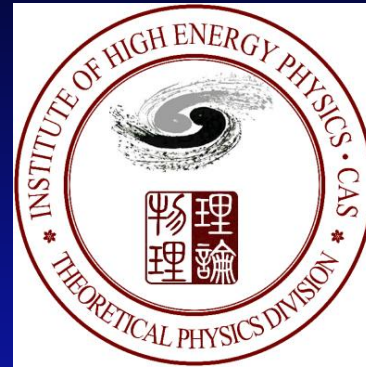


How to interpret a discovery or null result of the neutrinoless double beta decay

Zhi-zhong Xing
(IHEP, Beijing)



- ★ Introduction and motivation
- ★ The coupling-rod geometry
- ★ The 3-dimensional diagram
- ★ Why beyond the $0\nu 2\beta$ decay

Related theory talks:
S. Petcov: everything
J. Menendez: NMEs

...

@ Workshop on 2β Decays & Underground Science, 8-10/11, Osaka

1935: 2β decays

2β decay: certain **even-even** nuclei have a chance to decay into the second nearest neighbors via two simultaneous β decays (equivalent to the decays of two neutrons).

necessary conditions:

$$m(Z, A) > m(Z + 2, A)$$

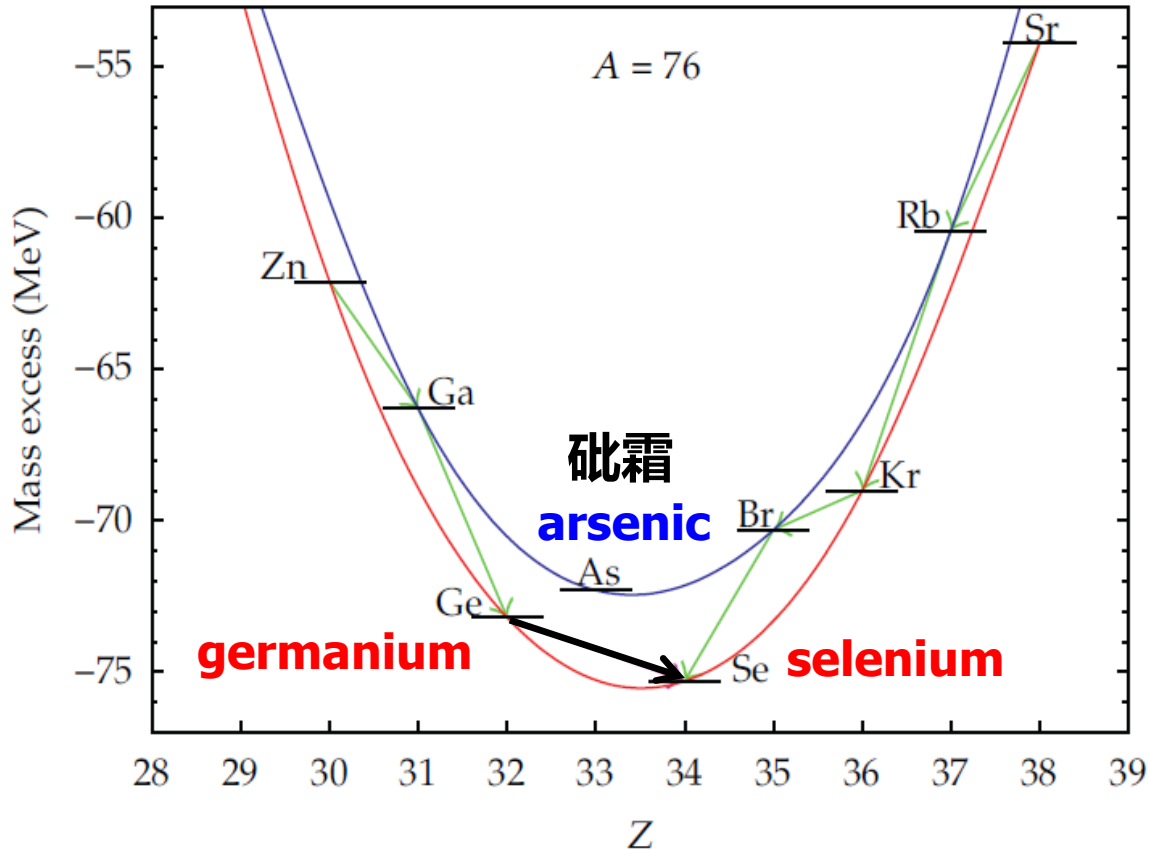
$$m(Z, A) < m(Z + 1, A)$$

$$(Z, A) \rightarrow (Z + 2, A) + 2e^- + 2\bar{\nu}_e.$$



1935

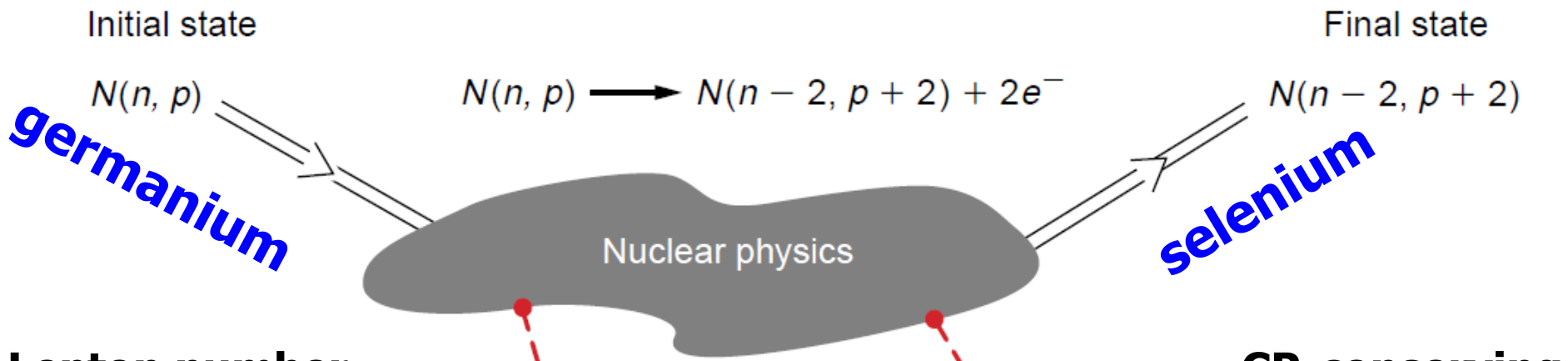
Maria Goeppert Mayer



1939: $0\nu 2\beta$ decays

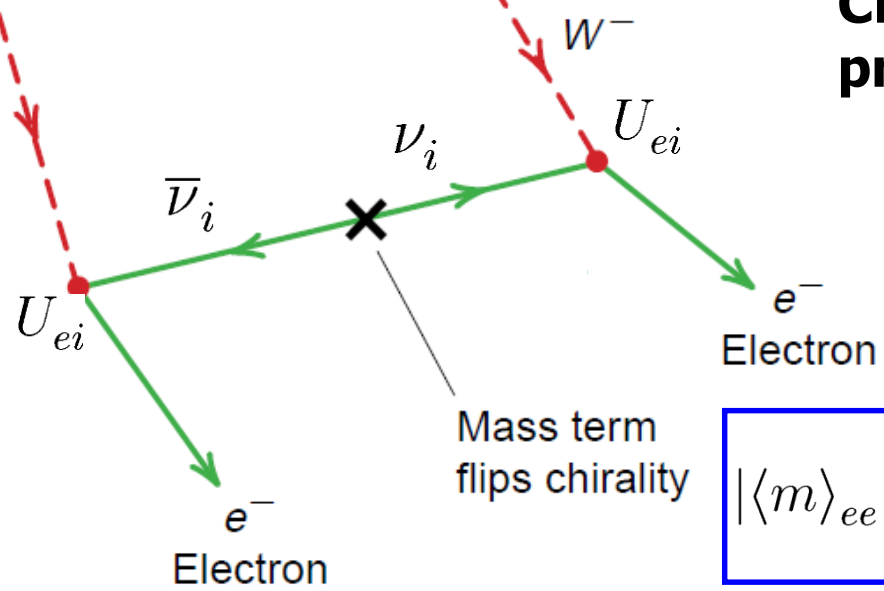
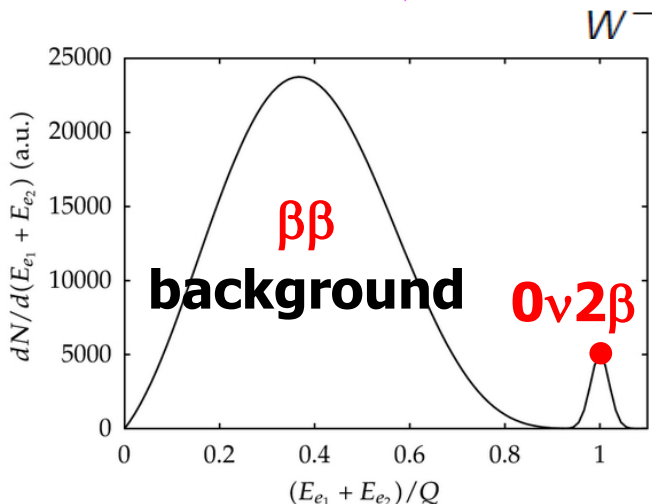
A $0\nu 2\beta$ decay can happen if massive ν 's have the Majorana nature (W.H. Furry 1939):

$$T_{1/2}^{0\nu} = (G^{0\nu})^{-1} |M^{0\nu}|^{-2} |\langle m \rangle_{ee}|^{-2}$$

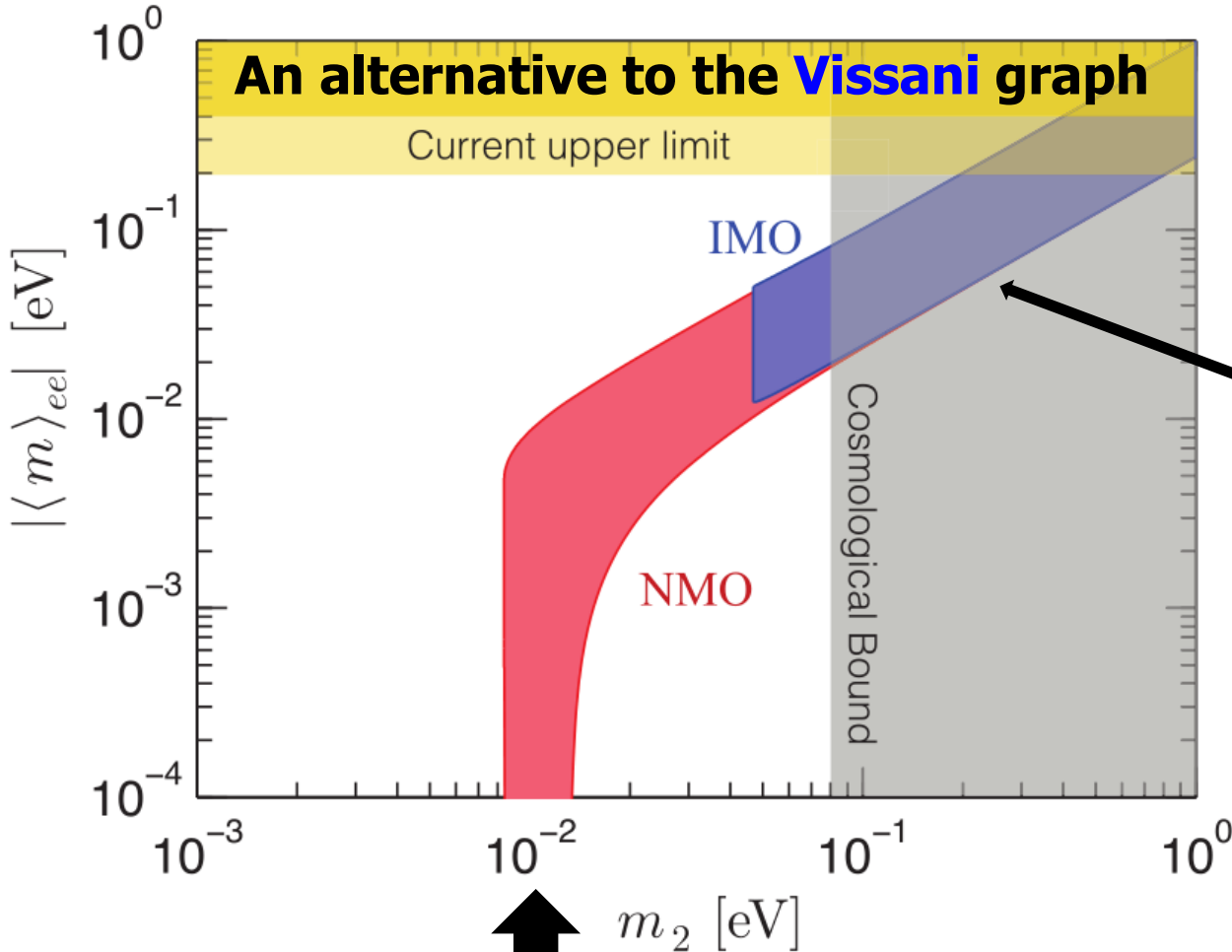


Lepton number violation \rightarrow

CP-conserving process \leftarrow



$$|\langle m \rangle_{ee}| = \left| \sum_i m_i U_{ei}^2 \right|$$



The effective mass

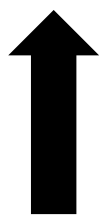
$$|\langle m \rangle_{ee}| = \left| \sum_i m_i U_{ei}^2 \right|$$

Maury Goodman asks:
An intelligent design?



Einstein (1921):
Subtle is the Lord, but
malicious He is not.

The dark well



Vanishing $0\nu 2\beta$ mass?
Xing, hep-ph/0305195

Burning Question:
how to interpret a discovery or a null
result of $0\nu 2\beta$?

Motivation of this talk

- to look at the effective $0\nu 2\beta$ mass in a geometric way;
- to show the effects of Majorana phases in a 3-d graph;
- to explain why we have to go beyond the $0\nu 2\beta$ decays.

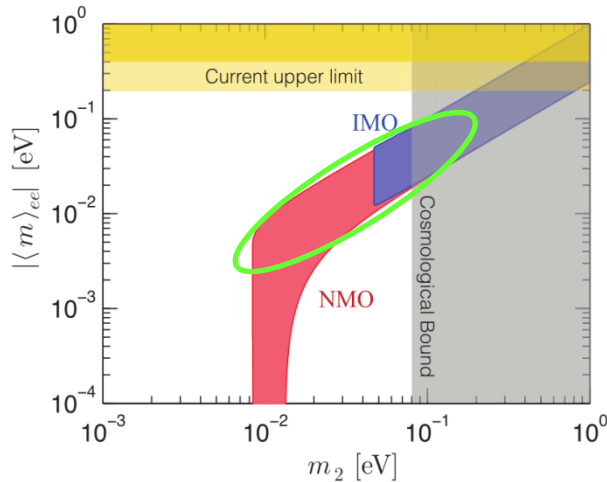
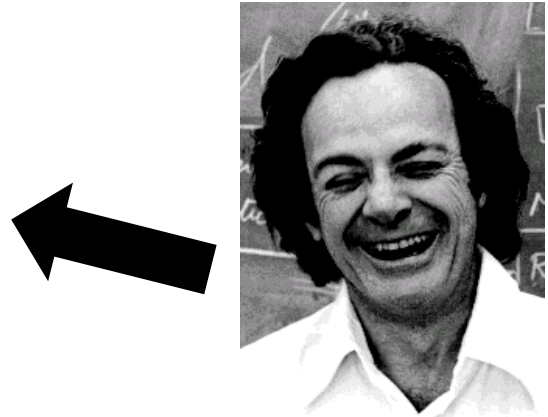
I won't tell you anything that you don't know. I'll tell you something that you have known in a slightly (**psychologically**) different way.

Example: the language of unitarity triangles in the quark sector proves to be very useful.

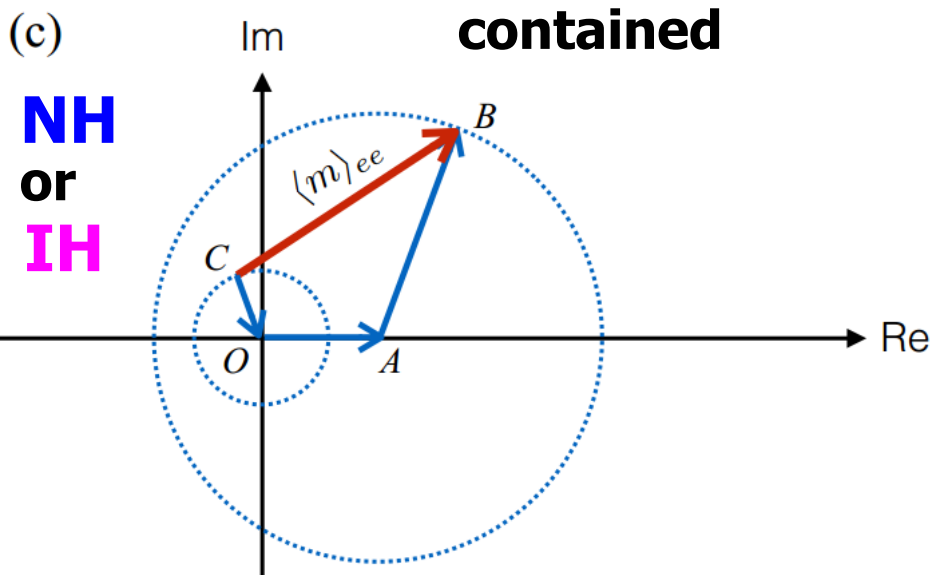
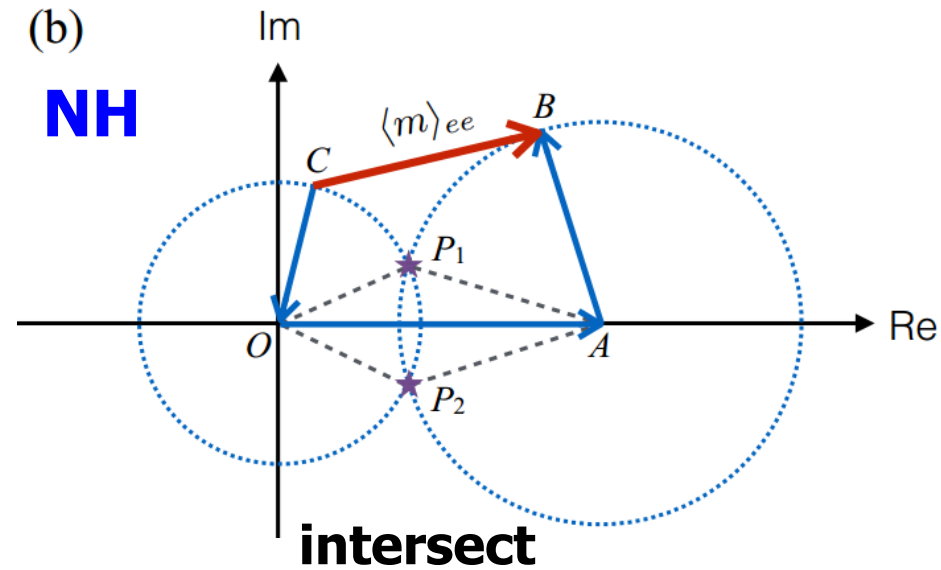
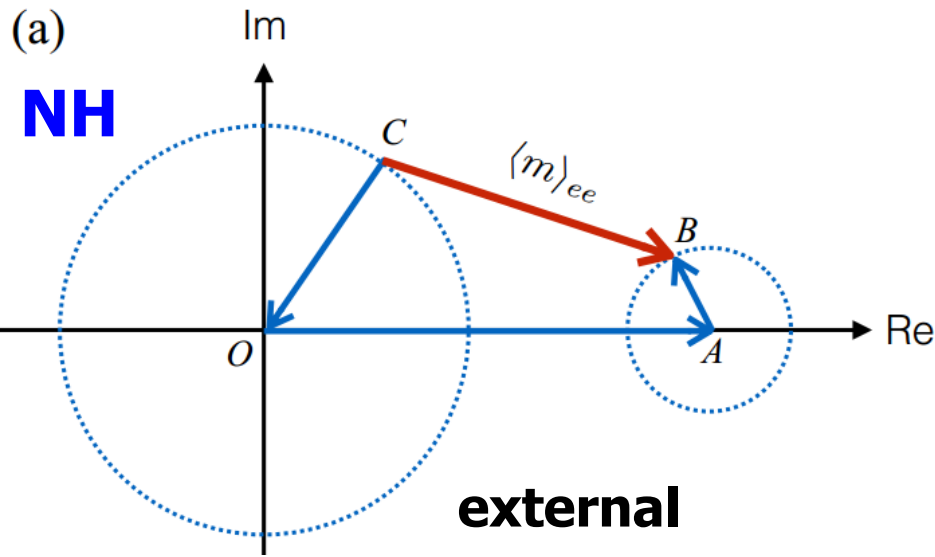
A naïve question: if the accelerator and reactor neutrino (antineutrino) oscillation experiments point to the normal neutrino mass hierarchy, is it still promising to look for the $0\nu 2\beta$ decays?

Sure!

I have bet 50 Euro (~5000 Yen) on the **normal mass hierarchy** ---- better nearly degenerate!



Coupling-rod diagram



$$\langle m \rangle_{ee} \equiv \overrightarrow{CB} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{CO}$$

$$\begin{aligned} \overrightarrow{OA} &\equiv m_2 U_{e2}^2 = m_2 |U_{e2}|^2, \\ \overrightarrow{AB} &\equiv m_1 U_{e1}^2 = m_1 |U_{e1}|^2 e^{i\rho} \\ \overrightarrow{CO} &\equiv m_3 U_{e3}^2 = m_3 |U_{e3}|^2 e^{i\sigma} \end{aligned}$$

The above diagrams allow us to obtain the **maximum/minimum** limit:

Maximum in either hierarchy: $|\langle m \rangle_{ee}|_{\max} = OA + AB + OC$

$$|\langle m \rangle_{ee}|_{\max} = m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} \left[1 + \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2} \cot^2 \theta_{12}} + \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2} + \frac{\Delta m_{31}^2}{m_2^2} \frac{\tan^2 \theta_{13}}{\sin^2 \theta_{12}}} \right]$$

Minimum in **normal** hierarchy (1): $|\langle m \rangle_{ee}|_{\min}^{(a)} = OA - AB - OC$

$$|\langle m \rangle_{ee}|_{\min}^{(a)} = m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} \left[1 - \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2} \cot^2 \theta_{12}} - \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2} + \frac{\Delta m_{31}^2}{m_2^2} \frac{\tan^2 \theta_{13}}{\sin^2 \theta_{12}}} \right]$$

Minimum in **normal** hierarchy (2): $|\langle m \rangle_{ee}|_{\min}^{(c)} = AB - OA - OC$

$$|\langle m \rangle_{ee}|_{\min}^{(c)} = m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} \left[\sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2} \cot^2 \theta_{12}} - 1 - \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2} + \frac{\Delta m_{31}^2}{m_2^2} \frac{\tan^2 \theta_{13}}{\sin^2 \theta_{12}}} \right]$$

Minimum in **inverted** hierarchy: $|\langle m \rangle_{ee}|_{\min}^{(c)} = AB - OA - OC$

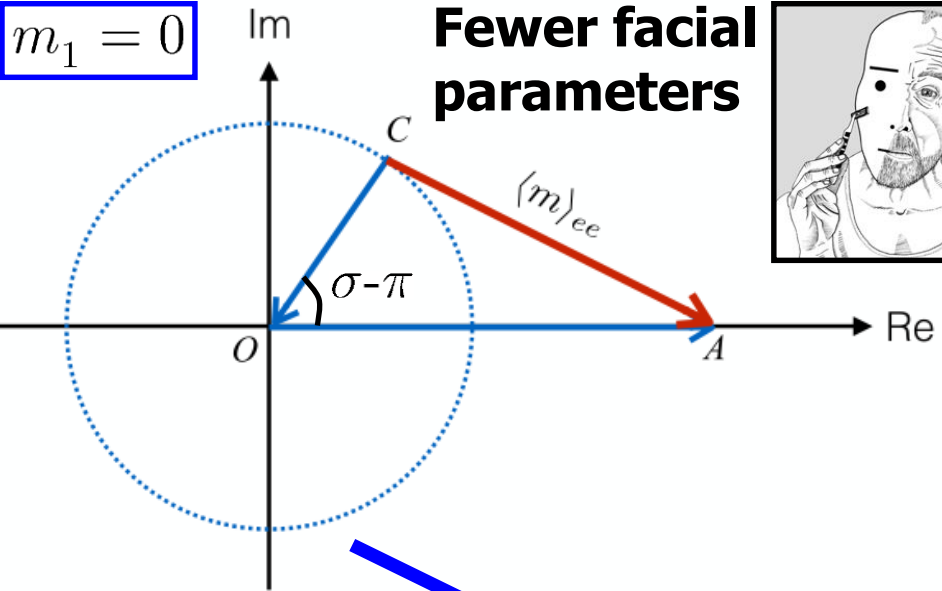
$$|\langle m \rangle_{ee}|_{\min}^{(c)} = m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} \left[\sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2} \cot^2 \theta_{12}} - 1 - \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2} + \frac{\Delta m_{31}^2}{m_2^2} \frac{\tan^2 \theta_{13}}{\sin^2 \theta_{12}}} \right]$$

Entities must not be multiplied beyond necessity.

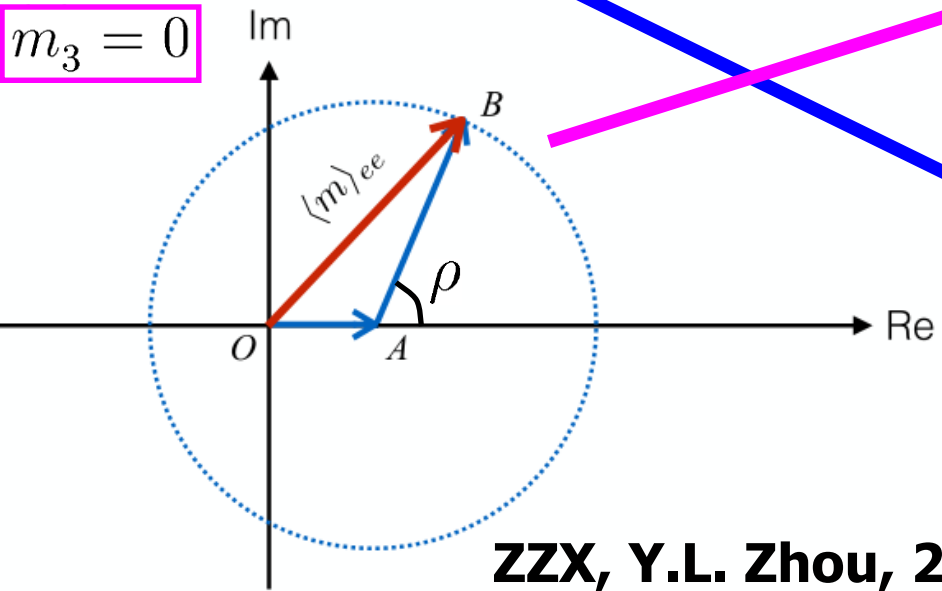


$m_1 = 0$

Fewer facial parameters



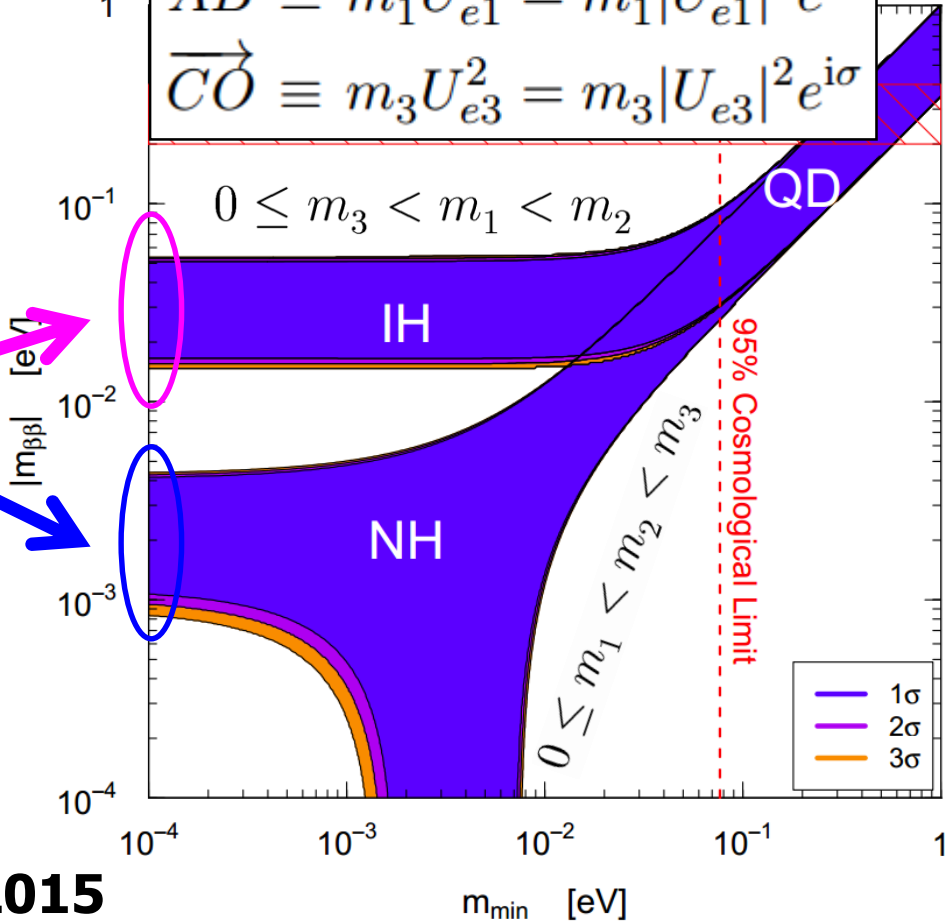
$m_3 = 0$

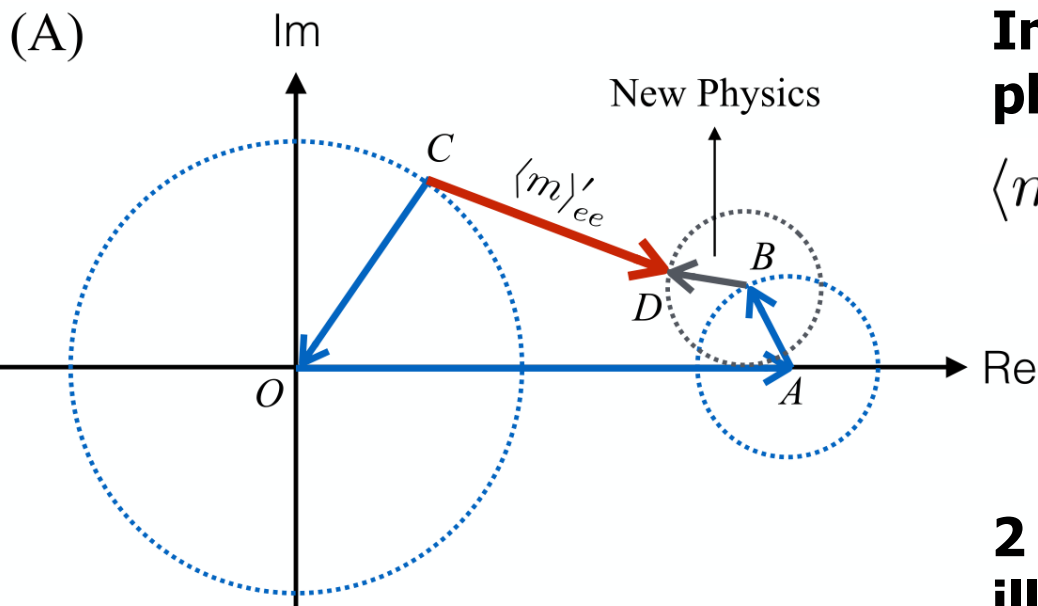


$$\vec{OA} \equiv m_2 U_{e2}^2 = m_2 |U_{e2}|^2,$$

$$\vec{AB} \equiv m_1 U_{e1}^2 = m_1 |U_{e1}|^2 e^{i\rho}$$

$$\vec{CO} \equiv m_3 U_{e3}^2 = m_3 |U_{e3}|^2 e^{i\sigma}$$



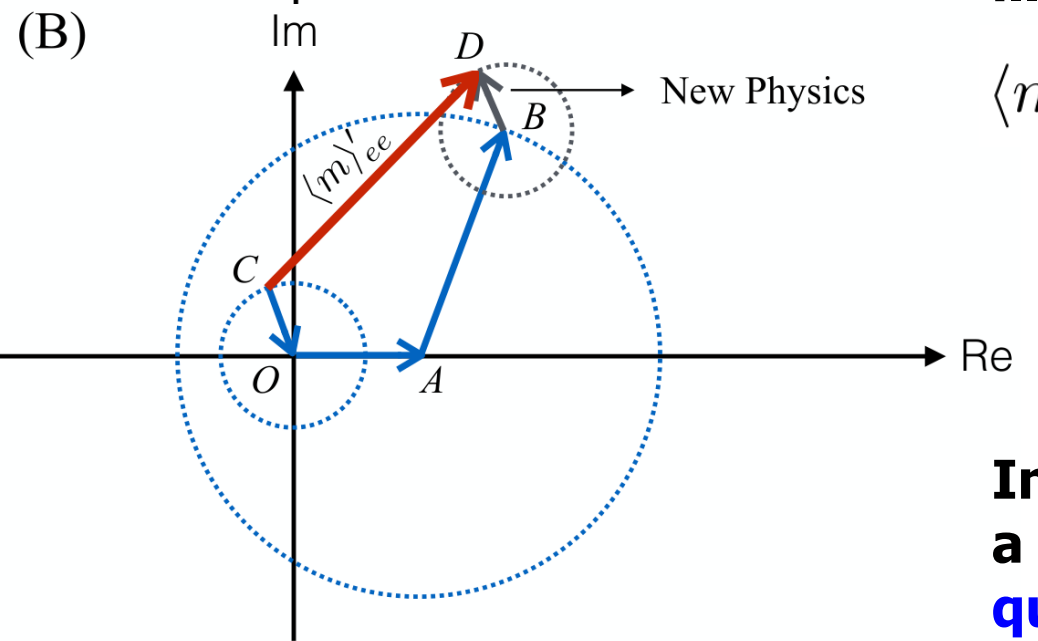


In the presence of a kind of new physics, we denote:

$$\langle m \rangle'_{\alpha\beta} = \langle m \rangle_{\alpha\beta} + \text{new physics}$$

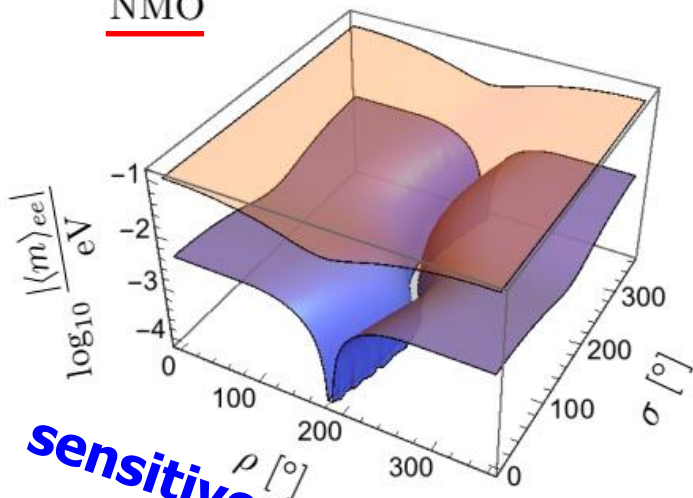
2 very simple configurations are illustrated on the left-hand side:

$$\begin{aligned} \langle m \rangle'_{\alpha\beta} &\equiv \overrightarrow{CD} = \langle m \rangle_{\alpha\beta} + \overrightarrow{BD} \\ &= \overrightarrow{CO} + \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BD} \end{aligned}$$

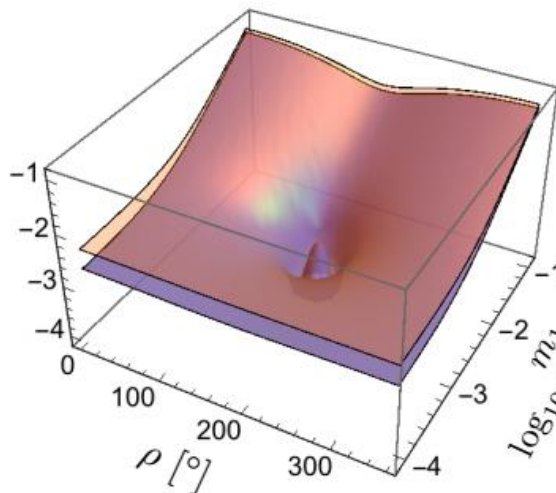


In the above Occam's razor case, a pentagon can be simplified to a quadrangle.

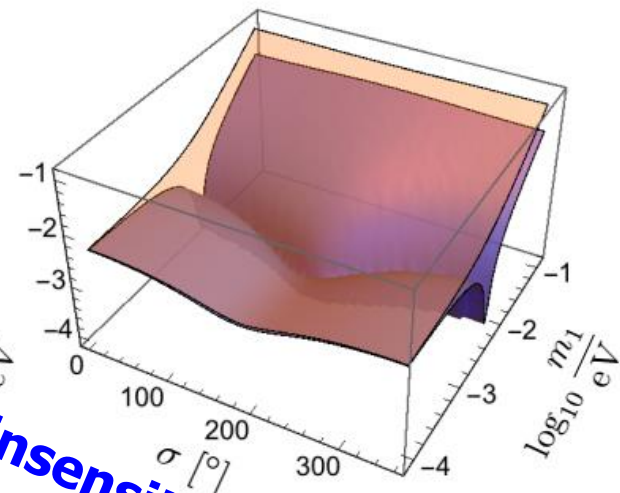
NMO



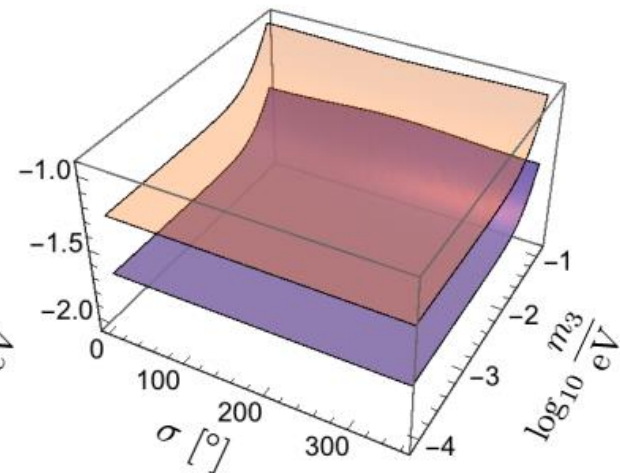
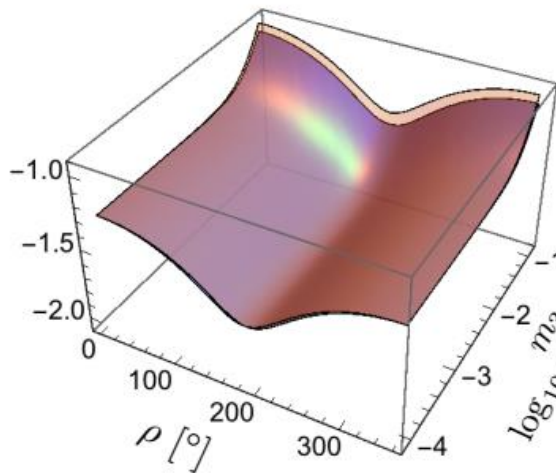
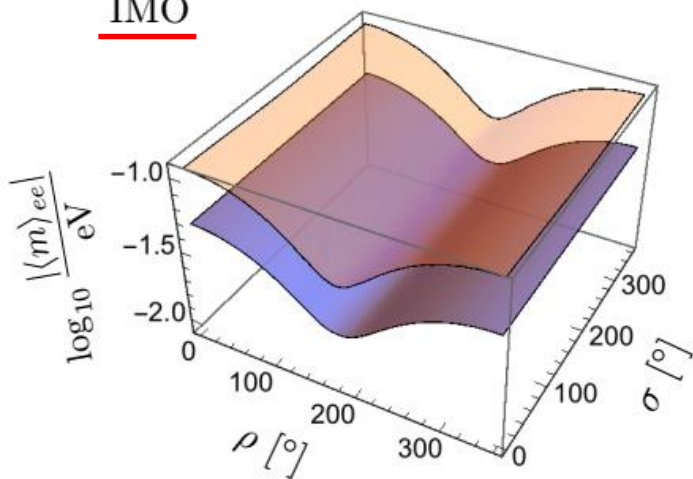
sensitive



insensitive



IMO



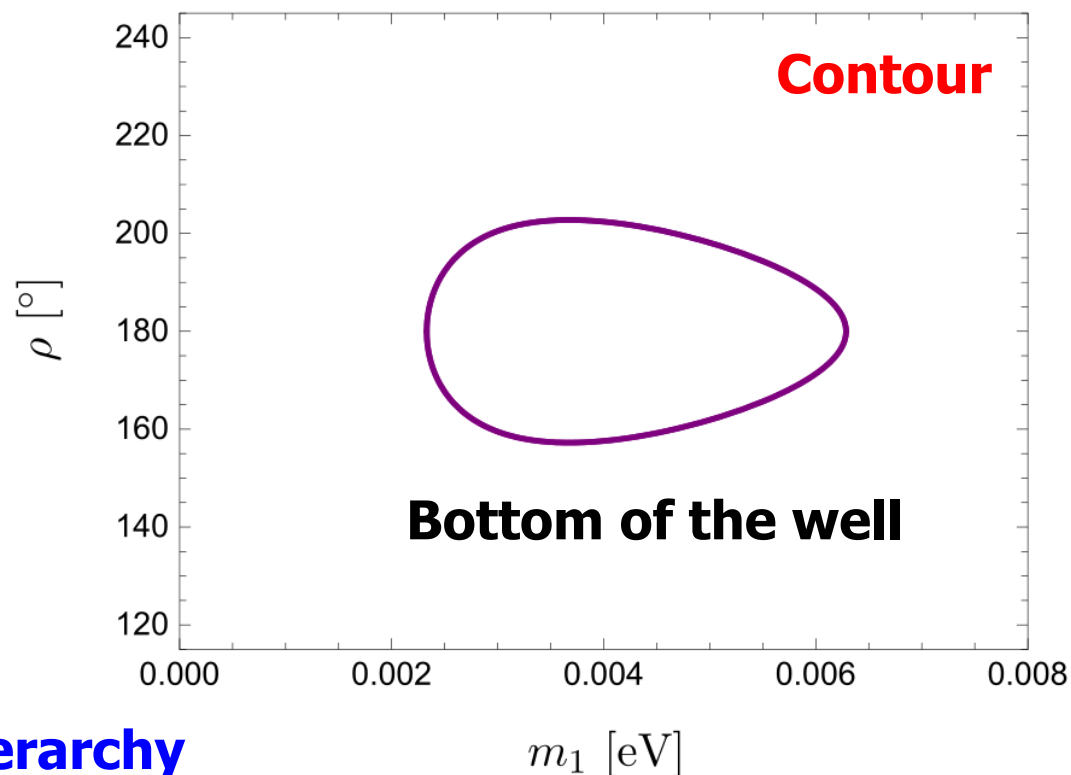
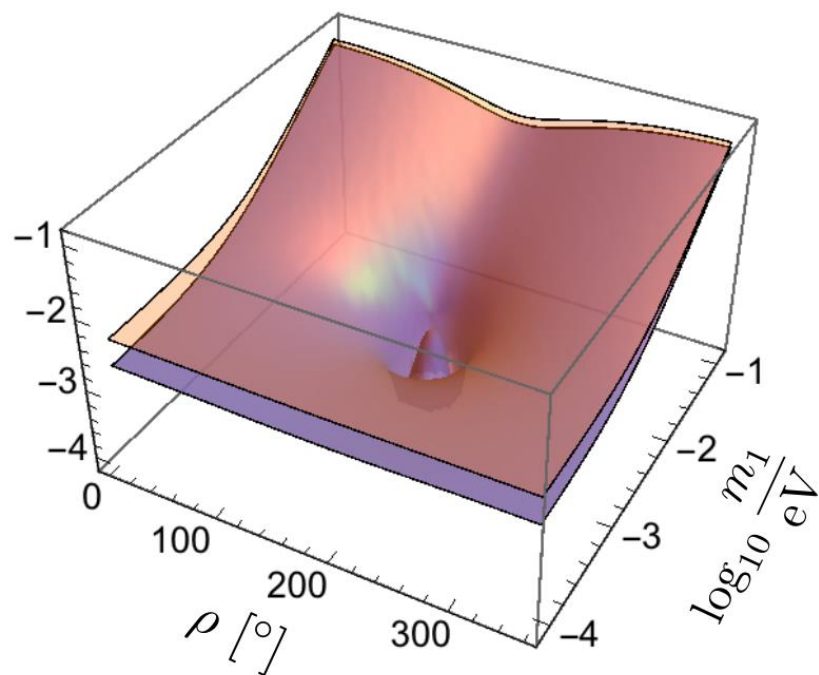
Lower bound: blue; upper bound: light orange. Clearer sensitivities to mass and phase parameters (Xing, Zhao, Zhou, arXiv:1504.05820)

Let us understand the champagne-bottle profile of the effective $0\nu 2\beta$ mass term in the **normal hierarchy** case:

$$\langle m \rangle_{ee} = m_1 c_{12}^2 c_{13}^2 e^{i\rho} + m_2 s_{12}^2 c_{13}^2 + m_3 s_{13}^2 e^{i\sigma} = 0$$



$$m_1^2 c_{12}^4 c_{13}^4 + 2m_1 m_2 c_{12}^2 s_{12}^2 c_{13}^4 \cos \rho + m_2^2 s_{12}^4 c_{13}^4 = m_3^2 s_{13}^4$$



The dark well in the **normal hierarchy**

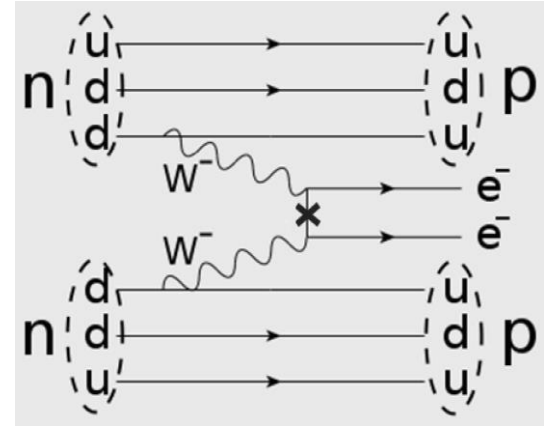
m_1 [eV]

Type (A): NP directly related to extra species of neutrinos.

Example 1: heavy Majorana neutrinos from type-I seesaw

$$-\mathcal{L}_{\text{lepton}} = \bar{l}_L Y_l H E_R + \bar{l}_L Y_\nu \tilde{H} N_R + \frac{1}{2} \bar{N}_R^c M_R N_R + \text{h.c.}$$

$$\Gamma_{0\nu\beta\beta} \propto \left| \sum_{i=1}^3 m_i U_{ei}^2 - \sum_{k=1}^n \frac{R_{ek}^2}{M_k} M_A^2 \mathcal{F}(A, M_k) \right|^2$$



In most cases the heavy contribution is negligible

Example 2: light sterile neutrinos from LSND etc

$$\langle m \rangle'_{ee} \equiv \sum_{i=1}^6 m_i U_{ei}^2 = \underline{\langle m \rangle_{ee}} (c_{14} c_{15} c_{16})^2 + m_4 (\hat{s}_{14}^* c_{15} c_{16})^2 + m_5 (\hat{s}_{15}^* c_{16})^2 + m_6 (\hat{s}_{16}^*)^2$$

In this case the new contribution might be constructive or destructive

Type (B): NP has little to do with the neutrino mass issue.

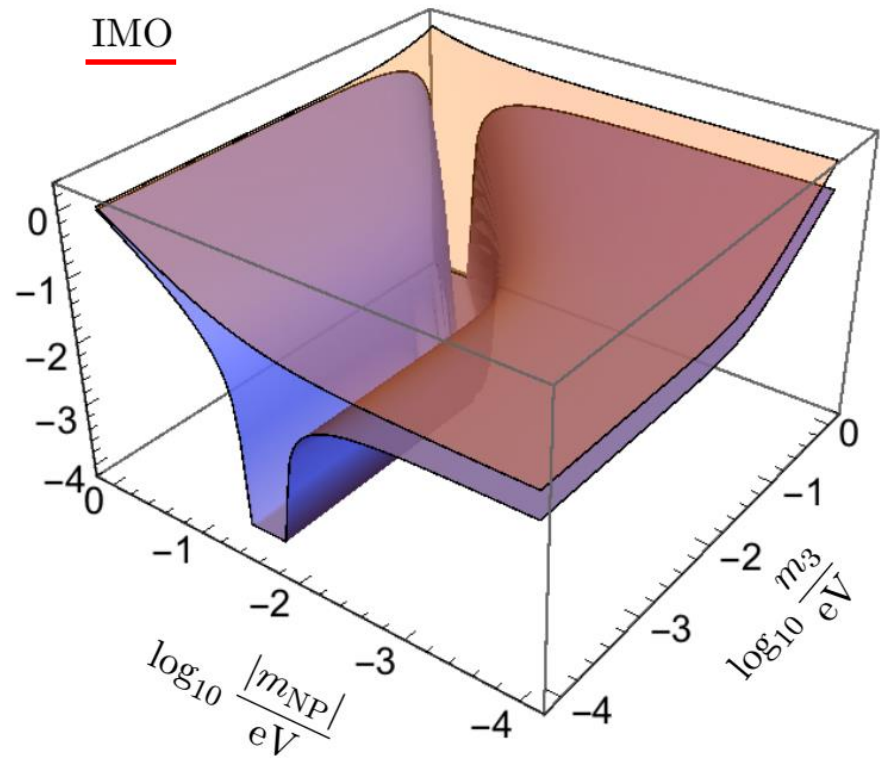
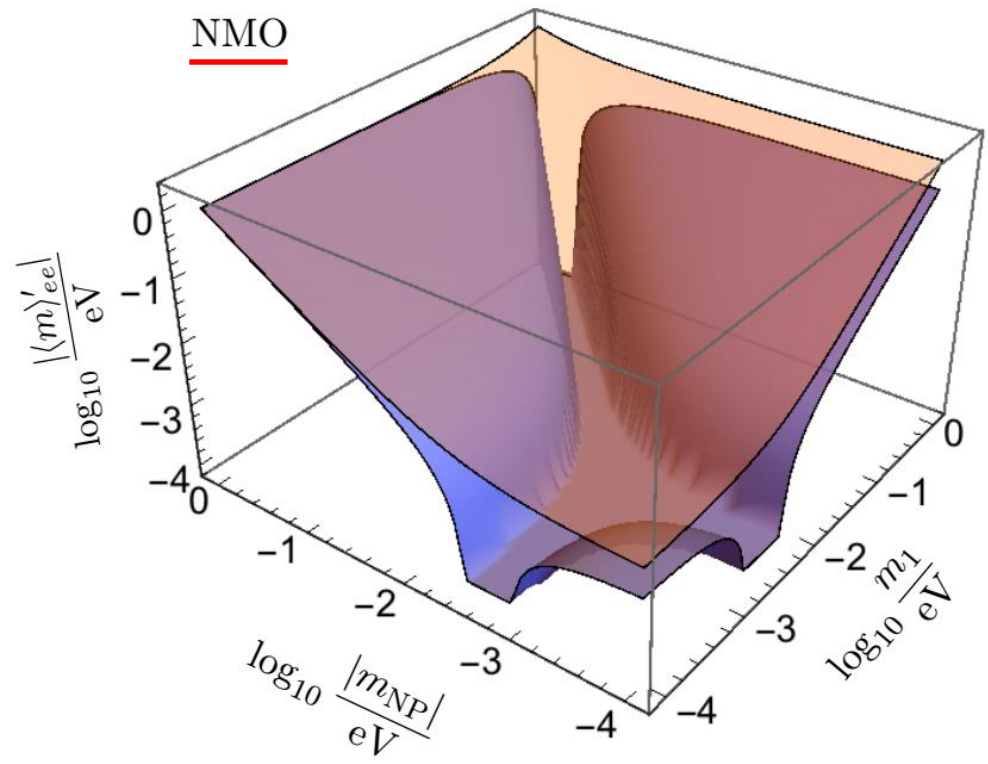
SUSY, Left-right, and some others that I don't understand

New physics effects:

$$\langle m \rangle'_{ee} = m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2 + m_{\text{NP}}$$

NMO

IMO



Lower bound: blue; upper bound: light orange. Clearer sensitivities to mass and phase parameters (Xing, Zhao, Zhou, arXiv:1504.05820)

$$|\langle m \rangle'_{ee}|_{\text{upper}} = m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 + m_3 |U_{e3}|^2 + |m_{\text{NP}}|,$$

$$|\langle m \rangle'_{ee}|_{\text{lower}} = \max \left\{ 0, 2m_i |U_{ei}|^2 - |\langle m \rangle'_{ee}|_{\text{upper}}, 2|m_{\text{NP}}| - |\langle m \rangle'_{ee}|_{\text{upper}} \right\}$$

It is hard to tell much

QUESTION: are massive neutrinos the **Majorana** particles?

One might be able to answer **YES** through a measurement of the $0\nu 2\beta$ decay or other **LNV** processes someday, but how to answer with **NO**?



YES
or
I don't know!



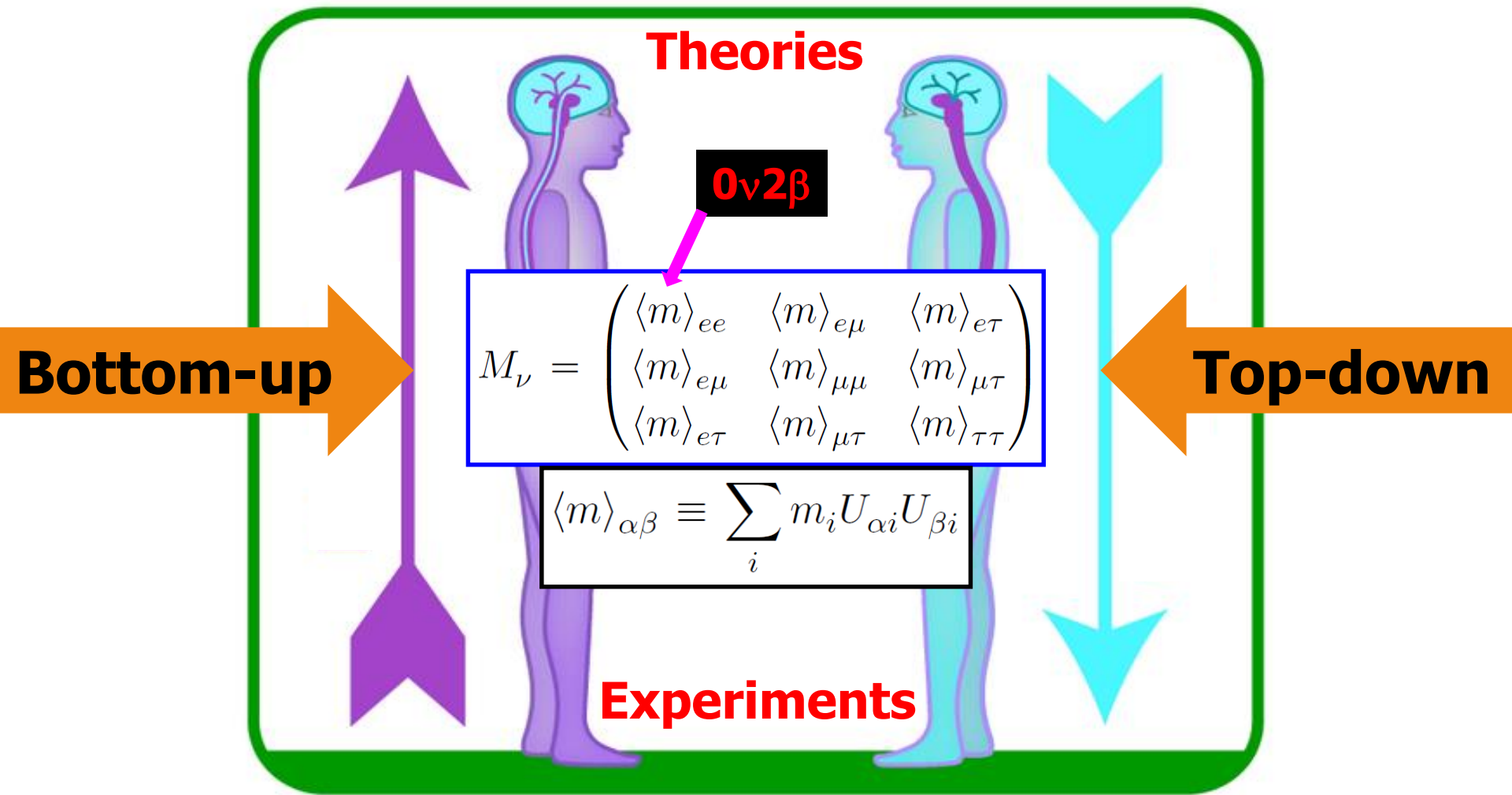
The same question: how to distinguish between **Dirac** and **Majorana** neutrinos in a realistic experiment?

Answer 1: The $0\nu 2\beta$ decay is currently the only possibility.

Answer 2: In principle their dipole moments are different.

Answer 3: They show different behavior if nonrelativistic.

Without information on the **nature of massive neutrinos** (**Majorana** or not) and **all the CP-violating phases**, one will have no way to establish a full theory of ν masses and flavor mixing. Give **$0\nu 2\beta$** a chance!

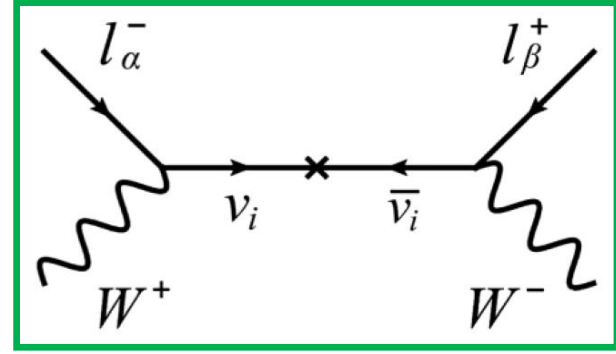


The neutrino-antineutrino oscillation can in principle help.

Neutrino-Antineutrino Oscillations:

$$A(\nu_\alpha \rightarrow \bar{\nu}_\beta) = \sum_i \left[U_{\alpha i}^* U_{\beta i} \frac{m_i}{E} \exp\left(-i \frac{m_i^2}{2E} L\right) \right] K$$

$$A(\bar{\nu}_\alpha \rightarrow \nu_\beta) = \sum_i \left[U_{\alpha i} U_{\beta i} \frac{m_i}{E} \exp\left(-i \frac{m_i^2}{2E} L\right) \right] \bar{K}$$



$$\phi_{ji} \equiv \Delta m_{ji}^2 L / (4E)$$

$$P(\nu_\alpha \rightarrow \bar{\nu}_\beta) = \frac{|K|^2}{E^2} \left[|\langle m \rangle_{\alpha\beta}|^2 - 4 \sum_{i<j} m_i m_j \mathcal{C}_{\alpha\beta}^{ij} \sin^2 \phi_{ji} + 2 \sum_{i<j} m_i m_j \mathcal{V}_{\alpha\beta}^{ij} \sin 2\phi_{ji} \right]$$

$$P(\bar{\nu}_\alpha \rightarrow \nu_\beta) = \frac{|\bar{K}|^2}{E^2} \left[|\langle m \rangle_{\alpha\beta}|^2 - 4 \sum_{i<j} m_i m_j \mathcal{C}_{\alpha\beta}^{ij} \sin^2 \phi_{ji} - 2 \sum_{i<j} m_i m_j \mathcal{V}_{\alpha\beta}^{ij} \sin 2\phi_{ji} \right]$$

mass

CP-conserving

CP-violating

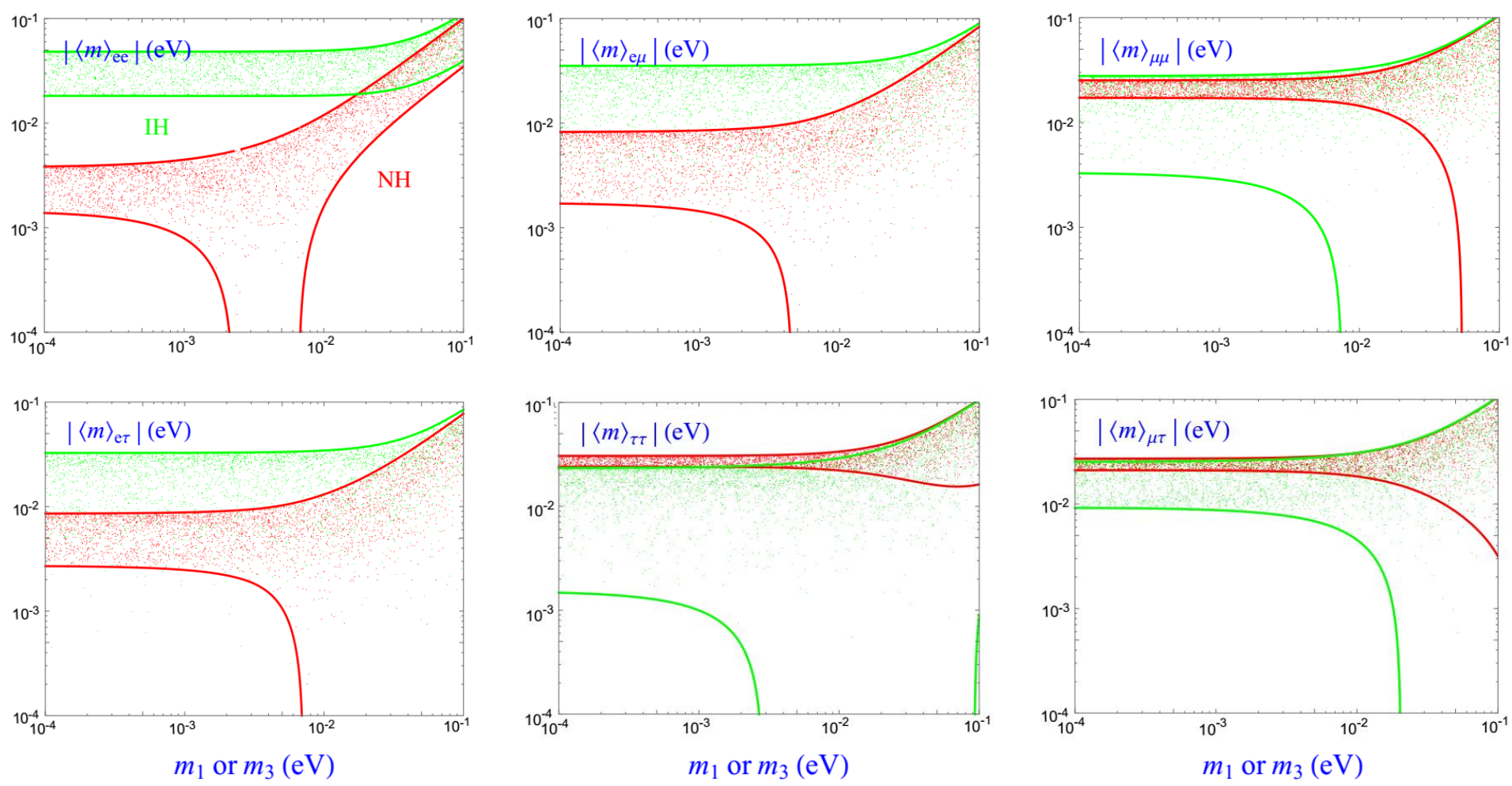
Effective mass terms:

$$\langle m \rangle_{\alpha\beta} \equiv \sum_i m_i U_{\alpha i} U_{\beta i}$$

Jarlskog-like parameters:

$$\mathcal{C}_{\alpha\beta}^{ij} \equiv \text{Re} (U_{\alpha i} U_{\beta i} U_{\alpha j}^* U_{\beta j}^*)$$

$$\mathcal{V}_{\alpha\beta}^{ij} \equiv \text{Im} (U_{\alpha i} U_{\beta i} U_{\alpha j}^* U_{\beta j}^*)$$



To identify the Majorana nature, CP-violating phases and new physics it is imperative to observe the $0\nu 2\beta$ decays and other lepton-number-violating processes (e.g., neutrino-antineutrino oscillations, the relic neutrino background, doubly-charged Higgs decays). **None is realistic**

All of us expect the massive neutrinos to be the **Majorana** particles. If this expectation comes true someday thanks to the $0\nu 2\beta$ decay, then we will be required to have some **new** or **good** or **right** ideas to probe the **Majorana phases**.



I.I. Rabi: Physics needs **new ideas**. But to have a **new idea** is a very difficult task....



C.S. Wu: It is easy to do the right thing once you have the **right ideas**.



L.C. Pauling: The best way to have a **good idea** is to have a lot of ideas.