Study of dark matter from lattice gauge theory

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Seminar @ RCNP, Osaka Univ.

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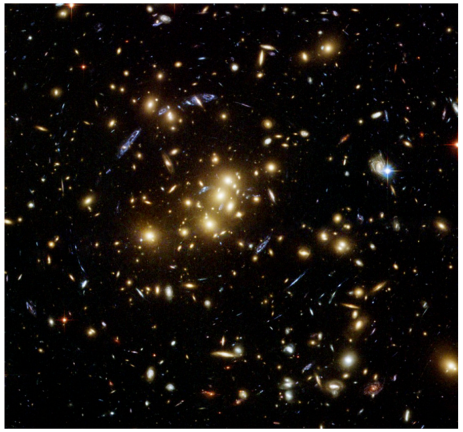
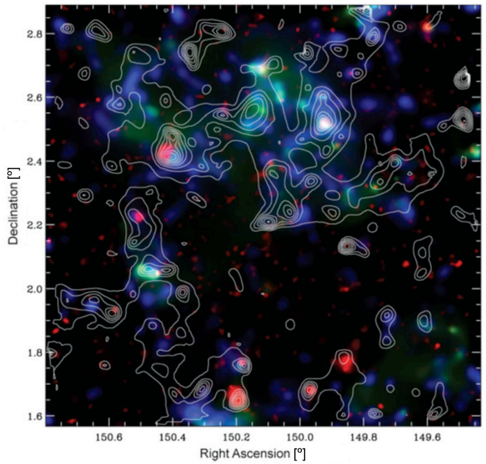
Dark matter

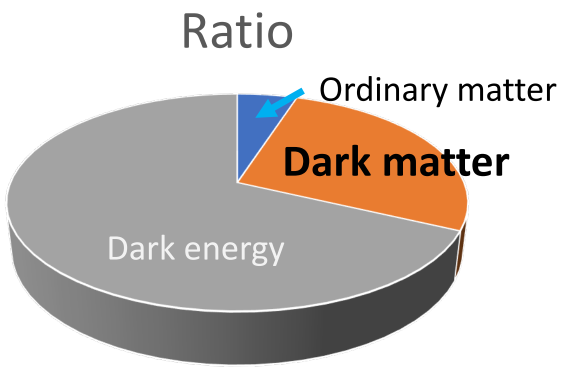
- Dark matter: there are so many evidence of existence
  - Velocity of rotation of galaxy:
    Distribution of rotation speed of galaxy … need DM
  - Gravitational lensing：DM bends light
  - Simulation：the galaxy cannot be made without DM



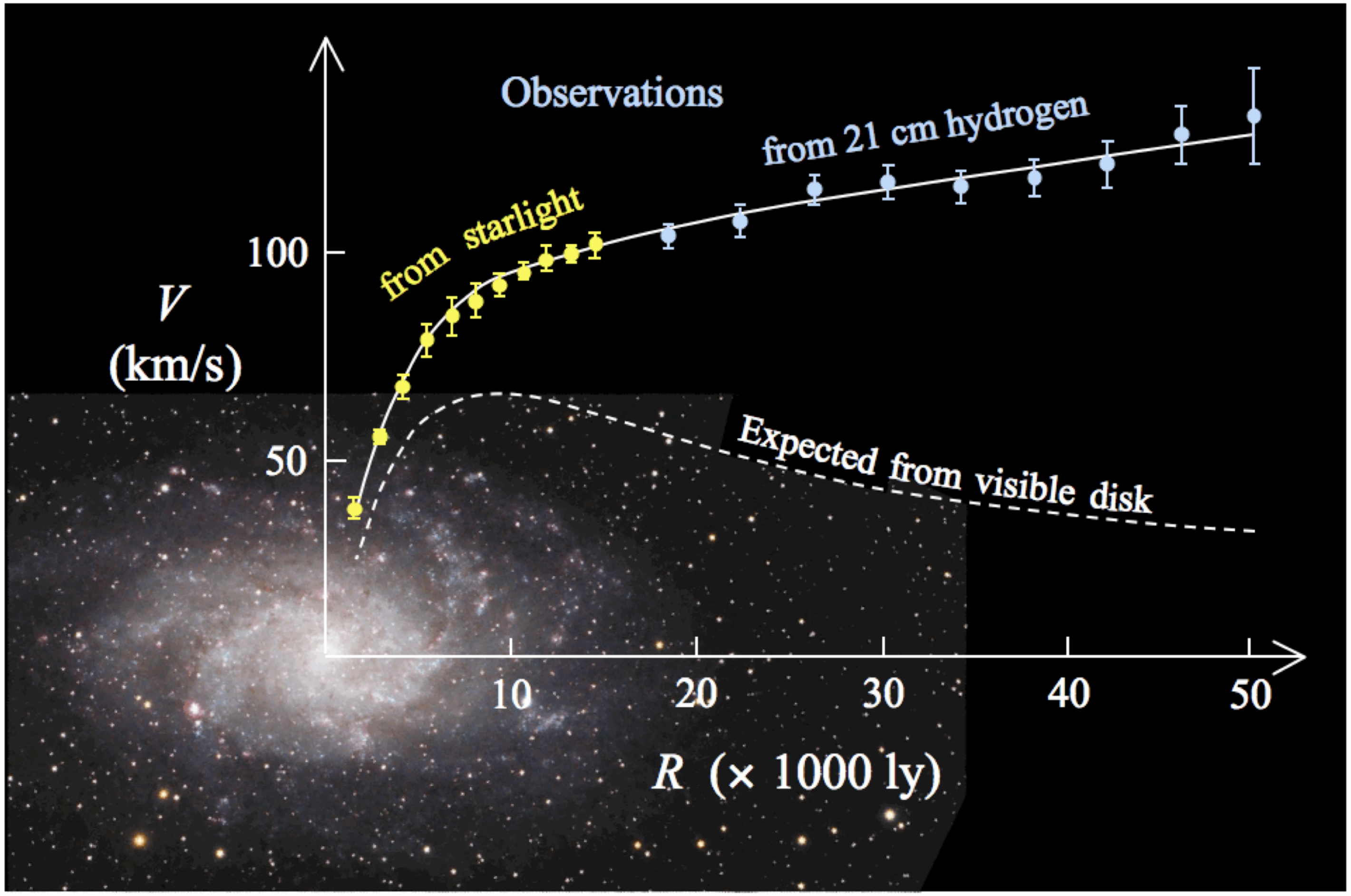
Glavitational lensing



DM distribution from GL



Ref: R.Massey, T.Kitching, J.Richard,
Rep.Prog.Phys.73(2010)086901



Velocity of rotation of galaxy

Figure from wikipedia.
Data taken from
Corbelli, E.; Salucci, P. (2000).
"The extended rotation curve and the dark matter halo of M33".
Monthly Notices of the Royal Astronomical Society.
311 (2): 441–447.

https://www.youtube.com/watch?v=rTHhMSE3DxA

Simulation：the galaxy cannot be made without DM

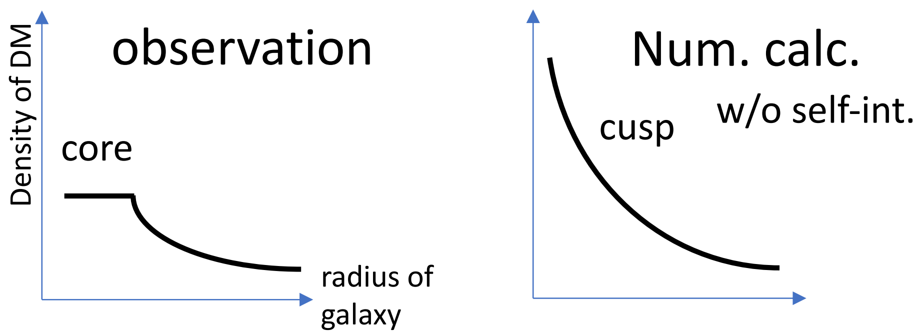
Formation of structure of dark matter

… N-body simulation on K computer @ RIKEN

https://www.youtube.com/watch?v=yk3VymoLl2o&index=3&list=FLoqzEd89WCLuaz-dylyaTGw

What is the identity of DM？

- DM is considered to be particles
  (Almost) No interaction with ordinary particles
- But there should be self-interaction! (Core-Cusp problem)



- Restriction of scat. cross section：
  cf: D.N.Spergel & P.J.Steinghardt, PRL84 (1999)

$$0.45\mathrm{cm}^2/\mathrm{g} < \sigma/m < 450\mathrm{cm}^2/\mathrm{g}$$

(σ: scat. cross section, m: mass of DM)

Important information of DM

What is DM？

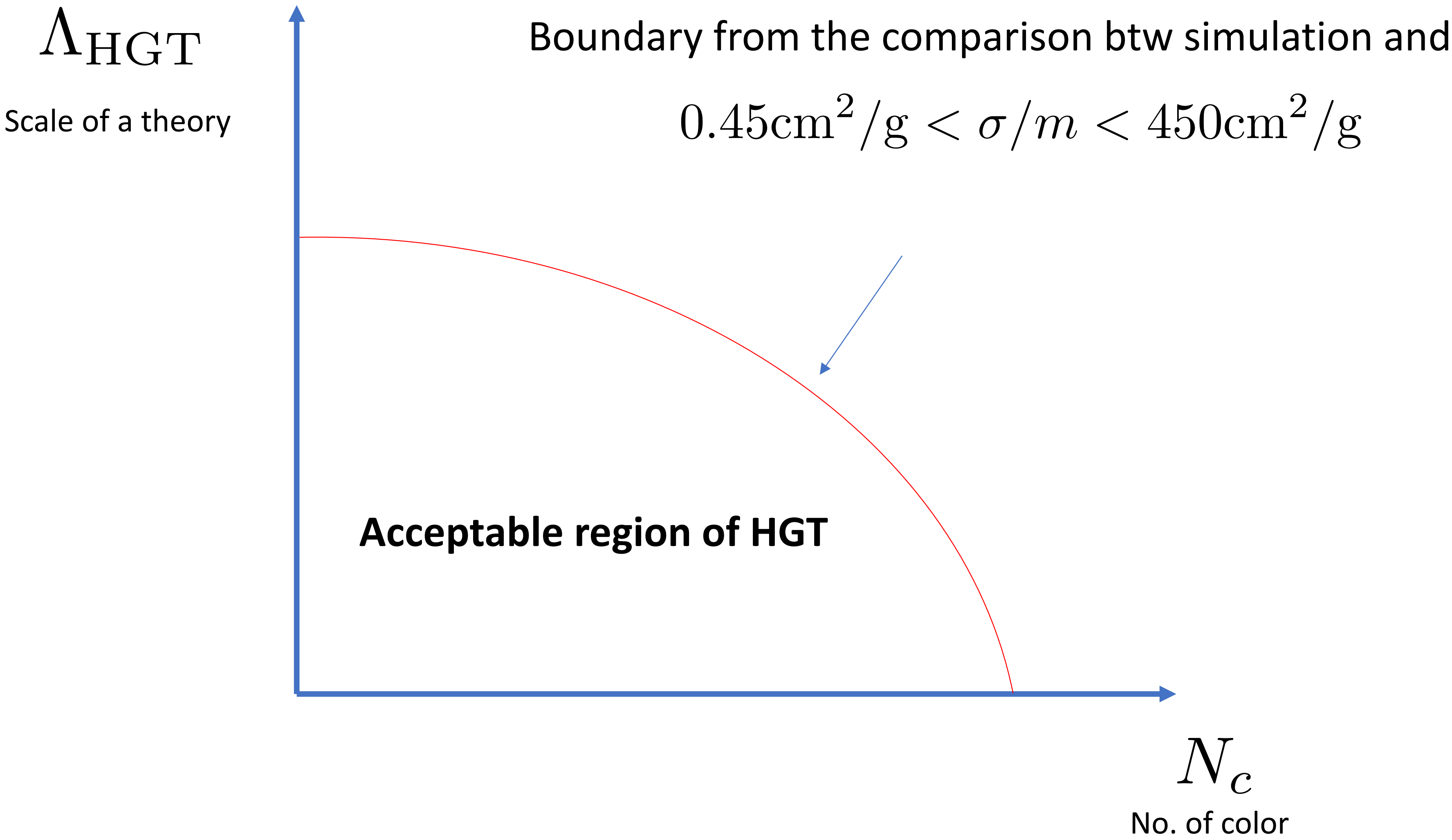
- **Candidate of DM … hadrons in Hidden Gauge Theory (HGT)**
  HGT: Gauge theory with particles which does not interact with ordinary particles
  → Lightest glueballs in HGT are the candidate of DM

- **Aim of the study:**
  Search for HGT which is consistent with the limitation of scat. cross sec., assuming that the lightest glueball in HGT is DM.

  Investigate the interaction btw glueballs, comparing σ/m with the limitation, and find the possible HGT as theories of DM (finding the allowed N of SU(N) and scale parameter Λ_{HGT})

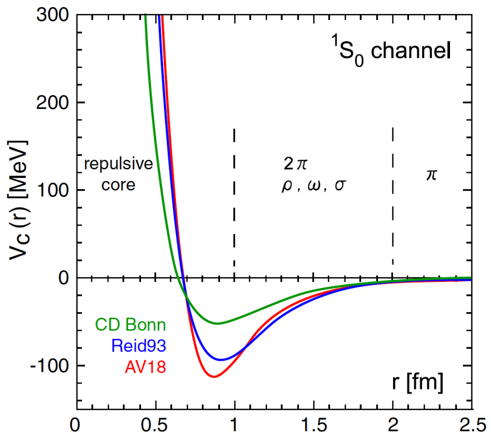
We first investigate the inter-glueball potential on lattice (SU(3),SU(2))

Making restriction of HGT (not yet done…)

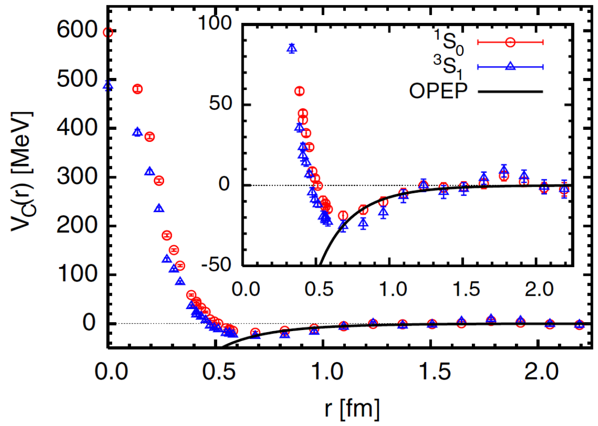


Measurement of inter-GB int. in lattice QCD

- HAL QCD method



Nuclear force based on experiment



Nucl. force from lattice QCD

Succeed to extract nucl. force from lat. QCD
Ishii, Hatsuda, Aoki（2007）

Current status:

- NN int. at the realistic quark mass in (2+1) flavor
- Interaction of strange baryon and charmed hadron

Extract glueball potential using HAL QCD method

HAL QCD Collaboration:

Yoichi Ikeda[1,2], Sinya Aoki[3,4], Takumi Doi[2], Shinya Gongyo[3], Tetsuo Hatsuda[2,5], Takashi Inoue[6], Takumi Iritani[7], Noriyoshi Ishii[1], Keiko Murano[1], Kenji Sasaki[3,4]
(HAL QCD Collaboration)

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[5] *iTHES Research Group, RIKEN, Saitama 351-0198, Japan*
[6] *Nihon University, College of Bioresource Sciences, Kanagawa 252-0880, Japan*
[7] *Department of Physics and Astronomy, Stony Brook University, New York 11794-3800, USA*

+ H.Nemura (RCNP), M.Yamada, F.Etminan (Tsukuba), B.Carron (Univ. of Tokyo), D.Kawai, T.Miyamoto (YITP),

Studies of **Strangeness S=-2 baryon-baryon int.**
Z_c(3900)
Λc-N interaction
Ω-Ω interaction
…
other than NN potential

HAL QCD method (N. Ishii, S. Aoki, and T. Hatsuda Phys. Rev. Lett. **99**, 022001)

Nambu-Bethe-Salpeter amplitude ("wave function") for multi-body particle system is calculated from lattice gauge theory, and obtain inter-particle potential V by inversely solving "Schroedinger equation".

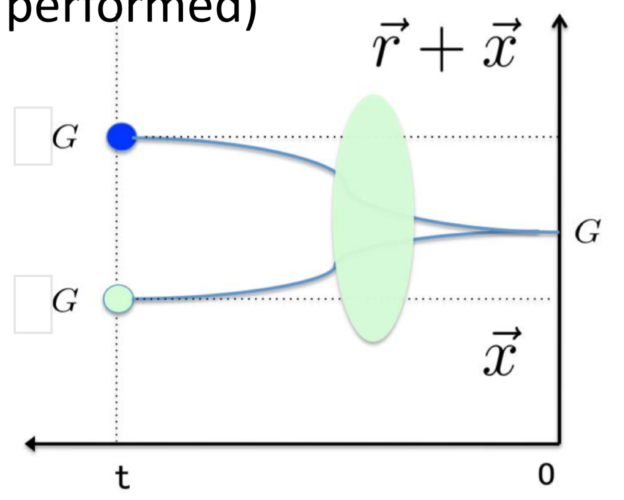
$$\psi(\vec{r}) \longrightarrow V(\vec{r})$$

This method is utilized for the extraction of inter-glueball pot.

Formalism

Equation for Nambu-Bethe-Salpeter amplitude for two-body glueballs:
(later mention the definition of ψ . A1 projection of cubic group is performed)

$$(E - H_0)\Psi_{\rm BS}^{S\text{-wave}}(\vec{r}) = \int d\vec{r} U(\vec{r}, \vec{r}')\Psi_{\rm BS}^{S\text{-wave}}(\vec{r}')$$



Velocity expansion and taking the leading term：

$$V_c^{\rm eff}(\vec{r}) - E = \frac{1}{m_{\rm GB}} \frac{\nabla^2 \Psi_{\rm BS}^{S\text{-wave}}(\vec{r})}{\Psi_{\rm BS}^{S\text{-wave}}(\vec{r})}$$

Time-independent HAL QCD method

Formalism

Ref) T.Miyamoto for HAL QCD Collaboration,
PoS Lattice2016 (arXiv:1602.0779)

- Potential→Scat. Cross section

  - Potential is fitted by some analytic function
    (superposition of gaussian for example)
  - Solve Schroedinger eq. … wave func. is obtained
  - Phase shift is determined by the asymptotic form of the wave func.
  - At low energies, only S-wave contributes to scat. cross sec.:

$$\lim_{k\to\infty}\sigma_{\mathrm{tot}} = \lim_{k\to\infty}\frac{4\pi}{k^2}\sin^2\delta_0(k)$$

Formalism

- Definition of BS amplitude

$$\Psi(\vec{r},T) \equiv \frac{1}{V^2 N_t} \left\langle \sum_t \left(\left\{ \sum_{\vec{x}} \left(\tilde{\phi}(\vec{x}+\vec{r},t+T)\tilde{\phi}(\vec{x},t+T) - \langle \phi(\vec{x}+\vec{r},t+T)\tilde{\phi}(\vec{x},t+T)\rangle \right) \right\} \sum_{\vec{y}} \tilde{\phi}(\vec{y},t) \right) \right\rangle$$

$$\tilde{\phi}(\vec{x},t) \equiv \phi(\vec{x},t) - \langle \phi(\vec{x},t) \rangle$$

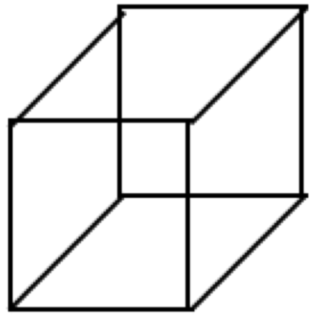
ϕ: glueball operator

Note:

1. <ϕϕ> is subtracted from ϕϕ to ensure that ψ becomes zero for large r
2. Initial operator is ϕ, not ϕϕ.
   This yields cleaner signal than the data from BS with initial operator ϕϕ,

Formalism

- Glueball operator



Operator which is invariant wrt rotation of Cubic group

- We adopt the following operator:

$$\phi_0^{0++}(t, x+a/2, y+a/2, z+a/2) = \frac{1}{2}\mathrm{Re}\,\mathrm{Tr}[P_{xy}(t,x,y,z) + P_{yz}(t,x,y,z) + P_{zx}(t,x,y,z)$$
$$+P_{xy}(t,x,y,z+a) + P_{yz}(t,x+a,y,z) + P_{zx}(t,x,y+a,z)]$$

Lattice setup

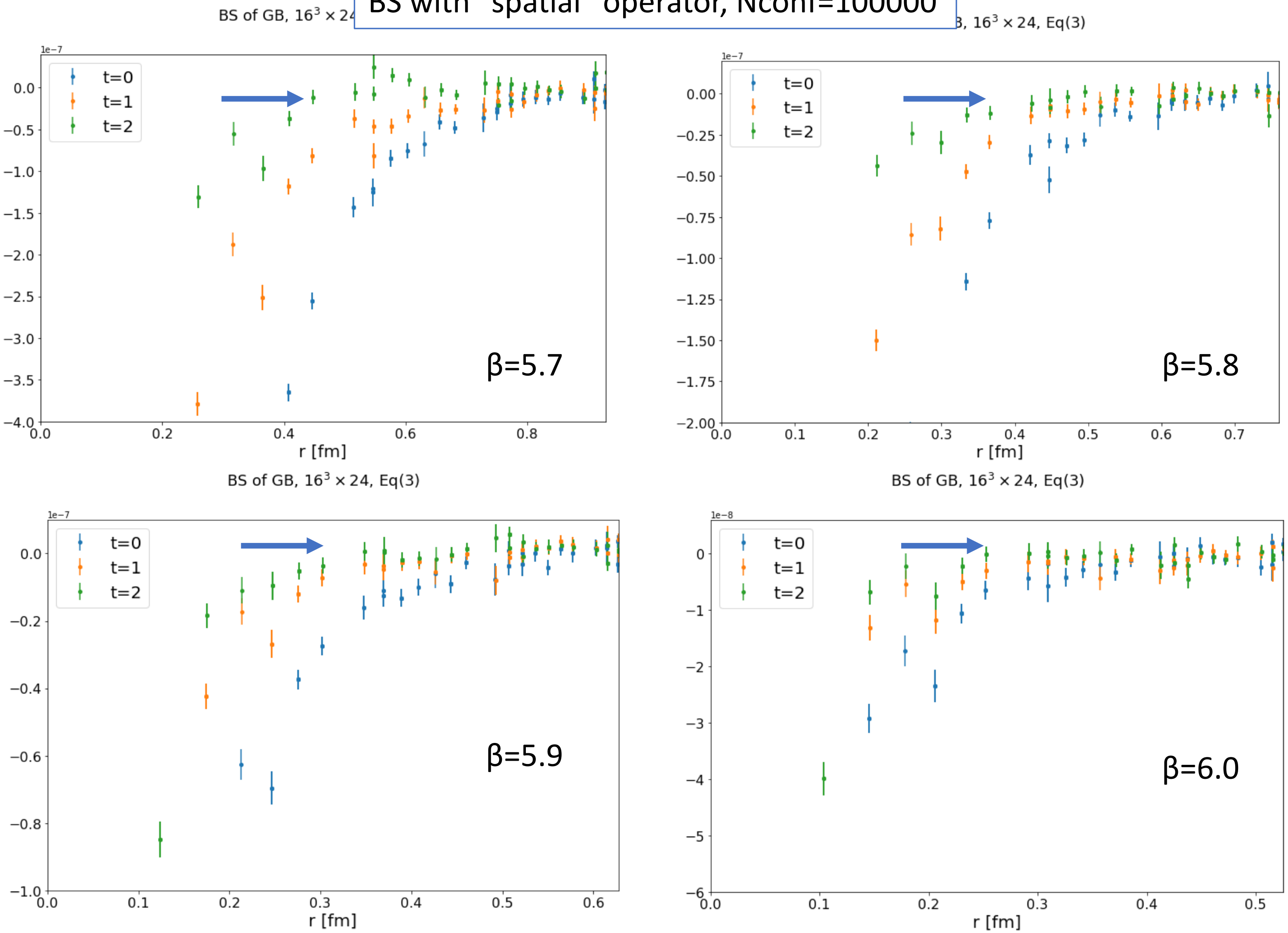
- Pure Yang-Mills, SU(3), standard gauge action
- APE smearing is performed for the source operator, not for sink operator.

Parameters:

β	a (fm)	a^{-1} (GeV)	volume	$N_{\rm therm}$	$N_{\rm sep}$	$N_{\rm smr}$ for src
5.70	0.182	1.082	$16^3 \times 24$	10000	100	8
5.80	0.149	1.322	$16^3 \times 24$	10000	100	13
5.90	0.123	1.602	$16^3 \times 24$	10000	100	18
6.00	0.103	1.913	$16^3 \times 24$	10000	100	26

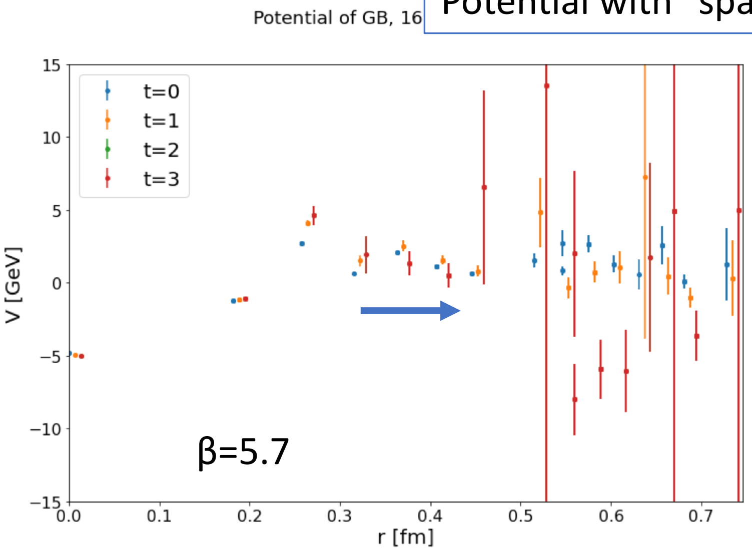
Numerical results

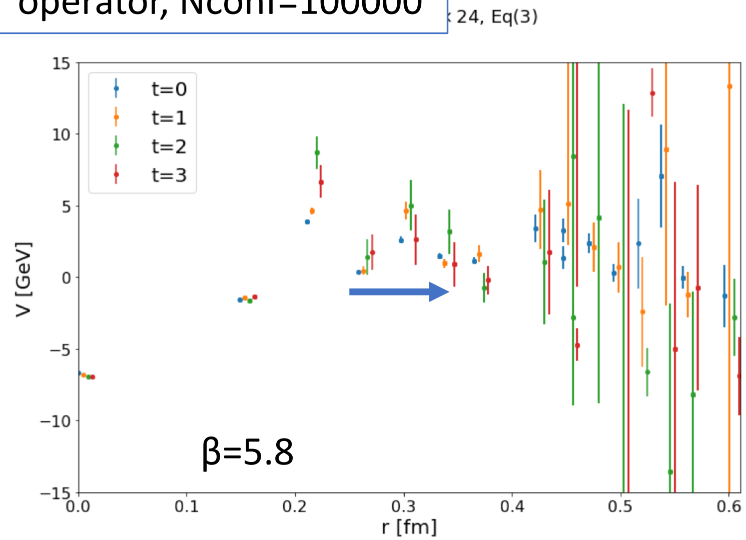
BS with “spatial” operator, Nconf=100000

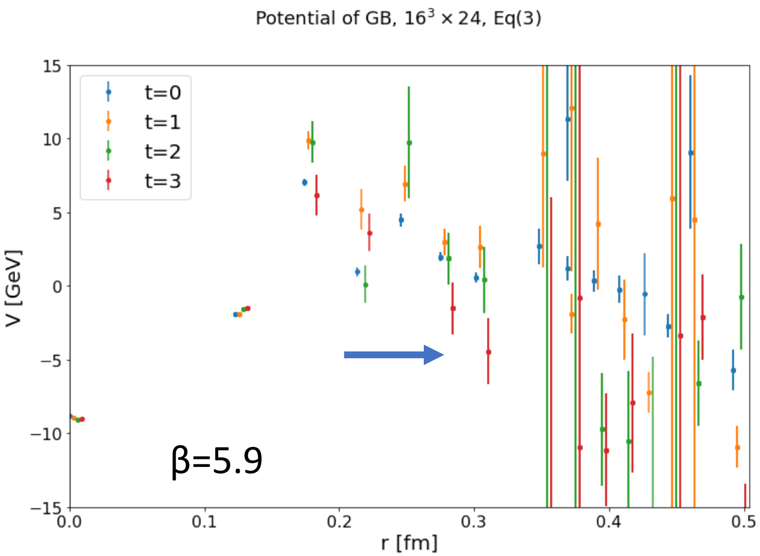


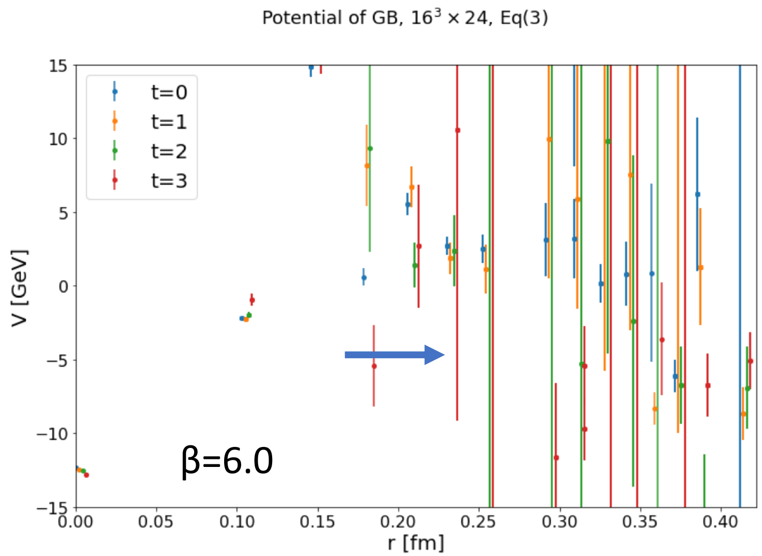
… the region where the overlap of the operators is absent.

Potential with “spatial” operator, Nconf=100000



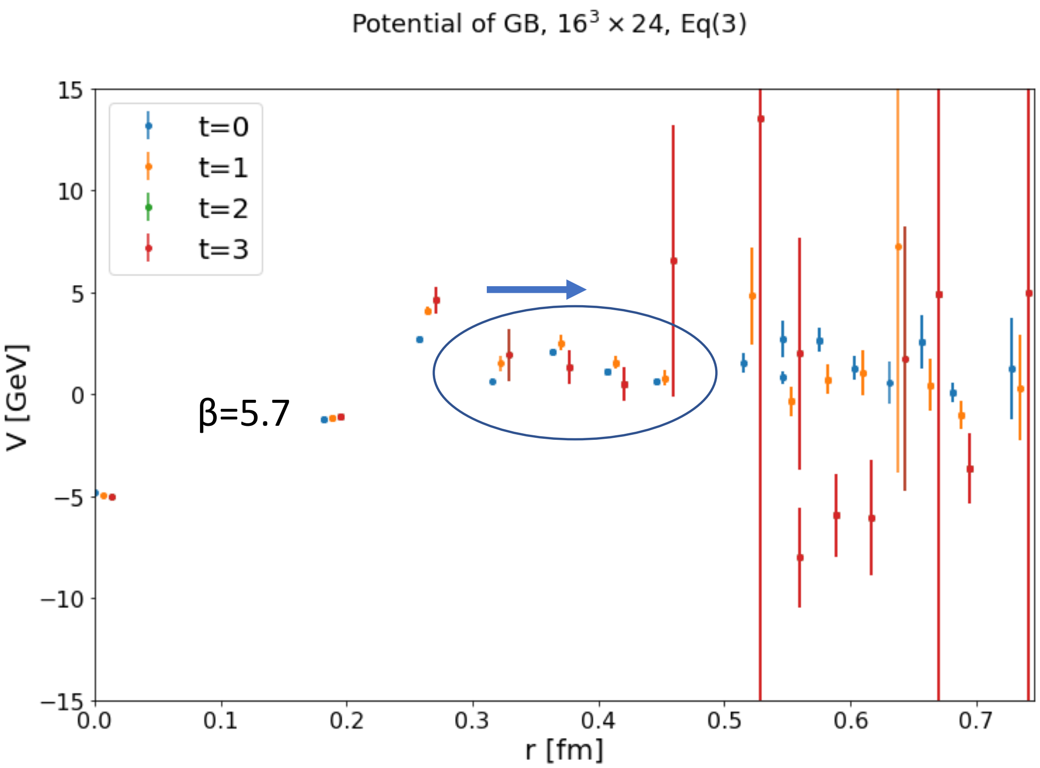


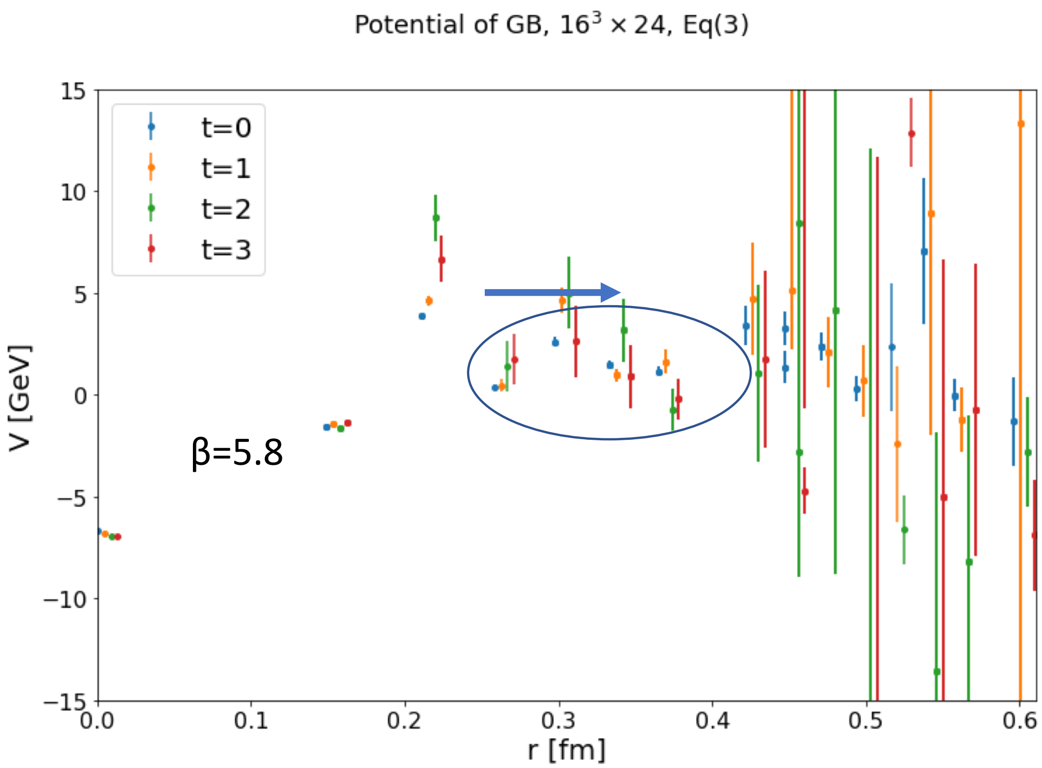




Repulsive behavior around 0.3fm??

Potential with “spatial” operator, Nconf=100000





Repulsive behavior around 0.3fm??

Time-dependent HAL method

Time-dependent HAL

- Difficulty of time-independent HAL
  We have to extract an energy eigen state
  ... Difficult for two-body state of hadrons
  $E_{1st\ excited\ state}$-$E_{ground\ state}$ $\sim$ several hundred MeV

  → Ground state saturation is hardly achieved for two-body system
- **Time-dependent HAL method**
  Potential can be extracted from time-dependent Schroedinger eq.
  → Ground state saturation is NOT needed!

Time-dependent HAL (for nucleon)

"Extended" time-dependent Schroedinger eq.:

$$\left\{ \frac{1}{4m_N} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right\} R(t,\vec{r}) = \int d^3r' U(\vec{r},\vec{r}') R(t,\vec{r}')$$

$$\left(\begin{array}{l} R(t,\vec{r}) \equiv C_{NN}(t,\vec{r})/(e^{-m_N t})^2 \\ C_{NN}(\vec{x}-\vec{y};t) \equiv \frac{1}{V} \sum_{\vec{r}} \langle 0|T[N(\vec{x}+\vec{r},t)N(\vec{y}+\vec{r},t) \cdot \bar{\mathcal{J}}(0)]|0\rangle \end{array} \right)$$

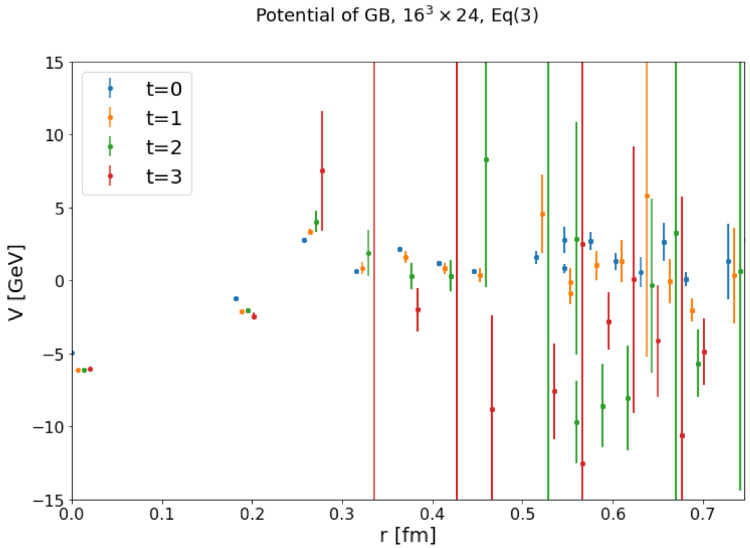
Velocity exp. of U and taking the leading term:

$$U(\vec{r},\vec{r}') = V(\vec{r},\vec{\nabla}_r)\delta^3(\vec{r}-\vec{r}') = \{V_C(r) + O(\nabla^2)\}\delta^3(\vec{r}-\vec{r}')$$

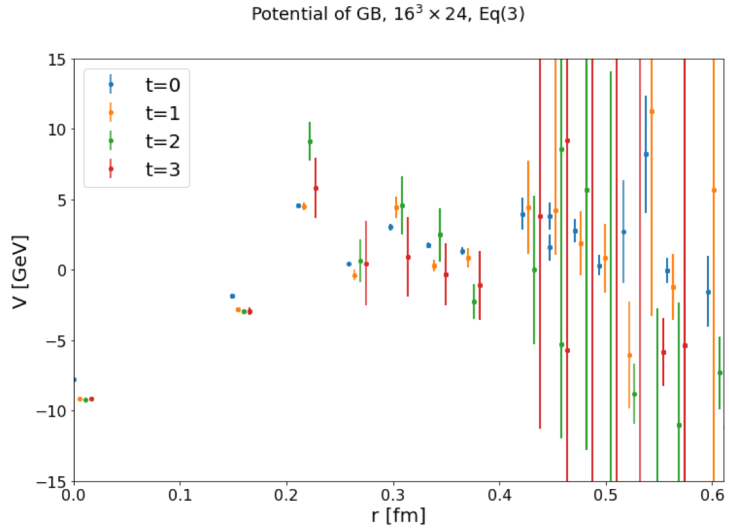
$$\boxed{V_C(r) = -\frac{H_0 R(t,\vec{r})}{R(t,\vec{r})} - \frac{(\partial/\partial t)R(t,\vec{r})}{R(t,\vec{r})} + \frac{1}{4m_N}\frac{(\partial/\partial t)^2 R(t,\vec{r})}{R(t,\vec{r})}}$$

Numerical results: potential from time-dependent HAL

β=5.7

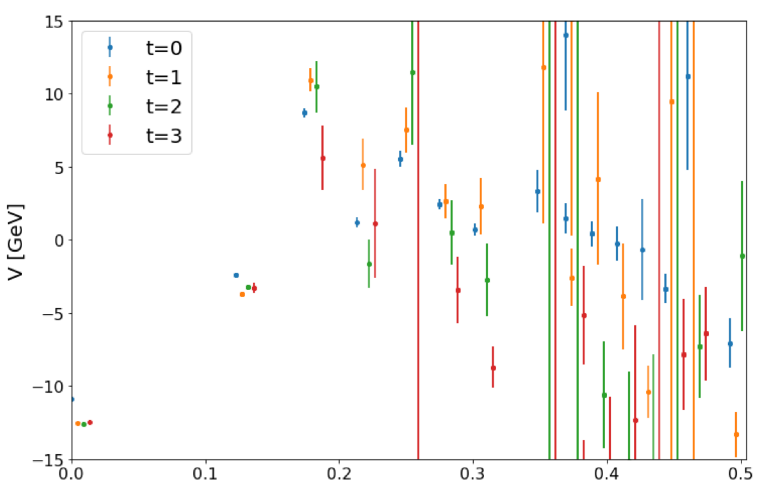


β=5.8



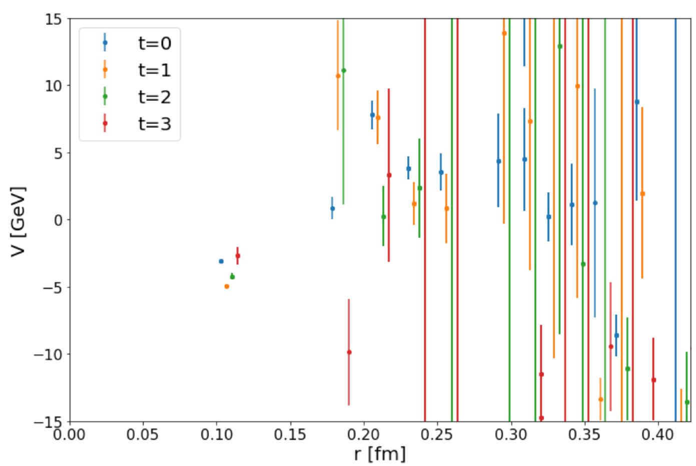
β=5.9

Potential of GB, $16^3 \times 24$, Eq(3)

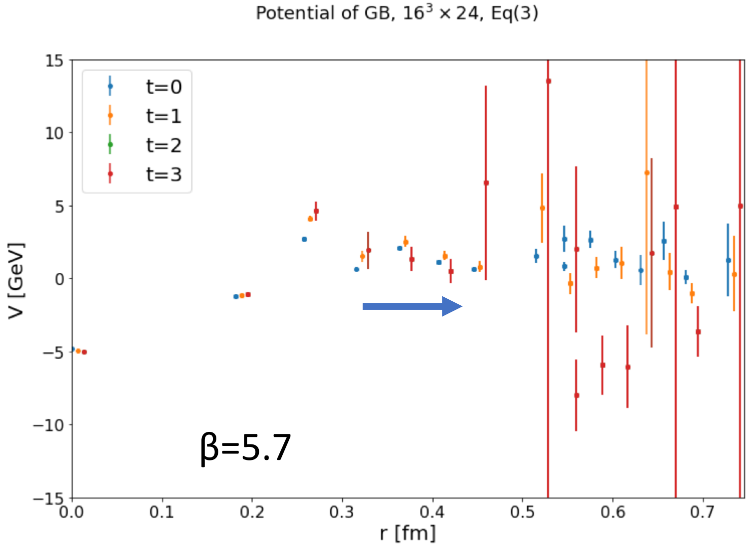


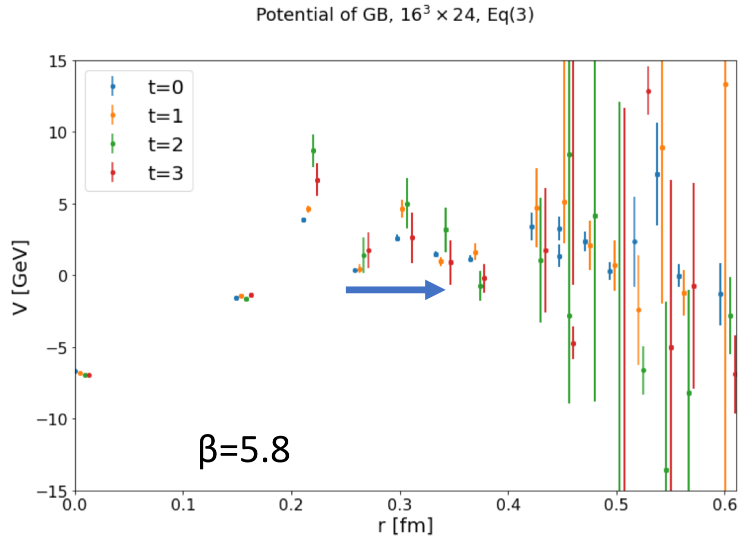
β=6.0

Potential of GB, $16^3 \times 24$, Eq(3)

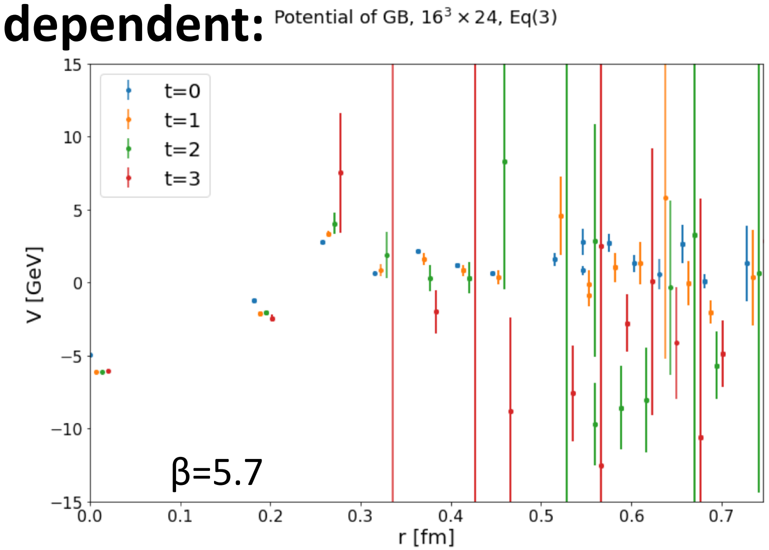


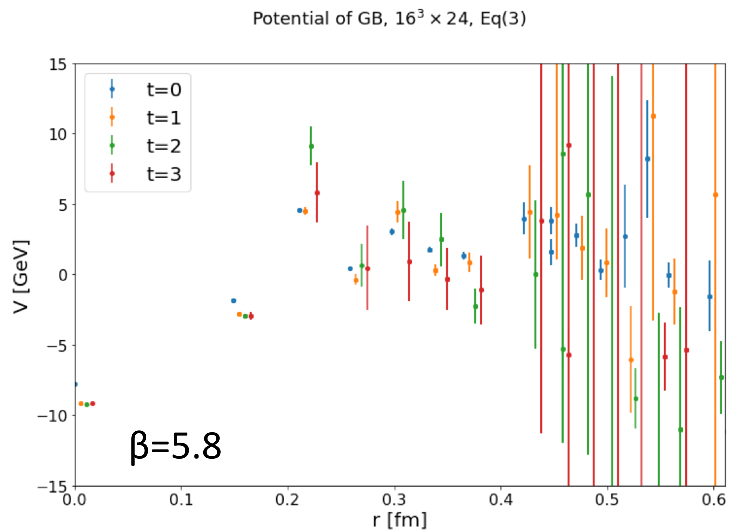
Time-dependent:



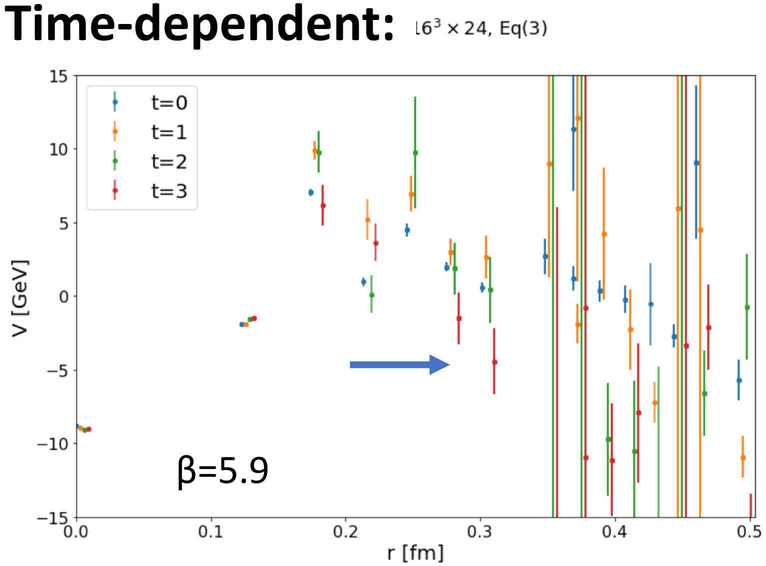


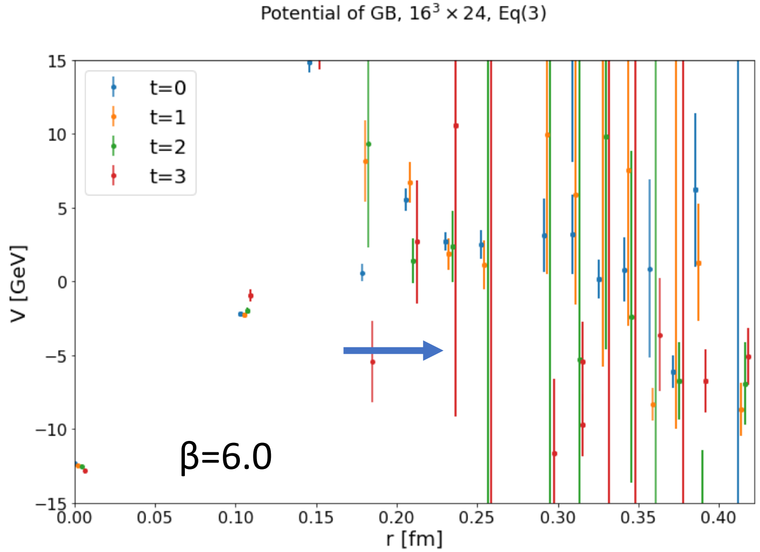
Time-independent:



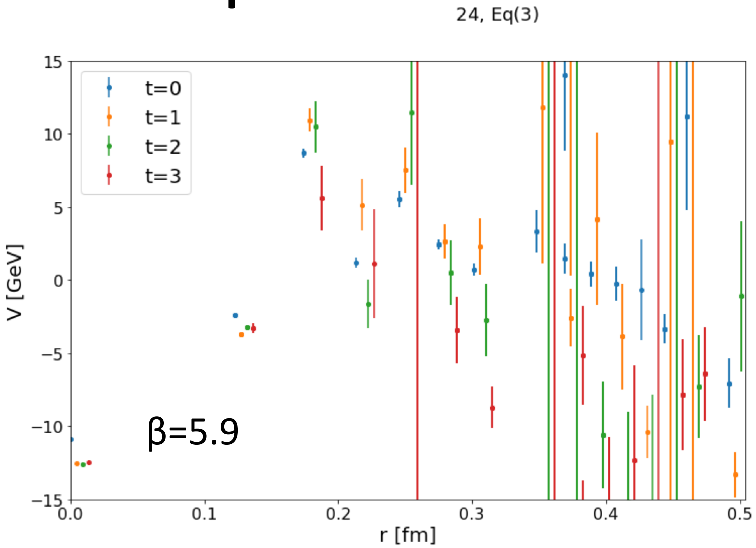


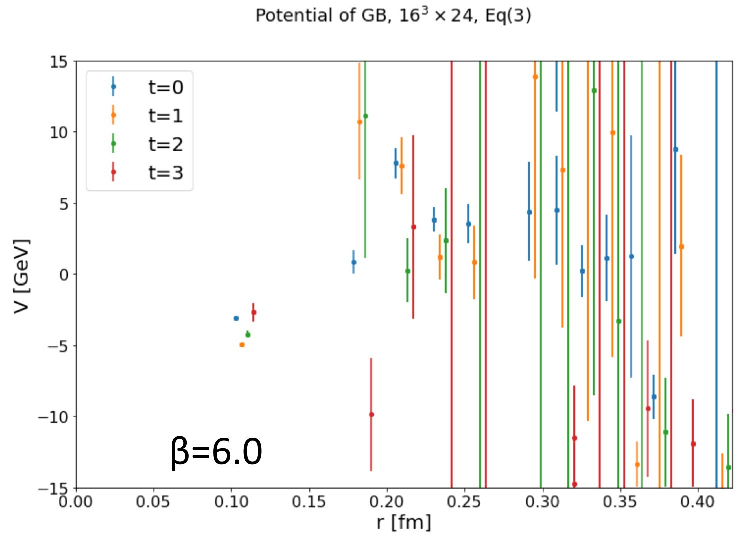
Time-dependent:





Time-independent:



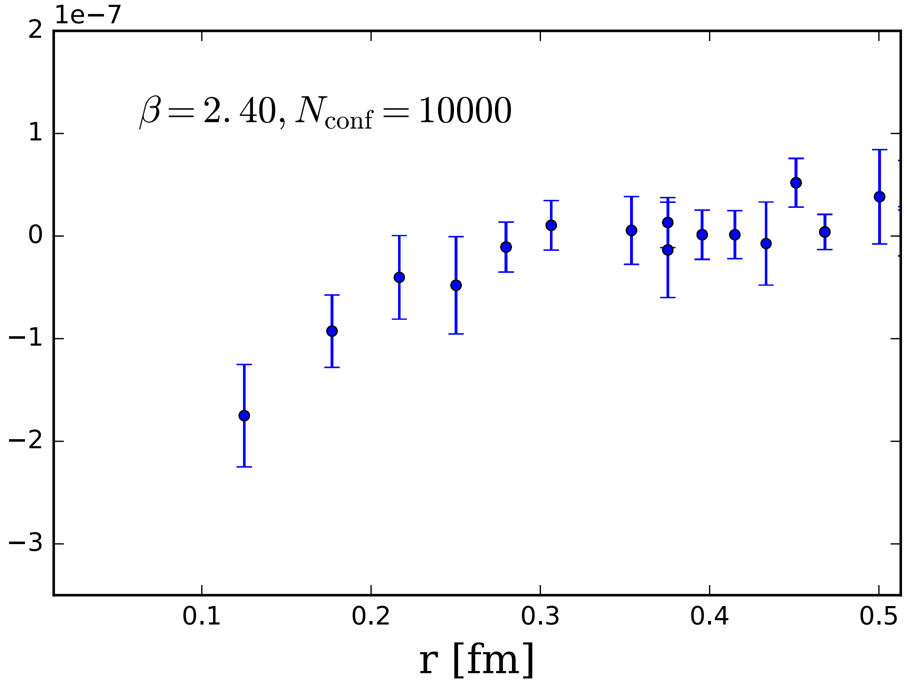


Not so big difference?
…ground-state saturation
is not so problematic?

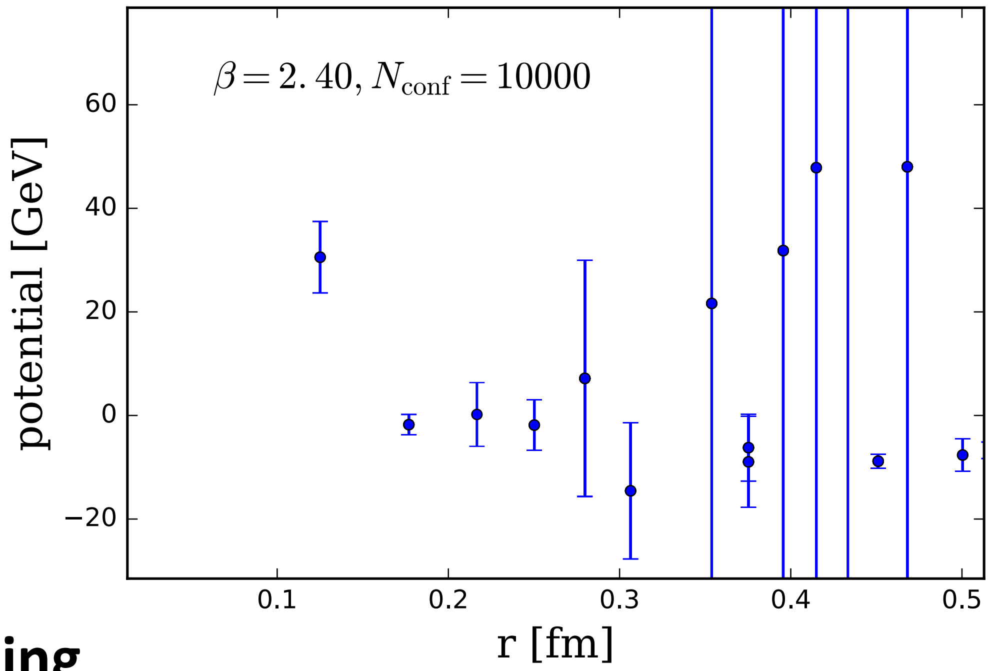
SU(N)

SU(2)

NBS wave func.



potential



On-going

Now we are preparing for the calculation of SU(4)

Next task: using smeared sink operator

(not yet done, sorry…)

Errorbars are so large for inter-glueball potentials
… improve operators with smearing

- D.Kawai et al. (HAL QCD Collaboration), arXiv:1711.01883

I=2 ππ scattering phase shift from the HAL QCD method with the LapH smearing

Potential is depending on the operator … not a physical quantity
↔ scattering length and phase shift are physical quantities … independent of operator choice

D.Kawai et al.:
Smeared operator is used for I=2 ππ scattering
… higher order terms of velocity expansion become important when smeared operator

The method may be useful for the inter-glueball potential

Extraction potentials with smeared op.

4pt correlator:

$$\mathcal{C}^{4,A_1^+,1}_{n_a,n_b}(\mathbf{r},t;|\mathbf{P}|,t_0)=\sum_{\mathbf{x}}\left\langle \pi^-_{n_a}(\mathbf{x},t)\pi^-_{n_a}(\mathbf{x}+\mathbf{r},t)(\pi_{n_b}\pi_{n_b})^{A_1^+,1}_{2,2}(|\mathbf{P}|,t_0)\right\rangle,\ (\pi_{n_s}\pi_{n_s})^{\Lambda,\mu}_{I,I_z}(|\mathbf{P}|,t)=\sum_{\substack{\mathbf{P}\\|\mathbf{P}|:\mathrm{fix}}}\sum_{I_1,I_2}\sum_{\mathbf{x},\mathbf{y}}C^{\Lambda,\mu}(\mathbf{P})D^{I,I_z}_{I_1,I_2}e^{-i\mathbf{P}\cdot\mathbf{x}}e^{i\mathbf{P}\cdot\mathbf{y}}\pi^{I_1}_{n_s}(\mathbf{x},t)\pi^{I_2}_{n_s}(\mathbf{y},t)$$

$R^{A_1^+,1}_{n_a,n_b}(\mathbf{r},t;|\mathbf{P}|,t_0)\equiv\mathcal{C}^{4,A_1^+,1}_{n_a,n_b}(\mathbf{r},t;|\mathbf{P}|,t_0)/\left\{\mathcal{C}^2_{n_a,n_b}(t,t_0)\right\}^2$ satisfies the following equation:

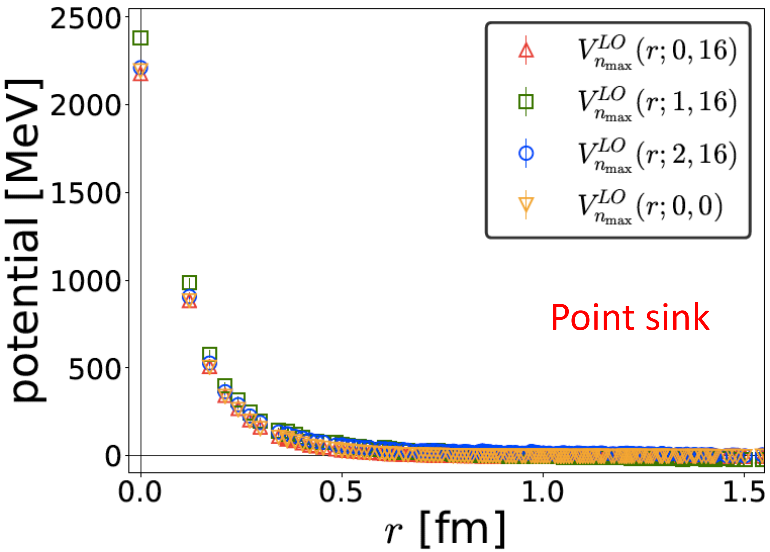
$$\left(\frac{1}{4m_\pi}\frac{\partial^2}{\partial t^2}-\frac{\partial}{\partial t}-H_0\right)R^{A_1^+,1}_{n_a,n_b}(\mathbf{r},t;|\mathbf{P}|,t_0)=\int d^3r'\,U_{n_a}(\mathbf{r},\mathbf{r}')\,R^{A_1^+,1}_{n_a,n_b}(\mathbf{r}',t;|\mathbf{P}|,t_0).$$

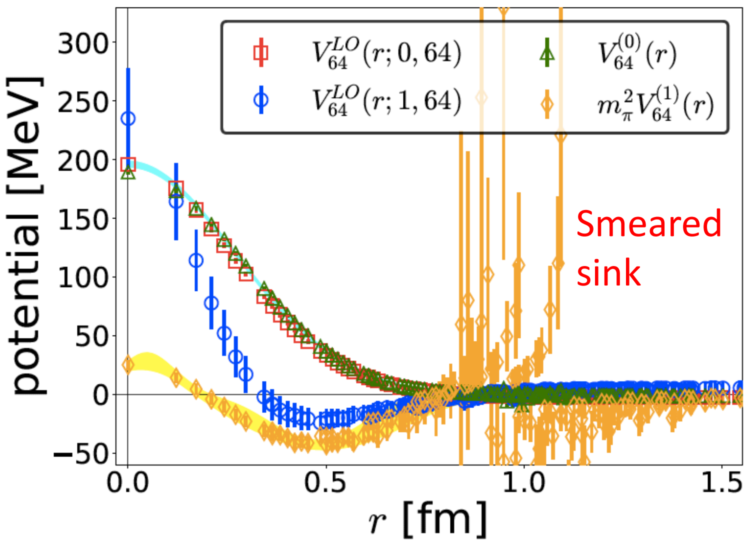
Nonlocal potential can be expanded by $U_{n_a}(\mathbf{r},\mathbf{r}')=\left\{V^{(0)}_{n_a}(r)+V^{(1)}_{n_a}(r)\nabla^2+\mathcal{O}(\nabla^4)\right\}\delta^{(3)}(\mathbf{r}-\mathbf{r}')$

Calculating $V^{\mathrm{LO}}_{n_a}(r;|\mathbf{P}|,n_b)\equiv\dfrac{\left(\frac{1}{4m_\pi}\frac{\partial^2}{\partial t^2}-\frac{\partial}{\partial t}-H_0\right)R^{A_1^+,1}_{n_a,n_b}(\mathbf{r},t;|\mathbf{P}|,t_0)}{R^{A_1^+,1}_{n_a,n_b}(\mathbf{r},t;|\mathbf{P}|,t_0)}$, the potentials are obtained by solving

$$\begin{pmatrix}1 & \dfrac{\nabla^2R^{A_1^+,1}_{n_a,n_b}(\mathbf{r},t;0,t_0)}{R^{A_1^+,1}_{n_a,n_b}(\mathbf{r},t;0,t_0)}\\[2ex] 1 & \dfrac{\nabla^2R^{A_1^+,1}_{n_a,n_b}(\mathbf{r},t;1,t_0)}{R^{A_1^+,1}_{n_a,n_b}(\mathbf{r},t;1,t_0)}\end{pmatrix}\begin{pmatrix}V^{(0)}_{n_a}(r)\\ V^{(1)}_{n_a}(r)\end{pmatrix}=\begin{pmatrix}V^{\mathrm{LO}}_{n_a}(r;0,n_b)\\ V^{\mathrm{LO}}_{n_a}(r;1,n_b).\end{pmatrix}$$

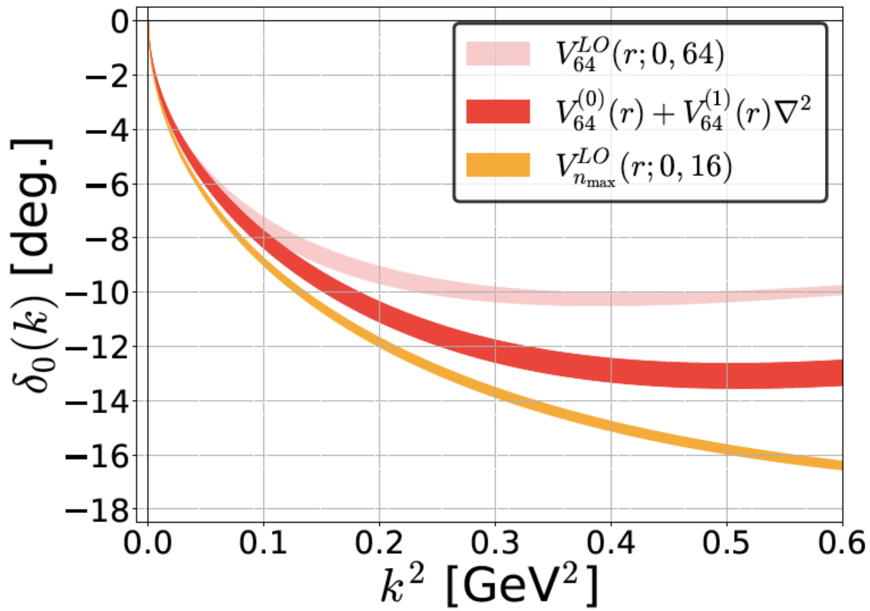
Potential of I=2 ππ scat.





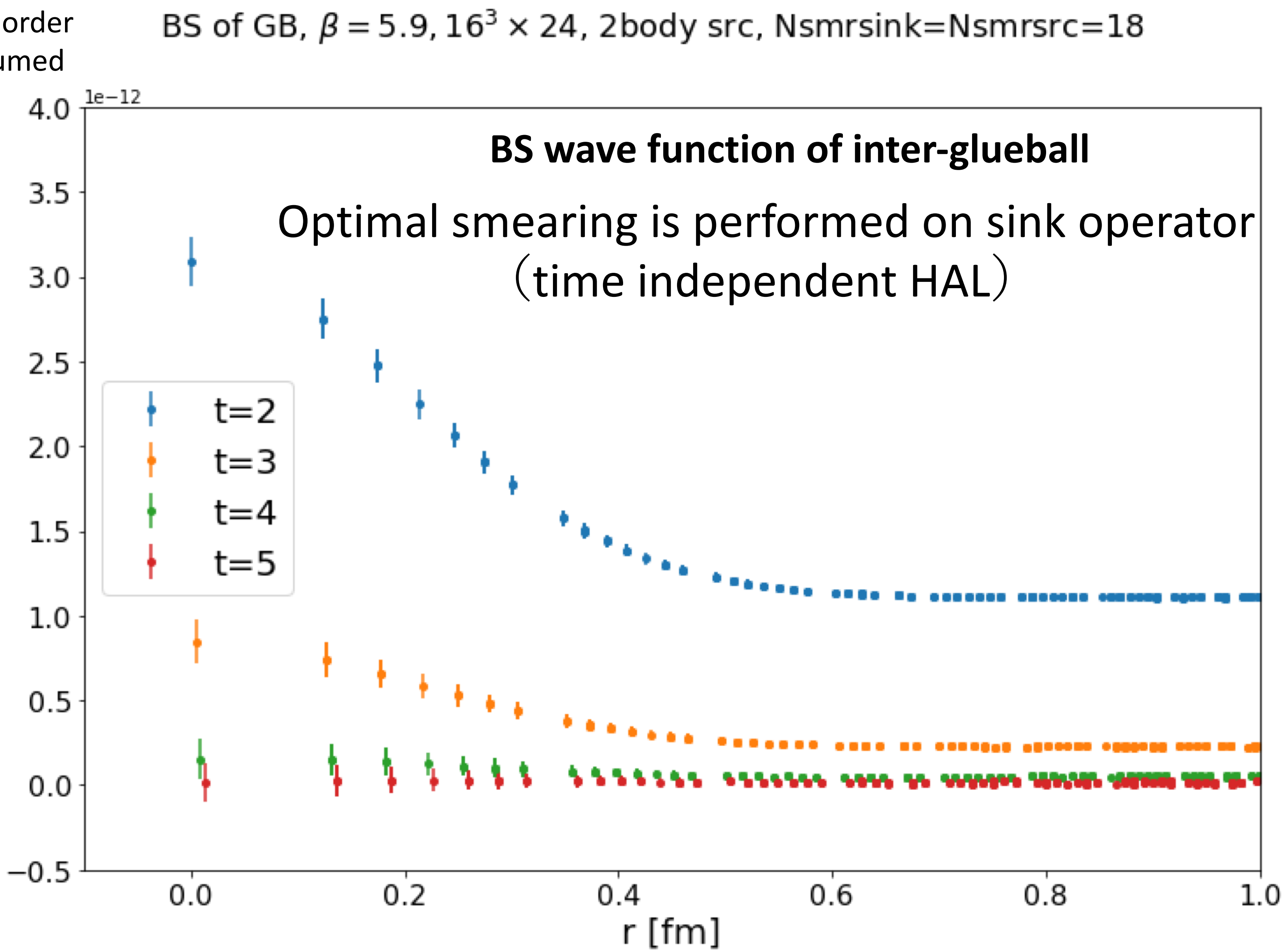
D.Kawai et al. (HAL QCD Collaboration),
Prog. Exp. Ther. Phys., Volume2018, Issue 4, 043B04.

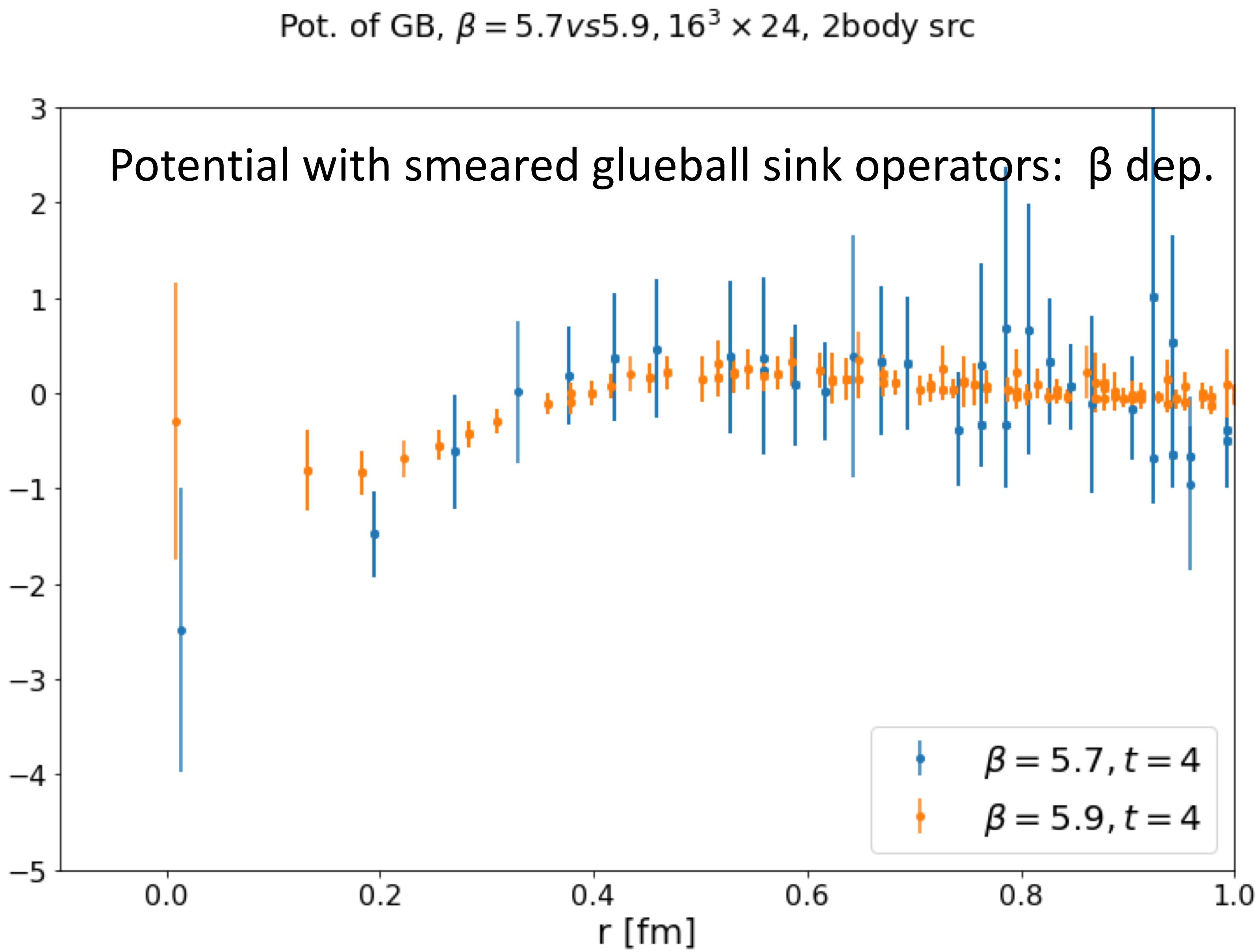
Scattering phase shift

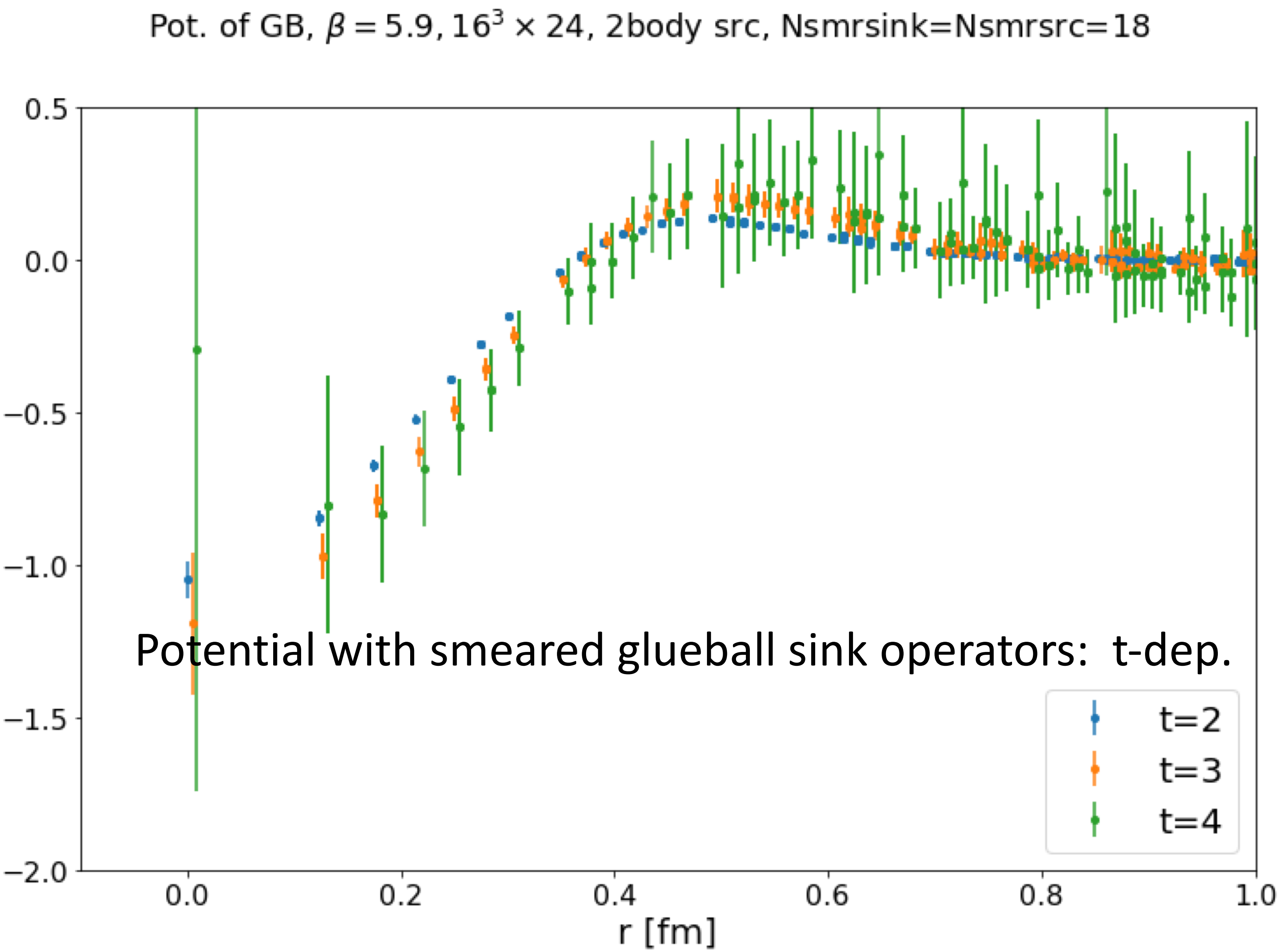


Next leading order (& higher order) are important

※ only leading order potential is assumed




Pot. of GB, $\beta = 5.7 vs 5.9, 16^3 \times 24$, 2body src
Potential with smeared glueball sink operators: β dep.
$\beta = 5.7, t = 4$
$\beta = 5.9, t = 4$
r [fm]
3
2
1
0
−1
−2
−3
−4
−5
0.0
0.2
0.4
0.6
0.8
1.0



Potential with smeared glueball sink operators: t-dep.

Summary and conclusion

- Lightest glueball in Hidden Gauge Theory is a candidate of DM.
- Investigating inter-glueball interaction and comparing the cross section with the limitation from observation
  -> limitation of possible HGT (Nc and Λ_{HGT})
- Interaction is investigated with HAL QCD method in lattice gauge theory
- For SU(3), repulsive around 0.3fm？
  … we have to find whether this is lattice artifact or not
- calculation of SU(N) (N=2&4) … ongoing & under preparation
- Time-dependent HAL method: not so large difference ??
- Smearing sink operator & potential with higher order terms

Thank you for your attention!