How to interpret a discovery or null result of the neutrinoless double beta decay

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★ Introduction and motivation
★ The coupling-rod geometry
★ The 3-dimensional diagram
★ Why beyond the $0\nu2\beta$ decay

Related theory talks:
S. Petcov: everything
J. Menendez: NMEs
...

@ Workshop on 2$\beta$ Decays & Underground Science, 8-10/11, Osaka
1935: $2\beta$ decays

$2\beta$ decay: certain even-even nuclei have a chance to decay into the second nearest neighbors via two simultaneous $\beta$ decays (equivalent to the decays of two neutrons).

necessary conditions:

\[
m(Z, A) > m(Z + 2, A)
\]

\[
m(Z, A) < m(Z + 1, A)
\]

\[
(Z, A) \rightarrow (Z + 2, A) + 2e^- + 2\bar{\nu}_e.
\]
1939: $0\nu2\beta$ decays

A $0\nu2\beta$ decay can happen if massive $\nu$'s have the Majorana nature (W.H. Furry 1939):

$$T_{1/2}^{0\nu} = (G^{0\nu})^{-1} |M^{0\nu}|^{-2} |\langle m \rangle_{ee}|^{-2}$$

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**Part A**

**Lepton number violation**

Initial state: $N(n, p) \rightarrow N(n-2, p+2) + 2e^-$

Final state: $N(n-2, p+2)$

**CP-conserving process**

$\beta\beta$ background

$$\sum_i m_i U_{ei}^2$$

**Germanium**

**Selenium**

Nuclear physics
The effective mass

An alternative to the Vissani graph

\[ |\langle m \rangle_{ee}| = \left| \sum_i m_i U_{ei}^2 \right| \]

Maury Goodman asks: An intelligent design?

Einstein (1921): Subtle is the Lord, but malicious He is not.

The dark well

Vanishing $0^{\nu}2\beta$ mass?

Xing, hep-ph/0305195

Burning Question:
how to interpret a discovery or a null result of $0^{\nu}2\beta$?
Part A

Motivation of this talk

- to look at the effective $0\nu2\beta$ mass in a geometric way;
- to show the effects of Majorana phases in a 3-d graph;
- to explain why we have to go beyond the $0\nu2\beta$ decays.

I won’t tell you anything that you don’t know. I’ll tell you something that you have known in a slightly (psychologically) different way.

**Example:** the language of unitarity triangles in the quark sector proves to be very useful.

**A naïve question:** if the accelerator and reactor neutrino (antineutrino) oscillation experiments point to the normal neutrino mass hierarchy, is it still promising to look for the $0\nu2\beta$ decays?

**Sure!**

I have bet 50 Euro (~5000 Yen) on the normal mass hierarchy ---- better nearly degenerate!
Coupling-rod diagram

(a) NH

(b) NH

(c) NH or IH

\[ \langle m \rangle_{ee} \equiv \overrightarrow{CB} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{CO} \]

\[ \overrightarrow{OA} \equiv m_2 U_{e2}^2 = m_2 |U_{e2}|^2 , \]
\[ \overrightarrow{AB} \equiv m_1 U_{e1}^2 = m_1 |U_{e1}|^2 e^{i\alpha} , \]
\[ \overrightarrow{CO} \equiv m_3 U_{e3}^2 = m_3 |U_{e3}|^2 e^{i\sigma} \]

Part B

Maximum + Minimum

The above diagrams allow us to obtain the maximum/minimum limit:

**Maximum in either hierarchy:** $|\langle m \rangle_{ee}\rangle_{\text{max}} = OA + AB + OC$

\[
|\langle m \rangle_{ee}\rangle_{\text{max}} = m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} \left[ 1 + \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2}} \cot^2 \theta_{12} + \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2} + \frac{\Delta m_{31}^2}{m_2^2} \tan^2 \theta_{13}} \right]
\]

**Minimum in normal hierarchy (1):** $|\langle m \rangle_{ee}\rangle_{\text{min}}^{(a)} = OA - AB - OC$

\[
|\langle m \rangle_{ee}\rangle_{\text{min}}^{(a)} = m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} \left[ 1 - \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2}} \cot^2 \theta_{12} - \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2} + \frac{\Delta m_{31}^2}{m_2^2} \tan^2 \theta_{13}} \right]
\]

**Minimum in normal hierarchy (2):** $|\langle m \rangle_{ee}\rangle_{\text{min}}^{(c)} = AB - OA - OC$

\[
|\langle m \rangle_{ee}\rangle_{\text{min}}^{(c)} = m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} \left[ \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2}} \cot^2 \theta_{12} - 1 - \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2} + \frac{\Delta m_{31}^2}{m_2^2} \tan^2 \theta_{13}} \right]
\]

**Minimum in inverted hierarchy:** $|\langle m \rangle_{ee}\rangle_{\text{min}}^{(c)} = AB - OA - OC$

\[
|\langle m \rangle_{ee}\rangle_{\text{min}}^{(c)} = m_2 \sin^2 \theta_{12} \cos^2 \theta_{13} \left[ \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2}} \cot^2 \theta_{12} - 1 \right] - \sqrt{1 - \frac{\Delta m_{21}^2}{m_2^2} + \frac{\Delta m_{31}^2}{m_2^2} \tan^2 \theta_{13}} \right]
\]
Occam’s razor

Entities must not be multiplied beyond necessity.

Fewer facial parameters

\[ m_1 = 0 \]

\[ m_3 = 0 \]

ZZX, Y.L. Zhou, 2015
New physics

In the presence of a kind of new physics, we denote:

\[ \langle m \rangle'_{\alpha\beta} = \langle m \rangle_{\alpha\beta} + \text{new physics} \]

2 very simple configurations are illustrated on the left-hand side:

\[ \langle m \rangle'_{\alpha\beta} \equiv \overrightarrow{CD} = \langle m \rangle_{\alpha\beta} + \overrightarrow{BD} \]
\[ = \overrightarrow{CO} + \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BD} \]

In the above Occam’s razor case, a pentagon can be simplified to a quadrangle.
Part C

3-d description

Part C

Contour of the well

Let us understand the champagne-bottle profile of the effective $0\nu 2\beta$ mass term in the **normal hierarchy** case:

$$\langle m \rangle_{ee} = m_1 c_{12}^2 c_{13}^2 e^{i\rho} + m_2 s_{12}^2 c_{13}^2 + m_3 s_{13}^2 e^{i\sigma} = 0$$

$$m_1^2 c_{12}^4 c_{13}^4 + 2m_1 m_2 c_{12}^2 s_{12}^2 c_{13}^4 \cos \rho + m_2^2 s_{12}^4 c_{13}^4 = m_3^2 s_{13}^4$$

The dark well in the **normal hierarchy**
What new physics?

**Part C**

**Type (A):** NP directly related to extra species of neutrinos.

**Example 1:** heavy Majorana neutrinos from type-I seesaw

\[
-\mathcal{L}_{\text{lepton}} = \bar{l}_L Y_l H E_R + \bar{l}_L Y_{\nu} \tilde{H} N_R + \frac{1}{2} N_R^c M_R N_R + \text{h.c.}
\]

\[
\Gamma_{0\nu\beta\beta} \propto \left| \sum_{i=1}^{3} m_i U_{ei}^2 - \sum_{k=1}^{n} \frac{R_{ek}^2}{M_k} M_A^2 \mathcal{F}(A, M_k) \right|^2
\]

In most cases the heavy contribution is negligible

**Example 2:** light sterile neutrinos from LSND etc

\[
\langle m' \rangle_{ee} \equiv \sum_{i=1}^{6} m_i U_{ei}^2 = \langle m \rangle_{ee} (c_{14} c_{15} c_{16})^2 + m_4 (\hat{s}_{14} c_{15} c_{16})^2 + m_5 (\hat{s}_{15} c_{16})^2 + m_6 (\hat{s}_{16})^2
\]

In this case the new contribution might be constructive or destructive

**Type (B):** NP has little to do with the neutrino mass issue.

SUSY, Left-right, and some others that I don’t understand
Possible effects

New physics effects:

$$\langle m \rangle'_{ee} = m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2 + m_{NP}$$


$$\langle m \rangle'_{ee} |_{\text{upper}} = m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2 + m_3 |U_{e3}|^2 + |m_{NP}|,$$

$$\langle m \rangle'_{ee} |_{\text{lower}} = \max \left\{ 0, 2m_i |U_{ei}|^2 - \langle m \rangle'_{ee} |_{\text{upper}}, 2 |m_{NP}| - \langle m \rangle'_{ee} |_{\text{upper}} \right\}$$

It is hard to tell much
**QUESTION:** are massive neutrinos the Majorana particles?

One might be able to answer **YES** through a measurement of the $0\nu2\beta$ decay or other LNV processes someday, but how to answer with **NO**?

**YES or NO?**

**YES or I don’t know!**

The same question: how to distinguish between Dirac and Majorana neutrinos in a realistic experiment?

**Answer 1:** The $0\nu2\beta$ decay is currently the only possibility.

**Answer 2:** In principle their dipole moments are different.

**Answer 3:** They show different behavior if nonrelativistic.
Remarks

Without information on the nature of massive neutrinos (Majorana or not) and all the CP-violating phases, one will have no way to establish a full theory of $\nu$ masses and flavor mixing. Give $0\nu2\beta$ a chance!

\[ M_{\nu} = \begin{pmatrix} \langle m \rangle_{ee} & \langle m \rangle_{e\mu} & \langle m \rangle_{e\tau} \\ \langle m \rangle_{e\mu} & \langle m \rangle_{\mu\mu} & \langle m \rangle_{\mu\tau} \\ \langle m \rangle_{e\tau} & \langle m \rangle_{\mu\tau} & \langle m \rangle_{\tau\tau} \end{pmatrix} \]

\[ \langle m \rangle_{\alpha\beta} \equiv \sum_i m_i U_{\alpha i} U_{\beta i} \]
**Part D**

\[ \nu \leftrightarrow \bar{\nu} \text{ oscillations?} \]

The neutrino-antineutrino oscillation can in principle help.

**Neutrino-Antineutrino Oscillations:**

\[
A(\nu_\alpha \rightarrow \bar{\nu}_\beta) = \sum_i \left[ U_{\alpha i}^* U_{\beta i}^* \frac{m_i}{E} \exp \left( -i \frac{m_i^2}{2E} L \right) \right] K
\]

\[
A(\bar{\nu}_\alpha \rightarrow \nu_\beta) = \sum_i \left[ U_{\alpha i} U_{\beta i} \frac{m_i}{E} \exp \left( -i \frac{m_i^2}{2E} L \right) \right] \bar{K}
\]

\[
P(\nu_\alpha \rightarrow \bar{\nu}_\beta) = \frac{|K|^2}{E^2} \left[ |\langle m \rangle_{\alpha\beta}|^2 - 4 \sum_{i<j} m_i m_j C_{\alpha\beta}^{ij} \sin^2 \phi_{ji} + 2 \sum_{i<j} m_i m_j \mathcal{V}_{\alpha\beta}^{ij} \sin 2\phi_{ji} \right]
\]

\[
P(\bar{\nu}_\alpha \rightarrow \nu_\beta) = \frac{|K|^2}{E^2} \left[ |\langle m \rangle_{\alpha\beta}|^2 - 4 \sum_{i<j} m_i m_j C_{\alpha\beta}^{ij} \sin^2 \phi_{ji} - 2 \sum_{i<j} m_i m_j \mathcal{V}_{\alpha\beta}^{ij} \sin 2\phi_{ji} \right]
\]

**Effective mass terms:**

\[ \langle m \rangle_{\alpha\beta} \equiv \sum_i m_i U_{\alpha i} U_{\beta i} \]

**Jarlskog-like parameters:**

\[ C_{\alpha\beta}^{ij} \equiv \text{Re} \left( U_{\alpha i} U_{\beta i} U_{\alpha j}^* U_{\beta j}^* \right) \]

\[ \mathcal{V}_{\alpha\beta}^{ij} \equiv \text{Im} \left( U_{\alpha i} U_{\beta i} U_{\alpha j}^* U_{\beta j}^* \right) \]
To identify the Majorana nature, CP-violating phases and new physics it is imperative to observe the $0\nu2\beta$ decays and other lepton-number-violating processes (e.g., neutrino-antineutrino oscillations, the relic neutrino background, doubly-charged Higgs decays). None is realistic.
All of us expect the massive neutrinos to be the Majorana particles. If this expectation comes true someday thanks to the $0\nu 2\beta$ decay, then we will be required to have some new or good or right ideas to probe the Majorana phases.

I.I. Rabi: Physics needs new ideas. But to have a new idea is a very difficult task....

L.C. Pauling: The best way to have a good idea is to have a lot of ideas.

C.S. Wu: It is easy to do the right thing once you have the right ideas.