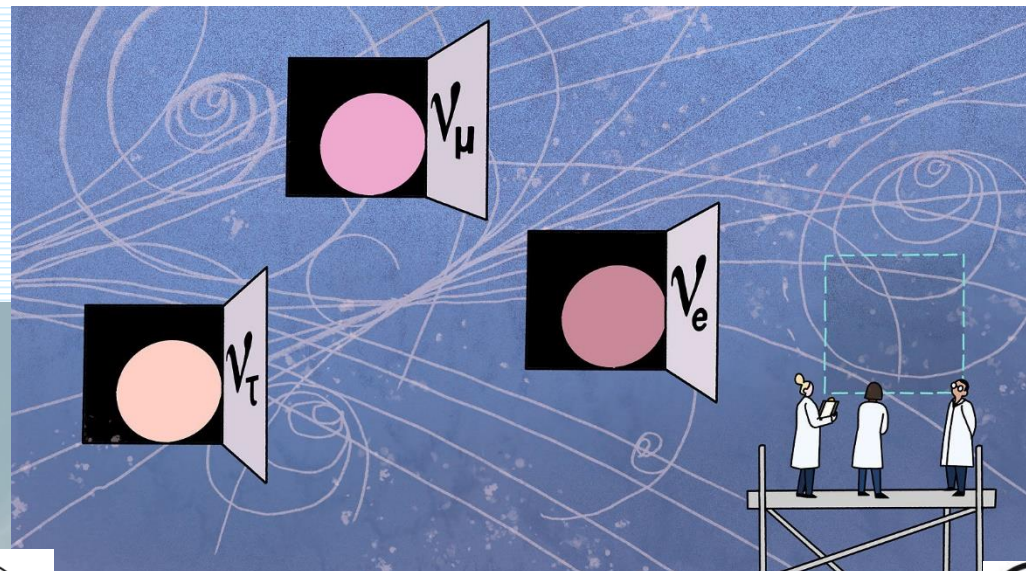
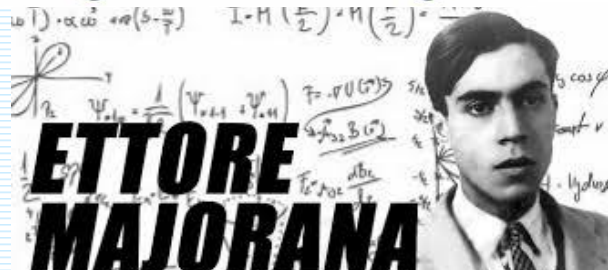
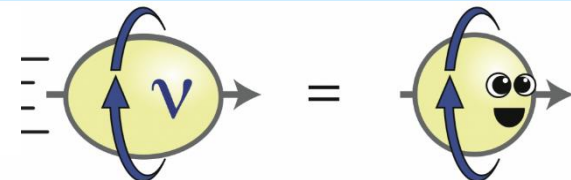


NEWS-Neutrinos Electro-Weak interactions and Symmetry

RCNP, Osaka, Japan, September 25, 2026



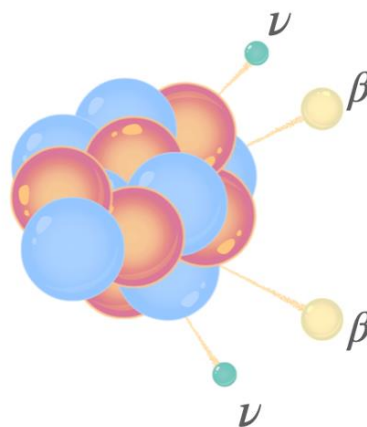
**ARE NEUTRINOS
THEIR OWN?
ANTIPARTICLES?**

ENDLESS DECAYING
WORLD

ENDLESS DECAYING
WORLD

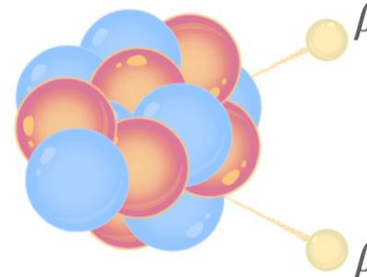


Double beta decay



$$(A, Z) \rightarrow (A, Z \pm 2) + 2\beta^{\mp} + 2(\bar{\nu})$$

Neutrinoless double beta decay



$$(A, Z) \rightarrow (A, Z \pm 2) + 2\beta^{\mp}$$



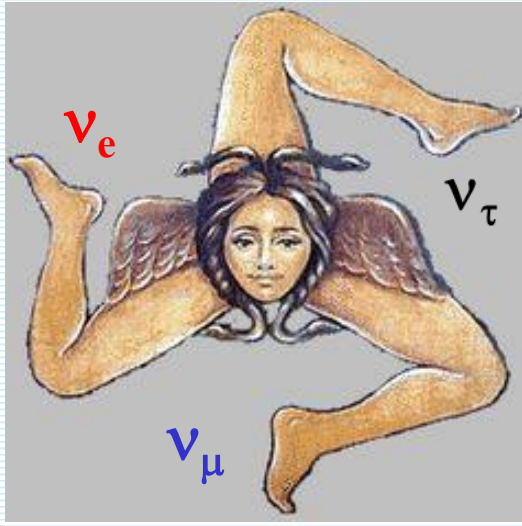
Modern Aspects of Double-Beta Decay

Fedor Šimkovic

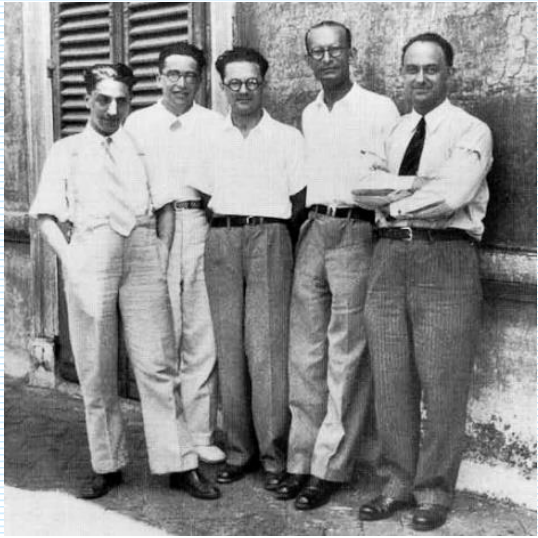
UNKNOWN WRITER

UNKNOWN WRITER

OUTLINE



- I. Introduction (ν -oscillations, DBD)*
- II. $0\nu\beta\beta$ -decay and tiny ν -masses (EFT, LNV ν -oscillations)*
- III. $0\nu\beta\beta$ -decay mechanisms with sterile ν 's (QD, see-saw, interpolating formula)*
- II. $2\nu\beta\beta$ -decay (phenomenological versus theoretical $2\nu\beta\beta$ -decay NMEs, role of the spin-isospin symmetry restoration)*
- V. Improved description of the $2\nu\beta\beta$ -decay (the effect of lepton energies in energy denominators, widths of intermediate nuclear states, effect of $p_{1/2}$ wave states, ...)*
- VI. A connection between $0\nu\beta\beta$ and $2\nu\beta\beta$ NMEs*
- VII. Supporting nuclear physics experiments (muon capture, DCE)*
- VIII. Outlook*



Oscar D'Agostino
Emilio Segrè
Edoardo Amaldi
Franco Rasetti
Enrico Fermi



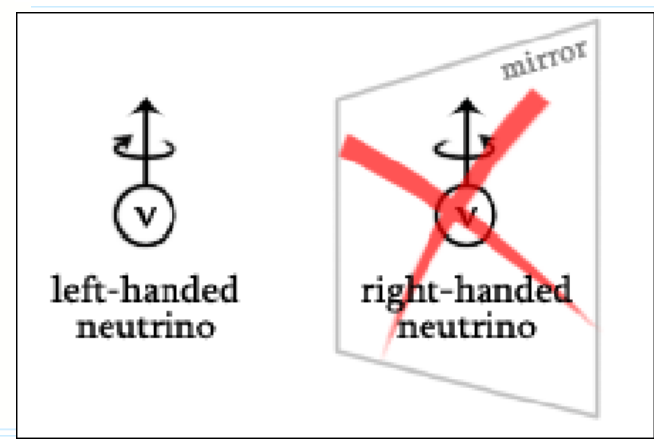
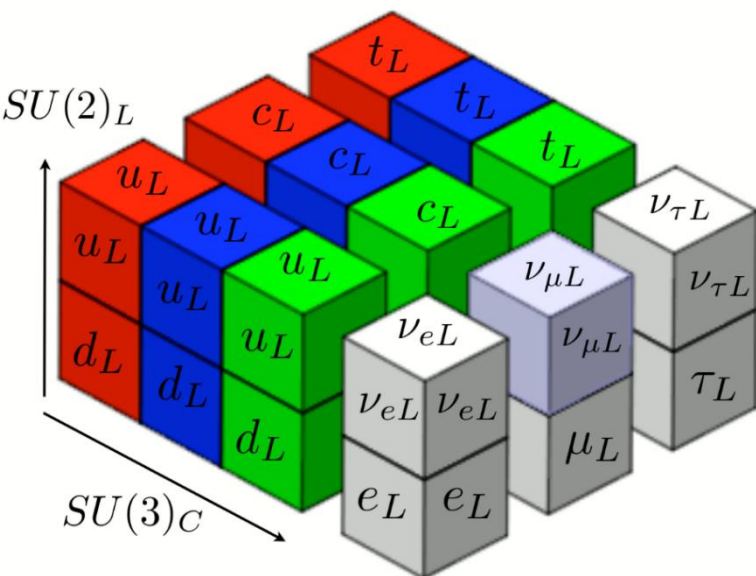
Ettore Majorana



Bruno Pontecorvo

Standard Model
(an astonishing successful theory,
based on few principles)

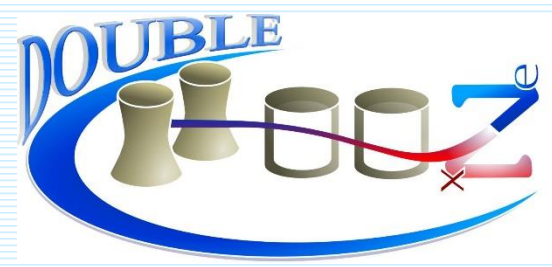
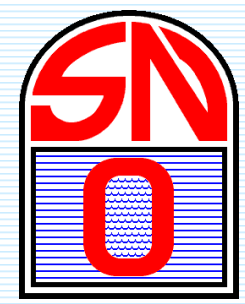
- ν is a **special particle** in SM:
- It is the only fermion that **does not carry electric charge** (like γ , g , H^0)
 - There are only **left-handed ν 's** (ν_{eL} , $\nu_{\mu L}$, $\nu_{\tau L}$)
 - **ν -mass** can not be generated with any renormalizable coupling with the Higgs fields through SSB



ν 's oscillations experiments
 $\Rightarrow$ tiny neutrino masses (!)
 $\Rightarrow$ Beyond SM physics (!)



9/25/2026

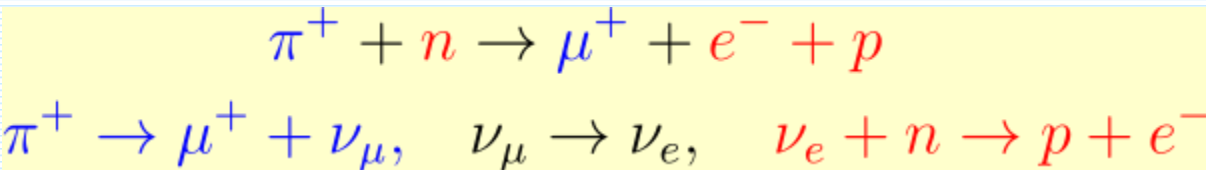


, etc



A comparison of standard and QFT approaches

J. Phys. G 51, 035202 (2024)



$$\Gamma_{osc}^{\pi^+n} = \int \frac{d\Phi_\nu(E'_\nu)}{dE'_\nu} \frac{\mathcal{P}_{\nu_\mu\nu_e}(E'_\nu)}{4\pi L^2} \sigma(E'_\nu) dE'_\nu$$

$$= \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{P_{\nu_\mu\nu_e}(E_\nu)}{4\pi L^2} (g_V^2 + 3g_A^2) p_e E_e$$

Standard QM approach

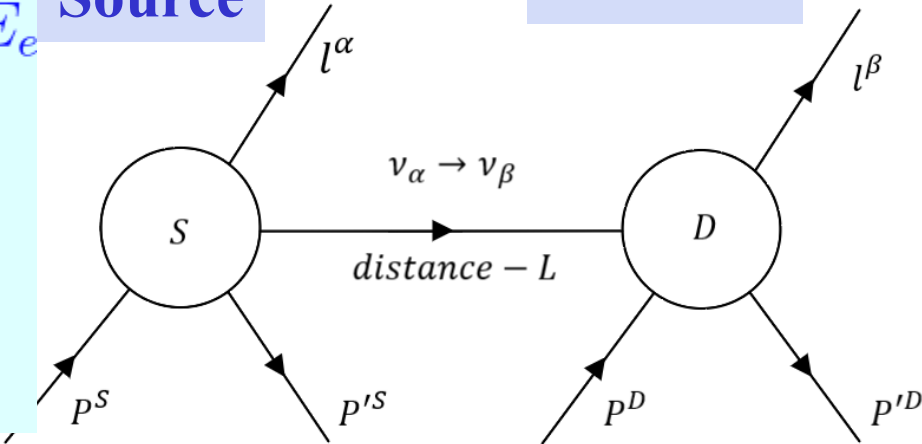
with

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \left| \sum_{j=1}^3 U_{\alpha j}^* U_{\beta j} e^{-im_j^2 L/(2E_\nu)} \right|^2$$

Neutrino oscillations as a single Feynman diagram

Source

Detector



New QFT approach

(no decoherence, no factorization of two processes)

$$\Gamma_{QFT}^{\pi^+n} = \frac{1}{2\pi^2} G_\beta^4 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{\mathcal{P}_{\mu e}^{QFT}(E_\nu)}{4\pi L^2} (g_V^2 + 3g_A^2) p_e E_e$$

with

$$\mathcal{P}_{\alpha\beta}^{QFT}(E_\nu) = \frac{1}{2} \sum_{km} U_{\beta k} U_{\alpha k}^* U_{\beta k}^* U_{\alpha k} e^{i(p_m - p_k)L} \left(1 + \frac{p_k p_m}{E_\nu^2} \right)$$

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{12} & s_{12} \\ 0 & -s_{12} & c_{12} \end{pmatrix}$$

$$\tilde{R}_{13} = \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

3 neutrino masses, 2 mass squared differences

$$\delta m^2 = m_2^2 - m_1^2, \quad \Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$$

$$U = R_{23} \tilde{R}_{13} R_{12}$$

3 mixing angles
CP-phase

$$|\nu_\alpha\rangle = \sum_{j=1}^3 U_{\alpha j}^* |\nu_j\rangle$$

($\alpha = e, \mu, \tau$)

Global neutrino
oscillations analysis
(PRD 101, 116013 (2020))

	best – fit	1σ	2σ	3σ
Normal hierarchy (NH)				
$\delta m^2/10^{-5} \text{ eV}^2$	7.34	7.20-7.51	7.05-7.69	6.92-7.90
$\Delta m^2/10^{-3} \text{ eV}^2$	2.485	2.453-2.514	2.419-2.547	2.2389-2.578
$\sin^2 \theta_{12}/10^{-1}$	3.05	2.92-3.19	2.78-3.32	2.65-3.47
$\sin^2 \theta_{13}/10^{-2}$	2.22	2.14-2.28	2.07-2.34	2.01-2.41
$\sin^2 \theta_{23}/10^{-1}$	5.45	4.98-5.65	4.54-5.81	4.36-5.95
δ/π	1.28	1.10-1.66	0.95-1.90	0-0.07 $\oplus$ 0.81-2.00
Inverted hierarchy (IH)				
$\delta m^2/10^{-5} \text{ eV}^2$	7.34	7.20-7.51	7.05-7.69	6.92-7.91
$-\Delta m^2/10^{-3} \text{ eV}^2$	2.465	2.434-2.495	2.404-2.526	2.374-2.556
$\sin^2 \theta_{12}/10^{-1}$	3.03	2.90-3.17	2.77-3.31	2.64-3.45
$\sin^2 \theta_{13}/10^{-2}$	2.23	2.17-2.30	2.10-2.37	2.03-2.43
$\sin^2 \theta_{23}/10^{-1}$	5.51	5.17-5.67	4.60-5.82	4.39-5.96
		$\oplus$ 5.31-6.10		
δ/π	1.52	1.37-1.65	1.23-1.78	1.09-1.90

*Towards an understanding
of the Universe*

Even though my friends and I are so small,
we can teach you the biggest things of all.

*Neutrino masses,
mixing,
oscillations,
 $0\nu\beta\beta$ -decay*

Dirac

$$L_{\text{mass}}^D = - \sum_{\alpha\beta} \bar{\nu}_{\alpha R} M_{\alpha\beta}^D \nu_{\beta L} + H.c.$$

$$= - \sum_{k=1}^3 m_k \bar{\nu}_k \nu_k$$

$$\alpha, \beta = e, \mu, \tau, \quad V^\dagger M^D U = M_{\text{diag}}^D$$

Neutrino Mass Term

Majorana

$$L_{\text{mass}}^M = \frac{1}{2} \sum_{\alpha\beta} \nu_{\alpha L}^T C^\dagger M_{\alpha\beta}^L \nu_{\beta L} + H.c.$$

$$= \frac{1}{2} \sum_{k=1}^3 m_k \nu_k^T C^\dagger \nu_k$$

$$\alpha, \beta = e, \mu, \tau$$

$$M_{\alpha\beta}^L = M_{\beta\alpha}^L \quad U^T M^M U = M_{\text{diag}}^M$$

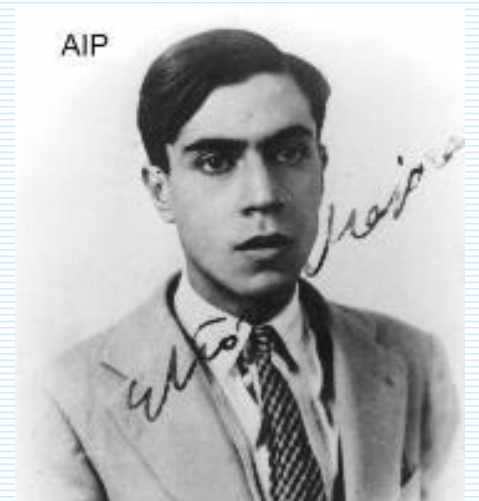
Dirac-Majorana

$$L_{\text{mass}}^{D+M} = - \sum_{\alpha\beta} \bar{N}_{\alpha R} M_{\alpha\beta}^{D+M} N_{\beta L} + H.c.$$

$$= - \sum_{k=1}^6 m_k \bar{\nu}_k \nu_k$$

$$N = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix}, \quad \alpha, \beta = e, \mu, \tau, s_1, s_2, s_3$$

$$U^T M^{D+M} U = M_{\text{diag}}^{D+M}$$





Majorana fermions

Ettore Majorana

Teoria simmetrica dell'elettrone e del positrone
(A symmetric theory of electrons and positrons).
Il Nuovo Cimento, 14: 171–184, 1937.) 171

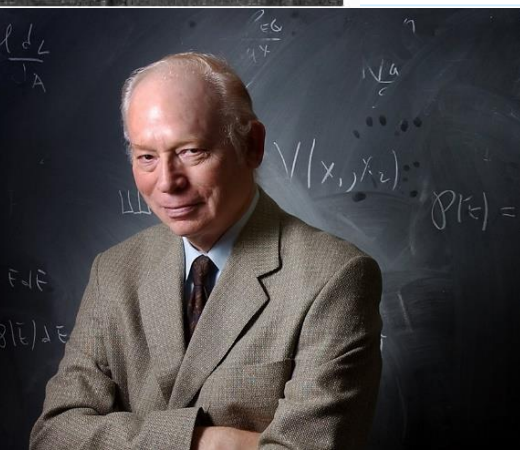
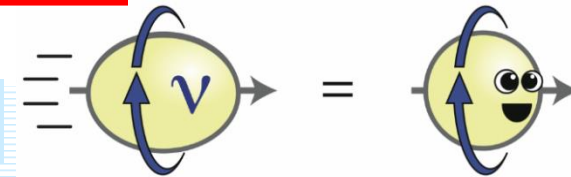
ν is its own
antiparticle



Bruno Pontecorvo
*Inverse beta processes and
nonconservation of lepton charge*
Zhur. Eksptl'. i Teoret. Fiz.
34, 247 (1958)

It follows from the above assumptions that in vacuum a neutrino can be transformed into an antineutrino and vice versa. This means that the neutrino and antineutrino are “mixed” particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity.⁵

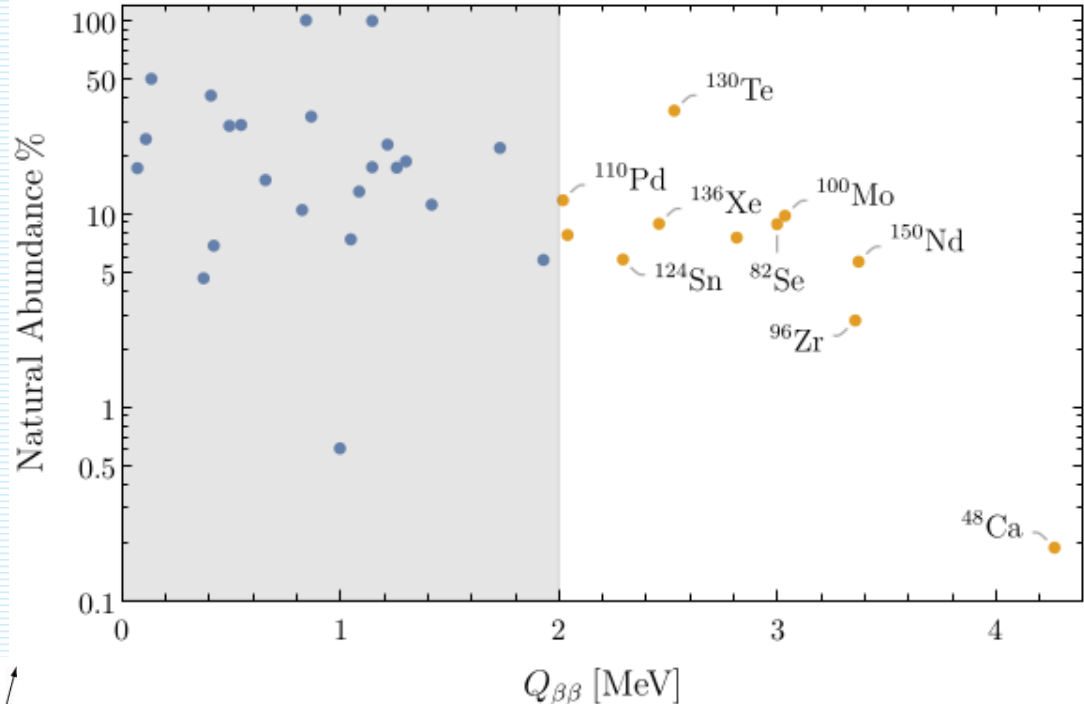
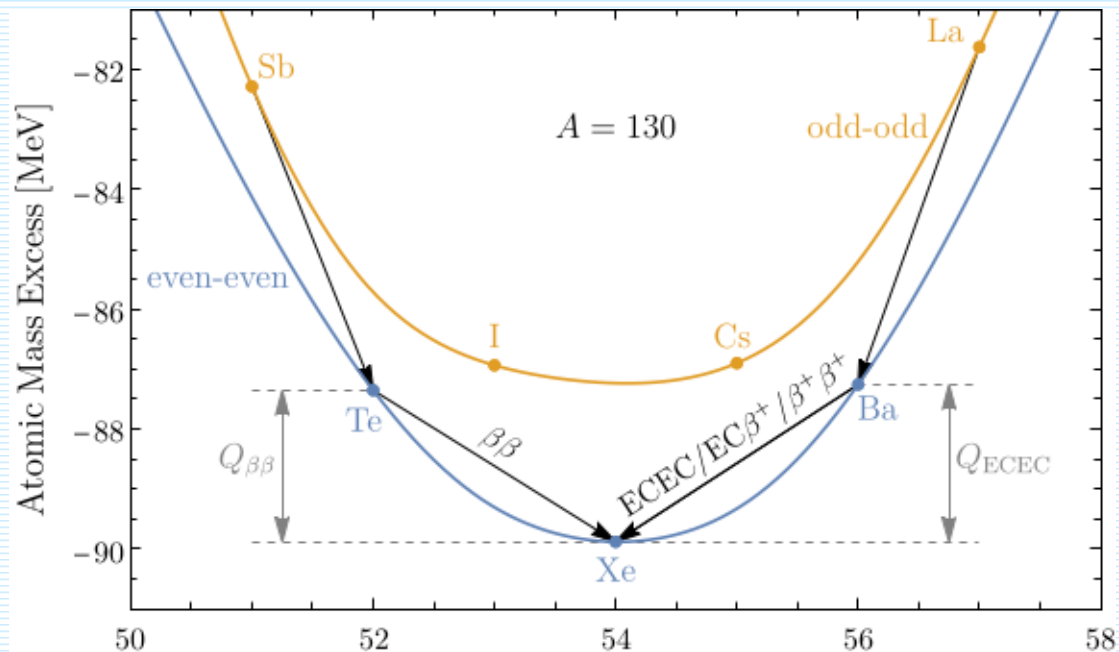
$\nu \leftrightarrow \text{anti-}\nu$ oscillation



Steve Weinberg
 ν -mass generation
via d=5 eff. oper.
related to unknown
high energy scale (GUT?)

thought massless back in 1979. Weinberg does not take credit for predicting neutrino masses, but he thinks it's the right interpretation. What's more, he says, the non-renormalisable interaction that produces the neutrino masses is probably also accompanied with non-renormalisable interactions that produce proton decay and other things that haven't been observed, such as violation of baryon-number conservations. “We don't know anything about the details of those terms, but I'll swear they are there.”

Nuclear double- β decay (even-even nuclei, pairing int.)

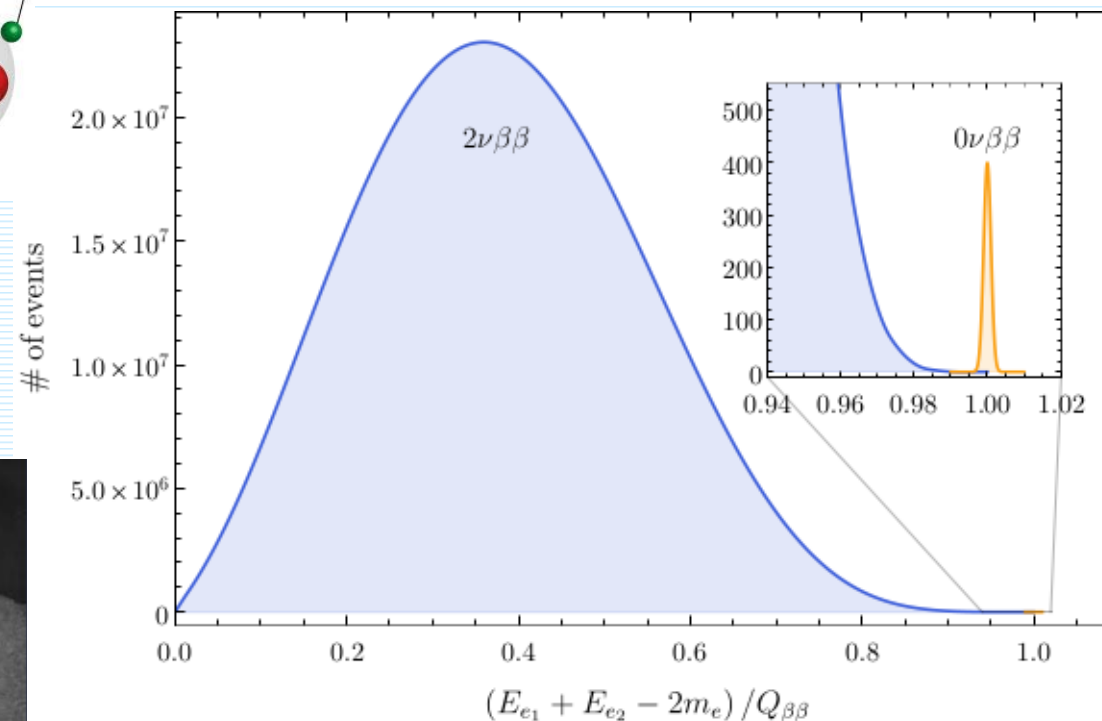
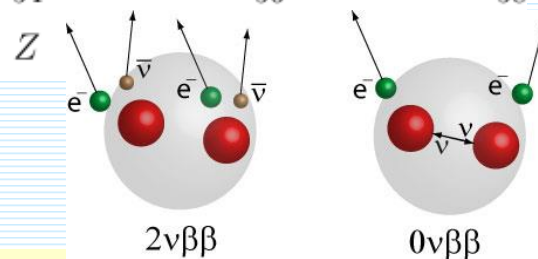


Phys. Rev. 48, 512 (1935)

Two-neutrino double- β decay – LN conserved

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

Goepert-Mayer – 1935. 1st observation in 1987



Nuovo Cim. 14, 322 (1937)

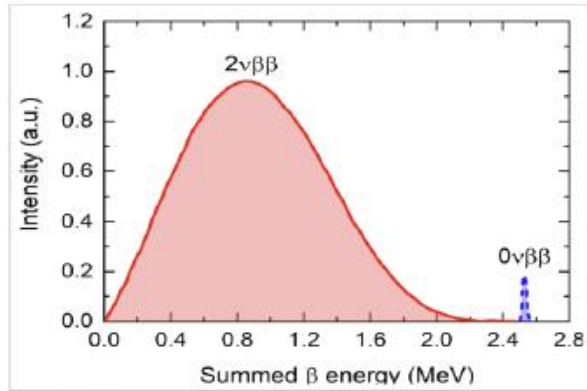
Phys. Rev. 56, 1184 (1939)

Neutrinoless double- β decay – LN violated

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^- \quad (\text{Furry 1937})$$

Not observed yet. Requires massive Majorana ν 's

$0\nu\beta\beta$ experiments – a worldwide competition of ideas and underground physics technologies



GERDA (^{76}Ge)

$$T_{1/2}^{0\nu} > 1.8 \times 10^{26} \text{ yrs}$$

Phys.Rev.Lett.125(2020)252502

GUORE (^{130}Te)

$$T_{1/2}^{0\nu} > 2.2 \times 10^{25} \text{ yrs}$$

Nature 604(2022)53

CUPID-0 (^{82}Se)

$$T_{1/2}^{0\nu} > 2.4 \times 10^{24} \text{ yrs}$$

Phys.Rev.Lett.120(2018)232502

CUPID (^{100}Mo)

$$T_{1/2}^{0\nu} > 1.5 \times 10^{24} \text{ yrs}$$

Phys.Rev.Lett.126(2021)181802

MAJORANA (^{76}Ge)

$$T_{1/2}^{0\nu} > 8.3 \times 10^{25} \text{ yrs}$$

Phys.Rev.Lett.130(2023)062501

SNO+

**NvDex, PandaX,
CDEX, CUPID-China**

NEMO-3

JUNO

KamLAND-Zen (^{136}Xe)

$$T_{1/2}^{0\nu} > 2.3 \times 10^{26} \text{ yrs}$$

★ Phys.Rev.Lett.130(2023)051801

EXO-200 (^{136}Xe)

$$T_{1/2}^{0\nu} > 3.5 \times 10^{25} \text{ yrs}$$

Phys.Rev.Lett.123(2019)161802

Isotope	$Q_{\beta\beta}$ (MeV)	N.A. (%)
^{48}Ca	4.268	0.187
^{76}Ge	2.039	7.8
^{82}Se	2.998	8.8
^{96}Zr	3.356	2.8
^{100}Mo	3.034	9.7
^{110}Pd	2.017	11.7
^{116}Cd	2.813	7.5
^{124}Sn	2.293	5.8
^{130}Te	2.528	34.1
^{136}Xe	2.458	8.9
^{150}Nd	3.371	5.6

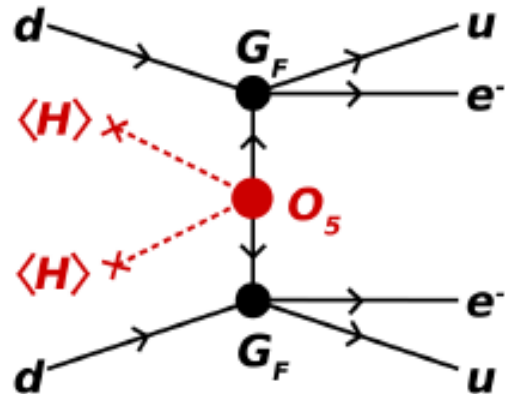
Beyond the SM physics

$$\mathcal{L} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_i c_i^{(5)} \mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} \mathcal{O}_i^{(6)} + O\left(\frac{1}{\Lambda^3}\right)$$

Amplitude for
(A,Z)→(A,Z+2)+2e⁻
can be divided into:

long range: d=7

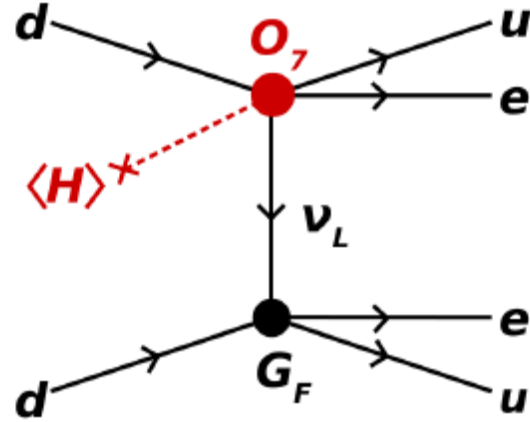
mass mechanism: d=5



$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda} (L_i H)(L_j H)$$

Weinberg, 1979

+



$$\mathcal{O}_2 \propto LLL e^c H$$

$$\mathcal{O}_3 \propto LLQ d^c H$$

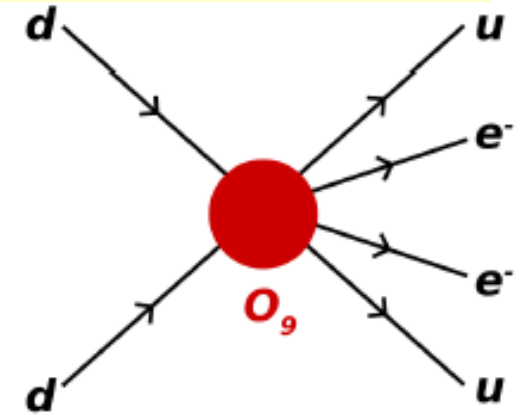
$$\mathcal{O}_4 \propto LL\bar{Q}\bar{u}^c H$$

$$\mathcal{O}_8 \propto L\bar{e}^c\bar{u}^c d^c H$$

Babu, Leung: 2001

de Gouvea, Jenkins: 2007

short range: d=9 (d=11)



+

$$\mathcal{O}_5 \propto LLQ d^c H H H^\dagger$$

$$\mathcal{O}_6 \propto LL\bar{Q}\bar{u}^c H H^\dagger H$$

$$\mathcal{O}_7 \propto LQ\bar{e}^c\bar{Q} H H H^\dagger$$

$$\mathcal{O}_9 \propto LLL e^c L e^c$$

$$\mathcal{O}_{10} \propto LLL e^c Q d^c$$

$$\mathcal{O}_{11} \propto LLQ d^c Q d^c$$

.....

Valle

$$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$$

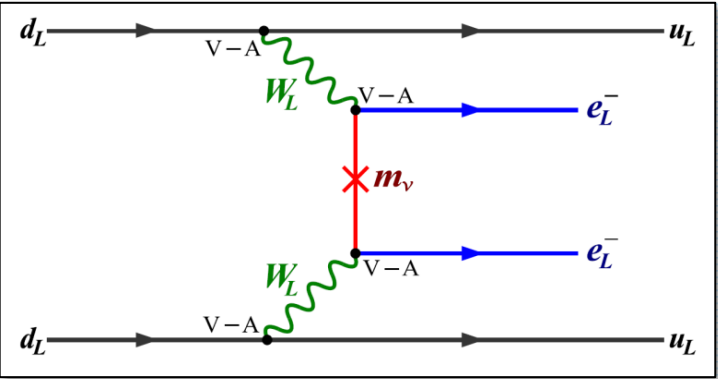
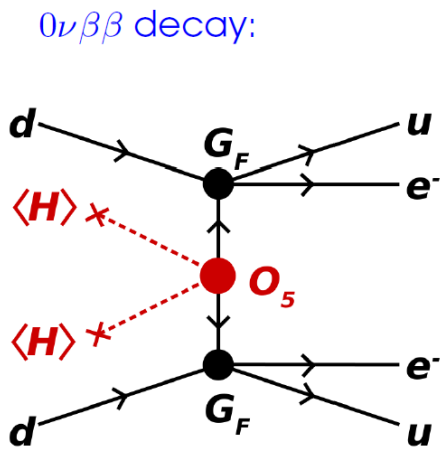
$0\nu\beta\beta$ -decay
(LNV at $\approx GUT$ scale, exchange of three light ν)

$$\left(T_{1/2}^{0\nu}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M_\nu^{0\nu}\right|^2 G^{0\nu}$$

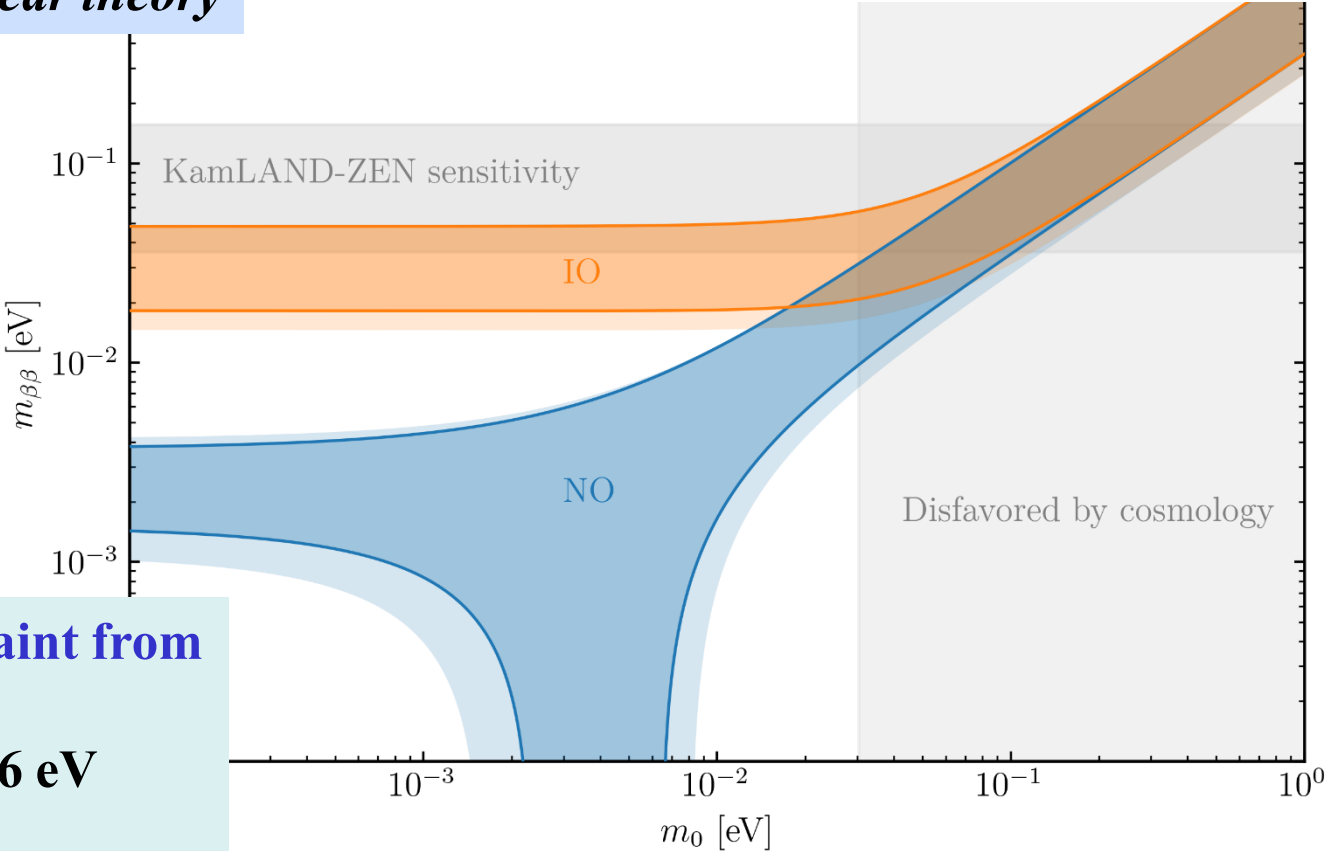
?

Phase space factor
well understood

*NME must be evaluated
using tools of nuclear theory*



$$m_{\beta\beta} = \left| c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3 \right|$$



Constraint from cosmology
 $\Sigma = m_1 + m_2 + m_3$
 $< 0.90 \text{ eV}$
 $< 0.26 \text{ eV (Planck coll.)}$
 $< 0.12 \text{ eV}$

**Contrary, the constraint from
 $0\nu\beta\beta$ -decay (KLZ)**
 $m_{\beta\beta} < 0.036\text{-}0.156 \text{ eV}$
implies
 $\Sigma < 0.12 \text{ eV}$

Quark Condensate Seesaw Mechanism for Neutrino Mass

PRD 103, 015007 (2021)

This operator contributes to the **Majorana-neutrino mass matrix** due to chiral symmetry breaking via the **light-quark condensate**.

The SM gauge-invariant effective operators

$$\mathcal{O}_7^{u,d} = \frac{\tilde{g}_{\alpha\beta}^{u,d}}{\Lambda^3} \overline{L}_\alpha^C L_\beta H \left\{ (\overline{Q} u_R), (\overline{d}_R Q) \right\}$$

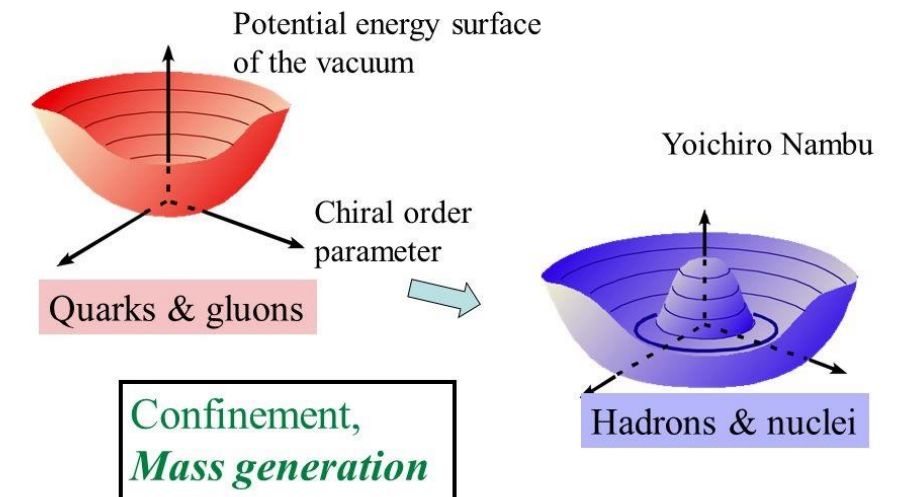
After the **EWSB** and **ChSB** one arrives at the Majorana mass matrix of active neutrinos

$$\begin{aligned} m_{\alpha\beta}^\nu &= g_{\alpha\beta} v \frac{\langle \overline{q} q \rangle}{\Lambda^3} \\ &= g_{\alpha\beta} v \left(\frac{\omega}{\Lambda} \right)^3 \end{aligned}$$

$$g_{\alpha\beta} = g_{\alpha\beta}^u + g_{\alpha\beta}^d, \quad v/\sqrt{2} = \langle H^0 \rangle$$

$$\omega = -\langle \overline{q} q \rangle^{1/3}, \quad \langle \overline{q} q \rangle^{1/3} \approx -283 \text{ MeV}_{\text{vic}}$$

Spontaneous breaking of **chiral (χ) symmetry**



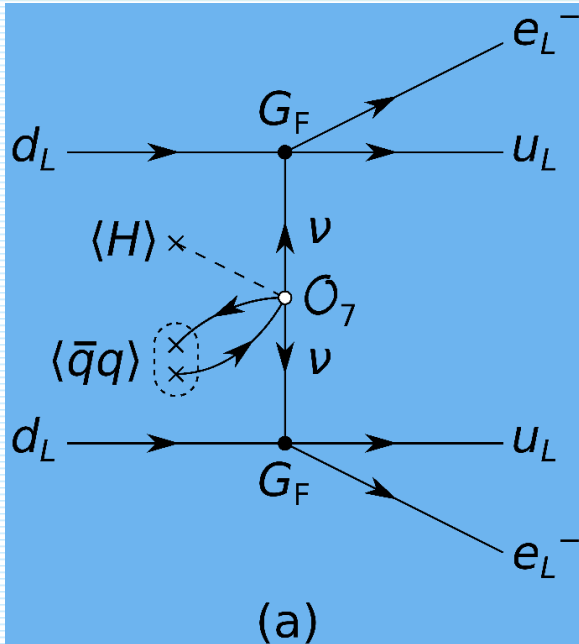
$\Lambda \sim$ **a few TeV**
we get the neutrino mass in the **sub-eV ballpark**

The genuine QCSS scenario (predicts NH and ν -mass spectrum)

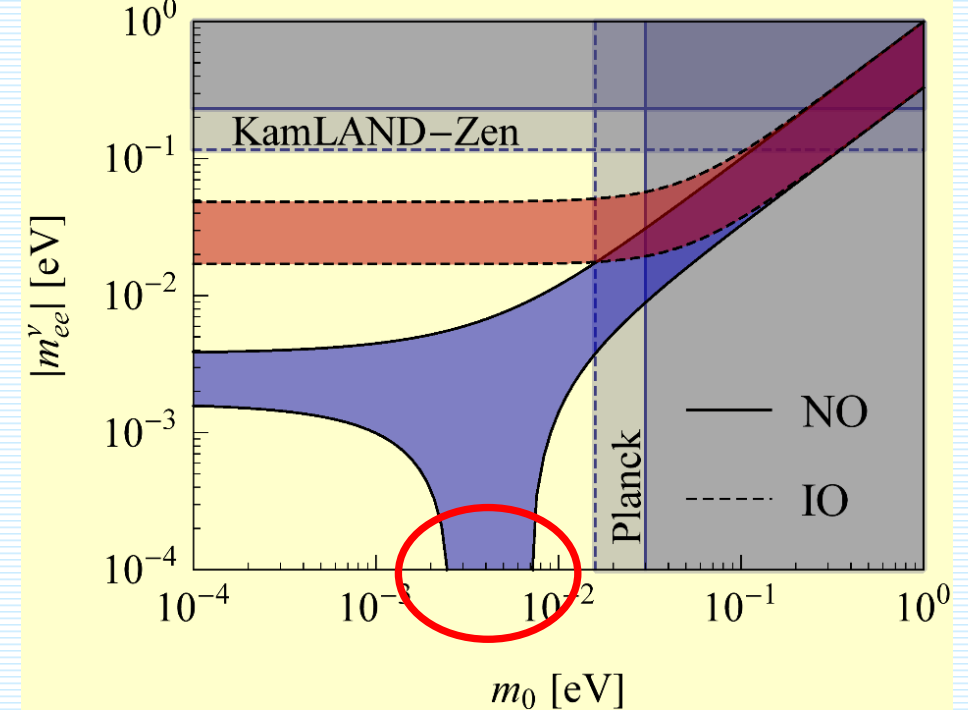
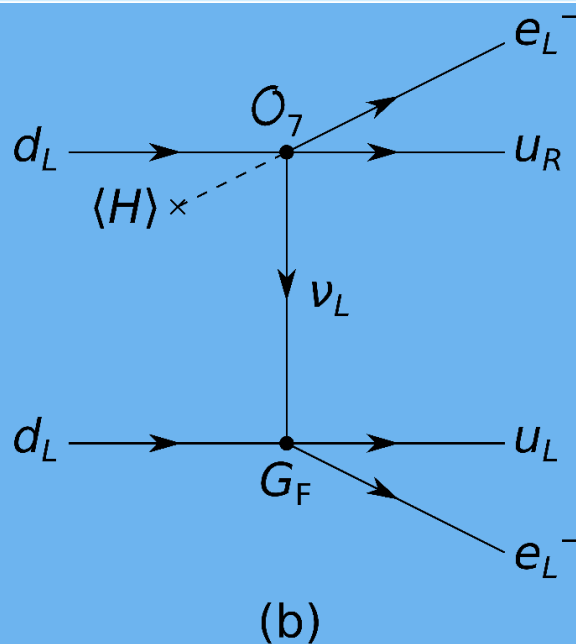
$$\mathcal{L}_7 = \frac{1}{\sqrt{2}} \sum_{\alpha\beta} \frac{v}{\Lambda^3} \bar{\nu}_{\alpha L}^C \nu_{\beta L} (g_{\alpha\beta}^u \bar{u}_L u_R + g_{\alpha\beta}^d \bar{d}_R d_L) + \text{H.c.}$$

$$m_{\alpha\beta}^\nu = -\frac{g_{\alpha\beta}}{\sqrt{2}} v \frac{\langle \bar{q}q \rangle}{\Lambda^3} = \frac{g_{\alpha\beta}}{\sqrt{2}} v \left(\frac{\omega}{\Lambda} \right)^3$$

(a) PRL 112, 142503 (2014).



(b) PLB 453, 194 (1999).



Neutrino spectrum (NH) !!!

(PRD 103, 015007 (2021))

$2 \text{ meV} < m_1 < 7 \text{ meV}$

$9 \text{ meV} < m_2 < 11 \text{ meV}$

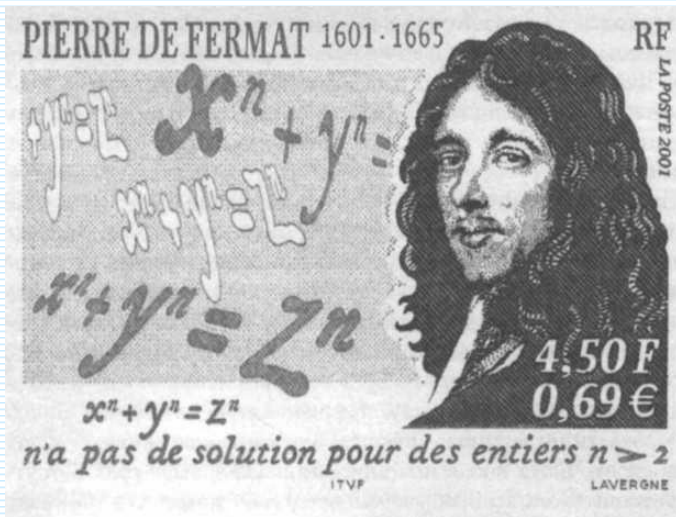
$50 \text{ meV} < m_3 < 51 \text{ meV}$

Prediction for m_β

$9 \text{ meV} < m_\beta < 12 \text{ meV}$

Prediction for cosmology (Σ)

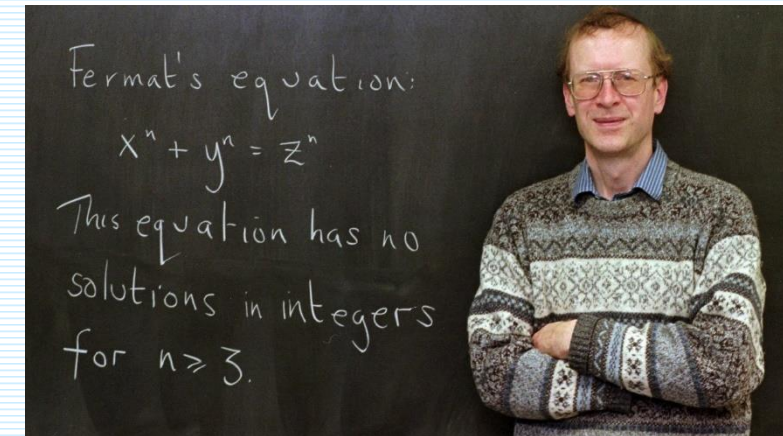
$62 \text{ meV} < m_1 + m_2 + m_3 < 69 \text{ meV}$



Around 1637, Pierre de Fermat wrote
in the margin of a book that
the more general equation

$$a^n + b^n = c^n$$

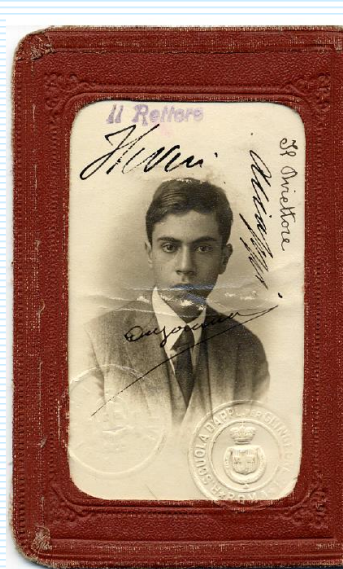
had no solutions in positive integers if n
is an integer greater than 2.



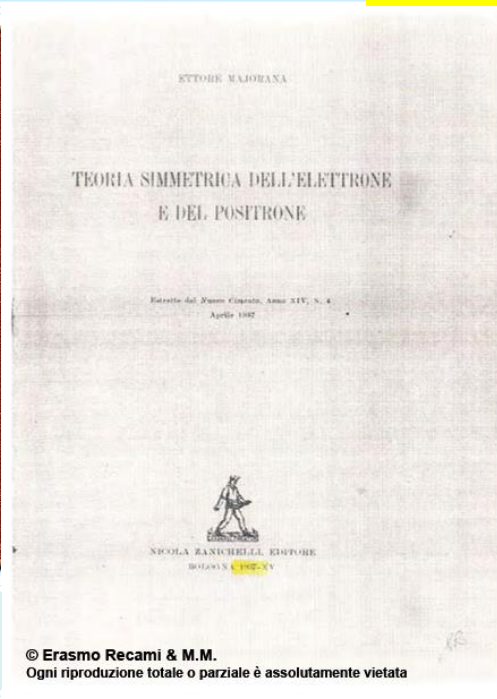
The proof was published by
Andrew Wiles in 1995.

After **358** years

Some long-standing tasks of humanity ...



1937



After
99
years

n-ton-class $0\nu\beta\beta$ exp.
with discovery potential

KamLAND-Zen 800

SNO+

LEGEND

nEXO

NEXT

CUPID

PandaX-xT

etc

After
?
years

If $m_{\beta\beta} < 1$ meV,
what
technology is
needed for
observation
of $0\nu\beta\beta$?

Nuovo Cim. 14,
322 (1937)



neutrino $\leftrightarrow$ antineutrino oscillations

Second order process
with real intermediate neutrinos

$$S + D \rightarrow \ell_{\alpha}^{+} + \ell_{\beta}^{+} + S' + D'$$

$$S \rightarrow S' + \ell_{\alpha}^{+} + \nu_{\alpha}, \quad \nu_{\alpha} \rightarrow \bar{\nu}_{\beta}, \quad \bar{\nu}_{\beta} + D \rightarrow D' + \ell_{\beta}^{+}$$

Amplitude proportional to **v-mass**

$$T_k^{\alpha\beta} = J_S^{\mu}(P'_S, P_S) J_D^{\nu}(P'_D, P_D) \times \\ \bar{v}(P_{\alpha}; \lambda_{\alpha}) \gamma_{\mu} (1 - \gamma_5) m_k \gamma_{\nu} u(P_{\beta}; \lambda_{\beta})$$

Replacement:

$$U_{\alpha k} \rightarrow U_{\alpha k}^{*} \\ U_{\beta m}^{*} \rightarrow U_{\beta m}$$

Particular process:

$$\pi^{+} + p \rightarrow \mu^{+} + e^{+} + n$$

Production rate

$$\Gamma_{QFT}^{\pi^{+}p} = \frac{1}{2\pi^2} G_{\beta}^4 \left(\frac{f_{\pi}}{\sqrt{2}} \right)^2 \frac{m_{\mu}^2}{m_{\pi}} E_{\nu}^2 \frac{P_{\nu_{\mu}\bar{\nu}_e}^{QFT}(E_{\nu}, L)}{4\pi L^2} (g_V^2 + 3g_A^2) p_e E_e$$

Oscillation probability

$$\mathcal{P}_{\alpha\bar{\beta}}^{QFT}(E_{\nu}, L) \equiv |\langle \nu_{\beta} | \bar{\nu}_{\alpha} \rangle|^2 \\ = \left| \sum_{j=1}^3 U_{\alpha j}^{*} U_{\beta j} \frac{m_j}{E_{\nu}} e^{-im_j^2 L/(2E_{\nu})} \right|^2$$

S. Kovalenko, F.Š., J. Phys. G 51, 035202 (2024)

A connection of neutrino-antineutrino oscillation with $0\nu\beta\beta$ -decay

$$m_{ee}^{L=0} = m_{\beta\beta}$$

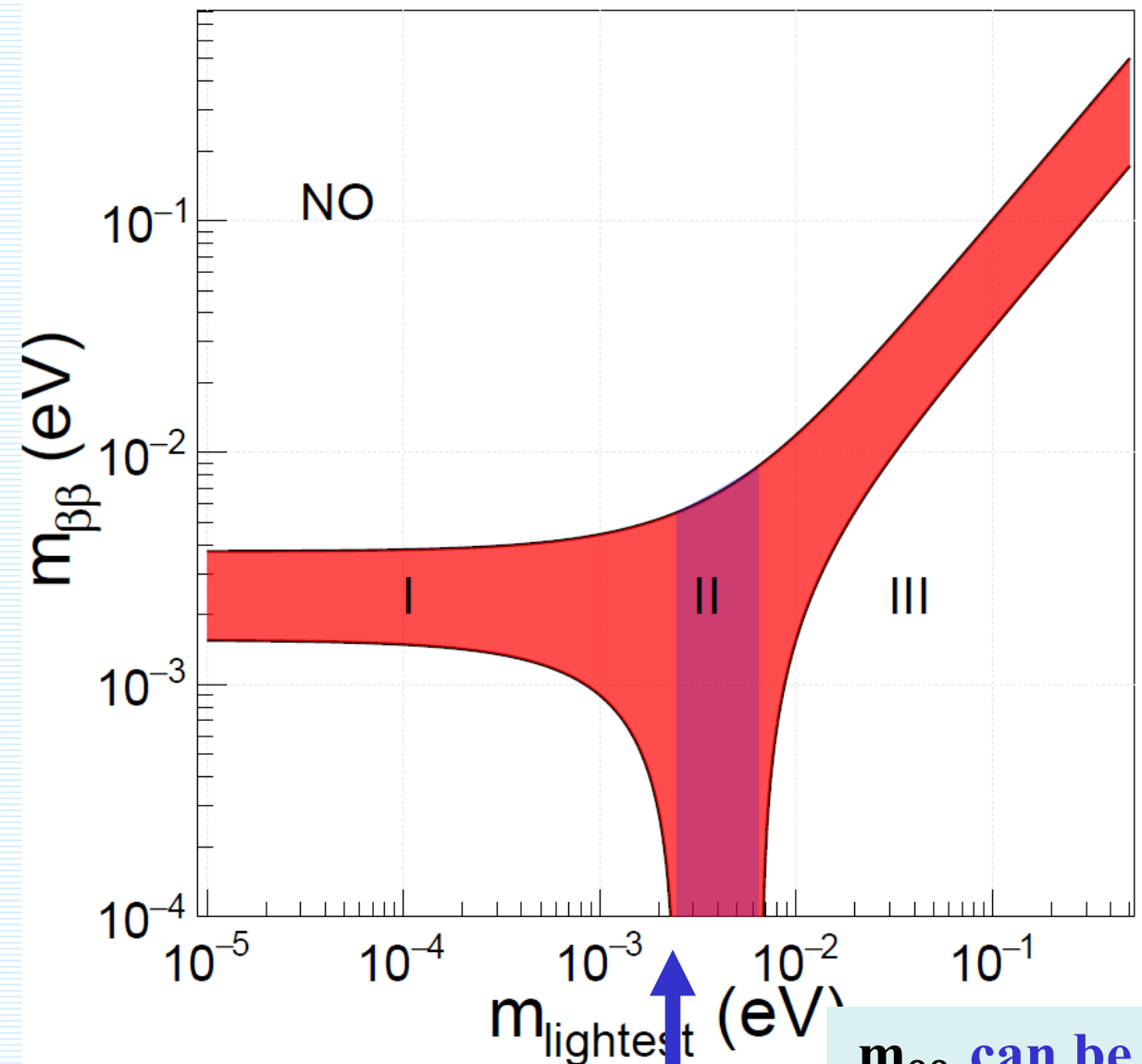
$$\begin{aligned} \mathcal{P}_{e\bar{e}}^{\text{QFT}}(E_\nu, L=0) &\equiv |\langle \nu_e | \bar{\nu}_e \rangle|^2 = \left| \sum_{k=1}^3 U_{ek}^* U_{ek} \frac{m_k}{E_\nu} \right|^2 \\ &= \frac{(m_{ee}^{L=0})^2}{E_\nu^2} = \frac{(m_{\beta\beta})^2}{E_\nu^2} \end{aligned}$$

$$m_{\beta\beta} = |\rho_1 e^{2i\phi_1} + \rho_2 e^{2i\phi_2} + \rho_3|$$

$$\rho_1 = c_{12}^2 c_{13}^2 m_1, \quad \rho_2 = s_{12}^2 c_{13}^2 m_2, \quad \rho_3 = s_{13}^2 m_3$$

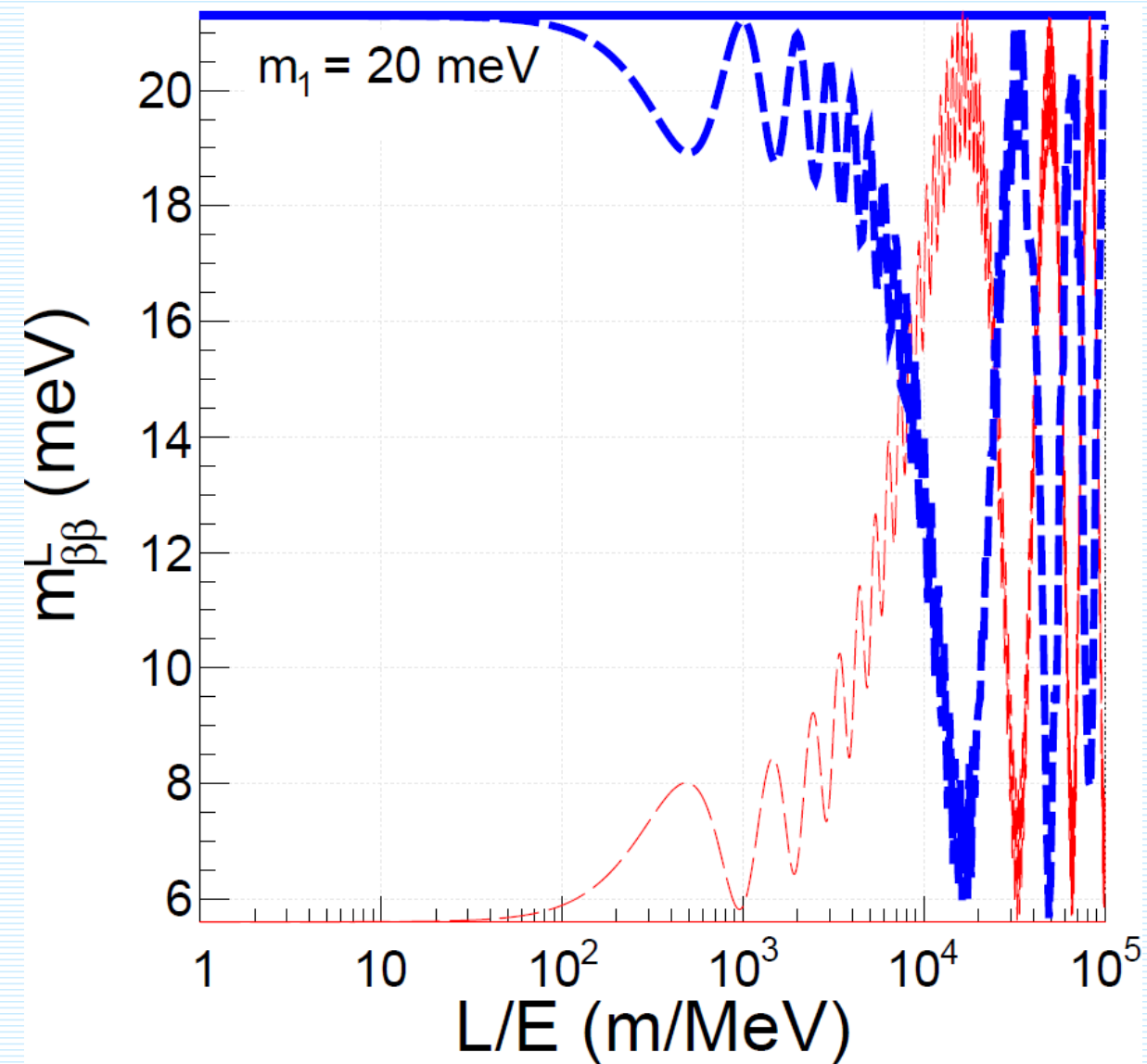
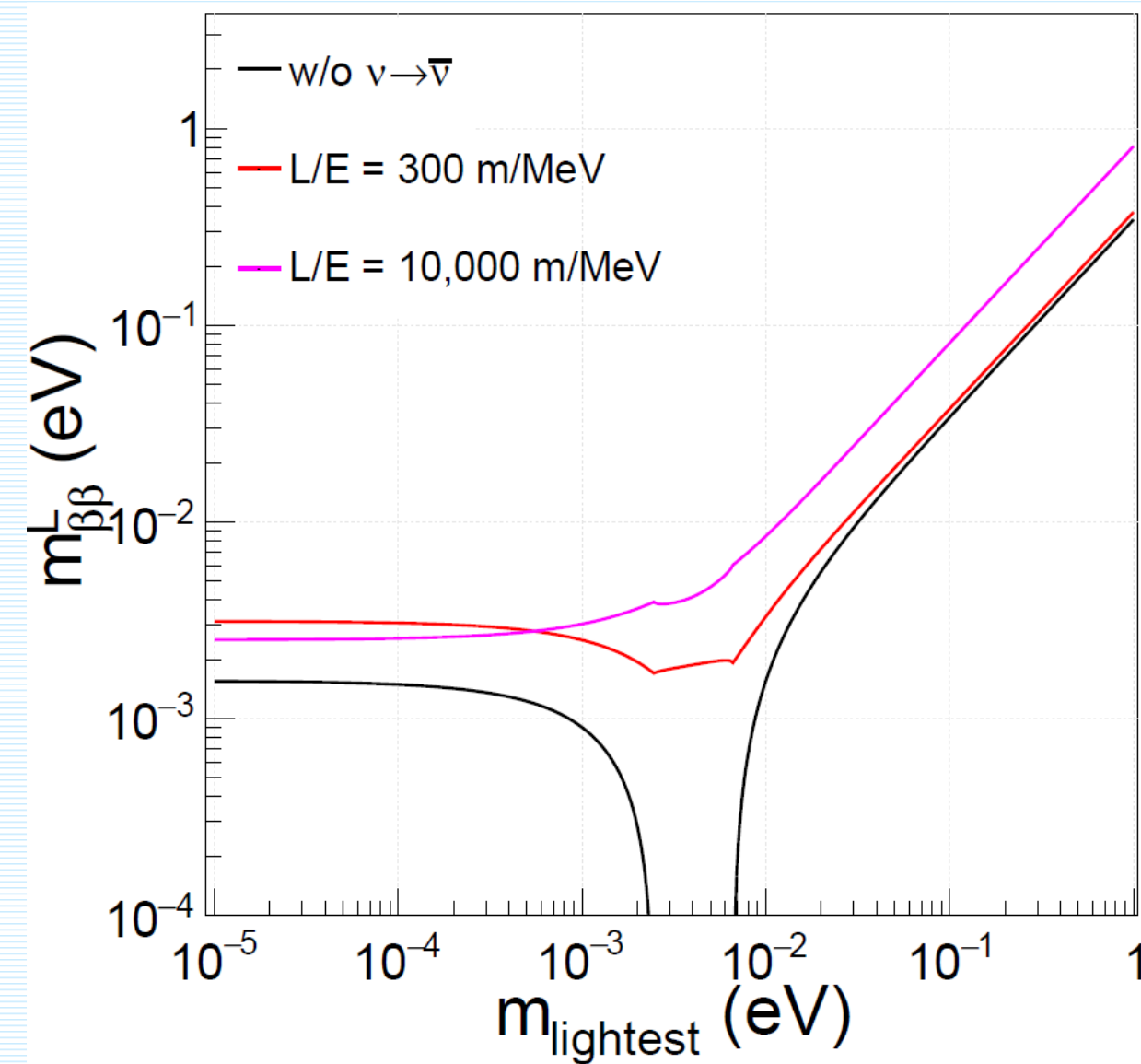
$$\min_{\phi_1, \phi_2} m_{\beta\beta} = \begin{cases} |\rho_2 - \rho_3| - \rho_1, & \text{if } \rho_1 < |\rho_2 - \rho_3| \\ 0, & \text{if } |\rho_2 - \rho_3| \leq \rho_1 \leq \rho_2 + \rho_3 \\ \rho_1 - (\rho_2 + \rho_3), & \text{if } \rho_2 + \rho_3 < \rho_1 \end{cases}$$

: region I,
: region II,
: region III.



$m_{\beta\beta}$ can be
strongly
\suppressed
(?!)

Dependence of m_{ee}^L on m_{lightest} and L/E



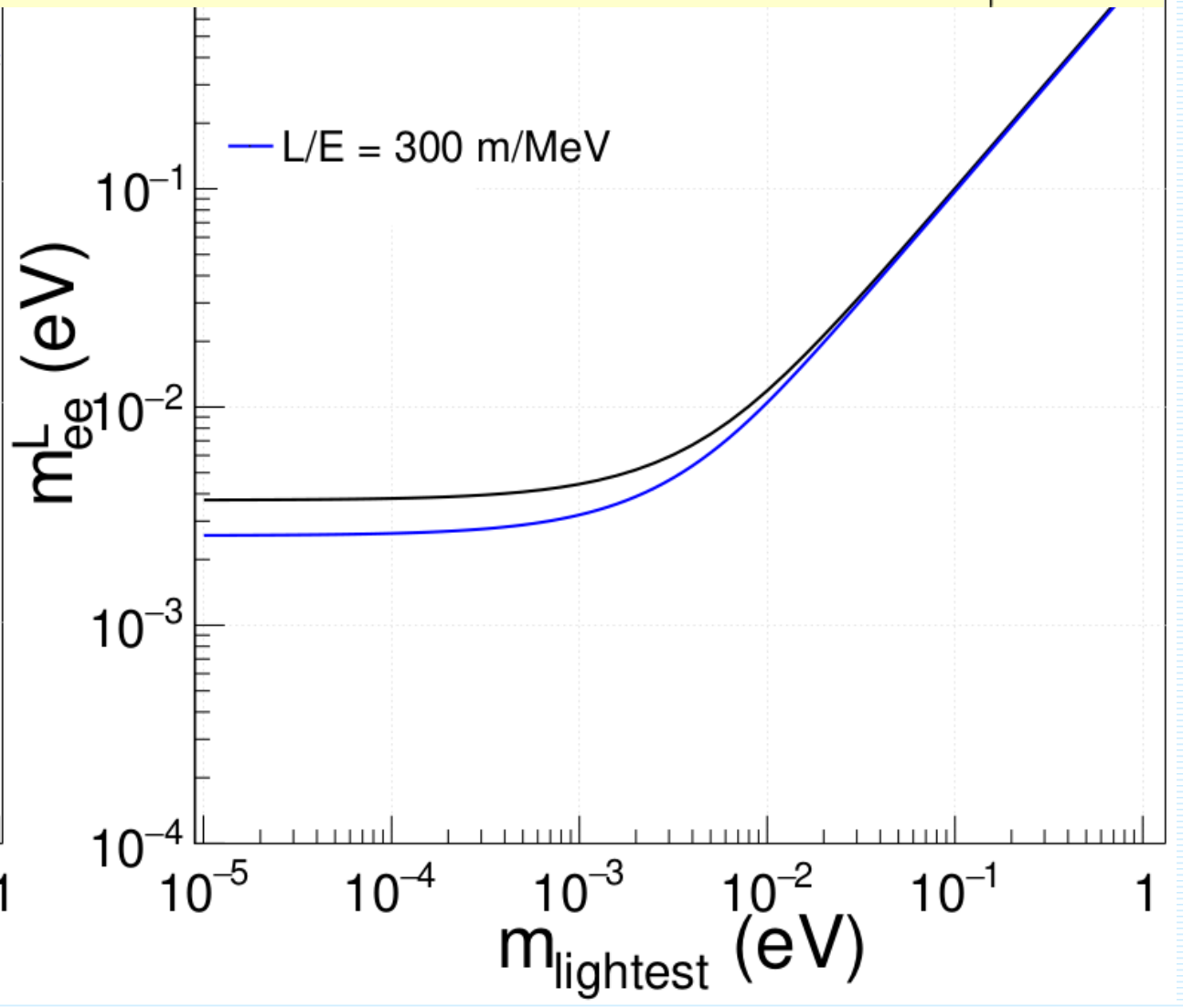
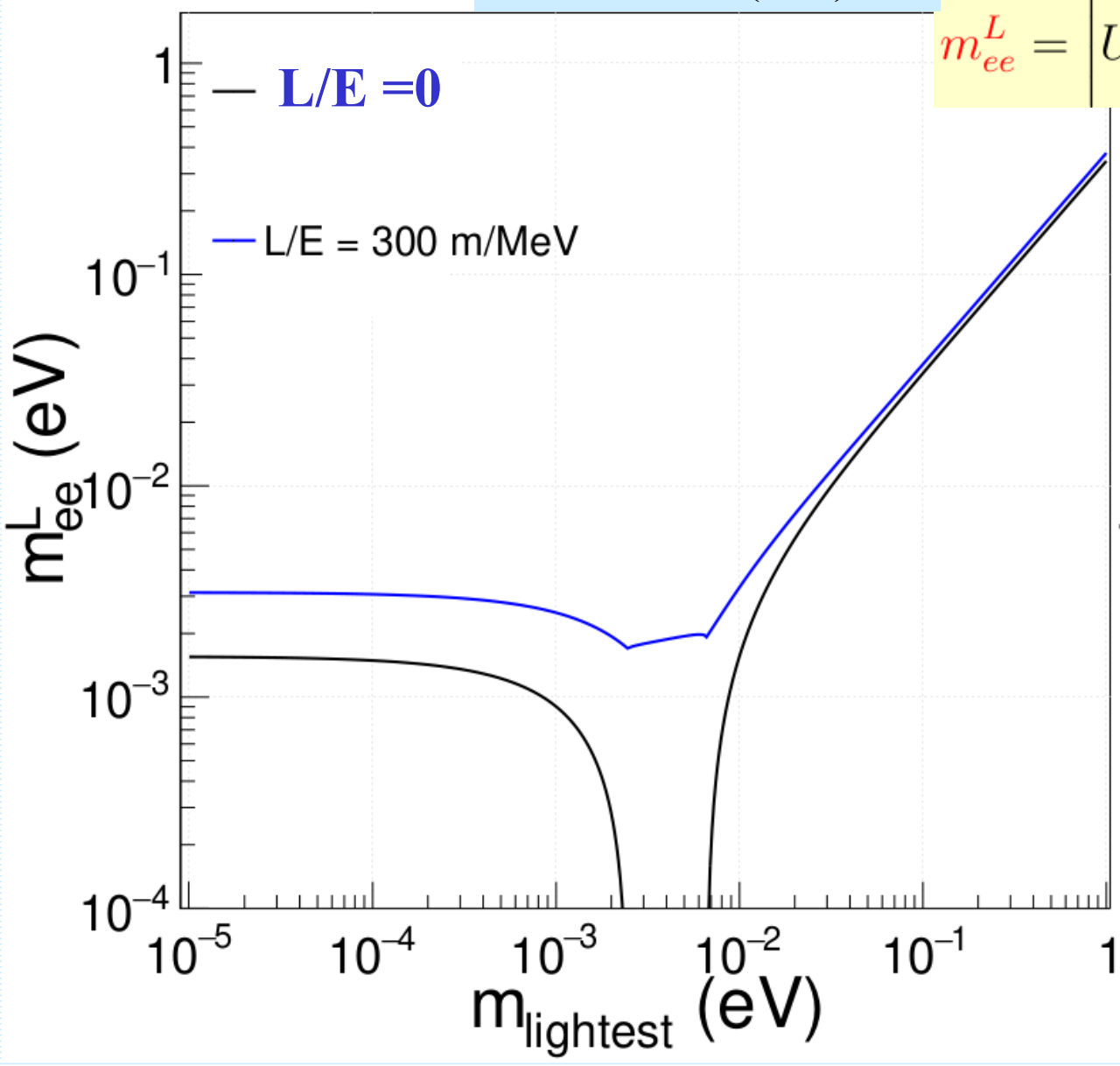
Dependence of m_{ee}^L on m_{lightest} and L/E

$$m_{ee}^{L=0} = m_{\beta\beta}$$

PoS TAUP2023 (2024) 346

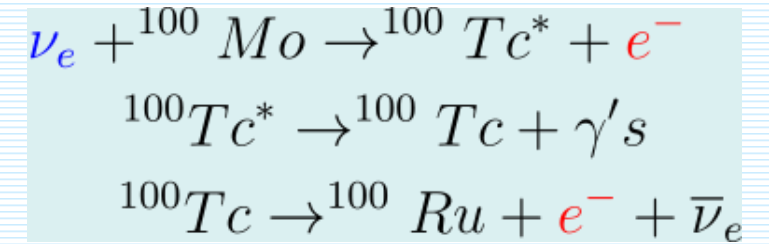
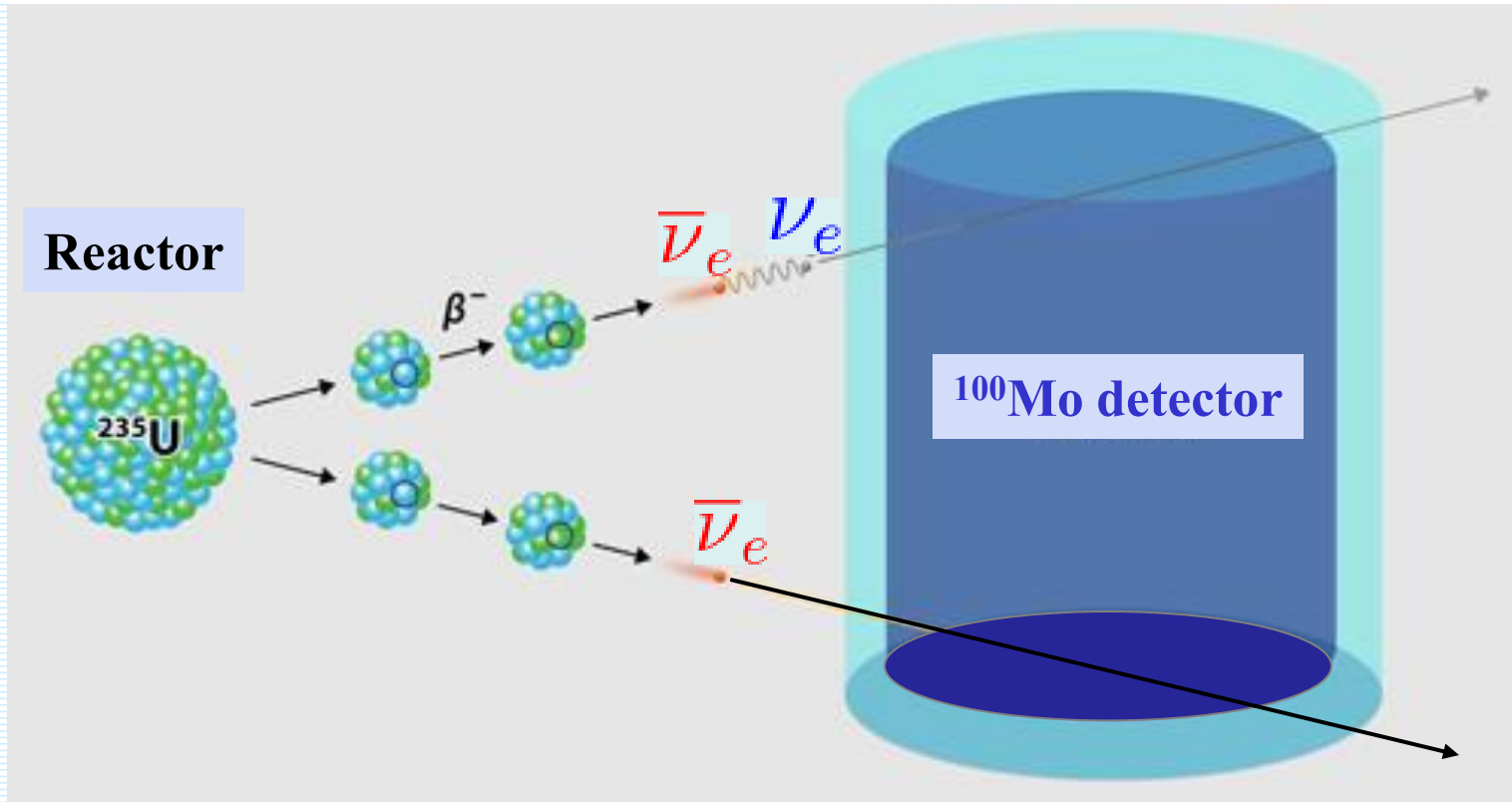
$$m_{\beta\beta} = \left| \sum_j U_{ej}^2 m_j \right| = \left| c_{12}^2 c_{13}^2 m_1 e^{2i\phi_1} + s_{12}^2 c_{13}^2 m_2 e^{2i\phi_2} + s_{13}^2 m_3 \right|$$

$$m_{ee}^L = \left| U_{e1}^2 m_1 + U_{e2}^2 m_2 e^{-i \frac{\Delta m_{21}^2}{2E} L} + U_{e3}^2 m_3 e^{-i \frac{\Delta m_{31}^2}{2E} L} \right|$$



Oscillations of reactor antineutrinos to neutrinos

PoS (ICHEP2024) 138



$$M_T = 1 \text{ ton}$$

$$W = 1 \text{ Gwatt}$$

$$L = 10 \text{ m}$$

$$F(Z, E_e) B_{GT} = 5 \times 0.33$$

$$\mu \approx 100 \text{ g mol}^{-1}$$

$$R_L^{\bar{\nu}\nu} = 5.33 \times 10^{-13} \left(\frac{M_T}{1\text{kg}} \right) \left(\frac{1\text{g} \cdot \text{mol}^{-1}}{\mu} \right) \times \left(\frac{m_{\beta\beta}}{1\text{eV}} \right)^2 \left(\frac{W}{1\text{GWatt}} \right) \left(\frac{1\text{m}}{L} \right)^2 F(Z, E_e) B_{GT} \left(\frac{\text{events}}{\text{s}} \right)$$

$$R_L^{\bar{\nu}\nu} \approx 10^{-6} \left(\frac{m_{\beta\beta}}{1\text{eV}} \right)^2 \left(\frac{\text{events}}{\text{year}} \right)$$

Mixing of 3 active and 3 sterile neutrinos

Mixing of 6 Majorana neutrinos

$$\begin{pmatrix} \nu_\alpha \\ \nu_{\alpha'} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} U^* & U^* \\ -V & V \end{pmatrix} \begin{pmatrix} \nu_i \\ \nu_{i'} \end{pmatrix}$$

Time evolution of flavor states

$$\begin{aligned} |\nu_\alpha\rangle(t) &= \frac{1}{\sqrt{2}} \sum_{i=1}^3 U_{\alpha i}^* e^{-itE_i} |\nu_i\rangle + \frac{1}{\sqrt{2}} \sum_{i'=1}^3 U_{\alpha i'}^* e^{-itE_{i'+3}} |\nu_{i'+3}\rangle \\ |\nu_{\alpha'}\rangle(t) &= -\frac{1}{\sqrt{2}} \sum_{i=1}^3 V_{\alpha' i} e^{-itE_i} |\nu_i\rangle + \frac{1}{\sqrt{2}} \sum_{i'=1}^3 V_{\alpha' i'} e^{-itE_{i'+3}} |\nu_{i'+3}\rangle \end{aligned}$$

Active-active and active-sterile oscillation amplitudes

$$\begin{aligned} \langle \nu_\beta | \nu_\alpha(t) \rangle &= \frac{1}{2} \sum_{i=1}^3 U_{\alpha i}^* U_{\beta i} e^{-itE_i} + \frac{1}{2} \sum_{i'=1}^3 U_{\alpha i'}^* U_{\beta i'} e^{-itE_{i'+3}} \\ \langle \nu_{\beta'} | \nu_\alpha(t) \rangle &= -\frac{1}{2} \sum_{i=1}^3 U_{\alpha i}^* V_{\beta' i} e^{-itE_i} + \frac{1}{2} \sum_{i'=1}^3 U_{\alpha i'}^* V_{\beta' i'} e^{-itE_{i'+3}} \end{aligned}$$

$$|\mathbf{M}_{L,R}| \ll |\mathbf{M}_D|$$

Close to a restoration of 3 Dirac neutrinos

Unitary mixing matrix
introduced

$$\mathcal{T} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\mathcal{U} = \mathcal{X} \cdot \mathcal{T} \cdot \mathcal{A} \cdot \mathcal{S}$$

Expansion of mixing matrix in X

$$\mathcal{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} (1 + X^\dagger)U^T & -(1 - X^\dagger)V^\dagger \\ (1 - X)U^T & (1 + X)V^\dagger \end{pmatrix} + \mathcal{O}(X^2)$$

Full restoration of Dirac neutrinos ($\mathbf{M}_{L,R}=0$)

$$\mathcal{M}_{\text{diag}}(\epsilon_i = 0) = \mathcal{U}^*(X = 0) \mathcal{M}(M_{L,R} = 0) \mathcal{U}^\dagger(X = 0)$$

$$U^\dagger U = 1 = V^\dagger V$$

$$M_{\text{diag}} = U^\dagger M_D V \equiv \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

Simplified QD-mixing scenario

$$\mathcal{U}_{\text{QD}} = \frac{1}{\sqrt{2}} \begin{pmatrix} U & U \\ -V^* & V^* \end{pmatrix}$$

Oscillation probability for QD neutrinos (active-active) (6x6 generalization of the PMNS matrix)

$$\mathcal{U}_{\text{QD}} = \frac{1}{\sqrt{2}} \begin{pmatrix} U & U \\ -V^* & V^* \end{pmatrix}$$

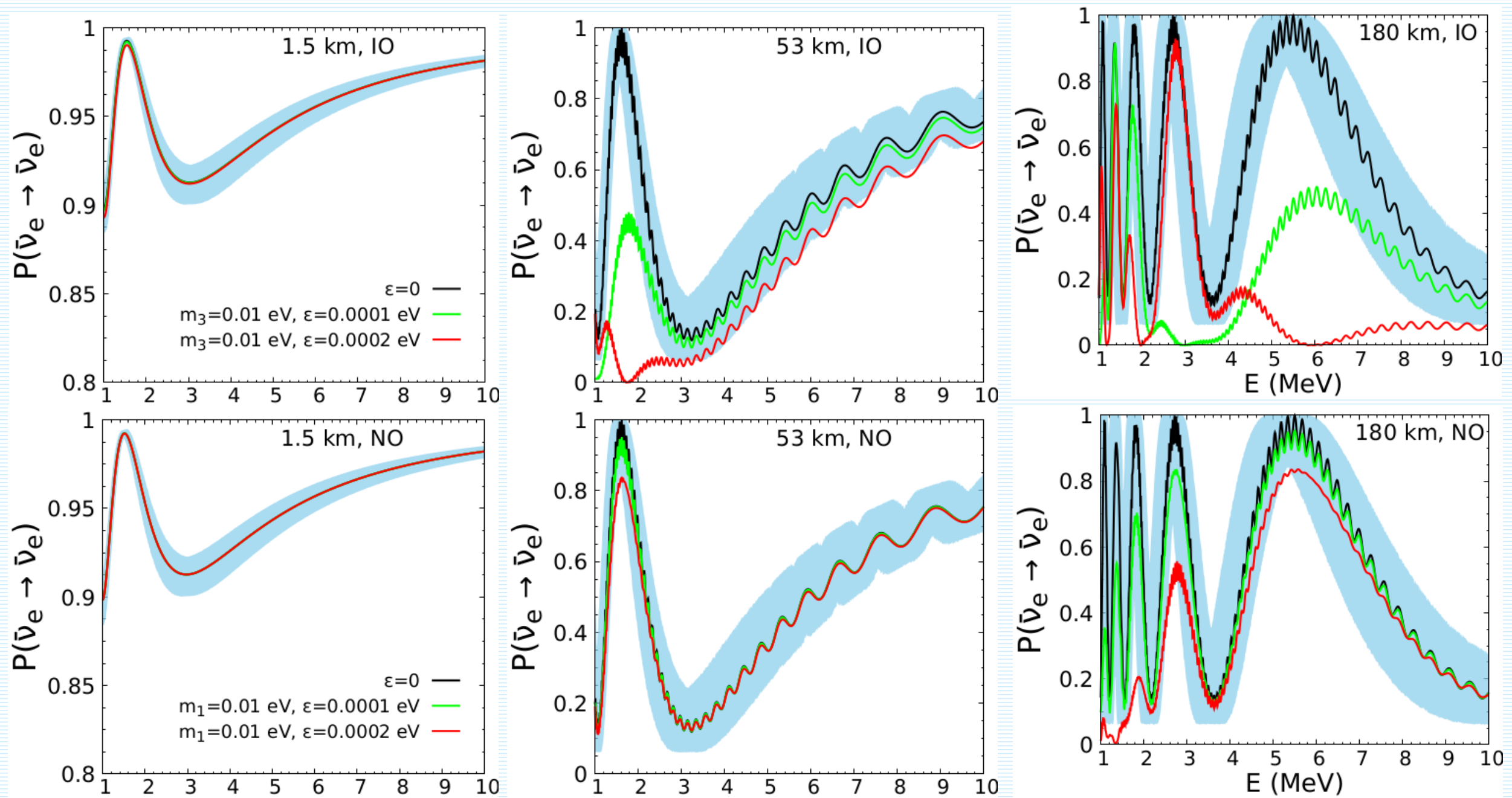
$$m_i^\pm = \pm m_i + \epsilon \quad (\epsilon > 0)$$

Oscillation probabilities among
active neutrinos

3 Dirac masses and 1 universal Majorana mass
splitting ϵ

$$\begin{aligned} P_{\alpha\beta} = & \delta_{\alpha\beta} - \sum_{i=1}^3 |U_{\alpha i}|^2 |U_{\beta i}|^2 \sin^2 \frac{m_i \epsilon}{E} L - \sum_{i>j=1}^3 \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \left(\sin^2 \frac{\Delta m_{ij}^2 + 2\epsilon \Delta m_{ij}}{4E} L \right. \\ & \left. + \sin^2 \frac{\Delta m_{ij}^2 - 2\epsilon \Sigma m_{ij}}{4E} L + \sin^2 \frac{\Delta m_{ij}^2 + 2\epsilon \Sigma m_{ij}}{4E} L + \sin^2 \frac{\Delta m_{ij}^2 - 2\epsilon \Delta m_{ij}}{4E} L \right) \\ & + \frac{1}{2} \sum_{i>j=1}^3 \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \left(\sin \frac{\Delta m_{ij}^2 + 2\epsilon \Delta m_{ij}}{2E} L + \sin \frac{\Delta m_{ij}^2 - 2\epsilon \Sigma m_{ij}}{2E} L \right. \\ & \left. + \sin \frac{\Delta m_{ij}^2 + 2\epsilon \Sigma m_{ij}}{2E} L + \sin \frac{\Delta m_{ij}^2 - 2\epsilon \Delta m_{ij}}{2E} L \right) \end{aligned}$$

Quasi-Dirac neutrino oscillations at different distances



The survival probability of electron antineutrinos

Quasi-Dirac neutrinos and constraints on neutrino masses

$$P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}(\epsilon \neq 0) = P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}(\epsilon = 0) - \frac{\epsilon^2 L^2}{E^2} \left[c_{13}^4 c_{12}^4 m_1^2 + c_{13}^4 s_{12}^4 m_2^2 + s_{13}^4 m_3^2 \right] \\ - \frac{\epsilon^2 L^2}{4E^2} \left[4 c_{13}^4 s_{12}^2 c_{12}^2 \Sigma m_{21}^2 \cos \frac{\Delta m_{21}^2 L}{2E} + 4 s_{13}^2 c_{13}^2 c_{12}^2 \Sigma m_{31}^2 \cos \frac{\Delta m_{31}^2 L}{2E} \right. \\ \left. + 4 s_{13}^2 c_{13}^2 s_{12}^2 \Sigma m_{32}^2 \cos \frac{\Delta m_{32}^2 L}{2E} \right] + \mathcal{O}(\epsilon^4),$$

Tritium β -decay

$$m_\beta = \sqrt{m_1^2 c_{12}^2 c_{13}^2 + m_2^2 c_{13}^2 s_{12}^2 + m_3^2 s_{13}^2 + \epsilon^2} \\ = m_\beta^{(0)} \left(1 + \frac{1}{2} \left(\epsilon / m_\beta^{(0)} \right)^2 + \dots \right)$$

Cosmology

$$\frac{1}{2} \sum_{i=1}^3 |\tilde{\mathcal{M}}_{ii}| = \sum_{i=1}^3 m_i$$

Restriction from **Daya-Bay** data (3σ):

Survival probabilities with non-zero ϵ are the same **3v** cases.

$0\nu\beta\beta$ -decay

$$m_{\beta\beta} = [M_L]_{ee} \\ = \epsilon \left| c_{12}^2 c_{13}^2 + e^{2i\alpha_{21}} c_{13}^2 s_{12}^2 + e^{2i\alpha_{31}} s_{13}^2 \right| \text{ for Simkovic}$$

$$m_{\beta\beta} \lesssim 30 \text{ meV for NO} \\ \lesssim 1 \text{ meV for IO}$$

*The sterile ν mechanism of the $0\nu\beta\beta$ -decay
(D-M mass term, V-A, SM int.)
Interpolating formula*

Dirac-Majorana
mass term

$$N = \sum_{\alpha=s,e,\mu,\tau} U_{N\alpha} \nu_{\alpha}$$

small ν masses due to see-saw mechanism

$$\begin{pmatrix} 0 & m_D \\ m_D & m_{LNV} \end{pmatrix}$$

Light ν mass $\approx (m_D/m_{LNV}) m_D$
Heavy ν mass $\approx m_{LNV}$

Different motivations for the LNV scale Λ

eV

light sterile ν
 10^{-6} GeV

keV

hot DM
 10^{-6} GeV

Fermi

10^{-6} GeV

TeV

LHC
 10^3 GeV

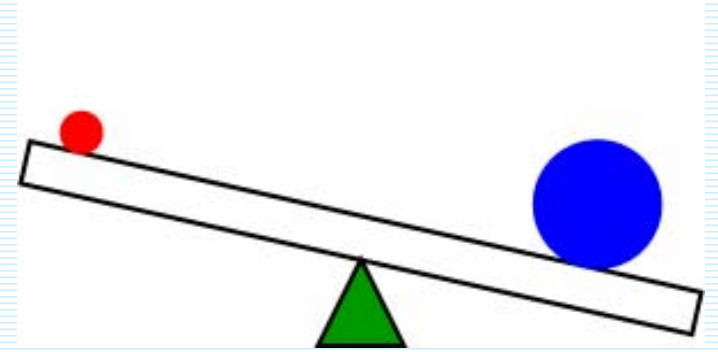
GUT

10^{16} GeV

Planck

10^{19} GeV

Mixing of active-sterile
neutrinos



Neutrinos masses offer a great
opportunity to jump
beyond the EW framework
via see-saw ...

Left-handed neutrinos: Majorana neutrino mass eigenstate N with arbitrary mass m_N

Faessler, Gonzales, Kovalenko,
F. Š., PRD 90 (2014) 096010]

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} g_A^4 \left| \sum_N \left(U_{eN}^2 m_N \right) m_p M'^{0\nu}(m_N, g_A^{\text{eff}}) \right|^2$$

General case

$$M'^{0\nu}(m_N, g_A^{\text{eff}}) = \frac{1}{m_p m_e} \frac{R}{2\pi^2 g_A^2} \sum_n \int d^3x d^3y d^3p$$

$$\times e^{ip \cdot (x-y)} \frac{\langle 0_F^+ | J^{\mu\dagger}(x) | n \rangle \langle n | J_\mu^\dagger(y) | 0_I^+ \rangle}{\sqrt{p^2 + m_N^2} (\sqrt{p^2 + m_N^2} + E_n - \frac{E_I - E_F}{2})}$$

light ν exchange

$$M'^{0\nu}(m_N \rightarrow 0, g_A^{\text{eff}}) = \frac{1}{m_p m_e} M_\nu'^{0\nu}(g_A^{\text{eff}})$$

$$M'^{0\nu}(m_N \rightarrow \infty, g_A^{\text{eff}}) = \frac{1}{m_N^2} M_N'^{0\nu}(g_A^{\text{eff}})$$

heavy ν exchange

Particular cases

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} g_A^4 \times$$

$$\times \begin{cases} \left| \frac{\langle m_\nu \rangle}{m_e} \right|^2 \left| M_\nu'^{0\nu}(g_A^{\text{eff}}) \right|^2 & \text{for } m_N \ll p_F \\ \left| \langle \frac{1}{m_N} \rangle m_p \right|^2 \left| M_N'^{0\nu}(g_A^{\text{eff}}) \right|^2 & \text{for } m_N \gg p_F \end{cases}$$

$$\langle m_\nu \rangle = \sum_N U_{eN}^2 m_N$$

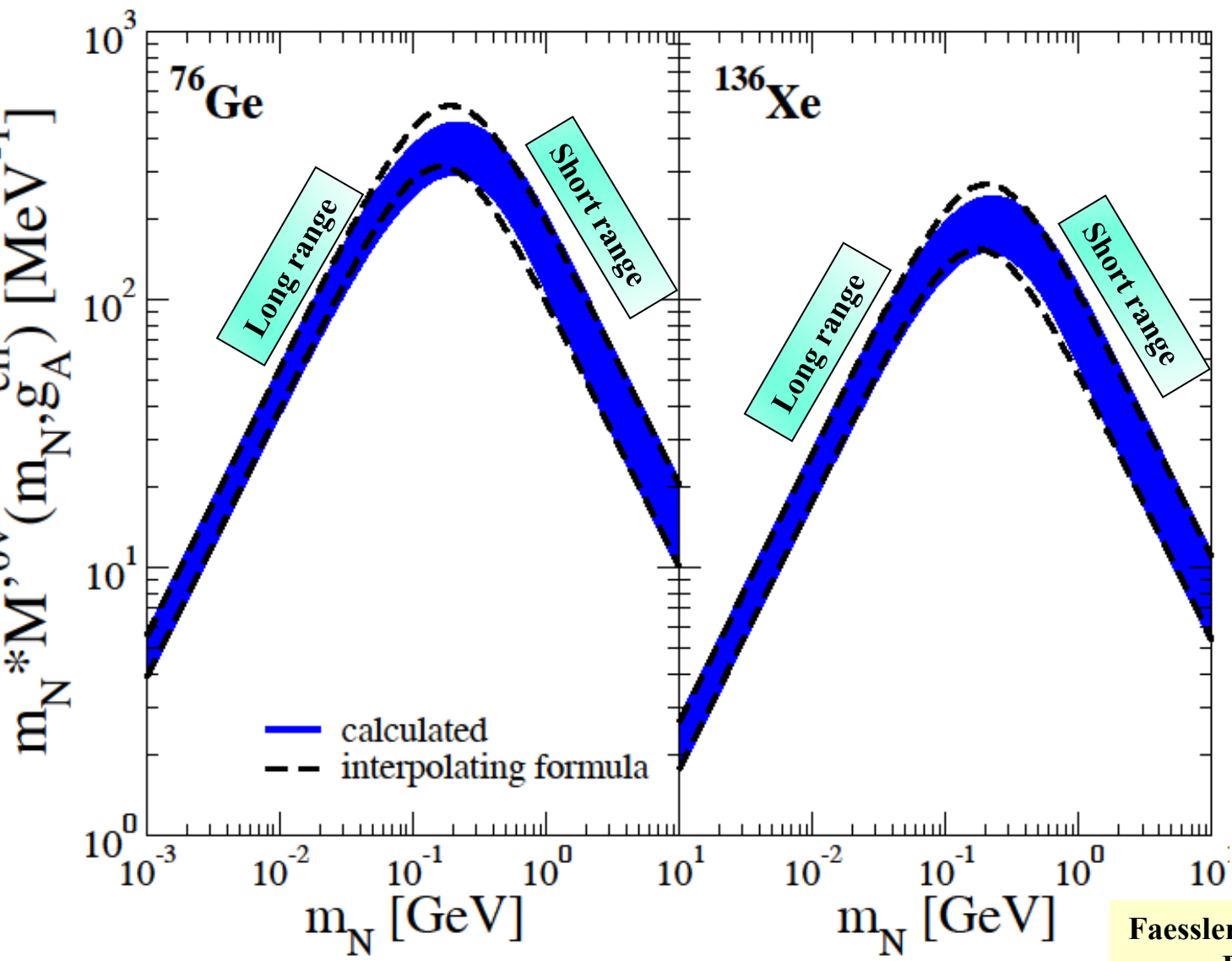
$$\left\langle \frac{1}{m_N} \right\rangle = \sum_N \frac{U_{eN}^2}{m_N}$$

Interpolating formula

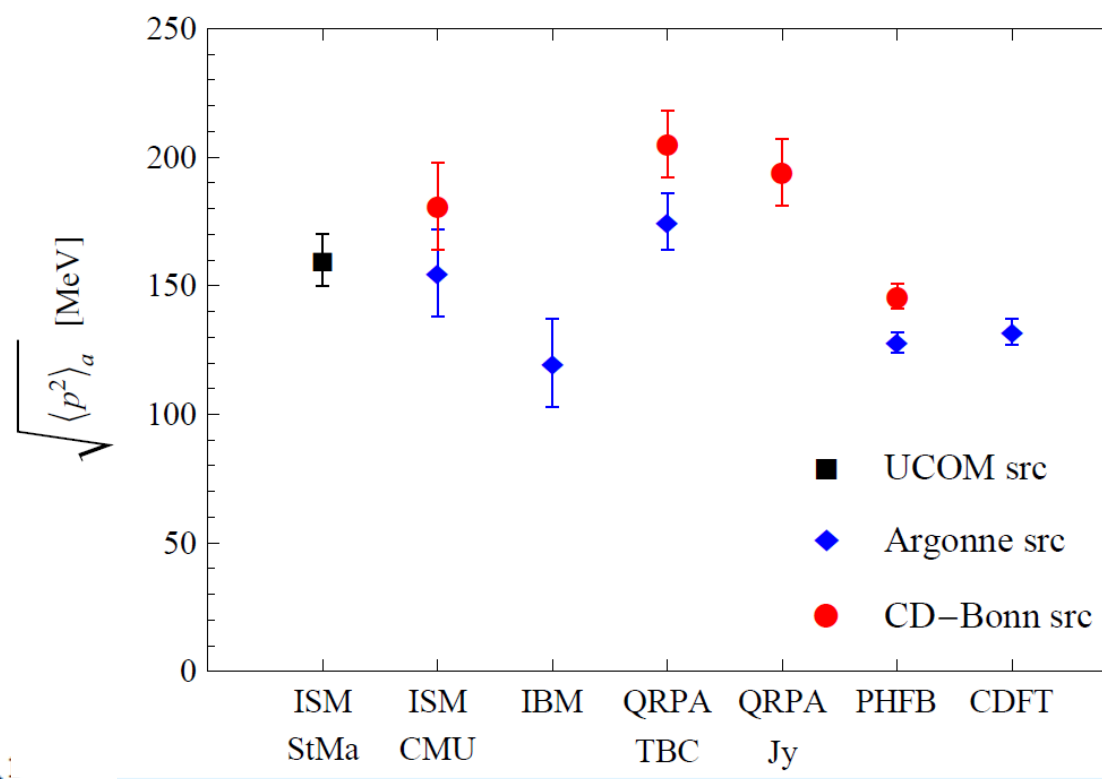
$$[T_{1/2}^{0\nu}]^{-1} = \mathcal{A} \cdot \left| m_p \sum_N U_{eN}^2 \frac{m_N}{\langle p^2 \rangle + m_N^2} \right|,$$

$$\mathcal{A} = G^{0\nu} g_A^4 \left| M_N^{0\nu}(g_A^{\text{eff}}) \right|^2,$$

$$\langle p^2 \rangle = m_p m_e \left| \frac{M_N^{0\nu}(g_A^{\text{eff}})}{M_\nu^{0\nu}(g_A^{\text{eff}})} \right|^2 \approx 200 \text{ MeV}$$

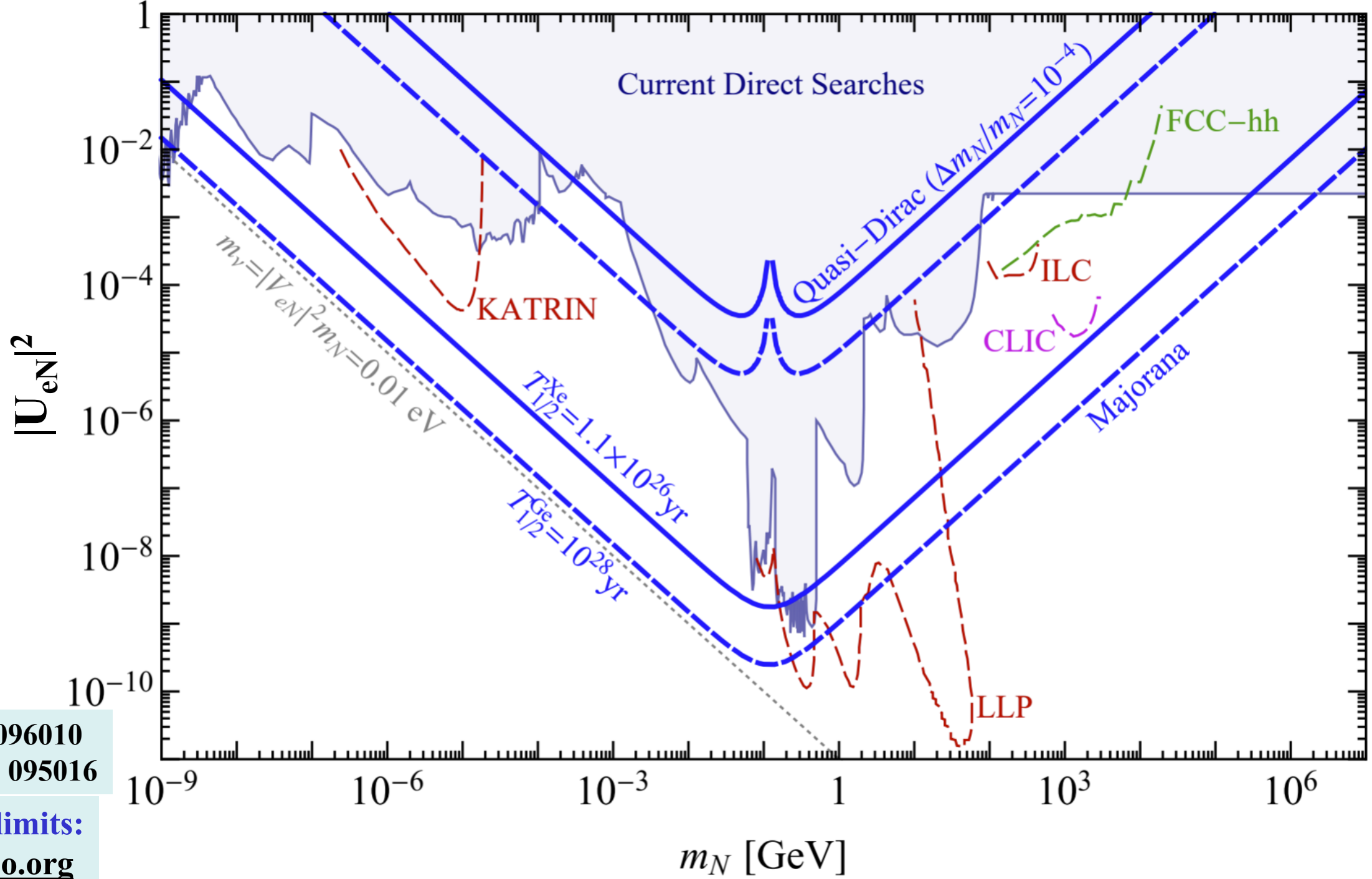


Interpolating formula is justified by practically no dependence $\langle p^2 \rangle$ on Λ



Sterile ν (U_{eN} , m_N)

Constraints from **Direct Searches** are less Stringent as those from **$0\nu\beta\beta$**



PRD 90 (2014) 096010
PRD 102 (2020) 095016

Direct search limits:
sterile-neutrino.org

The $0\nu\beta\beta$ -decay within L - R symmetric theories (interpolating formula)

(D - M mass term, see-saw, V - A and V + A int., exchange of heavy neutrinos)

A. Babič, S. Kovalenko, M.I. Krivoruchenko, F.Š., PRD 98, 015003 (2018)

$$[T_{1/2}^{0\nu}]^{-1} = \eta_{\nu N}^2 C_{\nu N}$$

6x6 matrix: 15 angles, 10+5 CP phases
3x3 matrix: 3 angles, 1+2 CP phases

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

Effective LNV parameter (interpolating formula)

$$\eta_{\nu N}^2 = \left| \sum_{j=1}^3 \left(U_{ej}^2 \frac{m_j}{m_e} + S_{ej}^2 \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M_j^2} \frac{M_j}{m_e} \right) \right|^2 + \lambda^2 \left| \sum_{j=1}^3 \left(T_{ej}^2 \frac{m_j}{m_e} + V_{ej}^2 \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M_j^2} \frac{M_j}{m_e} \right) \right|^2$$

$$\langle p^2 \rangle = m_p m_e \frac{M_N'^{0\nu}}{M_\nu'^{0\nu}}$$

$$\nu_{eL} = \sum_{j=1}^3 \left(U_{ej} \nu_{jL} + S_{ej} (N_{jR})^C \right),$$

$$\nu_{eR} = \sum_{j=1}^3 \left(T_{ej}^* (\nu_{jL})^C + V_{ej}^* N_{jR} \right)$$

Assumptions:

- i) the see-saw structure
- ii) mixing between different generations is neglected

$$\mathcal{U}_{\text{PMNS}} = \begin{pmatrix} U_{\text{PMNS}} & \zeta \mathbf{1} \\ -\zeta \mathbf{1} & U_{\text{PMNS}}^\dagger \end{pmatrix}$$

6x6 matrix $\Rightarrow$

3 angles, 1+2 CP phases, 1 see-saw par.

$$\mathcal{U}_{\text{PMNS}} \mathcal{U}_{\text{PMNS}}^\dagger = \mathcal{U}_{\text{PMNS}}^\dagger \mathcal{U}_{\text{PMNS}} = \mathbf{1}$$

$M_{\beta\beta}^R$ depends on
 “Dirac” CP phase δ
 unlike “Majorana”
 CP phases α_1 and α_2

$$m_i M_i \simeq m_D^2$$

$$M_{\beta\beta}^R = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^3 (U_0^\dagger)_{ej}^2 m_j \right|$$

see-saw
parameter

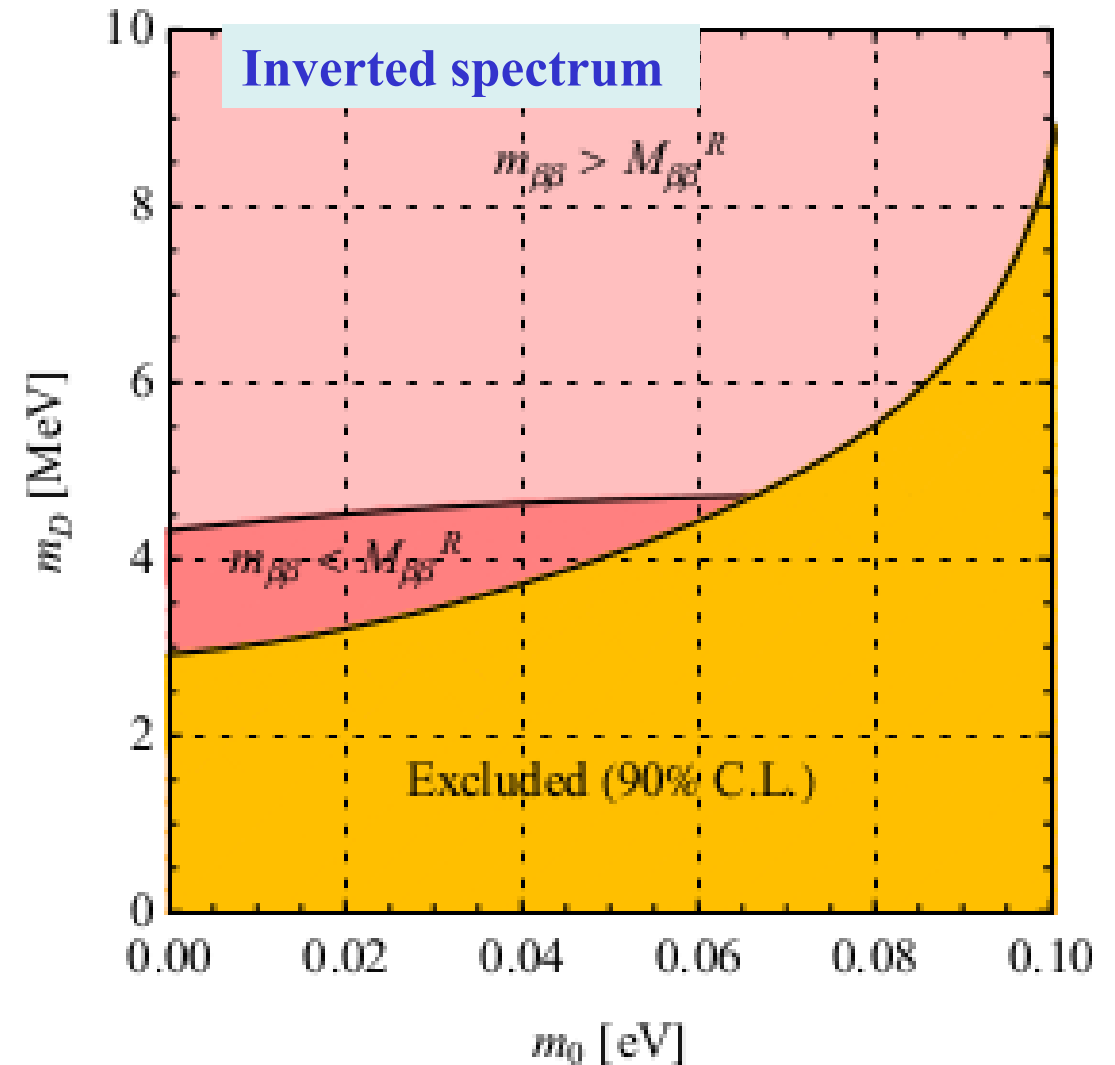
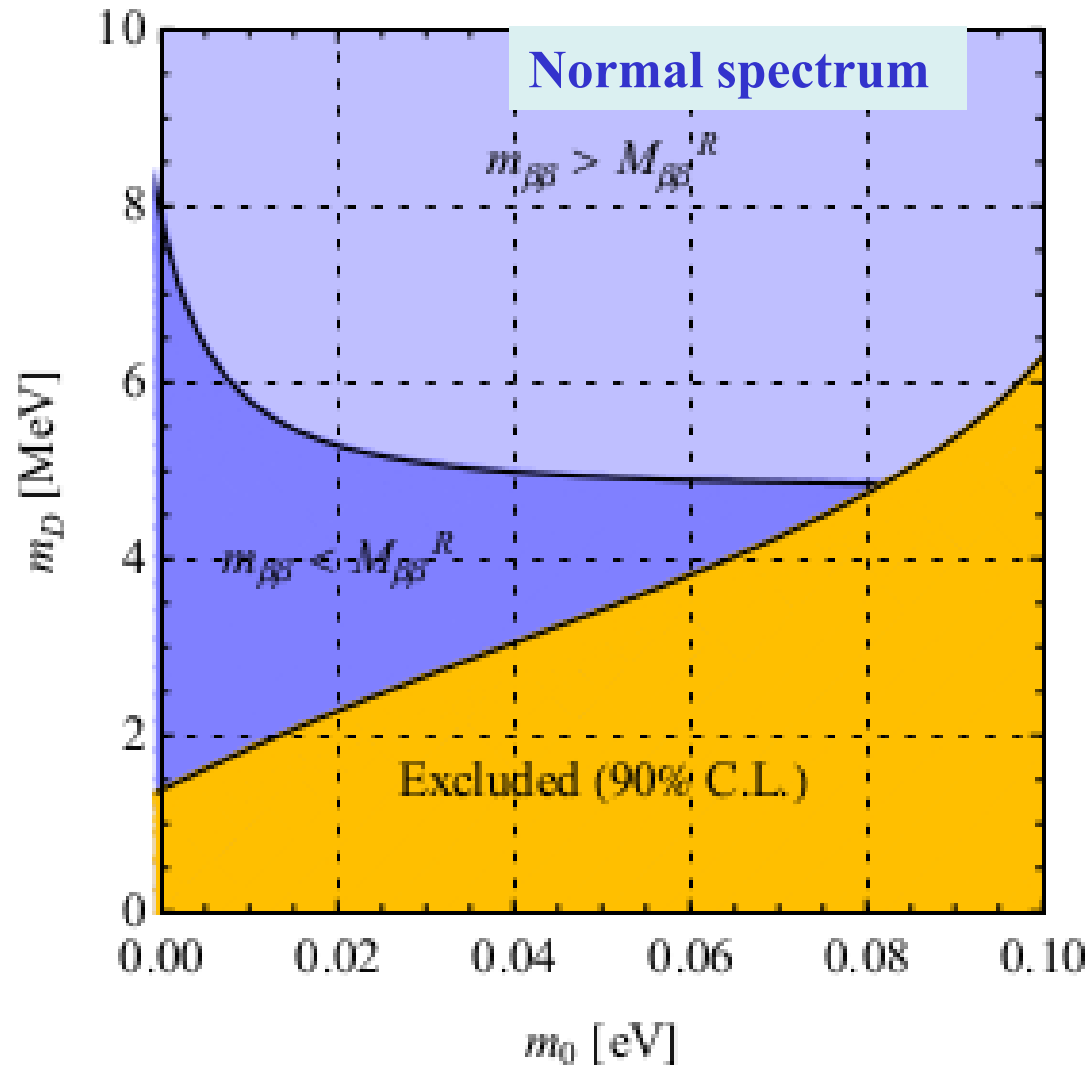
$$\zeta = \frac{m_D}{m_{\text{LNV}}}$$

$$\eta_{\nu N}^2 = \frac{1}{m_e^2} \left(m_{\beta\beta}^2 + (M_{\beta\beta}^R)^2 \right)$$

See-saw scenario

$$m_i M_i \simeq m_D^2$$

$$M_{\beta\beta}^R = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^3 (U_0^\dagger)_{ej}^2 m_j \right|$$



$0\nu\beta\beta$ -decay NME status 2023(25)

The nuclear w. f. of
(A,Z), (A,Z+1)*, (A,Z+2)
Many-body methods
of choice:

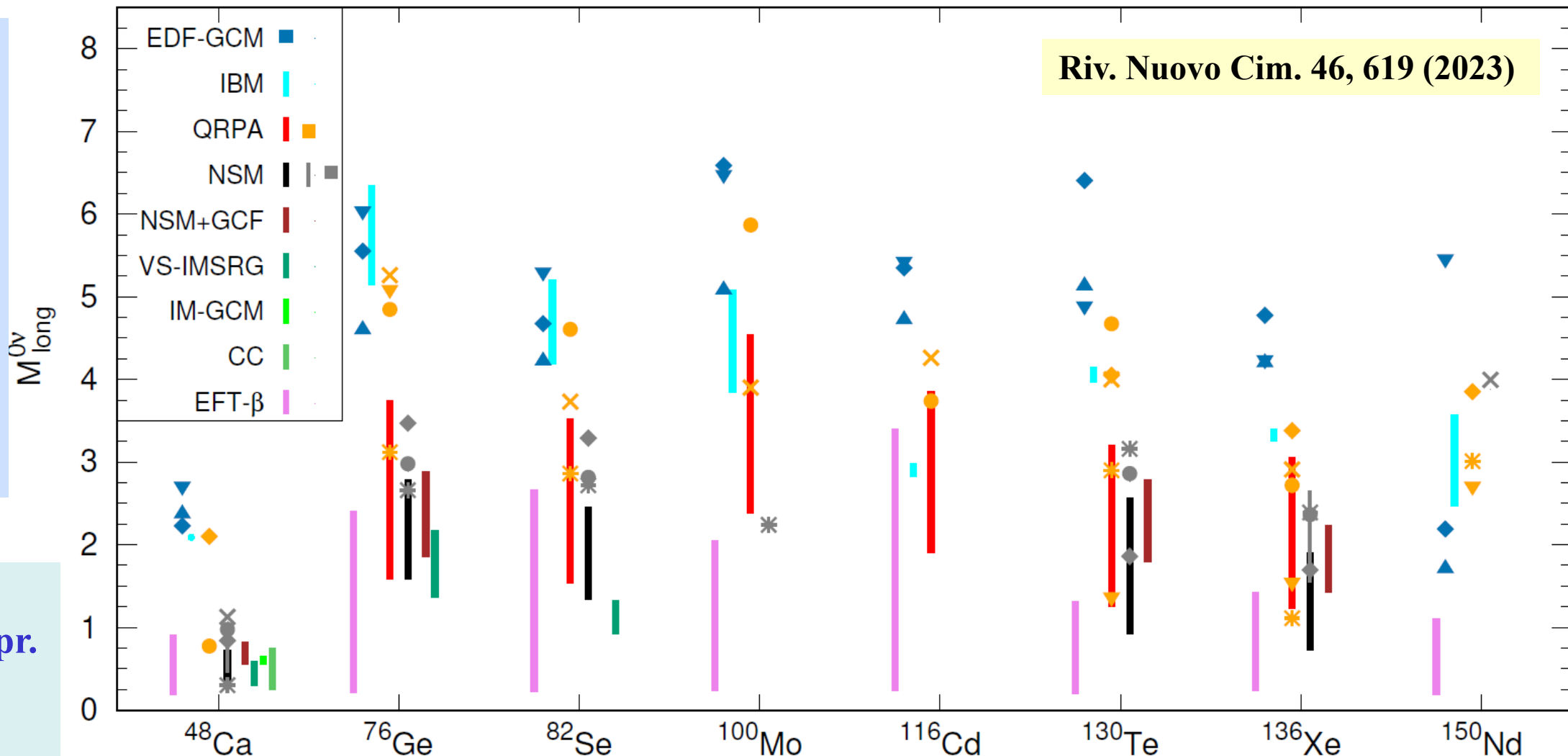
The $0\nu\beta\beta$ nuclear transition operators
(F, GT, and tensor type):

! Isospin, and spin-isospin symmetries
($M_{Fcl}\approx 0$, M_{GTcl} strongly suppressed):

All
models
missing
essential
physics

Impossible
to assign
rigorous
uncertainties

Differences:
Many-body appr.
Size of the m.s.
residual int.



Assuming that the $0\nu\beta\beta$ process is mediated by a light-Majorana-neutrino exchange, a systematic analysis in chiral effective field theory shows that already at leading order **a contact operator is required to ensure renormalizability of the amplitude for $nn \rightarrow pp + ee$ process**. Without the strong $^1S_0$ short range interaction (which appears universally in all nuclear potentials) there would be no need of contact term.

$$M^{0\nu} = -\frac{1}{g_A^2} M_F^{0\nu} + M_{GT}^{0\nu} + M_T^{0\nu} + 2 \frac{m_\pi^2 \mathbf{g}_\nu^{NN}}{g_A^2} M_{F, sd}^{0\nu}$$



Long-range



Short-range

Some questions:
A **correspondence** of the standard and the chiral field theory formalisms. Is the contact term involved in the standard mechanism (completeness ...)?
What is the **magnitude** of the contact term NME? **Can it be large?** Justification with other phenomenology needed – pion and heavy-ion DCX, etc.

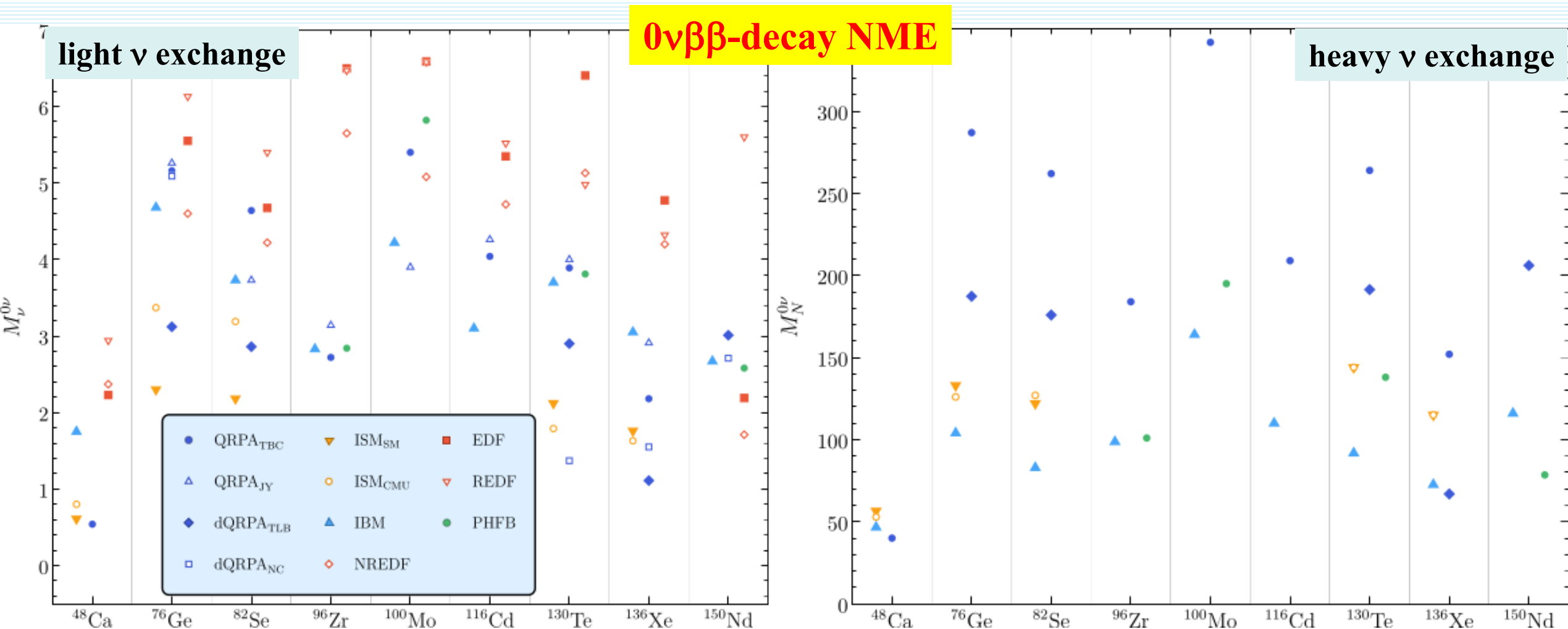
Nucleus	QRPA	Short-range
	$M_S^{0\nu} / M_L^{0\nu} (\%)$	$M_S^{0\nu} / M_L^{0\nu} (\%)$
^{48}Ca		23 – 62
^{76}Ge	32 – 73	15 – 42
^{82}Se	30 – 70	15 – 41
^{96}Zr	29 – 69	
^{100}Mo	49 – 108	
^{116}Cd	26 – 61	
^{124}Sn	36 – 81	17 – 46
^{128}Te	35 – 77	17 – 46
^{130}Te	34 – 77	17 – 47
^{136}Xe	30 – 70	17 – 47

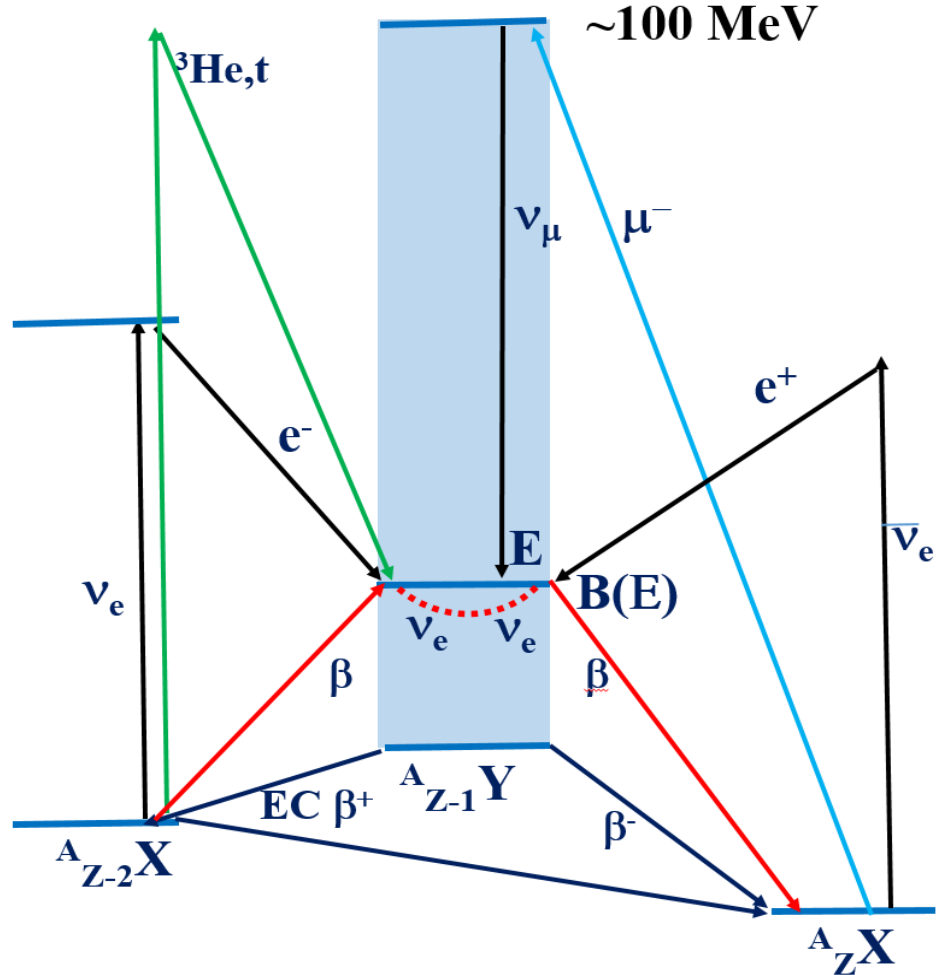
$$M'^{0\nu}(m_N, g_A^{\text{eff}}) = \frac{1}{m_p m_e} \frac{R}{2\pi^2 g_A^2} \sum_n \int d^3x d^3y d^3p$$

$$\times e^{ip \cdot (x-y)} \frac{\langle 0_F^+ | J^{\mu\dagger}(\mathbf{x}) | n \rangle \langle n | J_\mu^\dagger(\mathbf{y}) | 0_I^+ \rangle}{\sqrt{p^2 + m_N^2} (\sqrt{p^2 + m_N^2} + E_n - \frac{E_I - E_F}{2})}$$

$$M'^{0\nu}(m_N \rightarrow 0, g_A^{\text{eff}}) = \frac{1}{m_p m_e} M_\nu'^{0\nu}(g_A^{\text{eff}})$$

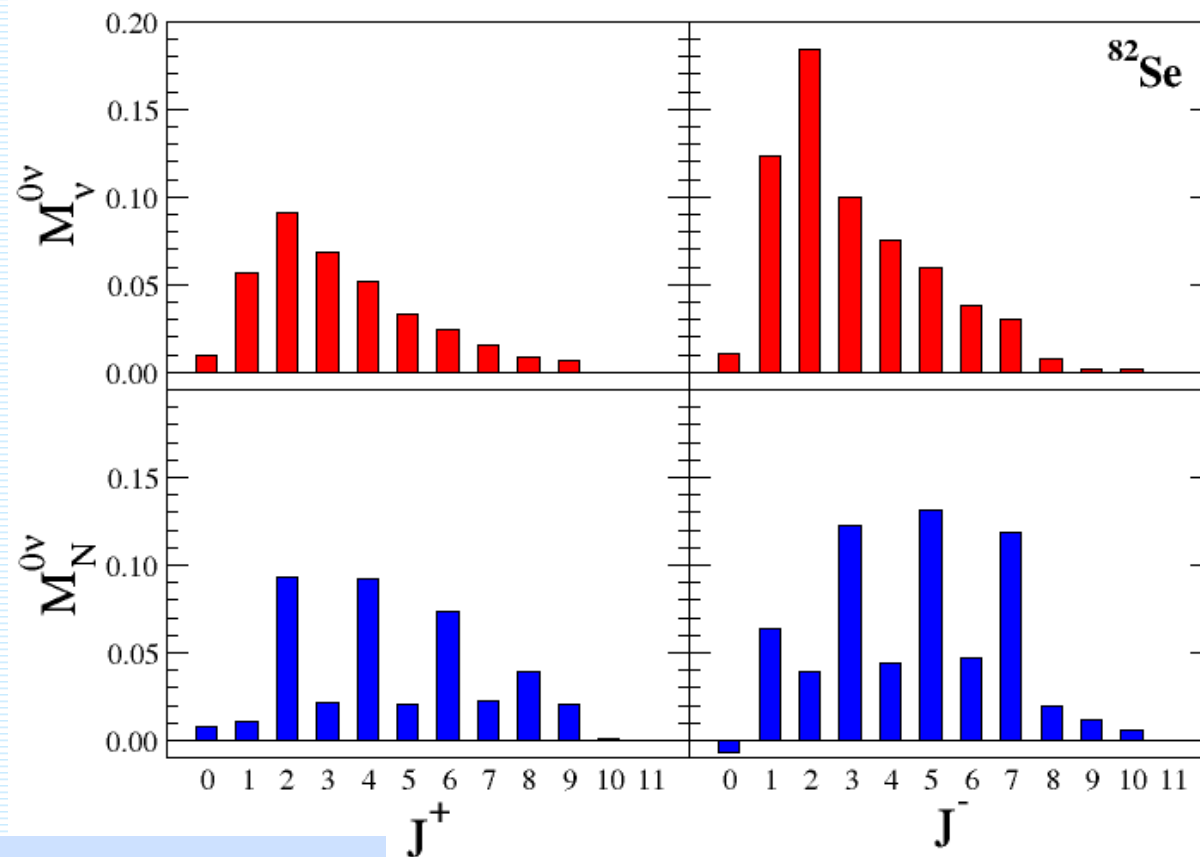
$$M'^{0\nu}(m_N \rightarrow \infty, g_A^{\text{eff}}) = \frac{1}{m_N^2} M_N'^{0\nu}(g_A^{\text{eff}})$$





- ✓ β -decay, EC and $2\nu\beta\beta$ decay
- ✓ μ -capture
- ✓ (π^+, π^-) , single charge exchange
- ✓ $(^3\text{He}, t)$, $(d, ^2\text{He})$, transfer reactions
- ✓ γ -ray spectroscopy, $\gamma\gamma$ -decay
- ✓ A promising experimental tool:
Heavy-Ion Double Charge-Exchange

Supporting nuclear physics experiments
(Measurements still not conclusive for $0\nu\beta\beta$ NME)



*Multipole
decomposition
of light and heavy
 $0\nu\beta\beta$ -decay NMEs
normalized to unity*

*Higher multipoles
are populated mostly due large ν -
momenta transfer*

Understanding of the $2\nu\beta\beta$ -decay NMEs is of crucial importance for correct evaluation of the $0\nu\beta\beta$ -decay NMEs



*Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states.
Both change two neutrons into two protons.*

Explaining $2\nu\beta\beta$ -decay is necessary but not sufficient

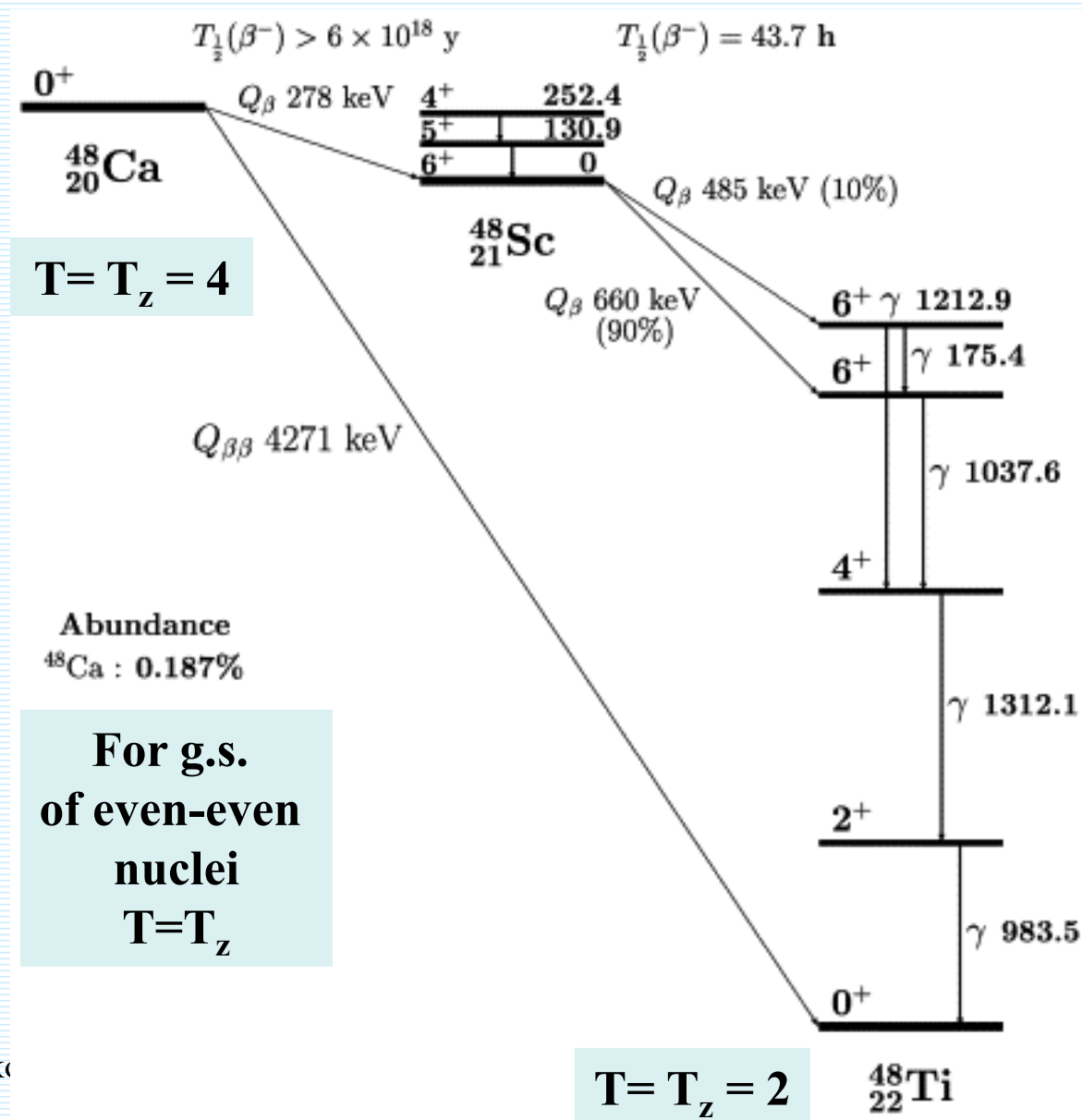
Let's put credit approaches that allow
to reproduce $2\nu\beta\beta$ -decay NMEs, in particular
without closure approximations

QRPA calculations at the forefront:

i) importance of pairing int.; **ii)** importance of isospin and spin-isospin symmetry restoration (isoscalar and isovector residual int.); **iii)** importance of relative deformation of the initial and final nuclei

There is no reliable calculation of
the $2\nu\beta\beta$ -decay NMEs yet

Fedor Simko



2νββ-decay rate

$$\left[T_{1/2}^{2\nu\beta\beta}(0^+) \right]^{-1} = \frac{m_e}{8\pi^7 \ln 2} (G_\beta m_e^2)^4 I^{2\nu}(0^+),$$

$$\begin{aligned} \mathcal{A}^{2\nu} = & g_V^4 \left[\frac{1}{4} |M_F^K + M_F^L|^2 + \frac{3}{4} |M_F^K - M_F^L|^2 \right] \\ & - g_V^2 g_A^2 \operatorname{Re} \{ M_F^{K*} M_{GT}^L + M_{GT}^{K*} M_F^L \} \\ & + \frac{g_A^4}{3} \left[\frac{3}{4} |M_{GT}^K + M_{GT}^L|^2 + \frac{1}{4} |M_{GT}^K - M_{GT}^L|^2 \right] \end{aligned}$$

$$\begin{aligned} M_F^K &= \sum_n \frac{K(0_n^+)}{2} F_n, & M_F^L &= \sum_n \frac{L(0_n^+)}{2} F_n, \\ M_{GT}^K &= \sum_n \frac{K(1_n^+)}{2} G_n, & M_{GT}^L &= \sum_n \frac{L(1_n^+)}{2} G_n, \end{aligned}$$

$$F_n = \langle 0_f^+ \| \sum_m \tau_m^- \| 0_n^+ \rangle \langle 0_n^+ \| \sum_m \tau_m^- \| 0_i^+ \rangle,$$

$$G_n = \langle 0_f^+ \| \sum_m \tau_m^- \sigma_m \| 1_n^+ \rangle \langle 1_n^+ \| \sum_m \tau_m^- \sigma_m \| 0_i^+ \rangle$$

$$\epsilon_K = E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1}$$

$$\epsilon_L = E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1}$$

In the closure approximation,
by assuming

$$2\bar{E}_n - E_i - E_f = 0$$

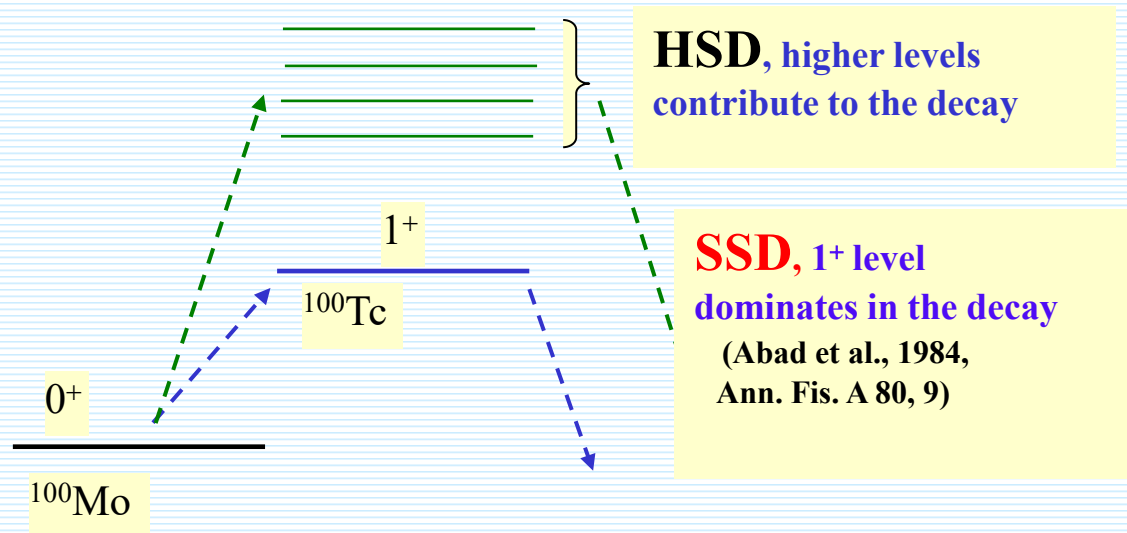
$$\mathcal{A}^{2\nu} = 0$$

$$\begin{aligned} I^{2\nu}(0^+) &= \frac{1}{m_e^9} \int_{m_e}^{E_i - E_f - m_e} F_0(Z_f, E_{e_1}) p_{e_1} E_{e_1} dE_{e_1} \\ &\times \int_{m_e}^{E_i - E_f - E_{e_1}} F_0(Z_f, E_{e_2}) p_{e_2} E_{e_2} dE_{e_2} \\ &\times \int_0^{E_i - E_f - E_{e_1} - E_{e_2}} E_{\nu_1}^2 E_{\nu_2}^2 \mathcal{A}^{2\nu} dE_{\nu_1}. \end{aligned}$$

$$\begin{aligned} K_n(J^+) &= \frac{2}{(2E_n(J^+) - E_i - E_f) + \epsilon_K} \\ &+ \frac{2}{(2E_n(J^+) - E_i - E_f) - \epsilon_K} \\ L_n(J^+) &= \frac{2}{(2E_n(J^+) - E_i - E_f) + \epsilon_L} \\ &+ \frac{2}{(2E_n(J^+) - E_i - E_f) - \epsilon_L} \end{aligned}$$

Is the real closure value close?
Cancellation of contributions from **higher**
and **lower** lying states
of intermediate nucleus

Single State Dominance (^{100}Mo , ^{106}Cd , ^{116}Cd , ^{128}Te ...)



Isotope	f.s.	$T_{1/2}(\text{SSD})[\text{y}]$	$T_{1/2}(\text{exp.})[\text{y}]$
$2\nu\beta\beta^-$			
^{100}Mo	$0_{\text{g.s.}}$	$7.3 \cdot 10^{18}$	$7.07 \cdot 10^{18}$
	0_1	$4.2 \cdot 10^{20}$	$6.1 \cdot 10^{20}$
^{116}Cd	$0_{\text{g.s.}}$	$1.1 \cdot 10^{19}$	$2.63 \cdot 10^{19}$
^{128}Te	$0_{\text{g.s.}}$	$1.1 \cdot 10^{25}$	$2.49 \cdot 10^{24}$
EC/EC			
^{106}Cd	$0_{\text{g.s.}}$	$>4.4 \cdot 10^{21}$	$>4.7 \cdot 10^{20}$
^{130}Ba	$0_{\text{g.s.}}$	$5.0 \cdot 10^{22}$	$2.2 \cdot 10^{21}$

$$M_{GT}^K = \sum_m \left(\frac{M_m^i(1^+)M_m^f(1^+)}{E_m - E_i + e_{10} + \nu_{10}} + \frac{M_m^i(1^+)M_m^f(1^+)}{E_m - E_i + e_{20} + \nu_{20}} \right)$$

$$\Rightarrow \text{SSD} \quad \frac{M_1^i(1^+)M_1^f(1^+)}{E_1 - E_i + e_{10} + \nu_{10}} + \frac{M_1^i(1^+)M_1^f(1^+)}{E_1 - E_i + e_{20} + \nu_{20}} \Rightarrow 2 \frac{M_1^i(1^+)M_1^f(1^+)}{E_1 - E_i + \Delta}$$

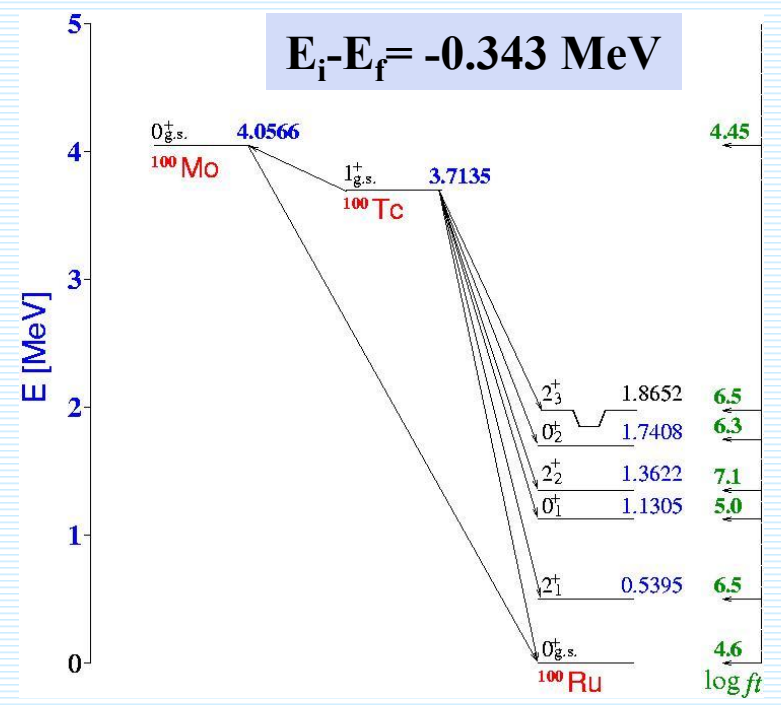
↑ **common approx.**

$$M_{GT}^K = M_{GT}^L(\nu_{10} \leftrightarrow \nu_{20})$$

$$e_{10} + \nu_{10} \approx e_{20} + \nu_{20} \approx (E_i - E_f)/2 \equiv \Delta$$

Domin, Kovalenko, F.Š., Semenov, NPA 753, 337 (2005)

Fedor Simkovic



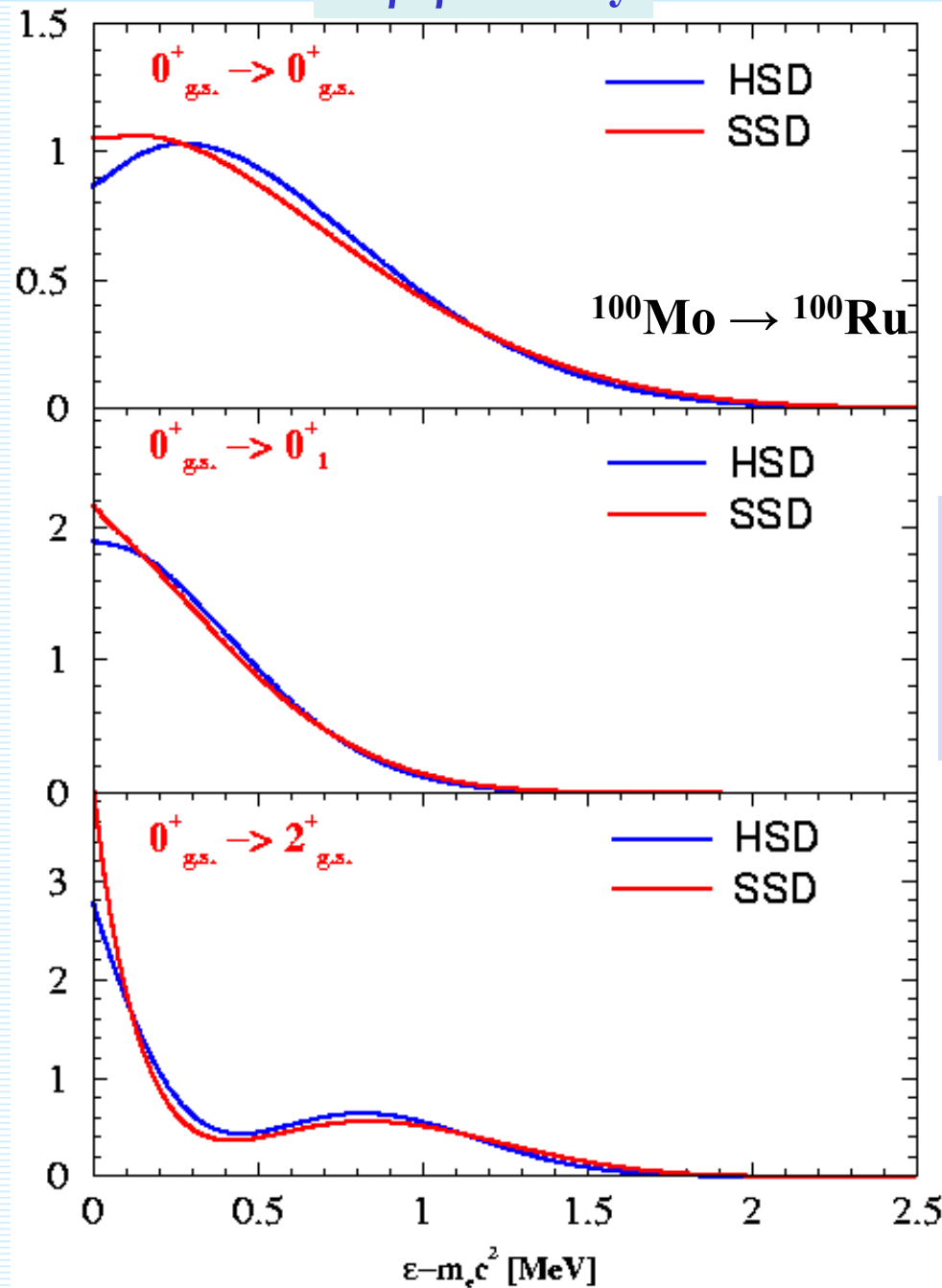
Study of differential characteristics

$E_1 - E_i \approx 0$ or neg. $\Rightarrow$ sensitivity to lepton energies in energy Denominators

$\Rightarrow$ SSD and HSD offer different differential characteristics

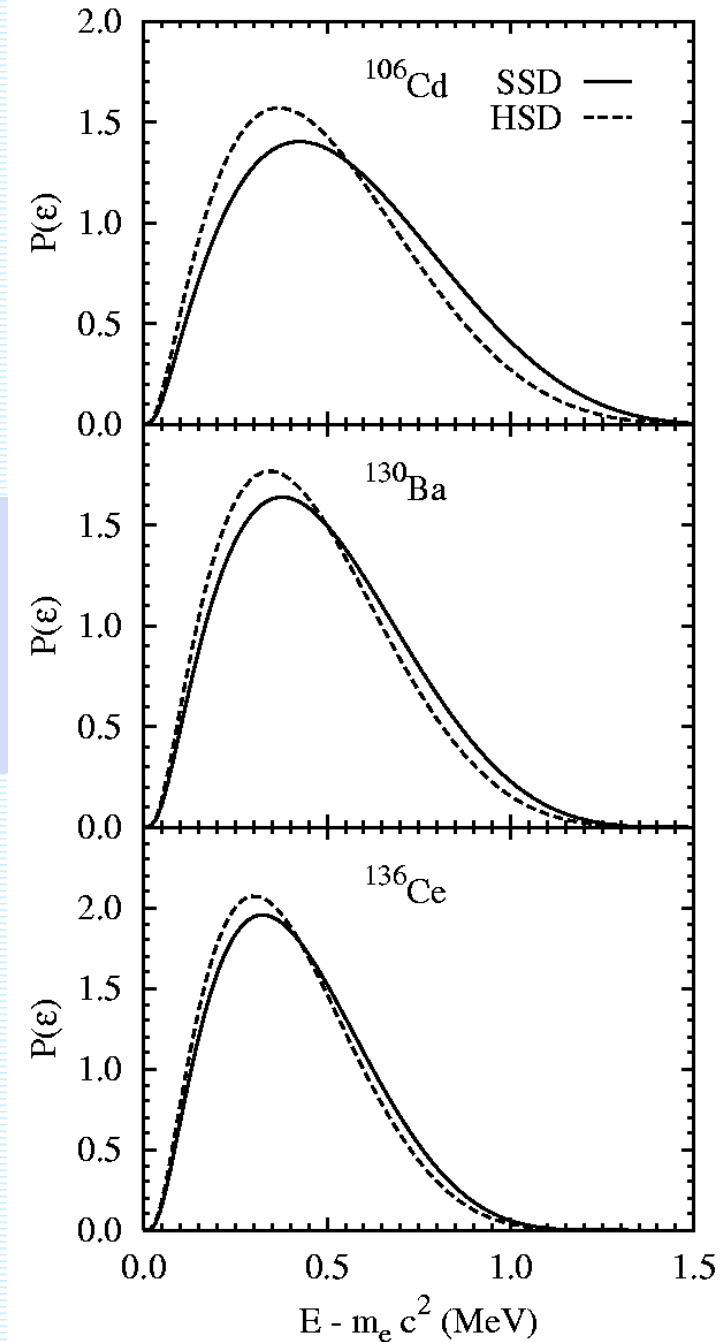
F.Š., Šmotlák, Semenov
J. Phys. G, 27, 2233, 2001

$2\nu\beta\beta$ -decay

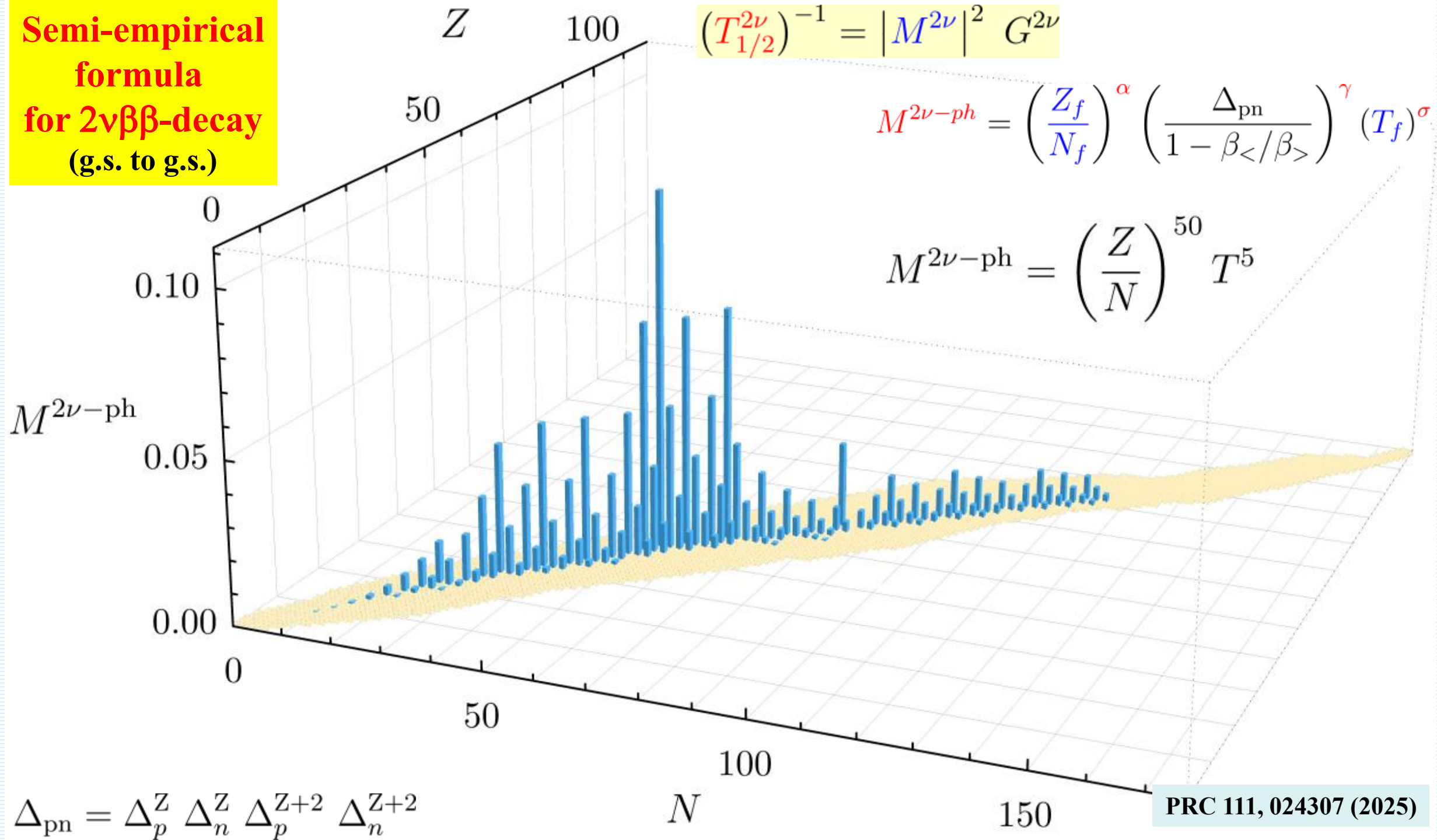


Do not depend on $M^i M^f$

$2\nu\text{EC}/\beta^+$ -decay



**Semi-empirical
formula
for $2\nu\beta\beta$ -decay
(g.s. to g.s.)**



$$\chi^2 = \sum_{i=1}^N \frac{(O_i - P_i)^2}{\sigma_i^2}$$

There is no reliable
calculation of the $2\nu\beta\beta$ -decay NMEs yet

PRC 111, 024307 (2025)

Nucleus	$M^{2\nu-\text{th}}$										$M^{2\nu-\text{exp}}$	$M^{2\nu-\text{ph}}$	
	QRPA	QRPA	IBM	IBM	IBM	NSM	NSM	PHFB	FSQP	ET		SEF	SSD
Fitted													
^{48}Ca	–	0.016	0.069	0.024	0.045	–	0.039	–	–	–	0.0314 ± 0.0030	0.022	–
^{76}Ge	–	0.063	0.083	0.018	0.085	–	0.097	–	0.083	0.085	0.0987 ± 0.0010	0.105	–
^{82}Se	–	0.058	0.072	0.024	0.063	0.099	0.105	–	0.103	0.156	0.0828 ± 0.0005	0.081	–
^{96}Zr	–	0.133	0.058	0.054	0.034	–	–	0.092	0.072	–	0.0770 ± 0.0040	0.063	–
^{100}Mo	0.105	0.251	0.197	0.157	0.045	–	–	0.164	0.154	0.179	0.2019 ± 0.0016	0.199	0.174
^{116}Cd	0.112	0.049	0.089	0.069	0.031	–	–	–	0.088	0.137	0.1142 ± 0.0027	0.120	0.148
^{130}Te	0.057	0.053	0.035	0.010	0.038	0.027	0.036	0.059	0.027	0.034	0.0265 ± 0.0003	0.028	–
^{136}Xe	0.036	0.030	0.056	0.022	0.032	0.024	0.021	–	–	–	0.0174 ± 0.0002	0.016	–
^{150}Nd	–	–	0.077	0.054	0.017	0.069	–	0.048	–	–	0.0527 ± 0.0019	0.032	0.023
^{238}U	–	–	–	–	0.023	–	–	–	–	–	0.0550 ± 0.0110	0.026	–
χ^2/N	5170	2100	2845	2828	1773	470	686	3442	480	4774	–	30	233
Predicted													
^{110}Pd	0.167	–	–	0.022	0.041	–	–	–	0.233	0.211	< 2.61	0.147	–
^{124}Sn	0.023	–	–	–	0.034	0.037	–	–	–	–	–	0.029	–
^{128}Te	0.051	0.063	0.022	0.018	0.044	0.013	0.049	0.058	0.030	0.050	0.0366 ± 0.0007	0.047	0.015
^{134}Xe	0.063	–	–	–	–	–	–	–	–	–	< 1.25	0.033	–

$$M_{F-cl}^{2\nu} = 0$$

*The DBD Nuclear Matrix Elements
and the SU(4) symmetry restoration)*

$$M_{GT-cl}^{2\nu} = 0$$

Suppression of the Two Neutrino Double Beta Decay by Nuclear Structure Effects

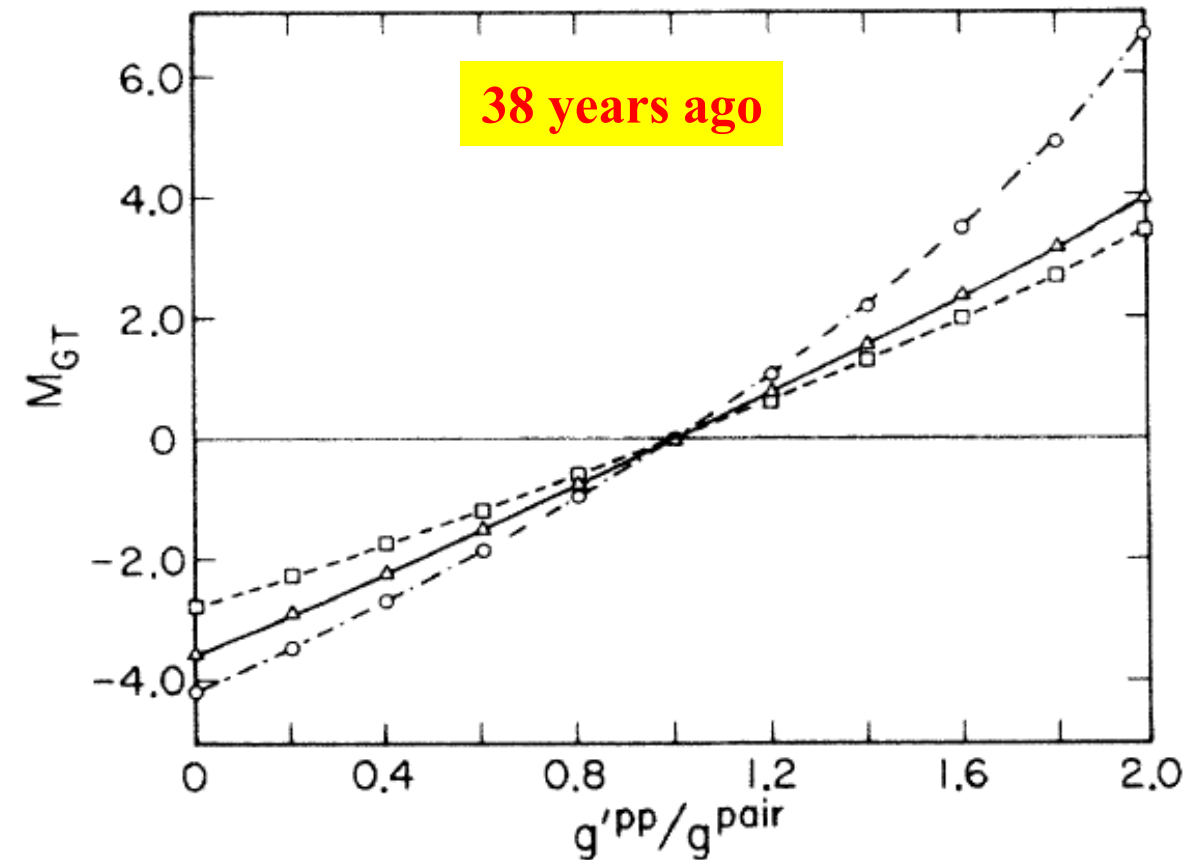
P. Vogel, M.R. Zirnbauer, PRL (1986) 3148

O. Civitarese, A. Faessler, T. Tomoda, PLB 194 (1987) 11
E. Bender, K. Muto, H.V. Klapdor, PLB 208 (1988) 53
...

The isospin is known to be a good approximation
in nuclei

In heavy nuclei the SU(4) symmetry is strongly
broken by the spin-orbit splitting.

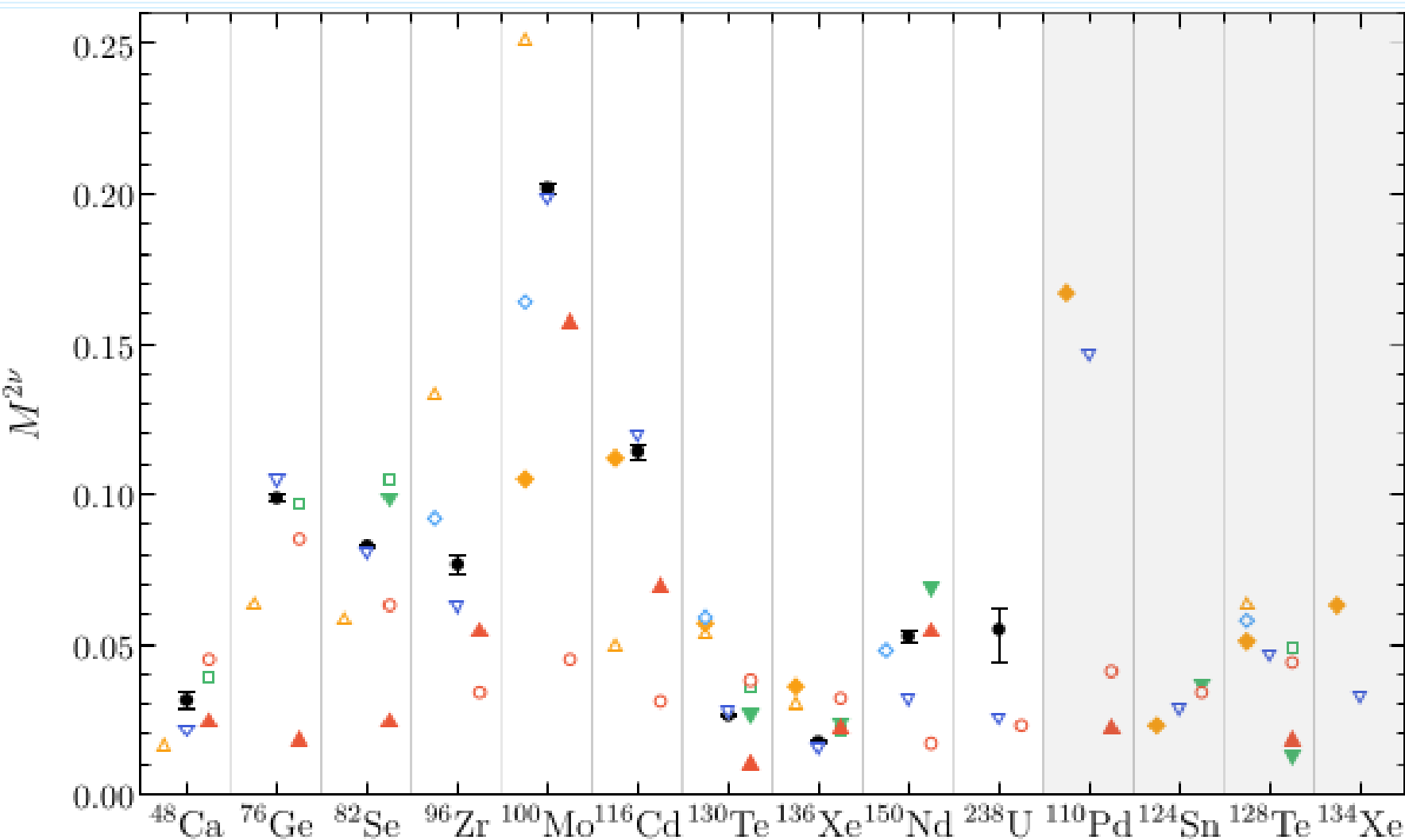
What is beyond this behavior?
Is it an artifact of the QRPA?



$$(T_{1/2}^{2\nu})^{-1} = G^{2\nu} |M_{GT}^{2\nu}|^2$$

2νββ-decay (theory versus experiment)

$$M_{GT}^{2\nu} = \sum_m \frac{\langle 0_f^+ || \tau^+ \sigma || 1_m^+ \rangle \langle 1_m^+ || \tau^+ \sigma || 0_i^+ \rangle}{E_m - E_i + \Delta}$$



- Experimental
- ▲ QRPA(Šimkovic *et al.*); $\chi^2/N = 2100$
- ◆ QRPA(Pirinen-Suhonen); $\chi^2/N = 5170$
- ISM(Caurier, *et al.*); $\chi^2/N = 686$
- ▼ ISM(Patel, *et al.*); $\chi^2/N = 470$
- IBM(Barea, *et al.*); $\chi^2/N = 1773$
- ▲ IBM(Nomura); $\chi^2/N = 2845$
- ◆ PHFB(Rath, *et al.*); $\chi^2/N = 3442$
- ▼ SEF(Nițescu-Šimkovic); $\chi^2/N = 30$

*Study of
2νββ NME
within the
schematic
model*

s.p. mean-field

Conserves SU(4) symmetry

$$H = \underbrace{e_n N_n + e_p N_p - g_{pair} \left(\sum_{M_T=-1,0,1} A_{0,1}^\dagger(0, M_T) A_{0,1}(0, M_T) + \sum_{M_S=-1,0,1} A_{1,0}^\dagger(M_S, 0) A_{1,0}(M_S, 0) \right)}_{H_0} + g_{ph} \sum_{a,b} E_{a,b}^\dagger E_{a,b}$$

$$+ \underbrace{(g_{pair} - g_{pp}^{T=0}) \sum_{M_S=-1,0,1} A_{1,0}^\dagger(M_S, 0) A_{1,0}(M_S, 0) + (g_{pair} - g_{pp}^{T=1}) A_{0,1}^\dagger(0, 0) A_{0,1}(0, 0)}_{H_I}.$$

H_I violates SU(4) symmetry

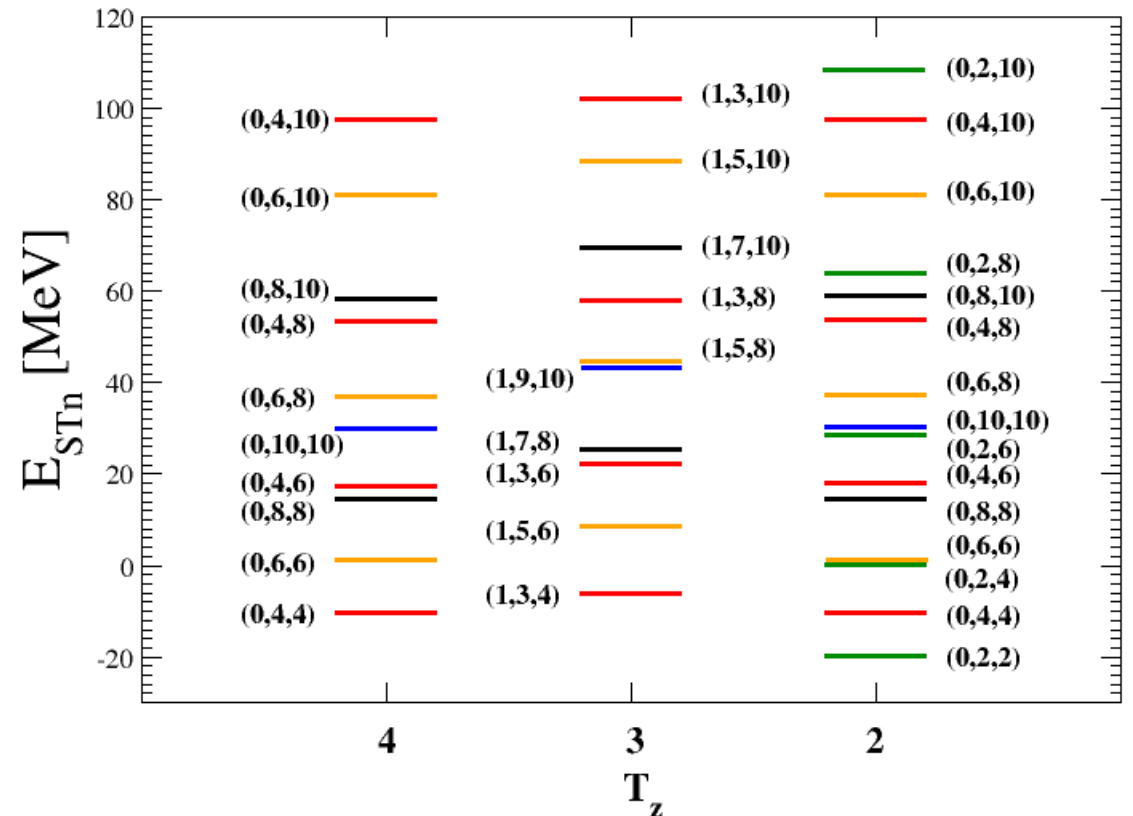
g_{pair}- strength of isovector like nucleon pairing
(L=0, S=0, T=1, M_T=±1)

g_{pp}^{T=1}- strength of isovector spin-0 pairing
(L=0, S=0, T=1, M_T=0)

g_{pp}^{T=0}- strength of isoscalar spin-1 pairing
(L=0, S=1, T=0)

g_{ph}- strength of particle-hole force

**Energies of excited states
for the case of conserved SU(4) symmetry
M_F=0, M_{GT}=0 (see SU(4) multiplets)**



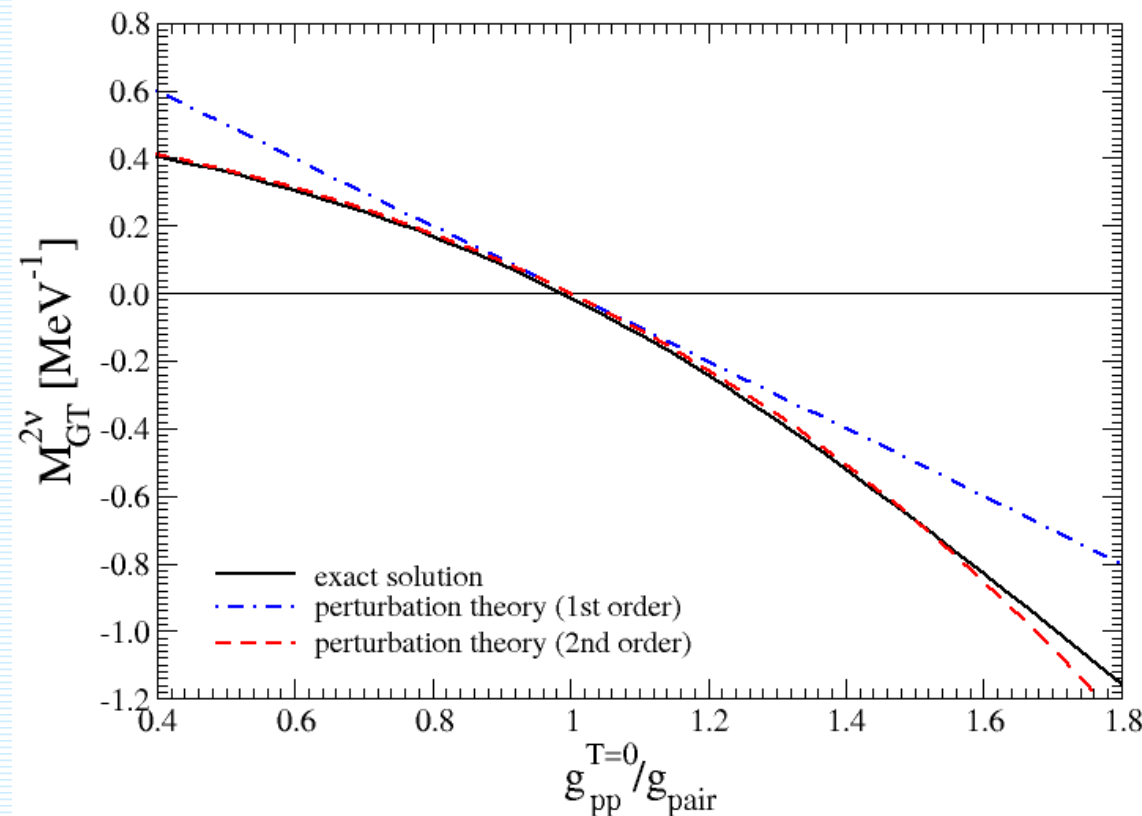
M_F and M_{GT} do not depend on the mean-field part of H

M_F and M_{GT} are governed by a weak violation of the $SU(4)$ symmetry by the particle-particle interaction of H

$$M_F^{2\nu} = -\frac{48\sqrt{\frac{33}{5}}(g_{pair} - g_{pp}^{T=1})}{(5g_{pair} + 3g_{ph})(10g_{pair} + 6g_{ph})}$$

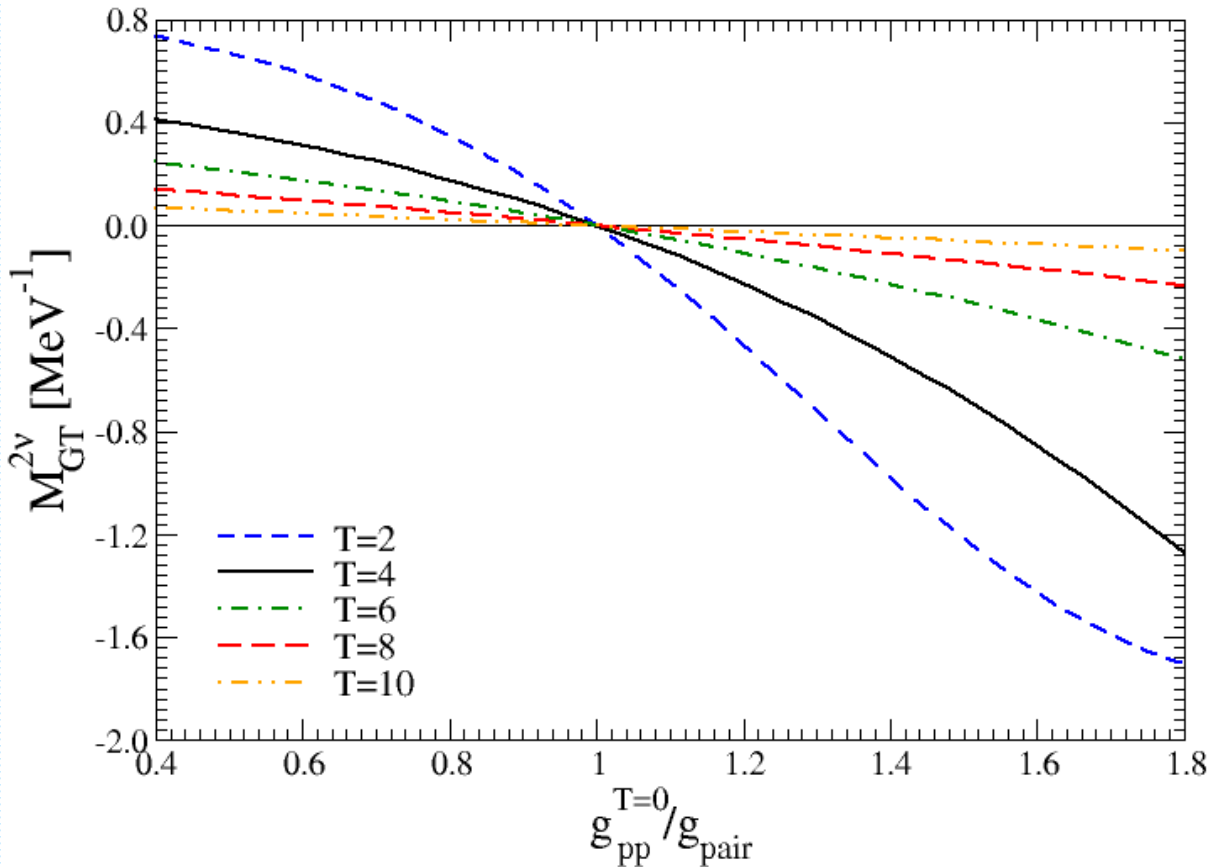
$$M_{GT}^{2\nu} = \frac{144\sqrt{\frac{33}{5}}}{5g_{pair} + 9g_{ph}} \left\{ \frac{(g_{pair} - g_{pp}^{T=0})}{(10g_{pair} + 20g_{ph})} + \frac{2g_{ph}(g_{pair} - g_{pp}^{T=1})}{(10g_{pair} + 20g_{ph})(10g_{pair} + 6g_{ph})} \right\}$$

M_{GT} up to the second order of perturbation theory due to the violation of the $SU(4)$ symmetry by the particle-particle interaction of H

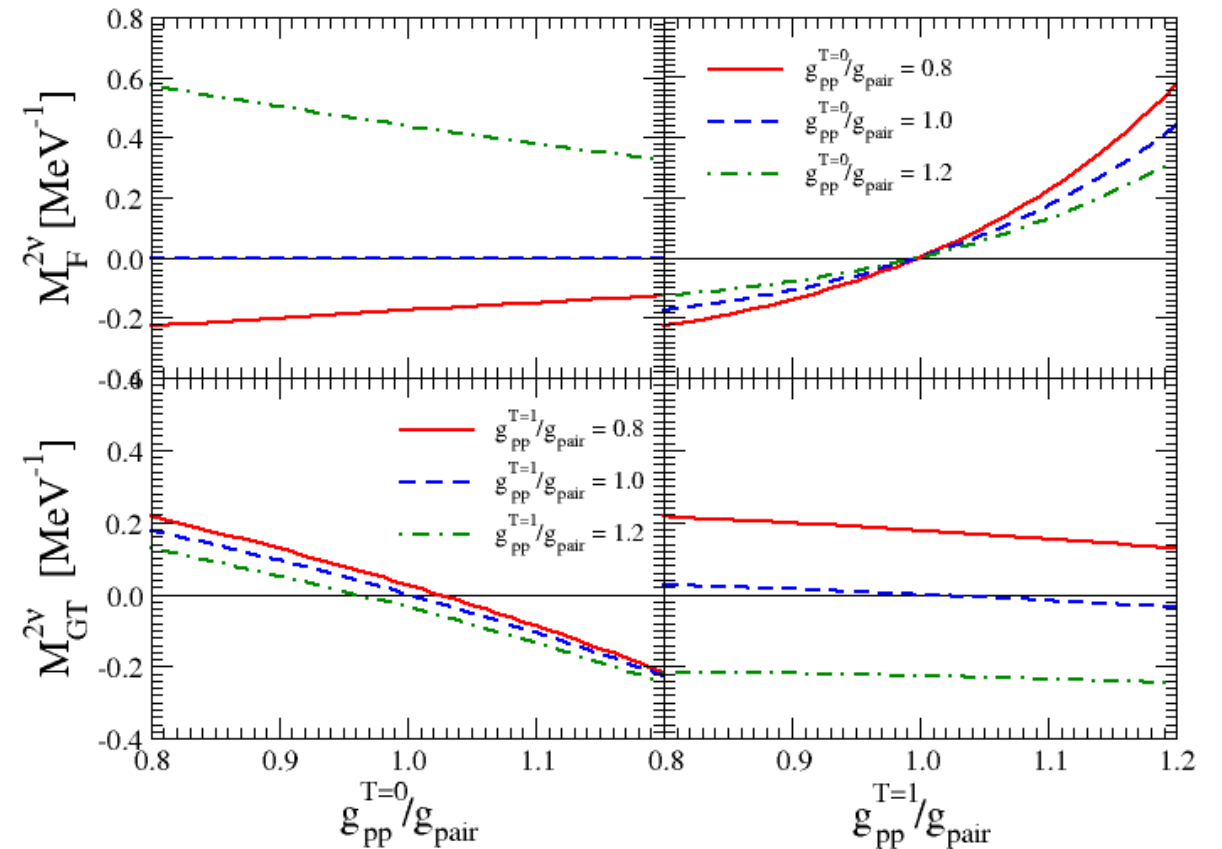


Results confirm the dependence of M_F and M_{GT} on $g_{pp}^{T=0}$ and $g_{pp}^{T=1}$ by the QRPA

By assuming a fixed violation of the **SU(4)** symmetry by particle-particle int. M_{GT} decreases by increase of **isospin** of the ground state



Results confirm dependence of M_F and M_{GT} on $g_{pp}^{T=0}$ and $g_{pp}^{T=1}$ by the QRPA



Improved description of the $0\nu\beta\beta$ -decay rate (a way to fix g_A^{eff})

PRC 97, 034315 (2018)

Taylor expansion

$$\frac{\epsilon_{K,L}}{E_n - (E_i + E_f)/2}$$

We get

$$\epsilon_{K,L} \in \left(-\frac{Q}{2}, \frac{Q}{2}\right)$$

Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states.
Both change two neutrons into two protons.
Explaining $2\nu\beta\beta$ -decay is necessary but not sufficient

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \epsilon_{K,L}^2}$$

$$\epsilon_K = (E_{e2} + E_{\nu2} - E_{e1} - E_{\nu1})/2$$

$$\epsilon_L = (E_{e1} + E_{\nu2} - E_{e2} - E_{\nu1})/2$$

$$\begin{aligned} [T_{1/2}^{2\nu}]^{-1} \simeq & (g_A^{\text{eff}})^4 |M_{GT-1}|^2 [G_0^{2\nu} + \xi_{31} G_2^{2\nu} \\ & + (\xi_{31})^2 G_{22}^{2\nu} + \left(\frac{1}{3}(\xi_{31})^2 + \xi_{51}\right) G_4^{2\nu} \\ & + \frac{1}{3}\xi_{31}\xi_{51} G_{42}^{2\nu} + \frac{2}{3}\xi_{31}\xi_{51} G_6^{2\nu}] \end{aligned}$$

$$\xi_{31} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

$$\xi_{51} = \frac{M_{GT-5}^{2\nu}}{M_{GT-1}^{2\nu}}$$

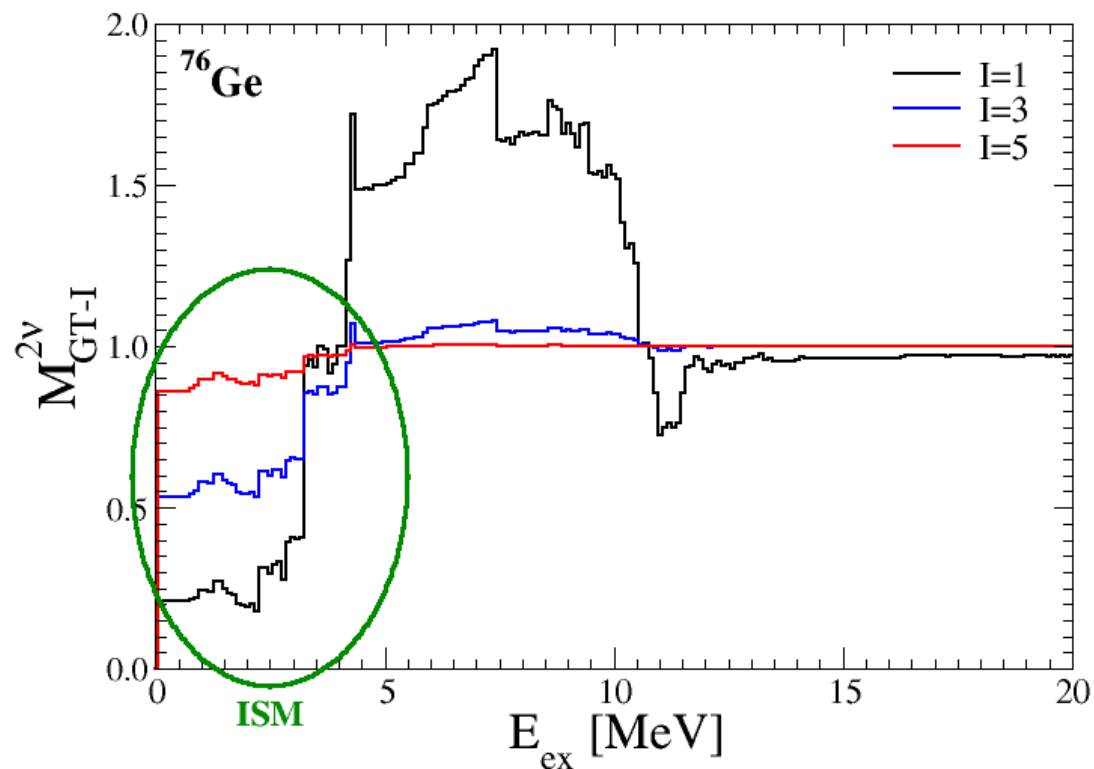
$$\begin{aligned} M_{GT-1}^{2\nu} &= \sum_n M_n(0^+) \frac{m_e}{E_n(1^+) - (E_i + E_f)/2} \\ M_{GT-3}^{2\nu} &= \sum_n M_n(0^+) \frac{4 m_e^3}{(E_n(1^+) - (E_i + E_f)/2)^3} \\ M_{GT-5}^{2\nu} &= \sum_n M_n(0^+) \frac{16 m_e^5}{(E_n(1^+) - (E_i + E_f)/2)^5} \end{aligned}$$

The g_A^{eff} can be determined with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM)

The running sum of the $2\nu\beta\beta$ -decay NMEs (QRPA)

$$M_{GT-1}^{2\nu} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2)} \quad \xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4 m_e^3}{(E_n - (E_i + E_f)/2)^3} \quad \xi_{15}^{2\nu} = \frac{M_{GT-5}^{2\nu}}{M_{GT-1}^{2\nu}}$$

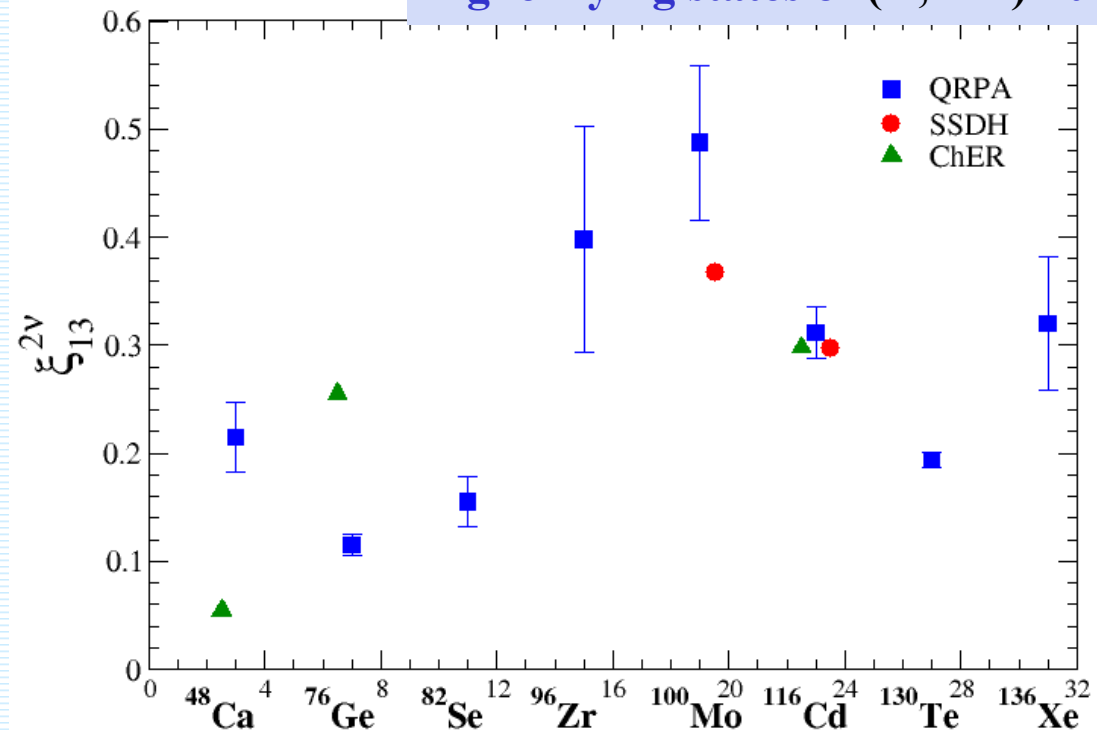


ξ_{13} tell us about importance of higher lying states of int. nucl.

HSD: $\xi_{13} \approx 0$

$\xi_{13} \approx 1$ (large)

Possible because of a large cancellation of contributions through lower and higher-lying states of $(A, Z+1)$ nucleus



ξ_{13} can be determined phenomenologically from the shape of energy distributions of emitted electrons

F.Š., Šmotlák, Semenov, J. Phys. G, 27, 2233, 2001

KamLAND-Zen Exp. : $\xi_{13} < 0.26$ (^{136}Xe)

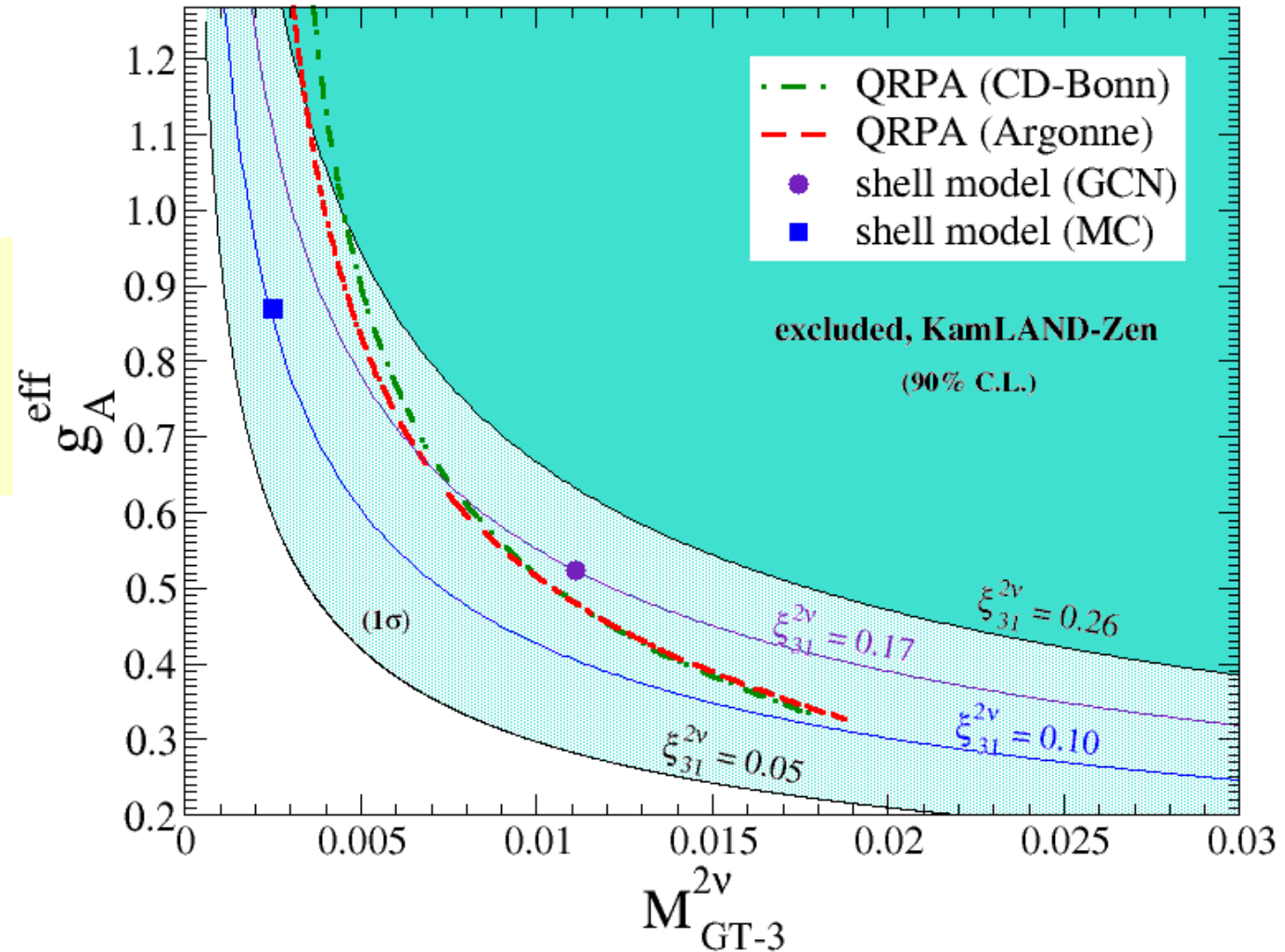
KamLAND-Zen Coll. (+J. Menendez, F.Š.),
Phys.Rev.Lett. 122, 192501 (2019)

ξ_{13} can be determined phenomenologically
from the shape of energy
distributions of emitted electrons

The g_A^{eff} can be determined with measured
half-life, ratio of NMEs ξ_{31} and calculated NME,
dominated by transitions through
low lying states of the intermediate nucleus.

$$\left(g_A^{\text{eff}}\right)^2 = \frac{1}{\left|M_{GT-3}^{2\nu}\right|} \frac{\left|\xi_{13}^{2\nu}\right|}{\sqrt{T_{1/2}^{2\nu-\text{exp}} \left(G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu}\right)}}$$

M_{GT-3} have to be calculated
by nuclear theory - ISM

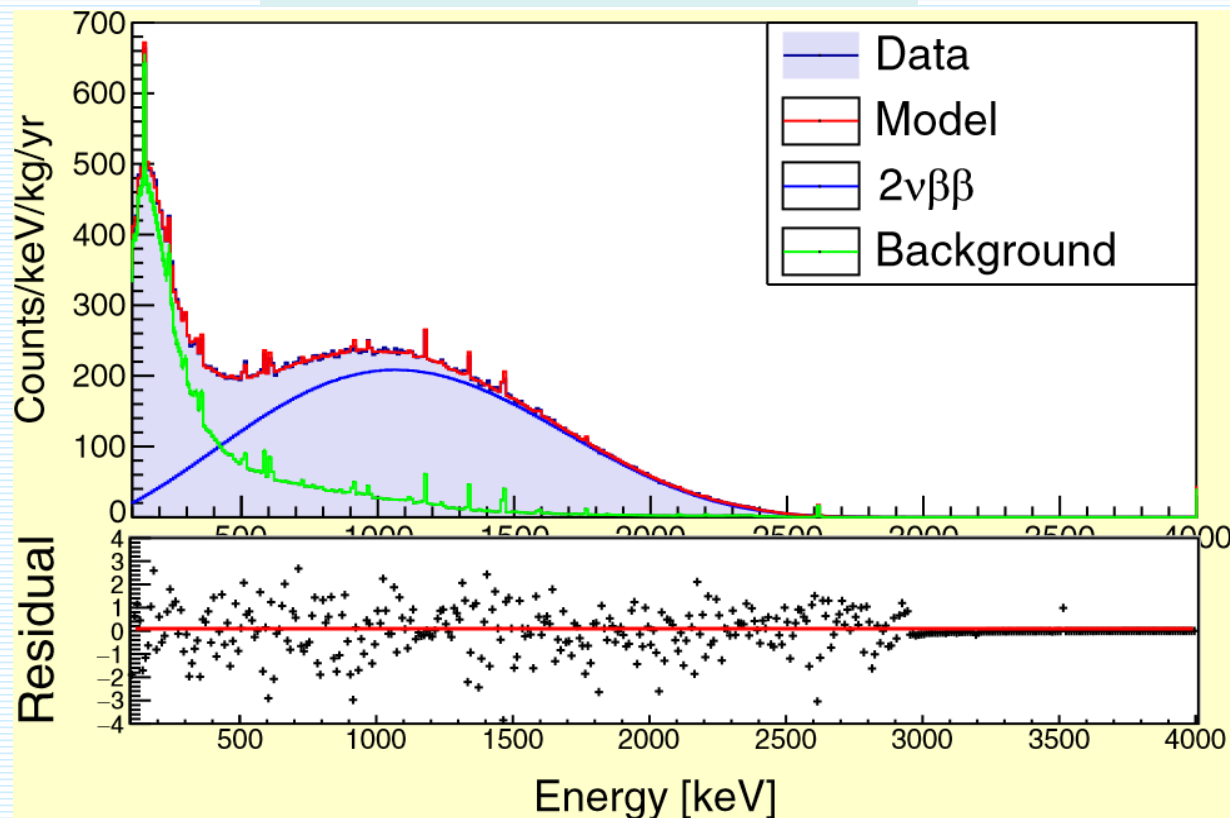


PandaX-4T data (37.8 kg y): $\xi_{13} = 0.59^{+0.41}_{-0.38}$ (^{136}Xe)

PandaX Coll.,
arXiv:2512.04849 [nucl-ex], (2025)

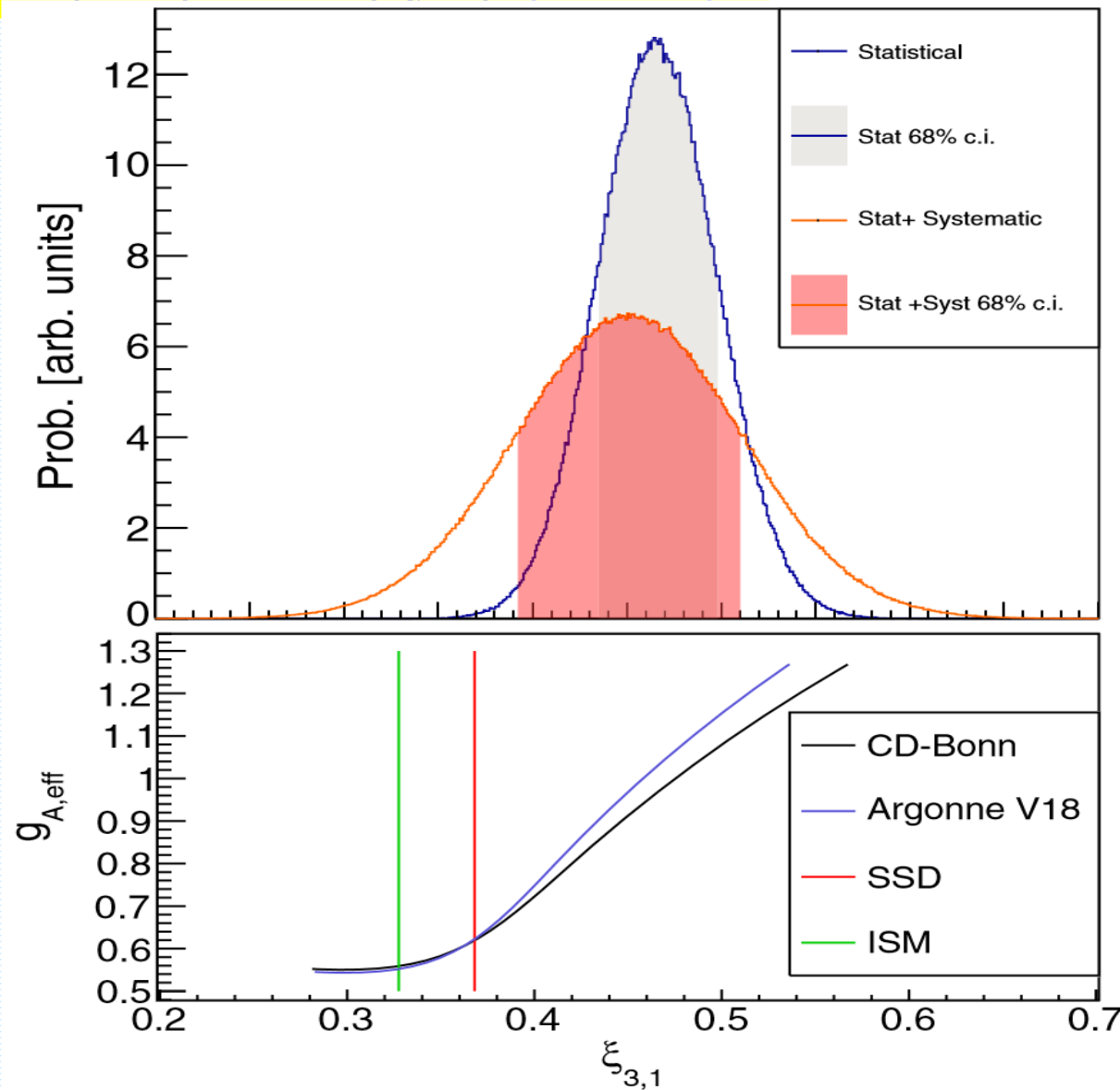
CUPID-Mo Exp. : $\xi_{13}=0.45\pm0.03$ (stat) ±0.05 (syst) (^{100}Mo)

Phys.Rev.Lett. 131, 162501 (2023)



$$T_{1/2} = [7.07\pm0.02(\text{stat})\pm0.11(\text{syst})] \times 10^{18} \text{ yr}$$

$$\xi_{51}/\xi_{31} = 0.364\text{--}0.368 \text{ (QRPA)}, 0.367 \text{ (SSD)}, 0.349 \text{ (ISM)}$$



9/25/2026

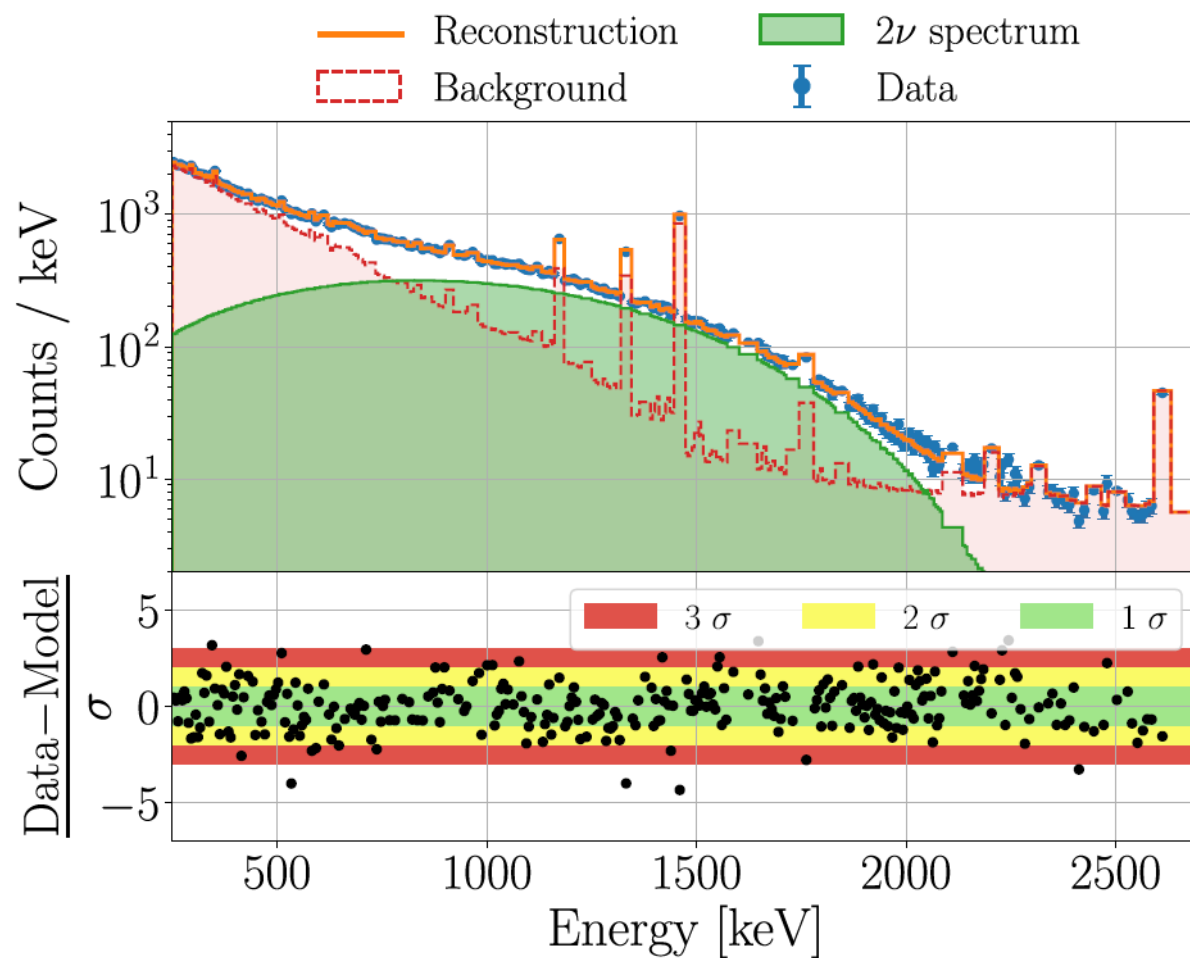
$$g_A^{\text{eff}} \text{ (pnQRPA)} = 1.0 \pm 0.1(\text{stat}) \pm 0.2(\text{syst})$$

$$g_A^{\text{eff}} \text{ (ISM)} = 1.11 \pm 0.03(\text{stat}) \pm 0.05(\text{syst})$$

CUORE Exp. (^{130}Te)

arXiv: 2503.24137 [nucl-ex]

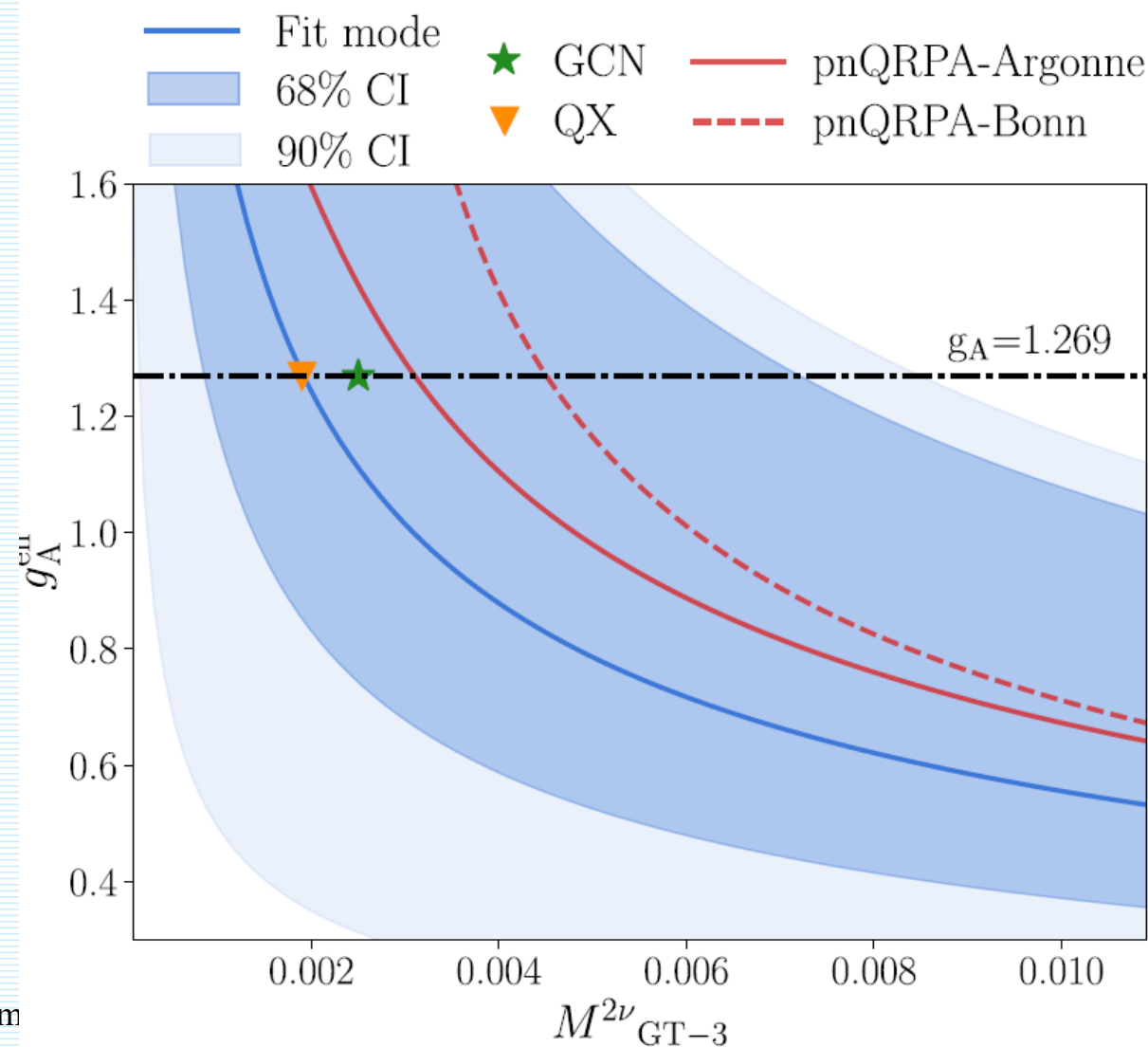
$$T_{1/2}^{2\nu} = (9.32_{-0.04}^{+0.05}\text{stat.} +_{-0.07}^{+0.07}\text{syst.}) \times 10^{20} \text{ yr}$$



$$\xi_{31} = 0.01_{-0.01}^{+0.31}$$

$$\xi_{51} = 1.46_{-0.62}^{+0.33}$$

It does not make sense(!?)
We are confused ...



The role of the width of the states of the intermediate nucleus ($E_n \rightarrow E_n + \Gamma_n/2$)

$$\left[T_{1/2}^{2\nu\beta\beta} \right]^{-1} = (g_A^{\text{eff}})^4 |M_{GT}^{R1}|^2 \left[(1 + \xi_{IR11}^2) G_0^{2\nu} + (\xi_{RR31} + \xi_{IR11}\xi_{IR31}) G_2^{2\nu} + \frac{1}{3} (\xi_{IR31}^2 + \xi_{RR31}^2) (G_{22}^{2\nu} + G_4^{2\nu}) \right]$$

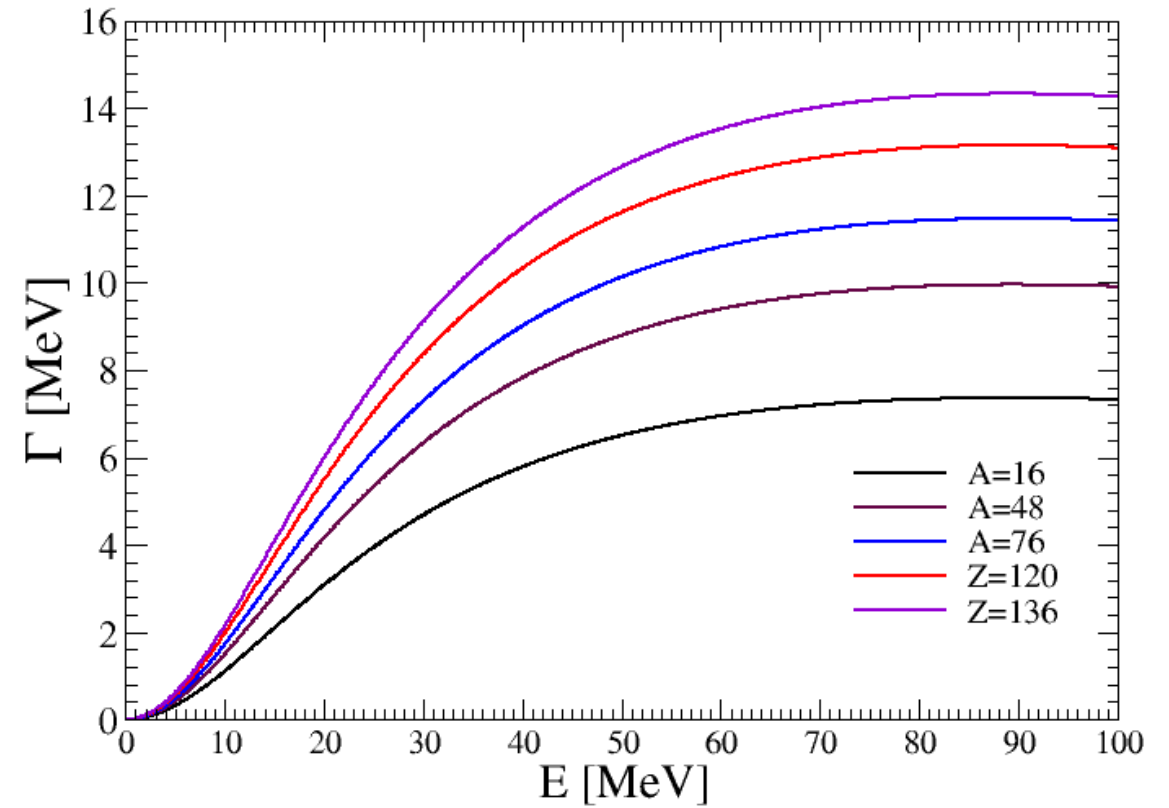
$$\xi_{RR31} = \frac{M_{GT}^{R3}}{M_{GT}^{R1}} \quad \xi_{IR11} = \frac{M_{GT}^{I1}}{M_{GT}^{R1}} \quad \xi_{IR31} = \frac{M_{GT}^{I3}}{M_{GT}^{R1}}$$

$$M_{GT}^{R1} = m_e \sum_n \frac{M_n D_n}{(D_n^2 + \Gamma_n^2/4)}$$

$$M_{GT}^{R3} = 4m_e^3 \sum_n \frac{M_n D_n (D_n^2 - 3\Gamma_n^2/4)}{(D_n^2 + \Gamma_n^2/4)^3}$$

$$M_{GT}^{I1} = m_e \sum_n \frac{M_n \Gamma_n/2}{(D_n^2 + \Gamma_n^2/4)}$$

$$M_{GT}^{I3} = 4m_e^3 \sum_n \frac{M_n \Gamma_n/2 (3D_n^2 - \Gamma_n^2/4)}{(D_n^2 + \Gamma_n^2/4)^3}$$



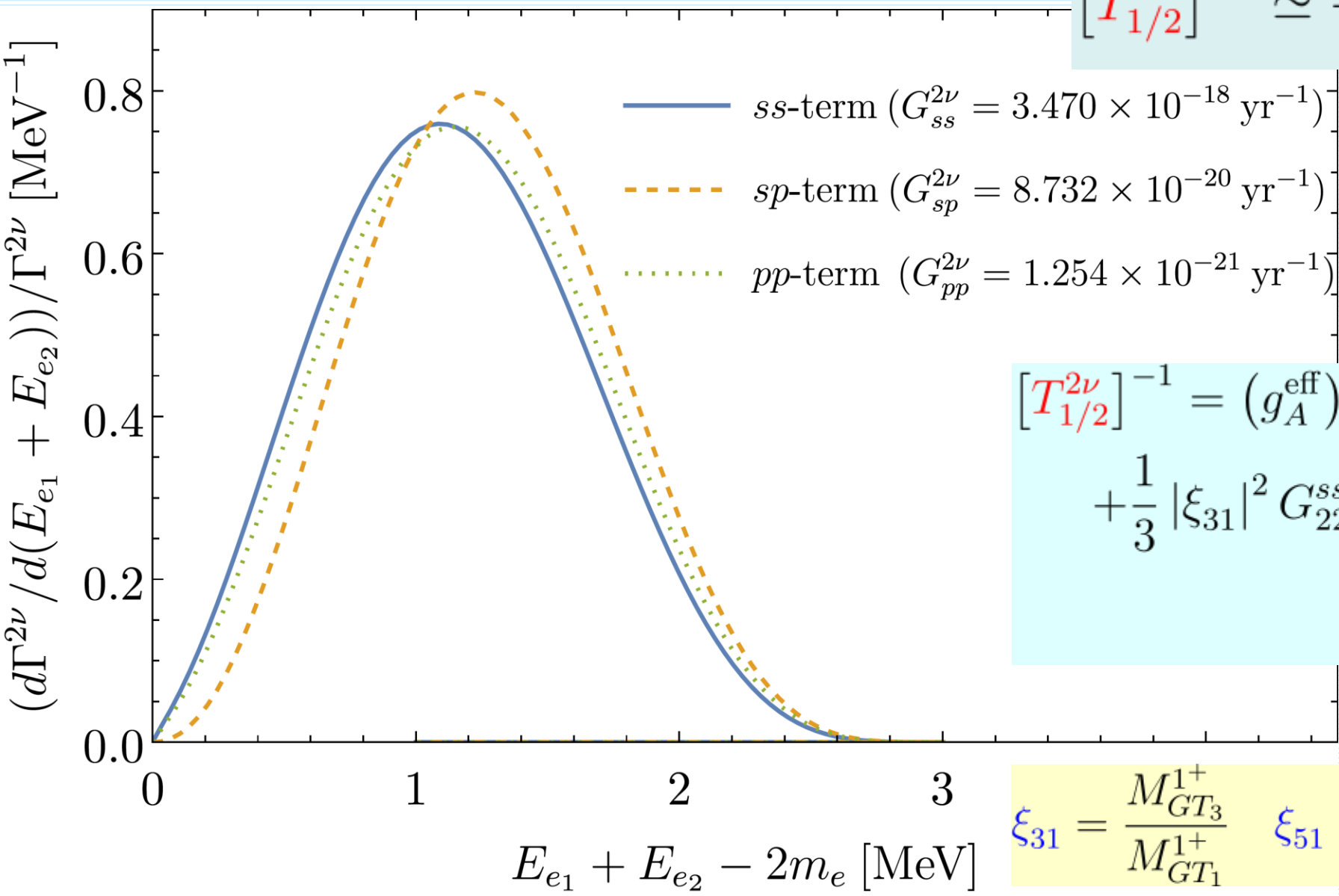
$$D_n = E_n - (E_i + E_f)/2$$

Fedor S. We found that ξ_{IR11} and ξ_{IR31} are negligibly small

Effect of the electron $p_{1/2}$ -wave state in $2\nu\beta\beta$ -decay (preliminary)

$$A^{2\nu} = A^{ss} + A^{p_{1/2}p_{1/2}} + \dots$$

$$[T_{1/2}^{2\nu}]^{-1} \simeq \frac{\Gamma_0^{ss} + \Gamma_2^{ss} + \Gamma_4^{ss} + \Gamma_0^{sp_{1/2}}}{\ln(2)}$$



$$[T_{1/2}^{2\nu}]^{-1} = (g_A^{\text{eff}})^4 |M_{GT_1}^{ss}|^2 (G_0^{ss} + \Re\{\xi_{31}\} G_2^{ss} + \frac{1}{3} |\xi_{31}|^2 G_{22}^{ss} + \left(\frac{1}{3} |\xi_{31}|^2 + \Re\{\xi_{51}\}\right) G_4^{ss} + G_0^{sp_{1/2}} \xi_{p_{1/2}s})$$

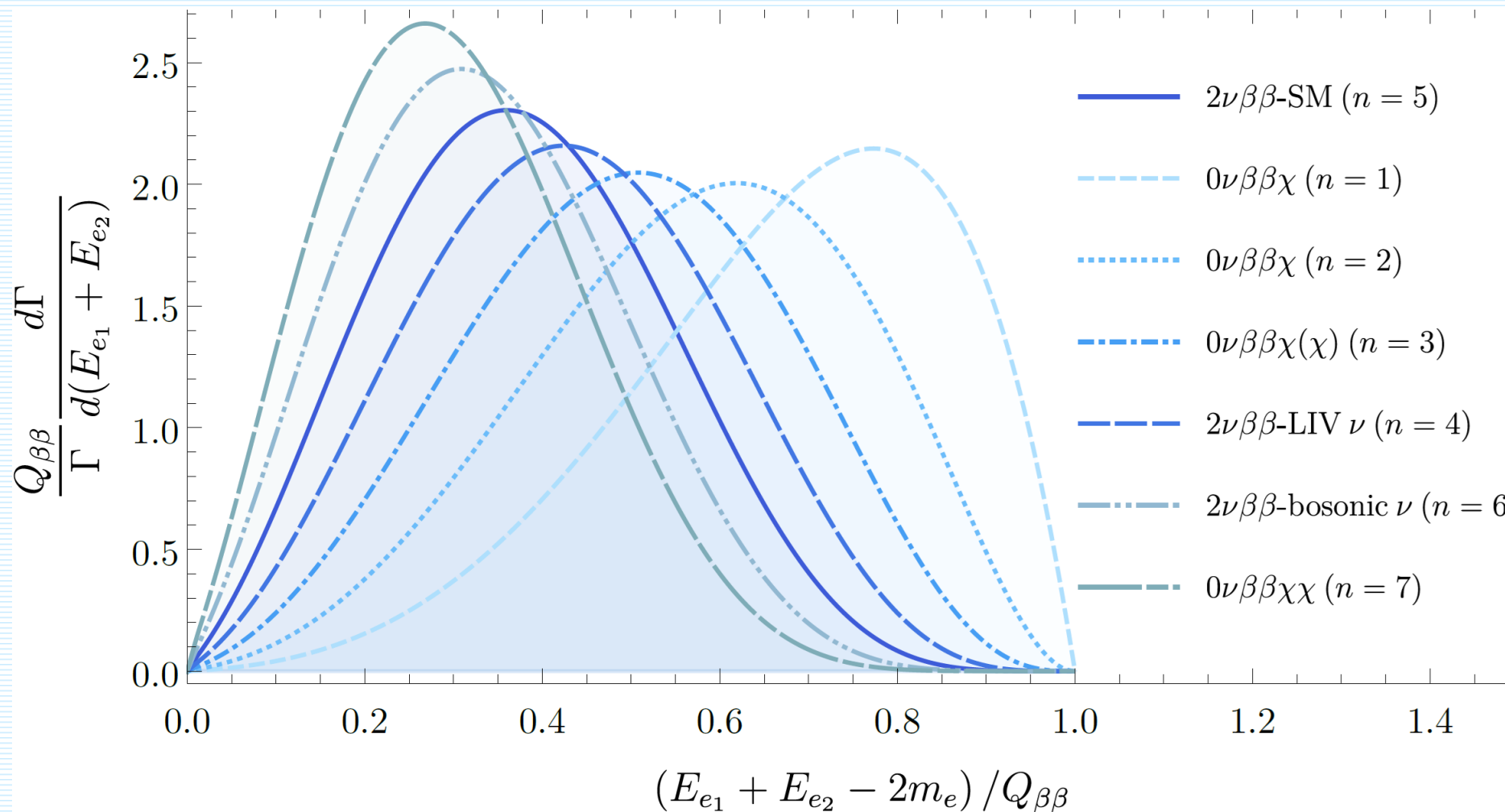
$$\xi_{31} = \frac{M_{GT_3}^{1+}}{M_{GT_1}^{1+}} \quad \xi_{51} = \frac{M_{GT_5}^{1+}}{M_{GT_1}^{1+}} \quad \xi_{p_{1/2}s} = \frac{M^{0-} + M^{1-}}{M_{GT_1}^{1+}}$$

**2νββ probes
New Beyond SM Physics**

All 100 kg- and ton-class 0νββ experiments can also study a diverse range of **exotic phenomena**, e.g. through **spectral distortion in 2νββ**.
Future searches will probe the 2νββ with **high statistics** about **10⁵-10⁶ events**.

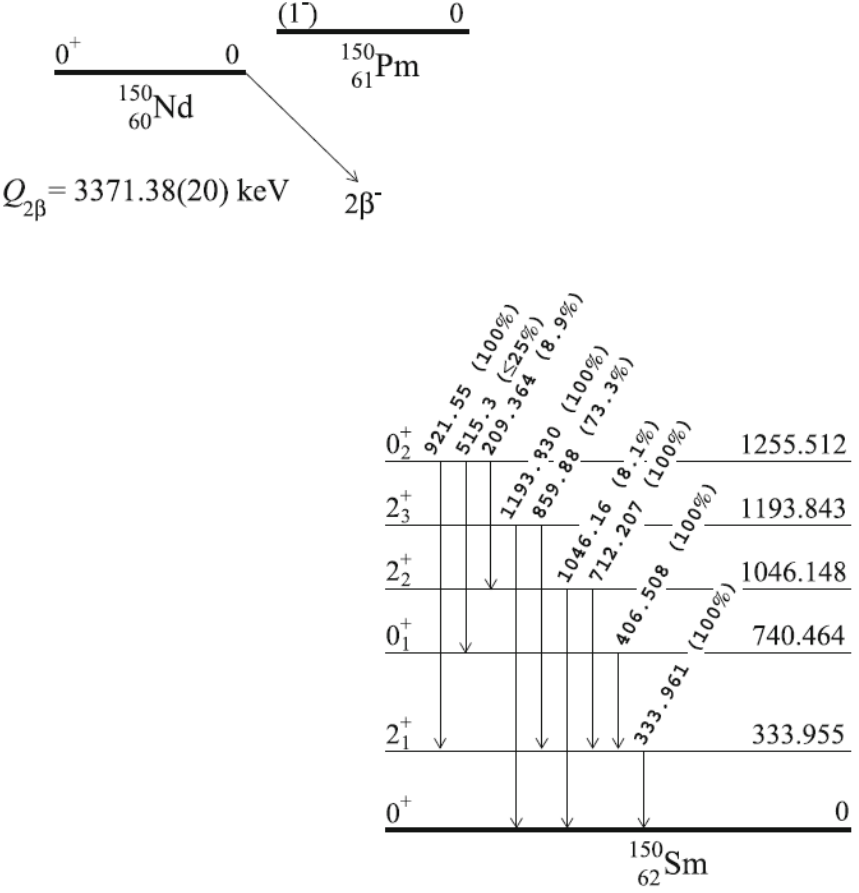
Common subjects:
Majoron(s) emission
(partly)bosonic neutrinos,
Lorentz invariance
violation

Recent subjects:
Lepton-number conserving
right-handed currents
(PRL 125 (2020) 17, 171801)
Neutrino self-interactions
(PRD 102 (2020) 5, 051701)
Sterile neutrino and light
fermion searches through
energy end point
(PRD 103 (2021) 5, 055019;
PLB 815 (2021) 136127)



$$\frac{d\Gamma}{d\varepsilon_1 d\varepsilon_2} = C (Q - \varepsilon_1 - \varepsilon_2)^n [p_1 \varepsilon_1 F(\varepsilon_1)] [p_2 \varepsilon_2 F(\varepsilon_2)]$$

Spectral index n

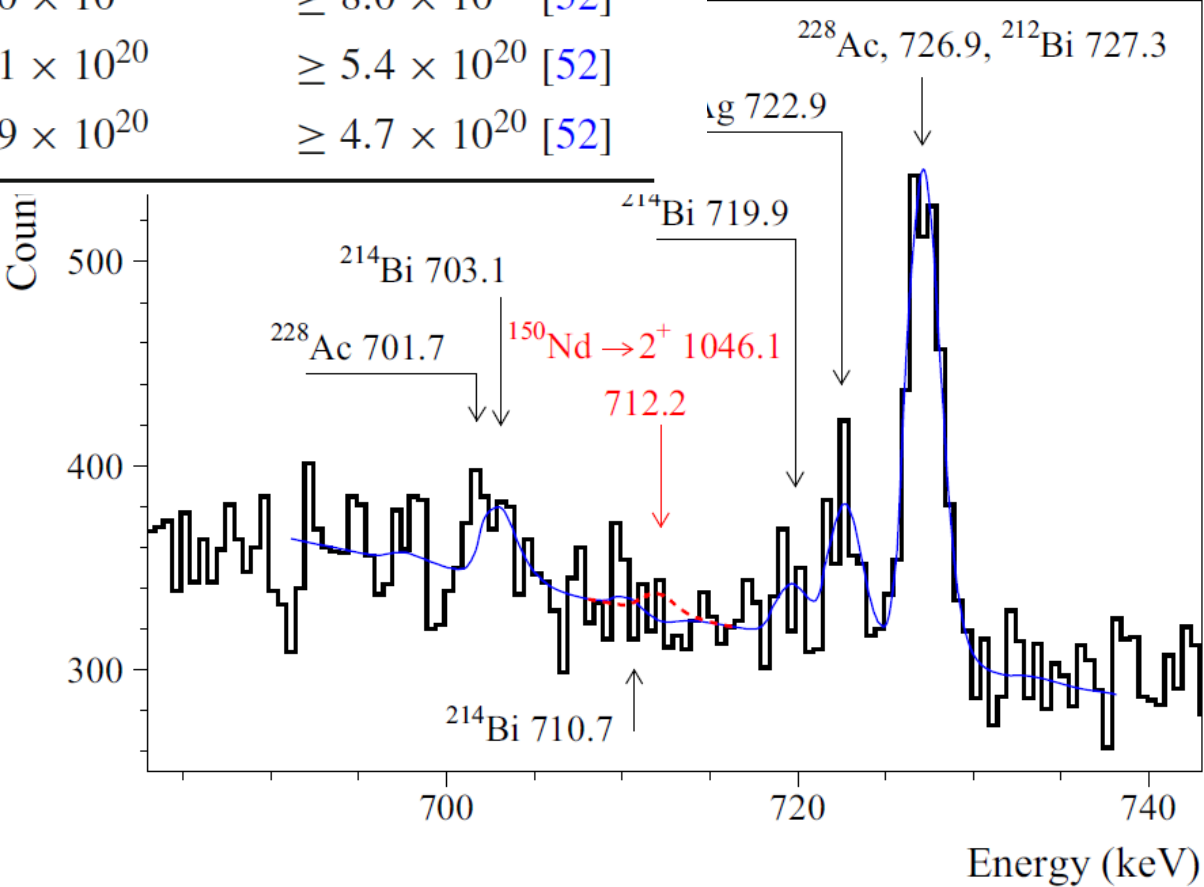


Level of ^{150}Sm (keV)	Experimental limit, $\lim T_{1/2}$ (year)	
	Present work	Previous results
334.0 2_1^+	$1.5^{+2.3}_{-0.7} \times 10^{20}$	$\geq 2.42 \times 10^{20}$ [28]
334.0 2_1^+	$\geq 7.3 \times 10^{19}$	$\geq 1.26 \times 10^{23}$ [28] ^a
740.5 0_1^+	$1.03^{+0.38}_{-0.29} \times 10^{20}$	See Table 1
740.5 0_1^+	$\geq 7.1 \times 10^{19}$	$\geq 1.36 \times 10^{22}$ [28] ^a
1046.1 2_2^+	$\geq 6.0 \times 10^{20}$	$\geq 8.0 \times 10^{20}$ [52]
1193.8 2_3^+	$\geq 5.1 \times 10^{20}$	$\geq 5.4 \times 10^{20}$ [52]
1255.5 0_2^+	$\geq 3.9 \times 10^{20}$	$\geq 4.7 \times 10^{20}$ [52]

**$2\nu\beta\beta$ -decay
to 2^+ state
(First observation)**

$$T_{1/2}^{2\nu 2\beta} (^{150}\text{Nd} \rightarrow ^{150}\text{Sm}(0_1^+))$$

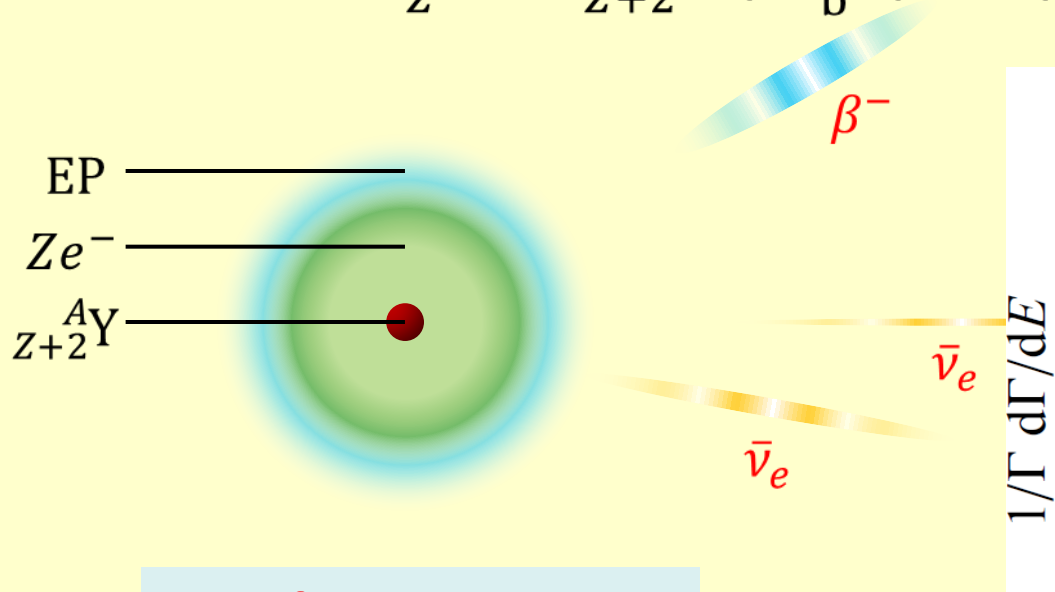
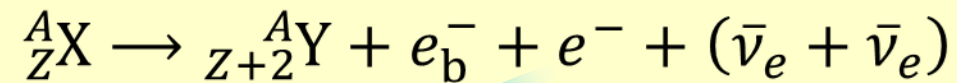
$$= [0.83^{+0.18}_{-0.13}(\text{stat})^{+0.16}_{-0.19}(\text{syst})] \times 10^{20} \text{ year}$$



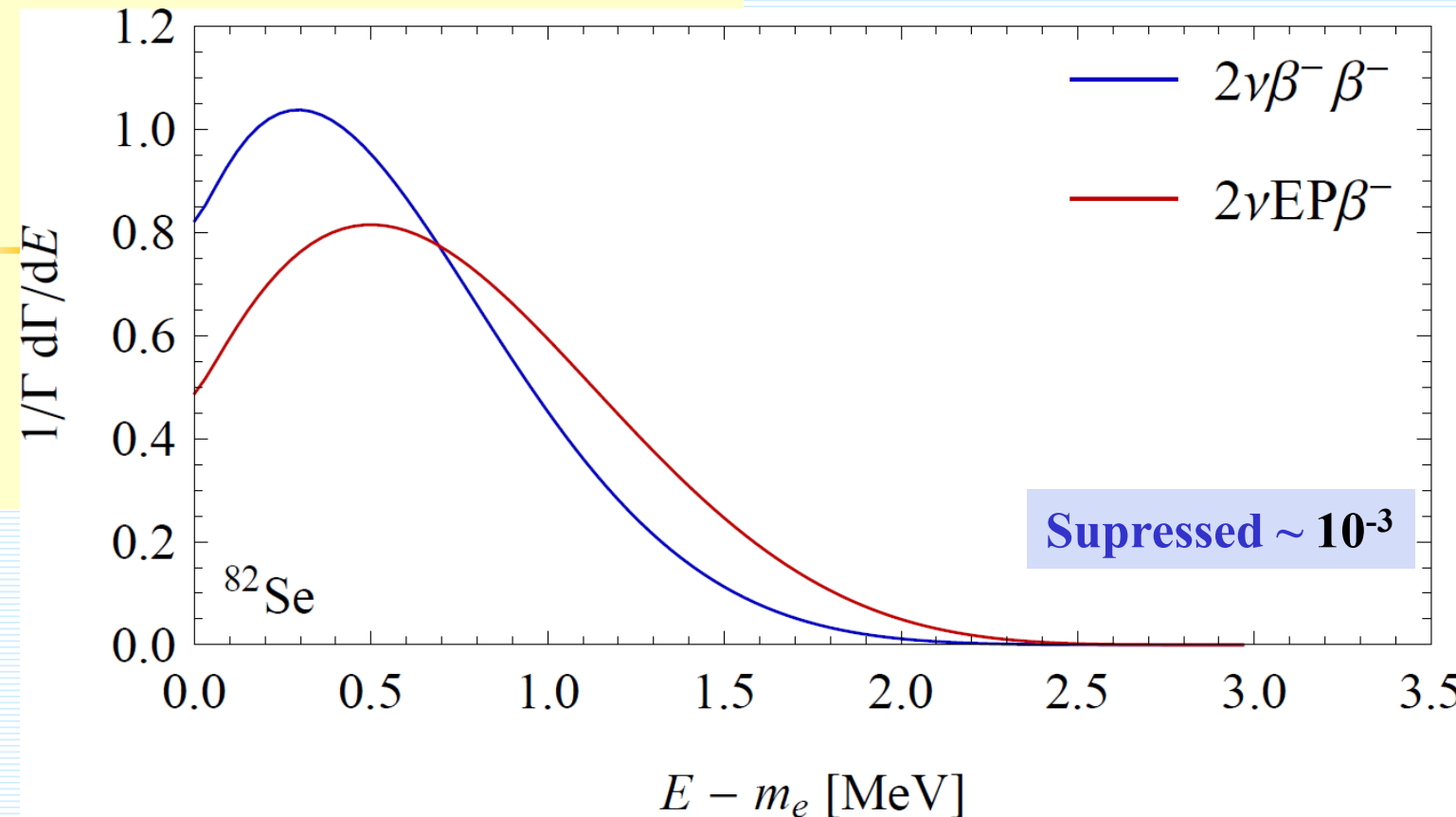
[Jung *et al.* (GSI), 1992] observed beta decay of $^{163}_{66}\text{Dy}^{66+}$ ions with Electron Production (EP) in K or L shells: $T_{1/2}^{\text{EP}} = 47$ d

Bound-state double-beta decay $0\nu\text{EP}\beta^-$ ($2\nu\text{EP}\beta^-$) with EP in available $s_{1/2}$ or $p_{1/2}$ subshell of daughter $2+$ ion:

**Double Beta Decay
with emission of
a single electron**



$2\nu\text{EP}\beta^-$ can be detected
Half-life predictions
are independent
on value of g_A and
Value of $2\nu\beta\beta$ NME

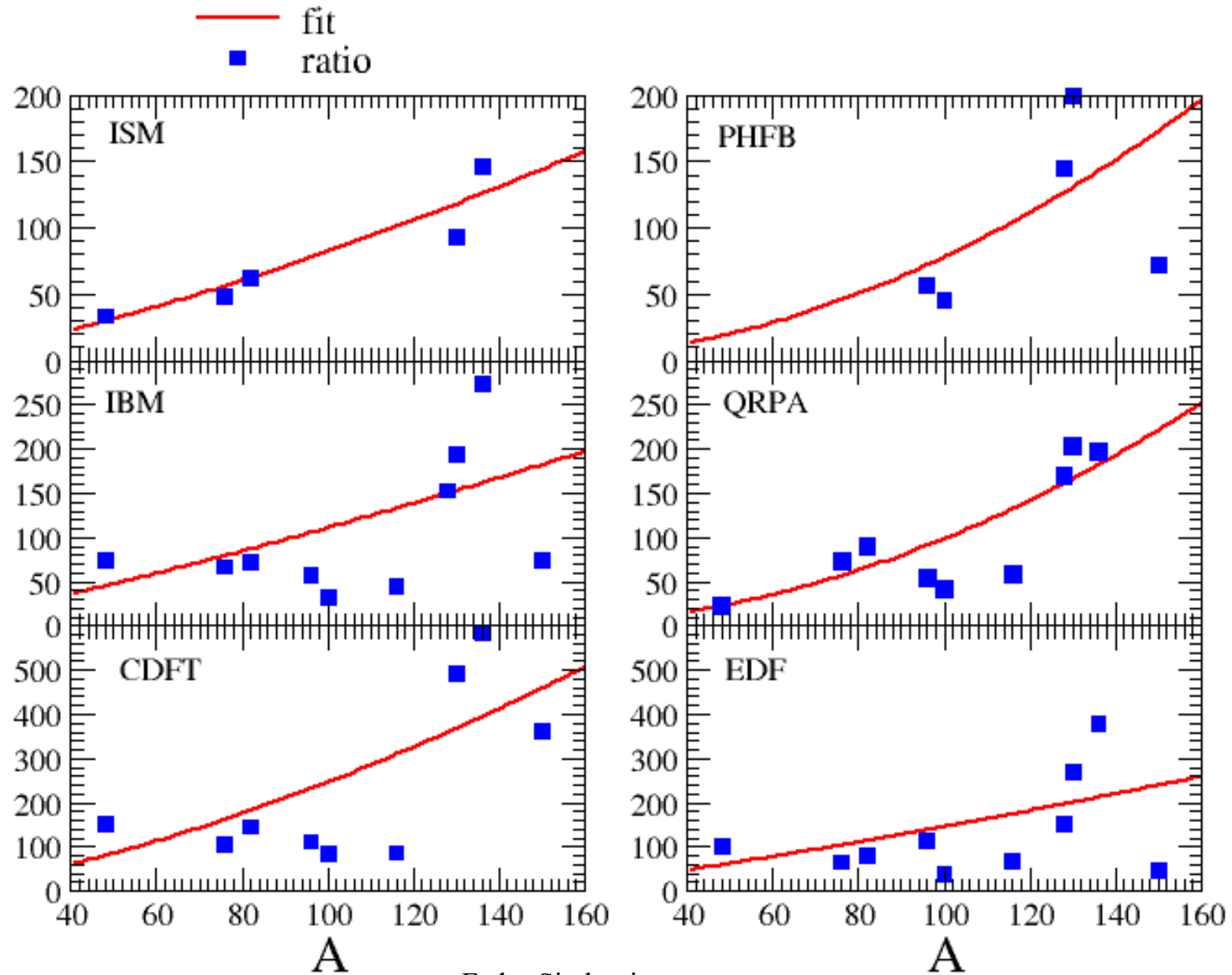


Is there a proportionality between $0\nu\beta\beta$ - and $2\nu\beta\beta$ -decay NMEs?

Known
from
measured
 $2\nu\beta\beta$ -
decay
half-life

$$M_V^{0\nu}/(m_e M_e^{2\nu\text{-exp}})$$

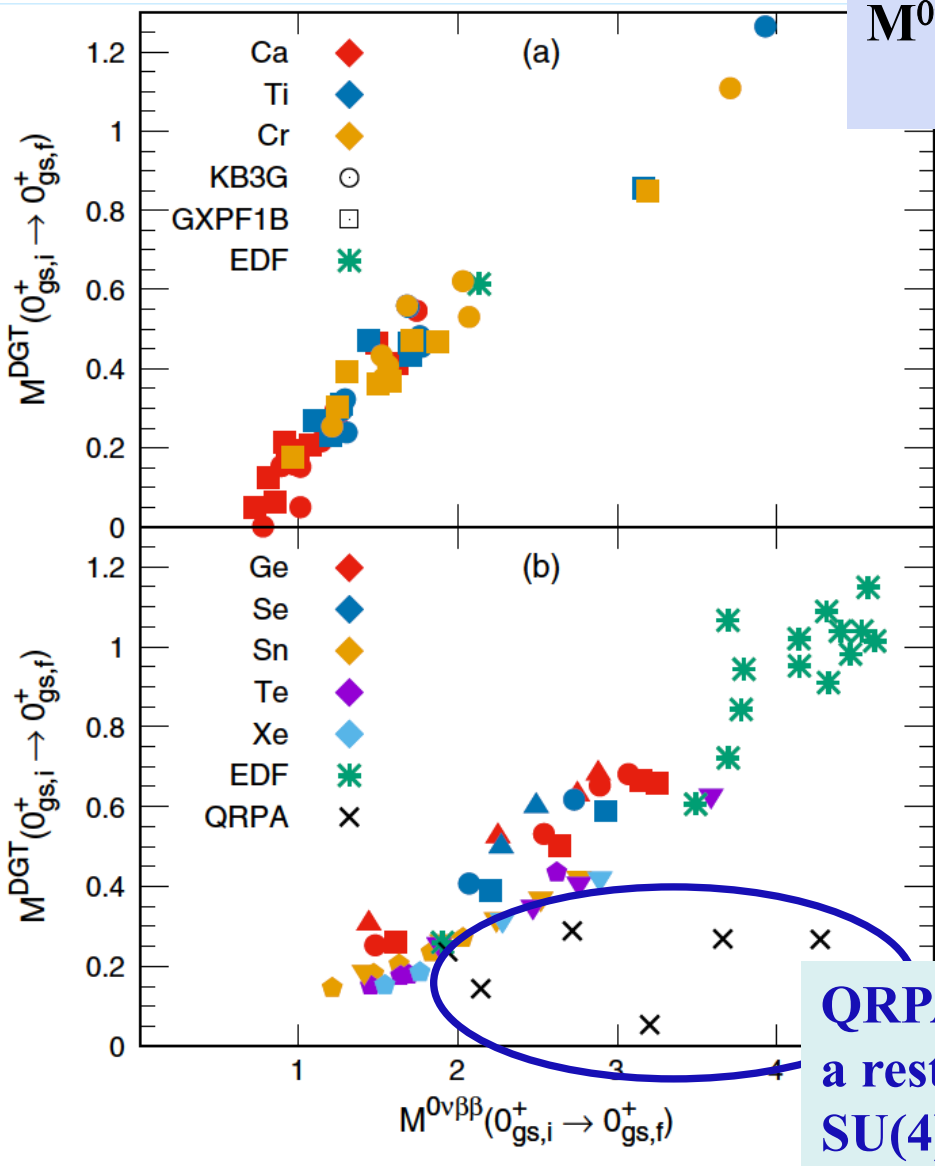
Calc.
within
nuclear
model



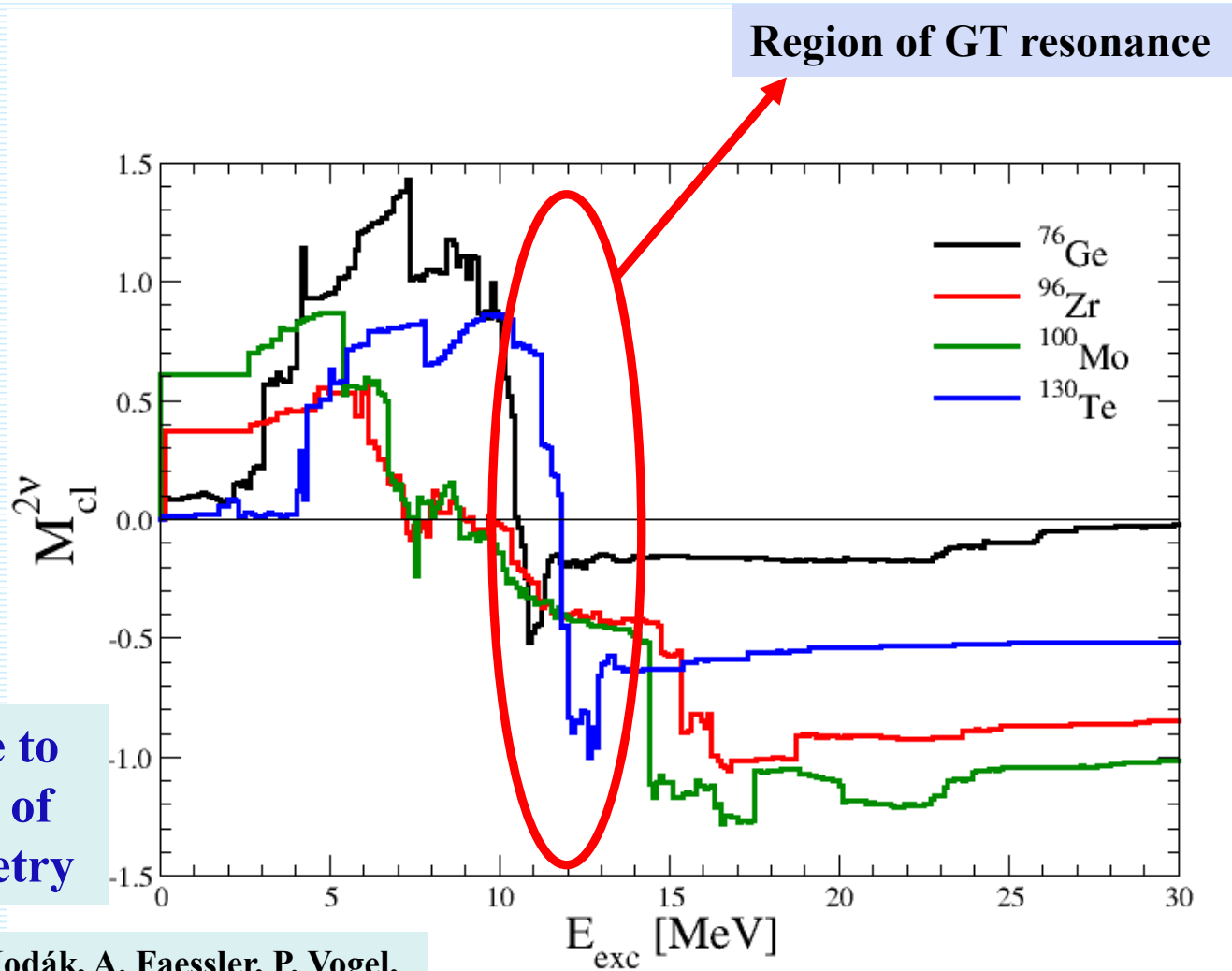
ISM, EDF : $M^{0\nu} \propto M^{2\nu}_{\text{GT-cl}}$

**$M^{\text{DGT}} \equiv M_{\text{GT-cl}} - \text{only } 1^+$
 $M^{0\nu}$ - contribution
 from many J^π (!)**

**QRPA: There is no proportionality between
 $0\nu\beta\beta$ - and $2\nu\beta\beta$ -decay NMEs
 (change of sign at GTR level)**



**QRPA - close to
 a restoration of
 SU(4) symmetry**



Region of GT resonance

**ISM: N. Shimizu, J. Menendez, K. Yako,
 PRL 120, 142502 (2018)**

**QRPA: F.Š., R. Hodák, A. Faessler, P. Vogel,
 PRC 83, 015502 (2011)**

A connection between *closure* $2\nu\beta\beta$ and $0\nu\beta\beta$ GT NMEs

ν - and N- exchange
potential ($\propto f_{src}^2(r)$)

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)
F. Š., A. Smetana, P. Vogel, PRC 98, 064325 (2018)

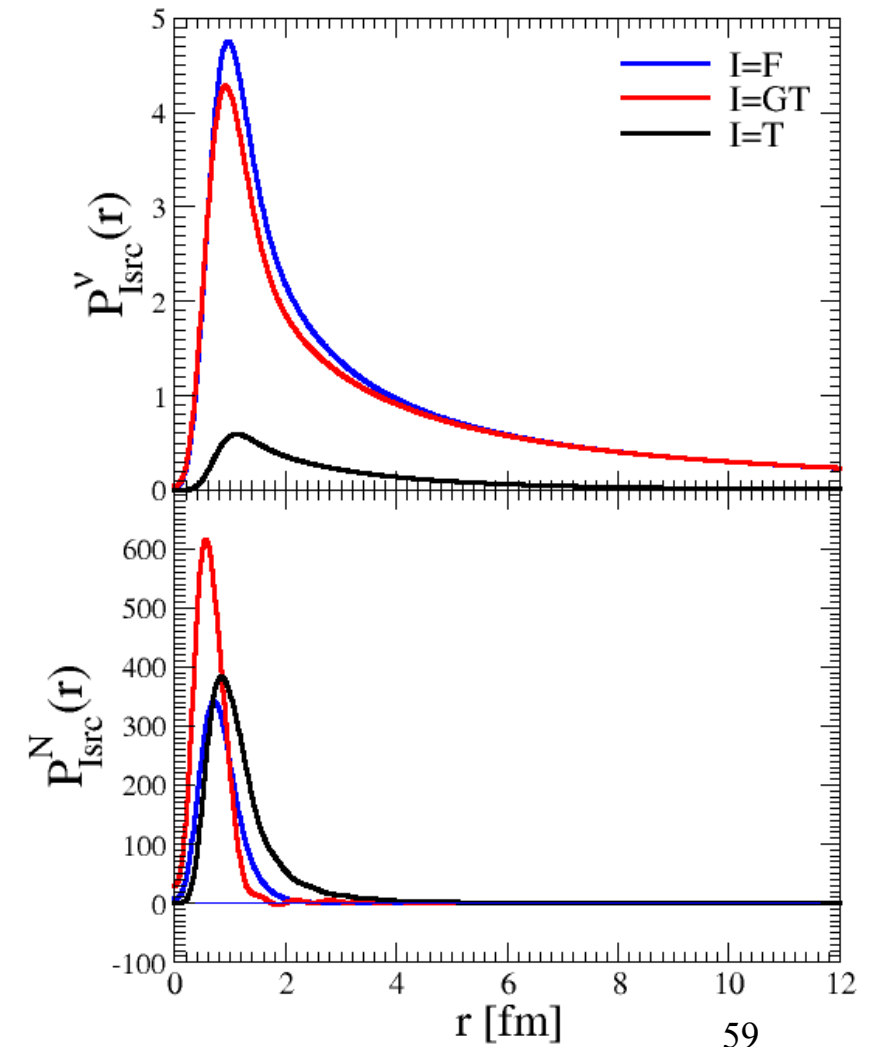
Going to relative
coordinates:

$$\begin{aligned} M_{\nu, N-I}^{0\nu} &= \int_0^\infty P_{I-src}^{\nu, N}(r) C_{I-cl}^{2\nu}(r) dr \\ &= \int_0^\infty f_{src}^2(r) P_I^{\nu, N}(r) C_{I-cl}^{2\nu}(r) dr \\ &\quad I = F, GT \text{ and } T \end{aligned}$$

r - relative distance
of two decaying nucleons

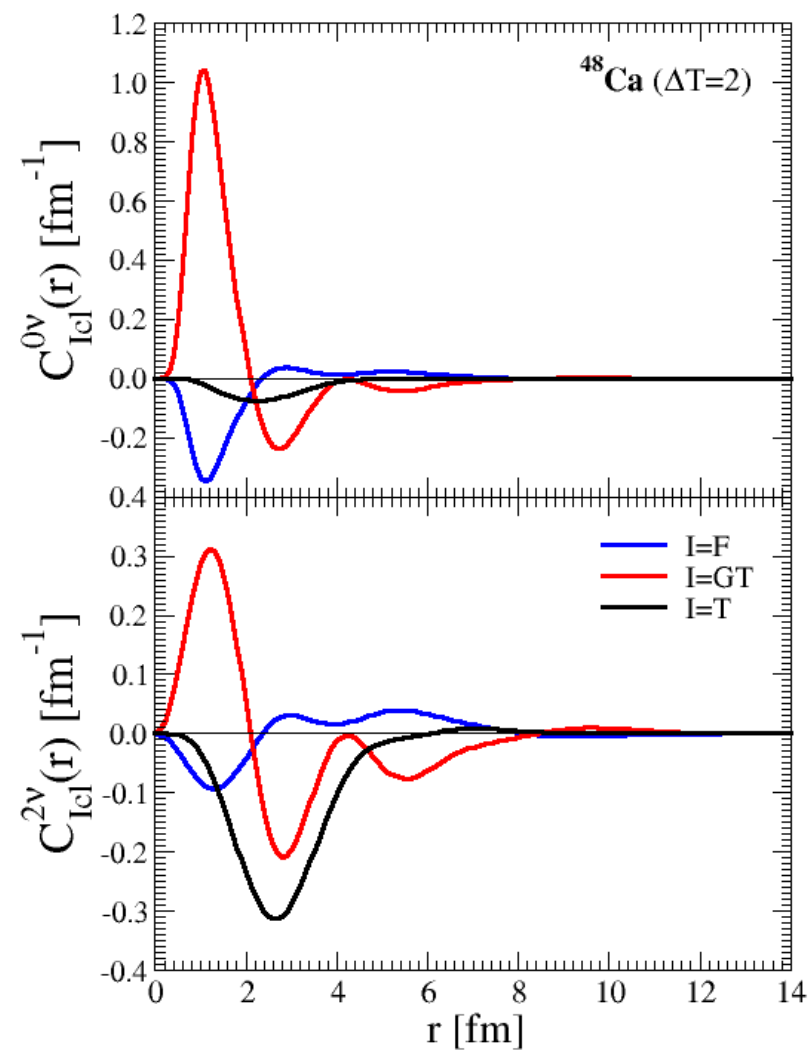
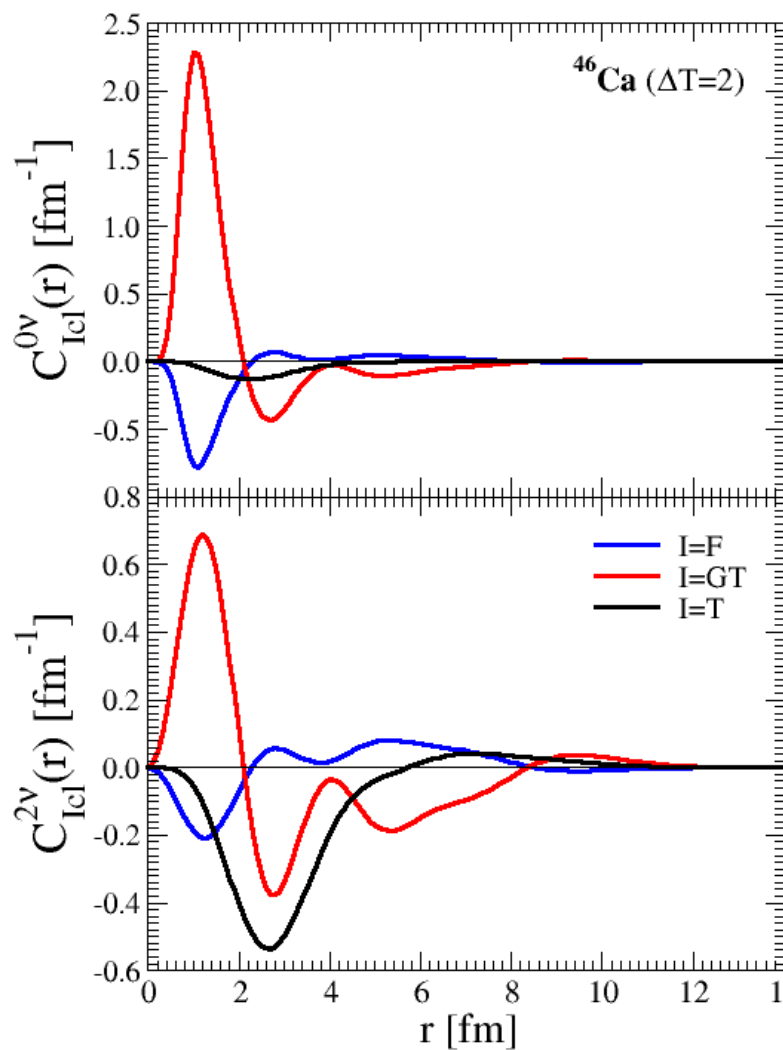
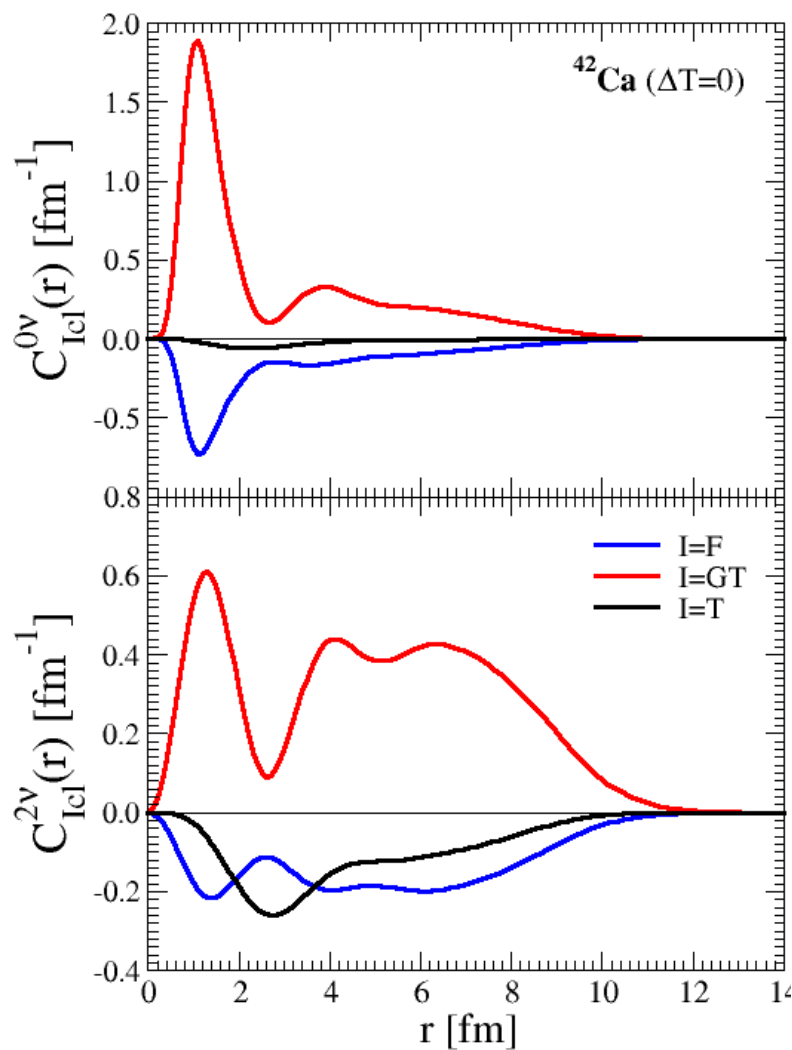
$$M_{GT-cl}^{2\nu} = \int_0^\infty C_{GT-cl}^{2\nu}(r) dr$$

$$\begin{aligned} M_{GT-cl}^{2\nu} &= \\ \sum_{J^\pi, m} &\langle 0_f^+ | \tau^+ \vec{\sigma} | J^\pi, m \rangle \cdot \langle J^\pi, m | \tau^+ \vec{\sigma} | 0_i^+ \rangle \\ \sum_m &\langle 0_f^+ | \tau^+ \vec{\sigma} | 1^+, m \rangle \cdot \langle 1^+, m | \tau^+ \vec{\sigma} | 0_i^+ \rangle \quad \text{mkovic} \end{aligned}$$



Neutrino potential prefers short distances

*The role of the difference in isospin between the initial and final nuclei
($\Delta T = 2$ versus $\Delta T = 0$)*



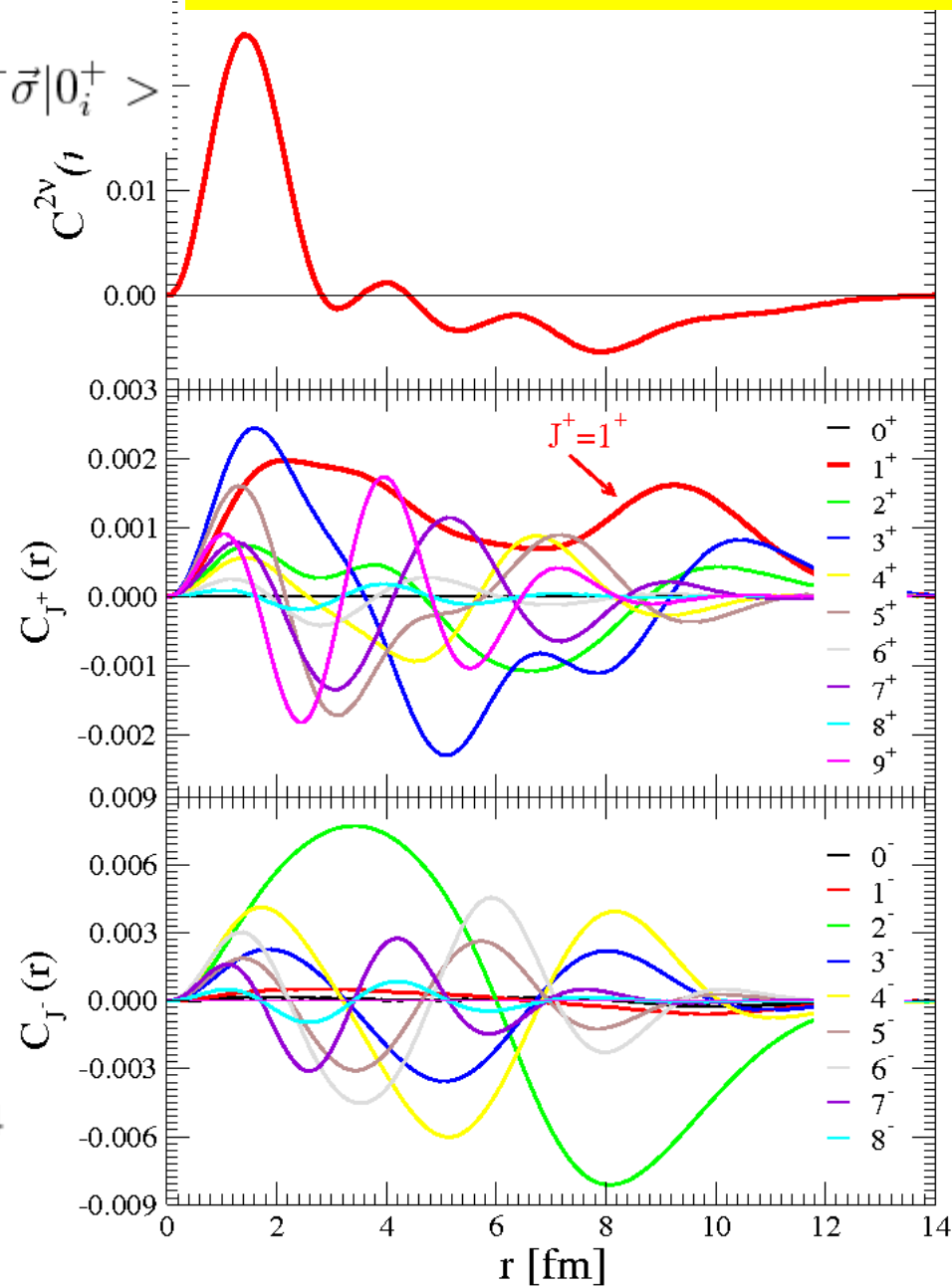
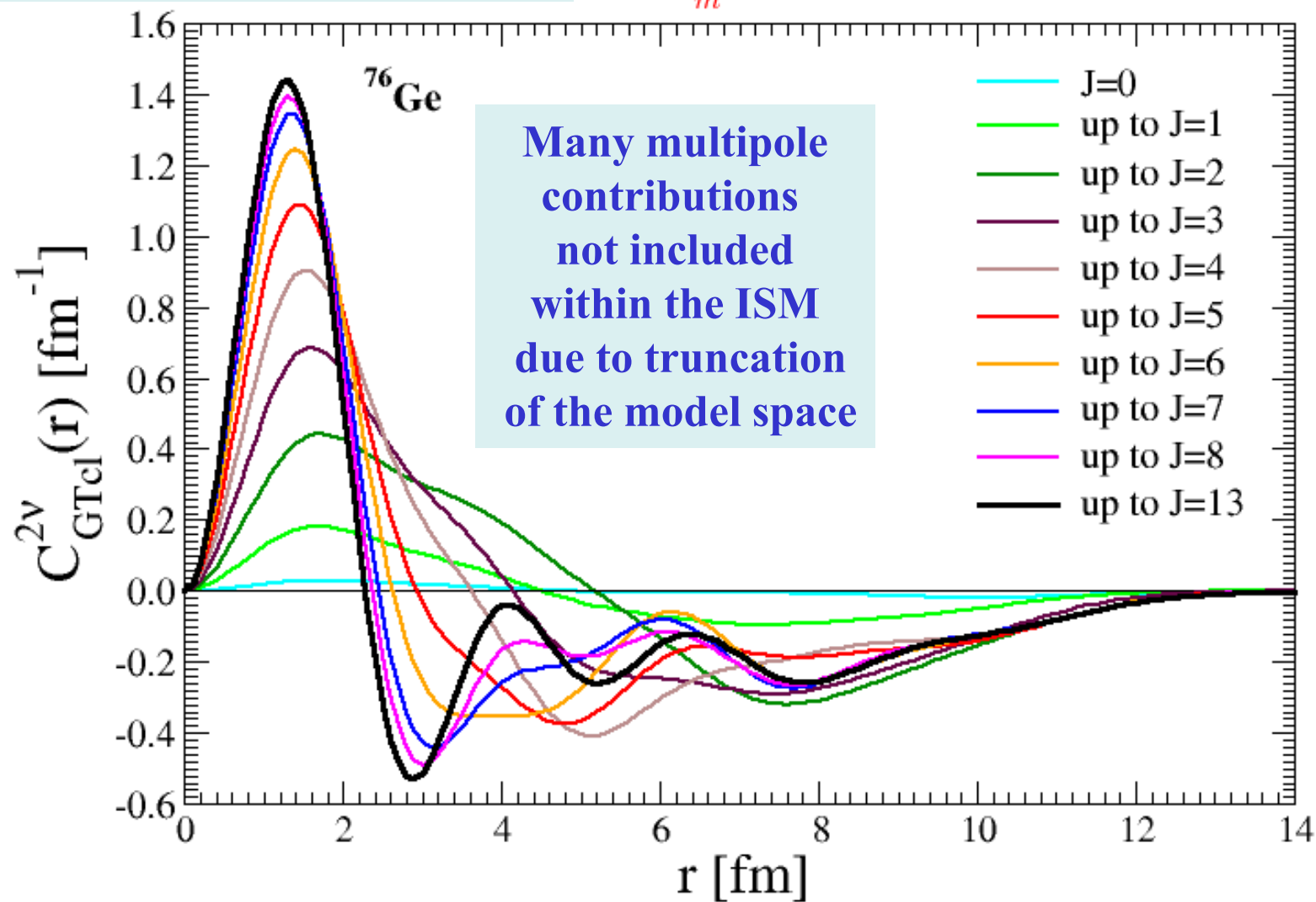
$$M_{GT-cl}^{2\nu} = \sum_{J^\pi} \int_0^\infty C_{GT-J^\pi}^{2\nu}(r) dr$$

*The only non-zero contribution
from $J^\pi=1^+$*

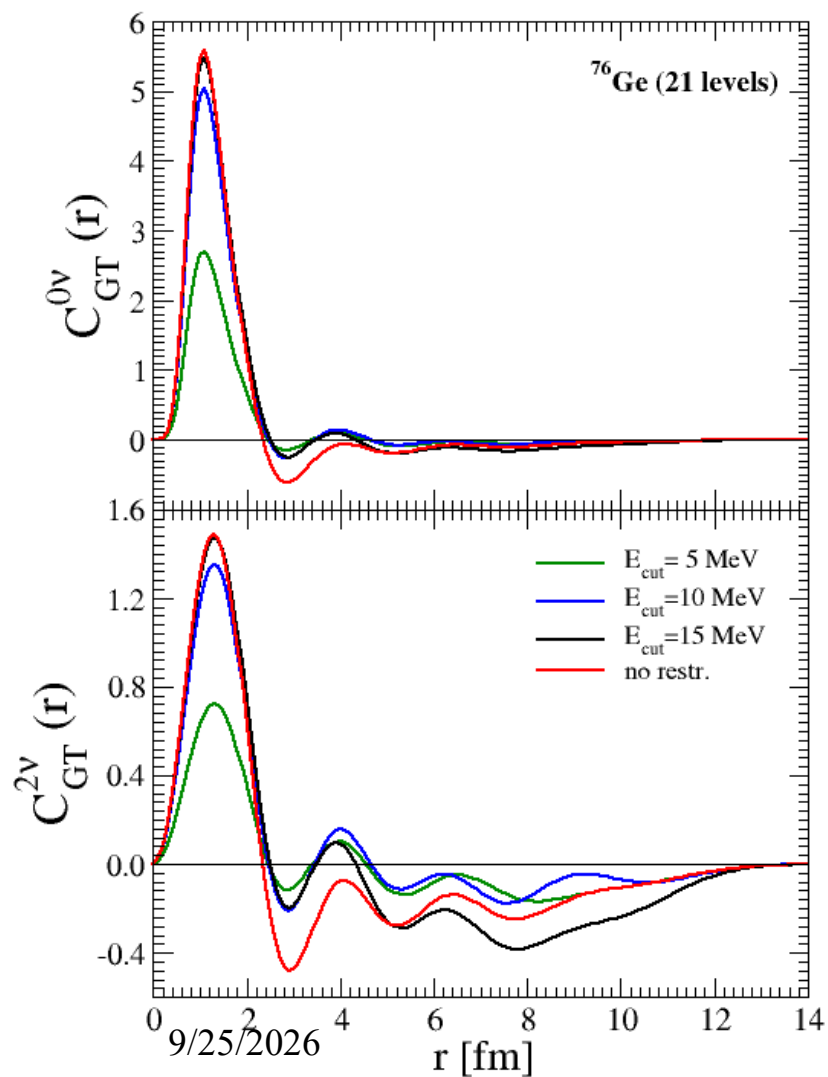
$$M_{GT-cl}^{2\nu} = \sum_{J^\pi, m} \langle 0_f^+ | \tau^+ \vec{\sigma} | J^\pi, m \rangle \cdot \langle J^\pi, m | \tau^+ \vec{\sigma} | 0_i^+ \rangle$$

$$\sum_m \langle 0_f^+ | \tau^+ \vec{\sigma} | 1^+, m \rangle \cdot \langle 1^+, m | \tau^+ \vec{\sigma} | 0_i^+ \rangle$$

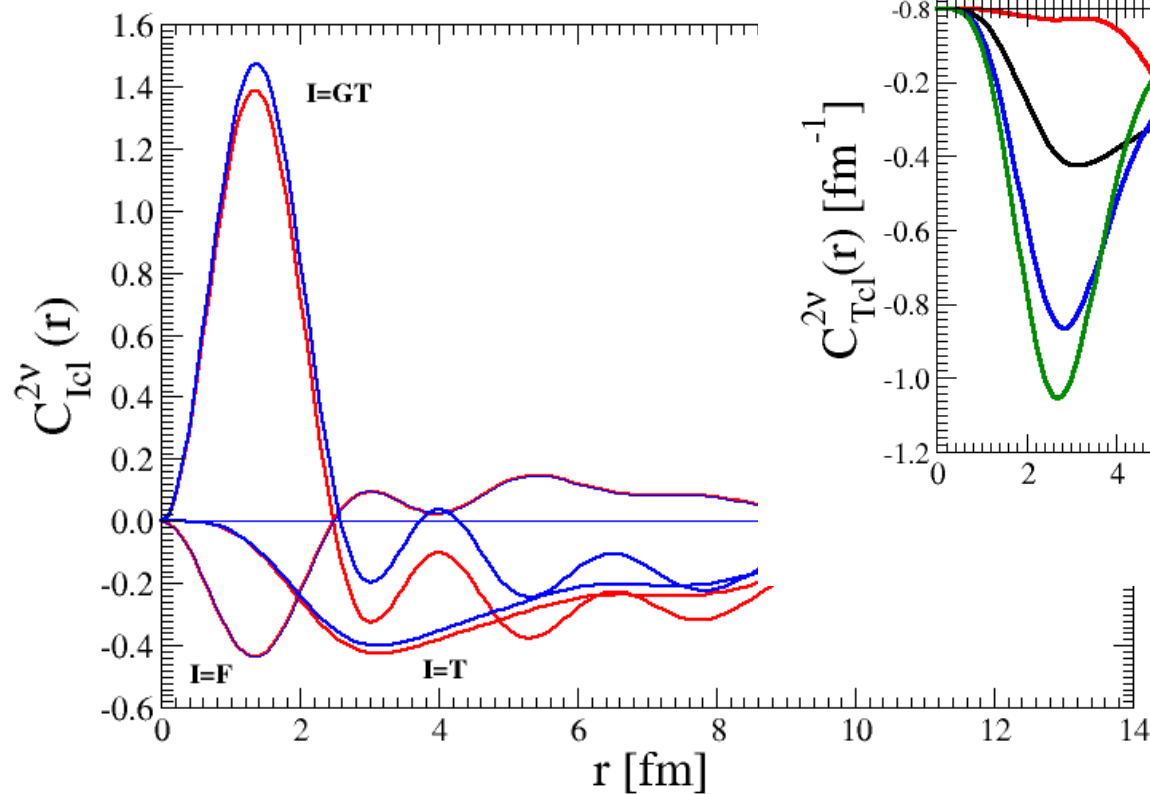
Closure $2\nu\beta\beta$ GT NME
(role of the higher multipolarities)



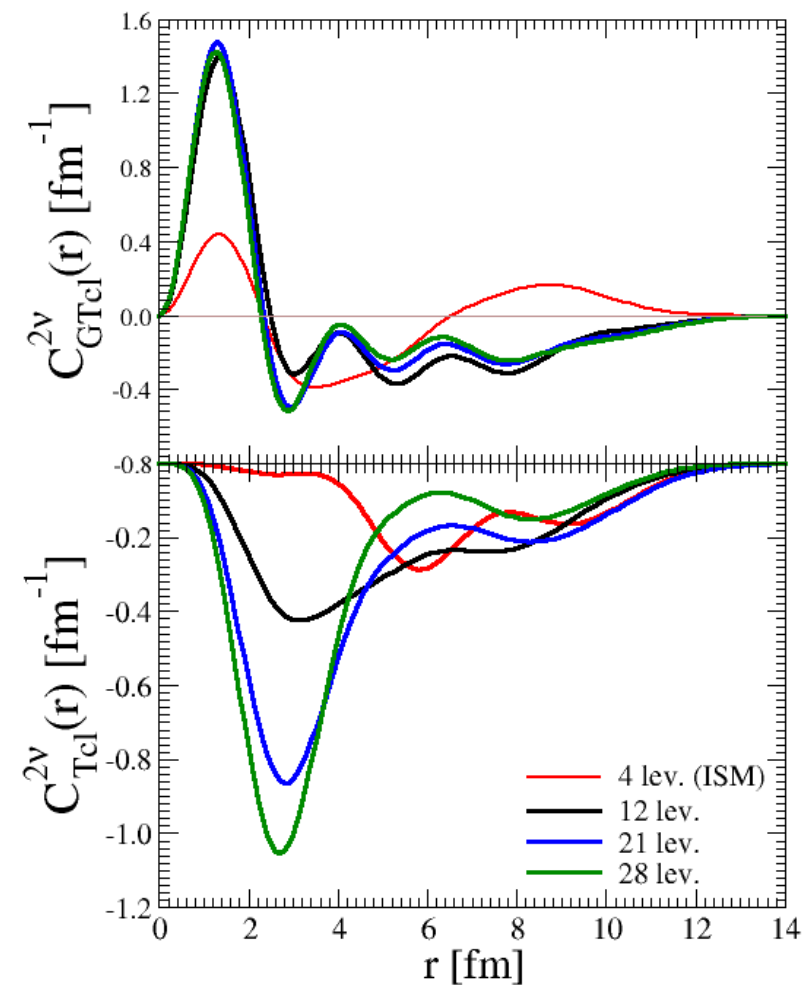
Restriction on the energy range of intermediate states



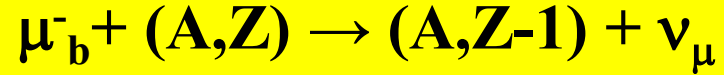
Dependence on $T=0$ particle-particle strength (for $g_A = 0.80$ and 1.27)



The size of model space



Measurement of GT strength via μ -capture



Contradicting QRPA results:

- Strong quenching ($g_A \approx 0.6$)

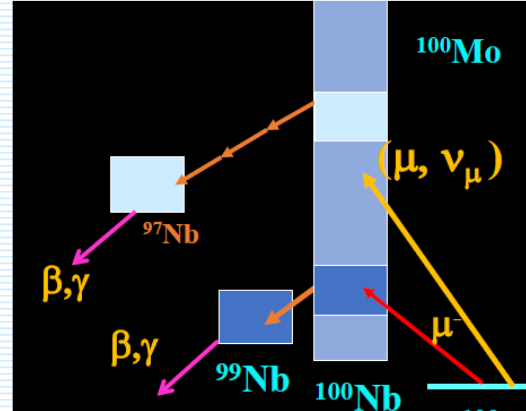
PRC 100, 014619 (2019)

- Weak quenching ($g_A \approx 1.1$)

PRC 74, 024326 (2006)

PRC 79, 054323 (2009)

PRC 102, 034301 (2020).



J-PARC 3-50 GeV p,

ν_μ, μ

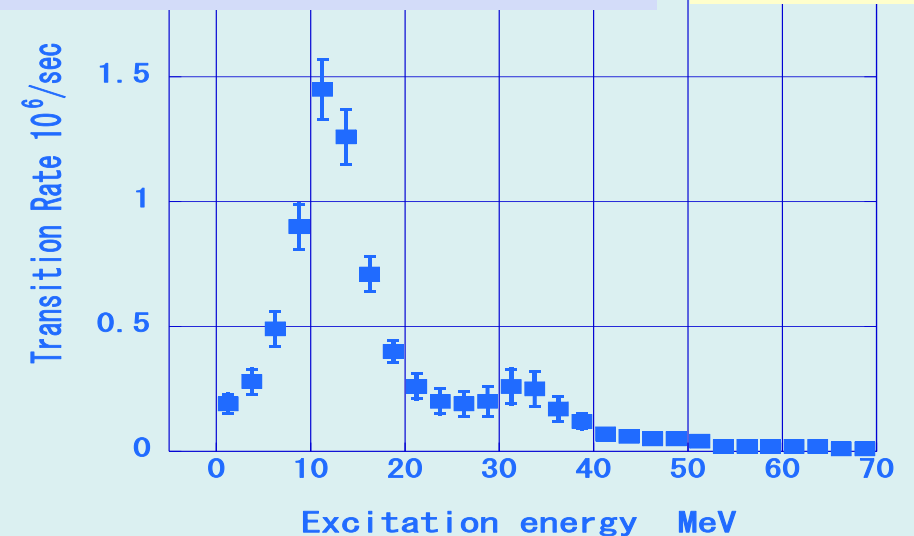


PSI

Monument
Exp.

⇒ Small basis nuclear structure calculations (NSM, IBM) are disfavored. ⇒

Momentum
transfer
 $q \sim 80$ MeV



I. Hashim H. Ejiri, MXG16, PR C 97 2018

$$\Gamma = m_\mu \frac{(G_\beta m_\mu^2)^2}{2\pi} \times (g_A^{\text{eff}})^2 \left(C_F \frac{B_{\Phi F}}{(g_A^{\text{eff}})^2} + C_{GT} B_{\Phi GT} + C_T B_{\Phi T} \right)$$

$$B_{\Phi K}^k(p_{v_k}) = \frac{1}{\hat{f}_i} \sum_{M_i M_k} \int \frac{d\Omega_v}{4\pi}$$

$$\times |\langle J_k M_k | \sum_j^A \tau_j^- e^{i\mathbf{p}_{v_k} \cdot \mathbf{r}_i} O_K \frac{\Phi_g(r_i)}{r_i^{3/2}} | J_i M_i \rangle|^2$$

- **New formalism (derivation as for $0\nu\beta\beta$ -decay)**
- **No factorization of muon w.f. and NME**

Muon capture rates evaluated within QRPA

F. Š, R. Dvornický,
P. Vogel,
PRC 102, 034301 (2020).

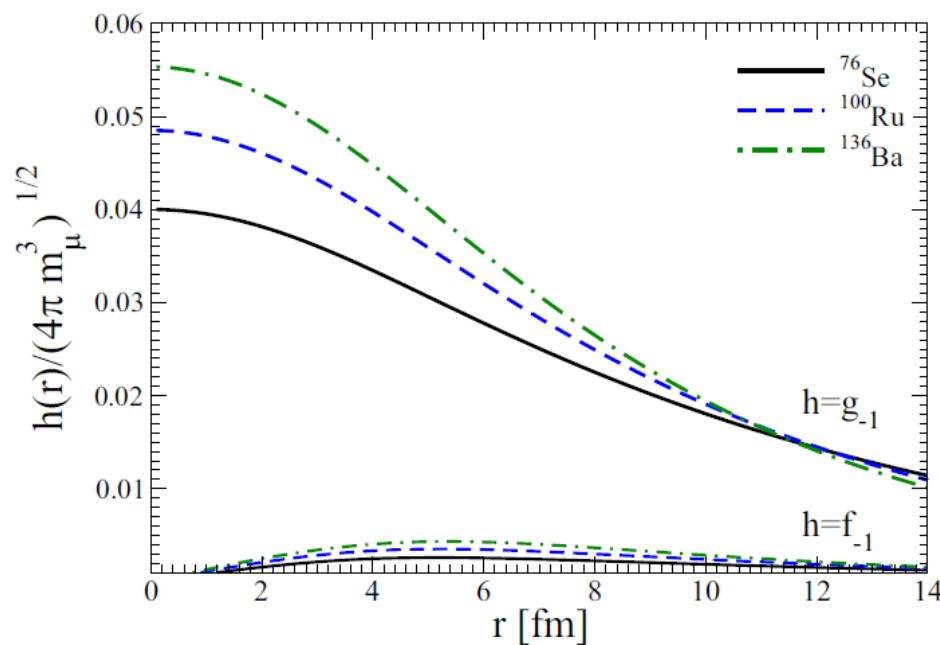
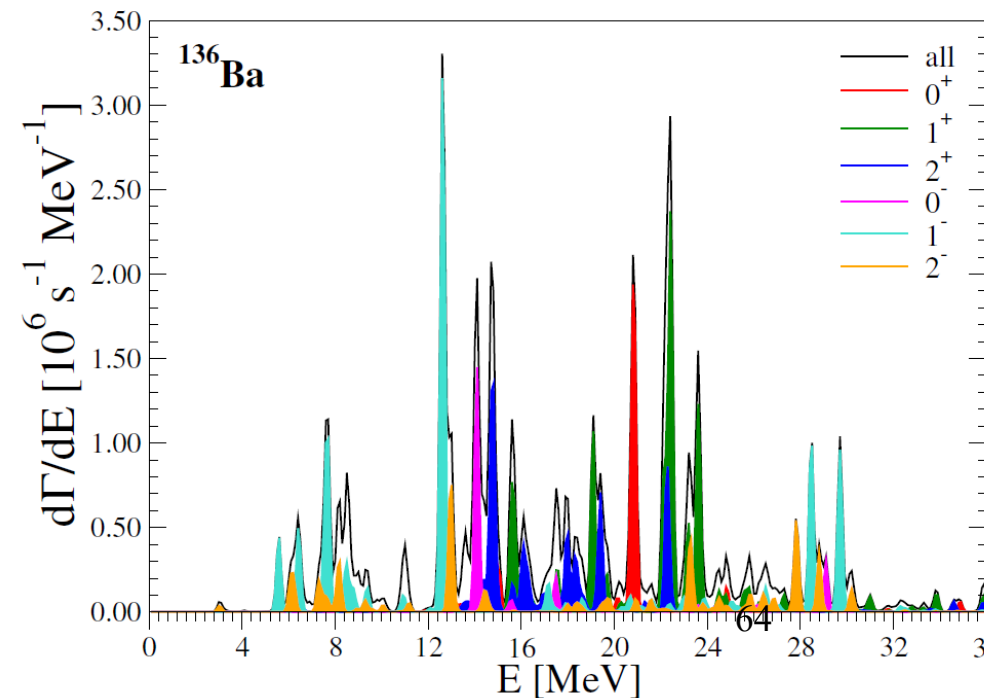
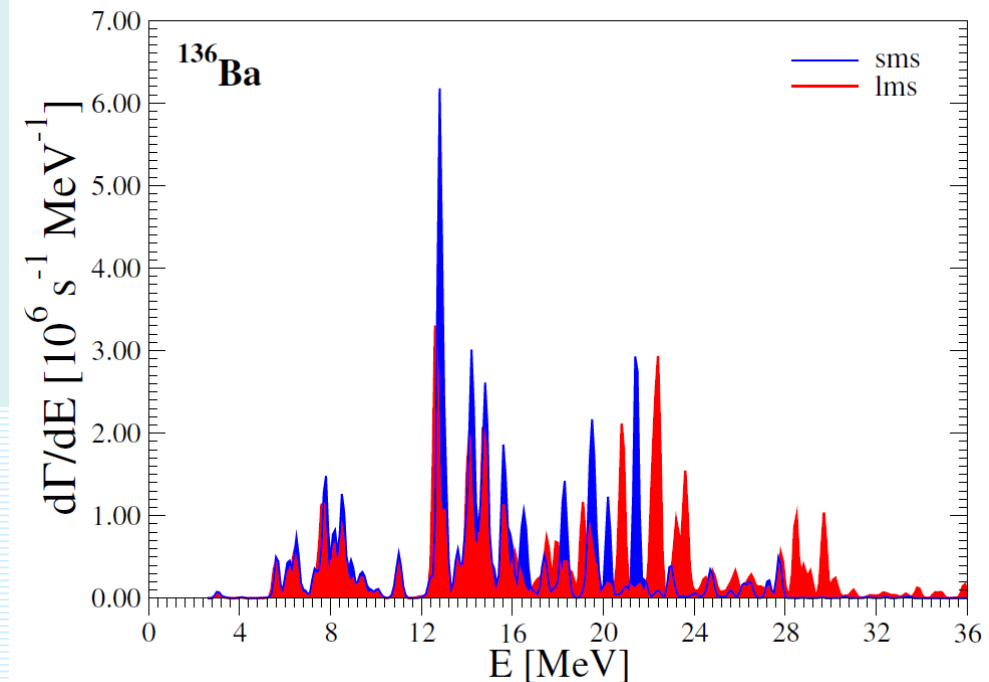
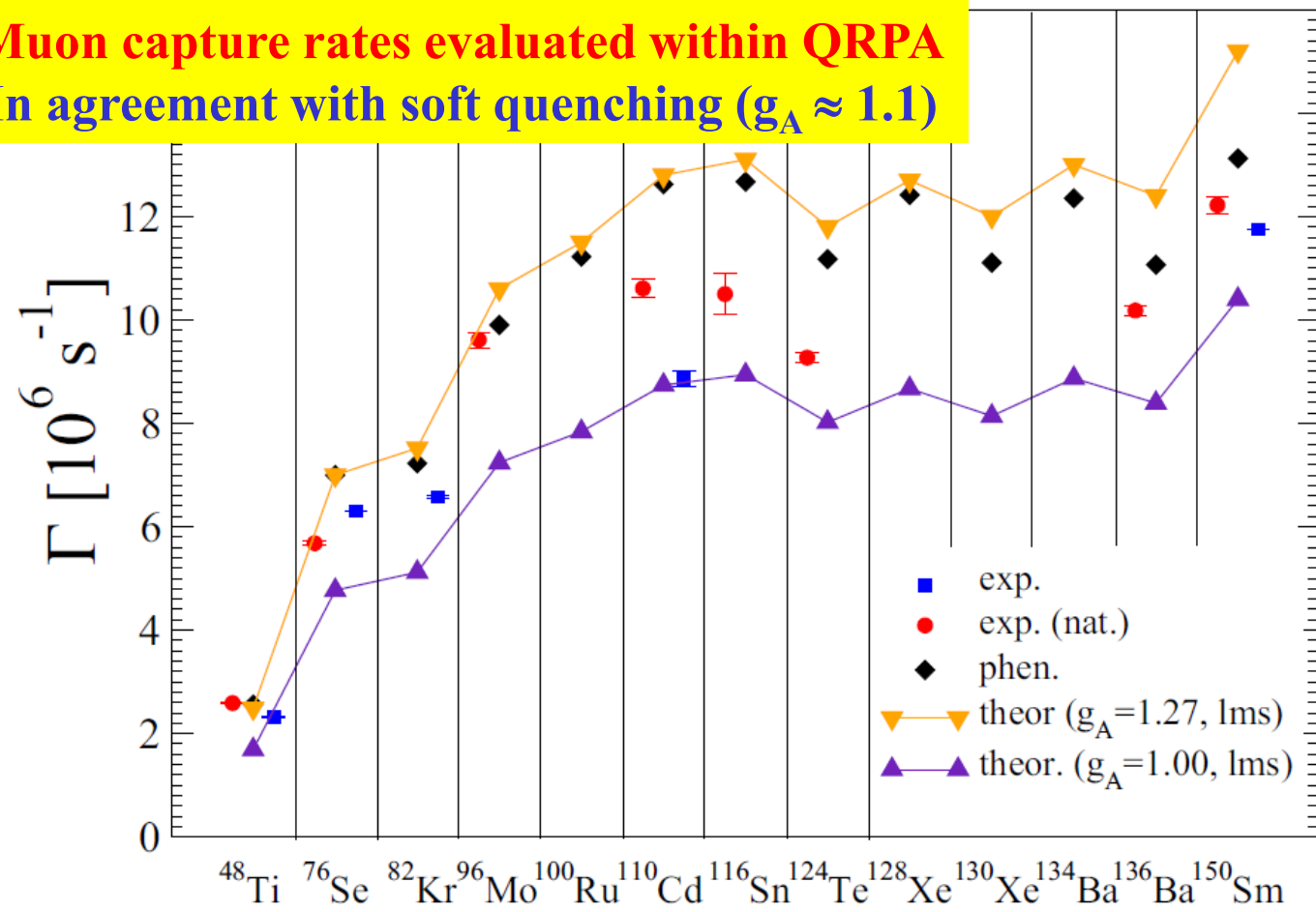


FIG. 1. The radial dependence of the bound muon wave functions $g_{-1}(r)$ and $f_{-1}(r)$ for ^{76}Se , ^{100}Ru , and ^{136}Ba .



Muon capture rates evaluated within QRPA In agreement with soft quenching ($g_A \approx 1.1$)



Monument - ^{76}Se (Phys.Rev.C 113 (2026) 6):

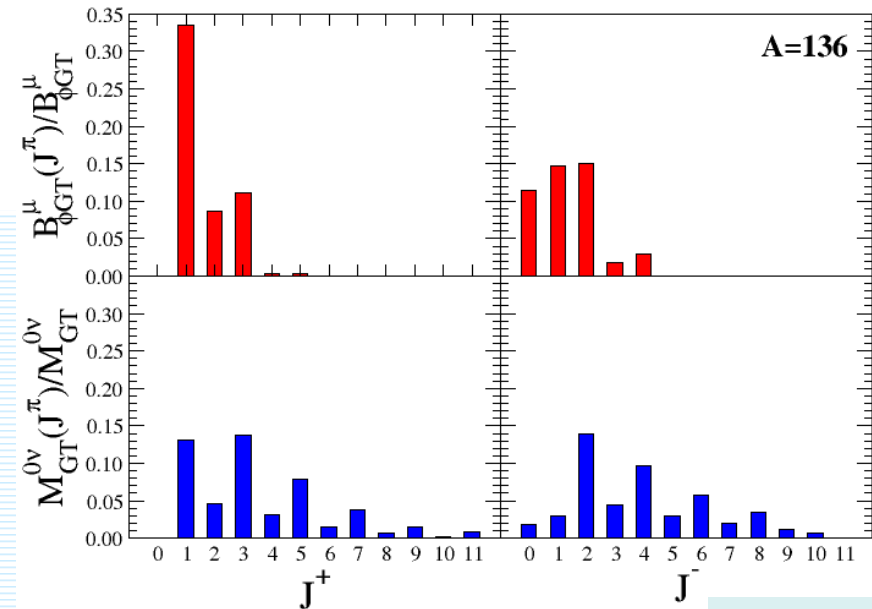
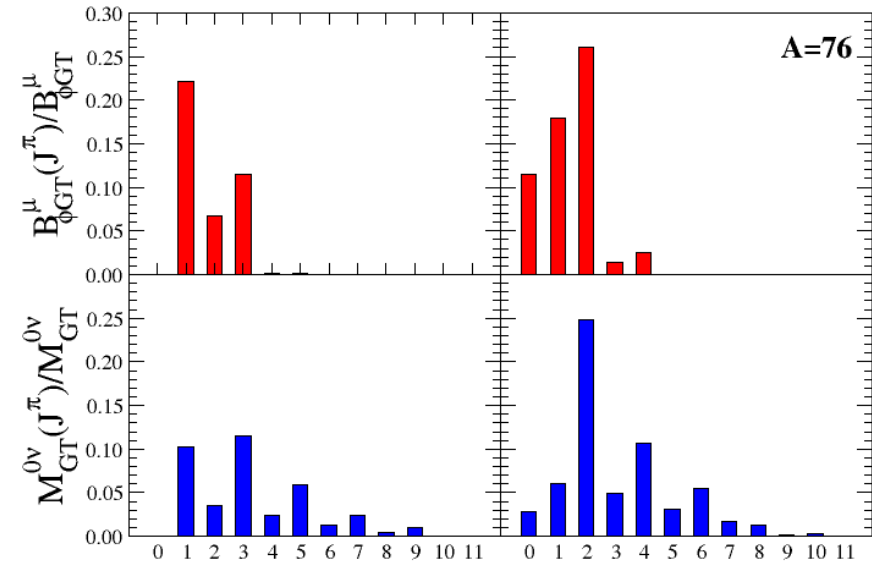
$\tau = 135.1$ ns

Our result - ^{76}Se (PRC 102, 034301 (2020)):

$\tau = 134.5$ ns ($g_A = 1.27$)

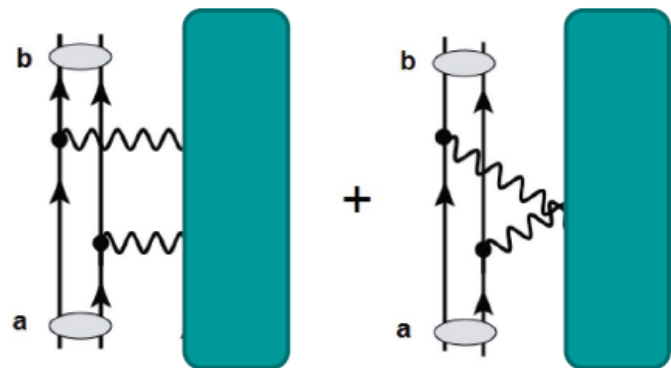
$\tau = 192.1$ ns ($g_A = 1.00$)

$\tau = 254.1$ ns ($g_A = 0.80$)

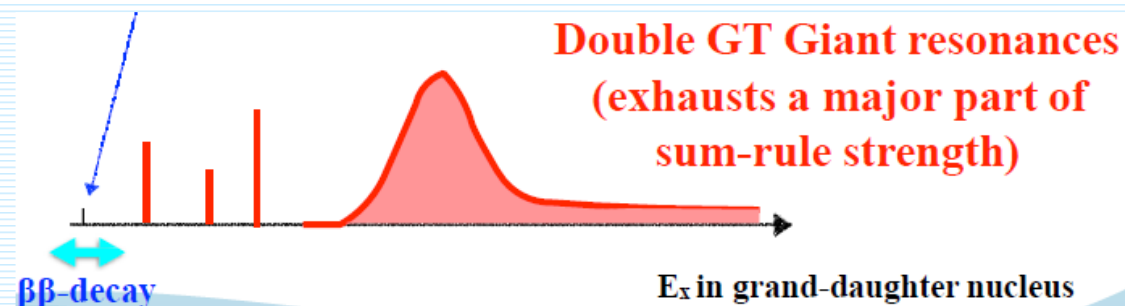


Multipole decomposition of $B_{\mu\text{GT}}$ and $M_{\mu\text{GT}}$

PRC 102, 034301 (2020).



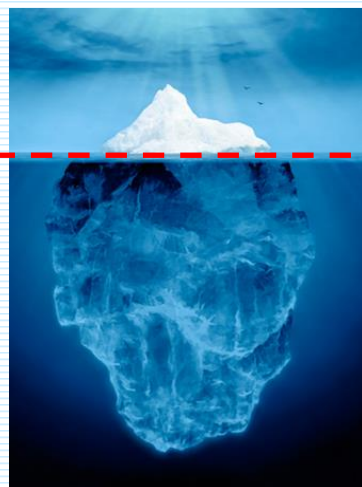
Heavy-ion DCE as surrogate processes of $\beta\beta$ -decay



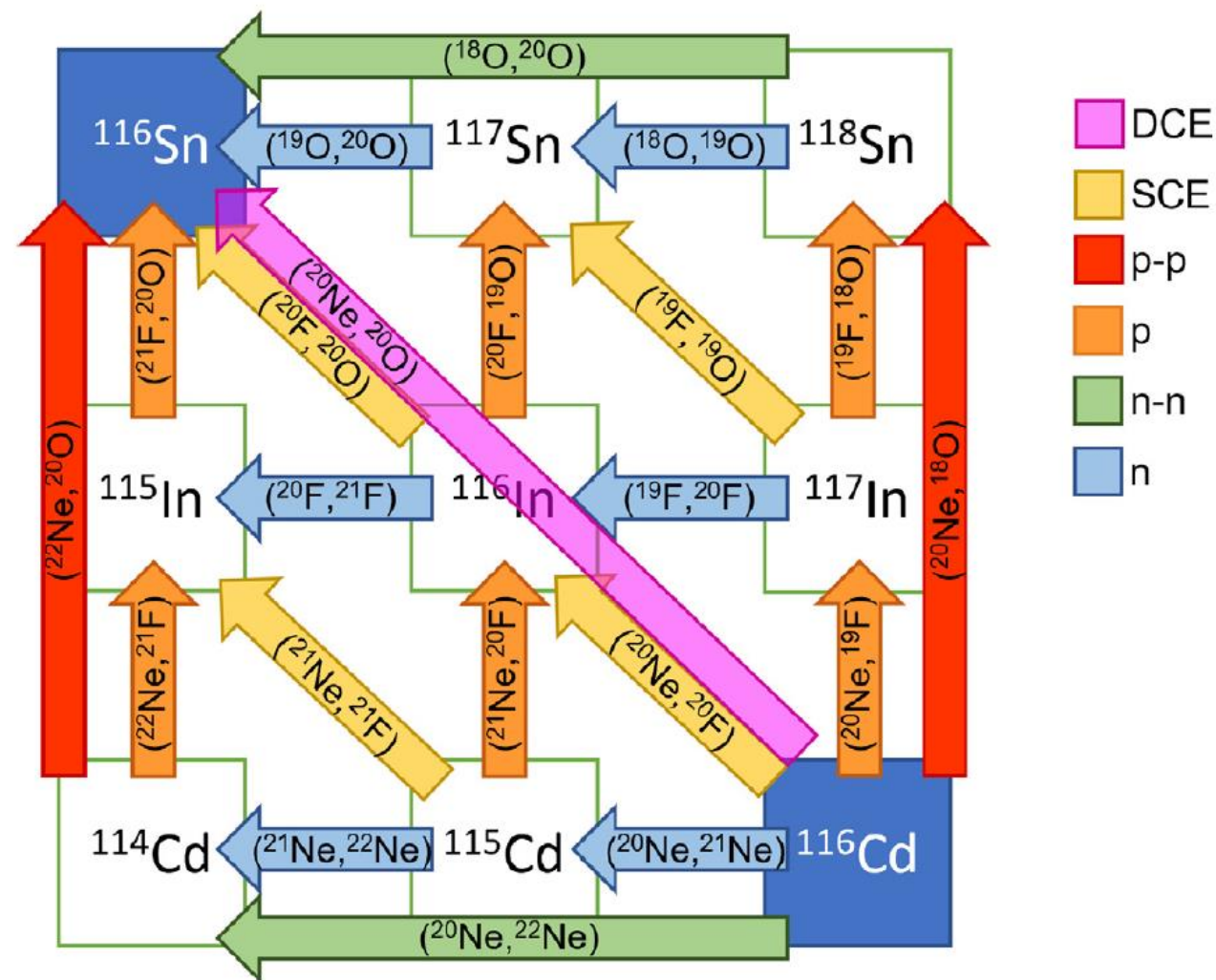
- ✓ Induced by strong interaction
- ✓ Sequential nucleon transfer mechanism 4th order: Kinematical matching
- ✓ Meson exchange mechanism 1st or 2nd order
- ✓ Possibility to go in both directions
- ✓ Low cross section

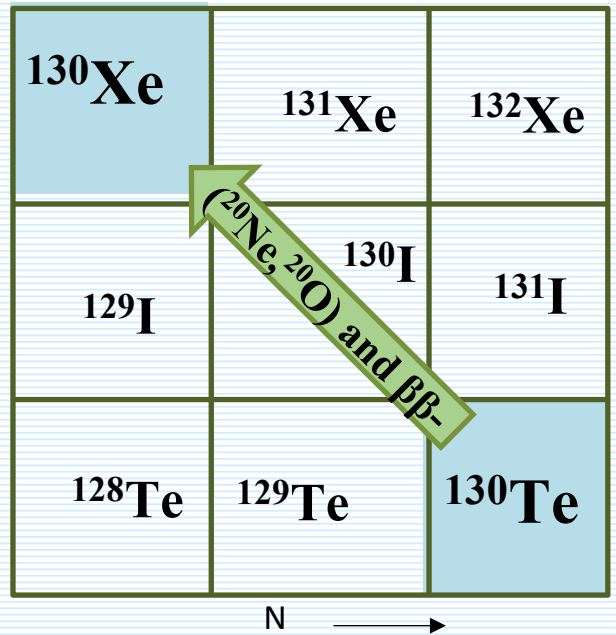
Tiny amount of
DGT strength for
low lying states

Sum rule almost
exhausted by
DGT Giant Mode,
still not observed



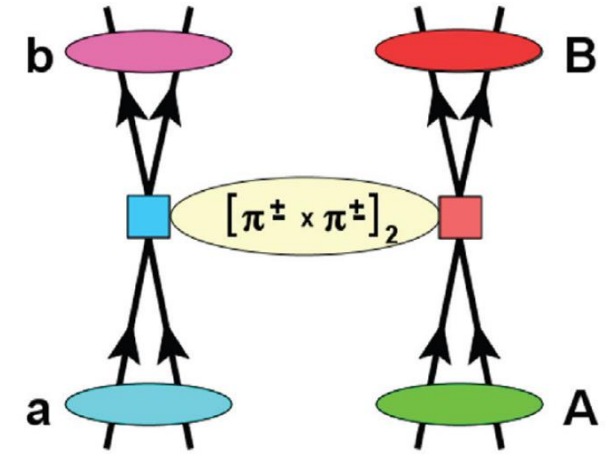
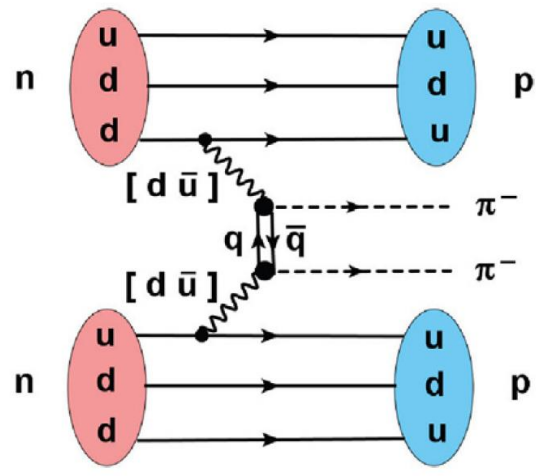
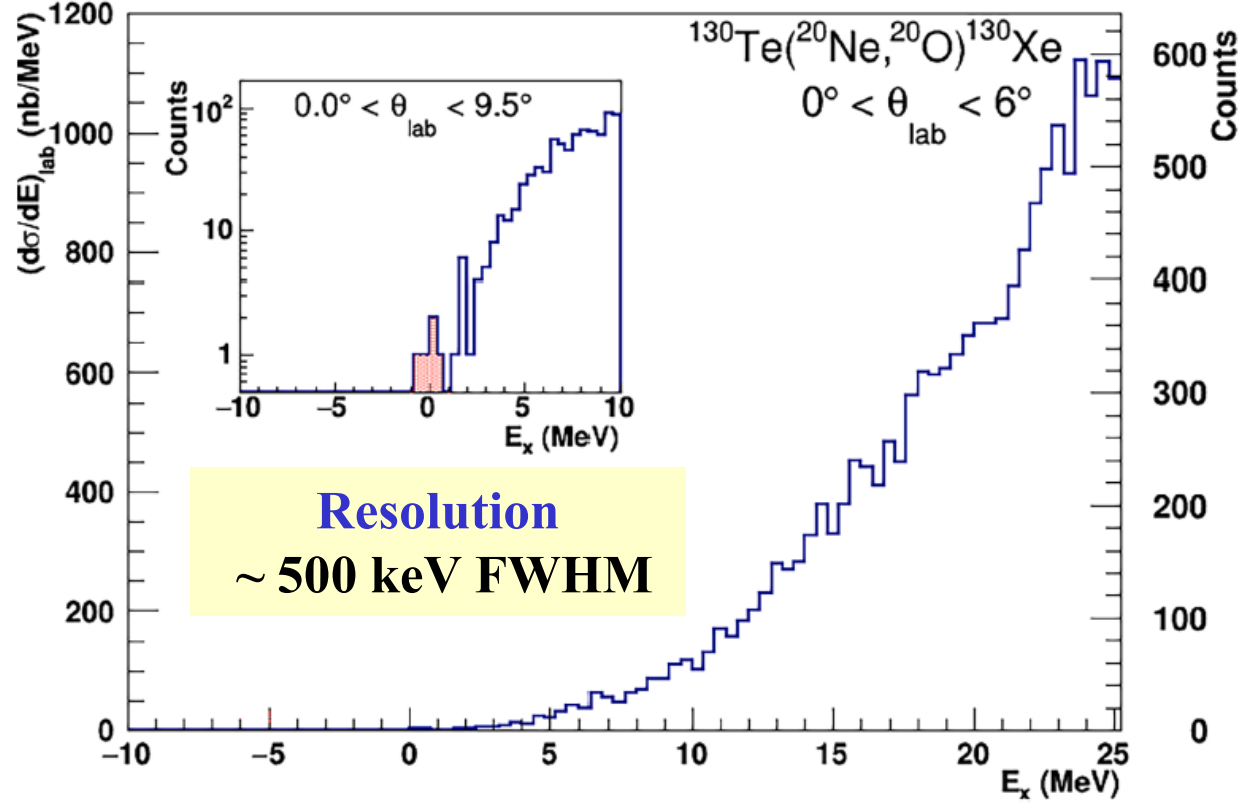
RIKEN
RCNP
Future:
INFN-LN





The $^{130}\text{Te}(^{20}\text{Ne}, ^{20}\text{O})^{130}\text{Xe}$ DCE reaction

- g.s. $\rightarrow$ g.s. transition can be isolated
- Absolute cross section measured



State (MeV)	Counts	Absolute cross section (nb)	Cross section 95% limit (nb)
g.s. (0^+) + 2^+ (536 keV)	5	13	[3--18]

Analysis of cross-section sensitivity < 0.1 nb in the Region Of Interest



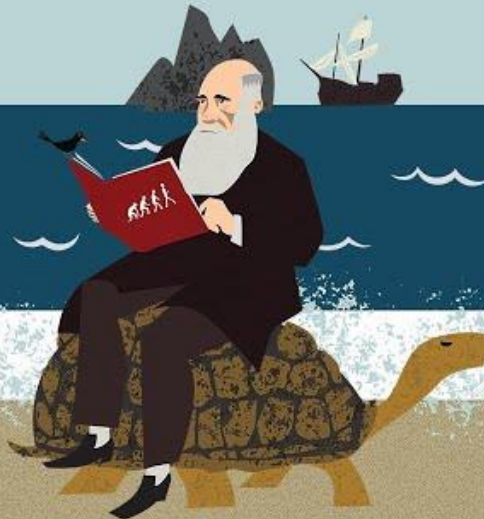
どうもありがとう

THANK YOU!



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A MOST INTERESTING
PROBLEM



Time flies when
you are having fun.

Albert Einstein

quotefancy