The Structure of Nuclear force in a chiral quark-diquark model

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Contents

1. Introduction

- Motivation: Understanding of hadron physics from the quark level

Nuclear force

1. long and medium range ; OBEP
2. short range ; internal structure (repulsive core)

To describe the nuclear force in a unified way, we need to include

1. chiral symmetry
2. hadrons as composites

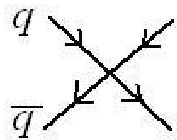
Hadronization of chiral quark-diquark model

Basic idea of hadronization

NJL model and Auxiliary field method (Bosonization)

(Eguchi, Sugawara(1974), Kikkawa (1976))

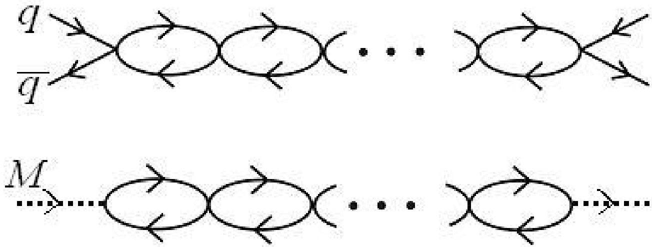
NJL model : chiral sym.



Auxiliary field method :

NJL(quark) model → meson Lagrangian

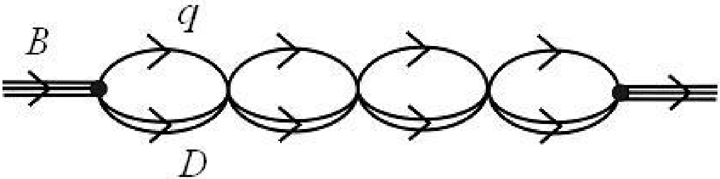
Path-integral identity



- Ebert, Jurke (1998)
- Abu-Raddad, Hosaka, Ebert, Toki (2002)

◇ Bosonization was extended to baryon by including diquark.

$$B = q + D$$



◇ GT relation, WT identity is satisfied.

1. chiral symmety → O.K
2. hadrons as composites → O.K

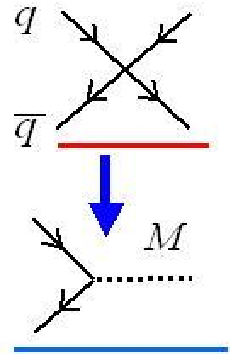
2. Lagrangian **(Abu-Raddad et al, PRC, 66, 025206 (2002))**

I. Quark-meson

$$L_{NJL} = \bar{q}(i\partial\!\!\!/ - m_0)q + \frac{G}{2}\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right]$$

$$\sigma = \bar{q}q,\ \vec{\pi} = i\bar{q}\vec{\tau}\gamma_5 q$$

$$L_{NJL} = \bar{q}(i\partial\!\!\!/ - m_0 - \sigma - i\gamma_5\vec{\tau}\cdot\vec{\pi})q - \frac{1}{2G}(\sigma^2 + \pi^2)$$



$\bullet$ Non-linear rep. $\rightarrow \chi = \xi_5 q,\ \xi_5 = \left(\frac{\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}}{f}\right)^{1/2}$

$$L_{NJL} = \bar{\chi}\left[i(\partial\!\!\!/ - i v\!\!\!/) - (\sigma' + m_q) - a\!\!\!/\gamma_5\right]\chi - \frac{1}{2G}(\sigma' + m_q)^2 + O(m_0)$$

chiral sym. breaking

$$v_\mu = \frac{1}{2i}\left(\xi_5\partial_\mu\xi_5^+ + \xi_5^+\partial_\mu\xi_5\right),\ a_\mu = \frac{1}{2i}\left(\xi_5\partial_\mu\xi_5^+ - \xi_5^+\partial_\mu\xi_5\right)$$

II. Introduction of diquark

- Pauli principle
  for quark
- Color singlet for baryon

(Spin, Isospin)

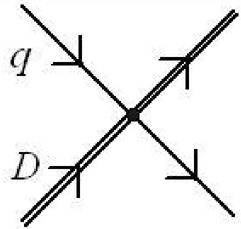
(0, 0) ···· scalar diquark D

(1, 1) ···· axial-vector diquark $\vec{D}_\mu$

◇As a first step, we take into account only scalar diquark

III. Quark-diquark interaction

→ local interaction



$\sim \quad \bar{\chi} D^\dagger D \chi$

◇ Microscopic(q, D, M) lagrangian

$$L = \bar{\chi}\left[i(\partial\!\!\!/ - i v\!\!\!/) - (\sigma' + m_q) - a\!\!\!/\gamma_5\right]\chi - \frac{1}{2G}(\sigma' + m_q)^2 + O(m_0)$$
$$+ D^+(\partial^2 + M_S^2)D + \tilde{G}\bar{\chi}D^+ D\chi$$

Hadronization

I. Baryons are introduced as auxiliary fields $B = D\chi$

$$L = \bar{\chi} S^{-1} \chi + D^{+} \Delta^{-1} D - \tilde{G}\left(BD\chi + \bar{\chi} D^{+} B\right) - \frac{1}{\tilde{G}} \bar{B} B + \cdots$$

$$S^{-1} = i(\not{\partial} - m_q) - M, \quad M = \sigma + \not{v} + \not{a}\gamma_5$$

$$\Delta^{-1} = \partial^2 + M_S^2$$

$$B = \begin{pmatrix} p \\ n \end{pmatrix}$$

II. q and D are integrated out by

$$\int D[\bar{\psi}\psi] \exp i \int d^4x\, \bar{\psi} A \psi = \exp\left(\pm i \operatorname{tr} \log A\right)$$

◇ Meson baryon lagrangian

meson   baryon

$$L = -i \operatorname{tr} \log\left(1 - SM\right) + i \operatorname{tr} \log\left(1 - \Delta^{T} \bar{B} S B\right) + \cdots$$

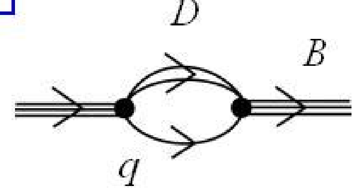
quark   diquark

$$L = i\,\mathrm{tr}\log\left(1 - \Delta^{\mathrm{T}}\bar{B}SB\right) + \cdots$$

$$\text{Expansion: } \mathrm{tr}\log(1 - A) = -\mathrm{tr}\left(A + \frac{A^2}{2} + \cdots\right)$$

$$\mathrm{tr}\ A = \mathrm{tr}\,\bar{B}\Delta SB$$

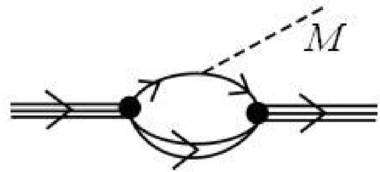
$$= \mathrm{tr}\,\bar{B}(x)\Delta(x-y)S(y-z)B(z)$$



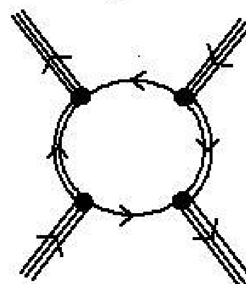
q-M interaction

$$S^{-1} = i\partial - m_q - M$$

$$\rightarrow S = S_0 + S_0 M S_0 + \cdots$$



$$\mathrm{tr}\ A^2 = \mathrm{tr}\,\bar{B}\Delta SB\,\bar{B}\Delta SB$$



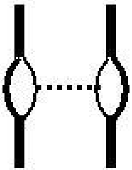
Nucleon properties (one N term)

- Magnetic moment, radius, g_A was calculated.
- GT relation, WT identity is satisfied.

Abu-Raddad et al (2002)

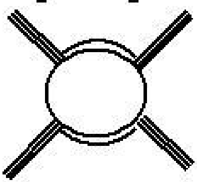
Nuclear force (two N term)

- Two types of NN force



OBEP with form factor

→ long and medium range

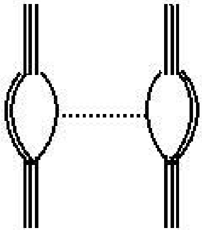


q-*D* loop

→ short range

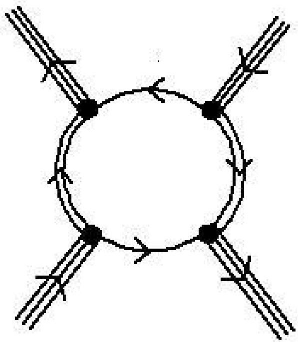
3. Structure of NN force

long and medium



chiral σ and π exchange

short range

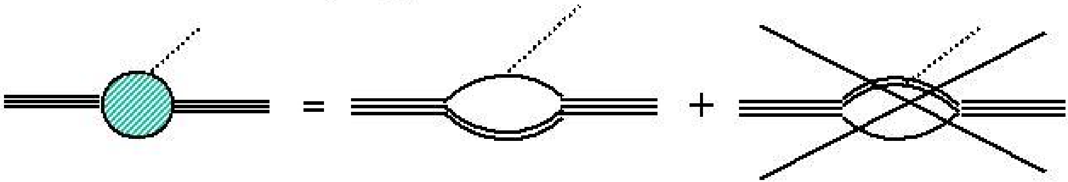


loop diagram in terms of

quark and diquark

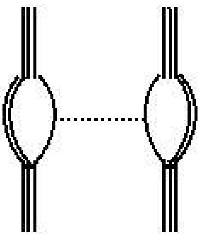
long range part ····σ, π exchange potential

πNN axial- coupling



- g_A =0.87
- GT relation is satisfied

OPEP potential



$$V_{OPEP}(\vec{q}) = \frac{g(\vec{q}^2)}{2M_N}\frac{\vec{\sigma}_1\cdot\vec{q}\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^2+m_\pi^2}\frac{g(\vec{q}^2)}{2M_N}$$

range ~ m_π

Short range part

We evaluate for various B.E.

1. the range, mass and strength of q-D loop
2. relation between these values and nulceon size

and compare our results to OBEP

- Fixed parameters

m_q=390, M_s=600 MeV, Λ=630 MeV

- Free parameter

$\tilde{G}$; quark – diquark coupling constant

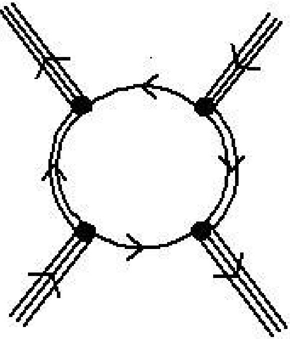
$\tilde{G}$ controls q - D binding energy, nulceon size, nucleon mass.

$(\mathrm{B.E.} = m_q + M_S - M_N)$

qDloop elastic case in the CoM system $\rightarrow |\vec{p}| = |\vec{p}\,'|$

$p_2 = (E_{\vec{p}\,'}, -\vec{p}\,')$

$p_2 = (E_{\vec{p}}, -\vec{p})$



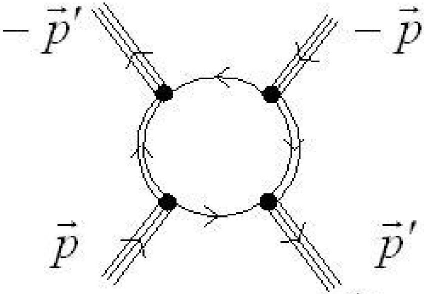
$p_1 = (E_{\vec{p}}, \vec{p})$

$p_1' = (E_{\vec{p}\,'}, \vec{p}\,')$

$$L_{NN} = -i\,\mathrm{tr}\,\overline{B}\Delta S B\,\overline{B}\Delta S B$$

$$= -i Z^2 N_c \int \frac{d^4 k}{(2\pi)^4}$$

$$\times \frac{\overline{B}(-\vec{p}\,')(\not{k} + m_q) B(\vec{p}) \overline{B}(\vec{p}\,')(p_2 - p_2' + \not{k} + m_q) B(-\vec{p})}{\left[(p_1 - k)^2 - M_S^2\right]\left[k^2 - m_q^2\right]\left[(p_2 - k)^2 - M_S^2\right]\left[(p_2 - p_1' + k)^2 - m_q^2\right]}$$



$$\begin{cases} \vec{q} = \vec{p}' - \vec{p} \\ \vec{P} = \vec{p}' + \vec{p} \end{cases}$$

$$L_{NN} = -i Z^2 N_c \int \frac{d^4 k}{(2\pi)^4}$$

$$\times \frac{\overline{B}(-\vec{p}')(\not{k} + m_q) B(\vec{p}) \overline{B}(\vec{p}')(\not{p}_2 - \not{p}_2' + \not{k} + m_q) B(-\vec{p})}{\left[(p_1 - k)^2 - M_S^2\right]\left[k^2 - m_q^2\right]\left[(p_2 - k)^2 - M_S^2\right]\left[(p_2 - p_1' + k)^2 - m_q^2\right]}$$

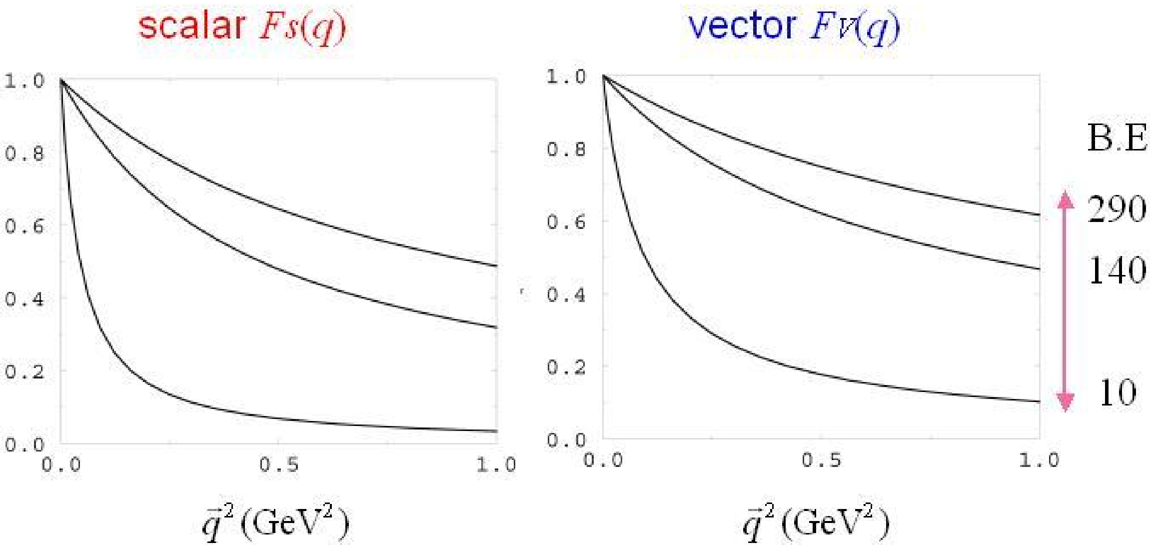
$$= F_S(\vec{q}^{\,2}, \vec{P}^2)\left(\overline{B}B\right)^2 - F_V(\vec{q}^{\,2}, \vec{P}^2)\left(\overline{B}\gamma^\mu B\right)^2 + \cdots$$

attractive　　　　repulsive

We investigat e in the case $\vec{P} = 0$

$$F_{S,V}(\vec{q}^{\,2}) \equiv F_{S,V}(\vec{q}^{\,2}, 0)$$

Form factors for various B.E



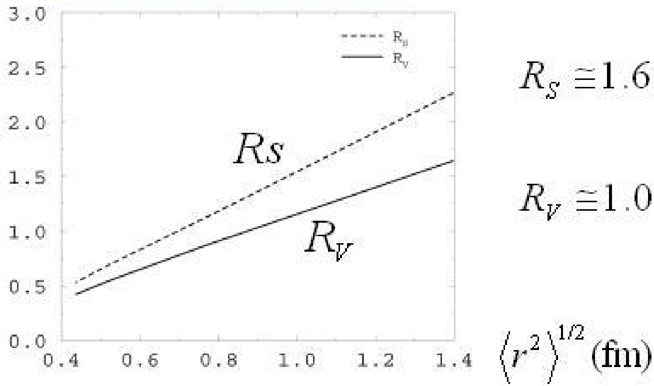
- B.E. becomes larger (nulceon becomes smaller), interaction ranges becomes shorter.

Interaction range, mass and strength

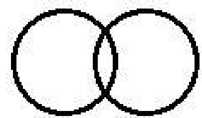
Range $$R^2_{S,V} = -6 \frac{1}{F_{S,V}(\vec{q}^{\,2})} \frac{\partial F_{S,V}(\vec{q}^{\,2})}{\partial \vec{q}^{\,2}} \bigg|_{\vec{q} \to 0}$$

Mass and Strength $$F_{S,V}(\vec{q}^{\,2}) = \frac{g^2_{S,V}}{\vec{q}^{\,2} + m^2_{S,V}}$$

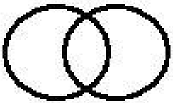
Range vs size



$$R_S \cong 1.6 \langle r^2 \rangle^{1/2}$$

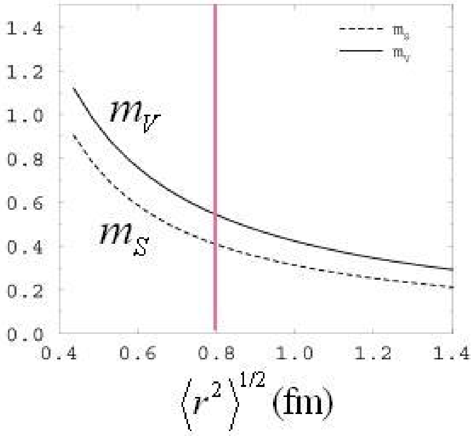


$$R_V \cong 1.0 \langle r^2 \rangle^{1/2}$$

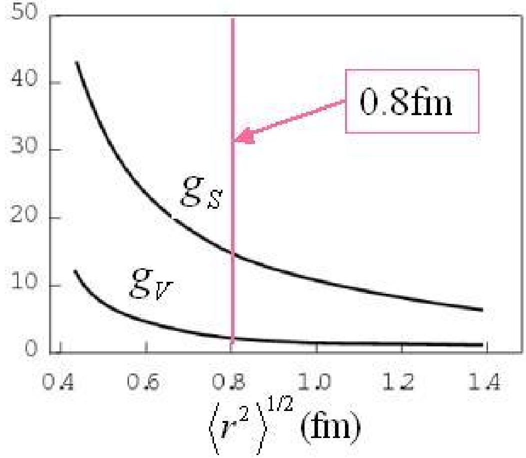


B.E. weak

Mass vs size



strength vs size



m_S = 610 MeV $\sim m_\sigma$

m_V = 790 MeV $\sim m_\omega$

$g_S = 23$ (10) (attractive)

$g_V = 4$ (13) (repusive)

4. Summary

- Hadronization of q-D model gives the composite meson- baryon Lagrangian.

- Nuclear force is composed of two part
  short range part → qD loop
  long and medium range part → chiral meson exchange

- qD loop is composed of
  scalar type (attraction) and vector type (repulsion).

- Interaction range and mass → good
  strength → scalar type is stronger than vector type.

- We need to include P-dependence, exchange term, a.v.diquark

  and calculate phase shifts, cross section…