原子核結合エネルギーに対する低運動量相互作用の切断運動量依存性

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1. はじめに

- 現実的核力から導かれる低運動量相互作用（$V_{\text{low-}k}$）
- $V_{\text{low-}k}$ を用いた最近の構造計算

2. ユニタリー変換による $V_{\text{low-}k}$ の導出

3. $V_{\text{low-}k}$ を用いた結合エネルギーの計算結果

- ^{3}H, ^{4}He … Faddeev-Yakubovsky
- ^{16}O (^{15}N, ^{15}O, ^{17}F, ^{17}O) … UMOA

4. まとめ

RCNP研究会「核力と核構造」
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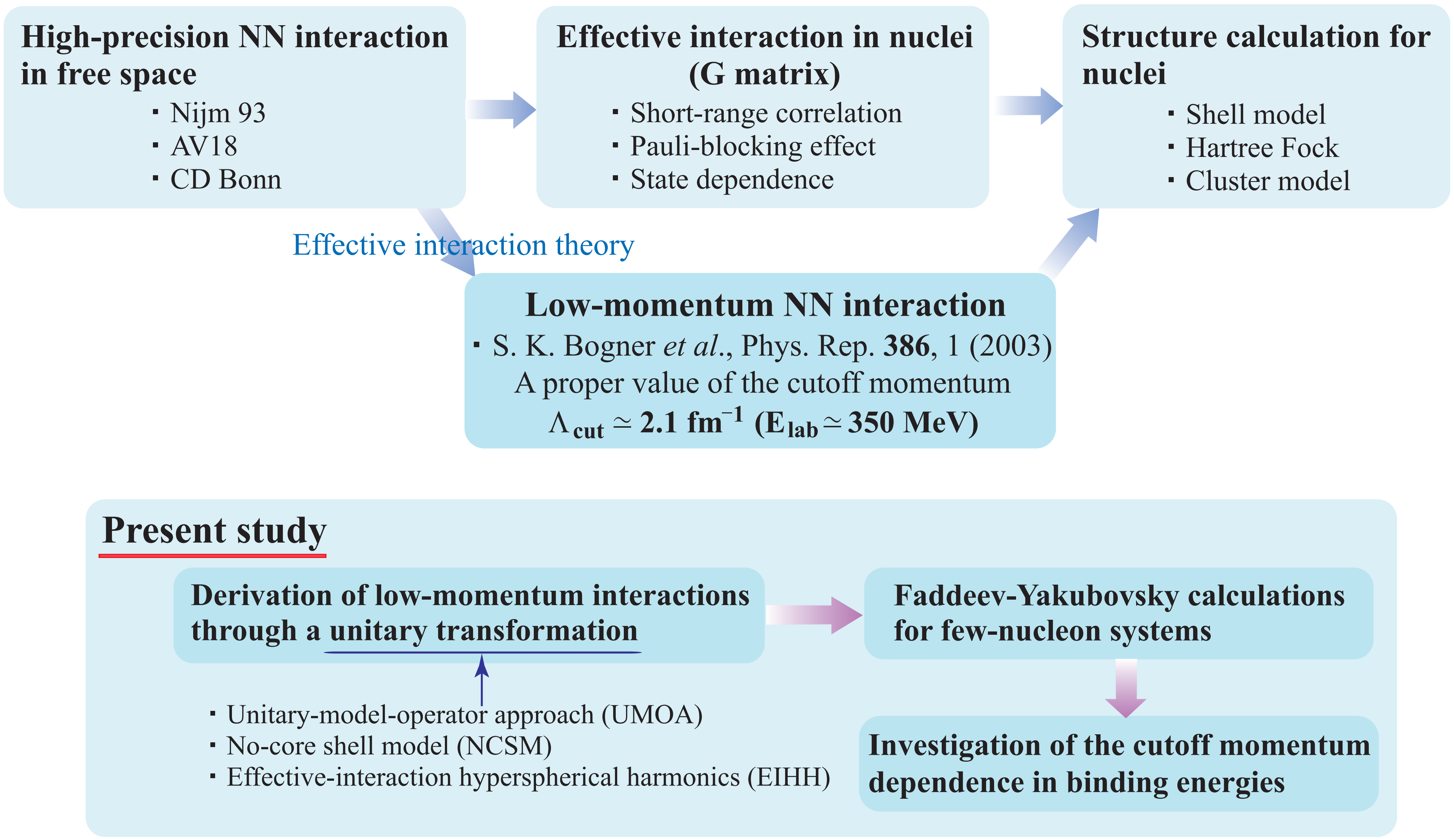
Microscopic description of nuclear structure



High-precision NN interaction in free space

- Nijm 93
- AV18
- CD Bonn

Effective interaction in nuclei (G matrix)

- Short-range correlation
- Pauli-blocking effect
- State dependence

Structure calculation for nuclei

- Shell model
- Hartree Fock
- Cluster model

Effective interaction theory

Low-momentum NN interaction

- S. K. Bogner *et al.*, Phys. Rep. **386**, 1 (2003)

A proper value of the cutoff momentum

$\Lambda_{\mathrm{cut}} \simeq \mathbf{2.1\ fm^{-1}}$ $\mathbf{(E_{lab} \simeq 350\ MeV)}$

Present study

Derivation of low-momentum interactions through a unitary transformation

- Unitary-model-operator approach (UMOA)
- No-core shell model (NCSM)
- Effective-interaction hyperspherical harmonics (EIHH)

Faddeev-Yakubovsky calculations for few-nucleon systems

Investigation of the cutoff momentum dependence in binding energies

Derivation of effective interaction (Hamiltonian) by means of unitary transformation

Hamiltonian

$$H = H_0 + V$$

Unitary transformation of H

$$\tilde{H} = U^{-1} H U$$

$$U = e^{S}, \quad (S : \text{anti-Hermitian}, \ S^{\dagger} = -S)$$

Decoupling equation

$$Q(e^{-S} H e^{S}) P = 0$$

Solution

$$S = \operatorname{arctanh}(\omega - \omega^{\dagger}), \quad \omega = Q\omega P$$

(with the restrictive condition $PSP = QSQ = 0$)

K. Suzuki, Prog. Theor. Phys. **68** (1982), 246

Effective Hamiltonian

$$\underline{H_{\text{eff}} = P\tilde{H}P}$$

Effective interaction

$$\underline{V_{\text{eff}} = P\tilde{H}P - PH_0P}$$

Unitary transformation operator U in terms of ω

$$U = (1 + \omega - \omega^{\dagger})(1 + \omega^{\dagger}\omega + \omega\omega^{\dagger})^{-1/2}$$

$$= \begin{pmatrix} P(1 + \omega^{\dagger}\omega)^{-1/2}P & -P\omega^{\dagger}(1 + \omega\omega^{\dagger})^{-1/2}Q \\ Q\omega(1 + \omega^{\dagger}\omega)^{-1/2}P & Q(1 + \omega\omega^{\dagger})^{-1/2}Q \end{pmatrix}$$

S. Ōkubo, Prog. Theor. Phys. **12** (1954), 603

Application of effective interaction theory to
the two-nucleon system in momentum space

Derivation of low-momentum interaction

$$H = \begin{array}{cc} & \begin{matrix} k_1 k_2 & \cdots\cdots\cdots & k_{max} \end{matrix} \\ \begin{matrix} k_1 \\ k_2 \\ \vdots \\ k_{max} \end{matrix} & \begin{pmatrix} & & \\ & H = T + V_{bare} & \\ & & \end{pmatrix} \end{array} \xrightarrow[]{\text{unitary transformation} \atop \widetilde{H} = U^{-1} H U} \widetilde{H} = \begin{array}{cc} & \begin{matrix} k_1 k_2 \cdots k_{cut} & \cdots\cdots & k_{max} \end{matrix} \\ \begin{matrix} k_1 \\ k_2 \\ \vdots \\ k_{cut} \\ \vdots \\ k_{max} \end{matrix} & \left(\begin{array}{c:c} P\widetilde{H}P & 0 \\ \hdashline 0 & Q\widetilde{H}Q \end{array}\right) \end{array}$$

Unitary transformation operator

$$U = (1 + \omega - \omega^{\dagger})(1 + \omega^{\dagger}\omega + \omega\omega^{\dagger})^{-1/2}, \quad \omega = Q\omega P$$

Low-momentum interaction

$$\underline{V_{low\text{-}k} = P(\widetilde{H} - T)P} \dashrightarrow$$

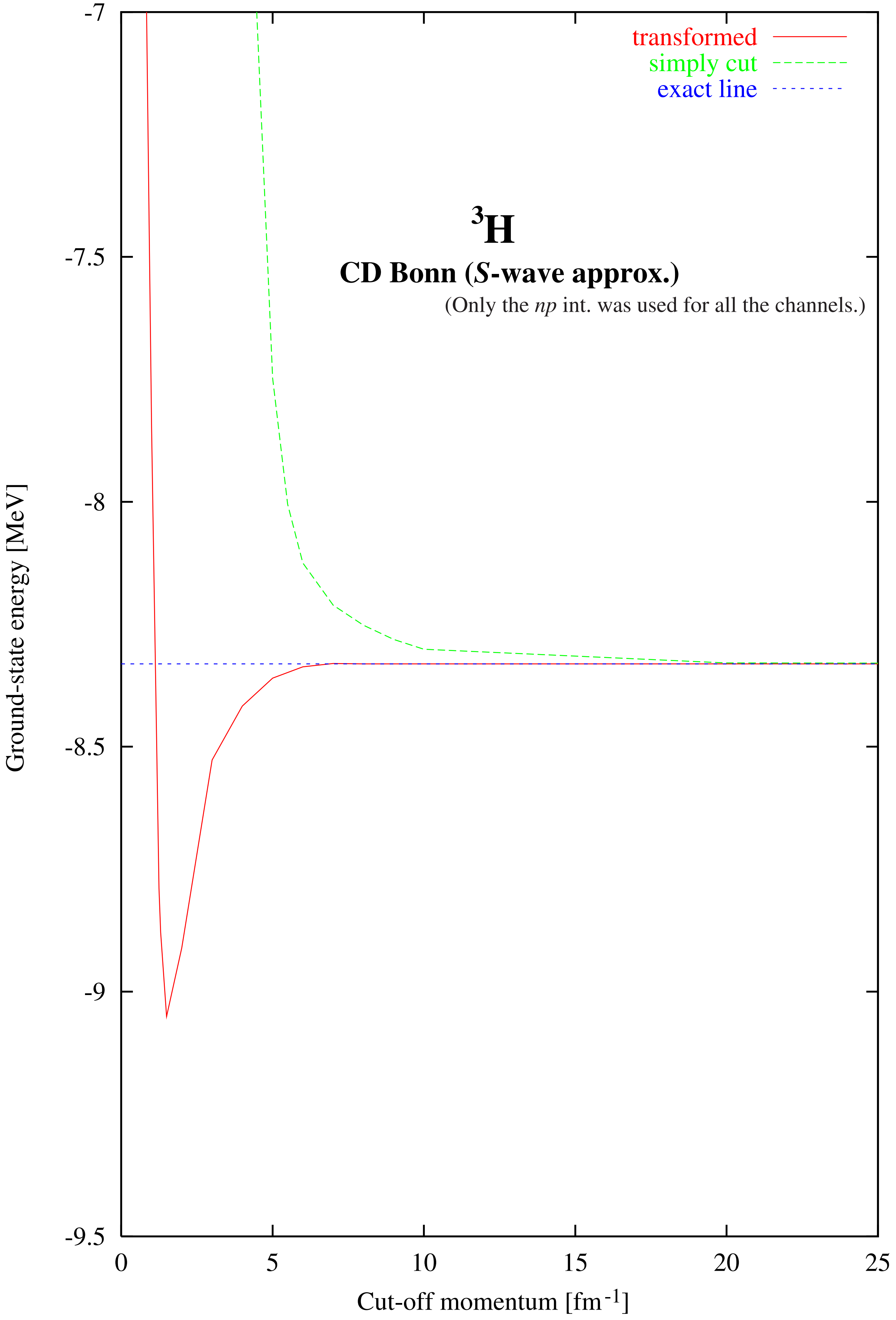
Properties of $V_{low\text{-}k}$

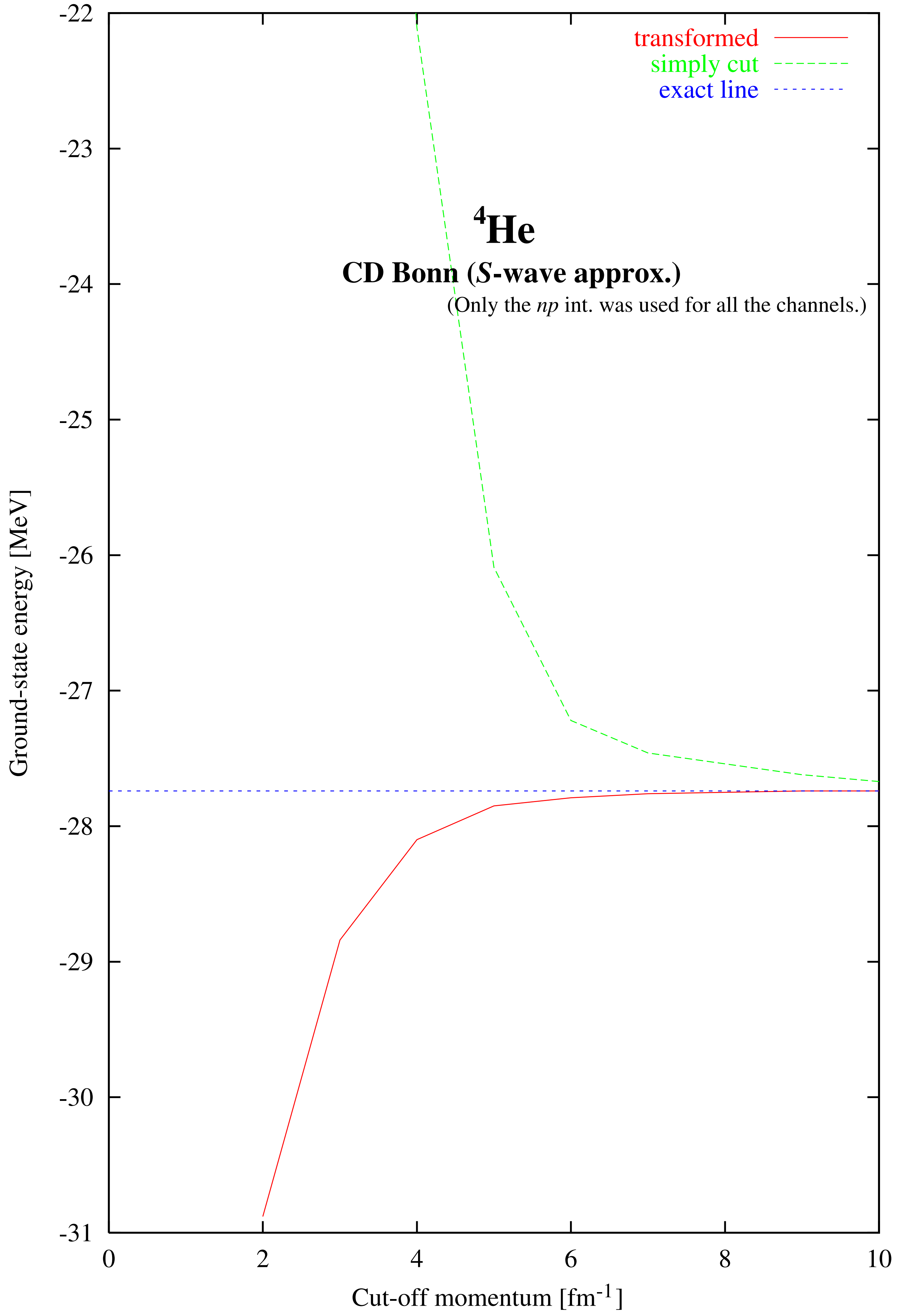
- $QV_{low\text{-}k}P = PV_{low\text{-}k}Q = 0$
- Energy independent
- Hermitian

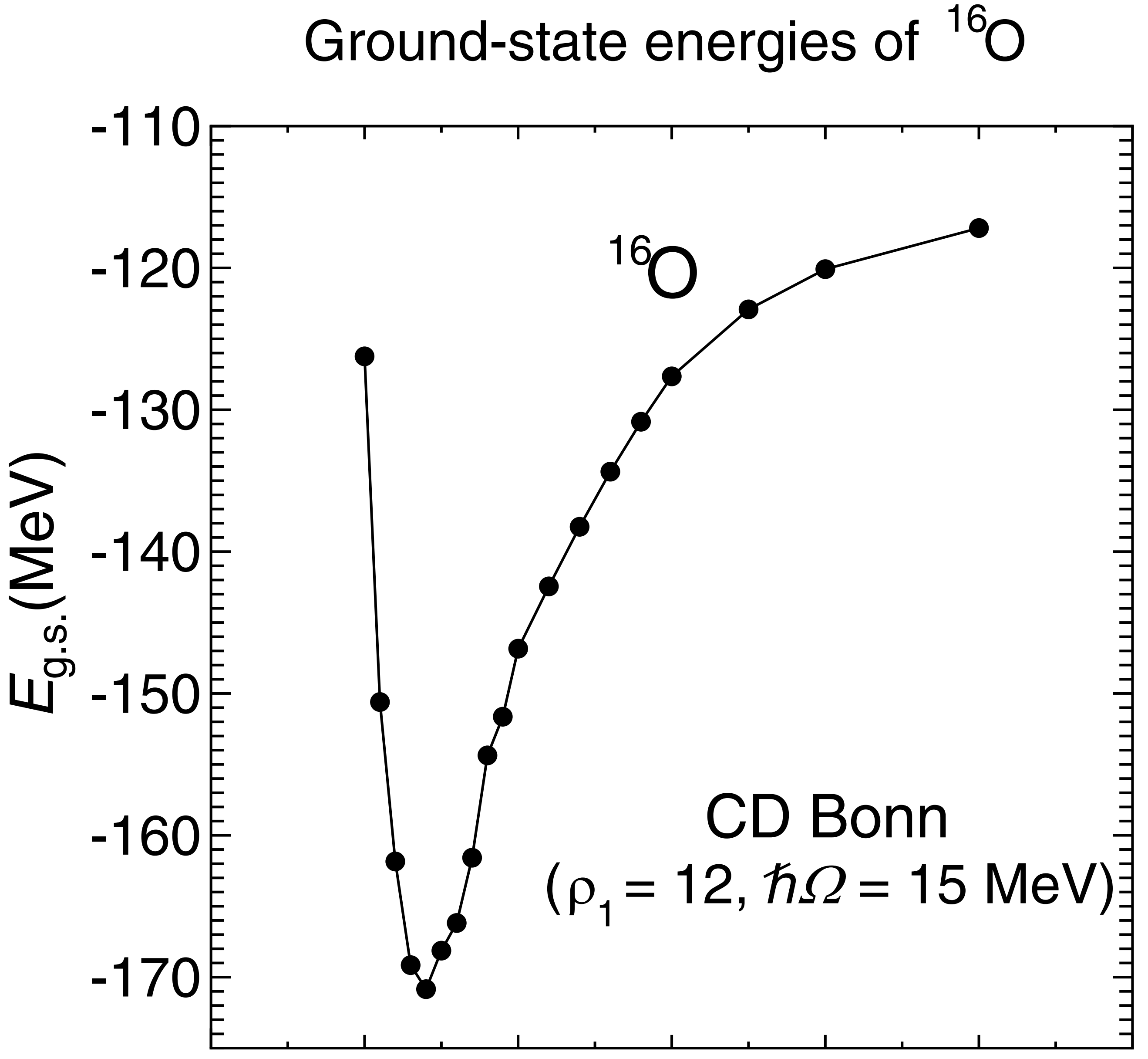
Accuracy of low-momentum interactions

Deuteron properties

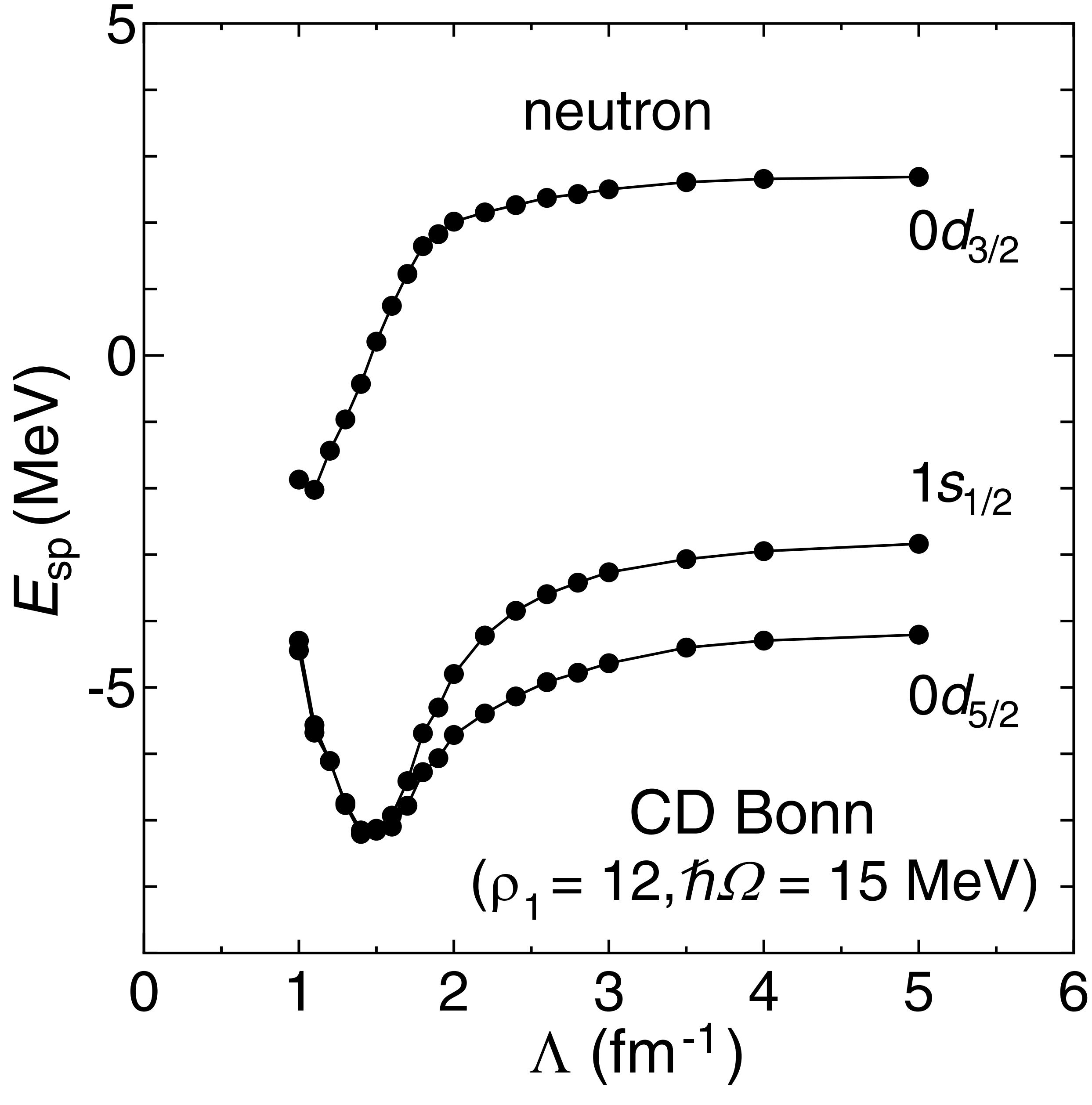
	CD Bonn		Nijm I	
Λ_{cut} (fm^{-1})	BE(MeV)	P_D (%)	BE(MeV)	P_D(%)
1.0	2.224576	1.212	2.224575	1.236
2.0	2.224576	3.549	2.224575	3.828
3.0	2.224576	4.546	2.224575	5.119
4.0	2.224576	4.789	2.224575	5.532
5.0	2.224576	4.830	2.224575	5.639
6.0	2.224576	4.834	2.224575	5.661
7.0	2.224576	4.833	2.224575	5.664
quoted	2.224575	4.83	2.224575	5.664


transformed
simply cut
exact line
^{3}H
CD Bonn (*S*-wave approx.)
(Only the *np* int. was used for all the channels.)
Ground-state energy [MeV]
-7
-7.5
-8
-8.5
-9
-9.5
0
5
10
15
20
25
Cut-off momentum [fm^{-1}]

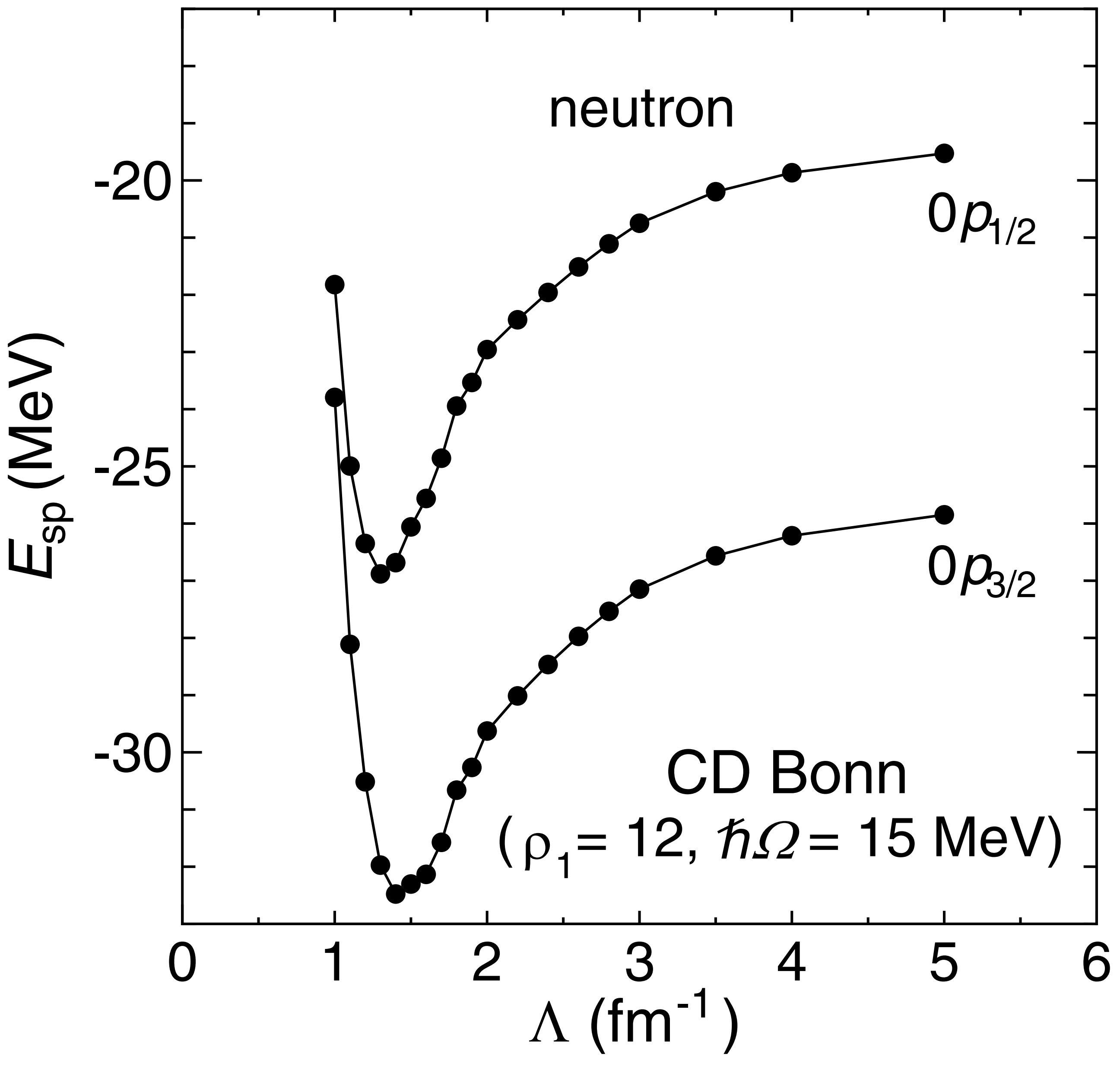

transformed
simply cut
exact line
^{4}He
CD Bonn (S-wave approx.)
(Only the np int. was used for all the channels.)
Ground-state energy [MeV]
-22
-23
-24
-25
-26
-27
-28
-29
-30
-31
0
2
4
6
8
10
Cut-off momentum [fm^{-1}]


Ground-state energies of ^{16}O
^{16}O
CD Bonn
($\rho_1 = 12$, $\hbar\Omega = 15$ MeV)
$E_{\mathrm{g.s.}}$(MeV)
Λ (fm^{-1})
-110
-120
-130
-140
-150
-160
-170
0
1
2
3
4
5
6

Single-particle energies for particle states in ^{17}O



Single-particle energies for hole states in ^{16}O



Summary

- Low-momentum interactions were derived from realistic nucleon-nucleon interactions through a unitary transformation which has also been used in the unitary-model-operator approach (UMOA), the no-core shell model (NCSM), and the effective-interaction hyperspherical harmonics (EIHH).

- The low-momentum interactions obtained have high accuracy numerically, which was confirmed by the calculations of the deuteron binding energy.

- The low-momentum interactions were successfully applied to the Faddeev-Yakubovsky calculations for three- and four-nucleon systems.

- The calculated ground-state energies of the few-nucleon systems using the low-momentum interactions vary considerably at $\Lambda < 4 \sim 5\,\mathrm{fm}^{-1}$, and there occurs the energy minimum at $\Lambda = 1 \sim 2\ \mathrm{fm}^{-1}$.

- A similar tendency of the energy curve was obtained also in the calculations for ^{16}O. However, the magnitudes of relative spacings of single-particle levels are not so changed in the area $\Lambda > 2\ \mathrm{fm}^{-1}$.

- The low-momentum interaction should be used with care especially in calculations of the total binding energy though the low-momentum interaction would be very useful in structure calculations as has been shown in shell-model calculations.