Solution to the uncertainty problem of nuclear matrix element for neutrinoless double-β decay

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Talk plan

1. **Introduction — background**
2. **Review of my ββ NME study**
3. **Our method to solve the uncertainty problem**
   Collaboration with Dr. Civitarese (Univ. La Plata)
   i) **NME with perturbed transition operator**
   ii) **Estimation of effective g_A for 0vββ NME**
4. **Application of our method to 0vββ NME of other groups**
5. **Summary**

NME: nuclear matrix element
g_A: axial-vector current coupling
0vββ: neutrinoless double-β; 2vββ: two-neutrino double-β
QRPA: quasiparticle random-phase approximation

Introduction — background

Majorana neutrino search.

Its antiparticle is identical with itself, not yet found, and it is a fermion.

The 0νββ decay ⇨ Evidence of the Majorana nature of neutrino

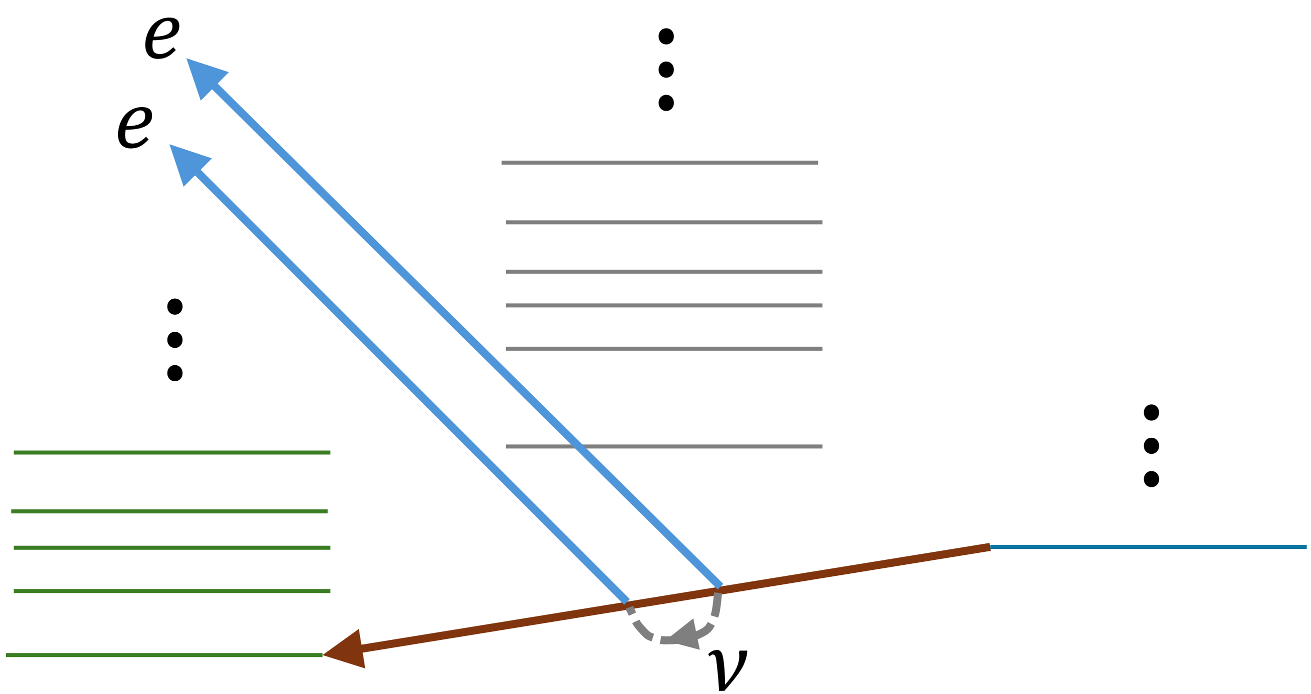
- Determination of neutrino mass scale

Around 2000, the massiveness of the neutrino was proven.
But the mass scale is *not yet established*.
One of the mass scale parameters is the **effective neutrino mass determined by the 0νββ decay**.

- Foundation of the theory to explain the matter prevalence in the universe

This theory, seesaw mechanism, needs the *left-handed and right-handed Majorana neutrinos*.
M. Fukugita, T. Yanagida, Phys. Lett. B, **174**, 45 (1986).

Appearance of 0vββ decay



$(N-2, Z+2)$
Final nucleus

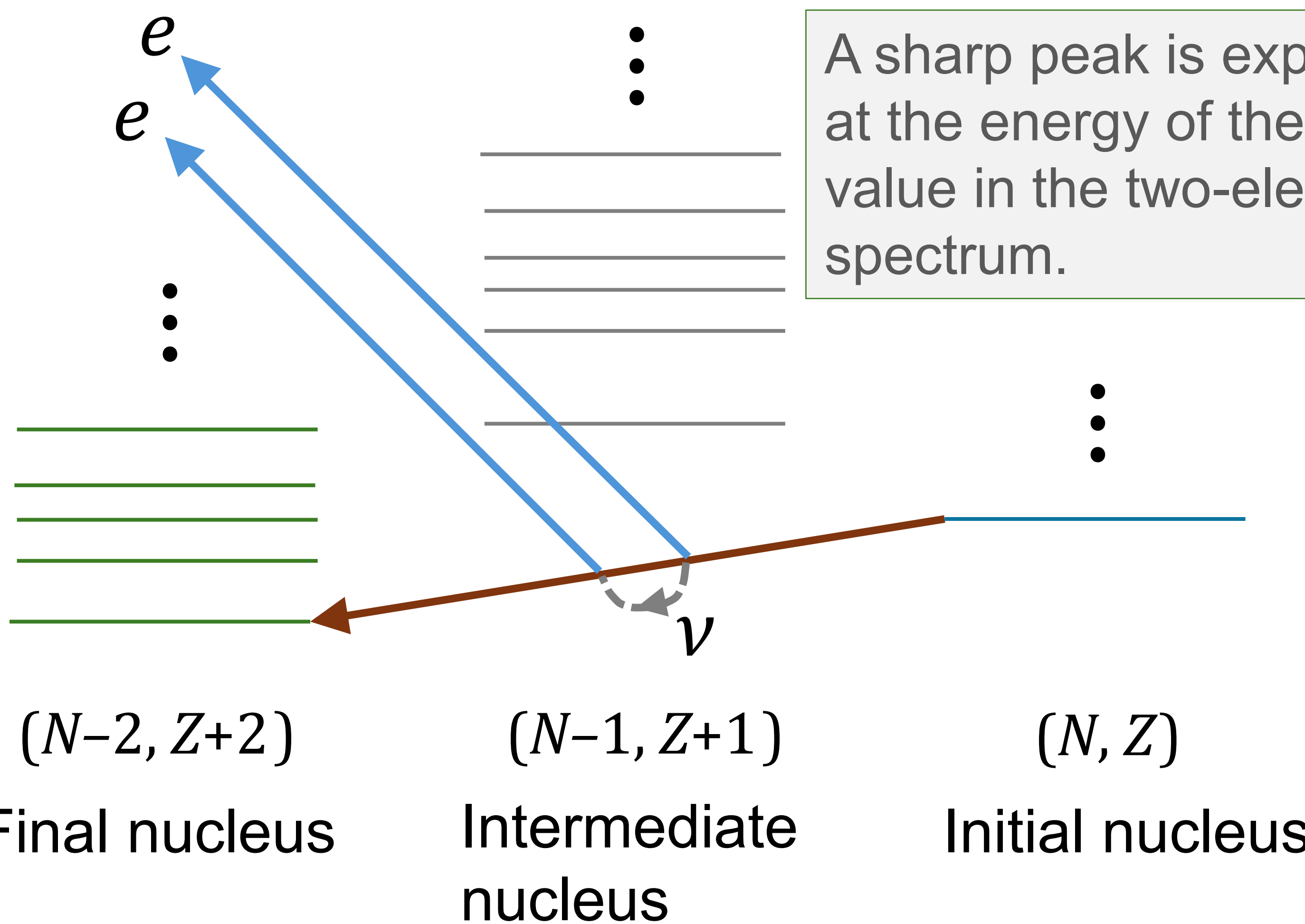
$(N-1, Z+1)$
Intermediate nucleus

(N, Z)
Initial nucleus

Appearance of 0νββ decay

Unlike usual weak decays, no antineutrino is emitted. The lepton number is not conserved.

A sharp peak is expected at the energy of the Q value in the two-electron spectrum.



Probability of 0νββ decay

$$1/T_{0\nu}(0^+ \to 0^+) = \left|M^{(0\nu)}\right|^2 G_{01} \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2$$

Half-life

Experimental measurement

Nuclear matrix element

Phase-space factor

Effective ν mass

Theoretical calculation

Effective neutrino mass

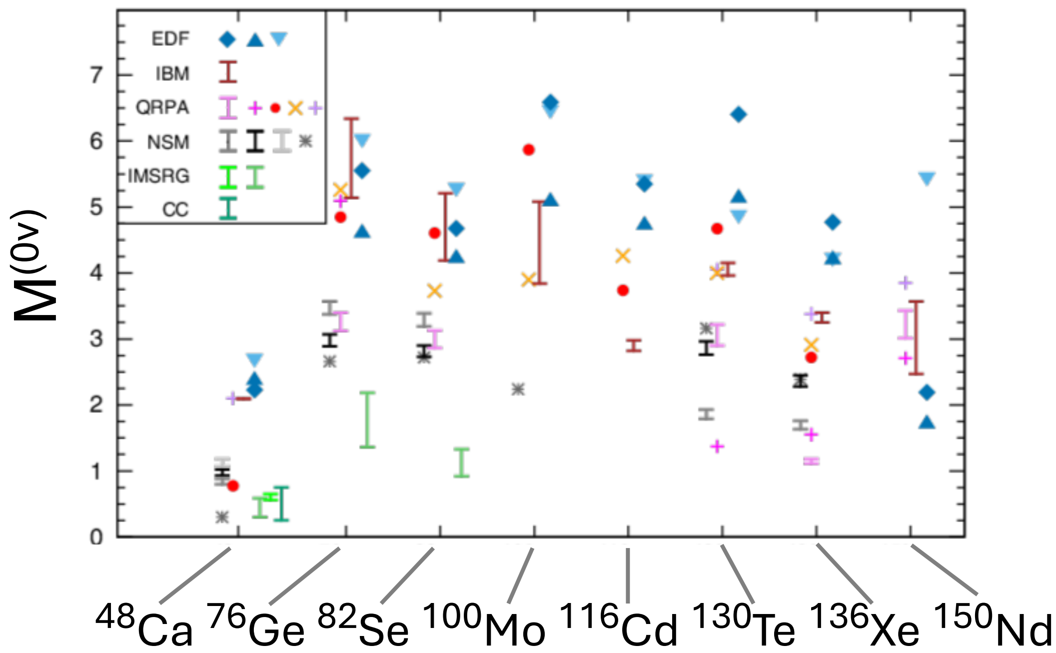
$$\langle m_\nu \rangle = \left| \sum_{i=1,2,3} U_{ei}^2 m_i \right|$$

U_{ei}: PMNS matrix element
m_i: neutrino eigen mass

PMNS matrix: Pontecorvo-Maki-Nakagawa-Sakata matrix.
Unitary transformation between the mass and flavor eigen bases of the neutrino.

Compilation of calculated $M^{(0\nu)}$

M. Agostini et al., Rev. Mod. Phys., **95,** 025002 (2023)



The uncertainty of the distribution has not changed since a few decades ago.
The number of calculations increased.

Review of my ββ NME study

Improvement of application of QRPA

1. The QRPA is a method to obtain the transition from the ground to excited states.

   In the QRPA approach to the ββ decay, the intermediate states are given by the excited states of the QRPA. There are two ways to obtain these states; one is obtained by the transition from the initial state, and the other is from the final states. To obtain the ββ NME, the QRPA approach calculates the **overlap** between the two QRPA wave functions obtained by the different transitions.

I introduced a mathematically rigorous calculation of the overlap. J. T. PRC **87**, 024316 (2013)

Review of my ββ NME study

2. How to determine strength of the isoscalar pairing intn. for the QRPA

The usual method

This intn. strength is determined so as to reproduce the 2νββ half-life with

$g_A = g_A^{\text{bare}} = 1.25\text{–}1.27$ or 1.0

This strength is close to the breaking point of QRPA.

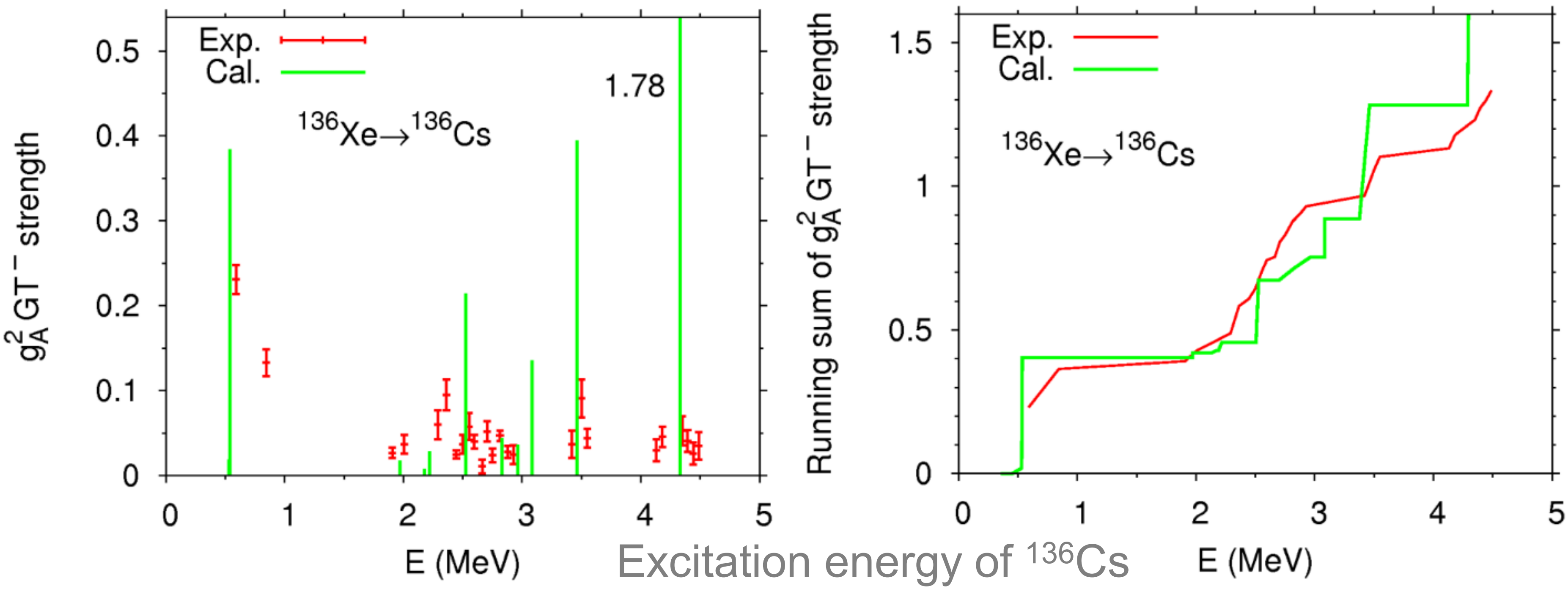
My method

That strength ← $\boxed{\text{0νββ NME = NME by two-neutron removal and two-proton addition under the closure approximation.}}$

➡ The strength is moderate. QRPA is OK.

J. T. PRC **102**, 044303 (2020)

Exp. data and calculation of g_A^2 x GT$^-$ strength



^{136}Xe(^{3}He,t)^{136}Cs and e capture

Used for the normalization

Exp. data from D. Frekers et al., Nucl. Phys. A **916**, 219 (2013);

J.T. used g_A^{eff} = 0.49.

to reproduce exp. half-life of 2νββ decay

J. T., Phys. Rev. C, **100**, 034325 (2019)

Exp. data and calculation of g_A^2 x GT$^-$ strength

I also checked

- binding energy. Skyrme SkM* intn. is excellent.
- β decay spectrum and strength of ^{138}Xe; acceptable.
- Ikeda sum rule → single-particle space large enough.
- Comparison of spectrum of ^{136}Cs obtained from ^{136}Xe g.s. and that from ^{136}Ba

and others.

J. T. and Y. Iwata, PRC **100**, 034325 (2019).

QRPA + $\underbrace{\text{SkM* is}}_{}$ ... + that pairing intn.

QRPA + SkM* is a good approximation for ^{136}Xe and ^{136}Ba

J.T. used g_A^{eff} = 0.49. ← to reproduce exp. half-life of 2νββ decay

J. T., Phys. Rev. C, **100**, 034325 (2019)

Experimental half-life of ^{136}Xe for 2νββ decay

$$2.18 \times 10^{21}\ \mathrm{yr}$$

A.S. Barabash, Proc. MEDEX'19 (AIP pub., Melville, NY, 2019) p.020002

My cal. (SkM*) value with $g_A = 1.267$ (bare value) is

$$0.03 \times 10^{21}\ \mathrm{yr}$$

Effective g_A to reproduce the exp. half-life is

$$0.422$$

J. T. and O. Civitarese, PRC **112**, 024304 (2025);
112, L051302 (2025)

My assumption:

If the calculation uses no approximation, the exp. half-life can be reproduced with the bare g_A.

Cf. the elecric transitions do not need an effective charge if the single-particle space is large enough.
J. T. et al., PRC **78**, 044311 (2008)

The QRPA is a good approximation for nuclear wave function of ^{136}Xe. My interaction strength is not close to the breaking point of the QRPA. My overlap calculation has no problem. The single-particle space is large enough.

But my calculation does not reproduce the exp. half-life without an effective g_A much smaller than the bare value.

What is missing?

The only possibility is that the transition operator is affected by the nucleon-nucleon interaction.

The decay takes time, during which interaction takes place between the nucleons.

Studies on the perturbed transition operator for β decay

A. Arima et al., *in* Adv. in Nucl. Phys. (1987) vol. 18, p. 1

I. S. Towner, Phys. Rep. **155**, 263 (1987)

P. Gysbers et al., Nat. Phys. **15**, 428 (2019) and others

NME with perturbed transition operator

Basic idea

We apply the Rayleigh-Schrödinger perturbation theory

Because the leading-order (usual) NME is derived by that perturbation formula.

J. T. and O. Civitarese, PRC **112**, 024304 (2025)

How?

1. $|\mathcal{J}\rangle = |I^{(0)}\rangle + |I^{(1)}\rangle + |I^{(2)}\rangle$

   : perturbed initial state of the system up to the 2nd-order due to an interaction $\mathcal{V}$ defined later.

   $|\tilde{\mathcal{F}}\rangle = |\tilde{F}^{(0)}\rangle + |\tilde{F}^{(1)}\rangle + |\tilde{F}^{(2)}\rangle$

: perturbed final state, prepared analogously.

Introduce the weak interaction H_W

Take the 2nd-order term of $\langle\tilde{\mathcal{F}}|H_W|\mathcal{J}\rangle$;

$$\langle\tilde{F}^{(0)}|H_W|I^{(2)}\rangle + \langle\tilde{F}^{(1)}|H_W|I^{(1)}\rangle + \langle\tilde{F}^{(2)}|H_W|I^{(0)}\rangle$$

2. Set $\mathcal{V} = H_W + :V:$, $:V:$ being the residual NN interaction. Take the terms with two H_W and one $:V:$.

Why do we do this?

Because the intermediate states are virtual states. For this, the 2nd order perturbation is necessary.

3. Remove the double-counting terms.

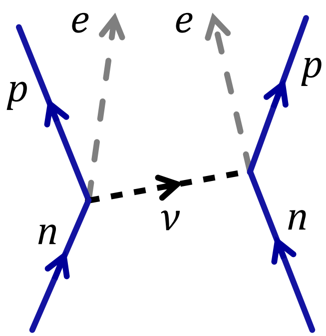
H_W of the transition operator and that of the perturbation interaction are not distinguished physically.

4. We derive the correction terms due to the perturbation of the transition operator.

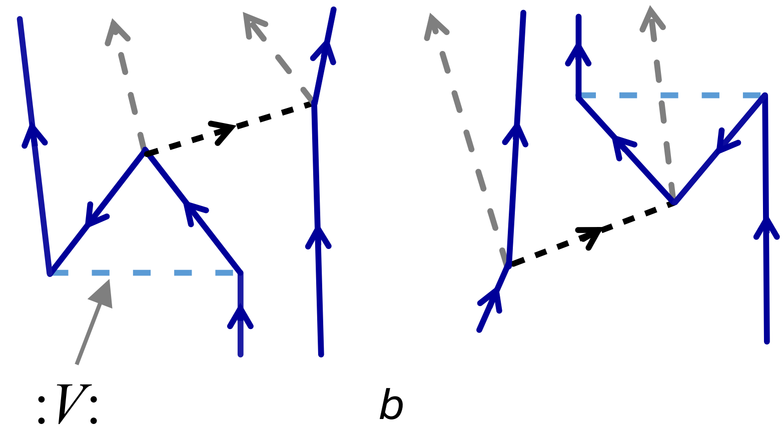
We pick up the terms referring to the diagrams. See next slide.

5. For 0νββ NME, the electron matrix element is moved to the phase space factor.

Diagrams for 0νββ NME



a



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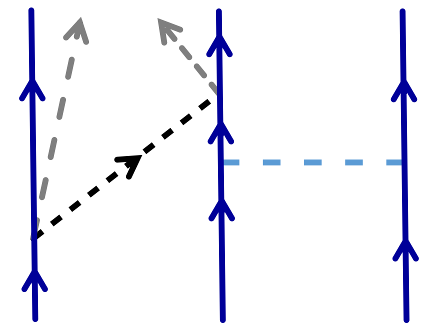
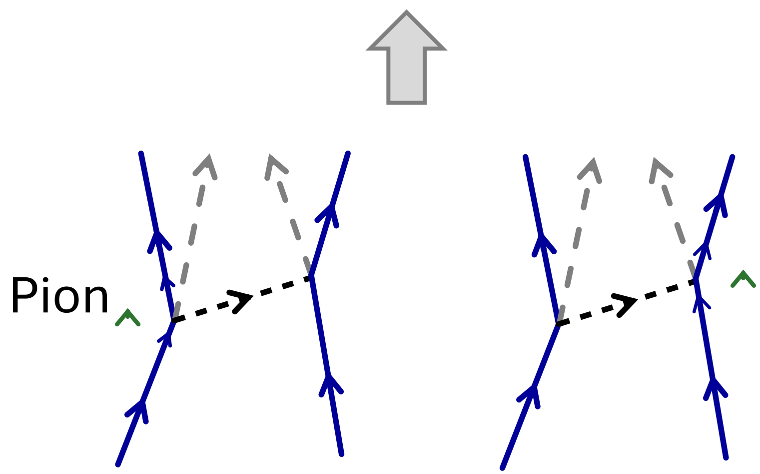


c

Leading -order

Vertex correction

Two-body current



Perturbed NMEs

$^{136}Xe \rightarrow {}^{136}Ba$, QRPA, Skyrme SkM* + Coulomb + pairing intn.

	0νββ NME		2νββ NME	
	GT	Fermi	GT	Fermi
Leading	3.095	–0.467	0.102	–0.002
VC	1.332	–0.984	0.055	–0.033
2BC	–2.731	1.758	–0.192	0.030
Perturbed (sum)	1.696	0.307	–0.035	–0.005

- The summed correction term (VC+2BC) of the NME is comparable and anti-coherent to the leading term except for the 2νββ Fermi NME, for which the summed correction is negligible.

Calculated half-life of ^{136}Xe for $2\nu\beta\beta$ decay

GT and F NME	g_A	Half-life (10^{21} y)
Leading	Bare g_A, 1.267	0.03
Perturbed	Bare g_A, 1.267	0.26
Experimental		2.18

The experimental value from A.S. Barabash, MEDEX'19 Proc. (AIP pub.)

Effective g_A to reproduce exp. $2\nu\beta\beta$ half-life

Used NME components	$g_A^{\mathrm{eff}}(\mathrm{exp})$	
	SkM*	SGII
Leading order	0.422	0.563
Perturbed	0.806	0.833

We obtained perturbed GT and Fermi NME components for 0νββ and 2νββ decays.

Our next step

To obtain effective g_A to reproduce the perturbed half-life with the bare g_A for both 0νββ and 2νββ decays.

Are the two effective g_A close or different?

Milestone to solve the uncertainty problem of 0νββ NME

Estimation of effective g_A for 0νββ NME

$$\frac{1}{T_{1/2}^{2\nu}} = G_{2\nu}\left|g_A^2 M_{2\nu}\right|^2$$

Effective g_A used with the leading $M_{2\nu}^{\mathrm{GT}(0)}$ and $M_{2\nu}^{\mathrm{F}(0)}$ to reproduce the experimental $T_{1/2}^{2\nu(\mathrm{exp})}$ is defined by

$$T_{1/2}^{2\nu}(M_{2\nu}^{\mathrm{GT}(0)}, M_{2\nu}^{\mathrm{F}(0)},\ g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{exp})) = T_{1/2}^{2\nu(\mathrm{exp})}.$$

leading

This is the usual effective g_A.

NEW

Effective g_A used with the leading $M_{2\nu}^{\mathrm{GT}(0)}$ and $M_{2\nu}^{\mathrm{F}(0)}$ to reproduce the half-life obtained from the perturbed NME components and the bare g_A

$$T_{1/2}^{2\nu}(M_{2\nu}^{\mathrm{GT}(0)}, M_{2\nu}^{\mathrm{F}(0)},\ g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt})) = T_{1/2}^{2\nu}(M_{2\nu}^{\mathrm{GT}},\ M_{2\nu}^{\mathrm{F}},\ g_{\mathrm{A}}^{\mathrm{bare}}).$$

perturbed

Analogously, for the 0νββ decay,

$$T_{1/2}^{0\nu}(M_{0\nu}^{\mathrm{GT}(0)}, M_{0\nu}^{\mathrm{F}(0)},\ g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt})) = T_{1/2}^{0\nu}(M_{0\nu}^{\mathrm{GT}},\ M_{0\nu}^{\mathrm{F}},\ g_{\mathrm{A}}^{\mathrm{bare}}).$$

Effective g_A numerically obtained

$^{136}Xe \rightarrow ^{136}Ba$, QRPA, Skyrme (SkM* and SGII)
Coulomb and contact pairing intn.

g_A specification	$g_{\mathrm{A}}^{\mathrm{eff}}$	
	SkM*	SGII
$g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld;pt})$	0.796	0.454
$g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld;pt})$	0.696	0.847

Most remarkable result

$$g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld;pt}) = 1.14 g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld;pt}), \qquad (\mathrm{SkM}^*)$$

The effective g_A for the 0vββ and 2vββ are close.

Usual discussion on $g_{A,0\nu}^{\text{eff}}$ and $g_{A,2\nu}^{\text{eff}}$

0νββ decay has a virtual neutrino with momentum

$$0 \leq q \leq \infty$$

⇨ Conjecture that $g_{A,0\nu}^{\text{eff}}$ and $g_{A,2\nu}^{\text{eff}}$ are quite different.

Argument 1

Major q for 0νββ NME is ≈ 100 MeV.

The 0νββ NME density in the r space conjugate to q has an outstandingly large peak at r = 1 fm.

Nucleon form factors are often included in the 0νββ NME density in the q space, *e.g.*,

$$F(q^2) = \frac{1}{(1 - q^2/M_A^2)^2}, \qquad M_A \approx 1\,\text{GeV}$$

J.D. Vergados, PRC **24**, 640 (1981)

⇨ Contribution of $F(q^2)$ to $g_{A,0\nu}^{\text{eff}}$ is small.

Argument 2

Two-body current (2bc) correction to the 0vββ NME

J. Menéndez et al., PRL **107**, 062501 (2011)

Another analysis of 2bc

J. Engel et al., PRC **89**, 064308 (2014)

They calculated the 0vββ and 2vββ NMEs with and without the 2bc for ^{76}Ge.

⇨ $g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt}) = 1.10 g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt})$

Argument 3

SGII does not results in similar $g_{\mathrm{A},0\nu}^{\mathrm{eff}}$ and $g_{\mathrm{A},2\nu}^{\mathrm{eff}}$.

$$g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt}) = 0.53 g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt})$$

Which is more reliable, SkM* or SGII?

⇨ That is SkM*. SGII overestimates the binding energy.

Argument

Two-body

J. Menénde

Another an

J. Engel et a

They calcu

without the

⇨ $g^{\mathrm{eff}}_{\mathrm{A},0\nu}$(l

	Binding E. /nucleon (MeV)	Discrepancy from exp. value (MeV)
Exp.	8.396	
SkM*	8.415	0.019
SGII	8.603	0.207

Exp. value is from `www.nndc.bnl.gov` (2025)
SkM* and SGII values are from the Hartree-Fock-Bogoliubov calculation of the ground state of ^{136}Xe.

Argument 3

SGII does not results in similar $g^{\mathrm{eff}}_{\mathrm{A},0\nu}$ and $g^{\mathrm{eff}}_{\mathrm{A},2\nu}$.

$$g^{\mathrm{eff}}_{\mathrm{A},0\nu}(\mathrm{ld};\mathrm{pt}) = 0.53\, g^{\mathrm{eff}}_{\mathrm{A},2\nu}(\mathrm{ld};\mathrm{pt})$$

Which is more reliable, SkM* or SGII?

⇨ That is SkM*. SGII overestimates the binding energy.

Idea to proceed

Half-life reproduced by the effective g_A

If the perturbed reference is replaced by the unperturbed one,

$$g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{ld}) = g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{ld}) = g_{A}^{\mathrm{bare}}$$

Referring to the half-life with 1st-order perturbation,

$$g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt}) \approx g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt})$$

The ratio of $g_{\mathrm{A},0\nu}^{\mathrm{eff}}$ and $g_{\mathrm{A},2\nu}^{\mathrm{eff}}$ is close to the convergence at the 1st order. Thus, I speculate

$$g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt}^{\infty}) \approx g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{exp})$$

I apply the phenomenological 2vββ g_A for the 0vββ NME.

The advantage: the exp. value nonperturbatively contains the many-body effects.

I will show two kinds of NMEs with $g_A{}^2$ multiplied.

- Previous (usual) one

$$\left(g_A^{\text{bare}}\right)^2 M_{0\nu}^{(0)} = \left(g_A^{\text{bare}}\right)^2 M_{0\nu}^{\text{GT}(0)} - g_V^2 M_{0\nu}^{\text{F}(0)}$$

- Modified NME with $\left(g_{A,0\nu}^{\text{eff}}\right)^2$

$$g_{A,0\nu}^{\text{eff}}(\text{ld; est}) = g_{A,2\nu}^{\text{eff}}(\text{ld; exp})\,\frac{g_{A,0\nu}^{\text{eff}}(\text{ld; pt})}{g_{A,2\nu}^{\text{eff}}(\text{ld; pt})},$$

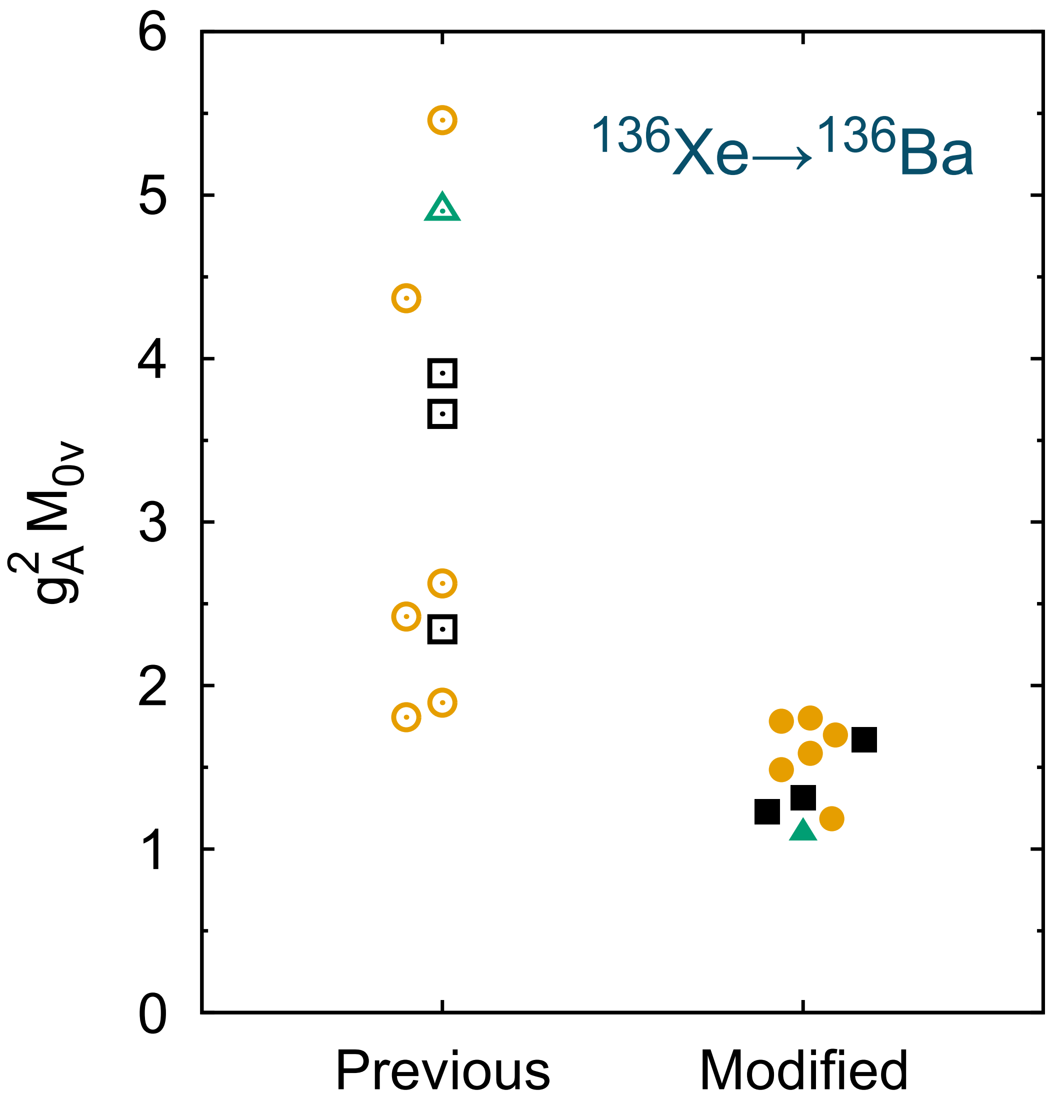
$$\left[g_{A,0\nu}^{\text{eff}}(\text{ld; est})\right]^2 M_{0\nu}^{\text{eff}} = \left[g_{A,0\nu}^{\text{eff}}(\text{ld; est})\right]^2 M_{0\nu}^{\text{GT}(0)} - g_V^2 M_{0\nu}^{\text{F}(0)}$$

This $g_{A,2\nu}^{\text{eff}}(\text{ld; exp})$ can be also obtained for the results of other groups.

g_A^{bare}: 1.25–1.27, depending on the group

We use our correction factor (1.14) for other groups because that of other groups not available.

Application of our method to 0vββ NME of other groups



Condition on the samples:

The results from the groups that obtained **both the 0vββ and 2vββ** NMEs on the same footing

Square: Shell model
Circle: QRPA
Triangle: IBM

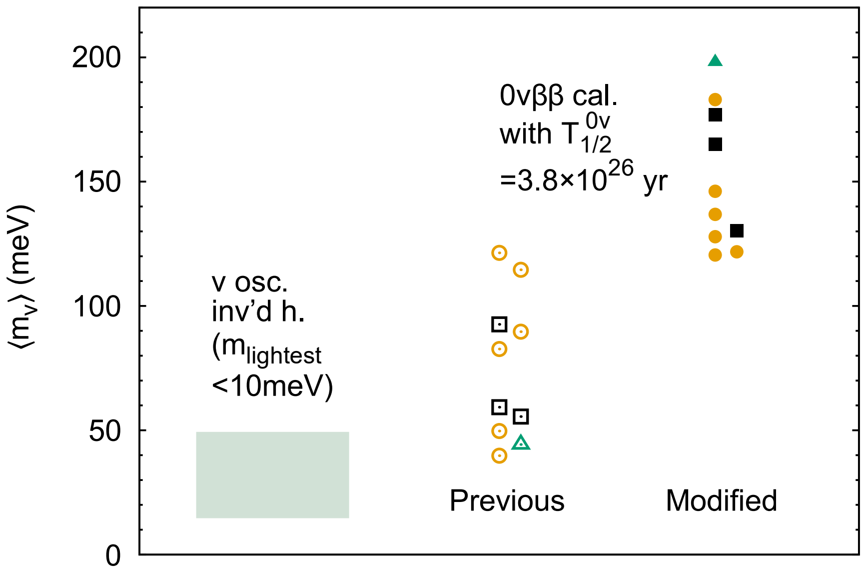
J. T. and O. Civitarese, arXiv: 2509.16605

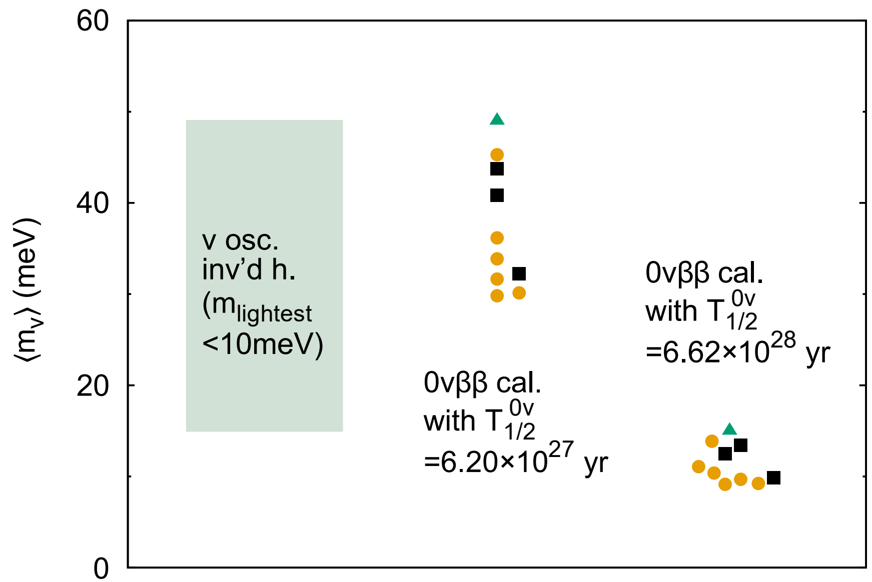
References of the calculations

Method		Reference
QRPA	1	Ours, SkM*
	2	M. Mustnen and J. Engel (2013), SkM*
	3	M. Mustnen and J. Engel (2013), Mod. SkM*
	4	Šimkovic et al. (2018)
	5	D.-L. Fang et al. (2018), AV18, w/o src
	6	D.-L. Fang et al. (2018), CD-Bonn, w/o src
Shell Model	1	J. Menéndez (2018), Argonne src
	2	J. Menéndez (2018), CD-Bonn src
	3	M. Horoi and B. A. Brown (2013), largest space
IBM		J. Barea et al. (2013)

src: short-range correlation

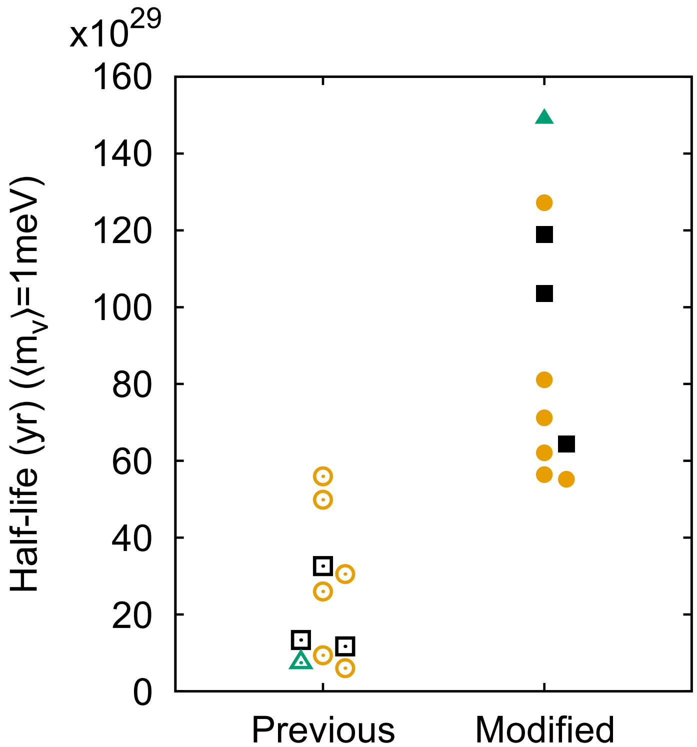
Together with neutrino oscillation





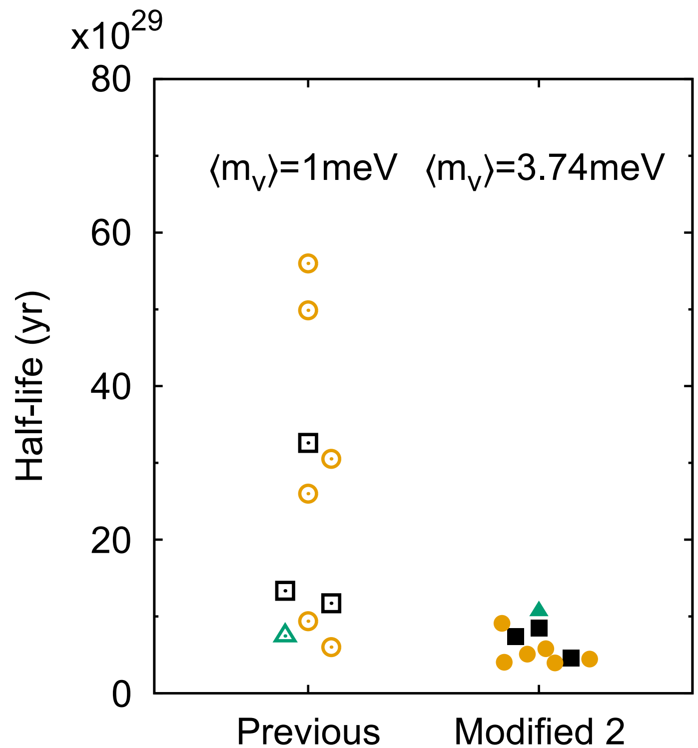
The upper and lower edges of the rectangular show the region of $\langle m_\nu \rangle$ allowed by the neutrino oscillation data with the assumption:
the inverted mass ordering,
the lightest neutrino eigen mass < 10 meV

The experimental lower limit of the half-life of ^{136}Xe for the 0νββ decay is 3.8×10^{26} yr (I. Shimizu, Int. Conf. Neutrino 2024)



$\langle m_\nu \rangle \rightarrow$ Half-life

1 meV is used as a sample.



$\langle m_\nu \rangle$ affects the half-life significantly.

Summary

- I showed the improvement of the QRPA approach to the ββ decay. I also discussed the validity of the QRPA to ^{136}Xe. ⇨ QRPA is a good approx. for ^{136}Xe.
- A small effective g_A is necessary for 2νββ decay. According to the assumption that the exact calculation does not need an effective g_A, something is missing in my calculation. That is not in the QRPA.
- We introduced the transition operator perturbed by the NN interaction by our own method.
- $g_{\mathrm{A},0\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt}) = 1.14 g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{pt}),\ (\mathrm{SkM}^*)$
- We introduced the idea to use $g_{\mathrm{A},2\nu}^{\mathrm{eff}}(\mathrm{ld};\mathrm{exp})$ for 0νββ NME.

- We applied this idea with a minor modification to the results of other groups, and the dramatic reduction of dispersion of the $g_A{}^2$ NME was obtained.
- The predicted half-life of ^{136}Xe is longer than many of the previous ones.

Thank you for
your attention.