Nuclear weak interaction rates with the cluster variational method for core-collapse supernova simulations

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Outline

1 : Introduction

2 : Supernova simulations with the variational EOS

3 : Nuclear weak interaction for core-collapse simulations

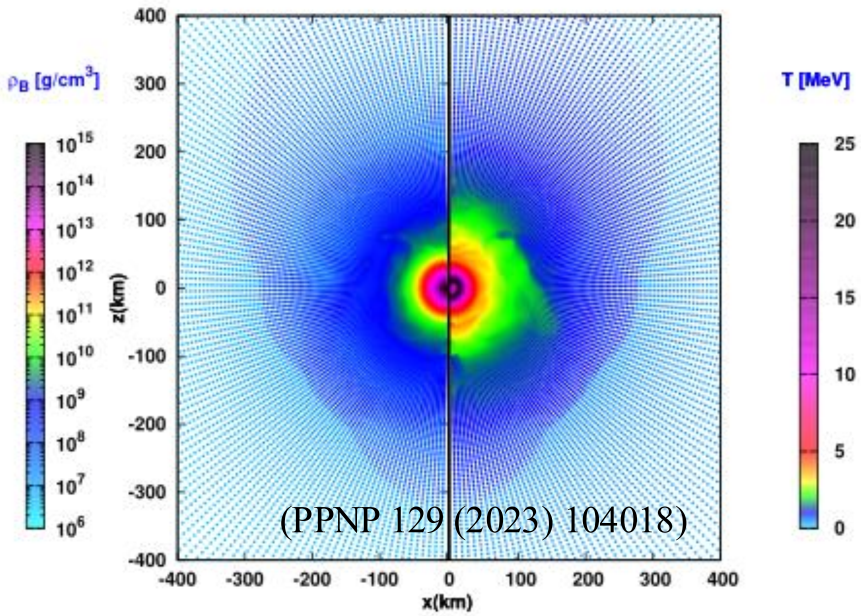
4: Summary

NEWS colloquium @ RCNP, Osaka, April 18, 2025

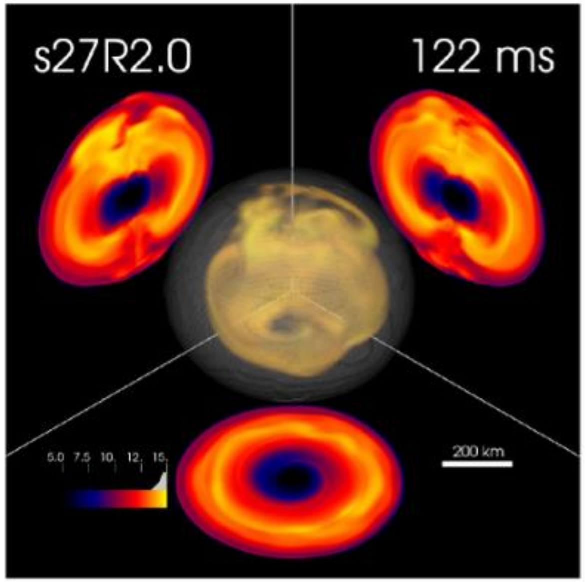
Important Notes (Excuse)

- While the word "*supernova*" appears in the title, the primary focus **is not** on the mechanism of supernova explosion itself. Instead, the real focus is on improving the **nuclear physics input data** required for simulating them accurately.

*Sorry for **not showing figures** like colorful explosion ones below in this talk.*



(PPNP 129 (2023) 104018)



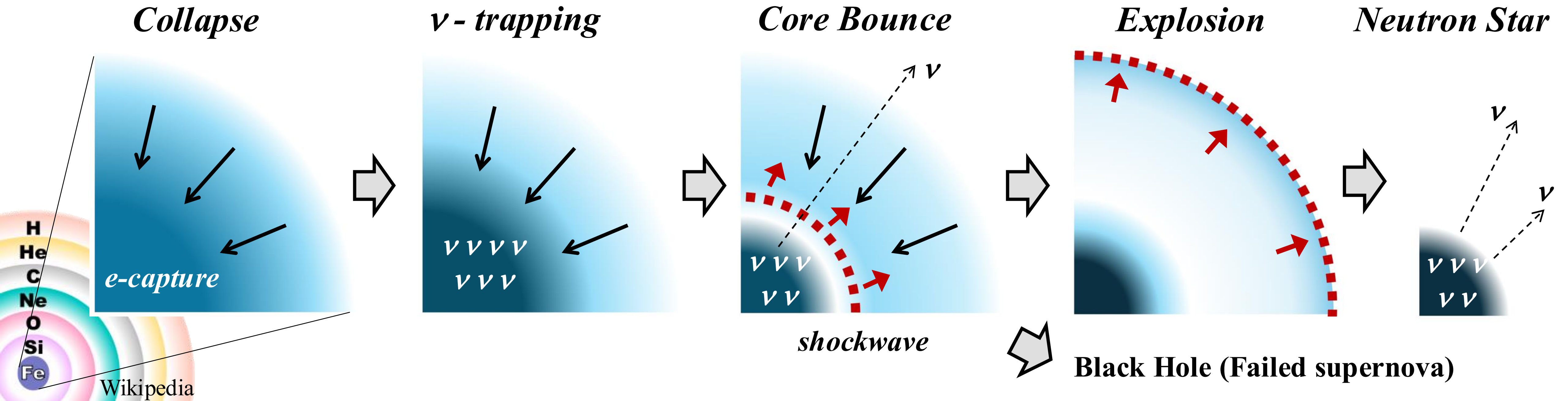
(MNRAS 508 (2021) 966)

1. Introduction

1. **Nuclear equation of state (EOS) for dense matter**
2. **Nuclear weak reactions with neutrino**

Those are essential ingredients that should be provided by Nuclear Physics for Compact Star studies.

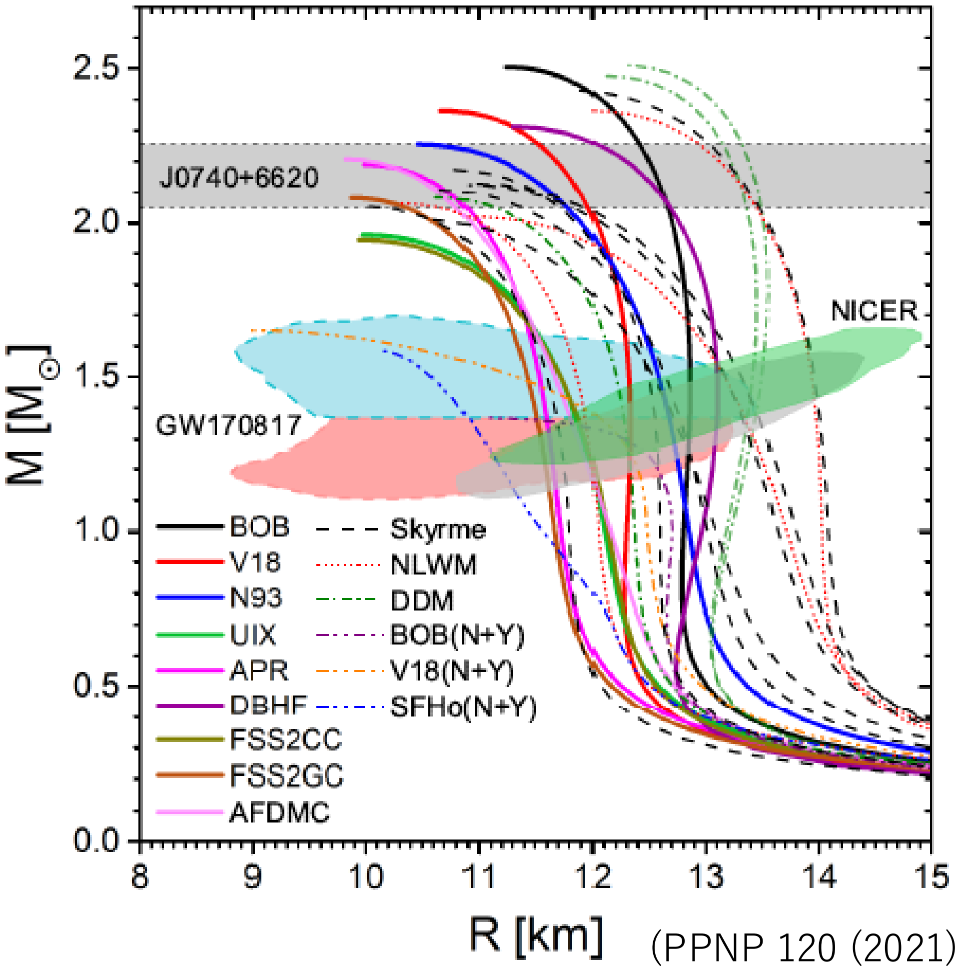
Core-collapse mechanism of massive stars



EOS for Neutron Stars & Supernovae

Neutron Stars

- T = 0 MeV, Y_p ~ 0.1
- Various EOS has been proposed.

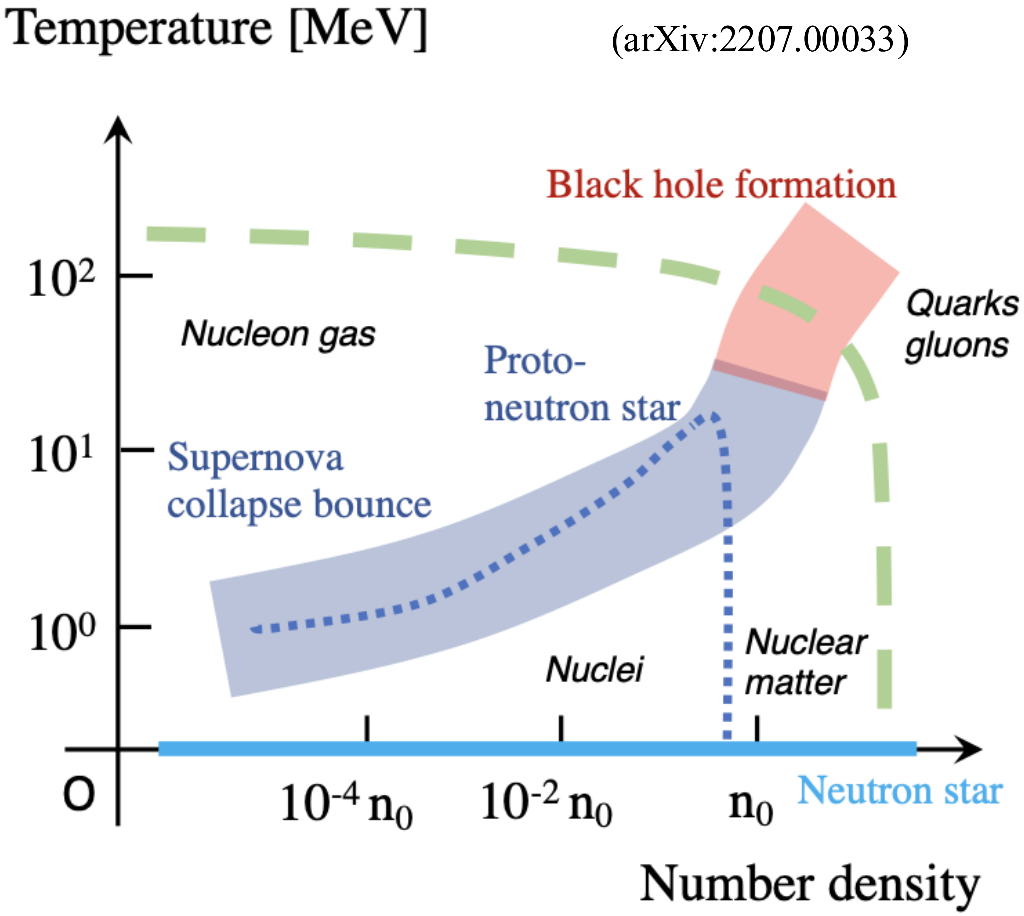


(PPNP 120 (2021) 103879)

Core-Collapse Supernovae

- Wide range of T, Y_p, n_B
- Limited number of EOSs are applicable.

(arXiv:2207.00033)



Current Status and Next Tasks

<u>Variational EOS with realistic nuclear forces</u>

(**HT**, K. Nakazato, Y. Takehara, S. Yamamuro, H. Suzuki, and M. Takano, NPA961 (2017) 78)

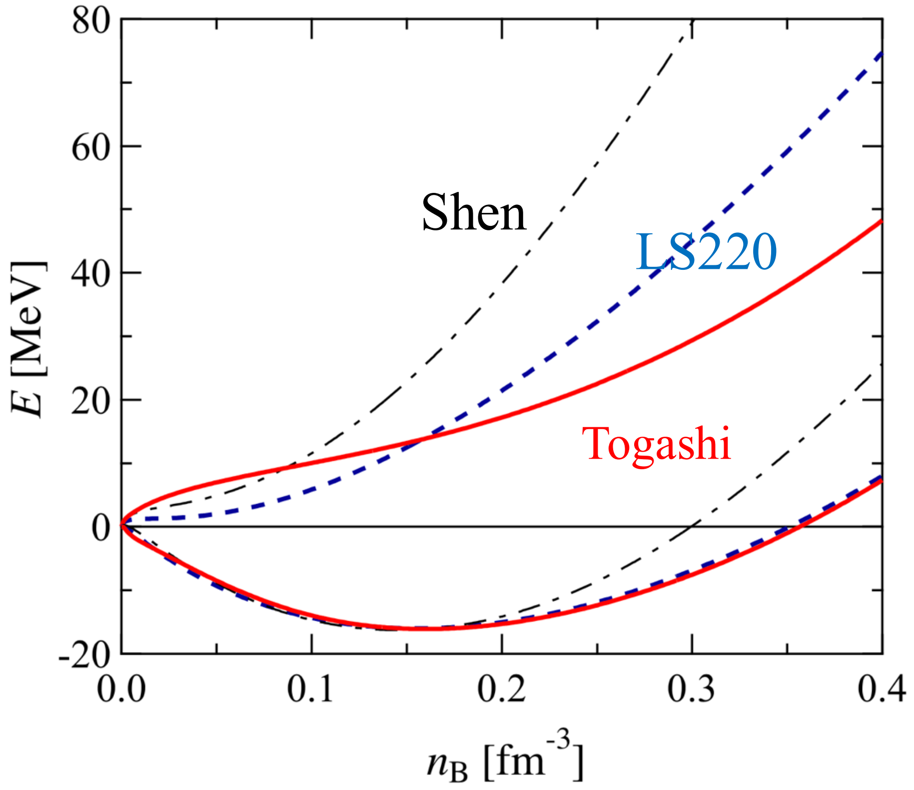
Constructed by ***the variational many-body theory*** *with bare nuclear forces (****AV18+UIX****)*

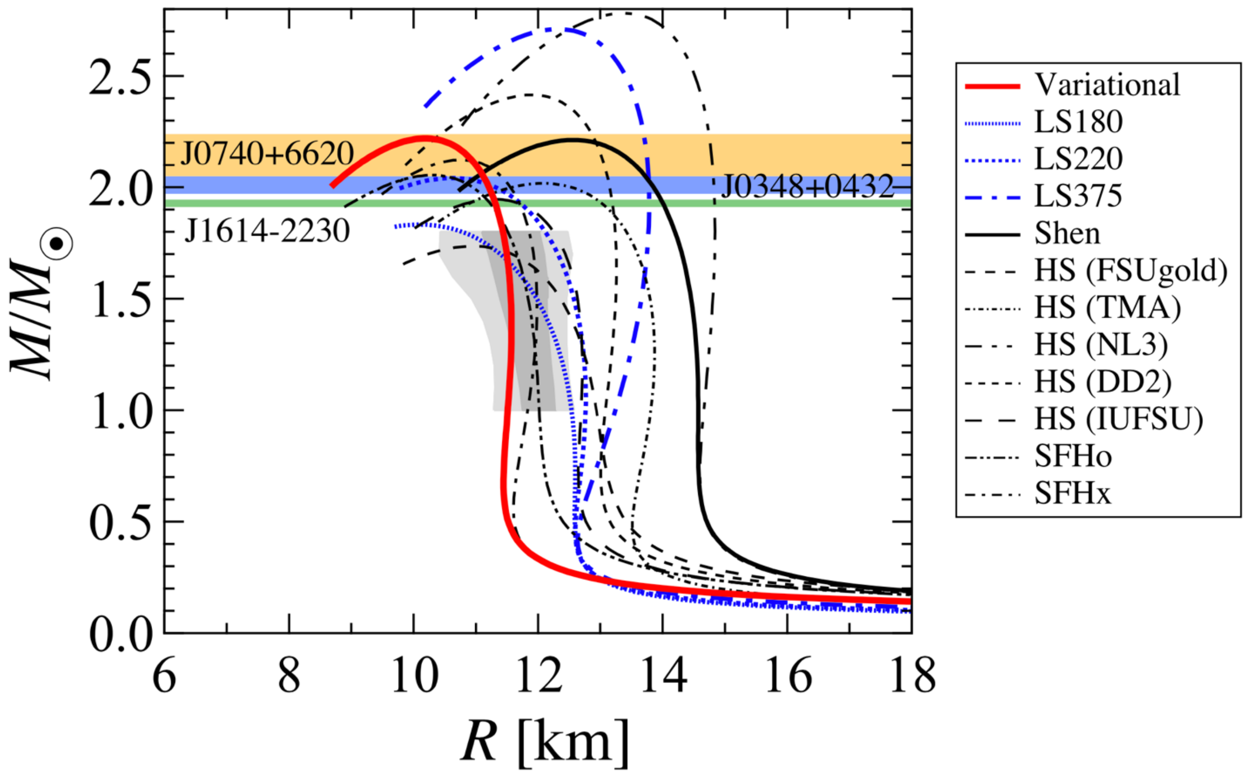
- This EOS is applicable to the studies for **<u>neutron stars</u>** <u>and</u> **<u>core-collapse supernovae</u>**<u>.</u>
- This EOS table is available on the web: **`http://www.np.phys.waseda.ac.jp/EOS/`**

EOS dependence in core-collapse simulations is gradually being investigated.

For a more sophisticated simulations to understand the supernova mechanism,
we aim to construct nuclear reaction rates with neutrino in a self-consistent manner.

2. Supernova simulation with the variational EOS





【Uniform phase】 【Non-uniform phase】

- **LS EOS**: Skyrme Hartree-Fock + Compressible Liquid Drop model
- **Shen EOS**: Relativistic Mean Field (TM1) + Thomas-Fermi calculation
- **Togashi EOS:** Variational method (AV18+UIX) + Thomas-Fermi calculation

Application to core-collapse simulations

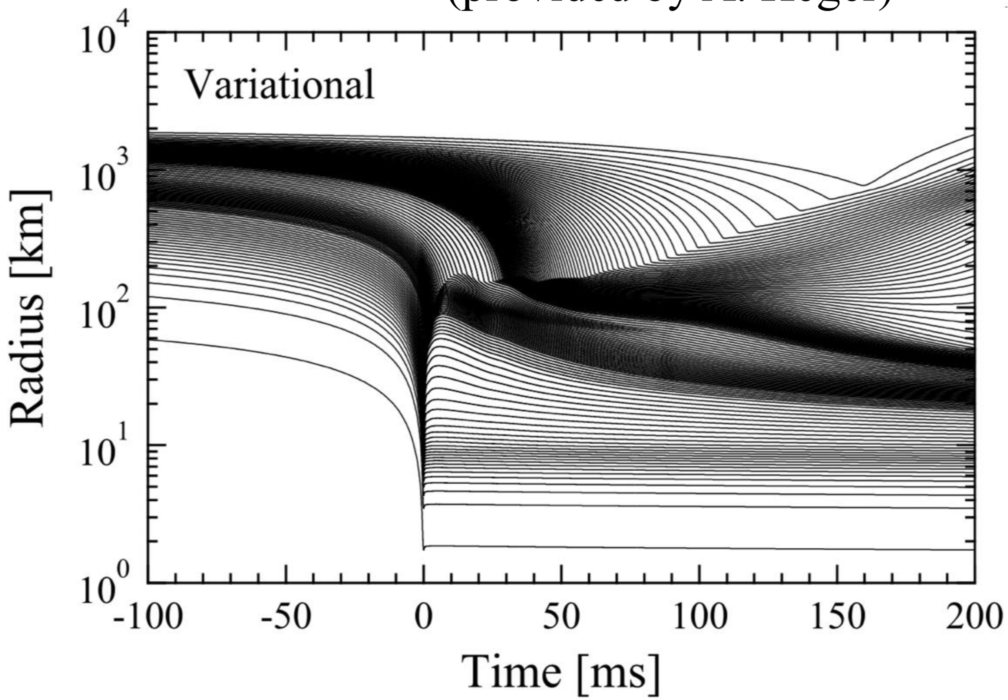
1D neutrino-radiation hydrodynamics simulations

(Nakazato, Sumiyoshi & HT, PASJ 73 (2021) 639)

- **EOS: Togashi / Shen / LS220 / LS180**
- Progenitor model : 9.6 $M_\odot$ / 15 $M_\odot$ / 30 $M_\odot$
- Neutrino Transport: Directly solve the Boltzmann equation

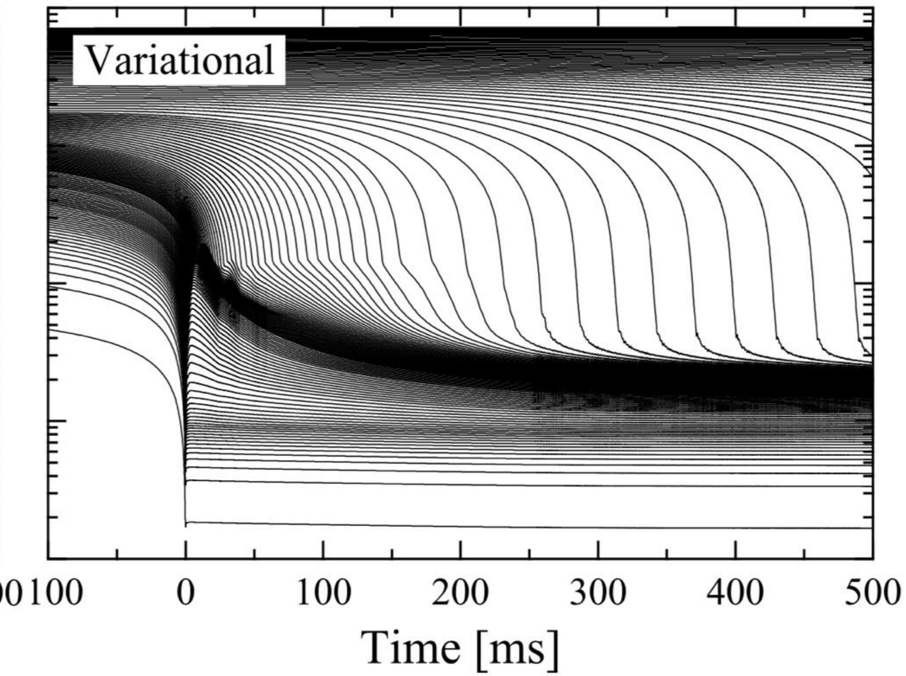
Progenitor model : 9.6 $M_\odot$

(provided by A. Heger)



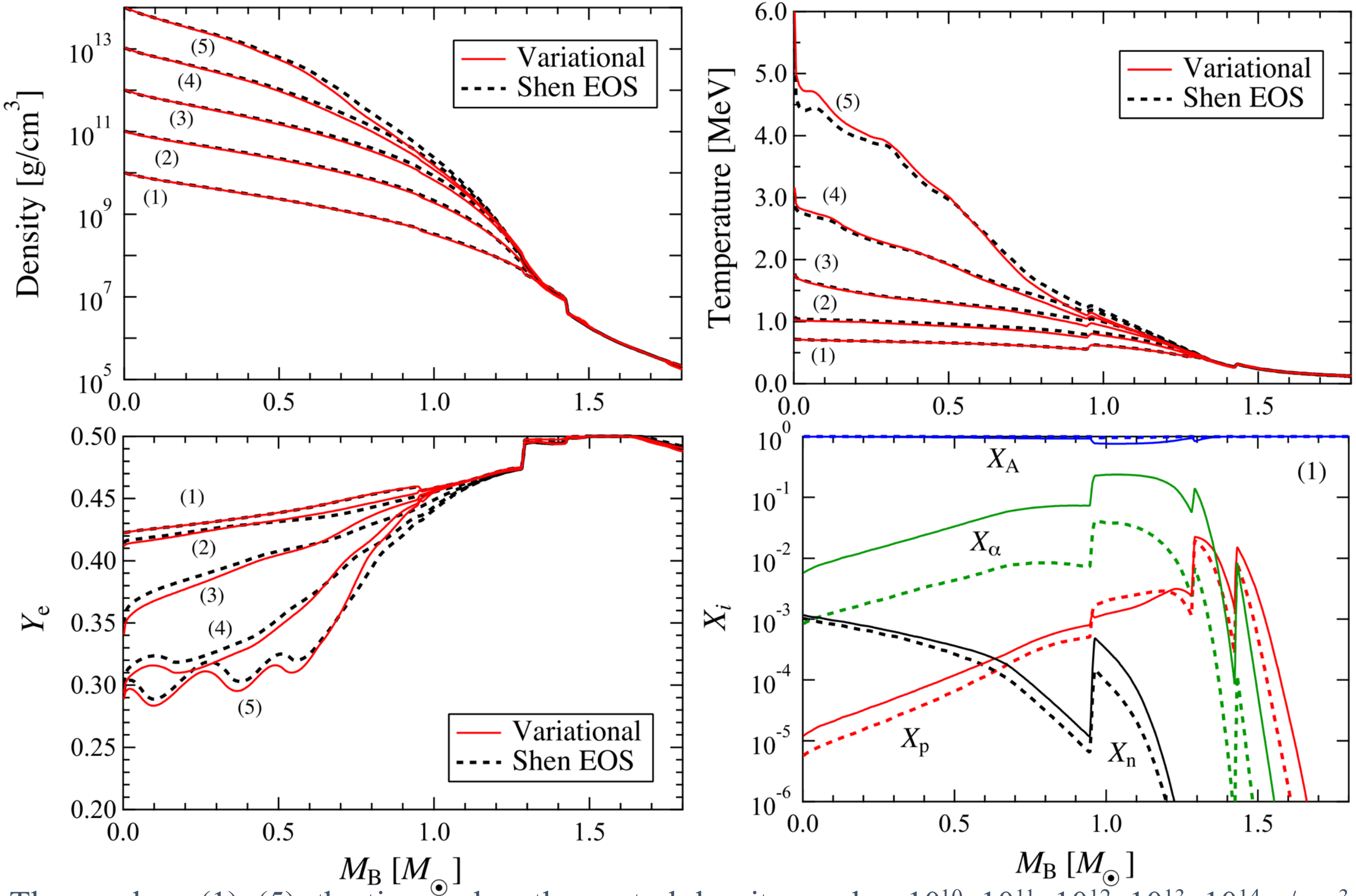
Progenitor model: WW 15$M_\odot$

(Astrophys. J. Suppl. 101 (1995) 181)



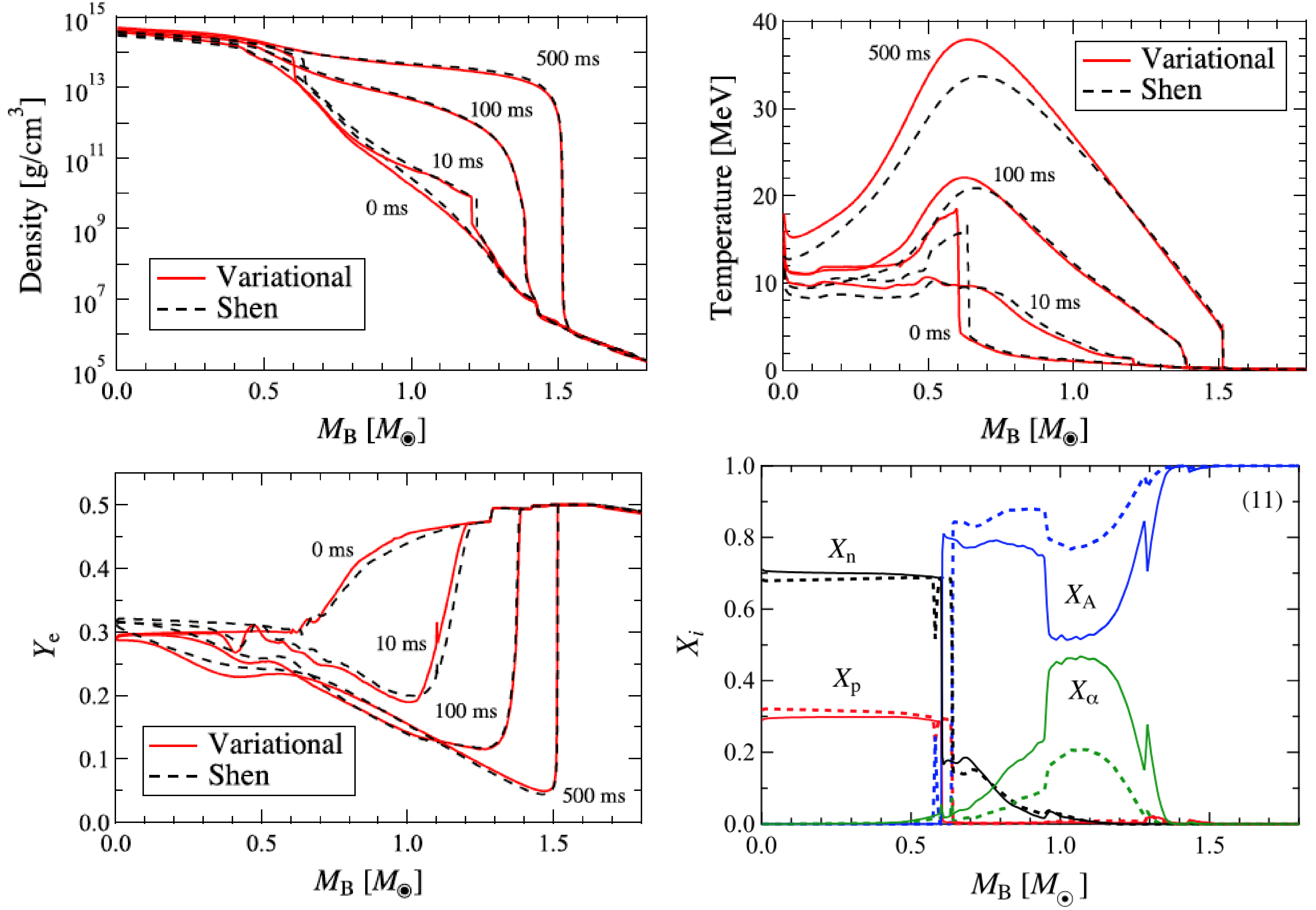
Radial trajectories of mass elements

Thermodynamic Profiles (Collapse Phase)



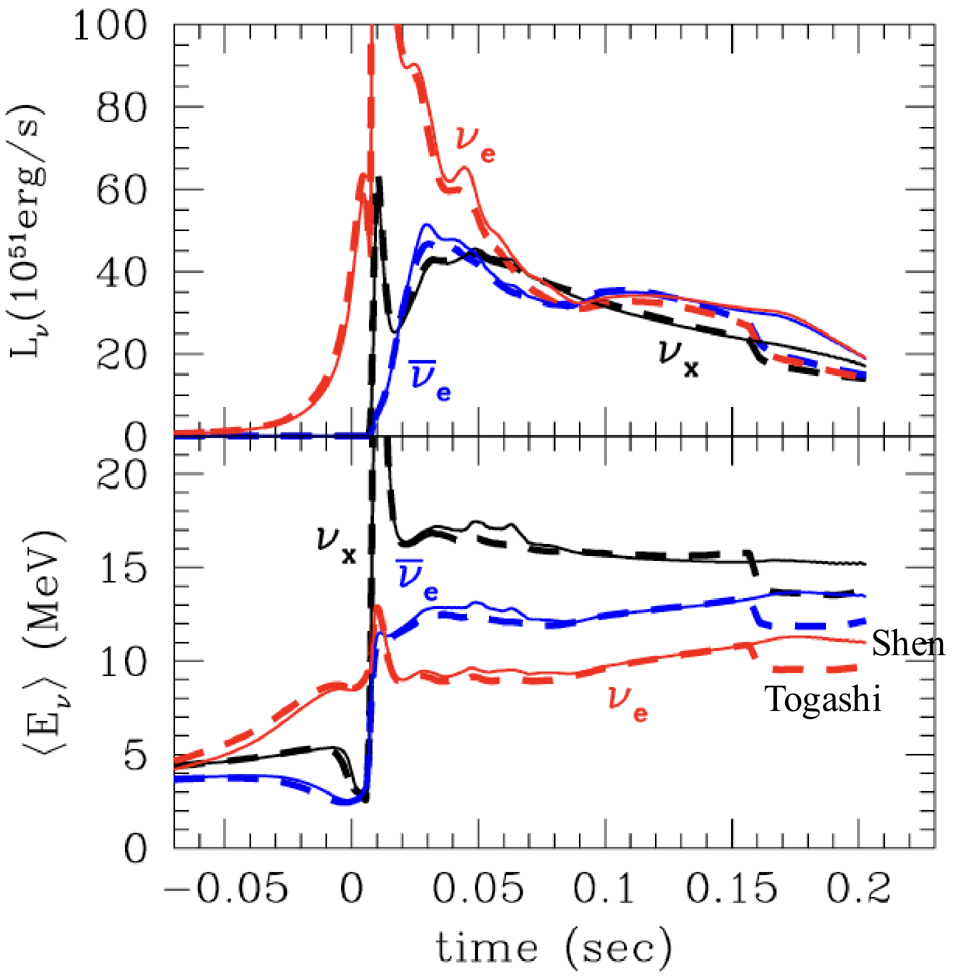
The numbers (1)–(5) :the times when the central density reaches 10^{10}, 10^{11}, 10^{12}, 10^{13}, 10^{14} g/cm^3

Thermodynamic Profiles (Postbounce Phase)

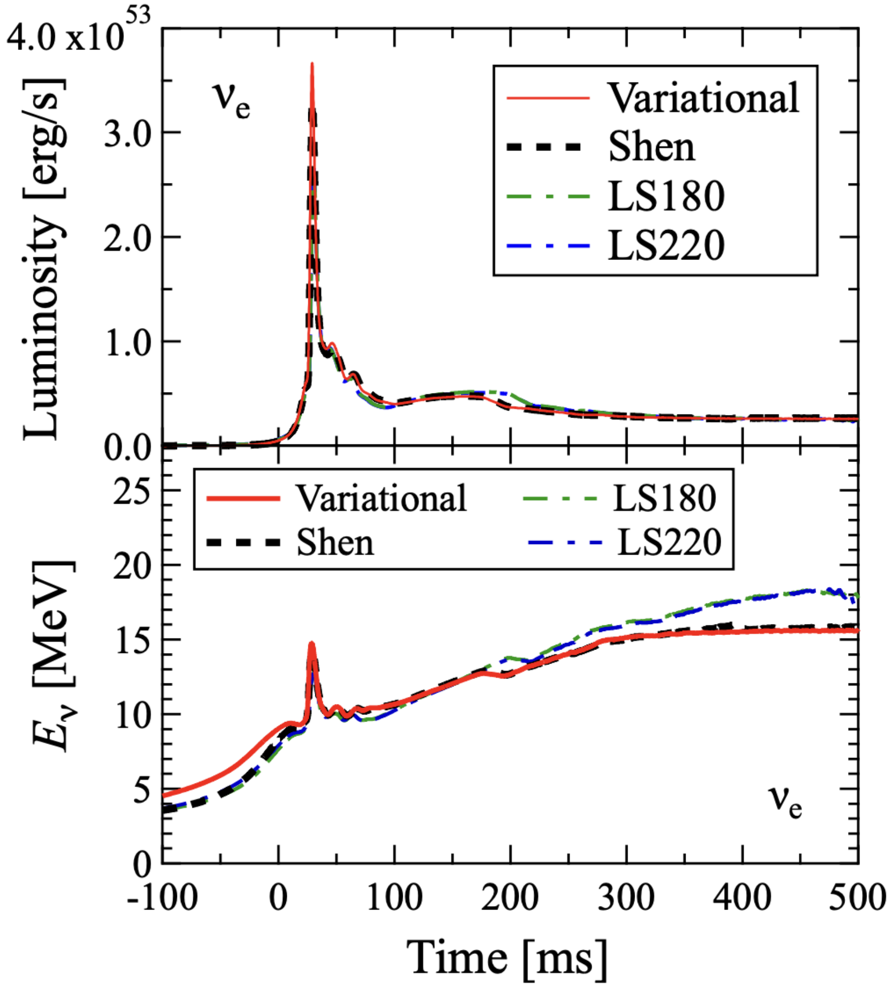


Neutrino Luminosity and average energy

Progenitor model : 9.6 $M_\odot$



Progenitor model : WW 15$M_\odot$



(Nakazato, Sumiyoshi & HT, PASJ 73 (2021) 639)

Effect of the EOS is small in emitted neutrino from supernova (!?)

3. Nuclear Weak Interaction for Core-Collapse Simulations

1. electron-type neutrino absorption on neutrons and its inverse,

$$\nu_e + n \longleftrightarrow e^- + p,$$

2. electron-type antineutrino absorption on protons and its inverse,

$$\bar{\nu}_e + p \longleftrightarrow e^+ + n,$$

3. neutrino scattering on nucleons,

$$\nu + N \longleftrightarrow \nu + N,$$

TODAY'S TALK

4. neutrino scattering on electrons,

$$\nu + e^- \longleftrightarrow \nu + e^-,$$

5. electron-type neutrino absorption on nuclei,

$$\nu_e + A \longleftrightarrow A + e^-,$$

6. neutrino coherent scattering on nuclei,

$$\nu + A \longleftrightarrow \nu + A,$$

7. electron-positron pair annihilation and creation,

$$e^- + e^+ \longleftrightarrow \nu + \bar{\nu},$$

8. plasmon decay and creation,

$$\gamma^* \longleftrightarrow \nu + \bar{\nu},$$

9. neutrino bremsstrahlung,

$$N + N' \longleftrightarrow N + N' + \nu + \bar{\nu}.$$

We aim to calculate the neutrino reaction rates in a nuclear medium by using the cluster variational method.

Neutrino Transport in Supernova Matter

We directly solve the Boltzmann equation for neutrino distributions.

- Neutrino distribution function: $f_\nu(t, r, E_\nu, \theta_\nu)$

$$\frac{\partial f_\nu}{c\,\partial t} + \mu \frac{\partial f_\nu}{\partial r} + \frac{(1-\mu^2)}{r}\frac{\partial f_\nu}{\partial \mu} = \left[\frac{\delta f_\nu}{c\,\delta t}\right]_{collision}$$

$\mu = \cos\theta_\nu$: angle

Neutrino source term which depends on various neutrino interaction rates

Difficulties in applying to astrophysical simulations

- Wide range of T, Y_p, n_B and neutrino energy ω
- Interaction rates need to be given in analytical forms!

(Computer memory is already fully occupied by the EOS table)

I. Neutrino Charged Current Reactions on Nucleons

$$\nu_e + n \longleftrightarrow e^- + p \qquad \bar{\nu}_e + p \longleftrightarrow e^+ + n$$

Fermi interaction: $M = \frac{G}{\sqrt{2}} \bar{u}_p(p_p)\gamma^\mu(g_V - g_A\gamma_5)u_n(p_n)\bar{u}_e(p_e)\gamma_\mu(1-\gamma_5)u_\nu(q)$

Emissivity:
$$Q(n_B, Y_p, T) = \frac{\hbar^8}{c^2}(2\pi)^4 \int \prod_{i=1}^{3}\left[\frac{d\boldsymbol{p}_i}{(2\pi\hbar^3)}\right]\frac{d\boldsymbol{q}_1}{(2\pi\hbar^3)2\omega}(\omega)$$
$$\times \quad \delta(E_{\mathrm{fin}} - E_{\mathrm{ini}})\delta^3(\boldsymbol{P}_{\mathrm{fin}} - \boldsymbol{P}_{\mathrm{ini}})f_1(\epsilon_1)f_2(\epsilon_2)[1 - f_3(\epsilon_3)]\sum_{\mathrm{spin}}|M|^2$$

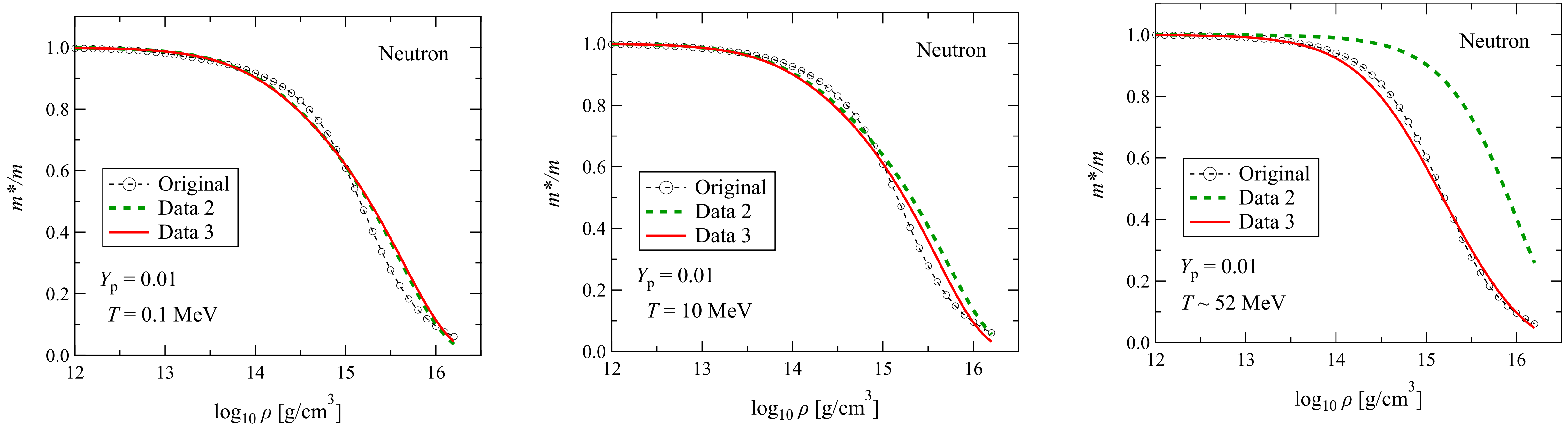
Occupation probabilities obtained from the variational method are applied.

$$f_i(k) = \left\{1 + \exp\left[\frac{\varepsilon_i(k) - \mu_{0i}}{k_B T}\right]\right\}^{-1} \qquad \varepsilon_i(k) = \frac{\hbar^2 k^2}{2m_i^*}$$

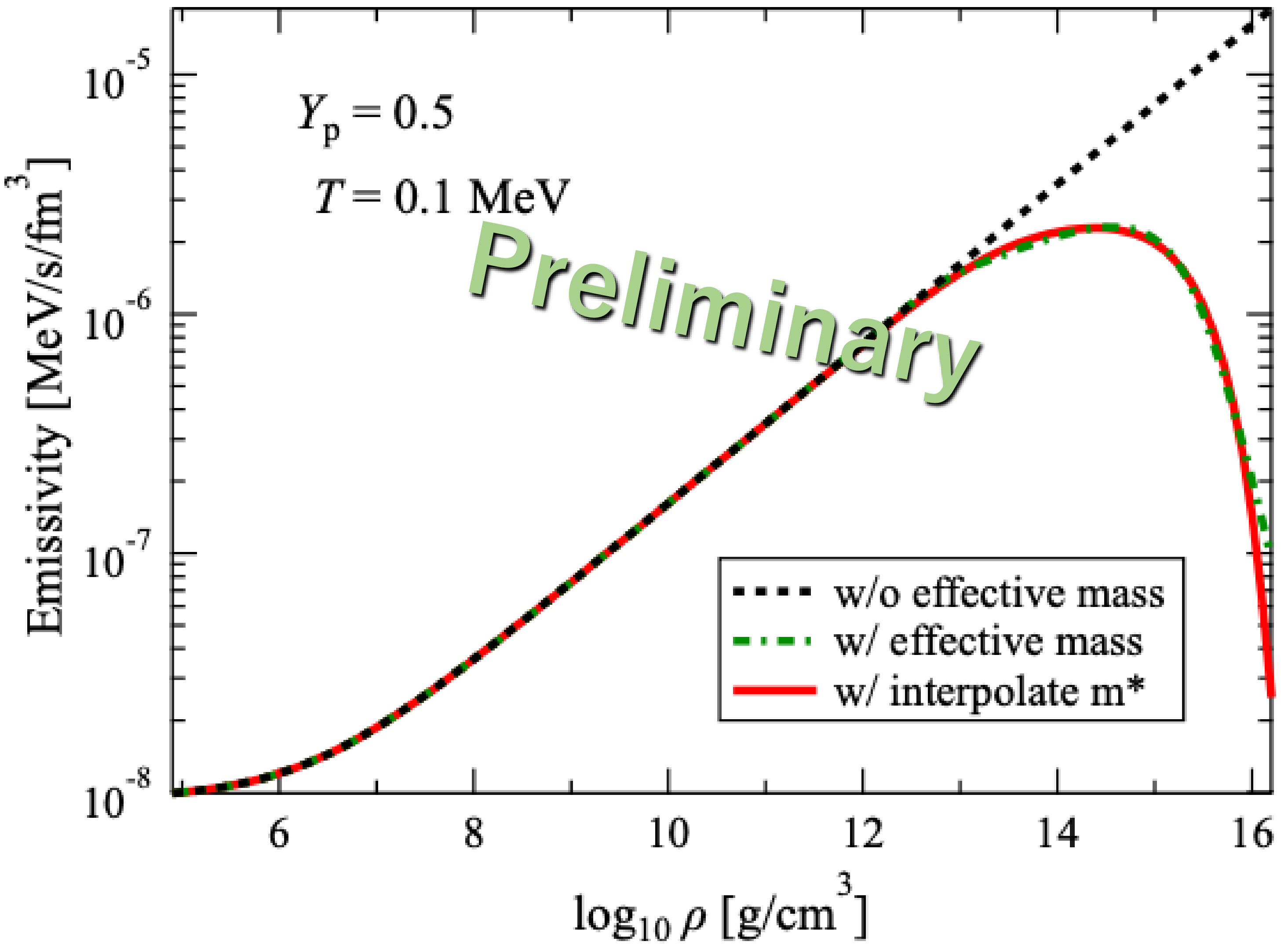
Fitting of the nucleon effective masses

$$\boxed{m_b^*/m = \left[1 + \left\{a_1(T) + a_2(T)Y_b + a_3(T)Y_b^2\right\} n_{\mathrm{B}} e^{n_{\mathrm{B}} a_4(T)}\right]^{-a_5(T)}}$$

$$a_i(T) = c_{i0} + c_{i1}T + c_{i2}e^{-\left(\frac{T-c_{i3}}{c_{i4}}\right)^2} + c_{i5}e^{-\left(\frac{T-c_{i6}}{c_{i7}}\right)^2} (i = 1, 2, 3, 4) \qquad a_5(T) = c_{50} + \sum_{j=1}^{8} c_{5j}T^j$$



Neutrino Emissivity with the variational method

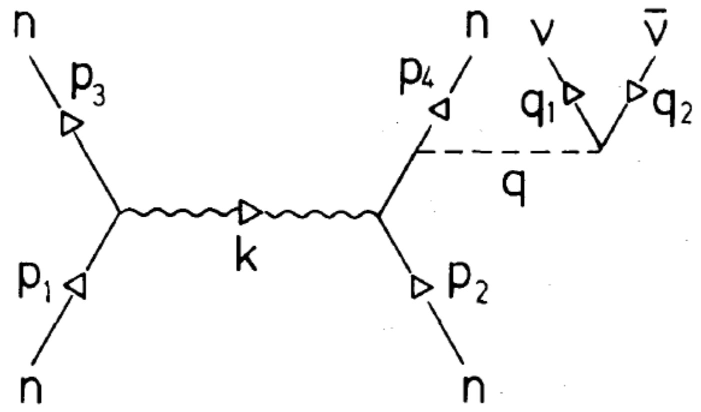


II. Nucleon Bremsstrahlung Process

One-Pion Exchange Potential

B. L. Friman and O. V. Maxwell, Astrophys. J. 232 (1979) 451.

O. V. Maxwell, Astrophys. J. 316 (1987) 691.



$$V_{12}(\mathbf{k}) = -\left(\frac{f}{m_\pi}\right)^2 (\tau_1 \cdot \tau_2) \frac{(\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k})}{k^2 + m_\pi^2}$$

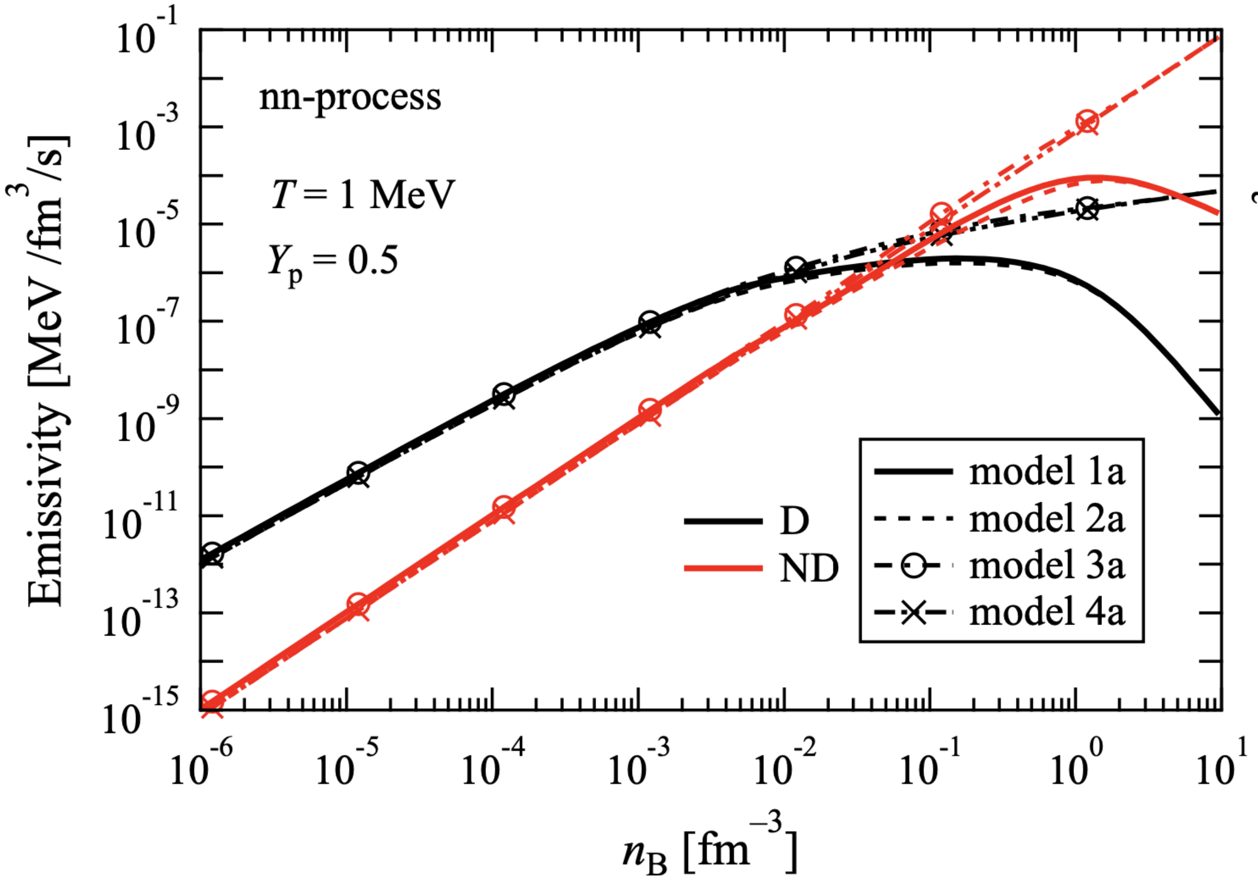
Emissivity

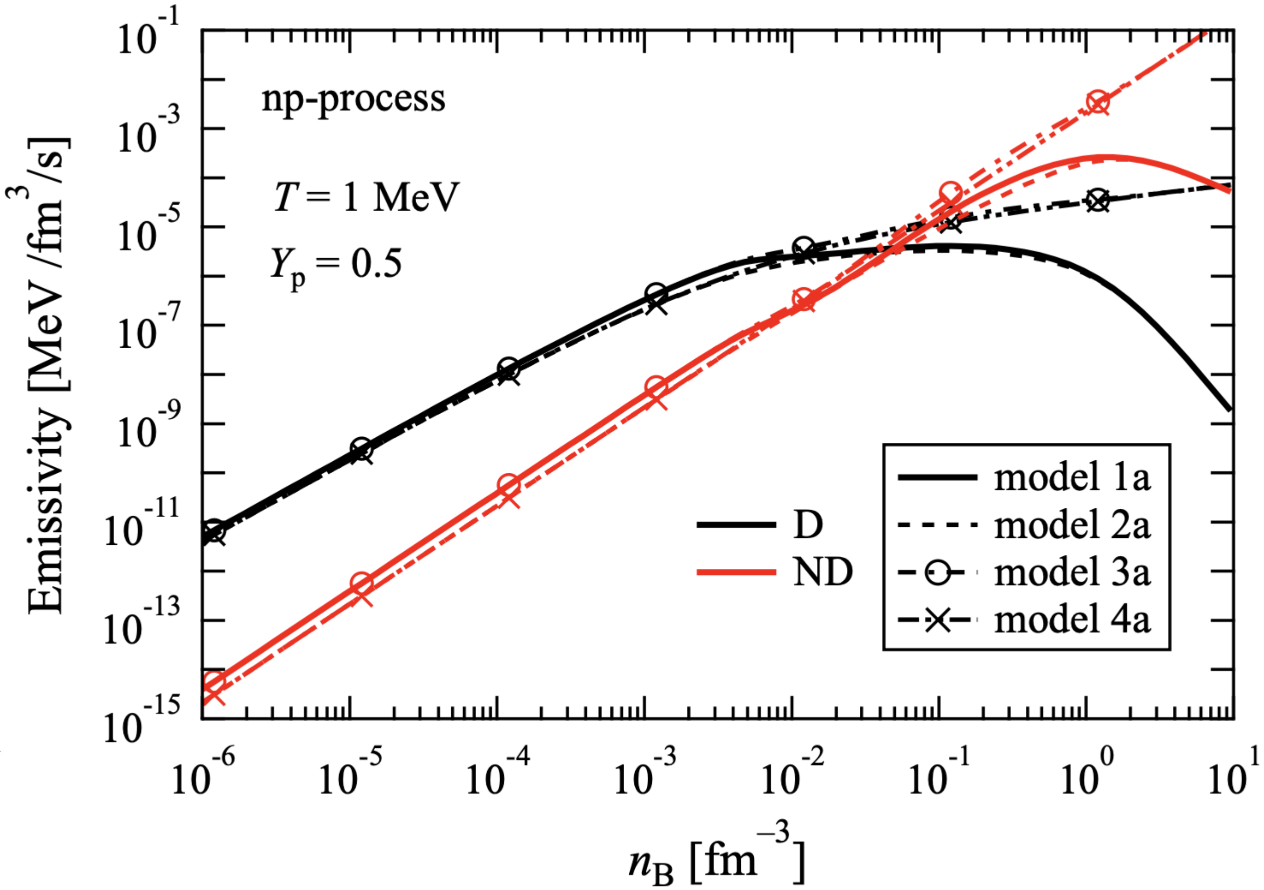
$$Q^{NN}(n_{\rm B}, Y_{\rm p}, T) = \frac{\hbar^8}{c^2}(2\pi)^4 \int \prod_{i=1}^{4} \left[\frac{d\boldsymbol{p}_i}{(2\pi\hbar^3)}\right] \frac{d\boldsymbol{q}_1}{(2\pi\hbar^3)2\omega_1} \frac{d\boldsymbol{q}_2}{(2\pi\hbar^3)2\omega_2} (\omega_1 + \omega_2)$$
$$\times \quad \delta(E_{\rm fin} - E_{\rm ini})\delta^3(\boldsymbol{P}_{\rm fin} - \boldsymbol{P}_{\rm ini}) f_1(\epsilon_1) f_2(\epsilon_2) \left[1 - f_3(\epsilon_3)\right] \left[1 - f_4(\epsilon_4)\right] s \sum_{\rm spin} |M_{NN}|^2$$

Occupation probabilities obtained from the variational method are applied.

Taking into account the correlation between two nucleons

Neutrino Emissivity





1a: f_i & $|M|$ are improved.

2a: f_i is improved.

3a: $|M|$ is improved.

4a: No improvement

III. Medium effects on neutrino-nucleon scattering

Neutrino scattering on nucleons in nuclear medium

Free-space: $$\frac{1}{N}\frac{d\sigma_0}{d\Omega} = \frac{G_F^2 E_\nu^2}{4\pi^2}\left(C_a^2(3-\cos\theta) + C_v^2(1+\cos\theta)\right)$$

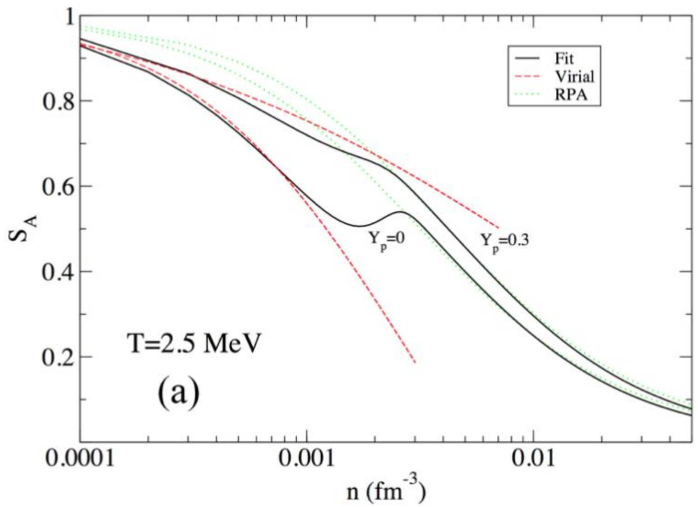
(C. J. Horowitz, PRD 65 (2002) 043001)
(C. J. Horowitz & A. Schwenk PLB 642 (2006) 326)
(C. J. Horowitz et al., PRC 95 (2017) 025801)

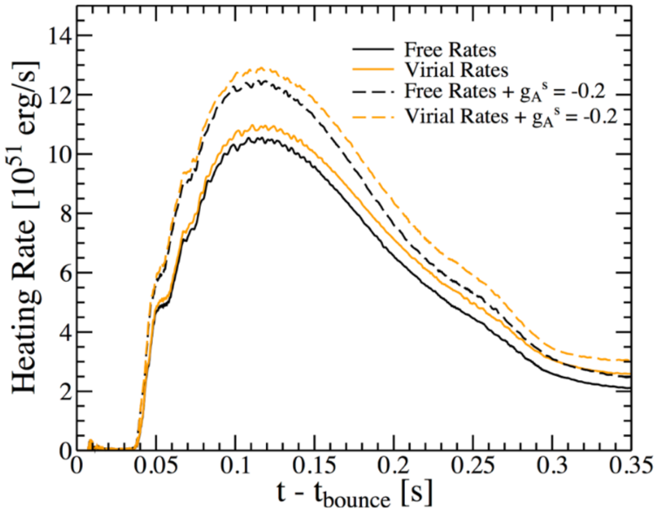
Scattering cross section In nucelar medium: $$\boxed{\frac{1}{N}\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_\nu^2}{16\pi^2}\left(g_a^2(3-\cos\theta)S_a(q) + (1+\cos\theta)S_v(q)\right)}$$

Spin susceptibility

Density vector response function: $S_V(q=0) = \frac{T}{(\partial P/\partial n)_T}$.

Spin axial response function: $S_a(q=0) = \frac{\chi}{\chi_F}$



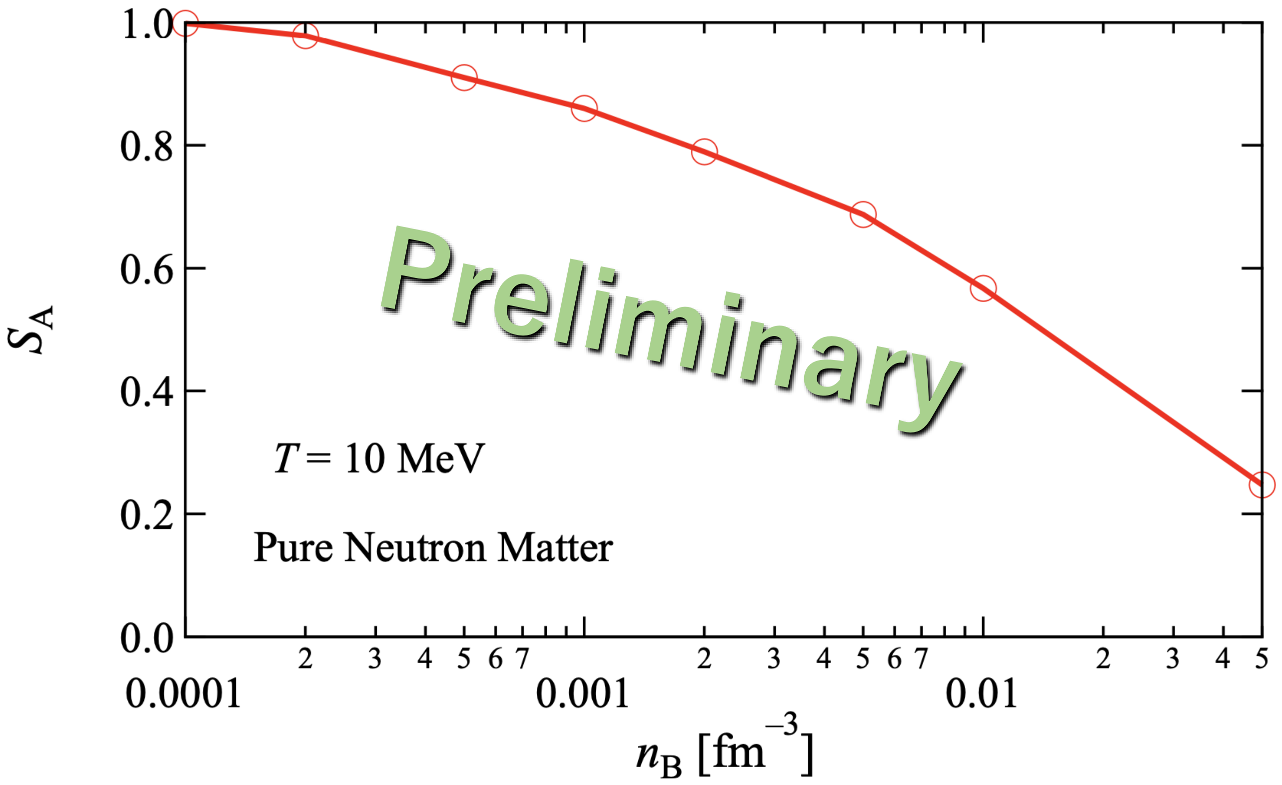


Axial response function for pure neutron matter

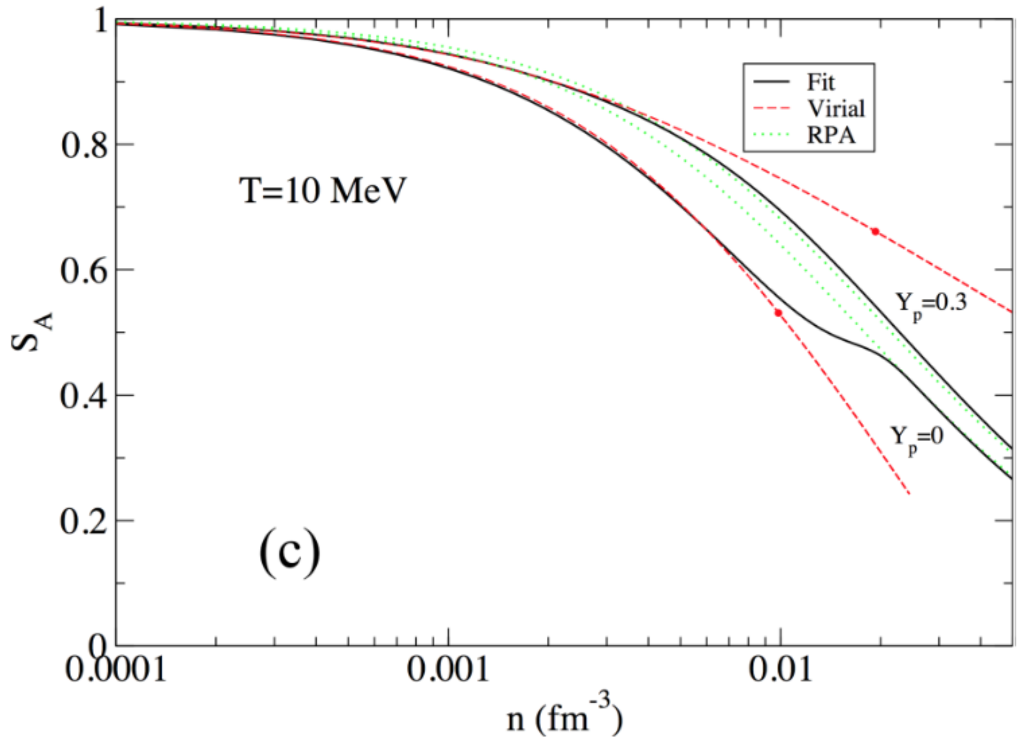
Neutrino scattering cross section in nuclear medium:

$$\frac{1}{N}\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_\nu^2}{16\pi^2}\left(g_a^2(3-\cos\theta)S_a(q) + (1+\cos\theta)S_v(q)\right),$$

【Cluster variational method】



【Virial & RPA method】



(C. J. Horowitz et al., PRC 95 (2017) 025801)

Summary

We have constructed the nuclear EOS for core-collapse supernovae by using the Argonne v 18 and Urbana IX potentials.

Effect of the nuclear EOS is small in emitted neutrino from supernova.

Nuclear weak reactions with neutrino are reformulated by using the cluster variational method.

→ **Consistent with the Togashi EOS**

Future Plans

- Supernova simulations with the obtained neutrino reaction rates
- Analytical expression of the interaction rates