Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$-Decay

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If the decay $(A,Z) \to (A,Z+2) + e^- + e^-$ ($(\beta\beta)_{0\nu}$-decay) will be observed, the question will inevitably arise:

Which mechanism is triggering the decay?

How many mechanisms are involved?

"Standard Mechanism": light Majorana ν exchange.

Fundamental parameter - the effective Majorana mass:

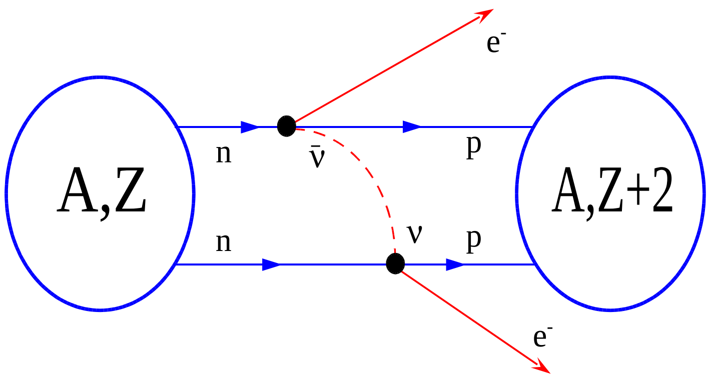
$$< m > = \sum_j^{light} \left(U_{ej}\right)^2 m_j \; , \text{ all } m_j \geq 0 \; ,$$

U - the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) neutrino mixing matrix, m_j - the light Majorana neutrino masses, $m_j \lesssim 1$ eV.

U - CP violating, in general: $(U_{ej})^2 = |U_{ej}|^2 e^{i\alpha_{j1}}$, $j = 2,3$, α_{21}, α_{31} - Majorana CPV phases.

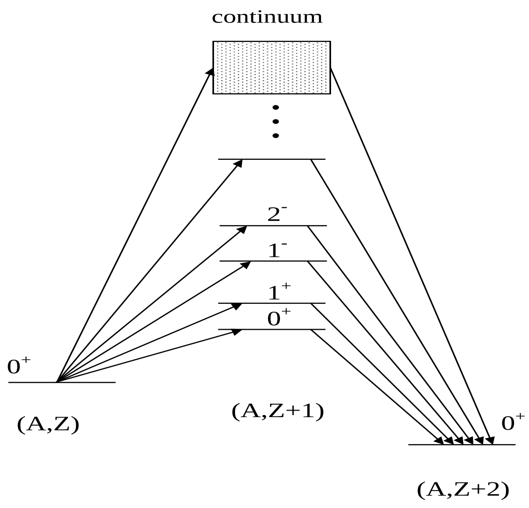
S.M. Bilenky, J. Hosek, S.T.P.,1980

Nuclear $0\nu\beta\beta$-decay



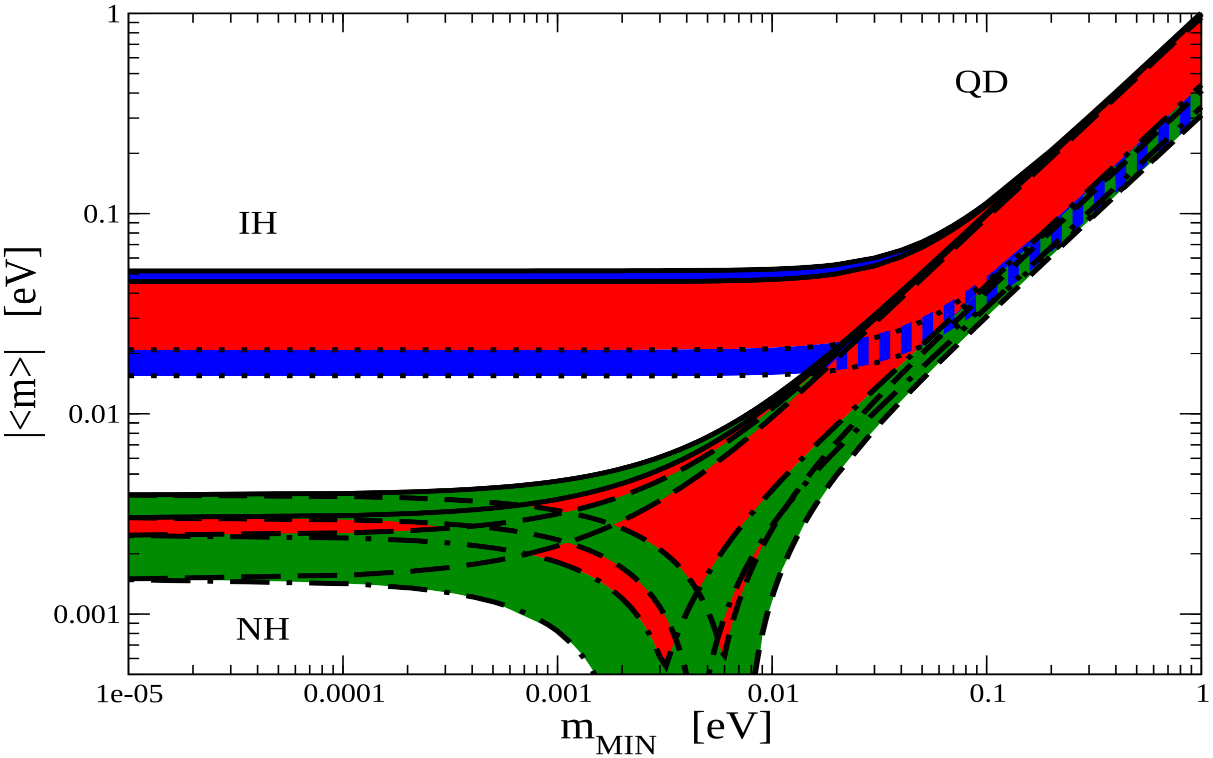
strong in-medium modification of the basic process

$dd \to uue^- e^- (\overline{\nu_e}\overline{\nu_e})$



virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

V. Rodin



S. Pascoli, S.T.P., 2007 (updated by S. Pascoli in 2010)

$\Delta m^2_{21} = 7.65 \times 10^{-5}$ eV^2, $1\sigma(\Delta m^2_{21}) = 3\%$;
$\sin^2\theta_{21} = 0.304$, $1\sigma(\sin^2\theta_{21}) = 7\%$;
$|\Delta m^2_{31}| = 2.4 \times 10^{-3}$ eV^2, $1\sigma(|\Delta m^2_{31}|) = 5\%$;
$\sin^2\theta_{13} = 0.01$.

$2\sigma(|<m>|\ )$ used.

A number of different mechanisms possible.

For a given mechanism κ we have in the case of $(A,Z) \to (A,Z+2) + e^- + e^-$:

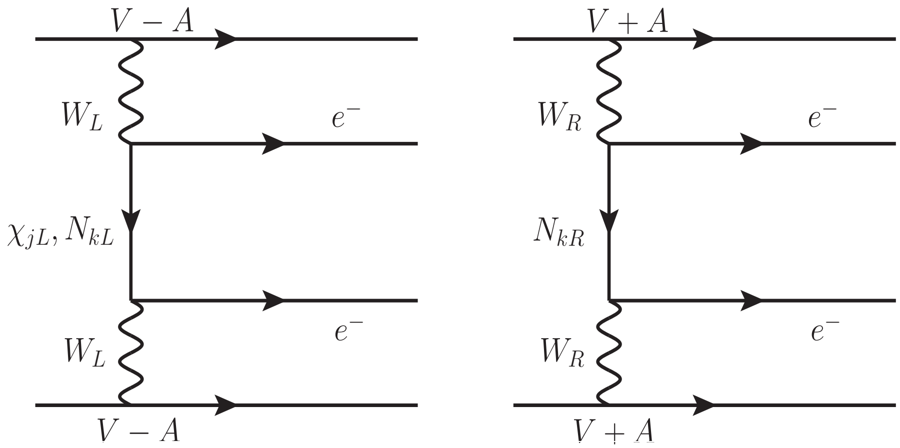
$$\frac{1}{T^{0\nu}_{1/2}} = |\eta^{LNV}_\kappa|^2 \; G^{0\nu}(E_0,Z)|{M'}^{0\nu}_\kappa|^2 \,,$$

η^{LNV}_κ - the fundamental LNV parameter characterising the mechanism κ,

$G^{0\nu}(E_0,Z)$ - phase-space factor (includes $g_A^4 = (1.25)^4$, as well as $R^{-2}(A)$, $R(A) = r_0 A^{1/3}$ with $r_0 = 1.1\ fm$),

${M'}^{0\nu}_\kappa = (g_A/1.25)^2 M^{0\nu}_\kappa$ - NME (includes $R(A)$ as a factor).

Different Mechanisms of $(\beta\beta)_{0\nu}$-Decay



Light Majorana Neutrino Exchange

$$\eta_\nu = \frac{<m>}{m_e}\,.$$

Heavy Majorana Neutrino Exchange Mechanisms

(V-A) Weak Interaction, LH N_k, $M_k \gtrsim$ 10 GeV:

$\eta^L_N \;=\; \sum_k^{heavy}\; U^2_{ek}\,\frac{m_p}{M_k}$, m_p - proton mass , U_{ek} - CPV .

(V+A) Weak Interaction, RH N_k, $M_k \gtrsim$ 10 GeV:

$$\eta_N^R \;=\; \left(\frac{M_W}{M_{WR}}\right)^4 \Sigma_k^{heavy} \; V_{ek}^2 \frac{m_p}{M_k}$$; V_{ek}: $N_k - e^-$ in the CC .

$M_W \cong$ 80 GeV; $M_{WR} \gtrsim$ 2.5 TeV; V_{ek} - CPV, in general.

A comment.

(V-A) CC Weak Interaction:

$\bar{e}(1+\gamma_5)e^c \equiv 2\bar{e_L}\,(e^c)_R$, $e^c = C(\bar{e})^T$,

C - the charge conjugation matrix.

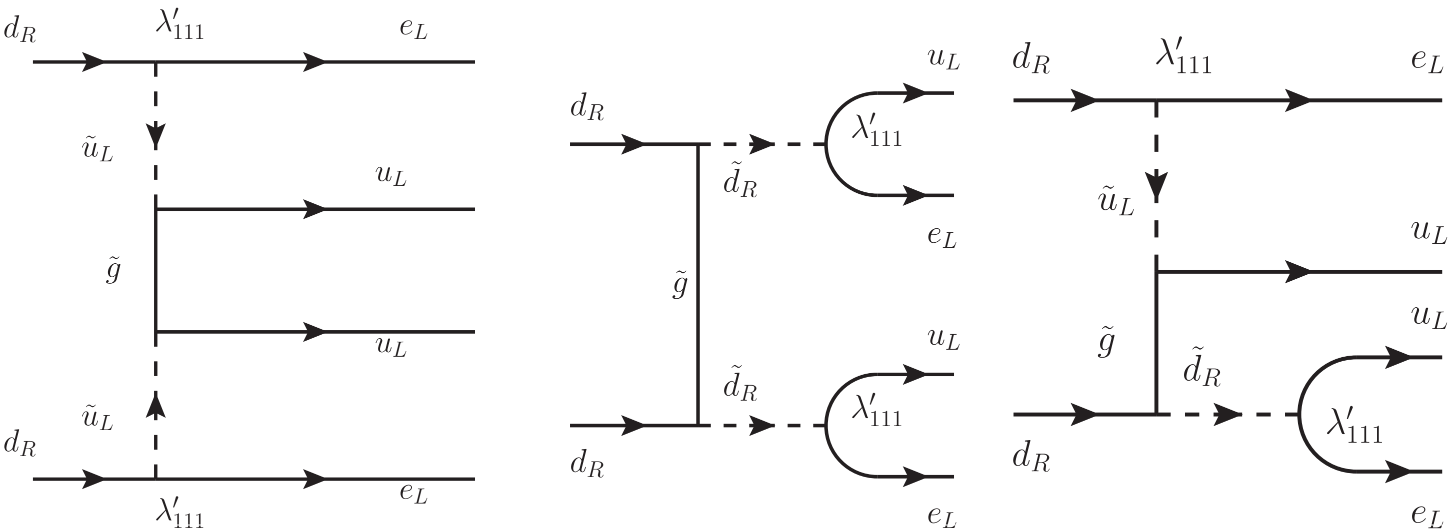
(V+A) CC Weak Interaction:

$\bar{e}(1-\gamma_5)e^c \equiv 2\bar{e_R}\,(e^c)_L$.

The interference term: $\propto m_e$, suppressed.

A. Halprin, S.T.P., S.P. Rosen, 1983

SUSY Models with R-Parity Non-conservation



$$
\begin{aligned}
\mathcal{L}_{R_p} \;&=\; \lambda'_{111}\Big[(\bar{u}_L\;\bar{d}_L)\begin{pmatrix} e^c_R \\ -\nu^c_{eR}\end{pmatrix}\tilde{d}_R + (\bar{e}_L\;\bar{\nu}_{eL})d_R\begin{pmatrix}\tilde{u}^*_L \\ -\tilde{d}^*_L\end{pmatrix} \\
&+\; (\bar{u}_L\;\bar{d}_L)d_R\begin{pmatrix}\tilde{e}^*_L \\ -\tilde{\nu}^*_{eL}\end{pmatrix}\Big] + h.c.
\end{aligned}
$$

The Gluino Exchange Dominance Mechanism

$$\eta_{\lambda'} = \frac{\pi\alpha_s}{6}\frac{\lambda'^2_{111}}{G_F^2 m^4_{\tilde{d}_R}}\frac{m_p}{m_{\tilde{g}}}\left[1+\left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}}\right)^2\right]^2 ,$$

G_F - the Fermi constant, $\alpha_s = g_3^2/(4\pi)$, g_3 - the SU(3)$_c$ gauge coupling constant, $m_{\tilde{u}_L}$, $m_{\tilde{d}_R}$ and $m_{\tilde{g}}$ - the masses of the LH u-squark, RH d-squark and gluino.

The Squark-Neutrino Mechanism

$$\eta_{\tilde{q}} = \sum_k \frac{\lambda'_{11k}\lambda'_{1k1}}{2\sqrt{2}G_F}\sin 2\theta^d_{(k)}\left(\frac{1}{m^2_{\tilde{d}_1(k)}} - \frac{1}{m^2_{\tilde{d}_2(k)}}\right) ,$$

$d_{(k)} = d, s, b$; θ^d: $\tilde{d}_{kL} - \tilde{d}_{kR}$ - mixing (3 light Majorana neutrinos assumed).

The $2e^-$ current in both mechanisms:

$\bar{e}(1+\gamma_5)e^c \equiv 2\bar{e_L}\,(e^c)_R$, as in the "standard" mechanism.

Example: $(\beta\beta)_{0\nu}$-Decay and TeV Scale See-Saw Mechanism

Type I see-saw mechanism, heavy Majorana neutrinos N_j at the TeV scale:

$m_\nu \simeq -\, M_D \widehat{M}_N^{-1} M_D^T$, $\widehat{M} = \text{diag}(M_1, M_2, M_3)$, $M_j \sim (100 - 1000)$ GeV.

$$\mathcal{L}_{CC}^N = -\frac{g}{2\sqrt{2}}\,\overline{\ell}\,\gamma_\alpha\,(RV)_{\ell k}(1-\gamma_5)\,N_k\,W^\alpha \;+\; \text{h.c.}\,,$$

$$\mathcal{L}_{NC}^N = -\frac{g}{2c_w}\,\overline{\nu_{\ell L}}\,\gamma_\alpha\,(RV)_{\ell k}\,N_{kL}\,Z^\alpha \;+\; \text{h.c.}$$

The exchange of virtual N_j gives a contribution to $|<m>|$:

$$|<m>| \;\cong \left|\sum_i (U_{PMNS})^2_{ei}\, m_i - \sum_k f(A, M_k)\,(RV)^2_{ek}\,\frac{(0.9\;\text{GeV})^2}{M_k}\right|\,,$$

$$f(A, M_k) \cong f(A)\,.$$

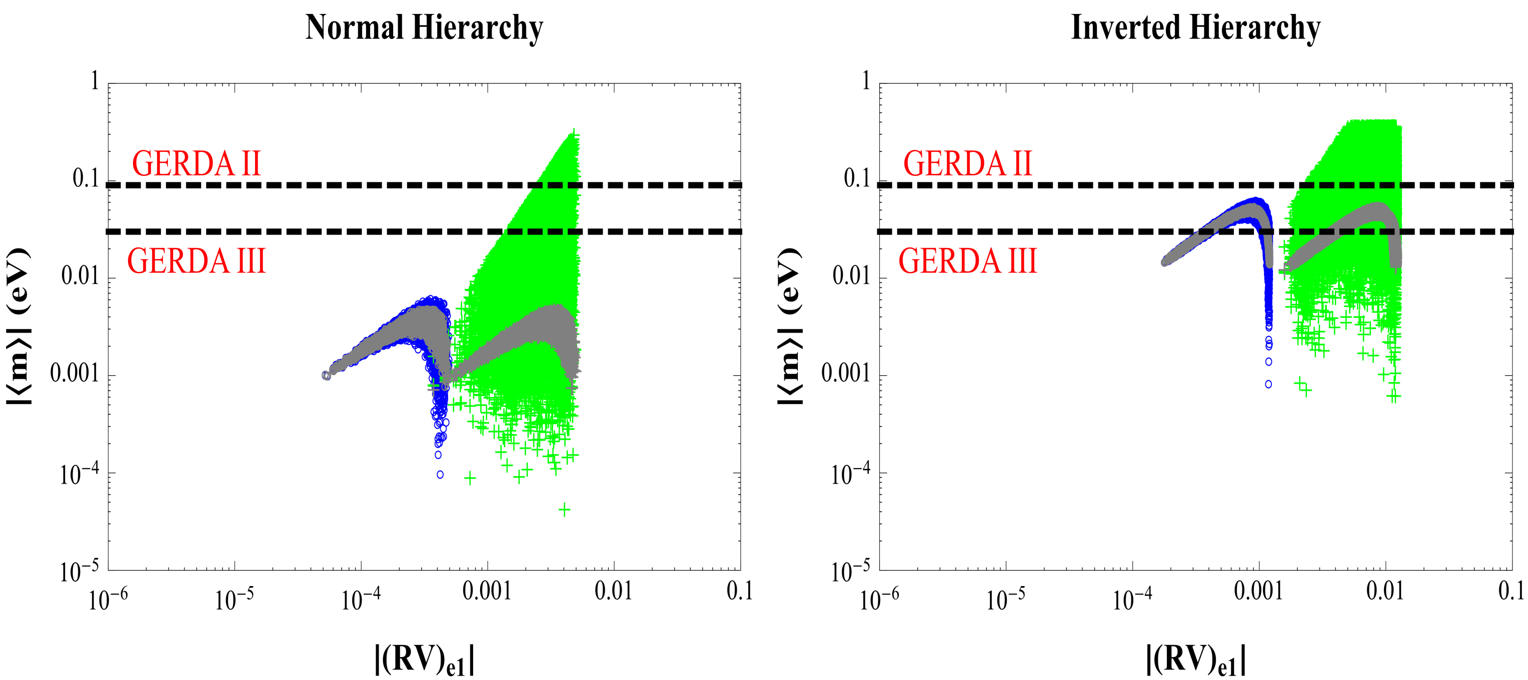
For, e.g., ^{48}Ca, ^{76}Ge, ^{82}Se, ^{130}Te and ^{136}Xe, the function $f(A)$ takes the values $f(A) \cong 0.033$, 0.079, 0.073, 0.085 and 0.068, respectively.

- **All low-energy constraints can be satisfied in a scheme with two heavy Majorana neutrinos $N_{1,2}$, which form a pseudo-Dirac pair:**
$M_2 = M_1(1+z)$, $0 < z \ll 1$.

- **Only NH and IH ν mass spectra possible.**

- **The Predictions for $|<m>|$ can be modified considerably.**

A. Ibarra, E. Molinaro, S.T.P., 2010 and 2011



$|<m>|$ vs $|(RV)_{e1}|$ for ^{76}Ge in the cases of NH (left panel) and IH (right panel) light neutrino mass spectrum, for $M_1 = 100$ GeV and $i)$ $y = 0.001$ (blue), $ii)$ $y = 0.01$ (green). The gray markers correspond to $|<m>^{\rm std}| = |\sum_i (U_{PMNS})^2_{ei}\, m_i|$.

A. Ibarra, E. Molinaro, S.T.P., 2010 and 2011

Illustrative examples:

$T^{0\nu}_{1/2}(^{76}Ge)$, $T^{0\nu}_{1/2}(^{100}Mo)$, $T^{0\nu}_{1/2}(^{130}Te)$ used as input,

$T^{0\nu}_{1/2}(^{76}Ge) \geq 1.9 \times 10^{25} y\,, T^{0\nu}_{1/2}(^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25} y$

(lower limit: Heidelberg-Moscow collab., 2001; value - Klapdor-Kleingrothaus et al., 2004.)

$5.8 \times 10^{23} y \leq T^{0\nu}_{1/2}(^{100}Mo) \leq 5.8 \times 10^{24} y$ (lower limit - NEMO3)

$3.0 \times 10^{24} y \leq T^{0\nu}_{1/2}(^{130}Te) \leq 3.0 \times 10^{25} y$ (lower limit-CUORICINO)

Constraints from ^{3}H β-decay data

Light ν exchange + "nonstandard" mechanisms

Moscow, Mainz: $m(\bar{\nu}_e) < 2.3$ eV; $|\eta_\nu|^2 \times 10^{10} < 0.21\,.$

KATRIN: $m(\bar{\nu}_e) < 0.2$ eV; $|\eta_\nu|^2 \times 10^{10} < 1.6 \times 10^{-3}\,.$

Calculation of the NMEs for ^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te

The NME: obtained within the Self-consistent Renormalized Quasiparticle Random Phase Approximation (SRQRPA) (takes into account the Pauli exclusion principle and conserves the mean particle number in correlated ground state).
Two choices of single-particle basis used:
i) the intermediate size model space has 12 levels (oscillator shells N=2-4) for ^{76}Ge and ^{82}Se, 16 levels (oscillator shells N=2-4 plus the f+h orbits from N=5) for ^{100}Mo and 18 levels (oscillator shells N=3,4 plus f+h+p orbits from N=5) for ^{130}Te;
ii) the large size single particle space contains 21 levels (oscillator shells N=0-5) for ^{76}Ge, ^{82}Se and ^{100}Mo, and 23 levels for ^{130}Te (N=1-5 and i orbits from N=6).
The single particle energies: obtained by using a Coulomb–corrected Woods–Saxon potential. Two-body G-matrix elements we derived from the Argonne and the Charge Dependent Bonn (CD-Bonn) one-boson exchange potential within the Brueckner theory. The calculations: for $g_{ph} = 1.0$. The particle-particle strength parameter g_{pp} of the SRQRPA is fixed by the data on the two-neutrino double beta decays.

Table

The phase-space factor $G^{0\nu}(E_0, Z)$ and the nuclear matrix elements ${M'}^{0\nu}_{\nu}$ (light Majorana neutrino exchange mechanism), ${M'}^{0\nu}_{N}$ (heavy Majorana neutrino exchange mechanism), ${M'}^{0\nu}_{\lambda'}$ (mechanism of gluino exchange dominance in SUSY with trilinear R-parity breaking term) and ${M'}^{0\nu}_{\tilde{q}}$ (squark-neutrino mechanism) for the $(\beta\beta)_{0\nu}$-decays of ^{76}Ge, ^{100}Se, ^{100}Mo and ^{130}Te. The nuclear matrix elements were obtained within the Self-consistent Renormalized Quasiparticle Random Phase Approximation (SRQRPA).

Nuclear transition	$G^{0\nu}(E_0, Z)$ $[y^{-1}]$	NN pot.	m.s.	$\lvert M'^{0\nu}_{\nu}\rvert$ $g_A =$ 1.0	$\lvert M'^{0\nu}_{\nu}\rvert$ $g_A =$ 1.25	$\lvert M'^{0\nu}_{N}\rvert$ $g_A =$ 1.0	$\lvert M'^{0\nu}_{N}\rvert$ $g_A =$ 1.25	$\lvert M'^{0\nu}_{\lambda'}\rvert$ $g_A =$ 1.0	$\lvert M'^{0\nu}_{\lambda'}\rvert$ $g_A =$ 1.25	$\lvert M'^{0\nu}_{\tilde{q}}\rvert$ $g_A =$ 1.0	$\lvert M'^{0\nu}_{\tilde{q}}\rvert$ $g_A =$ 1.25
$^{76}Ge \rightarrow {}^{76}Se$	7.98 10^{-15}	Argonne	intm.	3.85	4.75	172.2	232.8	387.3	587.2	396.1	594.3
			large	4.39	5.44	196.4	264.9	461.1	699.6	476.2	717.8
		CD-Bonn	intm.	4.15	5.11	269.4	351.1	339.7	514.6	408.1	611.7
			large	4.69	5.82	317.3	411.5	392.8	595.6	482.7	727.6
$^{82}Se \rightarrow {}^{82}Kr$	3.53 10^{-14}	Argonne	intm.	3.59	4.54	164.8	225.7	374.5	574.2	379.3	577.9
			large	4.18	5.29	193.1	262.9	454.9	697.7	465.1	710.2
		CD-Bonn	intm.	3.86	4.88	258.7	340.4	328.7	503.7	390.4	594.5
			large	4.48	5.66	312.4	408.4	388.0	594.4	471.8	719.9
$^{100}Mo \rightarrow {}^{100}Ru$	5.73 10^{-14}	Argonne	intm.	3.62	4.39	184.9	249.8	412.0	629.4	405.1	612.1
			large	3.91	4.79	191.8	259.8	450.4	690.3	449.0	682.6
		CD-Bonn	intm.	3.96	4.81	298.6	388.4	356.3	543.7	415.9	627.9
			large	4.20	5.15	310.5	404.3	384.4	588.6	454.8	690.5
$^{130}Te \rightarrow {}^{130}Xe$	5.54 10^{-14}	Argonne	intm.	3.29	4.16	171.6	234.1	385.1	595.2	382.2	588.9
			large	3.34	4.18	176.5	239.7	405.5	626.0	403.1	620.4
		CD-Bonn	intm.	3.64	4.62	276.8	364.3	335.8	518.8	396.8	611.1
			large	3.74	4.70	293.8	384.5	350.1	540.3	416.3	640.7

Important feature of the NMEs

For each mechanism κ discussed, the NMEs for the nuclei considered differ relatively little:

$|M'_{\kappa i} - M'_{\kappa j}| << M'_{\kappa i}, M'_{\kappa j}$, typically

$\frac{|M'_{\kappa i} - M'_{\kappa j}|}{0.5(M'_{\kappa i} + M'_{\kappa j})} \sim 0.1$, $i \neq j = ^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te$.

Two "Non-Interfering" Mechanisms

Example: light LH and heavy RH Majorana ν exchanges

The corresponding LNV parameters, $|\eta_\nu|$ and $|\eta_R|$ - from "data" on $T^{0\nu}_{1/2}$ of two nuclei:

$$\frac{1}{T_1 G_1} = |\eta_\nu|^2 |M'^{0\nu}_{1,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{1,N}|^2,$$
$$\frac{1}{T_2 G_2} = |\eta_\nu|^2 |M'^{0\nu}_{2,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{2,N}|^2.$$

The solutions read:

$$|\eta_\nu|^2 = \frac{|M'^{0\nu}_{2,N}|^2/T_1 G_1 - |M'^{0\nu}_{1,N}|^2/T_2 G_2}{|M'^{0\nu}_{1,\nu}|^2 |M'^{0\nu}_{2,N}|^2 - |M'^{0\nu}_{1,N}|^2 |M'^{0\nu}_{2,\nu}|^2},$$
$$|\eta_R|^2 = \frac{|M'^{0\nu}_{1,\nu}|^2/T_2 G_2 - |M'^{0\nu}_{2,\nu}|^2/T_1 G_1}{|M'^{0\nu}_{1,\nu}|^2 |M'^{0\nu}_{2,N}|^2 - |M'^{0\nu}_{1,N}|^2 |M'^{0\nu}_{2,\nu}|^2}.$$

Solutions giving $|\eta_\nu|^2 < 0$ and/or $|\eta_R|^2 < 0$ are unphysical. Given a pair (A_1, Z_1), (A_2, Z_2) of the three ^{76}Ge, ^{100}Mo and ^{130}Te we will be considering, and T_1, and choosing (for convenience) always $A_1 < A_2$, positive solutions for $|\eta_\nu|^2$ and $|\eta_R|^2$ - possible for the following range of values of T_2:

The positivity conditions

$$\frac{T_1 G_1 |M'^{0\nu}_{1,N}|^2}{G_2 |M'^{0\nu}_{2,N}|^2} \leq T_2 \leq \frac{T_1 G_1 |M'^{0\nu}_{1,\nu}|^2}{G_2 |M'^{0\nu}_{2,\nu}|^2}$$

($|M'^{0\nu}_{1,\nu}|^2 / |M'^{0\nu}_{2,\nu}|^2 > |M'^{0\nu}_{1,N}|^2 / |M'^{0\nu}_{2,N}|^2$ (from Table 1) used.)

Using $G_{1,2}$, and $M'^{0\nu}_{i,\nu}$, $M'^{0\nu}_{i,N}$, $i = 1, 2$, (Table 1, "CD-Bonn, large, $g_A = 1.25$ (1.0)"), we get the positivity conditions for the 3 ratios of pairs of $T^{0\nu}_{1/2}$:

$$0.15 \leq \frac{T^{0\nu}_{1/2}(^{100}Mo)}{T^{0\nu}_{1/2}(^{76}Ge)} \leq 0.18\ (0.17)\,,$$

$$0.17 \leq \frac{T^{0\nu}_{1/2}(^{130}Te)}{T^{0\nu}_{1/2}(^{76}Ge)} \leq 0.22\ (0.23)\,,$$

$$1.14\ (1.16) \leq \frac{T^{0\nu}_{1/2}(^{130}Te)}{T^{0\nu}_{1/2}(^{100}Mo)} \leq 1.24\ (1.30)\,.$$

Similar results with Argonne, large, g_A=1.25(1.0) NMEs:

$$0.15 \leq \frac{T^{0\nu}_{1/2}(^{100}Mo)}{T^{0\nu}_{1/2}(^{76}Ge)} \leq 0.18\,,$$
$$0.18 \leq \frac{T^{0\nu}_{1/2}(^{130}Te)}{T^{0\nu}_{1/2}(^{76}Ge)} \leq 0.24\ (0.25)\,,$$
$$1.22 \leq \frac{T^{0\nu}_{1/2}(^{130}Te)}{T^{0\nu}_{1/2}(^{100}Mo)} \leq 1.36\ (1.42)\,.$$

The physical solutions possible only for remarkably narrow intervals of T_2/T_1. If any of the ratios is shown to lie outside the relevant intervals, the case - excluded.

Conditions for only one mechanism being active:

$$|\eta_R|^2 = 0\ :\ |M'^{0\nu}_{1,\nu}|^2\, T_1\, G_1 = |M'^{0\nu}_{2,\nu}|^2\, T_2\, G_2\,,$$
$$|\eta_\nu|^2 = 0\ :\ |M'^{0\nu}_{1,N}|^2\, T_1\, G_1 = |M'^{0\nu}_{2,N}|^2\, T_2\, G_2\,.$$

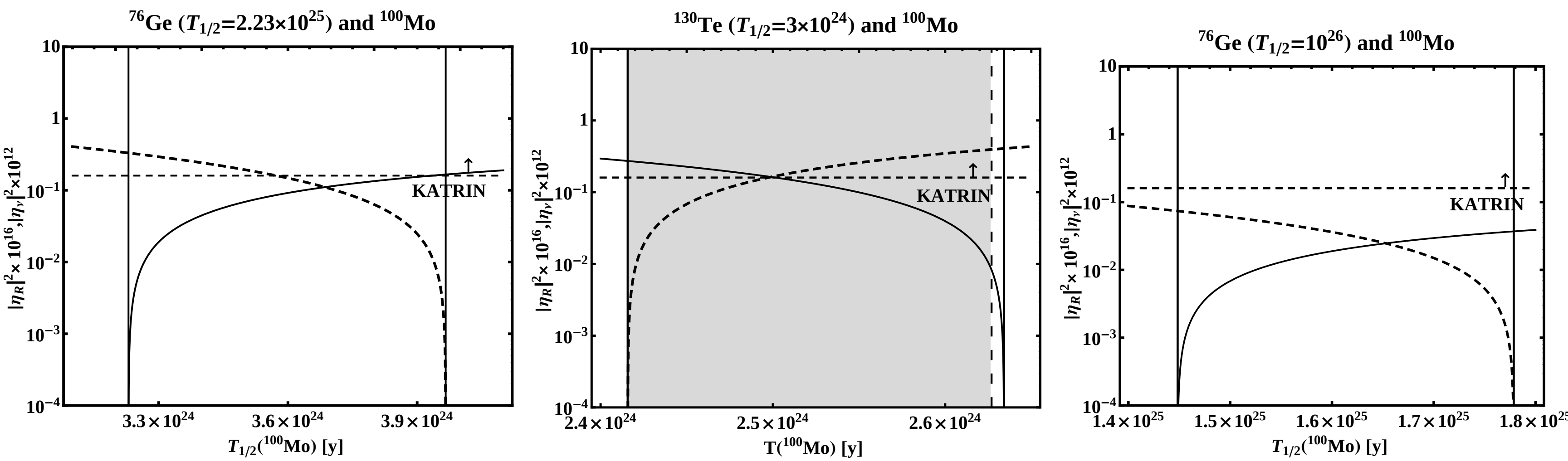
Comments.

- The feature discussed above - common to all cases of two "non-interfering" mechanisms considered.
- The indicated specific half-life intervals for the various isotopes, are stable with respect to the change of the NMEs.
- The intervals of T_2/T_1 depend on the type of the two "non-interfering" mechanisms. However, the differences in the cases of the exchange of heavy Majorana neutrinos coupled to (V+A) currents and i) light Majorana neutrino exchange, or ii) the gluino exchange mechanism, or iii) the squark-neutrino exchange mechanism, are extremely small.
- One of the consequences - if it will be possible to rule out one of them as the cause of $(\beta\beta)_{0\nu}$-decay, most likely one will be able to rule out all three of them.
- Using the indicated difference to get information about the specific pair of "non-interfering" mechanisms possibly operative in $(\beta\beta)_{0\nu}$-decay requires, in the cases considered by us, an extremely high precision in the measurement of the $(\beta\beta)_{0\nu}$-decay half-lives of the isotopes considered. The levels of precision required seem impossible to achieve in the foreseeable future.
- If it is experimentally established that any of the indicated intervals of half-lives lies outside the interval of physical solutions of $|\eta_\nu|^2$ and $|\eta_R|^2$, obtained taking into account all relevant uncertainties, one would be led to conclude that the $(\beta\beta)_{0\nu}$-decay is not generated by the two mechanisms considered.
- The constraints under discussion will not be valid, in general, if the $(\beta\beta)_{0\nu}$-decay is triggered by two "interfering" mechanisms with a non-negligible (destructive) interference term, or by more than two mechanisms none of which plays a subdominat role in $(\beta\beta)_{0\nu}$-decay.

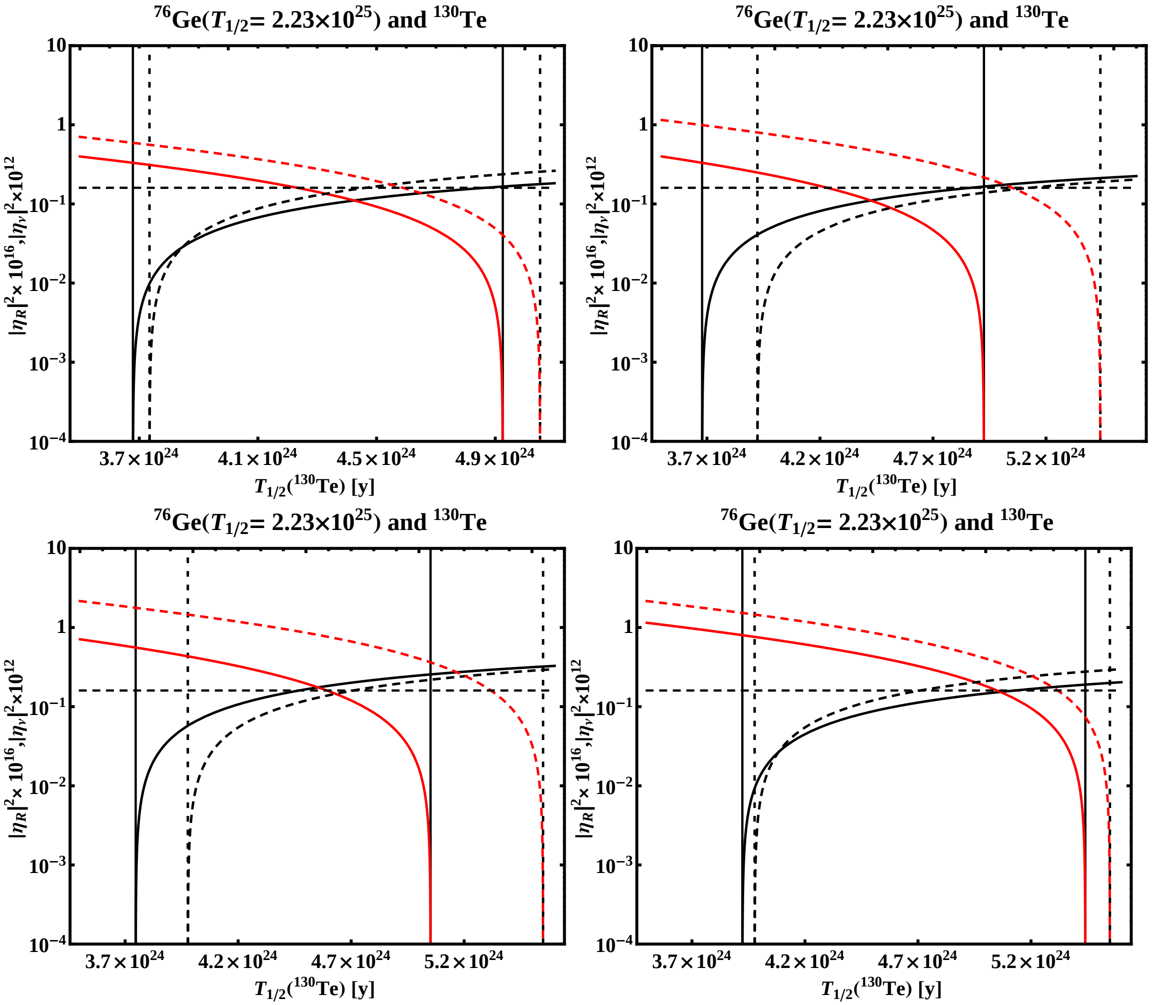
The predictions for the half-life of a third nucleus (A_3, Z_3), using as input in the system of equations for $|\eta_\nu|^2$ and $|\eta_R|^2$ the half-lives of two other nuclei (A_1, Z_1) and (A_2, Z_2). The three nuclei used are ^{76}Ge, ^{100}Mo and ^{130}Te. The results shown are obtained for a fixed value of the half-life of (A_1, Z_1) and assuming the half-life of (A_2, Z_2) to lie in a certain specific interval. The physical solutions for $|\eta_\nu|^2$ and $|\eta_R|^2$ are then used to derive predictions for the half-life of the third nucleus (A_3, Z_3). The latter are compared with the existing experimental lower limits. The results - obtained with "CD-Bonn, large, $g_A = 1.25$" NMEs (Table 1). One star beside the isotope pair whose half-lives are used as input indicates predicted ranges of half-lives of the nucleus (A_3, Z_3) that are not compatible with the existing lower bounds.

Pair	$T^{0\nu}_{1/2}(A_1, Z_1)$[yr]	$T^{0\nu}_{1/2}[A_2, Z_2]$[yr]	Prediction on $[A_3, Z_3]$[yr]
$^{76}Ge - ^{100}Mo$	$T(Ge) = 2.23 \cdot 10^{25}$	$3.23 \cdot 10^{24} \le T(Mo) \le 3.97 \cdot 10^{24}$	$3.68 \cdot 10^{24} \le T(Te) \le 4.93 \cdot 10^{24}$
$^{76}Ge - ^{130}Te$	$T(Ge) = 2.23 \cdot 10^{25}$	$3.68 \cdot 10^{24} \le T(Te) \le 4.93 \cdot 10^{24}$	$3.23 \cdot 10^{24} \le T(Mo) \le 3.97 \cdot 10^{24}$
$^{76}Ge - ^{100}Mo$	$T(Ge) = 10^{26}$	$1.45 \cdot 10^{25} \le T(Mo) \le 1.78 \cdot 10^{25}$	$1.65 \cdot 10^{25} \le T(Te) \le 2.21 \cdot 10^{25}$
$^{76}Ge - ^{130}Te$	$T(Ge) = 10^{26}$	$1.65 \cdot 10^{25} \le T(Te) \le 2.21 \cdot 10^{25}$	$1.45 \cdot 10^{25} \le T(Mo) \le 1.78 \cdot 10^{25}$
$^{100}Mo - ^{130}Te$ $\star$	$T(Mo) = 5.8 \cdot 10^{23}$	$6.61 \cdot 10^{23} \le T(Te) \le 7.20 \cdot 10^{23}$	$3.26 \cdot 10^{24} \le T(Ge) \le 4.00 \cdot 10^{24}$
$^{100}Mo - ^{130}Te$	$T(Mo) = 4 \cdot 10^{24}$	$4.56 \cdot 10^{24} \le T(Te) \le 4.97 \cdot 10^{24}$	$2.25 \cdot 10^{25} \le T(Ge) \le 2.76 \cdot 10^{25}$
$^{100}Mo - ^{130}Te$	$T(Mo) = 5.8 \cdot 10^{24}$	$6.61 \cdot 10^{24} \le T(Te) \le 7.20 \cdot 10^{24}$	$3.26 \cdot 10^{25} \le T(Ge) \le 4.00 \cdot 10^{25}$
$^{100}Mo - ^{130}Te$ $\star$	$T(Te) = 3 \cdot 10^{24}$	$2.42 \cdot 10^{24} \le T(Mo) \le 2.63 \cdot 10^{24}$	$1.36 \cdot 10^{25} \le T(Ge) \le 1.82 \cdot 10^{25}$
$^{100}Mo - ^{130}Te$	$T(Te) = 1.65 \cdot 10^{25}$	$1.33 \cdot 10^{25} \le T(Mo) \le 1.45 \cdot 10^{25}$	$7.47 \cdot 10^{25} \le T(Ge) \le 1.00 \cdot 10^{26}$
$^{100}Mo - ^{130}Te$	$T(Te) = 3 \cdot 10^{25}$	$2.42 \cdot 10^{25} \le T(Mo) \le 2.63 \cdot 10^{25}$	$1.36 \cdot 10^{26} \le T(Ge) \le 1.82 \cdot 10^{26}$

"CD-Bonn, large, $g_A = 1.0$" NMEs: intervals change by $\pm 5\%$;
"Argonne, large, $g_A = 1.25$ (1.0)" NMEs: intervals change by $\pm 10\%$ (± 14)%.



$|\eta_\nu|^2$: solid lines; $|\eta_R|^2$: dashed lines.
Physical solutions - between the two vertical lines;
the solutions in the grey area excluded by the lower limit $T^{0\nu}_{1/2}(^{76}Ge) \geq 1.9 \times 10^{25}$ y.



Solutions for $|\eta_\nu|^2$ (black lines) and $|\eta_R|^2$ (red lines), for given $T_1 = T^{0\nu}_{1/2}(^{76}Ge) = 2.23\times 10^{25}$ yr and $T_2 = T^{0\nu}_{1/2}(^{130}Te)$ and the "large basis" NMEs corresponding to: i) CD-Bonn p., $g_A = 1.25$ (solid lines), $g_A = 1$ (dashed lines) (u.l. panel); ii) CD-Bonn (solid lines) and Argonne (dashed lines) p. with $g_A = 1.25$ (u.r. panel); iii) CD-Bonn (solid lines) and Argonne (dashed lines) p. with $g_A = 1.0$ (l.l. panel); iv) Argonne p. with $g_A = 1.25$ (solid lines), $g_A = 1$ (dashed lines) (l.r. panel). The physical (positive) solutions shown with solid (dashed) lines - between the two vertical solid (dashed) lines. The horizontal dashed line - the prospective KATRIN limit $|\langle m \rangle| < 0.2$ eV.

Two "Interfering" Mechanisms

Example: light Majorana ν and gluino exchanges

In this case for a given $(\beta\beta)_{0\nu}$ decaying (A,Z),

$$\frac{1}{T^{0\nu}_{1/2,i}G^{0\nu}_i(E,Z)} =$$
$$|\eta_\nu|^2|M'^{0\nu}_{i,\nu}|^2 + |\eta_{\lambda'}|^2|M'^{0\nu}_{i,\lambda'}|^2 + 2\cos\alpha|M'^{0\nu}_{i,\lambda'}||M'^{0\nu}_{i,\nu}||\eta_\nu||\eta_{\lambda'}|$$

$\alpha = \arg(\eta_\nu\eta^*_{\lambda'})$ (NMEs - real).

The LNV parameters $|\eta_\nu|$, $|\eta_{\lambda'}|$ and $\cos\alpha$ -
from "data" on $T^{0\nu}_{1/2}$ of three nuclei.

The solutions read:

$$|\eta_\nu|^2 = \frac{D_1}{D}\,, \quad |\eta_{\lambda'}|^2 = \frac{D_2}{D}\,, \quad z \equiv 2\cos\alpha|\eta_\nu||\eta_{\lambda'}| = \frac{D_3}{D}\,,$$

$$\mathbf{D} = \begin{vmatrix} (M'^{0\nu}_{1,\nu})^2 & (M'^{0\nu}_{1,\lambda'})^2 & M'^{0\nu}_{1,\lambda'} M'^{0\nu}_{1,\nu} \\ (M'^{0\nu}_{2,\nu})^2 & (M'^{0\nu}_{2,\lambda'})^2 & M'^{0\nu}_{2,\lambda'} M'^{0\nu}_{2,\nu} \\ (M'^{0\nu}_{3,\nu})^2 & (M'^{0\nu}_{3,\lambda'})^2 & M'^{0\nu}_{3,\lambda'} M'^{0\nu}_{3,\nu} \end{vmatrix}, \quad \mathbf{D_1} = \begin{vmatrix} 1/T_1 G_1 & (M'^{0\nu}_{1,\lambda'})^2 & M'^{0\nu}_{1,\lambda'} M'^{0\nu}_{1,\nu} \\ 1/T_2 G_2 & (M'^{0\nu}_{2,\lambda'})^2 & M'^{0\nu}_{2,\lambda'} M'^{0\nu}_{2,\nu} \\ 1/T_3 G_3 & (M'^{0\nu}_{3,\lambda'})^2 & M'^{0\nu}_{3,\lambda'} M'^{0\nu}_{3,\nu} \end{vmatrix},$$

$$\mathbf{D_2} = \begin{vmatrix} (M'^{0\nu}_{1,\nu})^2 & 1/T_1 G_1 & M'^{0\nu}_{1,\lambda'} M'^{0\nu}_{1,\nu} \\ (M'^{0\nu}_{2,\nu})^2 & 1/T_2 G_2 & M'^{0\nu}_{2,\lambda'} M'^{0\nu}_{2,\nu} \\ (M'^{0\nu}_{3,\nu})^2 & 1/T_3 G_3 & M'^{0\nu}_{3,\lambda'} M'^{0\nu}_{3,\nu} \end{vmatrix}, \quad \mathbf{D_3} = \begin{vmatrix} (M'^{0\nu}_{1,\nu})^2 & (M'^{0\nu}_{1,\lambda'})^2 & 1/T_1 G_1 \\ (M'^{0\nu}_{2,\nu})^2 & (M'^{0\nu}_{2,\lambda'})^2 & 1/T_2 G_2 \\ (M'^{0\nu}_{3,\nu})^2 & (M'^{0\nu}_{3,\lambda'})^2 & 1/T_3 G_3 \end{vmatrix}.$$

Physical solutions ("positivity conditions"):

$$|\eta_\nu|^2 \geq 0,\ |\eta_{\lambda'}|^2 \geq 0,\ -|\eta_\nu||\eta_{\lambda'}| \leq \cos\alpha|\eta_\nu||\eta_{\lambda'}| \leq |\eta_\nu||\eta_{\lambda'}|.$$

Given (i.e. having data on) T_1, T_2 + using the condition on the interference term $z = 2\cos\alpha|\eta_\nu||\eta_{\lambda'}|$, determines an interval of allowed values of T_3.

Ranges of half-lives T_3 in the case of two interfering mechanisms: the light Majorana neutrino exchange and gluino exchange dominance.

$T^{0\nu}_{1/2}$[y] (fixed)	$T^{0\nu}_{1/2}$[y] (fixed)	Allowed
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Mo) = 5.8 \cdot 10^{24}$	$5.99 \cdot 10^{24} \leq T(Te) \leq 7.35 \cdot 10^{24}$
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Te) = 3 \cdot 10^{24}$	$2.46 \cdot 10^{24} \leq T(Mo) \leq 2.82 \cdot 10^{24}$
$T(Ge) = 10^{26}$	$T(Mo) = 5.8 \cdot 10^{24}$	$6.30 \cdot 10^{24} \leq T(Te) \leq 6.94 \cdot 10^{24}$
$T(Ge) = 10^{26}$	$T(Te) = 3 \cdot 10^{24}$	$2.55 \cdot 10^{24} \leq T(Mo) \leq 2.72 \cdot 10^{24}$
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Te) = 3 \cdot 10^{25}$	$2.14 \cdot 10^{25} \leq T(Mo) \leq 3.31 \cdot 10^{25}$
$T(Ge) = 10^{26}$	$T(Te) = 3 \cdot 10^{25}$	$2.38 \cdot 10^{25} \leq T(Mo) \leq 2.92 \cdot 10^{25}$

“CD-Bonn potential, large, $g_A = 1$” NMEs

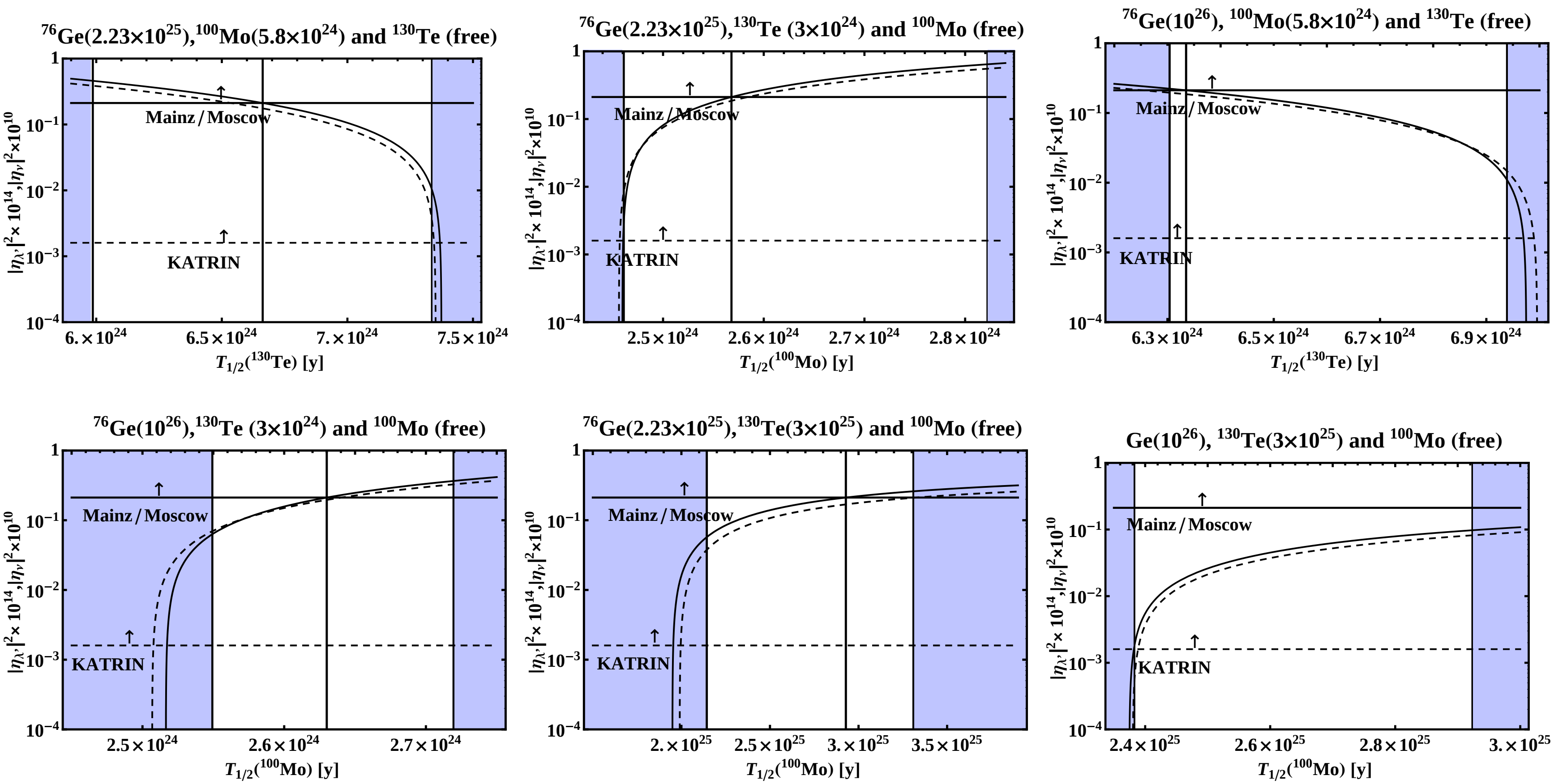
$T^{0\nu}_{1/2}$[y] (fixed)	$T^{0\nu}_{1/2}$[y] (fixed)	Allowed
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Mo) = 5.8 \cdot 10^{24}$	$3 \cdot 10^{24} \leq T(Te) \leq 8.62 \cdot 10^{24}$
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Te) = 3 \cdot 10^{24}$	$2.55 \cdot 10^{24} \leq T(Mo) \leq 6.18 \cdot 10^{24}$
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Te) = 3 \cdot 10^{25}$	$1.33 \cdot 10^{25} \leq T(Mo) \leq 3.88 \cdot 10^{26}$
$T(Ge) = 10^{26}$	$T(Mo) = 5.8 \cdot 10^{24}$	$3.62 \cdot 10^{24} \leq T(Te) \leq 6.04 \cdot 10^{24}$
$T(Ge) = 10^{26}$	$T(Te) = 3 \cdot 10^{24}$	$3.11 \cdot 10^{24} \leq T(Mo) \leq 4.70 \cdot 10^{24}$
$T(Ge) = 10^{26}$	$T(Te) = 3 \cdot 10^{25}$	$2.15 \cdot 10^{25} \leq T(Mo) \leq 8.29 \cdot 10^{25}$

“Argonne potential, large, $g_A = 1.25$” NMEs

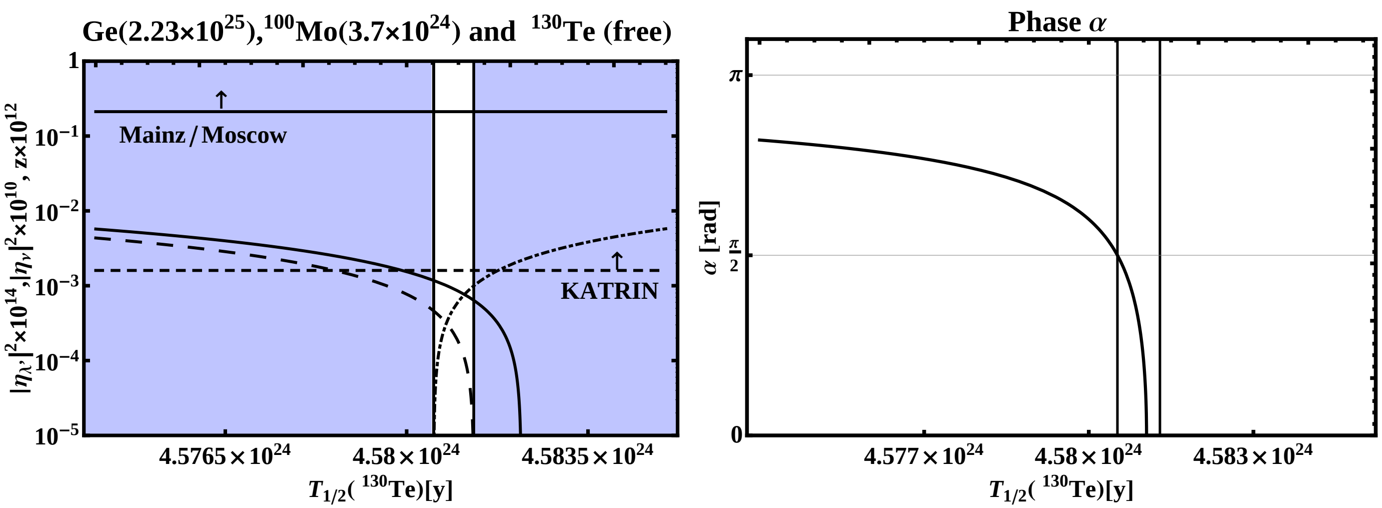
$T^{0\nu}_{1/2}$[y] (fixed)	$T^{0\nu}_{1/2}$[y] (fixed)	Allowed
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Mo) = 5.8 \cdot 10^{24}$	$3 \cdot 10^{24} \leq T(Te) \leq 9.22 \cdot 10^{24}$
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Te) = 3 \cdot 10^{24}$	$2.55 \cdot 10^{24} \leq T(Mo) \leq 7.92 \cdot 10^{24}$
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Te) = 3 \cdot 10^{25}$	$1.19 \cdot 10^{25} \leq T(Mo) \leq 2.55 \cdot 10^{27}$
$T(Ge) = 10^{26}$	$T(Mo) = 5.8 \cdot 10^{24}$	$3.15 \cdot 10^{24} \leq T(Te) \leq 5.85 \cdot 10^{24}$
$T(Ge) = 10^{26}$	$T(Te) = 3 \cdot 10^{24}$	$3.25 \cdot 10^{24} \leq T(Mo) \leq 5.49 \cdot 10^{24}$
$T(Ge) = 10^{26}$	$T(Te) = 3 \cdot 10^{25}$	$2.08 \cdot 10^{25} \leq T(Mo) \leq 1.20 \cdot 10^{26}$

“Argonne Potential, large, $g_A = 1$” NME

$T^{0\nu}_{1/2}$[y] (fixed)	$T^{0\nu}_{1/2}$[y] (fixed)	Allowed
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Mo) = 5.8 \cdot 10^{24}$	$3 \cdot 10^{24} \leq T(Te) \leq 1.11 \cdot 10^{25}$
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Te) = 3 \cdot 10^{24}$	$2.63 \cdot 10^{24} \leq T(Mo) \leq 2.04 \cdot 10^{25}$
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Te) = 3 \cdot 10^{25}$	$9.19 \cdot 10^{24} \leq T(Mo) \leq 2.36 \cdot 10^{26}$
$T(Ge) = 10^{26}$	$T(Mo) = 5.8 \cdot 10^{24}$	$3 \cdot 10^{24} \leq T(Te) \leq 5.07 \cdot 10^{24}$
$T(Ge) = 10^{26}$	$T(Te) = 3 \cdot 10^{24}$	$3.82 \cdot 10^{24} \leq T(Mo) \leq 9.44 \cdot 10^{24}$
$T(Ge) = 10^{26}$	$T(Te) = 3 \cdot 10^{25}$	$1.96 \cdot 10^{25} \leq T(Mo) \leq 6.54 \cdot 10^{26}$



$|\eta_\nu|^2 \times 10^{10}$ **: solid lines;** $|\eta_{\lambda'}|^2 \times 10^{14}$**: dashed lines. All solutions:** $\cos\alpha \cong -1$**.**
Allowed regions (physical solutions) - white areas;
the solutions in the grey (blue) areas - excluded.
The horizontal solid (dashed) line - the Moscow-Mainz limit $|<m>| < 2.3$ **eV**
(the prospective KATRIN limit $|<m>| < 0.2$ **eV).**



A case of constructive interference: $\cos\alpha > 0$.
$|\eta_\nu|^2 \times 10^{10}$ **(solid line),** $|\eta_{\lambda'}|^2 \times 10^{14}$ **(dashed line) and**
$z \times 10^{12} = 2\cos\alpha|\eta_\nu||\eta_{\lambda'}| \times 10^{12}$ **(dot-dashed line).**

Conditions for $|\eta_\nu|^2 > 0$, $|\eta_{\lambda'}|^2 > 0$ and $z = 0$ (no int.), or $z > 0$ (constructive int.), or $z < 0$ (distructive int.).

The general conditions were derived. Below - the conditions for "CD-Bonn, large, $g_a = 1.24$" NMEs.

Given T_1, $|\eta_\nu|^2 > 0$, $|\eta_{\lambda'}|^2 > 0$, $z > 0$:

$$z > 0 \; : \begin{cases} 0.14\,T_1 < T_2 \leq 0.16\,T_1\,, & \dfrac{4.44\,T_1\,T_2}{3.74\,T_1 - 0.93\,T_2} \leq T_3 \leq \dfrac{2.10\,T_1\,T_2}{1.78\,T_1 - 0.47\,T_2}\,; \\ 0.16\,T_1 < T_2 < 0.18\,T_1\,, & \dfrac{4.44\,T_1\,T_2}{3.74\,T_1 - 0.93\,T_2} \leq T_3 \leq \dfrac{4.10\,T_1\,T_2}{3.44\,T_1 - 0.81\,T_2}\,. \end{cases}$$

Given T_1, $z > 0$ only if T_2 lies in a relatively narrow interval and T_3 has a value in extremely narrow intervals; a consequence of the values of G_i and of the NMEs used: for the 3 nuclei considered, $|M_i - M_j| << M_i, M_j$, $|\Lambda_i - \Lambda_j| << \Lambda_i, \Lambda_j$, $i \neq j = 1, 2, 3$, and typically $|M_i - M_j|/(0.5(M_i + M_j)) \sim 10^{-1}$, $|\Lambda_i - \Lambda_j|/(0.5(\Lambda_i + \Lambda_j)) \sim (10^{-2} - 10^{-1})$, $M_i \equiv M'^{0\nu}_{i,\nu}$, $\Lambda_i \equiv M'^{0\nu}_{i,\lambda'}$, $i = ^{76}Ge, ^{100}Mo, ^{130}Te$.

Given T_1, $|\eta_\nu|^2 > 0$, $|\eta_{\lambda'}|^2 > 0$, $z < 0$:

$$z < 0 \; : \begin{cases} T_2 \leq 0.14\, T_1\,, & T_3 \leq \dfrac{2.10\, T_1\, T_2}{1.78 T_1 - 0.47 T_2}\,; \\ 0.14\, T_1 < T_2 \leq 0.18\, T_1\,, & T_3 \leq \dfrac{4.44\, T_1\, T_2}{3.74\, T_1 - 0.93\, T_2}\,; \\ 0.18\, T_1 < T_2 < 4.23\, T_1\,, & T_3 \leq \dfrac{4.10\, T_1\, T_2}{3.44\, T_1 - 0.81\, T_2}\,; \\ T_2 \geq 4.23\, T_1 & T_3 > 0\,. \end{cases}$$

The intervals of values of T_2 and T_3 - very different from those corresponding to the cases of two "non-interfering" mechanisms (the only exception - the second set of intervals which partially overlap with the latter).
This difference can allow to discriminate experimentally between the two possibilities of $(\beta\beta)_{0\nu}$-decay being triggered by two "non-interfering" mechanisms or by two "destructively interfering" mechanisms.

Given T_1, $|\eta_\nu|^2 = 0$, $|\eta_{\lambda'}|^2 > 0$ $(z = 0)$:

$$T_2 = 0.14 T_1\,, \quad T_3 = \frac{2.10\, T_1 T_2}{1.78\, T_1 - 0.47\, T_2} \cong 0.18\, T_1\,.$$

Given T_1, $|\eta_\nu|^2 > 0$, $|\eta_{\lambda'}|^2 = 0$ $(z = 0)$:

$$T_2 = 0.18 T_1\,, \quad T_3 = \frac{4.10\, T_1 T_2}{3.44\, T_1 - 0.81\, T_2} \cong 0.22\, T_1\,.$$

Additional consequence of "positivity" and "interference" conditions.

Given $T^{0\nu}_{1/2}$ of one isotope, say of ^{76}Ge (T_1) + an experimental lower bound on the $T^{0\nu}_{1/2}$ of a 2nd isotope, e.g., of ^{130}Te (T_3), the conditions imply a constraint on the $T^{0\nu}_{1/2}$ of any 3rd isotope, say of ^{100}Mo (T_2).

The constraint depends noticeably on the type of the two "interfering" mechanisms generating the $(\beta\beta)_{0\nu}$-decay and can be used, in principle, to discriminate between the different possible pairs of "interfering" mechanisms.

Example: $T_1 = 2.23 \times 10^{25}$ y (^{76}Ge), $T_3 > 3.0 \times 10^{24}$ y (^{130}Te), constraint on T_2 (^{100}Mo); "CD-bonn (Argonne), large, $g_A = 1.25$" NMEs used.

Light Neutrino and gluino exchange mechanisms:

$$T_2 \equiv T^{0\nu}_{1/2}(^{100}Mo) > 2.46\ (2.47) \times 10^{24} \text{ y.}$$

(Increasing the value of $T^{0\nu}_{1/2}(^{76}Ge)$ leads to the increasing of the value of the lower limit.)

Light Neutrino and LH Heavy neutrino exchanges:

$$T^{0\nu}_{1/2}(^{100}Mo) > 2.78\ (2.68) \times 10^{24} \text{ y.}$$

(The value of the lower limit increases with the increasing of the value of $T^{0\nu}_{1/2}(^{76}Ge)$.)

Squarks-neutrino and gluino exchange mechanisms:

$$T^{0\nu}_{1/2}(^{100}Mo) > 7.92\ (22.1) \times 10^{23} \text{ y.}$$

(For larger values of $T^{0\nu}_{1/2}(^{76}Ge)$, this lower bound assumes larger values.)

LH Heavy neutrino and gluino exchange mechanisms:

$$1.36 \times 10^{24}\ \text{y} < T^{0\nu}_{1/2}(^{100}Mo) < 3.42 \times 10^{24}\ \text{y}\,.$$

Increasing the value of $T^{0\nu}_{1/2}(^{76}Ge)$ leads to a shift of the interval to larger values; for a sufficiently large $T^{0\nu}_{1/2}(^{76}Ge) > 10^{26}$ y - only a lower bound on $T^{0\nu}_{1/2}(^{100}Mo)$. Using the NMEs derived with the Argonne potential - only a lower bound: $T^{0\nu}_{1/2}(^{100}Mo) > 5.97 \times 10^{23}$ y. The difference between the results obtained with the two sets of NMEs can be traced to fact that the determinant D, calculated with the second set of NMEs, has opposite sign to that, calculated with the first set of NMEs. As a consequence, the dependence of the physical solutions for $|\eta^L_N|^2$ and $|\eta_{\lambda'}|^2$ on T_1, T_2 and T_3 in the two cases of NMEs is very different.

The constraints thus obtained can be used, e.g., to exclude some of the possible cases of two "interfering" mechanisms inducing the $(\beta\beta)_{0\nu}$-decay: if, e.g., it is confirmed that $T^{0\nu}_{1/2}(^{76}Ge) = 2.23 \times 10^{25}$ y, and in addition it is established that $T^{0\nu}_{1/2}(^{100}Mo) \leq 10^{24}$ y, that combined with the experimental lower limit on $T^{0\nu}_{1/2}(^{130}Te)$ would rule out i) the light neutrino and gluino exchanges, and ii) the light neutrino and LH heavy neutrino exchanges, as possible mechanisms generating the $(\beta\beta)_{0\nu}$-decay.

Conclusions.

If the decay $(A,Z) \to (A,Z+2)+e^- +e^-$ ($(\beta\beta)_{0\nu}$-decay) will be observed, the questions will inevitably arise:

Which mechanism is triggering the decay?

How many mechanisms are involved?

Discussed how one possibly can answer these questions.

- The measurements of the $(\beta\beta)_{0\nu}$-decay half-lives with rather high precision and the knowledge of the relevant NMEs with relatively small uncertainties is crucial for establishing that more than one mechanisms are operative in $(\beta\beta)_{0\nu}$-decay.
- The method considered can be generalised to the case of more than two $(\beta\beta)_{0\nu}$-decay mechanisms.
- It allows to treat the cases of CP conserving and CP nonconserving couplings generating the $(\beta\beta)_{0\nu}$-decay in a unique way.