

# $M^{0\nu}$ for $^{48}\text{Ca}$ from charge-exchange reactions

Vadim Rodin

EBERHARD KARLS  
UNIVERSITÄT  
TÜBINGEN



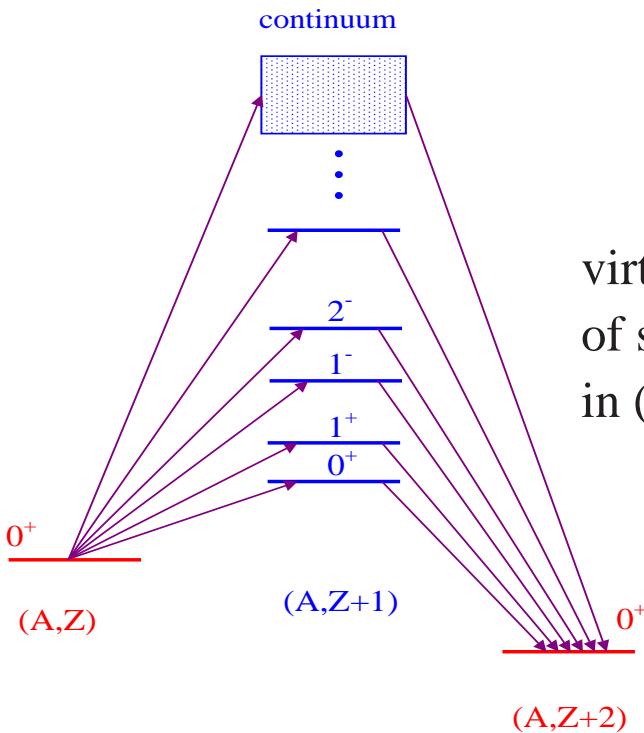
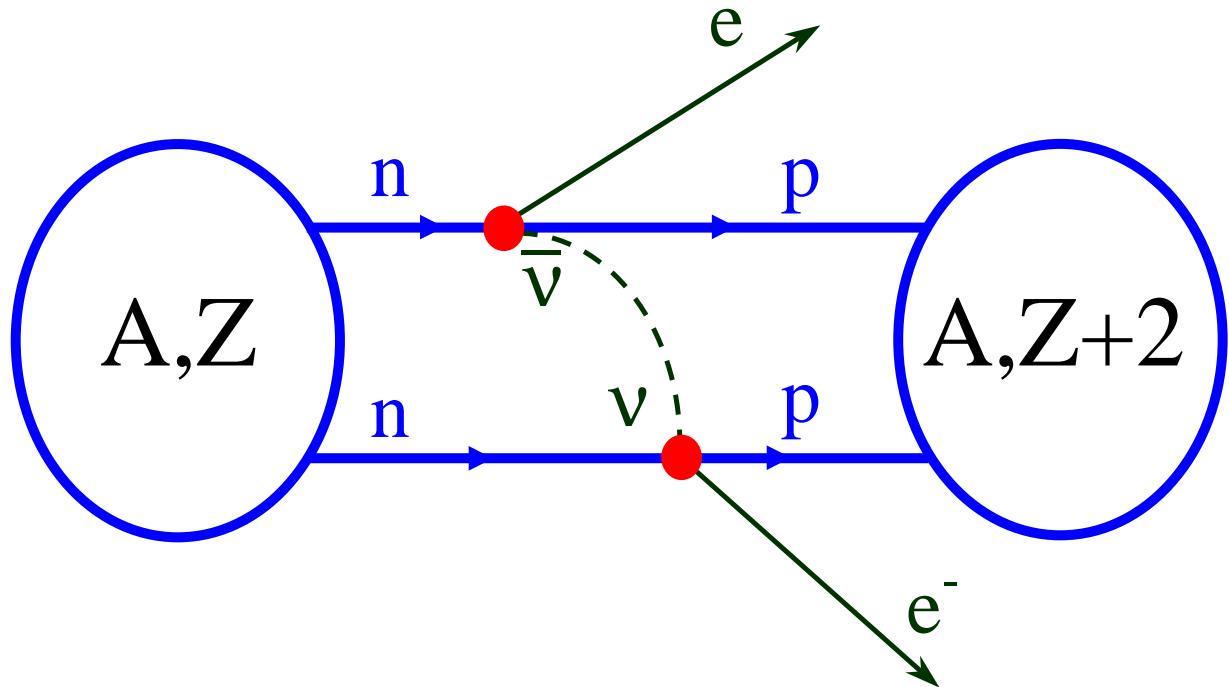
*DBD11, Osaka, 16/11/2011*

## Introduction

# Nuclear $0\nu\beta\beta$ -decay ( $\bar{\nu} = \nu$ )

strong in-medium modification of the basic process  $dd \rightarrow uue^-e^-(\bar{\nu}_e\bar{\nu}_e)$

Light neutrino  
exchange mechanism

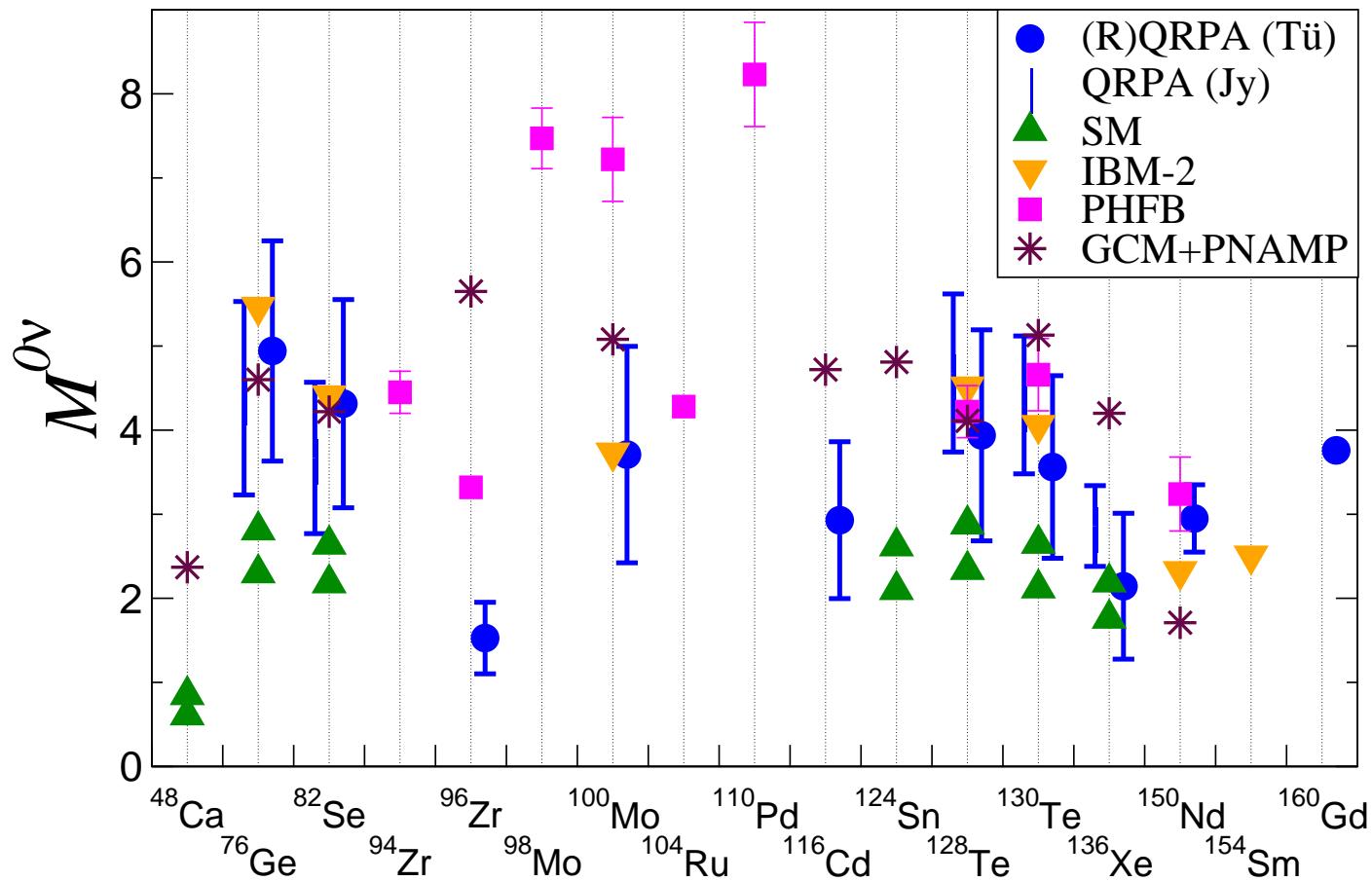


virtual excitation  
of states of all multipolarities  
in  $(A, Z+1)$  nucleus

GT amplitudes to  $1^+$  states  
— from charge-exchange reactions

(H. Ejiri, D. Frekers, H. Sakai, R. Zegers, et al.)

# World status of $M^{0\nu}$ , light neutrino mass mechanism



**QRPA: (Tü)** F. Šimkovic, A. Faessler, V.R., P. Vogel and J. Engel, PRC **77** (2008);

$^{150}\text{Nd}, ^{160}\text{Gd}$  with deformation: D. Fang, A. Faessler, V.R., F. Šimkovic, PRC **82** (2010); PRC **83**(2011)

**(Jy)** J. Suhonen, O. Civitarese, NPA **847** (2010)

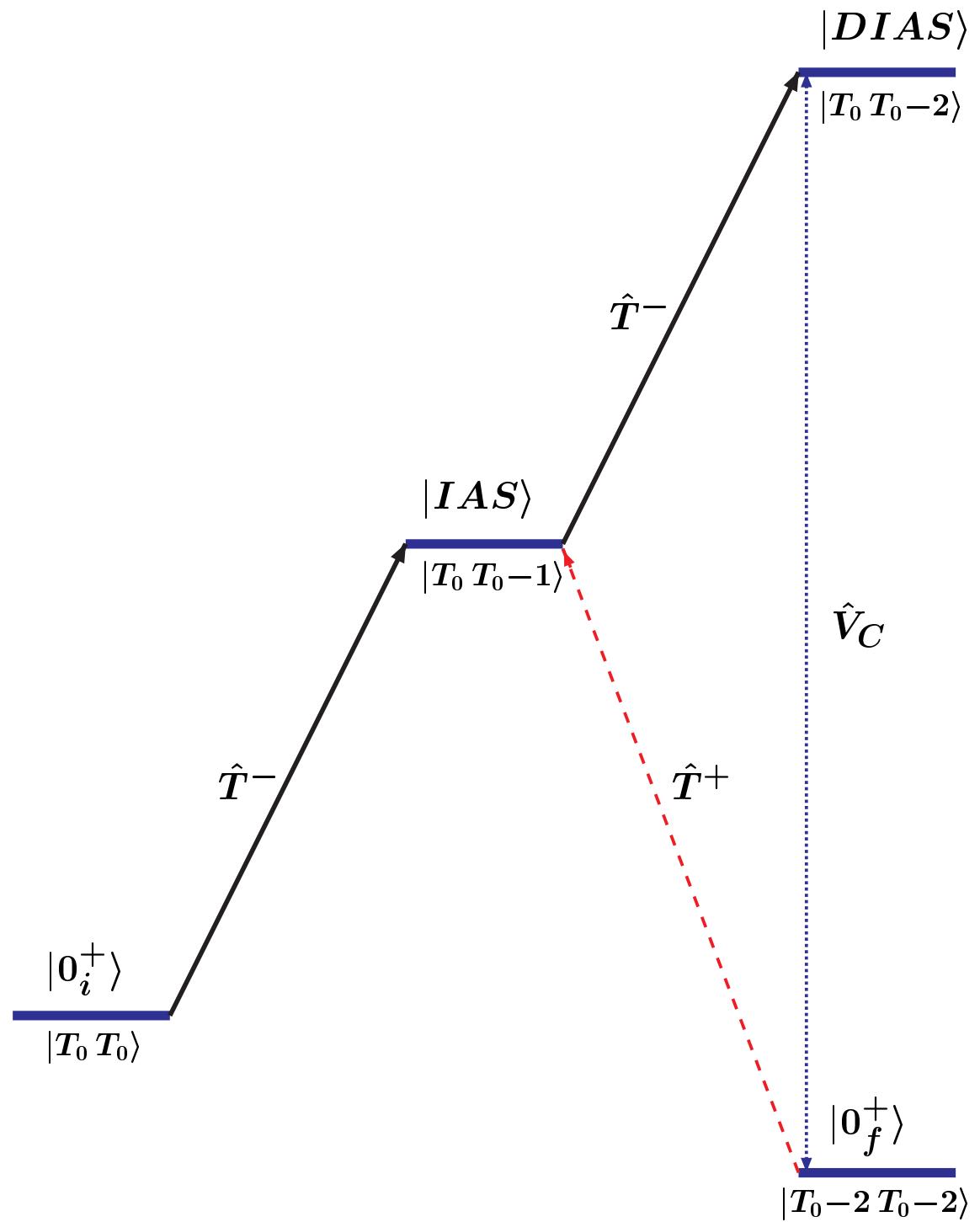
**SM** E. Caurier, J. Menendez, F. Nowacki, A. Poves, PRL **100** (2008) & NPA **818** (2009)    **IBM-2** J. Barea and F. Iachello, PRC **79** (2009);

**PHFB** P.K. Rath *et al.*, PRC **82** (2010);    **GCM+PNAMP** T. R. Rodriguez and G. Martinez-Pinedo, PRL **105** (2010)

# Measuring $M_F^{0\nu}$

Can one measure nuclear matrix elements of neutrinoless double beta decay?

V.R., A. Faessler, PRC **80**, 041302(R) (2009) [arXiv:0906.1759 [nucl-th]]  
PPNP **66**, 441 (2011); arXiv:1012.5176 [nucl-th]



## Measuring $M_F^{0\nu}$

$$\hat{W}_F^{0\nu} = \sum_{ab} P_\nu(r_{ab}) \tau_a^- \tau_b^- = \frac{1}{e^2} \left[ \hat{T}^-, [\hat{T}^-, \hat{V}_C] \right]$$

Isospin lowering operator  $\hat{T}^- = \sum_a \tau_a^-$ ; Coulomb interaction  $\hat{V}_C = \frac{e^2}{8} \sum_{a \neq b} \frac{(1 - \tau_a^{(3)})(1 - \tau_b^{(3)})}{r_{ab}}$ , isotensor Coulomb  $\hat{V}_C^{(2)} = \frac{e^2}{8} \sum_{ab} \frac{1}{r_{ab}} (\tau_a^{(3)} \tau_b^{(3)} - \frac{\boldsymbol{\tau}_a \boldsymbol{\tau}_b}{3})$

## Measuring $M_F^{0\nu}$

$$\hat{W}_F^{0\nu} = \sum_{ab} P_\nu(r_{ab}) \tau_a^- \tau_b^- = \frac{1}{e^2} \left[ \hat{T}^-, [\hat{T}^-, \hat{V}_C] \right]$$

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$$e^2 M_F^{0\nu} = \langle 0_f^+ | \left[ \hat{T}^-, [\hat{T}^-, \hat{V}_C] \right] | 0_i^+ \rangle$$

$$\begin{aligned} &\approx \langle T_0 - 2T_0 - 2 | V_C \left( \hat{T}^- \right)^2 | T_0 T_0 \rangle \\ &= \langle T_0 - 2T_0 - 2 | \hat{V}_C | T_0 T_0 - 2 \rangle \times \langle T_0 T_0 - 2 | \left( \hat{T}^- \right)^2 | T_0 T_0 \rangle \end{aligned}$$

## Measuring $M_F^{0\nu}$

$$M_F^{0\nu} = \frac{-2}{e^2} \sum_s \bar{\omega}_s \langle 0_f^+ | \hat{T}^- | 0_s^+ \rangle \langle 0_s^+ | \hat{T}^- | 0_i^+ \rangle$$

$$\bar{\omega}_s = E_s - (E_{0_i^+} + E_{0_f^+})/2$$

$$\text{used } \left[ \hat{T}^-, [\hat{T}^-, \hat{V}_C] \right] = \left[ \hat{T}^-, [\hat{T}^-, \hat{H}_{tot}] \right] \quad \text{assuming } \left[ \hat{T}^-, \hat{H}_{str} \right] = 0$$

$$\hat{H}_{tot} = \hat{K} + \hat{H}_{str} + \hat{V}_C$$

$$\begin{aligned} M_F^{0\nu} &\approx \frac{-2}{e^2} \bar{\omega}_{IAS} \langle 0_f^+ | \hat{T}^- | IAS \rangle \langle IAS | \hat{T}^- | 0_i^+ \rangle \\ &\approx \frac{1}{e^2} \langle 0_f^+ | \hat{V}_C | DIAS \rangle \langle DIAS | \left( \hat{T}^- \right)^2 | 0_i^+ \rangle \end{aligned}$$

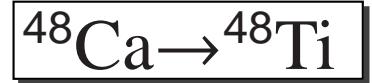
## Measuring $M_F^{0\nu}$

Measure the  $\Delta T = 2$  isospin-forbidden matrix element  $\langle 0_f^+ | \hat{T}^- | IAS \rangle$

charge-exchange ( $n, p$ )-type reaction

Challenge: 
$$\frac{\langle IAS | \hat{T}^+ | 0_f^+ \rangle}{\langle IAS | \hat{T}^- | 0_i^+ \rangle} \sim 0.001$$

$$\frac{M_F^{0\nu}(QRPA)}{M_F^{0\nu}(SM)} \approx 3 \div 5 \quad \text{and} \quad \frac{M_{GT}^{0\nu}}{M_F^{0\nu}} \approx -2.5$$



## IAS of ${}^{48}\text{Ca}$ ( $T = 4$ , $T_z = 3$ ) in ${}^{48}\text{Sc}$

1. is located at  $E_x = 6.678 \text{ MeV}$  ( $\bar{\omega}_{IAS} \approx 8.5 \text{ MeV}$ ) under threshold of particle emission
2. 100%  $\gamma$ -decay to  $1^+$  state at  $E_x = 2.517 \text{ MeV}$  ( $E_\gamma = 4.160 \text{ MeV}$ )
3. a single state — no fragmentation  
(too low density of  $T = 3$   $0^+$  states around the IAS)

Example Reaction:  ${}^{48}\text{Ti}(n,p){}^{48}\text{Sc}(\text{IAS})$

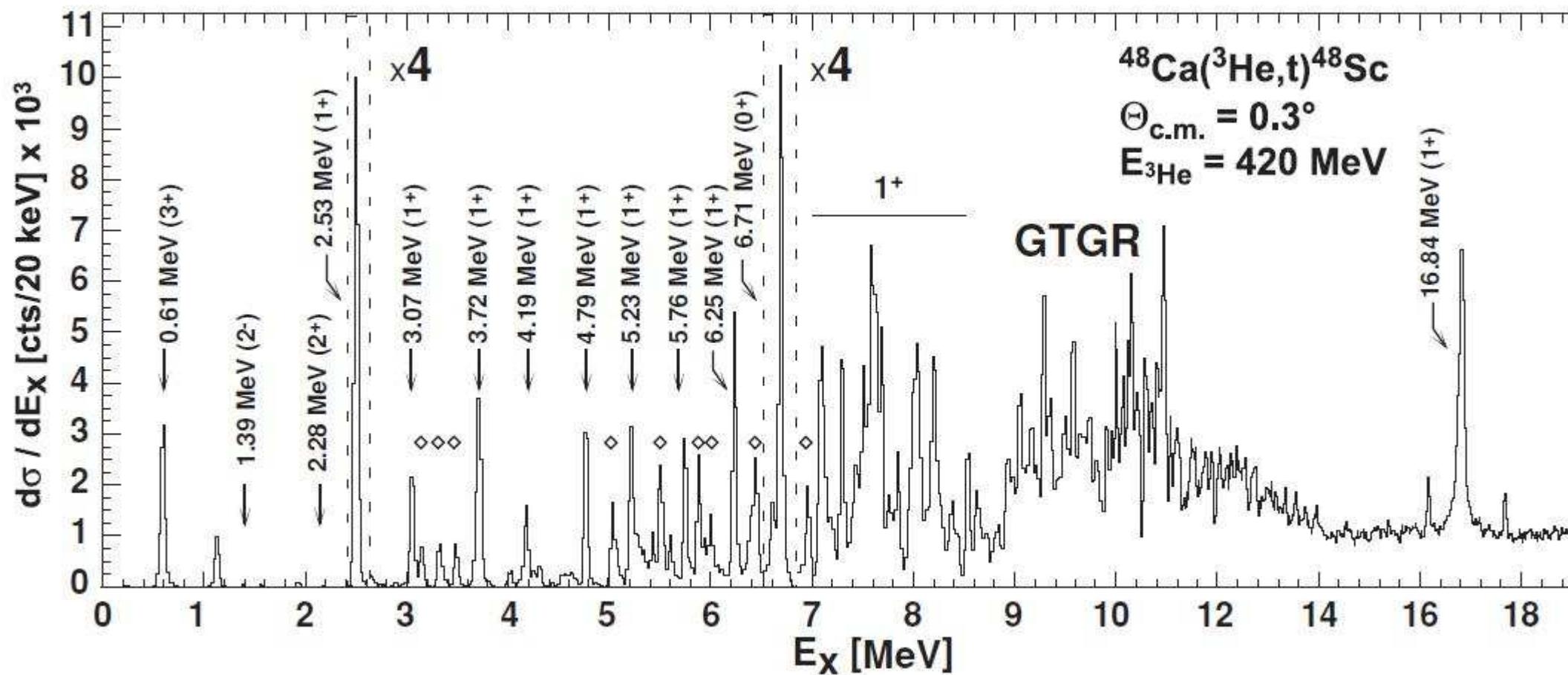
# $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$

PHYSICAL REVIEW C 76, 054307 (2007)

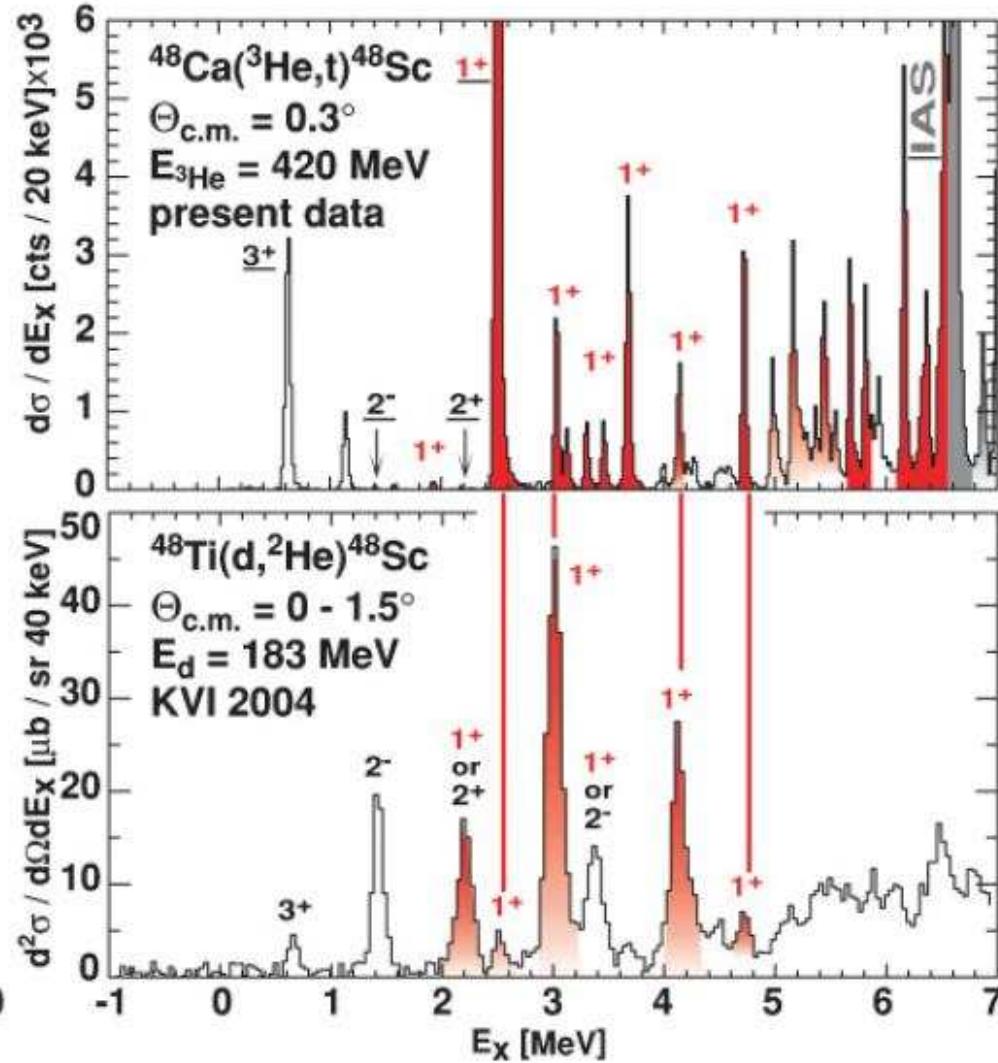
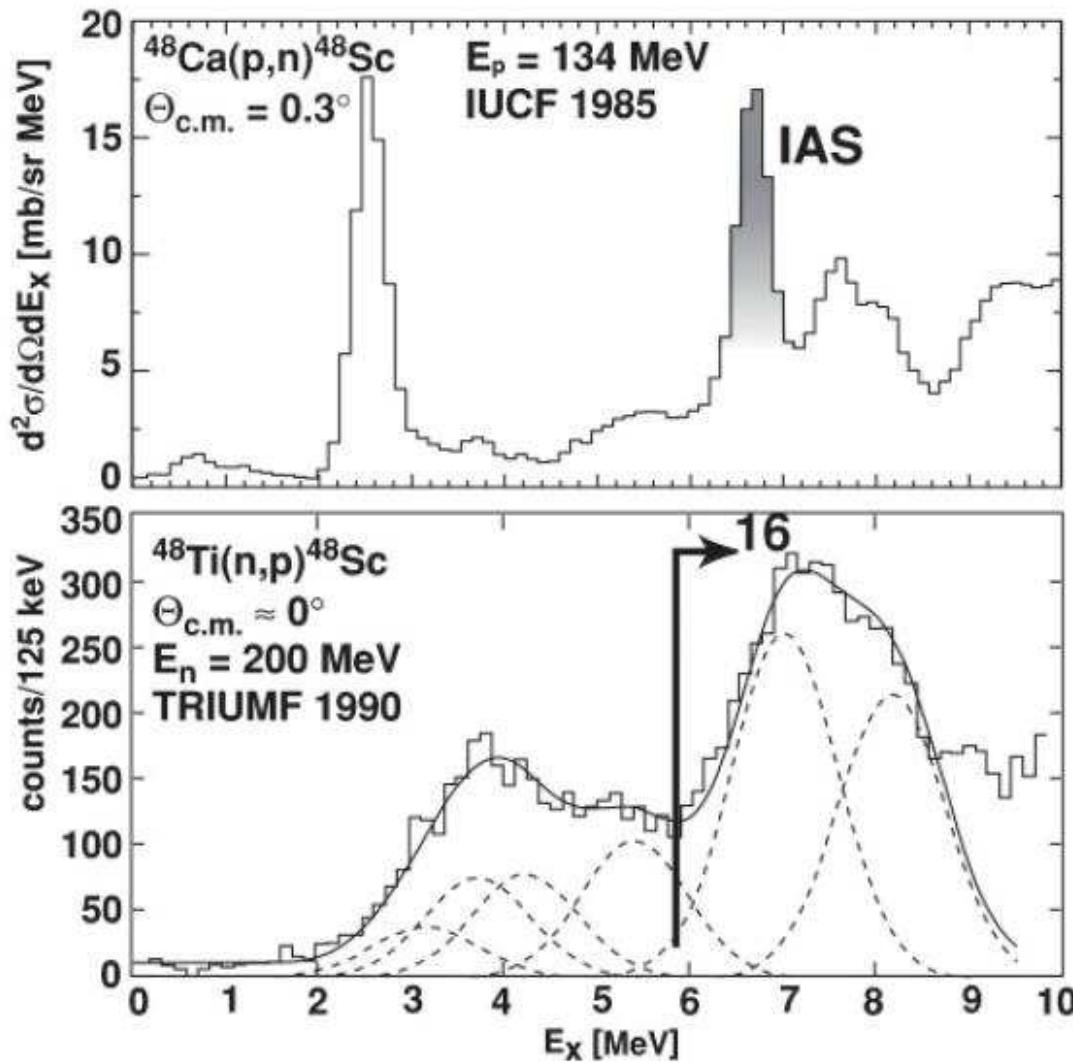
## $(^3\text{He},t)$ reaction on the double $\beta$ decay nucleus $^{48}\text{Ca}$ and the importance of nuclear matrix elements

E.-W. Grewe,<sup>1</sup> D. Frekers,<sup>1</sup> S. Rakers,<sup>1,\*</sup> T. Adachi,<sup>2,†</sup> C. Bäumer,<sup>1</sup> N. T. Botha,<sup>3</sup> H. Dohmann,<sup>1</sup> H. Fujita,<sup>4,5</sup> Y. Fujita,<sup>2</sup> K. Hatanaka,<sup>6</sup> K. Nakanishi,<sup>6,‡</sup> A. Negret,<sup>7,§</sup> R. Neveling,<sup>5</sup> L. Popescu,<sup>7,||</sup> Y. Sakemi,<sup>6,¶</sup> Y. Shimbara,<sup>2,\*\*</sup> Y. Shimizu,<sup>6,‡</sup> F. D. Smit,<sup>5</sup> Y. Tameshige,<sup>6</sup> A. Tamii,<sup>6</sup> J. Thies,<sup>1</sup> P. von Brentano,<sup>8</sup> M. Yosoi,<sup>9,††</sup> and R. G. T. Zegers<sup>10</sup>

IAS↓



## $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$



$$\boxed{{}^{48}\mathrm{Ca}{\rightarrow}{}^{48}\mathrm{Ti}}$$

$$\frac{\langle IAS|\hat{T}^+|0_f\rangle}{\langle IAS|\hat{T}^-|0_i\rangle}=-\frac{e^2M_F^{0\nu}}{2\bar{\omega}_{IAS}R}\cdot\frac{1}{N-Z}$$

$$\text{QRPA: } M_F^{0\nu}=0.6 \Rightarrow \left|\frac{\langle IAS|\hat{T}^+|0_f\rangle}{\langle IAS|\hat{T}^-|0_i\rangle}\right|^2 \approx 2\cdot 10^{-6}$$

$$\frac{d^2\sigma_{pn}}{d\Omega dE}\approx 10~\mathrm{mb}/(\mathrm{sr}~\mathrm{MeV}),\quad E_p=134~\mathrm{MeV}~(\mathrm{B.D.Anderson~et~al.,~PRC~31~(1985)})$$

$$\Rightarrow \frac{d^2\sigma_{np}}{d\Omega dE}\approx 20~\mathrm{nb}/(\mathrm{sr}~\mathrm{MeV})$$

$$\text{Unit cross section: }\hat{\sigma}_F\propto E_p^{-2}\quad E_p\rightarrow 0.5E_p\;\Rightarrow \sigma_{np}\rightarrow 4\sigma_{np}$$

## Reaction analysis

# Basic requirements for a charge-exchange probe

Measure cross section  $\equiv$  Know  $\langle IAS | \hat{T}^+ | 0_f^+ \rangle$   
???

## Reaction analysis

Any hadronic probe adds isospin to nuclear system  
(weak interaction probe would be ideal)

to probe small admixture of  $|DIAS\rangle$  to  $|0_f^+\rangle$   
⇒ must be forbidden to connect in reaction  
main components of  $|IAS\rangle$  and  $|0_f^+\rangle$  ( $\Delta T = 2$ )

Only  $T = \frac{1}{2}$  probes  $((n, p), (t, {}^3\text{He}), \dots)$

Reaction analysis

$$\sigma_{np}(0_f^+ \rightarrow IAS) \propto \langle IAS | \hat{T}^+ | 0_f^+ \rangle$$

???

$$|0_i^+\rangle = |T_0\ T_0\rangle; \quad |IAS\rangle = \frac{\hat{T}^-}{\sqrt{2T_0}}|0_i^+\rangle + \alpha |T_0-1\ T_0-1\rangle$$

$$|0_f^+\rangle = |T_0-2\ T_0-2\rangle + \beta |T_0-1\ T_0-2\rangle + \gamma \frac{(\hat{T}^-)^2}{\sqrt{4T_0(2T_0-1)}}|0_i^+\rangle$$

$$= |DIAS\rangle$$

## Reaction analysis

$^{48}\text{Ca}$ ,  $5\hbar\omega$  s.p. space, QRPA

$\sigma_{np}(\gamma D\text{IAS} \rightarrow I\text{AS})$  is 100 times larger than for the other mechanisms via admixtures of IVMR

Assumptions:

$$\sigma_{pn}(0_i^+ \rightarrow IVMR_s) = \sigma_0 \left| \langle IVMR_s | \hat{R}^- | 0_i^+ \rangle \right|^2, \quad \hat{R}^- = \sum_a \frac{r_a^2}{R^2} \tau_a^-$$

and

$$\sigma_{pn}(0_i^+ \rightarrow IVMR) \approx \frac{\sigma_{pn}(0_i^+ \rightarrow IAS)}{10}$$

## Conclusions

- $M_F^{0\nu}$  can be related to  $\Delta T = 2$  isospin admixture of the DIAS in the final g.s. and can be extracted from measured Fermi m.e.  $\langle IAS | \hat{T}^+ | 0_f \rangle$
- can help to discriminate between nuclear structure models (difference in  $M_F^{0\nu}$  as much as the factor of 5)
- Choice of a target: well-isolated IAS of  $^{48}\text{Ca}$  in  $^{48}\text{Sc}$  (weak Coulomb mixing applies)  
Reaction:  $^{48}\text{Ti}(n,p)^{48}\text{Sc}(\text{IAS})$   
Estimates:  $\sigma_{np} \approx 20 \text{ nb}/(\text{sr MeV})$  and  $\sigma_{np} \propto \langle IAS | \hat{T}^+ | 0_f \rangle$

## Conclusions

- Role of spread of IAS in heavy nuclei to be investigated

Supported by: DFG  TR27 “Neutrinos and beyond”

# Backup

I.E.1: 2.B

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## TOTAL (p, n) CROSS SECTIONS TO GROUND-STATE ANALOGUE STATES OF $^{48}\text{Ca}$ , $^{50}\text{Ti}$ , $^{52}\text{Cr}$ AND $^{58}\text{Ni}$

F. FOLKMANN and C. GAARDE

The Niels Bohr Institute, University of Copenhagen, Denmark

Received 24 June 1975

**Abstract:** Total (p, n) cross sections to isobaric analogues of  $0^+$  ground states are determined from the yield of  $\gamma$ -rays, de-exciting the IAS. Excitation functions are measured for targets  $^{48}\text{Ca}$ ,  $^{50}\text{Ti}$ ,  $^{52}\text{Cr}$  and  $^{58}\text{Ni}$  from threshold to 6.4, 4.0, 3.3 and 2.2 MeV above. The measured cross sections are compared with calculated cross sections for compound nuclear and direct reaction mechanisms. The decay scheme of the analogue state at 6.68 MeV in  $^{48}\text{Sc}$  is established.

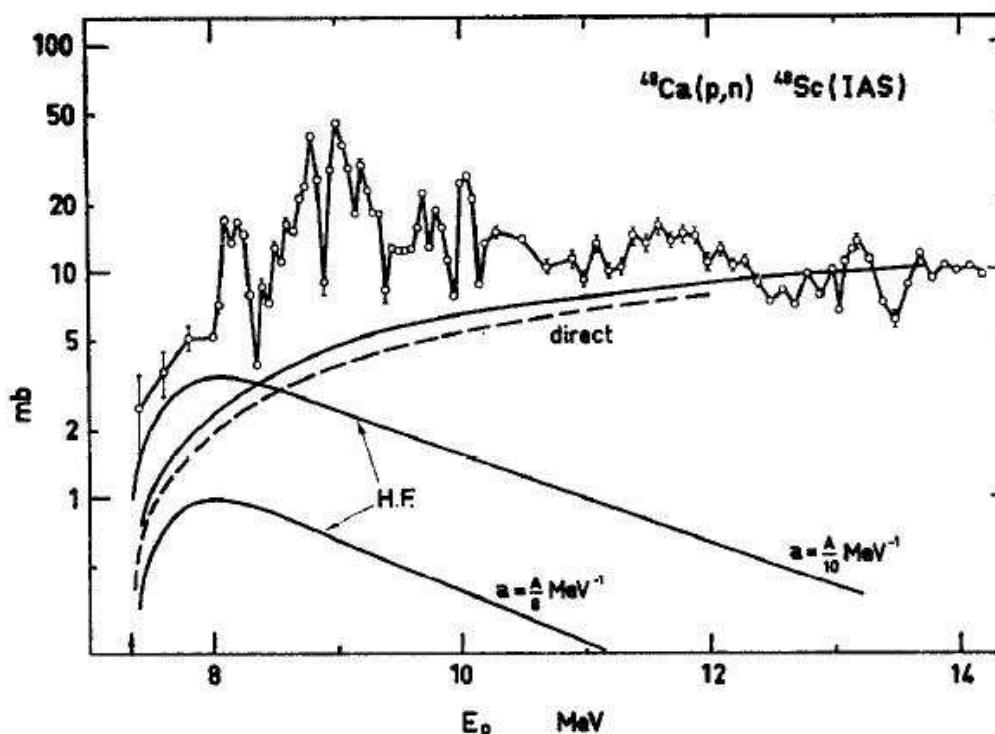


Fig. 1. Excitation function for the total cross section for the reaction  $^{48}\text{Ca}(p, n)^{48}\text{Sc}$  (IAS) where  $^{48}\text{Sc}$  (IAS) is the analogue state of the  $^{48}\text{Ca}$  g.s. The cross section is determined from the yield of the  $\gamma$ -decay of the IAS ( $E_{\text{ex}} = 6.679 \pm 0.002$  MeV) to the  $1^+$  state ( $E_{\text{ex}} = 2.5190 \pm 0.0015$  MeV). Also given are calculated cross sections.  $\Sigma$  and  $\Sigma_{\text{H.F.}}$  include calculations made with two different

# Backup

## 5. Summary

### 5.1. THE (p, n) CROSS SECTIONS

The  $^{48}\text{Ca}(p, n)^{48}\text{Sc}$  data are reasonably well described as a sum of compound and direct reaction contributions. For  $E_p > 11$  MeV the main cross section seems to come from the direct reaction.

### 5.2. THE $\gamma$ -DECAY OF ANALOGUE STATES

In all cases studied we only observe decay to a single  $1^+$  state. This is especially surprising in  $^{50}\text{V}$  and  $^{52}\text{Mn}$  where several  $1^+$  states are known. In  $^{48}\text{Sc}$  we have given quantitative limits on the M1 strength for  $\gamma$ -decay from the analogue state.

### 4.3. DECAY OF THE ANALOGUE STATE

In all the cases discussed above only a single transition from the analogue state is observed. In  $^{48}\text{Sc}$  we have given an upper limit of 8 % for other decay modes. This limit can be transformed into a limit on  $B(\text{M1})$ , and further to a limit of the GT strength for the  $\beta$ -decay of the  $^{48}\text{Ca}$  g.s. We could conclude that the GT strength to states below say 3.5 MeV excitation energy in  $^{48}\text{Sc}$  is less than 18 % of the strength to the  $1^+$  state at 2.519 MeV. Because of the  $E_\gamma^3$  dependence of the intensity for M1 transitions there is, however, very little sensitivity to the strength for states lying close to the analogue state.

## Spread of IAS

Why no fragmentation of IAS of  $^{48}\text{Ca}$ ?

### Density of $0^+$ states around IAS

back-shifted Fermi-gas (BSFG) model:

$$\rho(U, J, \pi) = \frac{1}{2} F(U, J) \rho(U)$$

$$\rho(U) = \frac{1}{12\sqrt{2}} \frac{1}{\sigma a^{1/4}} \frac{\exp(2\sqrt{aU})}{(U+t)^{5/4}}, \quad F(U, J) = \frac{2J+1}{2\sigma^2} \exp\left(\frac{-J(J+1)}{2\sigma^2}\right)$$

$$U = at^2 - t, \quad U = E - \delta,$$

the level density parameter  $a$ ; the spin cut-off parameter  $\sigma^2 = \frac{I_{\text{rigid}}}{\hbar^2} t \approx 0.015 A^{5/3} t$ ;  
 the backshift  $\delta$  ( $> 0$  even-even,  $\approx 0$  odd-A,  $< 0$  odd-odd);

## Spread of IAS

$$^{46}\text{Sc} \quad a = 5.96 \text{ MeV}^{-1}, \delta = -2.37 \text{ MeV} \quad (\text{W. Dilg et al. NPA 217 (1973)})$$

$$E_x = 6.8 \text{ MeV} \rightarrow \rho(0^+) + \rho(0^-) \approx 59 \text{ MeV}^{-1}$$

but at  $E_x = 3$  MeV  $\rightarrow \rho(0^+) + \rho(0^-) \approx 5$  MeV $^{-1}$   
 (no  $J = 0$  state is listed in ENSDF for  $^{48}\text{Sc}$  for  $E_x < 3$  MeV).

$$^{46}\text{Sc} \quad a = 5.74 \text{ MeV}^{-1}, \delta = -1.9 \text{ MeV} \quad (\text{RIPL-2})$$

$$E_x = 6.8 \text{ MeV} \rightarrow \rho(0^+) + \rho(0^-) \approx 33 \text{ MeV}^{-1}$$

at  $E_x = 3 \text{ MeV} \rightarrow \rho(0^+) + \rho(0^-) \approx 3 \text{ MeV}^{-1}$

## Spread of IAS

$$\begin{array}{ll} {}^{46}\text{Sc} & a = 5.96 \text{ MeV}^{-1}, \delta = -2.37 \text{ MeV} \\ & (W. Dilg et al. NPA 217 (1973)) \\ E_x = 6.8 \text{ MeV} & \rightarrow \rho(0^+) + \rho(0^-) \approx 59 \text{ MeV}^{-1} \end{array}$$

but at  $E_x = 3 \text{ MeV} \rightarrow \rho(0^+) + \rho(0^-) \approx 5 \text{ MeV}^{-1}$   
 (no  $J = 0$  state is listed in ENSDF for  ${}^{48}\text{Sc}$  for  $E_x < 3 \text{ MeV}$ ).

$$\begin{array}{ll} {}^{46}\text{Sc} & a = 5.74 \text{ MeV}^{-1}, \delta = -1.9 \text{ MeV} \\ & (RIPL-2) \\ E_x = 6.8 \text{ MeV} & \rightarrow \rho(0^+) + \rho(0^-) \approx 33 \text{ MeV}^{-1} \end{array}$$

at  $E_x = 3 \text{ MeV} \rightarrow \rho(0^+) + \rho(0^-) \approx 3 \text{ MeV}^{-1}$

$$\begin{array}{ll} {}^{76}\text{As} & \text{IAS at } E_x = 8.24 \text{ MeV} \\ a = 10.81 \text{ MeV}^{-1}, \delta = -1.45 \text{ MeV} & (W. Dilg et al. NPA 217 (1973)) \\ \rightarrow \rho(0^+) + \rho(0^-) \approx 7000 \text{ MeV}^{-1} & \end{array}$$

# Spread of IAS

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## Experimental High-Resolution Investigation and Shell-Model Interpretation of the $^{49}\text{Ca}$ Ground-State Analog

P. WILHJELM\*

*University of North Carolina, Chapel Hill, North Carolina, and Nuclear Structure Laboratory,  
Duke University, Durham, North Carolina 27706*

AND

G. A. KEYWORTH,† J. C. BROWNE, W. P. BERES,‡ M. DIVADEENAM, H. W. NEWSON, AND E. G. BILPUCH  
*Duke University, Durham, North Carolina 27706*

(Received 15 July 1968)

Excitation functions for  $^{48}\text{Ca}(p,p)$  at  $165^\circ$  and  $105^\circ$  have been measured from 1.93 to 2.01 MeV. In addition, the reactions  $^{48}\text{Ca}(p,n)^{48}\text{Sc}$  and  $^{48}\text{Ca}(p,n\gamma)^{48}\text{Sc}$  have been observed. Spins, parities, total and partial widths have been assigned. Eleven resonances are observed, eight of which have  $J^\pi = \frac{3}{2}^-$  and are associated with the  $^{49}\text{Ca}$  ground-state analog. These  $\frac{3}{2}^-$  levels are interpreted in terms of a detailed shell-model calculation.

TABLE I. Resonance parameters.

$\lambda$	Present work				Ref. 2					
	$E_{p\lambda}$ (keV)	$J^\pi$	$\Gamma_{p\lambda}$ (eV)	$\Gamma_{n\lambda}$ (eV)	$\gamma_{p\lambda}^2$ (keV)	$E_p$ (keV)	$J^\pi$	$\Gamma_p$ (eV)	$\Gamma_n$ (eV)	$\Gamma_\gamma$ (meV)
1	$1938 \pm 2$	$\frac{1}{2}^-$	10	20	0.297	...	...	...	...	...
2	$1947 \pm 2$	$\frac{1}{2}^+$	25	5	0.297	...	...	...	...	...
3	$1948 \pm 2$	$(\frac{3}{2}^-)$	30	70	0.862	$1950 \pm 2$	$(\frac{3}{2}^-)$	$\approx 80$	12	...
4	$1956 \pm 2$	$\frac{1}{2}^-$	15	120	0.421	...	...	...	...	...
5	$1959 \pm 2$	$\frac{3}{2}^-$	$100 \pm 25$	$500 \pm 100$	2.773	$1959 \pm 2$	$\frac{3}{2}^-$	$\approx 100$	24	10
6	$1962 \pm 2$	$(\frac{3}{2}^-)$	5	10	0.137	...	...	...	...	...
7	$1964 \pm 2$	$\frac{3}{2}^-$	$300 \pm 50$	$200 \pm 50$	8.193	$1964 \pm 2$	$\frac{3}{2}^-$	$400 \pm 50$	40	40
8	$1974 \pm 2$	$\frac{3}{2}^-$	$1300 \pm 100$	$600 \pm 100$	34.360	$1975 \pm 2$	$\frac{3}{2}^-$	$1500 \pm 200$	75	170
9	$1981 \pm 2$	$(\frac{3}{2}^-)$	40	80	1.035	$1982 \pm 2$	$(\frac{3}{2}^-)$	$\approx 70$	20	10
10	$1982 \pm 2$	$(\frac{3}{2}^-)$	50	50	1.290	...	...	...	...	...
11	$1991 \pm 2$	$(\frac{3}{2}^-)$	20	50	0.502	$1991 \pm 2$	$(\frac{3}{2}^-)$	$\approx 70$	9	7
...	...	...	...	...	...	$1.996 \pm 2$	$(\frac{3}{2}^-)$	$\approx 30$	5	...

TABLE III. Comparison of experimental and theoretical widths. All widths are given in keV.

$\Gamma_p = \sum \Gamma_{p\lambda}$	$\Gamma_n = \sum_\lambda \Gamma_{n\lambda}$	Experimental				Intermediate structure			Theoretical	
		Fine structure	$\Gamma_\gamma = \sum_\lambda \Gamma_{\gamma\lambda}$	$\Gamma^\dagger = \Gamma_p + \Gamma_n + \Gamma_\gamma$	$\Gamma_p$	$\Gamma_d$	$\Gamma^4 = \Gamma_d - \Gamma^\dagger$	$\Gamma_p$	$\Gamma^4$	
1.85	1.56	$\approx 0$		3.4	2.0	$4.7 \pm 0.4$	$1.3 \pm 0.4$	2.0	$\approx 0.6$	