Tensor correlation in neutron halo nuclei

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• Contents
  
  ○ **Tensor Correlation** for s-wave problem in $^{10,11}\text{Li}$ (cf. pairing).
    - Occurance of Halo structure in $^{11}\text{Li}$ ($N=8$).
    - Inversion problem in $^{10}\text{Li}$ ($N=7$).
  
  ○ Model: $^9\text{Li}+n+n$ with **Tensor correlation in $^9\text{Li}$**.
    
    Analogy: $^4\text{He}+n+n$ of $^6\text{He}$.
  
  ○ Results: 1. $^4\text{He}$ with tensor correlation using Shell Model basis.
    2. $^5\text{He}$ with $^4\text{He}+n$ ($3/2^--1/2^-$ splitting).
    3. Effective Interaction.
    4. $^8\text{He}$ with tensor and $p$-shell pairing correlations.
- Description of Halo nuclei based on the "core+n+n" model

  - $^6\text{He}$: Successful results for G.S. without core excitation.

  - $^{11}\text{Li}$: Ambiguity in $^9\text{Li}$-n interaction.
    - $^9\text{Li}$: $(0p_{3/2})_\nu$ -closed $\rightarrow$ Underbinding.
    - $^9\text{Li}$: p-shell pairing correlation for neutron
      - Inversion phenomena in $^{10}\text{Li}$.
      - $^p$-shell closed configuration in $^{11}\text{Li}$.

  - Effect of tensor correlation
    - $^9\text{Li}$: pairing corr. + tensor corr.
    - Degeneracy of p- and s-orbitals in both $^{10,11}\text{Li}$?
      - Inversion phenomena in $N=7$, halo production

[Ref]: K.Katō, K.Ikeda, PTP89('93)623.
T.Myo, S.Aoyama, K.Katō, K.Ikeda, PTP108('02)133.
• Tensor correlation in $^4$He and $^9$Li core

- $^4$He -

0p $\cdots \cdots \Leftrightarrow V_T$

0s $\begin{array}{c}
\pi \\
(0s)^4
\end{array}$

$\langle V_T \rangle$ ($^3E$) is large (comparable to $\langle V_C \rangle$)

2p-2h excitation from $(0s)^4$

P[D] ~ 10-13%

- $^9$Li -

$\begin{array}{c}
\pi \\
(0s)^2(0p)^2
\end{array}$

$\begin{array}{c}
\nu \\
\nu
\end{array}$

$\begin{array}{c}
1s_{1/2} \\
0p_{1/2} \\
0p_{3/2} \\
0s_{1/2}
\end{array}$

$\begin{array}{c}
\pi \\
\nu
\end{array}$

Lowest Pairing

Pairing

Tensor

H. Kamada et. al, PRC64(2001)044001
Effect of Tensor Correlation in $^{11}$Li

- **Energy Gain**
- **Pauli Blocking**
- **Tensor Pairing**
- **Pairing + Tensor**
Model to incorporate the tensor correlation

- Criterion: \(^4\text{He} \ (P[D] \sim 10-13\%)
  - Extension of Terasawa, Nagata’s works \(^5\text{He};\text{LS splitting}\)
  - Application to \(^6\text{He} = ^4\text{He}(\ast) + n + n\).
    \(\Rightarrow\) Review of Halo mechanism, Resonance structures.

- Wave Function for core part \((^4\text{He}, \ ^9\text{Li})\)
  - H.O.basis with different length parameters \(\{b_i\}\), such as \(b_{0s} \neq b_{0p}\) \(\ldots\)
    to include the higher shell effect.
  - for \(^4\text{He}, 0s_{1/2} + 0p_{1/2} + 0p_{3/2}\) up to \(2p-2h\).

\(\Phi(^4\text{He}) = \Sigma_\alpha C_\alpha \psi_\alpha(\{b_i\}) = C_1 \ (0s)^4 + C_2 \ (0s)^2 (0p_{1/2})^2 + \cdots\)

- \(\frac{\partial \langle H - E \rangle}{\partial b_i} = 0\), \(\frac{\partial \langle H - E \rangle}{\partial C_\alpha} = 0\)
○ Interaction :
  – Central : \textbf{Volkov No.2 with M=0.6}
  – Tensor : \textbf{Furutani} \ (^{3}\text{He}+p \text{ scattering})
  – LS : \textbf{G3RS}

[Ref]: H. Furutani, H. Horiuchi, R. Tamagaki, PTP62(’79)981
\( ^4\text{He G.S.}(0^+) \) with V2+Furu+G3RS

- **Amplitudes with** \( b_0p_{1/2} = b_0p_{3/2} = 0.8 \text{ fm} \)

<table>
<thead>
<tr>
<th>Term</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0s_{1/2})^4)</td>
<td>94.6 %</td>
</tr>
<tr>
<td>((0s_{1/2})^2(0p_{1/2})^2) ( (JT)=(10) )</td>
<td>4.5 %</td>
</tr>
<tr>
<td>((0s_{1/2})^2(0p_{3/2})^2)</td>
<td>0.3 %</td>
</tr>
<tr>
<td>((0s_{1/2})^2(0p_{1/2})(0p_{3/2}))</td>
<td>0.6 %</td>
</tr>
<tr>
<td>(P[D])</td>
<td>3.4 %</td>
</tr>
</tbody>
</table>

- \( 0^- \) coupling between \( 0s_{1/2} \) and \( 0p_{1/2} \)

\( \Rightarrow \) pion nature
- Coupling Matrix Element of Tensor force

\[ \langle (0s_{1/2})^2_1 (0p_{1/2})^2_1 | V | (0s)^4 \rangle \quad (b_{0s}=1.4 \text{ [fm]}) \]

Coupling from Tensor force is significant with a small \( b_{0p} \).
$^4\text{He G.S.}(0^+) \text{ with V2+Furu.} + \text{G3RS}$

- $^3E$ part of Central force is adjusted to reproduce the B.E. of $^4\text{He}$ (28.3 MeV).

<table>
<thead>
<tr>
<th>$b_{0p}$ [fm]</th>
<th>2.0</th>
<th>1.4 (= $b_{0s}$)</th>
<th>0.8</th>
<th>$(V_T \times 1.5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle\text{Kinetic}\rangle$ [MeV]</td>
<td>45.8</td>
<td>49.5</td>
<td>52.7</td>
<td>58.0</td>
</tr>
<tr>
<td>$\langle\text{Central}\rangle$</td>
<td>-74.3</td>
<td>-73.4</td>
<td>-66.0</td>
<td>-53.8</td>
</tr>
<tr>
<td>$\langle\text{Tensor}\rangle$</td>
<td>-0.6</td>
<td>-5.2</td>
<td>-16.4</td>
<td>-34.3</td>
</tr>
<tr>
<td>$\langle\text{LS}\rangle$</td>
<td>$2 \times 10^{-3}$</td>
<td>$1 \times 10^{-4}$</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>$R_m$ [fm]</td>
<td>1.52</td>
<td>1.48</td>
<td>1.48</td>
<td>1.48</td>
</tr>
<tr>
<td>2p-2h [%]</td>
<td>1.6</td>
<td>4.1</td>
<td>5.4</td>
<td>11.0</td>
</tr>
<tr>
<td>$(0p_{1/2})^2_{JT} \ (JT)=(10)$</td>
<td>0.6</td>
<td>2.9</td>
<td>4.5</td>
<td>9.6</td>
</tr>
<tr>
<td>$(0p_{1/2})^2_{JT} \ (JT)=(01)$</td>
<td>0.1</td>
<td>0.1</td>
<td>$3 \times 10^{-2}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$(0p_{3/2})^2$</td>
<td>0.8</td>
<td>0.2</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>$(0p_{1/2})(0p_{3/2})$</td>
<td>$6 \times 10^{-2}$</td>
<td>0.9</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>$P[D]$ [%]</td>
<td>0.5</td>
<td>2.3</td>
<td>3.4</td>
<td>7.2</td>
</tr>
</tbody>
</table>

Tensor force can be incorporated.
- $3/2^--1/2^-$ splitting in $^5\text{He}$ with $^4\text{He}+n$ (Preliminary)

\[ ^4\text{He}[2\text{p}-2\text{h}] + n(p_{3/2,1/2}) \]

- $^4\text{He}$: $0s_{1/2} + 0p_{1/2}$
  - ($b_{0s}=1.4 \text{ fm}, b_{0p}=0.8 \text{ fm}$)

- $^4\text{He}+n$ interaction (OCM)
  - Central: Folding potential with Volkov No.2
  - No LS part.

- $H(^5\text{He}) = H(^4\text{He}) + H_{\text{rel}}$

- $\Phi(^5\text{He}) = (0s)^4 \cdot \psi^1_{\text{rel}}$
  + $\left(0s\right)^2(0p_{1/2})^2 \cdot \psi^2_{\text{rel}}$
$E_R = (E_r, \Gamma)$ [MeV] of $^5$He using $^4$He with V2+Furu.+G3RS

<table>
<thead>
<tr>
<th></th>
<th>Exp.(KKNN)</th>
<th>Present ($V_T \times 1.0$)</th>
<th>Present ($V_T \times 1.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3/2^-$</td>
<td>(0.74, 0.60)</td>
<td>(0.74, 0.65)</td>
<td>(0.74, 0.65)</td>
</tr>
<tr>
<td>$1/2^-$</td>
<td>(2.13, 5.84)</td>
<td>(1.01, 1.05)</td>
<td>(1.37, 1.85)</td>
</tr>
<tr>
<td>$\Delta E$</td>
<td>1.47</td>
<td>0.27</td>
<td>0.63</td>
</tr>
</tbody>
</table>

⇒ Visible contribution of tensor correlation
Energy Levels of $^6$He without tensor correlation

\begin{align*}
\text{Energy \ [MeV]} & \quad \text{Exp.} \\
& (0.822 \pm 0.113 \pm 20) \\
& \alpha + n + n \\
\text{Theor.} (\alpha + n + n) & -0.975 \\
\end{align*}

Nakayama et. al

$E_r \sim 4$

$\text{Nakamura et. al}$

$(3 \pm 1, 4 \pm 1)$

$\Gamma \sim 2$

$\begin{align*}
0^+ & \quad (0p_{1/2})^2 \\
1^+ & \quad (0p_{3/2})(0p_{1/2}) \\
2^+ & \quad (0p_{3/2})^2 \\
\end{align*}$

$\begin{align*}
0^+ & \quad (2.35, 4.22) \\
1^+ & \quad (3.69, 9.14) \\
2^+ & \quad (0.81, 0.13) \\
\end{align*}$

$\begin{align*}
0^+ & \quad -0.975 \\
\end{align*}$

$\begin{align*}
\text{(0p}_{3/2}^2 \text{)}^2 \\
\end{align*}$
- **Effective Interaction**
  
  - **Akaishi potential**: G-matrix derived from AV8’ (Acknowledge to Prof. Akaishi)
  - **GPT potential** (Gogny-Pires-Tourreil).
    - C+LS+T, 3-range Gaussian to fit $d'$ properties, and NN phase shifts.

![Diagram](image)

Properties of $^4$He with $0s+0p$ up to $2p-2h$.

<table>
<thead>
<tr>
<th>Int.</th>
<th>$E \left( \langle V_T \rangle \right)$ [MeV]</th>
<th>$P[2p-2h]$</th>
<th>$R_m$ [fm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AK</td>
<td>$-19.0 \ (-30.9)$</td>
<td>13 %</td>
<td>1.23</td>
</tr>
<tr>
<td>GPT</td>
<td>$-17.4 \ (-11.2)$</td>
<td>8 %</td>
<td>1.45</td>
</tr>
</tbody>
</table>

$\Rightarrow$  
- Central, LS : GPT
- Tensor : Aakaishi
GPT+AK with modification to reproduce $^4$He properties

Central part of GPT (2nd range)

$$V_2 = v_2 \ e^{-(r/R_2)^2}$$

$$R_2 \rightarrow R_2 + \Delta R \quad (\Delta R=0.27 \ \text{fm})$$

$$v_2 \rightarrow v_2 + \Delta v$$

Properties of $^4$He using mod.GPT+AK

<table>
<thead>
<tr>
<th>E ($\langle V_T \rangle$) [MeV]</th>
<th>P[2p-2h]</th>
<th>R_m [fm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-28.3 \ (-16.9)$</td>
<td>10 %</td>
<td>1.49</td>
</tr>
</tbody>
</table>
- $^8\text{He} \ (0^+) \ : \ \text{same neutron number as } ^9\text{Li}$
  - Configuration with H.O. basis function:
    - $0s_{1/2} + 0p_{1/2} + 0p_{3/2}$ up to $2p-2h$.
    - Length parameters $\{b_i\}$ are determined variationally.
  - Interaction:
    - Central,LS : GPT with strengthening $\nu_2$ by 4%
    - Tensor : Akaishi
Energy of $^8$He $(0^+)$ with mod. GPT + AK $(b_{0s} = 1.6 \ [\text{fm}])$

- B.E. = 28.0 [MeV]
  - $(b_{0p\frac{1}{2}}, b_{0p\frac{3}{2}}) = (1.9, 1.9)$

- B.E. = 26.8 [MeV]
  - $(0.8, 2.0)$

Two minima

Energy of $^8$He [MeV]
Properties of two minima in $^8$He ($0^+$)

- Two minima:
  - Tensor correlation with small $b_{0p1/2}$ ($\sim b_{0s}/2$).
  - Pairing correlation with $b_{0p1/2} = b_{0p3/2}$. 

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Graphs showing:
- Energy of $^8$He [MeV] vs. $b_{0p1/2}$ [fm]
  - $\langle V_{LS} \rangle$
  - $\langle V_T \rangle$
  - $\langle V_{LS} \rangle$ and $\langle V_T \rangle$ are nearly constant.

- Probability [%] vs. $b_{0p1/2}$ [fm]
  - Lowest state
  - $\langle 0s \rangle^2 \langle 0p_{1/2} \rangle^2$ (tensor)$ \times 5$
  - $\langle 0p_{3/2} \rangle^2 \langle 0p_{1/2} \rangle^2$ (pairing)

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### Equations:

- $(b_{0s}, b_{0p3/2}) = (1.6, 2.0)$ [fm]
- $(b_{0s}, b_{0p3/2}) = (1.6, 2.0)$ [fm]
Summary

1. **Tensor correlation** is expected to give a contribution to lower the 1s-orbit in neutron drip line nuclei.

2. Effects of Tensor correlation in $^4$He and $^5$He.
   - $^4$He: p-wave is favored to shrink, Coupling between $0s_{1/2}$ and $0p_{1/2}$.
     (cf. Akaishi(HF), Sugimoto(HF), Doté(AMD))
   - $^5$He: Visible contribution to the $3/2^-$-$1/2^-$ splitting.

3. Effective interaction
   - modified GPT+AK tensor: Properties of $^4$He is reproduced.
   - Adequate interaction should be found such as for LS part.

4. For $^8$He and $^9$Li
   - **Tensor and Pairing correlations** produce the energy minima.
     (different $b_{0p1/2}$ values) $\implies$ superpose.
$^9\text{Li (3/2}^-, 1/2^-)$ with 0s+0p, $^8\text{He}(0^+)+p$

\[ (b_{0s}, b_{0p3/2}) = (1.6, 1.8) \text{ [fm]} \]

Energy of $^9\text{Li (3/2}^-)$ [MeV]

\[ \langle V_T \rangle = -6.3 \text{ MeV} \]

\[ \langle V_T \rangle = -0.3 \text{ MeV} \]

\[ \langle V_T \rangle = -0.7 \text{ MeV} \]
Tensor correlation (TC) in $^9$Li(3/2$^-$) for $^{11}$Li

$$|	ext{0}\rangle = (0s_{1/2})^4 (0p_{3/2})_π (0p_{3/2})^4_μ,$$

$$|^{9}\text{Li}\rangle = |\text{0}\rangle + |\text{TC}\rangle.$$ 

○ Nagata’s Method (PTP22(1959)274)

- Direct inclusion of D-state component in the relative motion.

$$|\text{TC}\rangle = \mathcal{F}_\text{T} |\text{0}\rangle$$

$$\mathcal{F}_\text{T} = \sum_{i<j} \mathcal{F}_{ij}$$

$$\mathcal{F}_{ij} = f(r_{ij}) \cdot \hat{a}_{ij}^r,$$

$$r_{ij} = r_i - r_j$$

$$f(r_{ij}) = f(r_{ij}) \cdot \left[ Y_2(r_{ij}) \otimes S_{2,ij} \right]_0$$

$$S_{2,ij} = [s_i \otimes s_j]_2$$

$$f(r) = \sum_{n=1}^{N} C_n \cdot \phi_n(r)$$
Effect of Tensor Correlation in $^{10}\text{Li}$

- **Pairing**
- **Tensor**
- **Pairing+Tensor**

$^{9}\text{Li}_{\text{g.s.}}$

$^{10}\text{Li}^{(p)}$

$^{10}\text{Li}^{(s)}$

Pauli blocking

energy gain
• Effect of Tensor Correlation in $^5\text{He}$, $^6\text{He}$, $^6\text{Li}$


• $3/2^--1/2^-$ splitting in $^5\text{He}$

Effect of Tensor Correlation in $^5\text{He}$, $^6\text{He}$, $^6\text{Li}$
Coupling Matrix Element of Tensor force

\[ \langle (0s_{1/2})_1 (0p_{1/2})_1 | V_T | (0s)^4 \rangle \quad (b_{0s}=1.4 \text{ [fm]}) \]

\[ b_{0s}=1.0 \text{ fm} \]
\[ b_{0s}=1.2 \text{ fm} \]
\[ b_{0s}=1.4 \text{ fm} \]
\[ b_{0s}=1.6 \text{ fm} \]
\[ b_{0s}=1.8 \text{ fm} \]

\( V_T \) becomes large with narrow 0s-wave.
Effect of tensor force on the energy surface of $^8$He ($0^+$)

With Tensor force

$(b_0s, b_{0p3/2}) = (1.4, 2.0) \text{ [fm]}$

Without Tensor force

$(b_0s, b_{0p3/2}) = (1.4, 2.0) \text{ [fm]}$