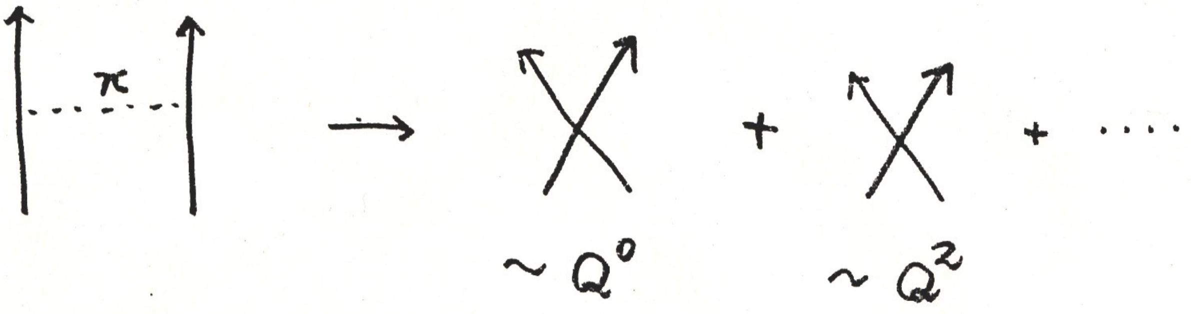
"How to integrate out heavy degrees of freedom in effective field theory"



Satoshi Nakamura

Introduction

What is effective field theory (EFT)?

EFT applied to few-nucleon system

S. Weinberg Phys. Lett. B **251**, 288 (1990)

- symmetries $\rightarrow$ general Lagrangian

- important degrees of freedom (d.o.f.)

  $\rightarrow$ explicit

  e.g. π, N for low-energy NN-scattering

- heavy d.o.f. $\xrightarrow[\text{integrate out}]{}$ contact interactions

- experimental data $\xrightarrow[\text{fixing couplings}]{}$ contact interactions

- counting rule

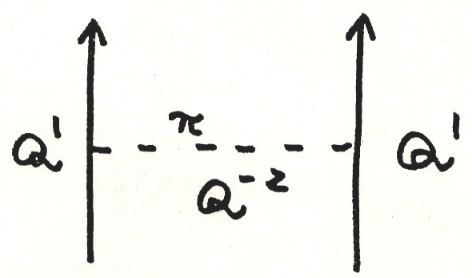
$$\text{diagram} \sim (Q/\Lambda)^{\nu}$$

$$\begin{cases} Q : \text{characteristic energy scale of system} \\ \Lambda : \text{energy scale of d.o.f. integrated out} \end{cases}$$

For few-nucleon system

counting rule $\longrightarrow$ $\begin{cases} \text{NN potential} \\ \text{transition operator} \end{cases}$

e.g. one-π-exchange potential



$\rightarrow \quad Q^0$

- cut off (Λ) is introduced

$\rightarrow$ high-momentum states are integrated out

$\rightarrow$ renormalization of contact interaction

☆ Construction of $\mathcal{L}_{eff}$

* assumed symmetries

$SU(2)_L \times SU(2)_R \times U(1)$

Lorentz invariance, hermitian

C, P, T

* building blocks

ψ, $D_\mu \psi$, U, $D_\mu U$, u_μ

ψ : nucleon field

D_μ : covariant derivative

$$U = \exp\left[i \frac{\boldsymbol{\tau} \cdot \boldsymbol{\pi}}{F_0} \right]$$

☆ Heavy - baryon formalism

$$P_{v\pm} = \frac{1 \pm \cancel{v}}{2} ,$$

v: arbitrary
$v^2 = 1$

$$N_v \equiv e^{imv\cdot x} P_{v+} \psi$$

$$H_v \equiv e^{imv\cdot x} P_{v-} \psi$$

$$\psi = e^{-imv\cdot x} (N_v + H_v)$$

For $v^0 = 1$, $\vec{v} = \vec{0}$, positive energy solution

$$N_v \propto \begin{pmatrix} \chi \\ 0 \end{pmatrix} e^{-i(E-m)t + i\vec{p}\cdot\vec{x}}$$

$$H_v \propto \begin{pmatrix} 0 \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi \end{pmatrix} e^{-i(E-m)t + i\vec{p}\cdot\vec{x}}$$

Dirac eq.

$$\left(i\not{D} - m_N + \frac{g_A}{2}\not{u}\gamma_5\right)\psi = 0$$

$$\left(i\not{D} + \frac{g_A}{2}\not{u}\gamma_5\right)N_v + \left(i\not{D} - 2m_N + \frac{g_A}{2}\not{u}\gamma_5\right) \times H_v = 0$$

Multiply P_{v+}, P_{v-} → two equations

eliminate H_v,

$$\mathcal{L}_{eff} = \bar{N}_v\left(i\, v\cdot D + g_A S_v \cdot u\right)N_v + \sum_{n=1}^{\infty}\frac{1}{(2m)^n}\mathcal{L}_{eff,n}$$

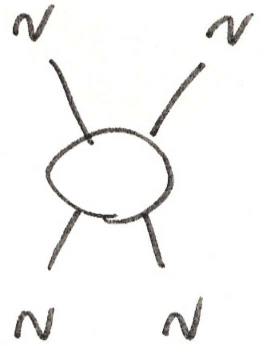
$$S_v^\mu \equiv \frac{i}{2}\gamma_5\sigma^{\mu\nu}v_\nu$$

for $v^\mu = (1, \vec{0})$, $S_v^0 = 0$ $\vec{S}_v = \begin{pmatrix}\vec{\sigma}/2 & 0 \\ 0 & \vec{\sigma}/2\end{pmatrix}$

$$
\begin{aligned}
\mathcal{L}_{eff} = \bar{N}_v \Big[& i\partial_0 - \frac{1}{4F_0^2} \boldsymbol{\tau} \cdot \boldsymbol{\pi} \times \dot{\boldsymbol{\pi}} \\
& - \frac{g_A}{F_0} \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} \boldsymbol{\pi} \cdot \boldsymbol{\tau} + \frac{\boldsymbol{\nabla}^2}{2m_N} \Big] N_v \\
& - \frac{1}{2} C_S \, \bar{N}_v N_v \, \bar{N}_v N_v \\
& - \frac{1}{2} C_T \, \bar{N}_v \boldsymbol{\sigma} N_v \cdot \bar{N}_v \boldsymbol{\sigma} N_v \\
& - C_1' \Big[(\bar{N}_v \boldsymbol{\nabla} N_v)^2 + (\overline{\boldsymbol{\nabla} N_v} \, N_v)^2 \Big] \\
& + \cdots
\end{aligned}
$$

☆ Counting rule

* a la Weinberg.

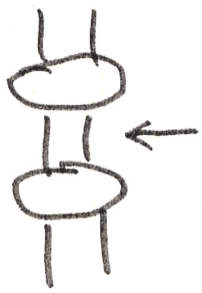


$\sim Q^{\nu}$

• reducible ? or irreducible ?

No pure nucleonic intermediate states.

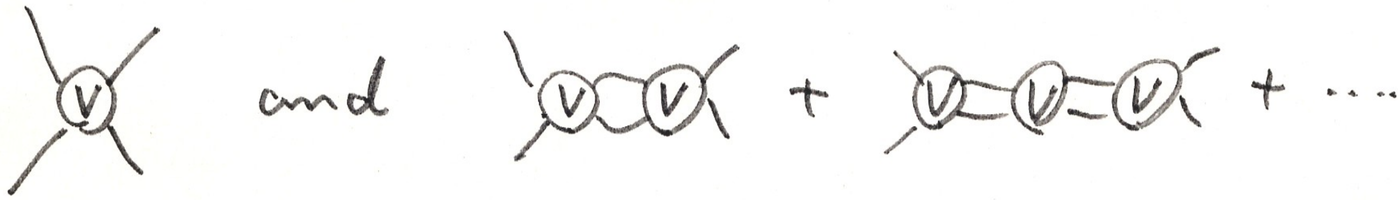
enhanced counting in reducible counting.



$\sim \frac{M}{Q^2} > \frac{1}{Q}$

The point is the large scattering length for $^1S_0$, $^3S_1$ scattering.

The large cancellation between



gives the large scattering length.

Resummation is essential.

- sum of irreducible graph → potential

- Solving Schrödinger eq

  → scattering amplitude.

* How to count irreducible graph ?

 $\sim Q^{\nu}$

$$\nu = \sum_i V_i \left(d_i - \frac{1}{2} p_i\right) - D + 3L$$

V_i : number of vertices of type i

d_i : " " derivatives "

p_i : " pion fields in type i

($1/\sqrt{E}$)

D : " intermediate states

L : " loops

- topological identities

$$D = \sum_i V_i - 1$$

$$L = I - \sum_i V_i + 1$$

$$2I + 2N = \sum_i V_i (p_i + n_i)$$

I : # of internal lines

n_i : 〃 nucleon fields on type i

$2N$: 〃 external nucleon lines

$$\nu = 2 - N + 2L$$

$$+ \sum_i V_i \underbrace{\left(d_i + \tfrac{1}{2} n_i - 2 \right)}$$

non-negative for all interactions allowed by chiral symmetry !

V_{NN}^{LO} $=$  $+$ 

$\sim Q^0$

V_{NN}^{NLO} $=$  $+$ 

$+$  $+$ 

$+$  $+ \cdots\cdots$

$\sim Q^2$

• 3 N force

$\sim Q^{-1}$ $\qquad$ $\sim Q^{0}$

3 N force is more important ?

δ- function in the disconnected graph

$\rightarrow Q^{-3}$

$\Rightarrow \sim Q^{-1}$ $\qquad$ $\sim Q^{-3}$

2 N force is more important !

$$\nu = 4 + 2L - 2C + \sum_i V_i \left(d_i + \frac{n_i}{2} - 2 \right)$$

∘ formal problem in the Weinberg counting.

At LO,

$\sim m_\pi^2 I_D$ $\qquad\qquad$ $\sim p^2 I_D$

I_D : divergent integral.

These divergences must be absorbed by counter terms which, however, appear in NLO. Renormalisation cannot be done order by order.

* Other counting

- KSW counting.

Kaplan, Savage, Wise PLB <u>424</u> 390 (1998)

NPB <u>534</u> 329 (1998)

dimensional regularization

plus power divergence subtraction (PDS)

- Unitary transformation.

Epelbaum, Glöckle, Meißner

NPA <u>637</u> 107 (1998)

EFTs for different energy scales and relation between them

		d.o.f.
◦ low-energy NN-scattering	:	π, N
◦ very low-energy NN-scattering	:	N

$$\frac{1}{m_\pi^2 + Q^2} \sim \underbrace{\frac{1}{m_\pi^2}}_{\sim Q^0} - \underbrace{\frac{1}{m_\pi^2}\frac{Q^2}{m_\pi^2}}_{\sim Q^2} + \cdots$$

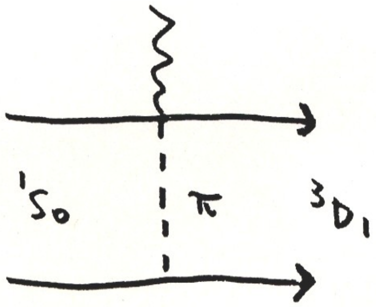
Why EFT (π) $\longrightarrow$ EFT ($\not\pi$) ?

* quantitative test of basic idea of EFT

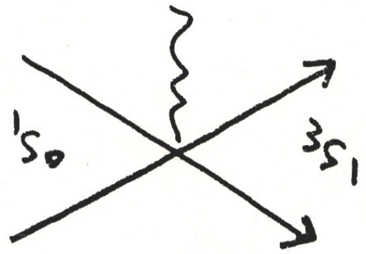
* understanding difference between

  EFT (π) and EFT ($\not\pi$)

  - electroweak current (two-body current)



EFT (π)



EFT ($\not\pi$)

Is the difference a result of integrating out π ?

How $\mathrm{EFT}(\pi) \to \mathrm{EFT}(\not\pi)$?

NOTE !

Two EFTs are defined

on different model - spaces

$$\Lambda_{\mathrm{EFT}(\not\pi)} < m_\pi < \Lambda_{\mathrm{EFT}(\pi)}$$

For $\mathrm{EFT}(\pi) \longrightarrow \mathrm{EFT}(\not\pi)$,

1. model-space reduction

   integrating out high-momentum states

2. integrating out π

How to reduce model - space ?

In this work !

model-space reduction $\xrightarrow{\text{applied}}$ NN potential

$\Downarrow$

NN potential with model-space for EFT ($\not\pi$)

[Is the potential well simulated by contact
interactions in accordance with basic of EFT ?]

$\Downarrow$ we address

* What is favorable model-space reduction ?

* How should EFT-based potential be ?

In the following :

- model-space reduction schemes
- result
- conclusion

Model - space reduction

Paths for model-space reduction

$$\begin{tikzcd} \mathcal{L}_H \arrow[r] \arrow[d] & \mathcal{L}_L \arrow[d] \\ V_H \arrow[r] & V_L \end{tikzcd}$$

<u>$\mathcal{L}_H \to \mathcal{L}_L$</u>

$$Z = \int \mathcal{D}\psi_L \mathcal{D}\psi_H \, e^{i\int \mathcal{L}_H} = \int \mathcal{D}\psi_L \, e^{i\int \mathcal{L}_L}$$

$\mathcal{L}$: cutoff for single particle momentum

V : cutoff for NN relative momentum

$V_H \to V_L$

$*$ consistency with $\mathcal{L}_H \to \mathcal{L}_L$

$\Downarrow$

Green's function is invariant under $\delta\Lambda$

$$\frac{1}{E-H+i\epsilon} = \frac{1}{E-H_0+i\epsilon} + \frac{1}{E-H_0+i\epsilon} V \frac{1}{E-H_0+i\epsilon} + \cdots$$

$$= \frac{1}{E-H_0+i\epsilon} + \frac{1}{E-H_0+i\epsilon} T \frac{1}{E-H_0+i\epsilon}$$

$\Rightarrow$ full off-shell T-matrix is invariant under $\delta\Lambda$

$\Rightarrow$ renormalization group equation for V

M. Birse et al. Phys. Lett. B 464, 169 (1999)

$$\frac{\partial V(k',k;p^2)}{\partial\Lambda} = \frac{V(k',\Lambda;p^2)\,V(\Lambda,k;p^2)}{1-(p/\Lambda)^2}$$

$\langle$ on-shell momentum (p) enters $\rangle$

Lippmann - Schwinger eq. for

model - space NN scattering

(full off-shell)

$$T(k', k; p^2) = V(k', k)$$

$$+ \int_0^\Lambda d\bar{p}\, \bar{p}^2\; V(k' \bar{p}) \frac{1}{E_p - E_{\bar{p}} + i\epsilon} T(\bar{p} k; p^2)$$

E_p : on-shell energy

$$\frac{\partial T(k', k; p^2)}{\partial \Lambda} = 0$$

$$\Rightarrow \frac{\partial V(k', k; p^2)}{\partial \Lambda} = \frac{V(k' \Lambda; p^2)\, V(\Lambda k; p^2)}{1 - (p/\Lambda)^2}$$

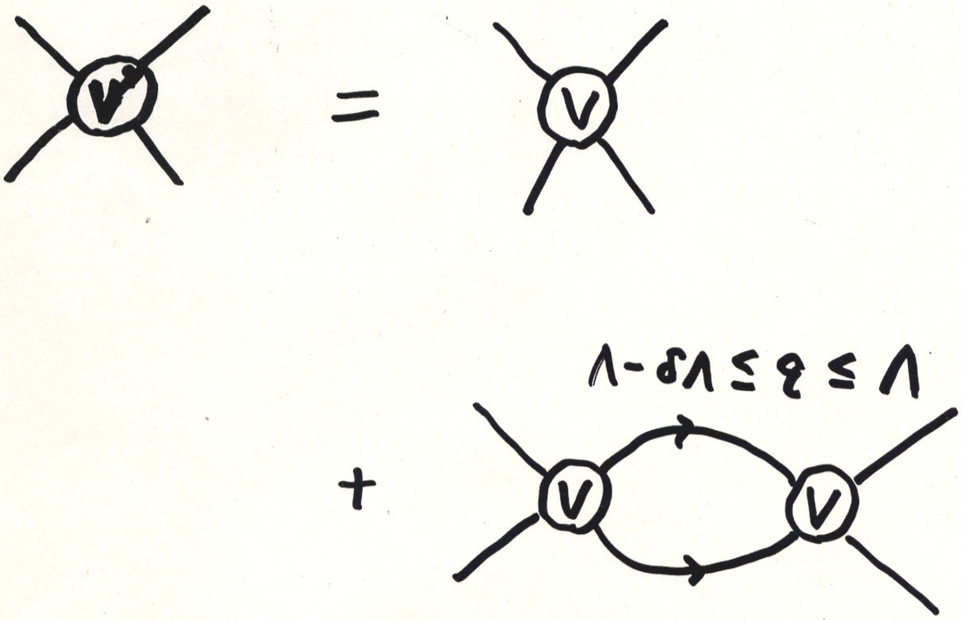
<on-shell energy dependence enters>

* Evolution of NN potential

Changing cutoff

$$\Lambda \longrightarrow \Lambda' = \Lambda - |\delta\Lambda|$$

$$V \longrightarrow V' = V + \delta V$$



* possible alternatives for $V_H \to V_L$

- $V_{low\text{-}k}$ S. Bogner et al.

Nucl. Phys. A684, 432c (2001)

$$\frac{\partial T(k', p; p^2)}{\partial \Lambda} = 0$$

$$\Rightarrow \quad \frac{\partial V_{low\text{-}k}(k', k)}{\partial \Lambda} = \frac{V_{low\text{-}k}(k' \Lambda)\, T(\Lambda\, k; \Lambda^2)}{1 - (k/\Lambda)^2}$$

⟨ non-Hermite Hamiltonian ⟩

- V_{FSTO} E. Epelbaoum et al.

Phys. Lett. B 439, 1 (1998)

$$\mathcal{H} = U^\dagger H U$$

$$P = \int_0^\Lambda dp\ |p\rangle\langle p| \qquad Q = \int_\Lambda^\infty dp\ |p\rangle\langle p|$$

$$P \mathcal{H} Q = Q \mathcal{H} P = 0$$

$$V_{FSTO} = P \mathcal{H} P - P H_0 P$$

⟨ mixing of nucleon states ⟩

Strategy

$$V_{EFT(\pi)} \xrightarrow[\text{reduction}]{\text{model-space}} V_{EFT(\not\pi)}$$

- formal problem in $V_{EFT(\pi)}$

○ $V_{ph} \longrightarrow V_{\text{"}EFT(\pi)\text{"}} \longrightarrow V_{\text{"}EFT(\not\pi)\text{"}}$

(phenomenological NN potential)

Why does it make sense ?

$$\boxed{\text{model-independent model-space potential } (V_{MM}) \text{ for } \Lambda \lesssim 400 \text{ MeV}}$$

$\Rightarrow$ V_{EFT} should reproduce V_{MM} !

- $\mathcal{L}_{eff}$ and a counting rule just give a parametrization of V_{MM}

- usefulness of EFT

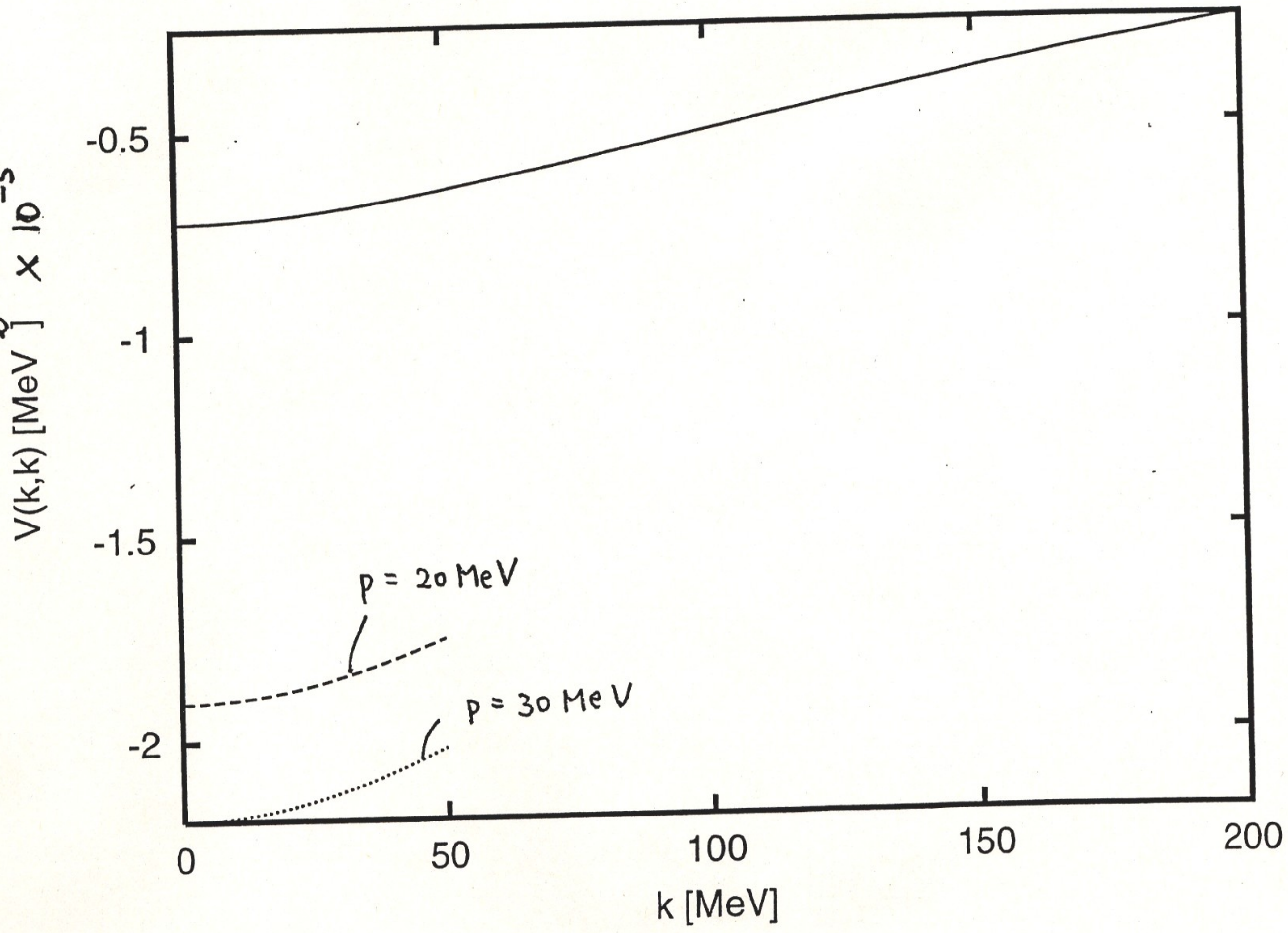
= How well V_{MM} can be simulated by EFT-based parametrization

Result

$$V_{EFT(\pi)} \longrightarrow V_{EFT(\not\pi)}$$

following the RG (full off-shell T-matrix invariance)

$^1S_0$ proton - neutron scattering

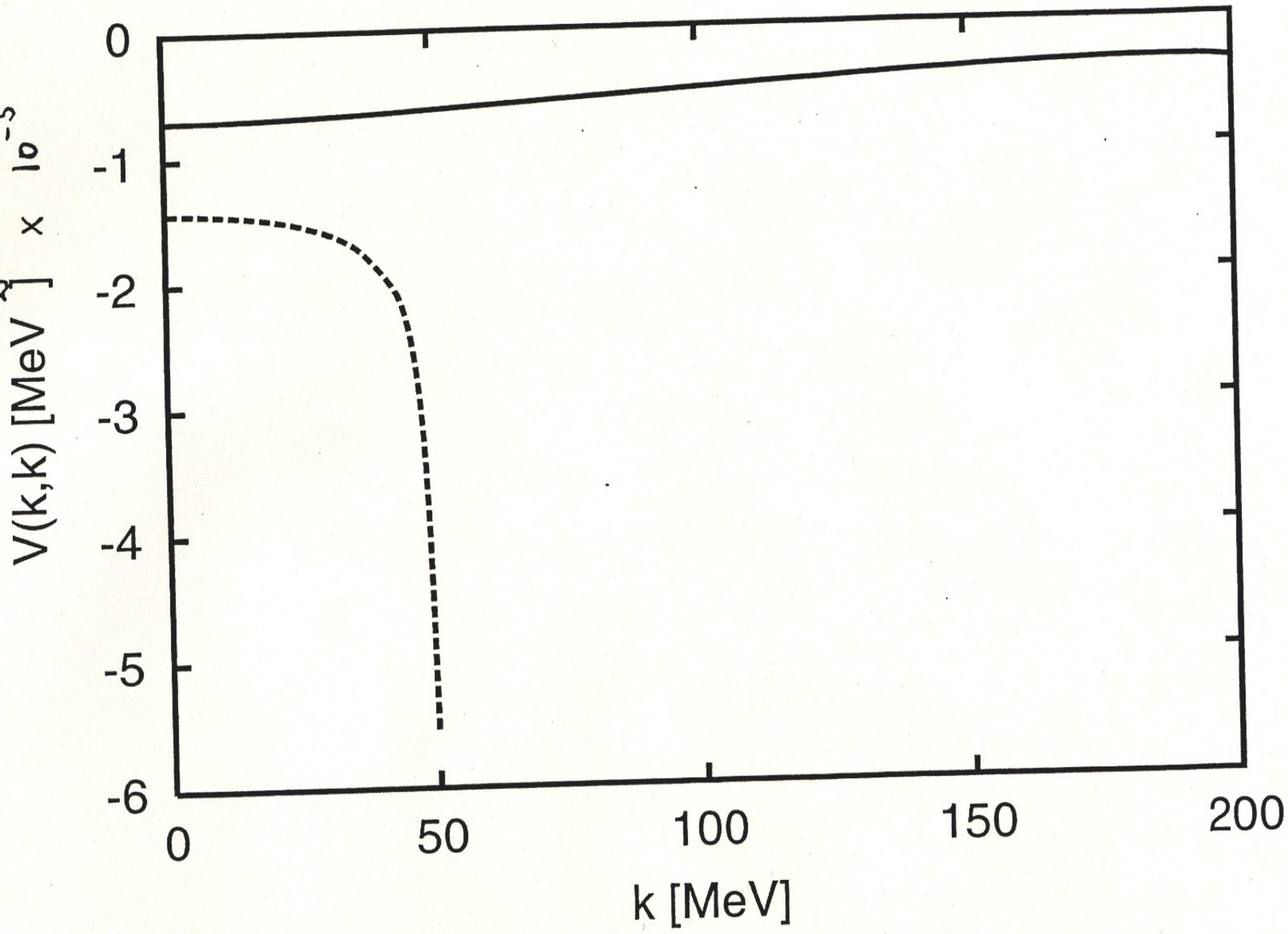


- large on-shell energy dependence for small Λ
- good simulation of V"$_{EFT(\not\pi)}$" by contact interactio

$V_{EFT(\pi)} \longrightarrow V_{EFT(\cancel{\pi})}$

following $V_{low\text{-}k}$ (half on-shell T-matrix invariance)

$^1S_0$ proton - neutron scattering



- $V_{\text{"}EFT(\cancel{\pi})\text{"}}$ cannot be simulated by contact interactions

- V_{FSTO} gives a similar result

$\boxed{\text{Discussion \& Conclusion}}$

favorable model-space reduction ?

RG eq. (full off-shell T-matrix invariance)

satisfies the basics of EFT :

- consistency with $\mathcal{L}_H \to \mathcal{L}_L$
- good description by contact interactions

$V_{low\text{-}k}$ and V_{FSTO} fail to satisfy the basics

The RG eq. is favorable model-space reduction scheme.

Is on-shell energy dependence OK ?

How V_{EFT} should be ?

On-shell energy dependence is a natural consequence of integrating out the nucleon high-momentum states !

* previous study on energy-dependence

- nucleon recoil correction

  C. Ordóñez et al. Phys. Rev. C <u>53</u> 2086 (1996)

  E. Epelbaoum et al. Nucl. Phys. A<u>637</u> 107 (1998)

- nucleon states integrated out

  M. Birse et al. Phys. Lett. B <u>464</u> 169 (1999)

  K.G. Richardson hep-ph/0008118

* In this work,

importance of energy-dependence in V_{EFT} is clarified by connecting two V_{EFT}'s defined on different model-spaces.