Neutrino Mass Hierarchy and the Right-Handed Weak Current in the Neutrinoless Double Beta Decay.

Mini-workshop on neutrino nuclear responses for double beta decays and astro neutrinos

Oct.23

Takeshi Fukuyama

Research Center for Nuclear Physics (RCNP), Osaka University,

Ibaraki, Osaka, 567-0047, Japan

This talk is based on the work with Gang Li (Univ. Masachusetts)

The outline of this talk

This is a short talk in connection with $(\beta\beta)_{0\nu}$ by a GUT theorist. This talk consists of two contents.
(1) Neutrino hierarchy, which is crucial for $\langle m\rangle_{ee} \equiv |\sum U_{ei}^2 m_i|$ since $|U_{11}|^2/|U_{13}|^2 \approx 30$ and $\Delta m_{12}^2 \ll \Delta m_{23}^2$
(2) The neutrino mass comes from the right-handed Majorana neutrino (N_R) (and scalar Fermions (S) for the inverted seesaw mechanism) together with the Dirac neutrino via $m_\nu = m_D M_R^{-1} m_D^T$ (Type I) or $m_D M^{-1}\mu(m_D M^{-1})^T$ (Inverse Seesaw), which also indicates the right-handed weak currents and their gauge boson W_R. The halflife time is

$$\frac{1}{T_{1/2}} = G_{0\nu}\|\frac{M_\nu}{m_e}\|^2 \left(\|\langle m\rangle_{ee}\|^2 + \|p^2\frac{\zeta M_W^2}{M_{WR}^2}\frac{U_{ei}U_{eNi}}{M_{N_i}}\|^2 + \|p^2\frac{M_W^4}{M_{W_R}^4}\frac{U_{eNi}^2}{M_{N_i}}\|^2\right). \tag{1}$$

Here p^2 is the averaged momentum square of the intermediate neutrino and $\approx -(200\mathrm{MeV})^2$, and $G_{0\nu}$ and M_ν are a phase space factor and the nuclear matrix element, respectively. Also,

$$\nu_\alpha = U_{\alpha i}\nu_i + U_{\alpha Ni}(N_{Ri})^c + U_{\alpha Si}S_i. \tag{2}$$

for the inverse seesaw mechanism. The first, second, third contributions comes from (c), (d), (a) of Fig.1, respectively. (b) is negligible.

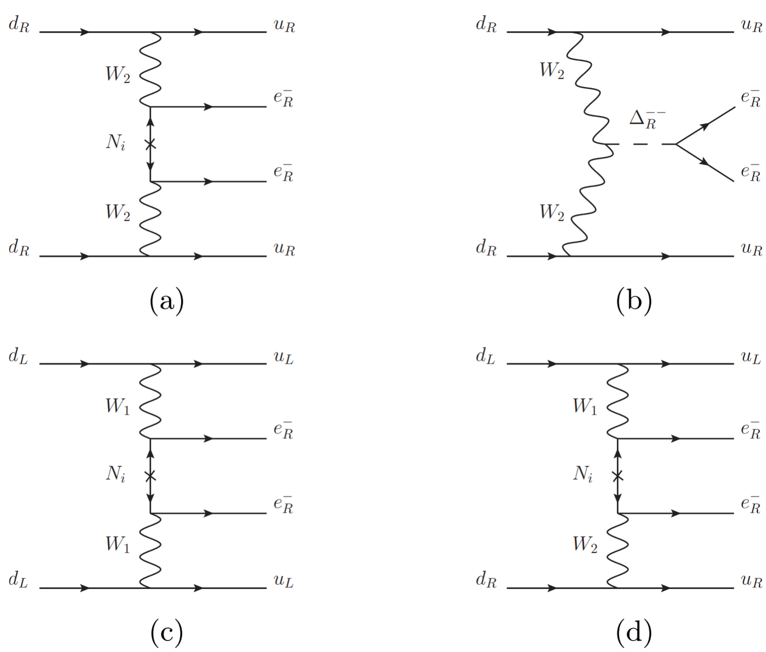

Figure 1: Diagrams contributing to the $(\beta\beta)_{0\nu}$. Here W_i are the mass eigenstates of $W_{L,R}^{+\mu}$. See Eqs.(17)(18) for the details.

1 Introduction

There are two conditions to realize neutrinoless double beta decay $(\beta\beta)_{0\nu}$. [1]
1. ν_e should be the same as its anti-particle

$$\nu_e = \overline{\nu_e}. \tag{3}$$

and
2. The connecting neutrinos should have the same helicity. The latter condition is satified if neutrinos are massive and/or if the right-handed current coexists with the left-handed current. The first case of 2. is described as

$$\langle m_\nu \rangle = \langle m_\nu \rangle_{ee} = |\sum_j U_{ej}^2 m_j|. \tag{4}$$

Here $U_{\alpha i}$ (Greek (Latin) indicates flavour (mass) eigen state) is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [2] in left-handed current. Normal Hierarchy (NH) indicates

$$m_1 < m_2 < m_3 \tag{5}$$

and the Inverted Hierarchy (IH) does

$$m_3 < m_1 < m_2. \tag{6}$$

The central values and relative uncertaities of PMNS mixing angles and the masss square differences are [3]

$$\begin{aligned}
&\theta_{12} = 0.5903, 2.3\% \quad \theta_{23} = 0.866, 4.1\%(\mathrm{NH}),\ 0.869, 4.0\%(\mathrm{IH}), \\
&\theta_{13} = 0.150, 1.5\%(\mathrm{NH}),\ 0.151, 1.5\%(\mathrm{IH}) \\
&\Delta m_{21}^2 = 7.39\times 10^{-5}\mathrm{eV}^2, 2.8\%,\ \Delta m_{32}^2 = 2.451\times 10^{-3}\mathrm{eV}^2, 1.3\%(\mathrm{NH}),\ \ -2.512\times 10^{-3}\mathrm{eV}^2, 1.3\%(\mathrm{IH})
\end{aligned} \tag{7}$$

and the PMNS matrix elements have the values

$$U_{11} = c_{12}c_{13},\ U_{12} = s_{12}c_{13},\ U_{13} = s_{13}e^{-i\delta} \tag{8}$$

up to the Majorana phases diag$(e^{i\alpha},\ e^{i\beta},\ 1)$. Here $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ as usual. Substituting these values

$$|U_{11}|^2/|U_{13}|^2 \approx 30. \tag{9}$$

Then IH case enhances $\langle m_\nu \rangle$ relative to NH case. Though the final answer to the hierarchy problem is obtained from the observation, theoretical predictions have been given by many models. The typical one is due to the predictive minimal SO(10) model [4]. Based on the SO(10) model, they fitted low energy spectra of all quark lepton masses and the CKM and the PMNS mixing angles and phases.

We assume the normal hierarchy of the neutrino masses, $m_1 < m_2 < m_3$, in the above discussion. We also search for χ^2 minimum for the inverted hierarchy case, which can be done since the neutrino mass matrix is given as input in our formula. We find that the fit does not lead to a competitive result within the energy scale from 10^{13} GeV to 10^{16} GeV, which gives $\chi^2 > 200$ (See the following tabe cited from [4]).

Type II				
	Best fit		$R = 0.001$	
θ_{23}	0.710	$\pi/4$	0.710	$\pi/4$
R	5.1582	7.2861	0.001	0.001
α_e	-0.81968	-1.8785	-0.66648	-0.25301
α_μ	-1.0015	1.5169	-2.8148	2.8177
α_τ	1.4632	2.8853	-0.53961	-0.84287
α_2	-2.8481	2.8306	-2.8709	-3.1146
α_3	-0.30601	1.1731	-1.9809	-2.8604
δ_{PMNS}	-0.28115	1.4476	-2.3550	-3.1131
$\log_{10}(m_1/\mathrm{GeV})$	-11.592	-11.467	-11.207	-11.173
$\lvert\delta\rvert$	75.056	100.11	15.545	16.156
$\mathrm{Arg}(\delta)$	0.11782	0.11535	0.43912	0.51567
m_c (GeV)	0.1931	0.1929	0.1978	0.1989
m_d (GeV)	0.0009278	0.0009309	0.0004138	0.0003936
m_s (GeV)	0.01785	0.01767	0.01980	0.02028
m_b (GeV)	0.9897	0.9898	0.9903	0.9901
V_{uc}	0.2240	0.2240	0.2240	0.2241
V_{sb}	0.003698	0.003698	0.003765	0.003724
V_{ub}	0.03700	0.03699	0.03694	0.03695
δ_{KM}	1.180	1.180	1.195	1.160
Pull				
m_c	0.004	-0.005	0.191	0.236
m_d	-0.010	-0.002	-1.363	-1.416
m_s	0.036	0.000	0.426	0.522
m_b	-0.002	0.000	0.017	0.010
V_{uc}	0.002	-0.012	-0.000	0.027
V_{sb}	-0.005	-0.005	0.144	0.054
V_{ub}	-0.004	-0.009	-0.044	-0.039
δ_{KM}	-0.001	-0.001	0.075	-0.099
r	0.0140	0.0138	0.0230	0.0233
r_2	$0.506 + 0.0252i$	$0.502 + 0.00628i$	$2.15 + 0.227i$	$2.22 - 0.160i$
$c_R v_R$ (GeV)	1.19×10^{13}	0.861×10^{13}	8.86×10^{16}	9.22×10^{16}
χ^2	0.001	0.0003	2.10	2.35

Table 1: *The fit result for type II.*

Here

$$\chi^2 = \sum_i \frac{(\chi_i - \hat{\chi}_i)}{\hat{\sigma}_i^2}, \tag{10}$$

where $\hat{\chi}_i$ and $\hat{\sigma}_i$ are the experimental measurements of the parametersand their standard deviations of errors, respectively. Using the fitted data, the effective neutrino mass is also predicted as

$$\langle m_\nu \rangle = 1 \text{ meV}. \tag{11}$$

Whereas, the recent $(\beta\beta)_{0\nu}$ experiment in the KamLAND-Zen provides the most stringent upper limit on it, $61-165$ meV.[5]. However, we have another contribution by the right-handed current, which is indispensable if we consider N_R. We will discuss on it in the next section.

2 Right-handed Weak Current

In the Grand Unified Theories (GUTs) whose rank ≤ 5 have the intermediate energy scale between GUTand the Standard Model (SM). For instance, in the SO(10) model, the $SU(2)_R$ (V+A) current coexists together with $SU(2)_L$ (V-A) current in the intermediate energy scale as we discuss below. All SM fermions + ν_R belongs to 16-dim. The Yukawa coupling with the Higgs field ϕ are

$$Y_{ij} 16_i 16_j \phi, \tag{12}$$

where i, j are generation numbers. Here

$$16 \times 16 = 10 + 120 + 126 \tag{13}$$

and Higgs field ϕ can be 10 and 120 or $\overline{126}$ to form SO(10) invariants. Under the breaking pattern, $SO(10) \to SU(4)_c \times SU(2)_L \times SU(2)_R$(Pati-Salam (PS) phase [6]) $\to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \to$ SM, Higgs fields are broken to

$$10 \to (1,2,2) + (6,1,1), \quad \overline{126} \to (6,1,1) + (10,3,1) + (\overline{10},1,3) + (15,2,2) \tag{14}$$

We describe the former bi-doublet by Φ and the latter triplet by Δ. Namely the bi-doublet is

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \tag{15}$$

and the $L-$ and $R-$ handed triplets are

$$\Delta_{L,R} = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}_{L,R}. \tag{16}$$

In $(\beta\beta)_{0\nu}$ the Hamiltonian is given by

$$H_W = \frac{G_F}{\sqrt{2}} \bar{e}\gamma^\alpha \left[(1-\gamma^5) + \eta(1+\gamma^5)\right] \nu_e J_{L\alpha}^\dagger + h.c. \tag{17}$$

Here $J_{L\alpha}$ is the $V-A$ hadronic current. The generalized form is given by [1]. The mass eigenvalues of the weak bosons in the left- and right-handed gauge sectors($W_L,\ W_R$) as follows:

$$
\begin{aligned}
W_L &= W_1 \cos\zeta + W_2 \sin\zeta, && (18)\\
W_R &= -W_1 \sin\zeta + W_2 \cos\zeta, && (19)
\end{aligned}
$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8}\cos^2\zeta \frac{M_1^2\tan^2\zeta + M_2^2}{M_1^2 M_2^2}. \tag{20}$$

Here M_1 and M_2 are the masses of the mass eigenstates W_1 and W_2, respectively, and ζ is the mixing angle which relates the mass eigenstates and the flavor eigenstates. ζ is given by the vevs of ϕ_1, ϕ_2 and $\Delta_{L,R}$ denoted by κ, κ', and $v_{L,R}$, respectively. The gauge mass matrix is given by

$$M_W^2 = \begin{pmatrix} \frac{1}{2}g^2(\kappa^2 + \kappa'^2 + 2v_L^2 & g^2\kappa\kappa' \\ g^2\kappa\kappa' & \frac{1}{2}g^2(\kappa^2 + \kappa'^2 + 2v_R^2 \end{pmatrix} \tag{21}$$

and the mixing angle ζ becomes

$$\tan 2\zeta = \frac{2\kappa\kappa'}{v_R^2 - v_L^2} \approx \frac{2\kappa\kappa'}{v_R^2}. \tag{22}$$

If we assumed type I seesaw scenario, the energy scale of the PS phase is very large. This is because

$$m_\nu \approx m_D^T M_R^{-1} m_D \approx 0.1\text{eV} \tag{23}$$

m_D is naively of order 100 GeV (which is estimated from the mass formula and data-fittings) and M_R is of oprder of 10^{14} GeV. However, if we consider the inverse seesaw mechanism, we can realize much lower M_R than that. Namely we introduce, for instance, SO(10) singlet Fermions S_i and consider 9×9 mass matrix,

$$\begin{pmatrix} \overline{nu_L^c} & \overline{N_R} & \overline{S_L^c} \end{pmatrix} \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & M_R & M \\ 0 & M^T & \mu \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^c \\ S_L \end{pmatrix} \tag{24}$$

Assuming $\mu < m_D < M_R, M$, the active light neutrino mass is given by

$$m_\nu = -(m_D M^{-1})^T \mu (m_D M^{-1}) \tag{25}$$

The other mass eigen values are

$$
\begin{aligned}
M_N &= M_R + M^T M_R^{-1} M, && (26)\\
M_S &= \mu - M^T M_R^{-1} M. && (27)
\end{aligned}
$$

In this case, M and μ are free parameters and even if $m_D \approx 10^2$ GeV, $M, M_R \approx 1$TeV [1] and $\mu \approx 1$eV is compatible with the observations. Then the effect of the right-handed current can be measured in the near future experiments.

[1]There is a constraint $M_R > 4.4$ TeV in Large Hadronic Collider at Cern [7]

3 $(\beta\beta)_{0\nu}$ in the Inverse Seesaw Mechanism

We have shown that the right-handed neutrino masses M_{Ni} (or M_i for short) can be lower in the inverse seesaw mechanism than those in the standard type I seesaw. In this case, the extended PMNS mixing matrices are

$$\nu_\alpha = U_{\alpha i}\nu_i + U_{\alpha Ni}N^c_{Ri} + U_{\alpha Si}S_i. \tag{28}$$

Here the mass eigenstates ν', N', S' are given by

$$\begin{pmatrix} \nu \\ N^c \\ S \end{pmatrix} = U \begin{pmatrix} \nu' \\ N' \\ S' \end{pmatrix} \tag{29}$$

and

$$U = \begin{pmatrix} U_{\nu\nu} & U_{\nu N} & U_{\nu S} \\ U_{N\nu} & U_{NN} & U_{NS} \\ U_{S\nu} & U_{SN} & U_{SS} \end{pmatrix}, \tag{30}$$

where all $U_{\alpha\beta}$ are 3×3 matrices, and $U_{\alpha Ni}$ in (28), for instance, indicates (α, i) component of $U_{\alpha N}$. The mass matrix of (24) is diagonalized as

$$U^T M U = \mathrm{diag}(m_1, m_2, m_3, M_1, M_2, M_3, S_1, S_2, S_3). \tag{31}$$

The half-life time of $(\beta\beta)_{0\nu}$ of $(\beta\beta)_{0\nu}$, then, can be expressed

$$\frac{1}{T_{1/2}} = G_{0\nu}\|\frac{M_\nu}{m_e}\|^2 \left(\|m_{ee}\|^2 + \|p^2\frac{\zeta M_W^2}{M_{WR}^2}\frac{U_{ei}U_{eNi}}{M_{N_i}}\|^2 + \|p^2\frac{M_W^4}{M_{W_R}^4}\frac{U_{eNi}^2}{M_{N_i}}\|^2 \right), \tag{32}$$

Here the averaged neutrino momentum square $p^2 \approx -\left(\frac{1}{r}\right)^2$ (r is the distance between decaying neutrons [8]) and $p^2 \approx -(180\mathrm{MeV})^2$, and $G_{0\nu}$ and M_ν are a phase space factor and the nuclear matrix element, respectively and their explicit values are given in [9]. The contribution of S is implicitly involved in mass and the unitary matrices.

Unfortunately, the ambiguities in nuclear matrix elements is still large and they are different by a factor two or three in different nuclear models [10, 11] and it may be difficult to distinguish these three contributions.

References

[1] M. Doi, T. Kotani and E. Takasugi, Prog. Theor. Phys. Supplement **83** 1 (1985).

[2] B. Pontecorvo, Sov. Phys. JETP, **7**, 172 (1958); Z.Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[3] B. Abi et al. (DUNE Collaboration), arXiv:2006.16043[hep-ex].

[4] T. Fukuyama, K. Ichikawa, and Y. Mimura, Phys. Rev. **D94**, 075018 (2016).

[5] A. Gando et al. (KamLAND-Zen Collaboration), Phys. Rev. Lett. **117**, 082503 (2016).

[6] J.C. Pati and A. Salam, Phys.Rev. **D10**, 275 (1974).

[7] A.M. Sirunyan et al. (CMS Collaboration), JHEP **05**, 148 (2018).

[8] T. Tomoda, Rep. Prog. Phys. **54**, 53 (1991).

[9] F. Simkovic, J. Vergados, and A. Faessler, Phys. Rev. **D82**, 113015 (2010).

[10] A. Faessler, J. Phys.: Conf. Ser. **337**, 012065 (2012)

[11] J. Engel and J. Menendez, Rep. Prog. Phys. **80** 046301 (2017).