

# Magnetic hexadecapole gamma transitions in the Microscopic Quasiparticle- Phonon Model (MQPM)

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# Motivation

- With the help of reduced matrix elements one can study the probability of transitions
- Earlier studies have revealed that the experimental reduced matrix elements of GT(1+) and SD(2-)  $\beta$ -transitions are reduced much in comparison with the theoretical ones
  - Why?
  - How about the transitions with higher multipolarities?



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# Introduction

- L. Jokiniemi, J. Suhonen and H. Ejiri. *Magnetic hexadecapole  $\gamma$  transitions and neutrino-nuclear responses in medium-heavy nuclei*. Adv. High Energy Phys. 2016 (2016) 8417598.
- Highly forbidden ( $\lambda=4$ )  $\beta$ -transitions are rare
  - Not enough data
  - Instead the M4 transitions of the isomeric states were studied
- Transitions examined were stretched M4 transitions in the mass regions of DBD,  $A=85-115$  and  $A=135-143$
- Transitions either  $0g_{9/2} \leftrightarrow 1p_{1/2}$  or  $0h_{11/2} \leftrightarrow 1d_{3/2}$

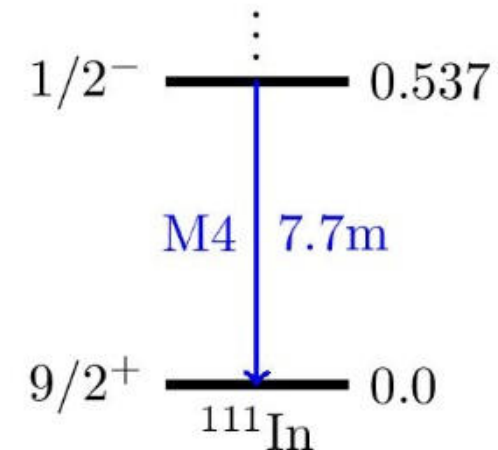


# Introduction

- Example nuclei either proton or neutron odd
- Reduced matrix elements calculated using BCS and MQPM theories
  - Comparison with experimental values

$$|M_{4,exp.}| = \sqrt{\frac{(2J_i + 1)\ln 2}{1.899 \times 10^{-6} E^9 t_{1/2} (1 + \alpha)}}$$

- Effect of gyromagnetic ratios?



# Theories used in the work

- BCS
- QRPA
- MQPM



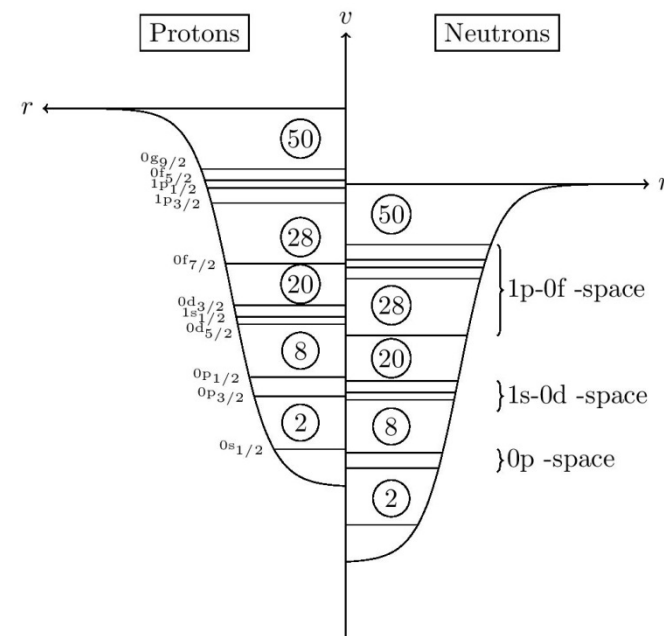
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# Bardeen-Cooper-Schrieffer theory (BCS)

■ Is used to describe the quasiparticle energies of the nuclei

- H.O. single-particle energies with Woods-Saxon potential
- No spin-isospin correlation



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# BCS

- Bogoliubov-Valatin transformation

$$\begin{cases} a_{\alpha}^{\dagger} = u_{\alpha} c_{\alpha}^{\dagger} + v_{\alpha} \tilde{c}_{\alpha} \\ \tilde{a}_{\alpha} = u_{\alpha} \tilde{c}_{\alpha} - v_{\alpha} c_{\alpha}^{\dagger} \end{cases}$$

- Quasiparticle matrix element

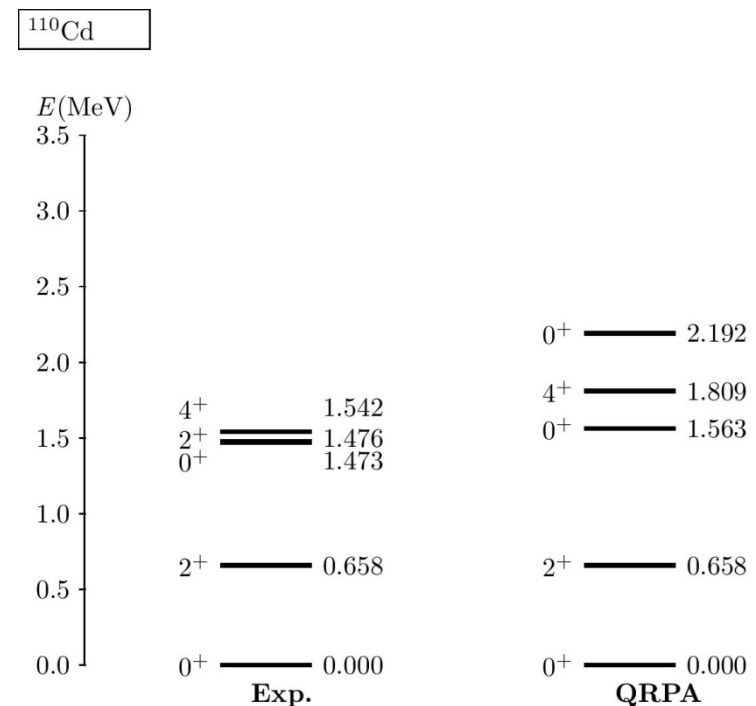
$$M_{QP}(M4) = M_{sp}(M4)P_{ij},$$

where  $P_{ij} = u_i u_j + v_i v_j$ .



# Quasiparticle Random Phase Approximation (QRPA)

- Describes the excited states of spherical, even-even, open-shell nuclei as two-quasiparticle configurations, QRPA phonons
- Takes into account the spin-isospin correlation



**Natural parity phonon:** Phonon with parity  $\pi = (-1)^J$  ( $2^+, 3^-, \dots$ )  
**Unnatural parity phonon:** Phonon with parity  $\pi = (-1)^{J+1}$  ( $2^-, 3^+, \dots$ )



# Microscopic Quasiparticle-Phonon Model (MQPM)

- Couples the QRPA phonons with the BCS quasiparticles
- Solves the three different parts of the microscopic Hamiltonian: The particle, The phonon, and the particle-phonon parts
- Schematically the MQPM matrix element can be written as

$$M_{MQPM} = g_s^{(p)} M_{sp} + g_l^{(p)} M_{lp} + g_s^{(n)} M_{sn} + g_l^{(n)} M_{ln}$$



# Methods

- Woods-Saxon single-particle basis
- BCS quasiparticles
  - Pairing factors of protons and neutrons were adjusted to the experimental pairing gaps
- QRPA phonons
  - Particle-hole factor was adjusted to the experimental energies if possible
  - Particle-particle factor was kept as  $g_{pp} = 1.0$



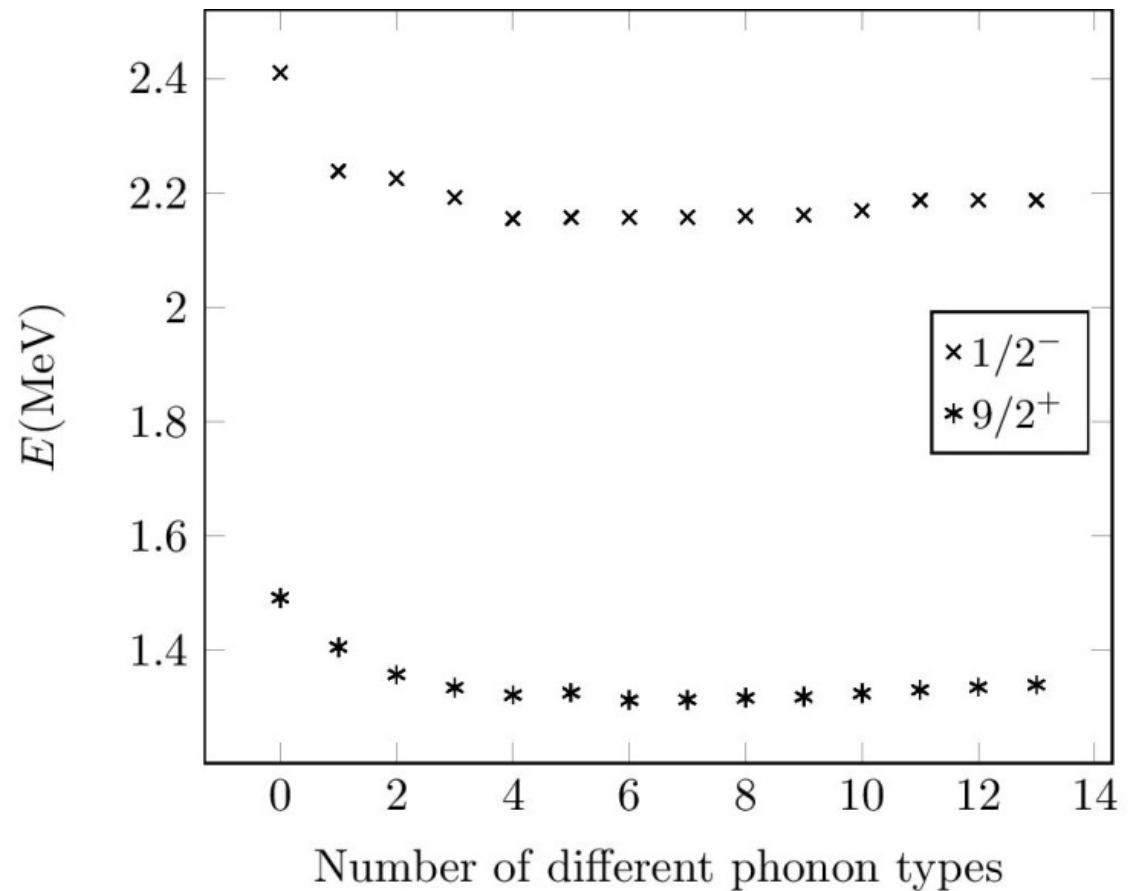
# Methods

- MQPM couples the quasiparticles and phonons
  - Odd-state = even-even –state + p/n quasiparticle  
(For example:  $^{111}_{49}\text{In} = ^{110}_{48}\text{Cd} \otimes p$ )
  - First the natural and then the unnatural parity phonons were added
  - Phonons were added until the convergence was reached



## Convergence of the MQPM energies

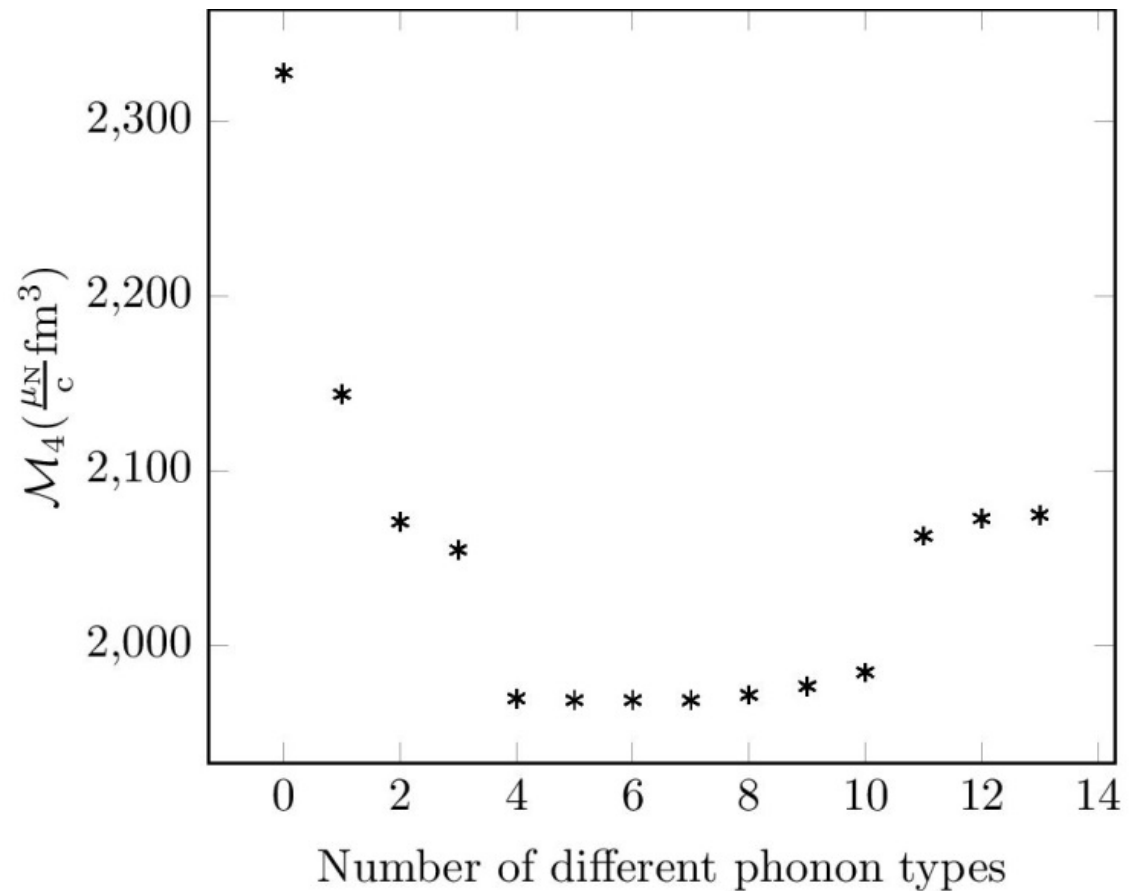
The MQPM energies for  $^{111}\text{In}$  as a function of the added phonon types. The phonons were added in the order 2+, 3-, 4+, 5-, 6+, 7-, 0+, 1+, 2-, 3+, 4-, 5+ and 6-. The cut-off energy was 15 MeV.



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## Convergence of the MQPM NMEs

The MQPM NME for  $^{111}\text{In}$  as a function of the added phonon types. The phonons were added in the order 2+, 3-, 4+, 5-, 6+, 7-, 0+, 1+, 2-, 3+, 4-, 5+ and 6-. The cut-off energy was 15 MeV.



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$^{111}\text{In}$

$E(\text{MeV})$

3.5

3.0

2.5

2.0

1.5

1.0

0.5

0.0

$11/2^+$	1.15285
$5/2^+$	1.10180
$3/2^-$	0.80292
$1/2^-$	0.53699
$9/2^+$	0.000

Exp.

$5/2^+$	3.04271
$5/2^-$	3.01992
$3/2^-$	2.32499
$1/2^-$	0.86239
$9/2^+$	0.000

QP

$13/2^+$	1.011
$5/2^+$	0.889
$7/2^+$	0.508
$1/2^-$	0.269
$9/2^+$	0.000

MQPM

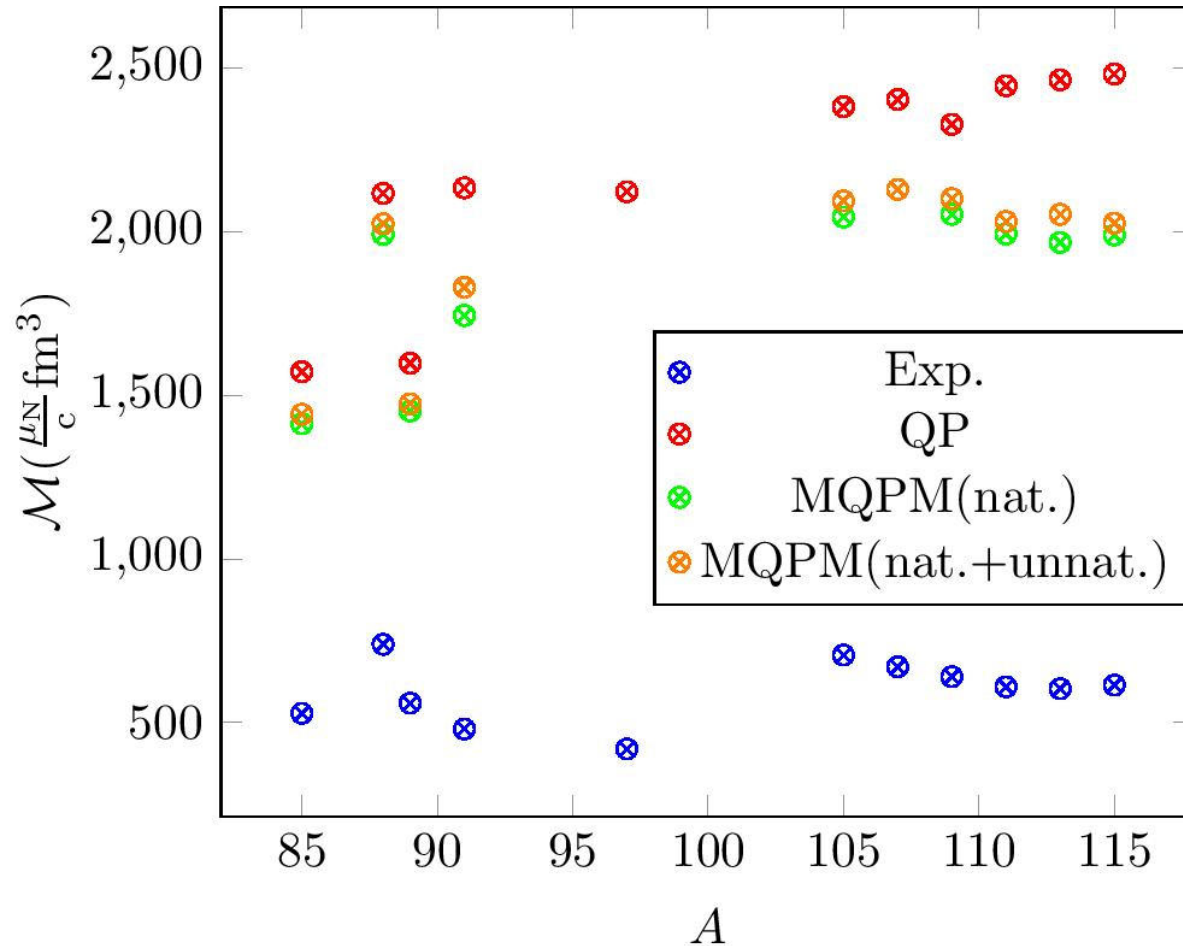
## Spectra of $^{111}\text{In}$

The experimental, quasiparticle, and MQPM spectra of  $^{111}\text{In}$ .



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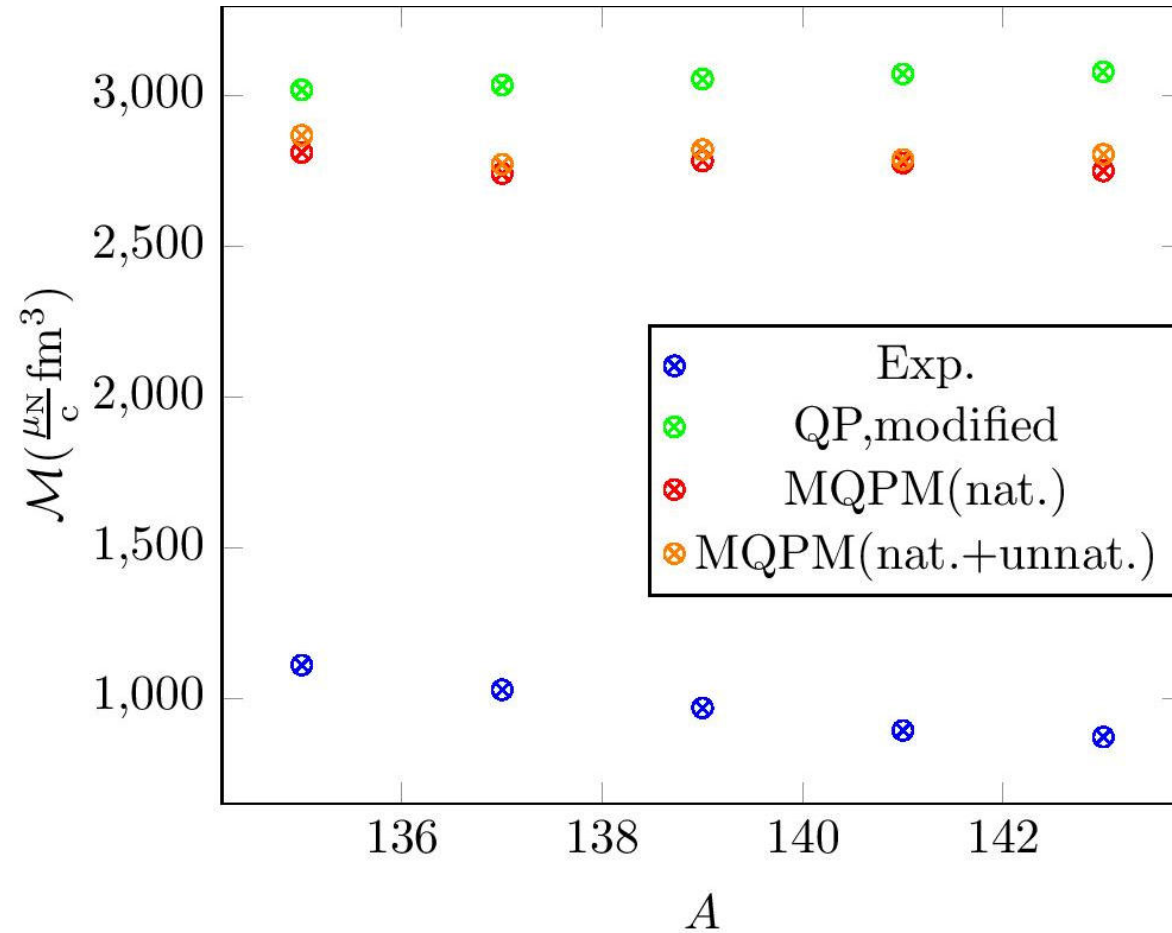
## Results for the region A=85-115

The experimental, quasiparticle and MQPM NMEs of the nuclei in the region A=85-115. The MQPM NMEs with only natural parity and both natural and unnatural parity phonons are presented separately.



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## Results for the region $A=135-143$

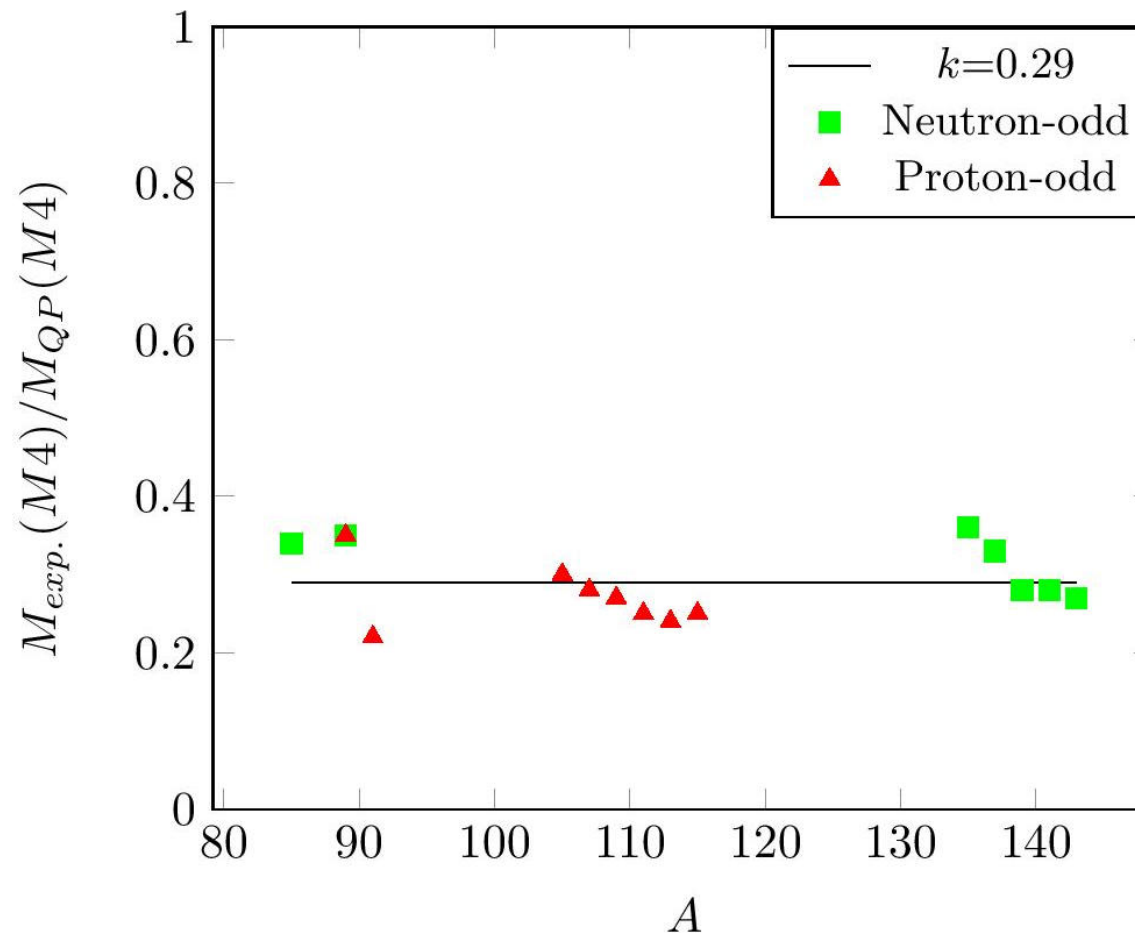
The experimental, quasiparticle and MQPM NMEs of the nuclei in the region  $A=135-143$ . The quasiparticle NMEs are calculated with the modified single-particle bases. The MQPM NMEs with only natural parity and both natural and unnatural parity phonons are presented separately.



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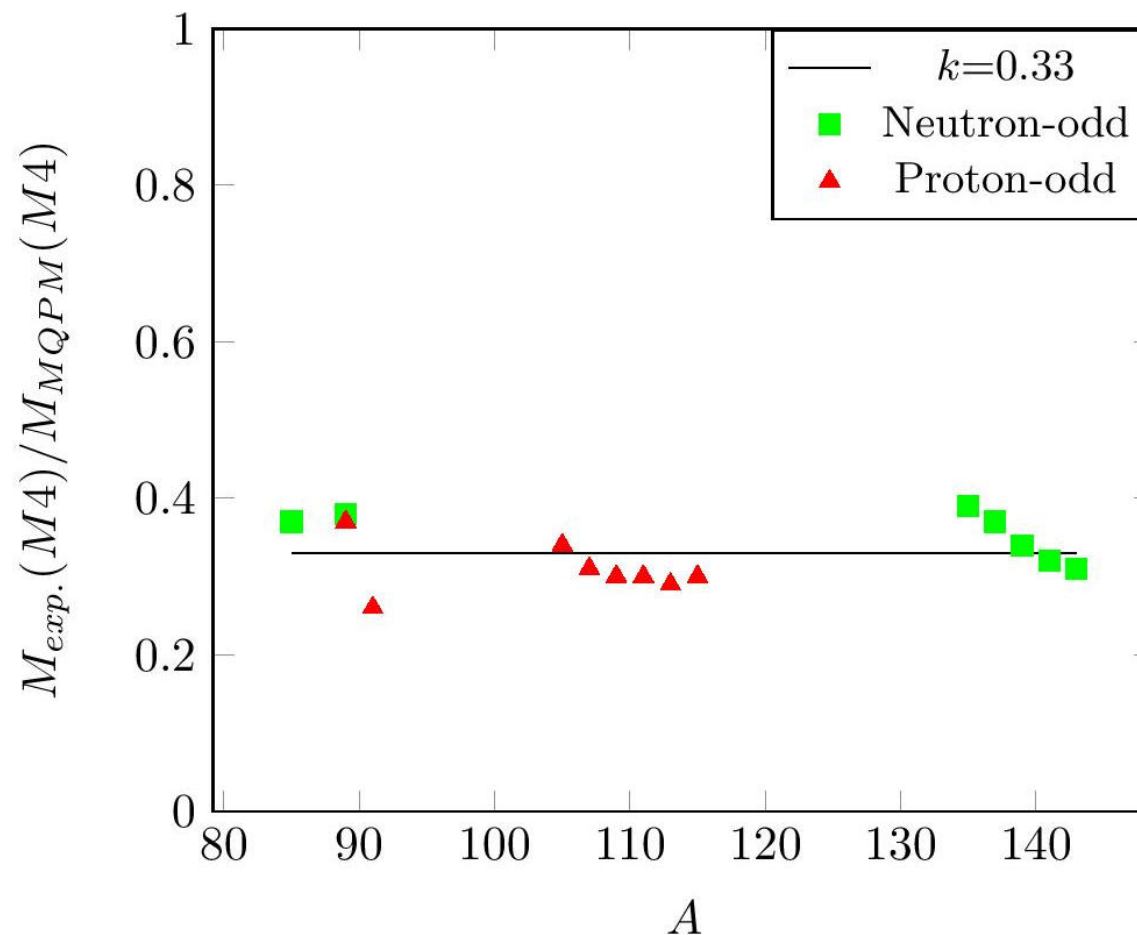
## Reduction coefficient for the QP NMEs

The reduction coefficients defined by  $k = M_{exp.}(M4)/M_{QP}(M4)$  for the nuclei in the regions  $A=85-115$  and  $A=135-143$ . The green squares refer to the neutron-odd and red triangles to the proton-odd nuclei. The average reduction coefficient  $k = 0.29$  is presented as a solid line.



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## Reduction coefficient for the MQPM NMEs



The reduction coefficients defined by  $k = M_{exp.}(M4)/M_{MQPM}(M4)$  for the nuclei in the regions  $A=85-115$  and  $A=135-143$ . The green squares refer to the neutron-odd and red triangles to the proton-odd nuclei. The average reduction coefficient  $k = 0.33$  is presented as a solid line.



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# Results


$$\begin{cases} M_{exp.}(M4)/M_{QP}(M4) = 0.29 \\ M_{exp.}(M4)/M_{MQPM}(M4) = 0.33 \end{cases}$$

 In earlier studies

$$- \begin{cases} M_{exp.}(SD2)/M_{QP}(SD2) = 0.2 \\ M_{exp.}(SD2)/M_{pnQRPA}(SD2) = 0.5 \end{cases}_1$$

$$- \begin{cases} M_{exp.}(GT1)/M_{QP}(GT1) = 0.23 \\ M_{exp.}(GT1)/M_{pnQRPA}(GT1) = 0.6 \end{cases}_2$$

 How about the effect of the gyromagnetic ratios?

<sup>1</sup> H. Ejiri, N. Soukouti and J. Suhonen. *Phys.Lett.B.* 729:27-32, 2014

<sup>2</sup> H. Ejiri and J. Suhonen. *J. Phys. G, Nucl. Part. Physics.* 42:055201, 2015

# Effect of gyromagnetic ratios

$$M_{MQPM} = g_s^{(p)} M_{sp} + g_l^{(p)} M_{lp} + g_s^{(n)} M_{sn} + g_l^{(n)} M_{ln}$$

	$M_{MQPM}(M4)(^{89}\text{Zr})$	$M_{MQPM}(M4)(^{109}\text{In})$
Bare $g$ factors	1474	2101
$g_l^{(p)} = 1.5, g_l^{(n)} = -0.5$	1388	2007
$g_s^{(p)} = 4.586$	1473	1690
$g_s^{(p)} = 6.586$	1475	2512
$g_s^{(n)} = -4.826$	1857	2098
$g_s^{(n)} = -2.826$	1091	2104
Experimental values	559	640

Bare  $g$  factors:

$$g_l^{(p)} = 1, \quad g_s^{(p)} = 5.586,$$

$$g_l^{(n)} = 0, \quad g_s^{(n)} = -3.826$$



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# Discussion

- Experimental NMEs reduced a lot in comparison with the theoretical ones
- Quasiparticle NMEs reduction coefficient of the same order that in the earlier studies
- MQPM NMEs not as close to the experimental ones as pnQRPA NMEs in the earlier studies



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# Discussion

- Reduction of the experimental NMEs partly due to the spin-isospin correlations but also other nuclear-medium effects
- Changing the spin  $g$  factor affected the NMEs a little
  - Effective spin  $g$  factor?



# What's next?

- Analysis of the strength and energy of isovector spin-dipole and spin-quadrupole resonances of DBD nuclei
  - Study of the key multipoles  
 $J^\pi = 0^-, 1^+, 1^-, 2^+, 2^-$  and  $3^+$  in the intermediate odd-odd nuclei for  $0\nu\beta\beta$ -decay triplets
  - In the earlier work <sup>3</sup>, the states were studied for closed-shell nuclei by the pnRPA model
  - In the ongoing work, the states are studied for open-shell nuclei by the pnQRPA model

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<sup>3</sup> N. Auerbach and A. Klein, Structure of isovector spin excitations in nuclei. Phys. Rev. C 30 (1984) 1032.



Thank you!



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