q,Z)\left|m_{\beta\beta}\right|^2 |M^{0\nu}|^2$$




**Phase Space Factor($G^{0\nu}$):**

$$G^{0\nu}(Q,Z) = \frac{1}{2(2\pi)^5} G_F^4 \frac{1}{R^2} g_A^4 \int_0^Q dT_1 \int_0^\pi \sin\theta d\theta \,(E_1E_2 - p_1p_2\cos\theta)p_1p_2F(E_1,Z+2)F(E_2,Z+2)$$

$$T_1 = E_1 - m_e,\ Q = M_i - M_f - 2m_e$$




Phase space of the nine more favorable double-beta decay isotopes. https://doi.org/10.1155/2012/857016

# Absolute Majorana Neutrino Mass

**Decay rate of $0\nu\beta\beta$**

$$\Gamma^{0\nu} = \frac{1}{T^{0\nu}_{\frac{1}{2}}} = G^{0\nu}(Q,Z)\left|m_{\beta\beta}\right|^2\left|M^{0\nu}\right|^2$$

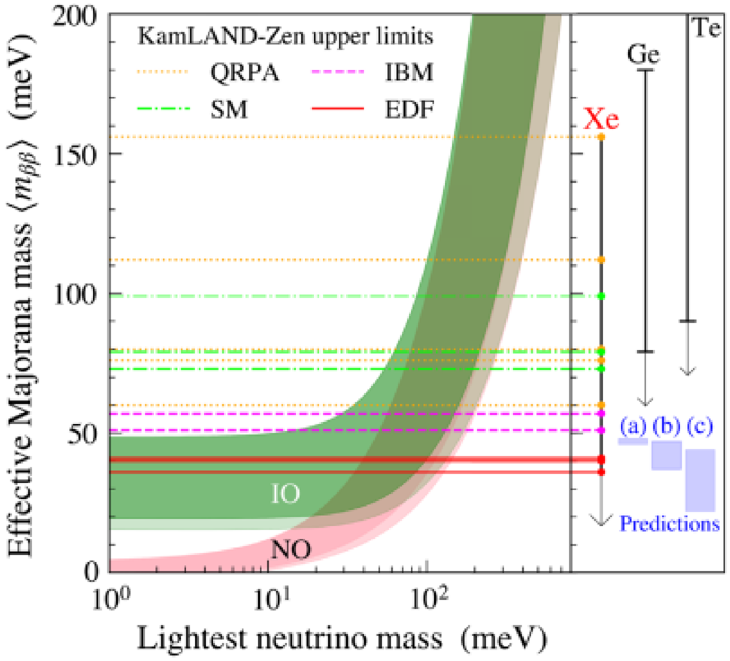
**Majorana neutrino mass($m_{\beta\beta}$):**

$$m_{\beta\beta} = \sum_i U_{ei}^2\, m_i$$

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

$$U = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix} \times \begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta_{13}}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta_{13}} & 0 & c_{13}\end{pmatrix}$$

$$\times \begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix} \times \begin{pmatrix}e^{1/2i\alpha_1} & 0 & 0\\ 0 & e^{1/2i\alpha_2} & 0\\ 0 & 0 & 1\end{pmatrix}$$




Where, $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$,
$0 \le \theta_{ij} \le \pi/2, 0 \le \delta_{13} \le 2\pi$, $\alpha_1$ and $\alpha_2$ are Majorana CP-violating phases.

Upper limit for $m_{\beta\beta}$ of 36-156 meV has been determined from $0\nu\beta\beta$ decay experiment of $^{136}$Xe at KamLAND-Zen ( PHYSICAL REVIEW LETTERS 130, 051801 (2023) ) with lower limit of $T^{0\nu}_{1/2}$ $2.3 \times 10^{26}$ yr using different nuclear matrix elements

# Nuclear Matrix Element(NME) $\mathbf{M^{0\nu}}$: our research interest

$$M^{0\nu} = M^{0\nu}_{GT} - \frac{g_V^2}{g_A^2} M^{0\nu}_{F} + M^{0\nu}_{T}$$

- F- Fermi
- GT- Gamow-Teller
- T- Tensor
- $g_V$ and $g_A$ are vector and axial vector constant

$$M^{0\nu}_{\alpha} = \langle f|\tau_{-1}\tau_{-2}\, O^{\alpha}_{12}|i\rangle \quad \alpha = (F, GT, T)$$

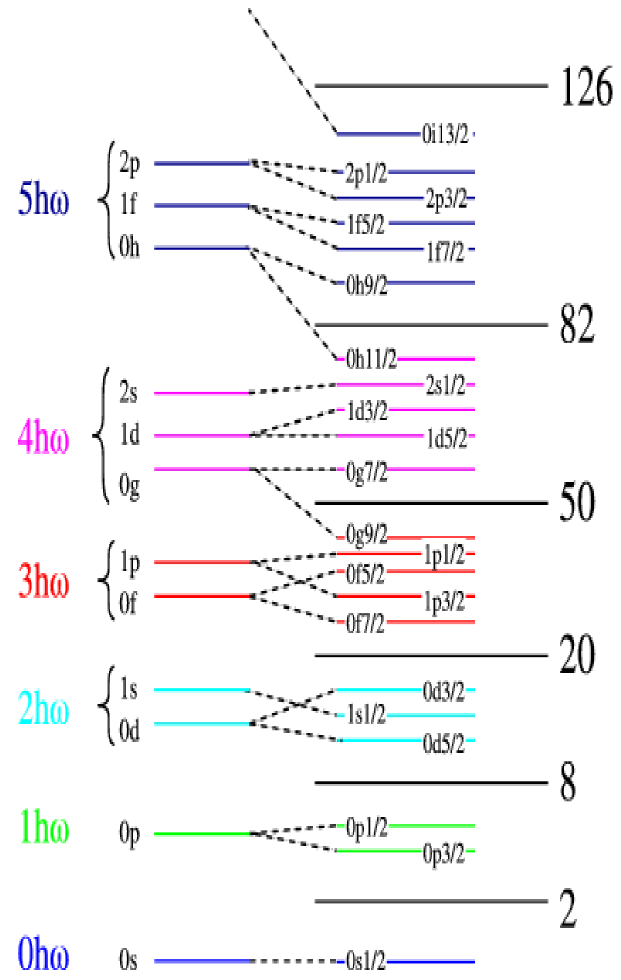
## $0\nu\beta\beta$ transition operators

- Fermi Type: $O^{F}_{12} = S_F H_F(r) = H_F(r)$
- Gamow Teller type:
  $$O^{GT}_{12} = S_{GT} H_{GT}(r) = \vec{\sigma}_1.\vec{\sigma}_2 H_{GT}(r)$$
- Tensor Type:
  $$O^{T}_{12} = S_T H_T(r) = [3(\vec{\sigma}_1.\hat{r})(\vec{\sigma}_2.\hat{r}) - \vec{\sigma}_1.\vec{\sigma}_2] H_T(r)$$

## Models of Nuclear Matrix Element calculations

- **Nuclear Shell Model (NSM) (We use this)**
- Quasiparticle Random Phase Approximation (QRPA)
- Projected Hartree Fock Bogliovob Method (PHFB)
- Interacting Boson Model 2 (IBM2)
- Energy Density Functional Theory (EDF)

## Shell structure of nucleus




https://oer.physics.manchester.ac.uk/NP/Notes/Notes/shells.png

Nuclear shell structure

# Approximations of NME calculations: closure vs nonclosure approximation

**Nonclosure Approximation**

$^{136}Xe \rightarrow {}^{136}Cs \rightarrow {}^{136}Ba$

$$H_\alpha(r, E_k^*) = \frac{2R}{\pi} * \int_0^\infty \frac{1}{q + E_0 + E_k^*} j_p(qr) g_\alpha(q) q dq$$

Where, $E_0 = \frac{Q_{\beta\beta}}{2} + \Delta M$

**Closure Approximation**

One replaces $E_k^* + E_0 = \langle E \rangle$

$$H_\alpha(r) = \frac{2R}{\pi} * \int_0^\infty \frac{1}{q + \langle E \rangle} j_p(qr) g_\alpha(q) q dq$$

# Method: Running nonclosure and closure

Here….Neutrino potential are calculated explicitly in terms of excitation energy of $^{136}$Cs

$$H_\alpha(r, E_k^*) = \frac{2R}{\pi} * \int_0^\infty \frac{1}{q + E_0 + E_k^*} j_p(qr) g_\alpha(q) q dq$$

$$\langle n'l' | H_\alpha(r, E_k^*) | nl \rangle = \int_0^\infty R_{n'l'} R_{nl} r^2 dr * H_\alpha(r, E_k^*)$$

**Closure approximation**

$$E_0 + E_k^* = \langle E \rangle$$

**Nonclosure Closure approximation**

$$E_0 + E_k^* \rightarrow 1.74 \text{ MeV} + E_k^*$$

**Explicit Form of NME in running nonclosure method**

$$M^{0\nu}_{\alpha-\text{running nonclosure}}(E) = \sum_{k_1' k_2' k_1 k_2 J J_k} \sum_{E_k^* \le E_C} \sqrt{(2J_k + 1)(2J_k + 1)(2J + 1)} \times (-1)^{j_{k1} + j_{k2} + J} \begin{Bmatrix} j_1 & j_2 & J_k \\ j_4 & j_3 & J \end{Bmatrix} \times \text{OBTD}(k, f, k_1', k_2', J_k) \times \text{OBTD}(k, i, k_1, k_2, J_k) \langle k_1' k_2' : J \| \tau_{-1} \tau_{-2} \mathcal{O}_{12}^{0\nu} \| k_1 k_2 \rangle$$

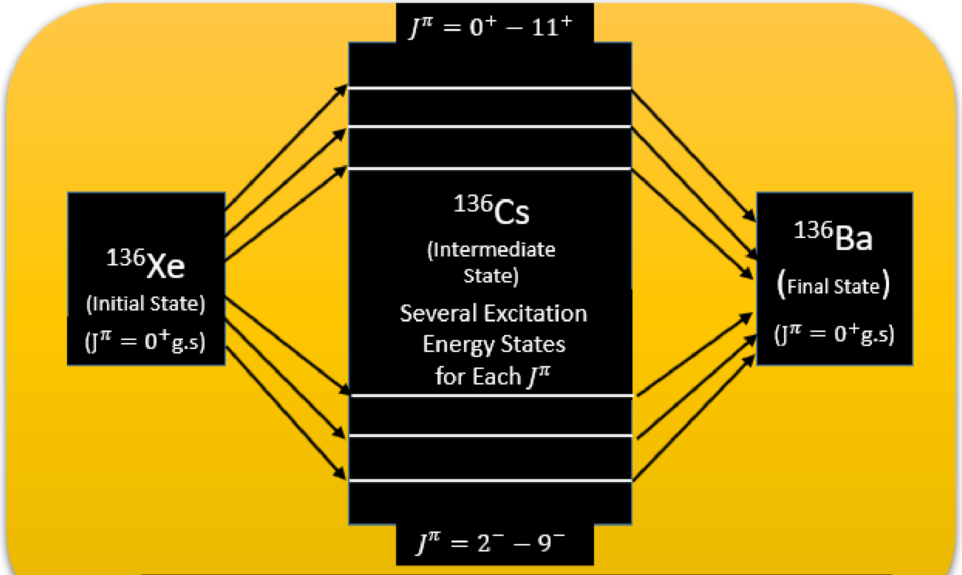



Diagram to Calculate OBTD for $0\nu\beta\beta$ of $^{136}$Xe

Schematic diagram to calculate OBTD

$$\text{OBTD}(k, f, k_1', k_2', J_k) = \frac{\langle k \| [a_{k_1}^+ \otimes \tilde{a}_{k_1'}]_{J_k} \| f \rangle}{\sqrt{2J_k + 1}}$$

# Two Body Matrix Elements of 0νββ decay

## 0νββ transition operators

- Fermi Type:
$$O_{12}^{F} = S_F H_F(r) = H_F(r)$$
- Gamow Teller type:
$$O_{12}^{GT} = S_{GT} H_{GT}(r) = \vec{\sigma}_1 . \vec{\sigma}_2 H_{GT}(r)$$
- Tensor Type:
$$O_{12}^{T} = S_T H_T(r) = [3(\vec{\sigma}_1 . \hat{r})(\vec{\sigma}_2 . \hat{r}) - \vec{\sigma}_1 . \vec{\sigma}_2] H_T(r)$$

## Explicit Form of Two Body Matrix Elements

$$\langle n_{p1} l_{p1} j_{p1}, n_{p2} l_{p2} j_{p2}, J_m^{\pi} | \tau_{-1} \tau_{-2} O_{12}^{\alpha} | n_{n1} l_{n1} j_{n1}, n_{n2} l_{n2} j_{n2} J_m^{\pi} \rangle$$

$$= \sum_{S',\lambda',S,\lambda} \begin{Bmatrix} l_{p1} & \frac{1}{2} & j_{p1} \\ l_{p2} & \frac{1}{2} & j_{p2} \\ \lambda' & S' & J_m \end{Bmatrix} * \times \begin{Bmatrix} l_{n2} & \frac{1}{2} & j_{n2} \\ l_{n1} & \frac{1}{2} & j_{n1} \\ \lambda & S & J_m \end{Bmatrix}$$

$$\times \frac{1}{\sqrt{2S+1}} \langle l_{p1} l_{p2} \lambda' \frac{1}{2} \frac{1}{2} S'; J_m | S_{12}^{\alpha} | l_{n2} l_{n1} \lambda \frac{1}{2} \frac{1}{2} S; J_m \rangle$$

$$\times \langle n_{p1} l_{p1} n_{p2} l_{p2} | H_{\alpha}(r) | n_{n1} l_{n1} n_{n2} l_{n2} \rangle$$

SPIN PART

Radial Part

$$\langle n_{p1} l_{p1}, n_{p2} l_{p2} | H_{\alpha}(r) | n_{n1} l_{n1}, n_{n2} l_{n2} \rangle$$ → Individual Coordinate

$$= \sum_{n',l',N',L'} \sum_{n,l,N,L} < n'l', N'L' | n_{p1} l_{p1}, n_{p2} l_{p2} >_{\lambda'}$$

$$\times < n'l', N'L' | n_{p1} l_{p1}, n_{p2} l_{p2} >_{\lambda'} \times \langle n'l' | H_{\alpha}(r) | nl \rangle$$

Harmonic Oscillator Bracket

Relative and COM coordinate

# Neutrino Potential Integral $\langle n'l'|H_\alpha(r)|nl\rangle$

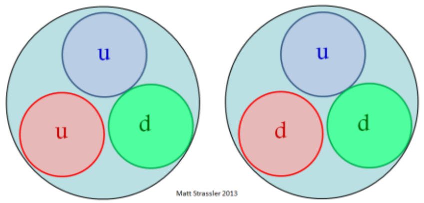
$$< n'l'|H_{Type}(r)|nl >= \int_0^\infty R_{n'l'}R_{nl}r^2dr * H_\alpha(r)$$

Where, $H_\alpha(r) = \frac{2R}{\pi} * \int_0^\infty \frac{1}{q+\langle E\rangle} j_p(qr)g_\alpha(q)qdq$

## Effects of Finite nucleon size (FNS) and higher order current(HOC)

FNS + HOC

https://profmattstrassler.com/




+ Weak interaction is not pure V-A type at nucleon level

$$J_L^\mu = \bar{\psi}\tau_-\left(g_V(q^2)\gamma^\mu - g_A(q^2)\gamma^\mu\gamma_5 - ig_M(q^2)\frac{\sigma^{\mu\nu}}{2m_p} + g_P(q^2)q^\mu\gamma_5\right)\psi$$

$$g_F(q) = g_V^2(q)$$

$$g_{GT}(q) = \frac{g_A^2(q)}{g_A^2}\left(1 - \frac{2}{3}\frac{q^2}{q^2+m_\pi^2} + \frac{1}{3}\left(\frac{q^2}{q^2+m_\pi^2}\right)^2\right) + \frac{2}{3}\frac{g_M^2(q)}{g_A^2} * \frac{q^2}{4m_p^2}$$

$$g_T(q) = \frac{g_A^2(q)}{g_A^2}\left(\frac{2}{3}\frac{q^2}{q^2+m_\pi^2} - \frac{1}{3}\left(\frac{q^2}{q^2+m_\pi^2}\right)^2\right) + \frac{1}{3}\frac{g_M^2(q)}{g_A^2} * \frac{q^2}{4m_p^2}$$

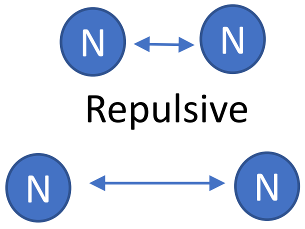
$g_A(q^2) = g_A/(1 + q^2/\Lambda_A^2)$

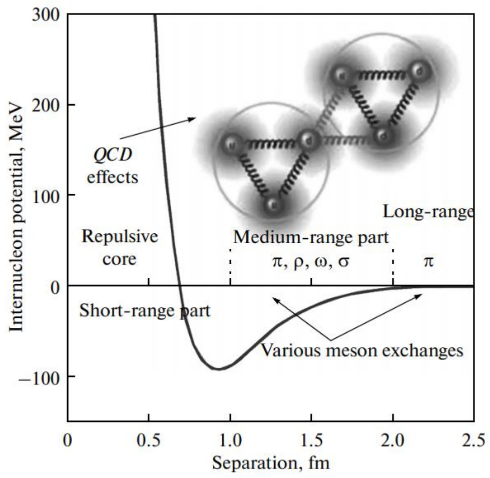
$g_M(q^2) = (\mu_p - \mu_n)g_V(q^2)$

$g_P(q^2) = 2m_p g_A(q^2)/(q^2 + m_\pi^2)$

$g_V(q^2) = g_V/(1 + q^2/\Lambda_V^2)$

## Short Range Correlation Effect (SRC)




Physics of Particles and Nuclei, 2014, Vol. 45, No. 5, pp. 924–971.

A general scheme for nucleon-nucleon potential

$$H_\alpha(r) \rightarrow H_\alpha(r)(1 + f(r))$$

$$f(r) = -ce^{-ar^2}(1 - br^2)$$

a, b, c are SRC parameters

| SRC Type | a | b | c |
|---|---|---|---|
| Miller-Spencer | 1.10 | 0.68 | 1.00 |
| CD-Bonn | 1.52 | 1.88 | 0.46 |
| AV18 | 1.59 | 1.45 | 0.92 |

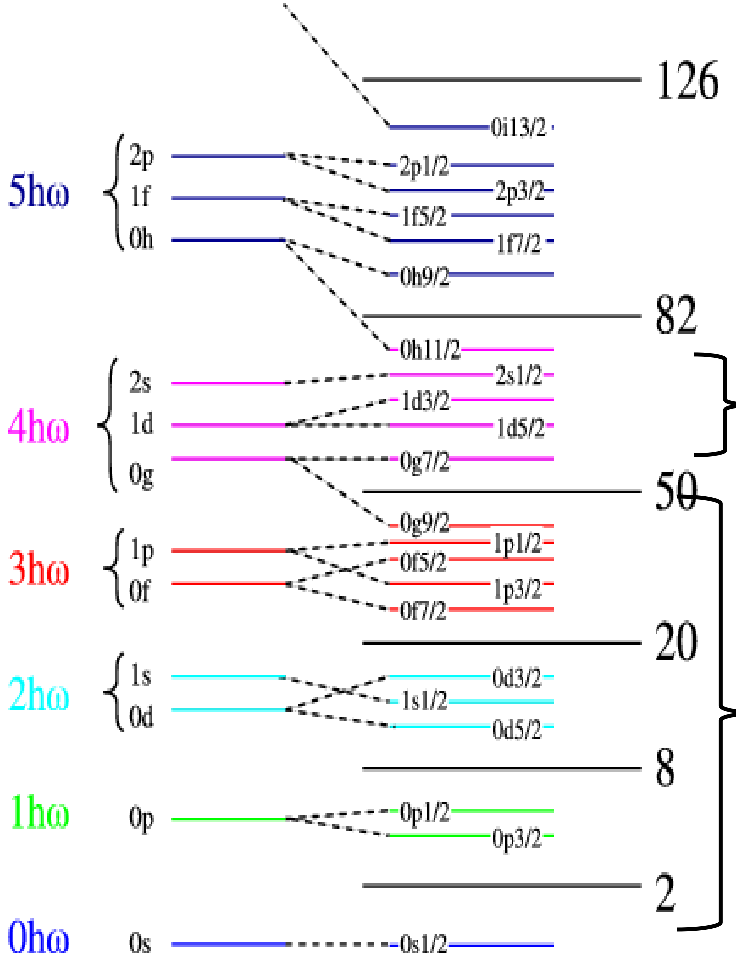
# Interacting Nuclear Shell Model and Effective Interactions

$^{136}Xe \rightarrow {}^{136}Cs \rightarrow {}^{136}Ba + e^- + e^-$

$^{136}Xe \rightarrow {}^{100}Sn$ (Core)+32 valence neutrons and 4 valence protons

$^{136}Cs \rightarrow {}^{100}Sn$ (Core)+31 valence neutrons and 5 valence protons

$^{136}Ba \rightarrow {}^{100}Sn$ (Core)+ 30 valence neutrons and 6 valence protons




Nuclear Shell structure

jj55-valence space

$0h_{11/2}$

$2s_{1/2}$

$1d_{3/2}$

$1d_{5/2}$

$0g_{7/2}$

ORBITS

$^{100}$**Sn Core**

Valence protons and neutrons occupy the above orbits of jj55-model space

For $^{136}$Xe, shell model diagonalization is performed with GCN5082 effective interactions to calculate initial, intermediate, and final nucleus using shell model code KSHELL [Shimizu et al., Comput. Phys. Commun. 2019, 244, 372–384]


ELSEVIER

Computer Physics Communications

Volume 244, November 2019, Pages 372-384


**Thick-restart block Lanczos method for large-scale shell-model calculations ☆, ☆☆**


Noritaka Shimizu [a], Takahiro Mizusaki [b], Yutaka Utsuno [c a], Yusuke Tsunoda [a]


GCN5082 Hamiltonian

```
!i jj55 model space (good isospin) g7,d5,d3,s1,h11
! gcn5082 Hamiltonian from Phys. Rev. C 82 (2010) 064304
0 -10.4165  -10.6023  -8.91581  -9.05719  -8.20568
   1  1  1  1   0  1    -0.445856
   1  1  1  1   1  0    -0.865073
   1  1  1  1   2  1     0.006127
   1  1  1  1   3  0    -0.221061
   1  1  1  1   4  1     0.262192
   1  1  1  1   5  0    -0.210628
   1  1  1  1   6  1     0.183064
   1  1  1  1   7  0    -0.918383
   1  1  1  2   1  0    -0.175292
   1  1  1  2   2  1    -0.307574
   1  1  1  2   3  0    -0.110939
```

Calculated Wavefunctions are further used to calculate the OBTD for nonclosure approach

# Results for $^{136}Xe$ $0\nu\beta\beta$ Decay

# Calculated NMEs for $0\nu\beta\beta$ decay of $^{136}Xe$ in Nonclosure Shell Model

$$M^{0\nu} = M_{GT} - \frac{g_V^2}{g_A^2} M_F + M_T$$

| NME Type | SRC Type | Nonclosure NME |
|---|---|---|
| $M_F$ | None | -0.458 |
| $M_F$ | Miller-Spencer | -0.322 |
| $M_F$ | CD-Bonn | -0.489 |
| $M_F$ | AV18 | -0.451 |
| $M_{GT}$ | None | 1.693 |
| $M_{GT}$ | Miller-Spencer | 1.225 |
| $M_{GT}$ | CD-Bonn | 1.743 |
| $M_{GT}$ | AV18 | 1.611 |
| $M_T$ | None | 0.013 |
| $M_T$ | Miller-Spencer | 0.012 |
| $M_T$ | CD-Bonn | 0.012 |
| $M_T$ | AV18 | 0.012 |
| $M^{0\nu}$ | None | 1.990 |
| $M^{0\nu}$ | Miller-Spencer | 1.437 |
| $M^{0\nu}$ | CD-Bonn | 2.058 |
| $M^{0\nu}$ | AV18 | 1.903 |

GT Type NME dominates over Fermi and Tensor Type Matrix Elements

The SRC effects are most visible for Miller-Spencer type SRC

# Comparison of Nonclosure and Closure NMEs in Different Models

| Nuclear Model | Reference | Approximation | $g_A$ | SRC Type | Total NME ($M^{0\nu}$) |
|---|---|---|---|---|---|
| ISM | Current Study | Nonclosure | 1.270 | CD-Bonn | 2.06 |
| ISM | Ref. [29] | Closure | 1.270 | CD-Bonn | 1.74 |
| ISM | Ref. [43] | Closure | 1.254 | CD-Bonn | 1.76 |
| ISM | Ref. [24] | Closure | 1.250 | UCOM | 2.19 |
| QRPA | Ref. [52] | Non Closure | 1.260 | CD-Bonn | 2.91 |
| QRPA | Ref. [53] | Non Closure | 1.269 | CD-Bonn | 2.46 |
| QRPA | Ref. [53] | Non Closure | 1.269 | AV18 | 2.18 |
| GCM | Ref. [28] | Closure | 1.254 | CD-Bonn | 2.35 |
| IBM2 | Ref. [54] | Closure | 1.269 | AV18 | 3.05 |
| EDF | Ref. [13] | Closure | 1.250 | UCOM | 4.20 |
| REDF | Ref. [15] | Closure | 1.254 | None | 4.32 |


[29] M. Horoi and A. Neacsu, Phys. Rev. C 98, 035502 (2018)
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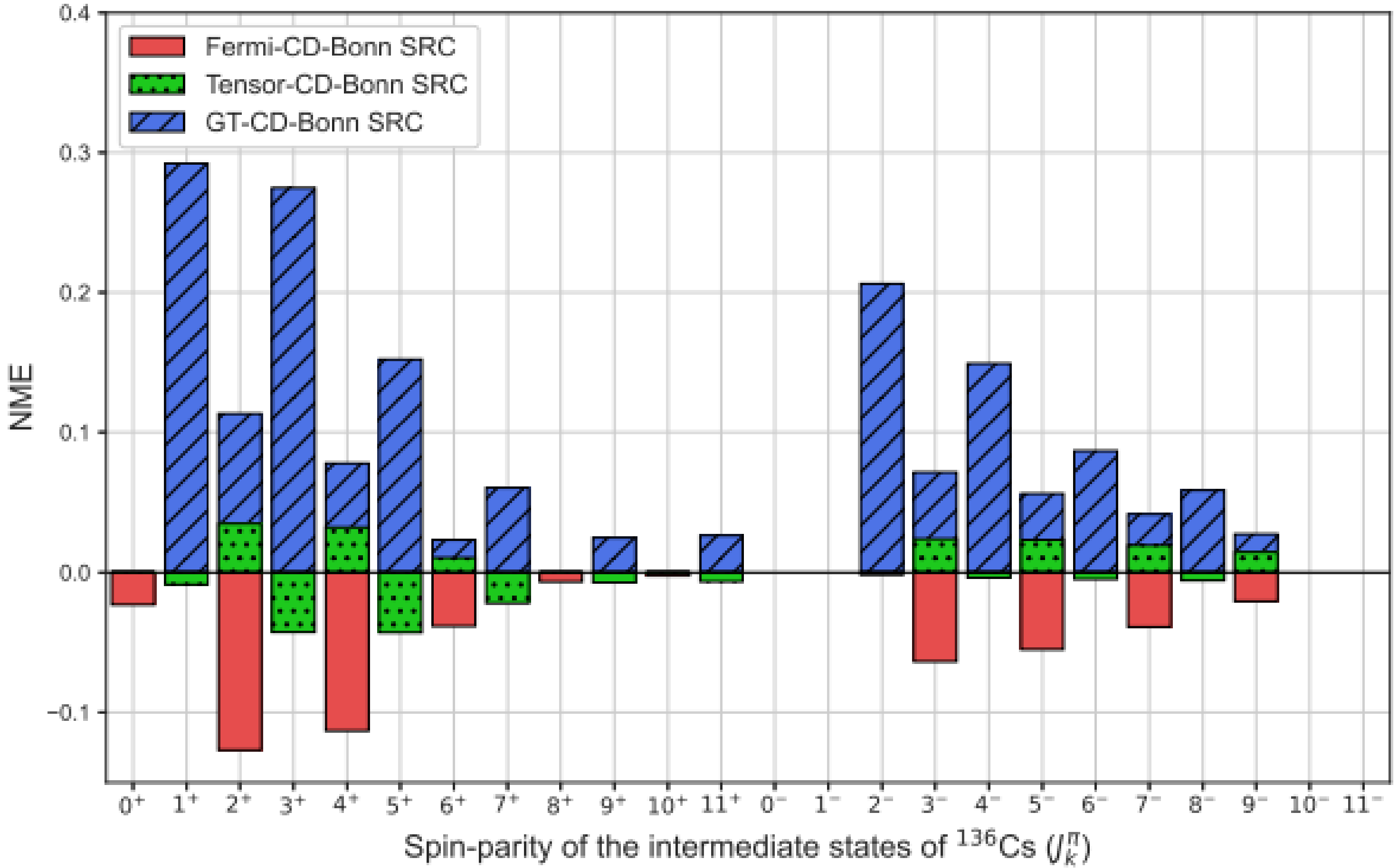
[54] J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C 91, 034304 (2015).
[13] T. R. Rodr´ıguez and G. Mart´ınez-Pinedo, Phys. Rev. Lett. 105, 252503 (2010).
[15] J. M. Yao, L. S. Song, K. Hagino, P. Ring, Phys. Rev. C 91, 024316 (2015).


We see 6-18% difference in nonclosure and closure NME in shell model

The Difference may arise from choice of Hamiltonian and closure energy used in closure approximation

Overall there for different nuclear models NME variation is large and it is an open problem to explore

# Contributions of Individual Spin-Parity of Intermediate Nucleus on $0\nu\beta\beta$ NME

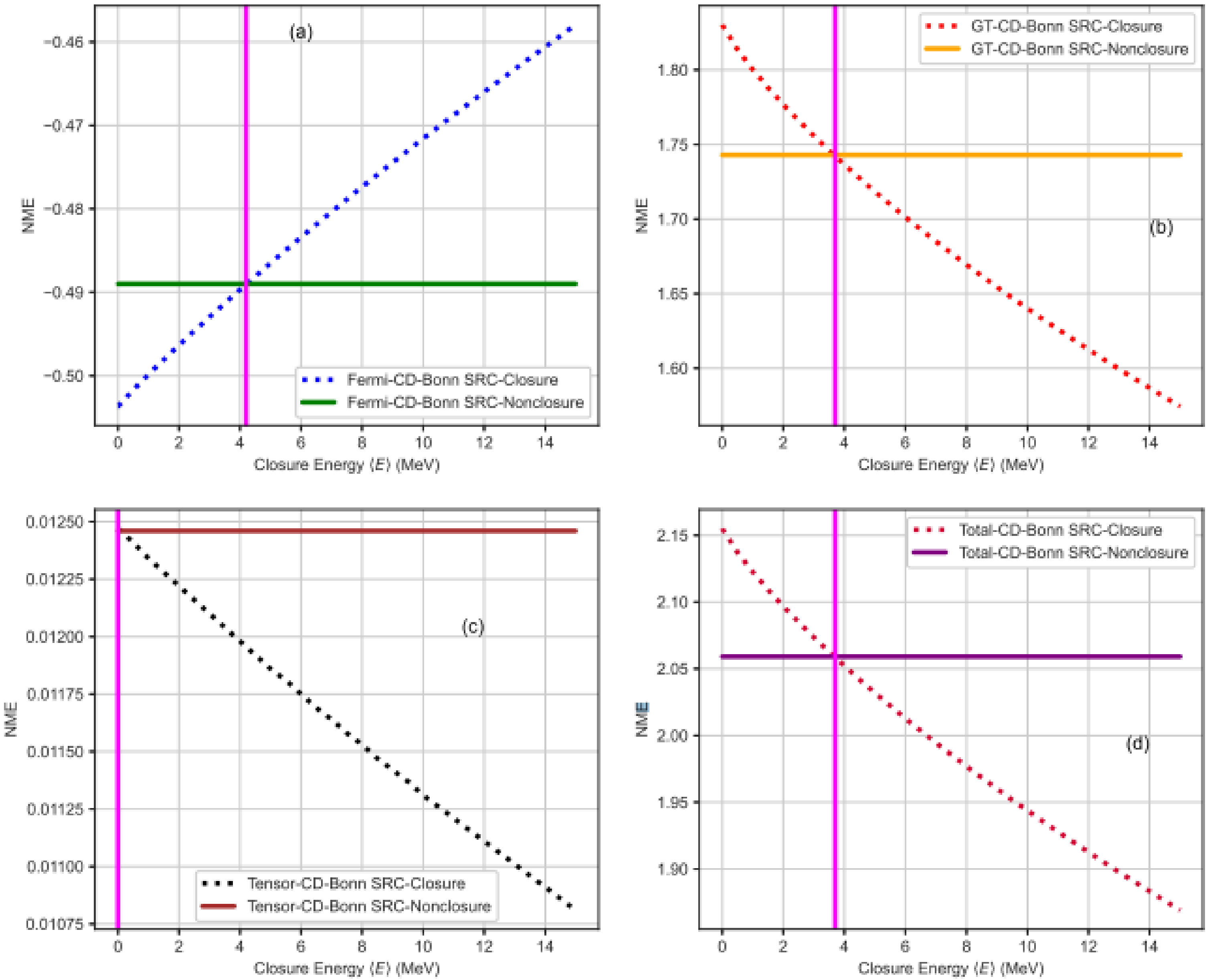



$^{136}$Cs is the Intermediate Nucleus for $0\nu\beta\beta$ decay of $^{136}$Xe

Partial GT NMEs are all positive and Fermi NMEs are all negative for different allowed spin-parity of $^{136}$Cs. Tensor NMEs have both positive and negative contributions

The 1+ state contributes most on GT NME 2+ for Fermi NMEs.

# Finding Optimal Closure Energy Where Nonclosure and Closure NME Overlaps

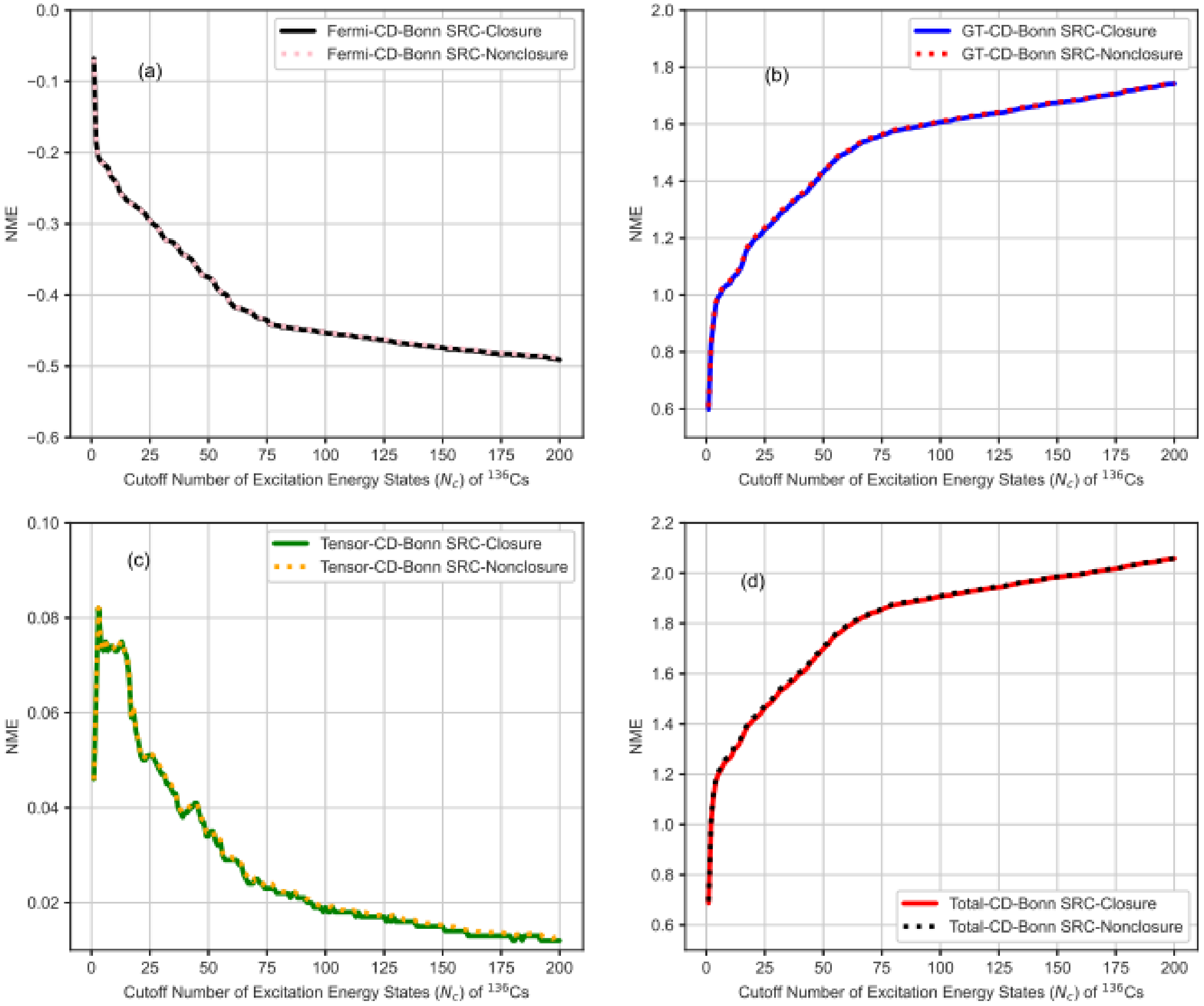



The goal here is to find appropriate closure energy for which nonclosure and closure NME overlaps

Optimal Closure Energies are 4.1, 3.7, Near 0, and 3.7 MeV for Fermi, GT, Tensor, and Total NMEs respectively.

Determined Optimal Closure Energy Can Help to Achieve a Accurate NME Using Closure Approach Which Requires Lesser Computational Resources

# Reproducing Nonclosure NMEs Using Closure Approach With Optimal Closure Energy With Smaller Computational Resources




Here we show that if we use determined optimal closure energy, we can accurately reproduce nonclosure NMEs using closure approach across all number of intermediate states considered with much smaller computational power

Here solid lines are closure NMEs and dotted lines are nonclosure NMEs

In addition, we showed how the convergence depends on choice of number of states for each spin-parity of $^{136}$Cs. Here we could consider up to 200 states for each spin-parity, with reasonable accuracy.

# Studying Non-standard Mechanisms of $0\nu\beta\beta$ Decay Using Nonclosure Approach

Apart from standard light neutrino-exchange mechanisms, the other popular BSM physics mechanisms are

**Left-Right Symmetric Mechanisms of $0\nu\beta\beta$ Decay (Will Discuss this Briefly)**

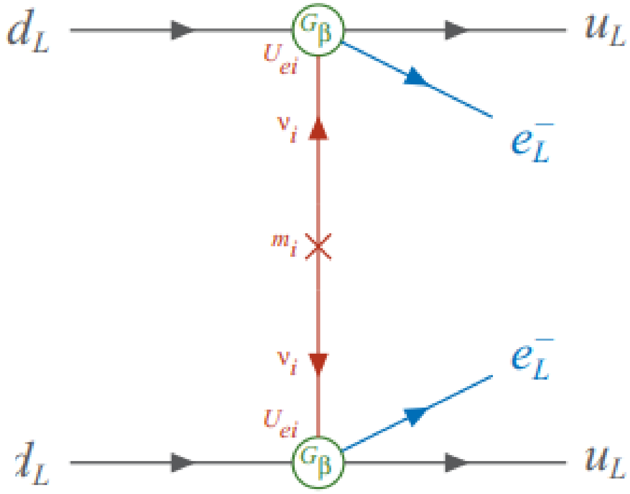
**Supersymmetric Particles Exchange Mechanisms of $0\nu\beta\beta$ Decay**

**Effective Field Theory Approach of $0\nu\beta\beta$ Decay**

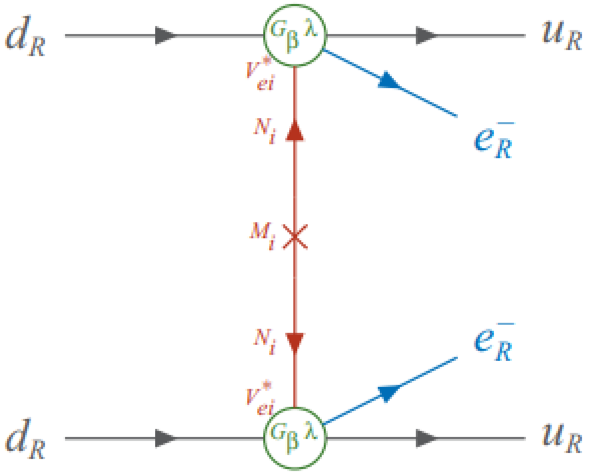
**Extra Dimensional Mechanisms of $0\nu\beta\beta$ Decay**

# Nuclear Matrix Element Calculation for 0νββ Decay in Left-Right Symmetric Model

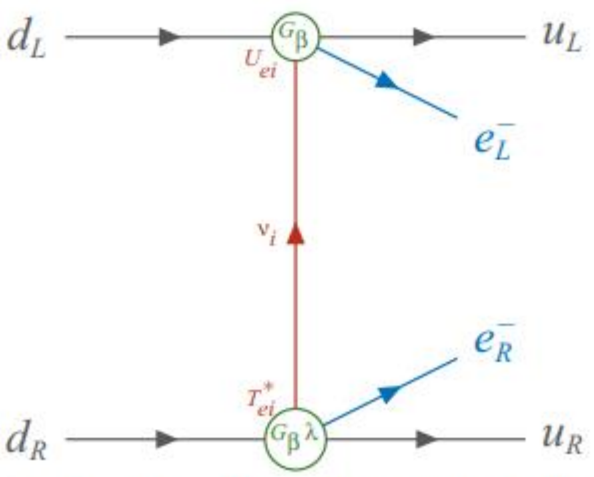
PHYSICAL REVIEW C 101, 035504 (2020)



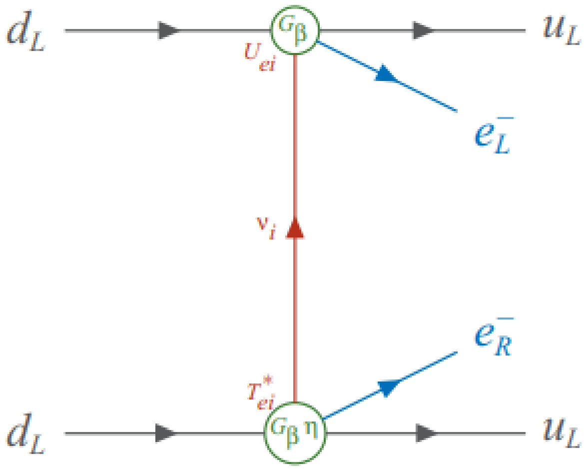

(a) Light neutrino exchange for purely LH currents. Diagram $\propto \eta_m$ arising from $j_L J_L^\dagger j_L J_L^\dagger$ term.




(b) Heavy neutrino exchange for purely RH currents. Diagram $\propto \eta_N$ arising from $j_R J_R^\dagger j_R J_R^\dagger$ term.




(a) $\lambda$-diagram due to both LH and RH currents. Diagram $\propto \eta_\lambda$ arising from $j_L J_L^\dagger j_R J_R^\dagger$ term.




(b) $\eta$-diagram due to gauge boson mixing. Diagram $\propto \eta_\eta$ arising from $j_L J_L^\dagger j_R J_L^\dagger$ term.

**If we consider all the diagrams of $0\nu\beta\beta$ decay in left-right symmetric model, the decay rate can be written as**

$$
\begin{aligned}
\left[T_{1/2}^{0\nu}\right]^{-1} = & g_A^4 \Big[C_m |\eta_m|^2 + C_N |\eta_N|^2 + C_\lambda |\eta_\lambda|^2 + C_\eta |\eta_\eta|^2 \\
& + C_{mN}|\eta_m||\eta_N|\cos(\phi_m - \phi_N) + C_{m\lambda}|\eta_m||\eta_\lambda|\cos(\phi_m - \phi_\lambda) \\
& + C_{m\eta}|\eta_m||\eta_\eta|\cos(\phi_m - \phi_\eta) + C_{N\lambda}|\eta_N||\eta_\lambda|\cos(\phi_N - \phi_\lambda) \\
& + C_{N\eta}|\eta_N||\eta_\eta|\cos(\phi_N - \phi_\eta) + C_{\lambda\eta}|\eta_\lambda||\eta_\eta|\cos(\phi_\lambda - \phi_\eta)
\end{aligned}
$$

**Nuclear Matrix Element (NME) (Our Interest)**

$$
C_m = G_{01}\left[M_{GT} - \left(\frac{g_V}{g_A}\right)^2 M_F + M_T\right]^2 ,
$$

…Similarly for other terms of decay rate equation

**Phase Space Factors which are Calculated Accurately**

# Nuclear Matrix Elements Calculation in Left-Right Symmetric Model

**NME of 0νββ decay is written as**

$$M_\alpha = \langle f|\tau_{-1}\tau_{-2}\mathcal{O}^{\alpha}_{12}|i\rangle$$

**Transition Operators of 0νββ in left-Right Symmetry**

$$\mathcal{O}_{12}^{GT,\omega GT,qGT,GTN} = (\sigma_1.\sigma_2)H_{GT,\omega GT,qGT,GTN}(r,E_k)$$

$$\mathcal{O}_{12}^{GTR} = \vec{\sigma}_1\cdot\vec{\sigma}_2 H_{GTR}(r,E_k). \qquad \mathcal{O}_{12}^{F,\omega F,qF,FN} = H_{F,\omega F,qF,FN}(r,E_k)$$

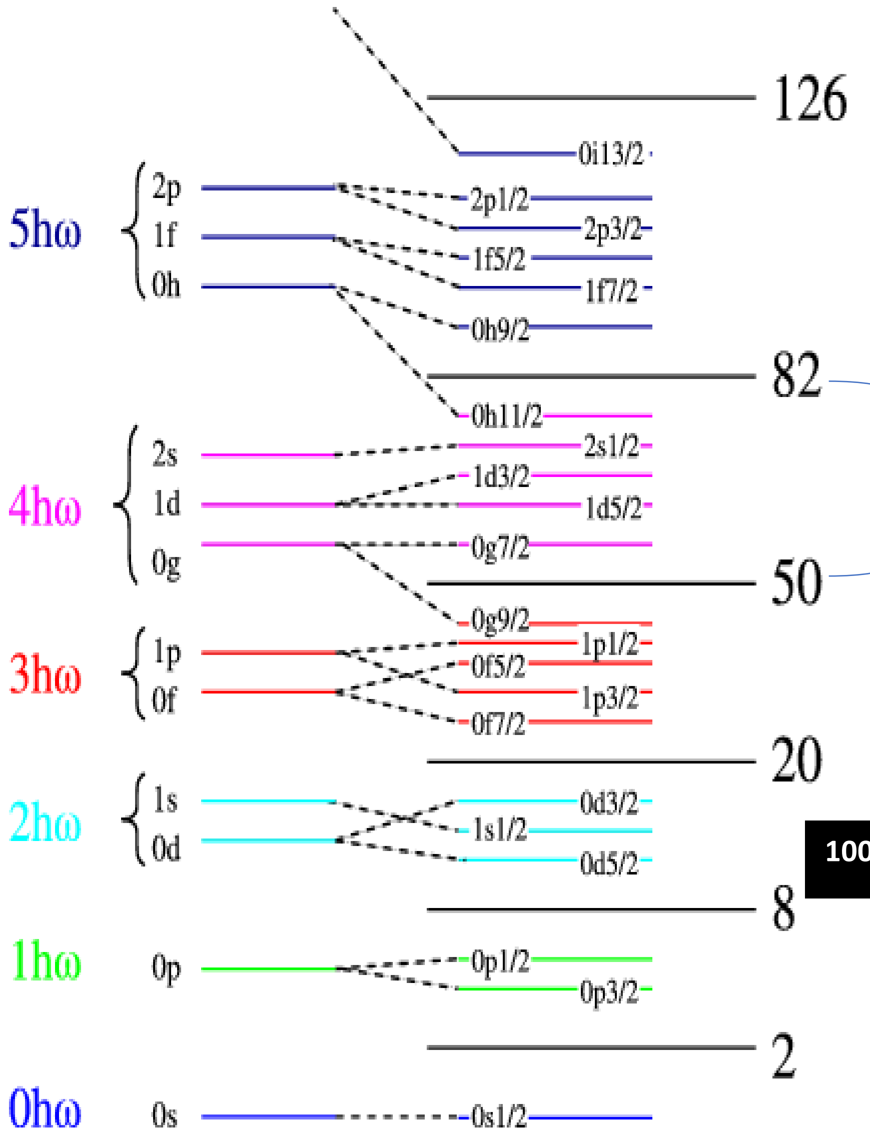
$$\mathcal{O}_{12}^{TR} = S_{12}H_{TR}(r,E_k) \qquad \mathcal{O}_{12}^{T,\omega T,qT,TN} = S_{12}H_{T,\omega T,qT,TN}(r,E_k)$$

$$\mathcal{O}_{12}^{P} = (\vec{\sigma}_1-\vec{\sigma}_2)\,H_P(r,E_k),$$

**Nuclear Matrix Element are Calculated in Shell Model as**

$$M_\alpha(J_k,J,E_k^*) = \sum_{k_1'k_2'k_1k_2}\sqrt{(2J_k+1)(2J_k+1)(2J+1)}$$
$$\times(-1)^{j_{k1}+j_{k2}+J}\begin{Bmatrix} j_{k1'} & j_{k1} & J_k \\ j_{k2} & j_{k2'} & J\end{Bmatrix}\mathrm{OBTD}(k,f,k_2',k_2,J_k)$$
$$\times\mathrm{OBTD}(k,i,k_1',k_1,J_k)\langle k_1',k_2':J||\tau_{-1}\tau_{-2}\mathcal{O}^{\alpha}_{12}||k_1,k_2:J\rangle,$$

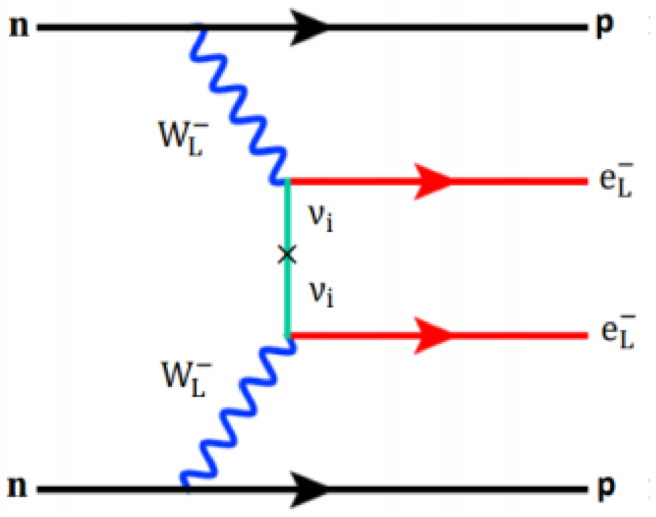
**Shell Model Calculation for $^{136}$Xe 0νββ Decay**



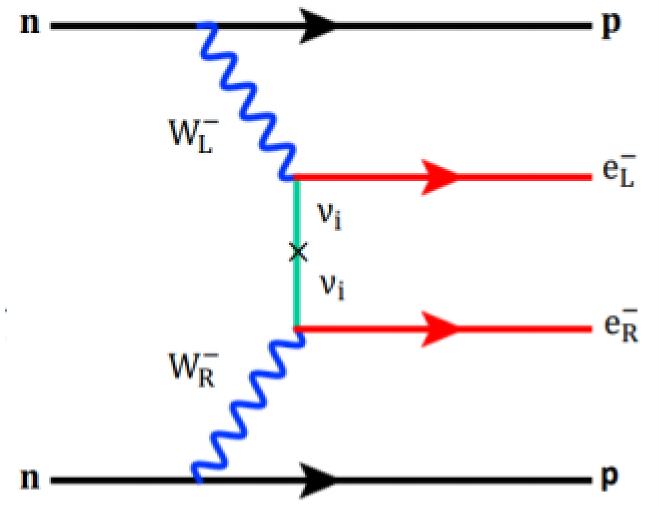

**GCN5082 Hamiltonian**

**$^{100}$Sn as inert Core**

# Study of λ mechanism of 0νββ in Nuclear Shell Model




The Feynman diagrams for light $m_{\beta\beta}$ mechanism




The Feynman diagrams for λ mechanism

## Motivation of Studying λ mechanism

PHYSICAL REVIEW C **98**, 035502 (2018)

**Shell model study of using an effective field theory for disentangling several contributions to neutrinoless double-β decay**

Mihai Horoi* and Andrei Neacsu†

(I) Shell Model was used in paper for closure approximation to study λ mechanism of 0νββ

PHYSICAL REVIEW C **92**, 055502 (2015)

**Reexamining the light neutrino exchange mechanism of the 0νββ decay with left- and right-handed leptonic and hadronic currents**

Dušan Štefánik,[1] Rastislav Dvornický,[1,2] Fedor Šimkovic,[1,3,4] and Petr Vogel[5]

(II) Exploited the revised formalism for λ mechanism

frontiers in Physics

**The λ Mechanism of the 0νββ-Decay**

*Fedor Šimkovic[1,2,3]*, Dušan Štefánik[1] and Rastislav Dvornický[1,4]*

(III) QRPA calculations with revised formalism for λ mechanism

# Decay rate and NME for λ mechanism

## Decay Rate

$$\left[T^{0\nu}_{\frac{1}{2}}\right]^{-1} = \eta_\nu^2 C_{mm} + \eta_\lambda^2 C_{\lambda\lambda} + \eta_\nu \eta_\lambda \cos\psi \, C_{m\lambda}$$

$$C_{mm} = g_A^4 M_\nu^2 G_{01},$$
$$C_{m\lambda} = -g_A^4 M_\nu (M_{2-} G_{03} - M_{1+} G_{04}),$$
$$C_{\lambda\lambda} = g_A^4 (M_{2-}^2 G_{02} + \frac{1}{9} M_{1+}^2 G_{011} - \frac{2}{9} M_{1+} M_{2-} G_{010})$$

## NMEs

$$M_\nu = M_{GT} - \frac{1}{g_A^2} M_F + M_T$$
$$M_{\nu\omega} = M_{\omega GT} - \frac{1}{g_A^2} M_{\omega F} + M_{\omega T}$$
$$M_{1+} = M_{qGT} + 3\frac{1}{g_A^2} M_{qF} - 6 M_{qT}$$
$$M_{2-} = M_{\nu\omega} - \frac{1}{9} M_{1+}$$
$$M_\alpha = \langle f | \tau_{1-}\tau_{2-} \mathcal{O}_{12}^{\alpha} | i \rangle$$

## Transition Operator

$$\mathcal{O}_{12}^{GT,\omega GT,qGT} = \tau_{1-}\tau_{2-} (\sigma_1 . \sigma_2) H_{GT,\omega GT,qGT}(r, E_k)$$
$$\mathcal{O}_{12}^{F,\omega F,qF} = \tau_{1-}\tau_{2-} H_{F,\omega F,qF}(r, E_k)$$
$$\mathcal{O}_{12}^{T,\omega T,qT} = \tau_{1-}\tau_{2-} (S_{12}) H_{T,\omega T,qT}(r, E_k)$$

## Radial neutrino potentials

**Nonclosure approximation**

$$H_\alpha(r, E_k) = \frac{2R}{\pi} \int_0^\infty \frac{f_\alpha(q, r) dq}{q + E_k + (E_i + E_f)/2}$$

**Closure approximation**

$$[E_k + (E_i + E_f)/2] \rightarrow \langle E \rangle$$

$$H_\alpha(r, E_k) = \frac{2R}{\pi} \int_0^\infty \frac{f_\alpha(q, r) dq}{q + \langle E \rangle}$$

**Revised Approach**

PHYSICAL REVIEW C **92**, 055502 (2015)

**Reexamining the light neutrino exchange mechanism of the $0\nu\beta\beta$ decay with left- and right-handed leptonic and hadronic currents**

Dušan Štefánik,[1] Rastislav Dvornický,[1,2] Fedor Šimkovic,[1,3,4] and Petr Vogel[5]

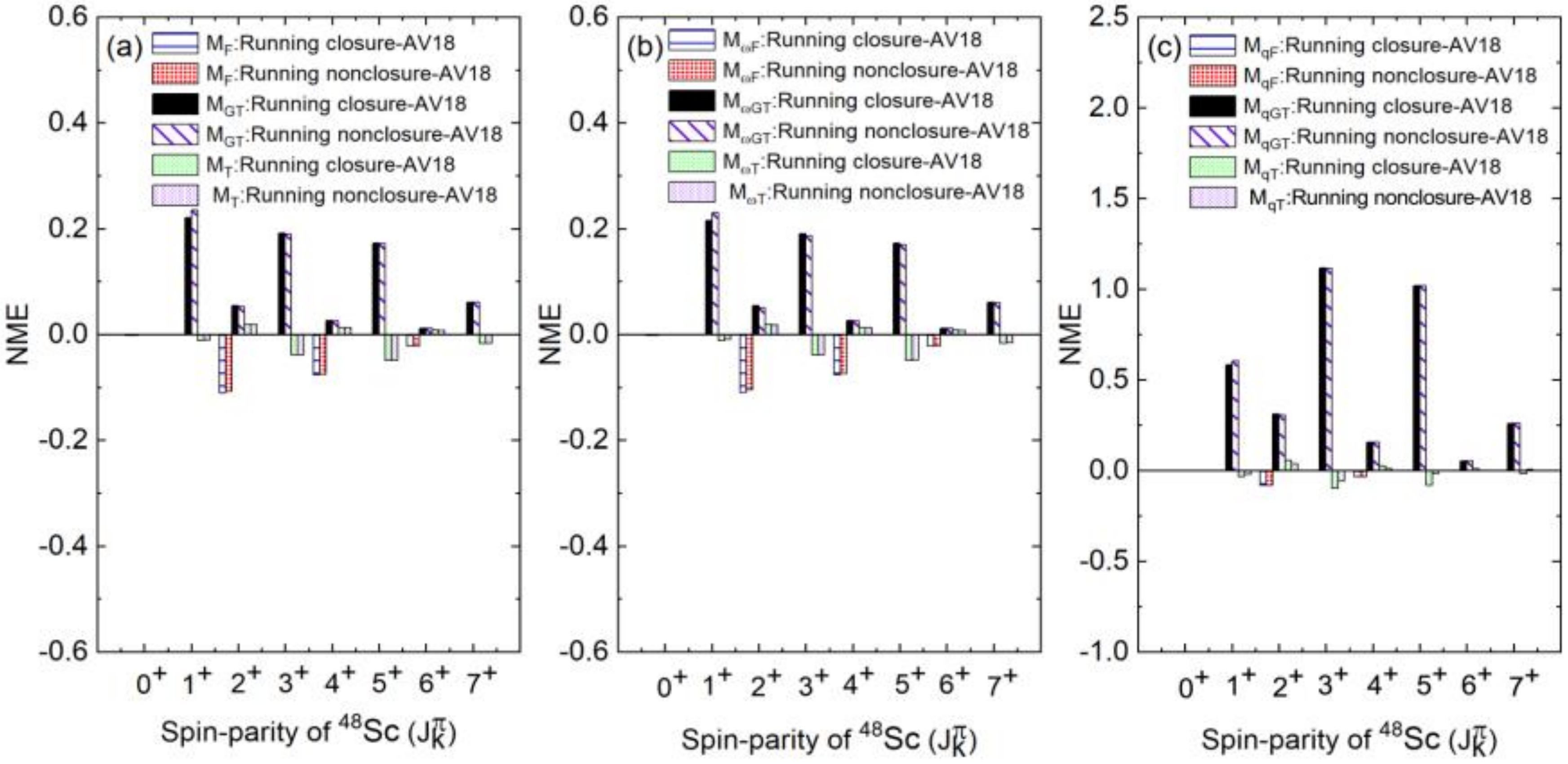
$$f_{qGT}(q, r) = \frac{1}{\left(1 + \frac{q^2}{\Lambda_A^2}\right)^4} q j_1(qr)$$

(Old)

$$f_{qGT}(q, r) = \left( \frac{g_A^2(q^2)}{g_A^2} q + 3 \frac{g_P^2(q^2)}{g_A^2} \frac{q^5}{4 m_N^2} + \frac{g_A^2(q^2) g_P^2(q^2)}{g_A^2} \frac{q^3}{m_N} \right) r j_1(qr)$$

(Revised)

# Results for λ mechanism of 0νββ decay of $^{48}$Ca




PHYSICAL REVIEW C **102**, 034317 (2020)

**Nuclear matrix elements for the λ mechanism of 0$\nu\beta\beta$ decay of $^{48}$Ca in the nuclear shell-model: Closure versus nonclosure approach**

Shahariar Sarkar,[1,*] Y. Iwata,[2] and P. K. Raina[1]

# Results for λ mechanism of $0\nu\beta\beta$ decay of $^{82}Se$

**TABLE 2 |** NMEs for $0\nu\beta\beta$ (light neutrino-exchange and $\lambda$ mechanism) of $^{82}Se$.

| NME Type | SRC Type | | | |
|---|---|---|---|---|
| | **None** | **Miller-Spencer** | **CD-Bonn** | **AV18** |
| $M_F$ | −0.633 | −0.442 | −0.674 | −0.621 |
| $M_{GT}$ | 3.681 | 2.536 | 3.247 | 3.068 |
| $M_T$ | −0.020 | −0.020 | −0.020 | −0.020 |
| $M_\nu$ | 3.529 | 2.790 | 3.645 | 3.433 |
| $M_{\omega F}$ | −0.630 | −0.441 | −0.671 | −0.618 |
| $M_{\omega GT}$ | 3.075 | 2.453 | 3.165 | 2.986 |
| $M_{\omega T}$ | −0.020 | −0.020 | −0.020 | −0.020 |
| $M_{\nu\omega}$ | 3.485 | 2.751 | 3.599 | 3.388 |
| $M_{qF}$ | −0.330 | −0.274 | −0.384 | −0.372 |
| $M_{qGT}$ | 11.667 | 10.167 | 12.538 | 12.184 |
| $M_{qT}$ | −0.097 | −0.097 | -0.097 | −0.097 |
| $M_{1+}$ | 11.636 | 10.241 | 12.409 | 12.076 |
| $M_{2-}$ | 2.192 | 1.613 | 2.220 | 2.046 |

**We some large enhancement of $\mathbf{M_{qGT}}$ type NME for including the recent nucleon current term as mentioned earlier**

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ORIGINAL RESEARCH
published: 19 November 2021
doi: 10.3389/fspas.2021.727880

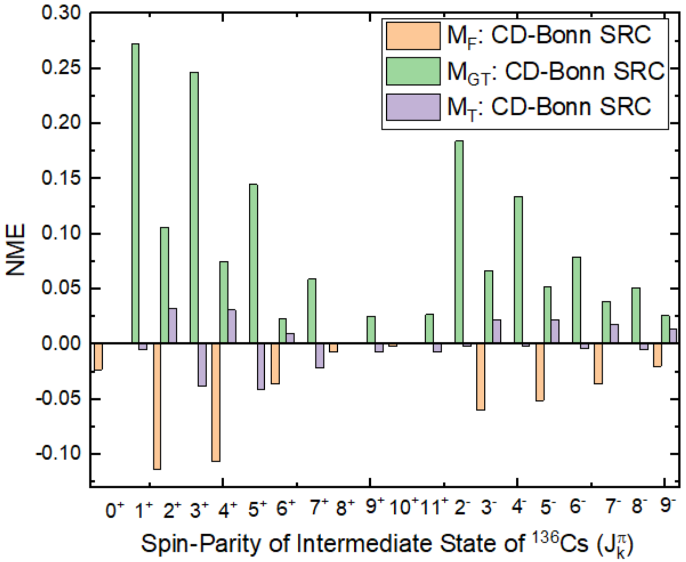
Check for updates

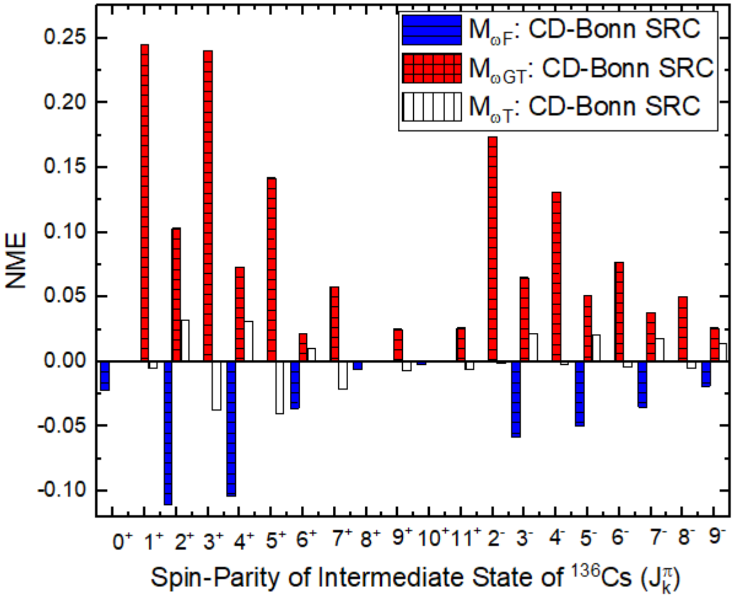
## Interacting Shell Model Calculations for Neutrinoless Double Beta Decay of $^{82}Se$ With Left-Right Weak Boson Exchange

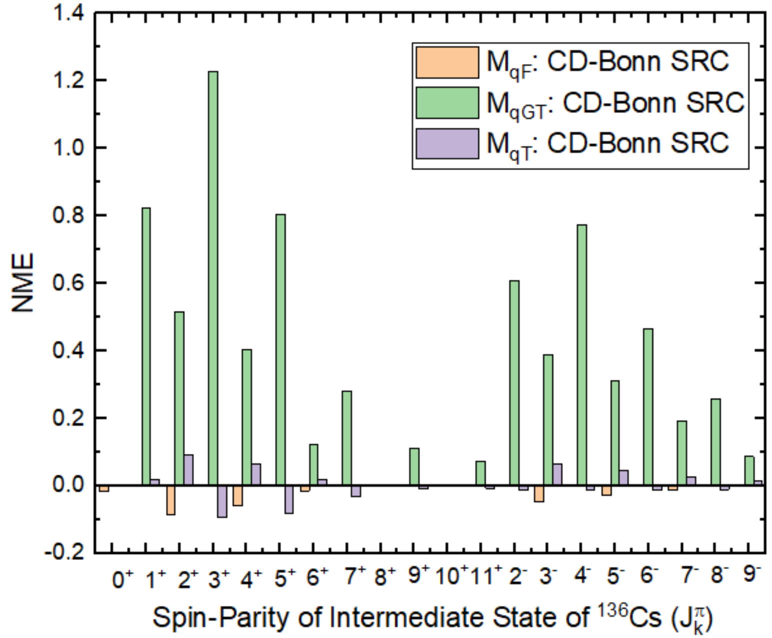
*Yoritaka Iwata*[1]* *and Shahariar Sarkar*[2]

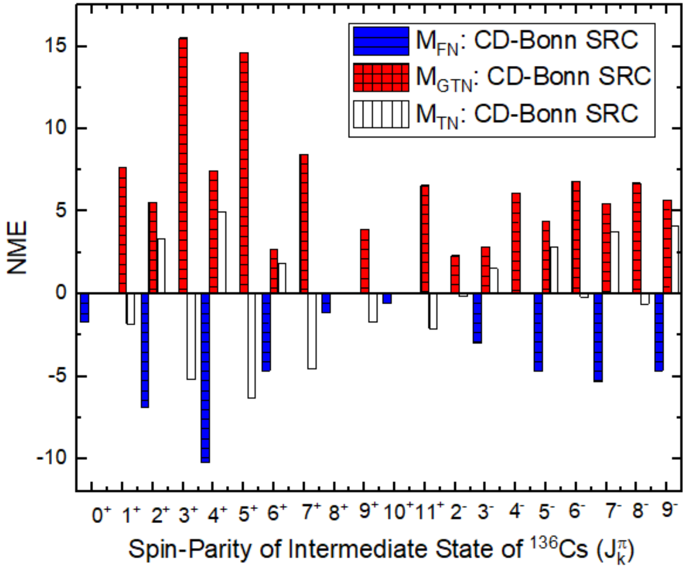
[1]*Faculty of Chemistry, Materials and Bioengineering, Kansai University, Osaka, Japan,* [2]*Indian Institute of Technology Ropar,*

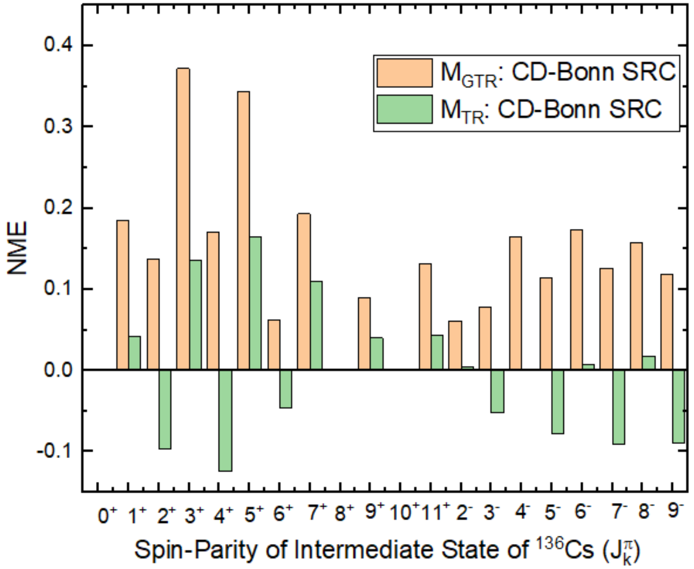
# Results: Contribution of different Spin-Parity States of Intermediate Nucleus $^{136}$Cs to the Nuclear Matrix Element of $^{136}$Xe $0\nu\beta\beta$ in Left-Right Symmetric Model



# Additional Methods of NME Calculations in Shell Model

# Method: Pure Closure

One of the Simplest that Requires Smallest Computational Resources is Pure closure Method

Pure Closure Method Only Requires Initial and Final States. So No Intermediate States are Required to Calculated

$$M^{0\nu}_{\alpha-\mathrm{closure}} = \langle f|\tau_{-1}\tau_{-2}\, O^{\alpha}_{12}|i\rangle = \sum_{J,k'_1k'_2k_1k_2} \mathrm{TBTD}(f,i,J)\langle k'_1k'_2 : JT|\tau_{-1}\tau_{-2}\, O^{\alpha}_{12}|k_1k_2 : JT\rangle_A$$

$$\mathrm{TBTD}(fikJ_oJ'_o\lambda) = \frac{< n\omega J||[A^+(k_\alpha k_\beta J_o) \otimes \tilde{A}(k_\gamma k_\delta J'_o)]^\lambda||n\omega' J' >}{\sqrt{2\lambda+1}} .$$

Lecture Notes in Nuclear Structure Physics

B. Alex Brown

Shell Model Code Like KSHELL Provides Option for TBTD Calculations

# Method: The 0νββ decay through (n-2) Channel

PRL 113, 262501 (2014) PHYSICAL REVIEW LETTERS

**Nuclear Structure Aspects of Neutrinoless Double-$\beta$ Decay**

B. A. Brown,[1] M. Horoi,[2] and R. A. Sen'kov[2,3]

**Two Nucleon Transfer Amplitudes (TNA)**

$$\mathrm{TNA}(f, m, k_1', k_2', J_m) = \frac{\langle f || A^{+}(k_1', k_2', J) || m \rangle}{\sqrt{2J_0 + 1}}$$

$^{136}Xe \rightarrow {}^{134}Xe \rightarrow {}^{136}Ba + e^{-} + e^{-}$

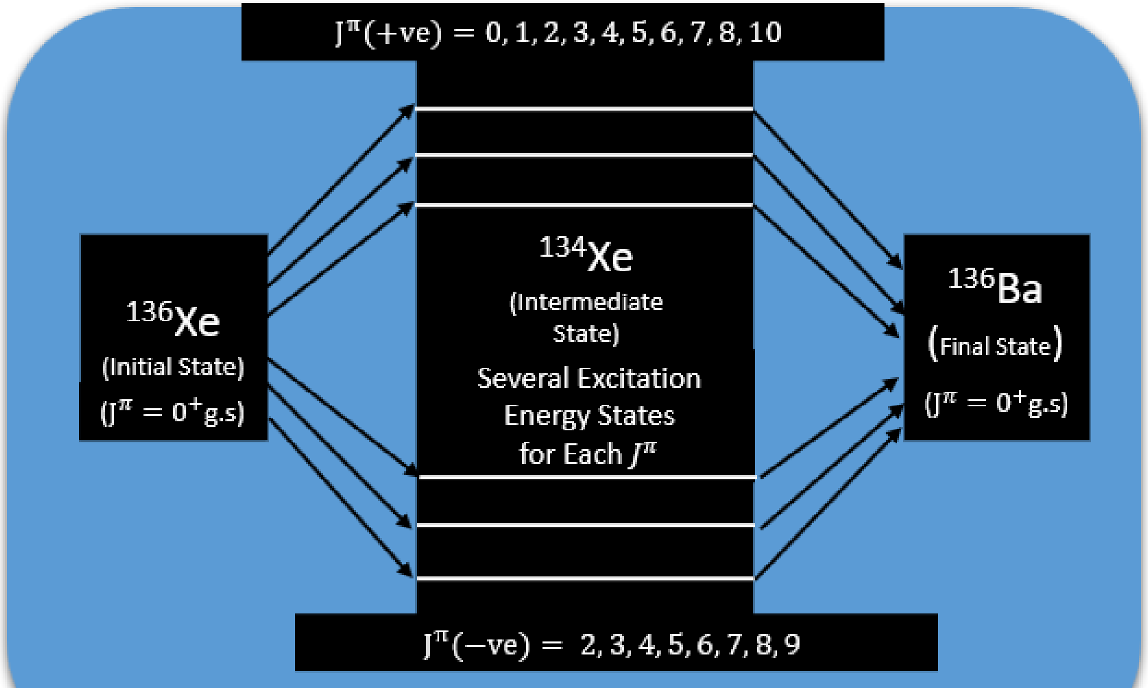



Diagram to Calculate TNA for $0\nu\beta\beta$ of $^{136}$Xe

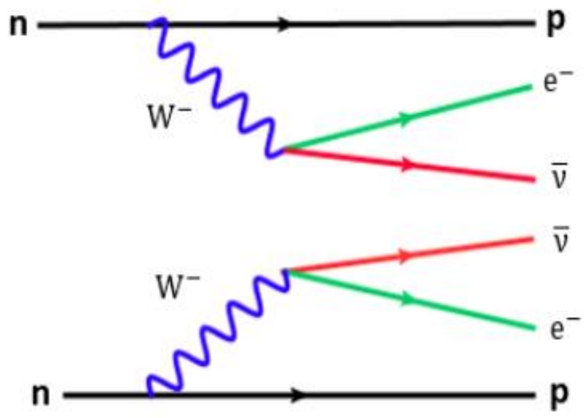
**Final Matrix Elements in Closure method**

$$M^{0\nu}_{\alpha-\mathrm{closure}} = \langle f | \tau_{-1}\tau_{-2}\, O^{\alpha}_{12} | i \rangle$$
$$= \sum_{J_m, k_1' k_2' k_1 k_2} \mathrm{TNA}(f, m, k_1', k_2', J_m)\,\mathrm{TNA}(m, i, k_1', k_2', J_m)\langle k_1' k_2' : JT | \tau_{-1}\tau_{-2}\, O^{\alpha}_{12} | k_1 k_2 : JT \rangle_A$$

**Shell model codes KSHELL and NushellX@MSU both provides option for TNA calculations**

# Finally Some Results of $^{136}Xe$ Two-Neutrino Double Beta Decay

# Two Neutrino Double Beta Decay Study of $^{136}$Xe in Shell Model

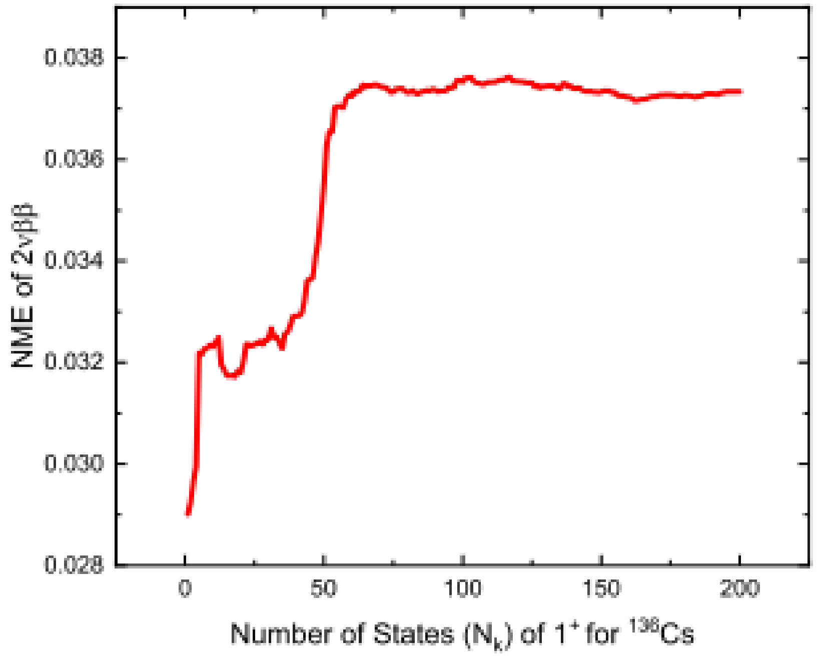


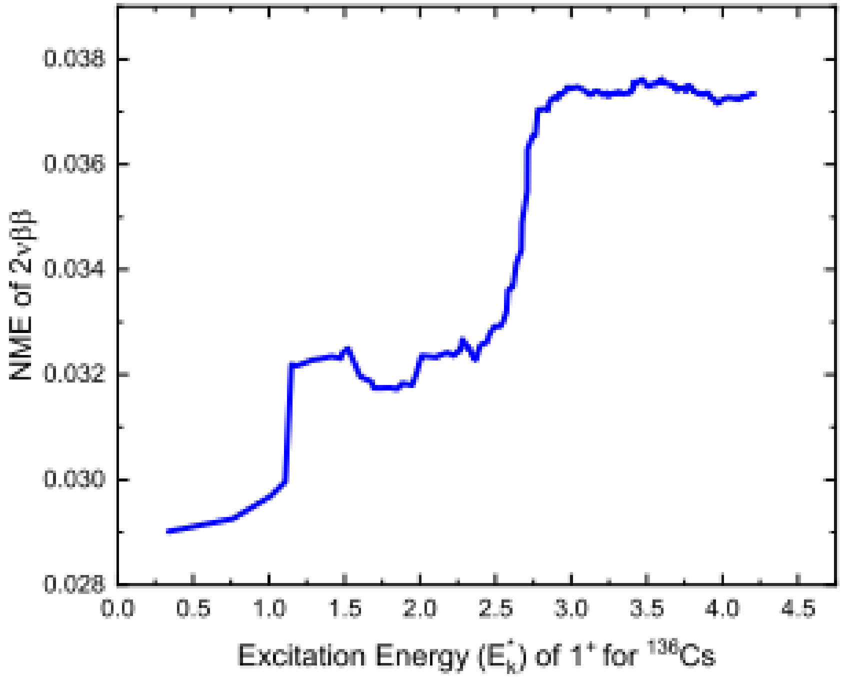

Feynman Diagram for Two Neutrino Double Beta (2νββ) Decay

$$^{136}\mathrm{Xe} \rightarrow {}^{136}\mathrm{Ba} + e^- + e^- + \overline{\nu}_e + \overline{\nu}_e.$$

$$[T^{2\nu}_{\frac{1}{2}}]^{-1} = G^{2\nu} g_A^4 |m_e c^2 M^{2\nu}_{GT}|^2,$$

$$M^{2\nu}_{GT} = \sum_{k, E_k \leqslant E_c} \frac{\langle f||\sigma\tau_2^-||k\rangle\langle k||\sigma\tau_1^-||i\rangle}{E_k^* + E_0},$$



# Summary

Study of $0\nu\beta\beta$ decay is important to know the Majorana nature of neutrino and absolute neutrino mass

↓

Our interest is to calculate nuclear matrix elements in nuclear shell model using both closure and nonclosure approximation

↓

We discussed some results for $0\nu\beta\beta$ decay $^{136}Xe$, $^{48}Ca$, and 82Se

↓

In future we plan to explore other beyond the standard model physics mechanisms of $0\nu\beta\beta$ decay such as left-right symmetric mechanisms

# Useful References for Nonclosure Approach of NME Calculations

PHYSICAL REVIEW C **88**, 064312 (2013)

**Neutrinoless double-$\beta$ decay of $^{48}$Ca in the shell model: Closure versus nonclosure approximation**

R. A. Sen'kov and M. Horoi
*Department of Physics, Central Michigan University, Mount Pleasant, Michigan 48859, USA*
(Received 14 October 2013; published 10 December 2013)

**The study of $0\nu\beta\beta$ decay of $^{136}$Xe using nonclosure approach in nuclear shell model**

S. Sarkar,[1,*] Y. Iwata,[2] K. Jha,[3] and R. Chatterjee[1]
[1]*Department of Physics, Indian Institute of Technology Roorkee, Roorkee - 247667, Uttarakhand, India*
[2]*Osaka University of Economics and Law, Yao, Osaka 581-0853, Japan*
[3]*Department of Physics, Medi-Caps University, Pigdamber, Rau, Indore - 453331, Madhya Pradesh, India*
(Dated: June 21, 2024)

PHYSICAL REVIEW C **109**, 024301 (2024)

**Calculation of nuclear matrix elements for $0\nu\beta\beta$ decay of $^{124}$Sn using the nonclosure approach in the nuclear shell model**

Shahariar Sarkar,[1,*] P. K. Rath,[2] V. Nanal,[3] R. G. Pillay,[1] Pushpendra P. Singh,[1] Y. Iwata,[4]

PHYSICAL REVIEW C **102**, 034317 (2020)

**Nuclear matrix elements for the λ mechanism of $0\nu\beta\beta$ decay of $^{48}$Ca in the nuclear shell-model: Closure versus nonclosure approach**

Shahariar Sarkar,[1,*] Y. Iwata,[2] and P. K. Raina[1]

# We Thank

**NEWS Colloquium Organizers**

Prof. Hiro Ejiri (RCNP)

Prof. Atsushi Tamii (RCNP)

Prof. Tatsushi Shima (RCNP)

**And all Audiences**