

The Nuclear Physics of Dark Matter Direct Detection

- ❑ *WIMP Dark Matter Detection*
- ❑ *Nonrelativistic Effective Theory Description*
- ❑ *Implications for experiment: number and type*

Wick Haxton

DBD+DM: Hawaii

October 22, 2018



I. Basic properties

- matter inventory

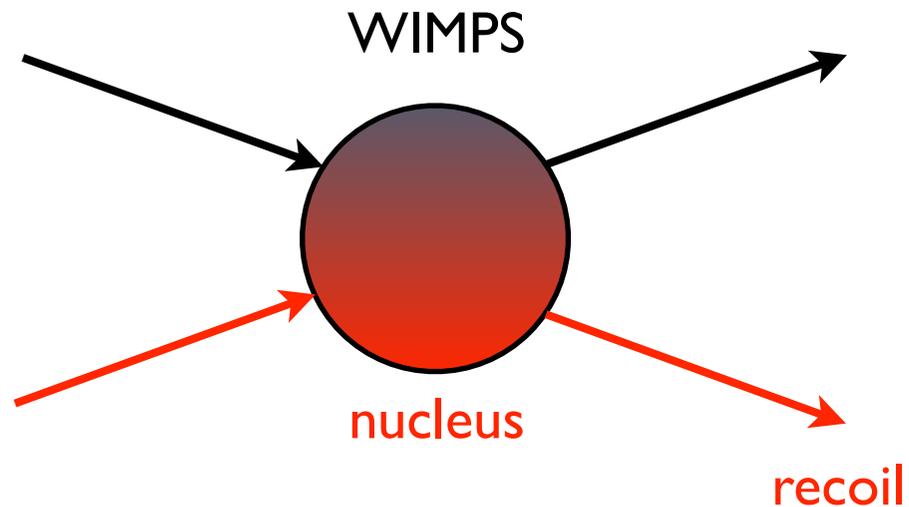
$$\Omega_m = \Omega_B + \Omega_{DM} \sim 0.314 \pm 0.020 \qquad \Omega_{DM}/\Omega_B \sim 5$$

- underlying particle is long-lived or stable
- cold or warm (slow enough to seed structure formation)
- gravitationally active
- lacks strong couplings to itself or to baryons

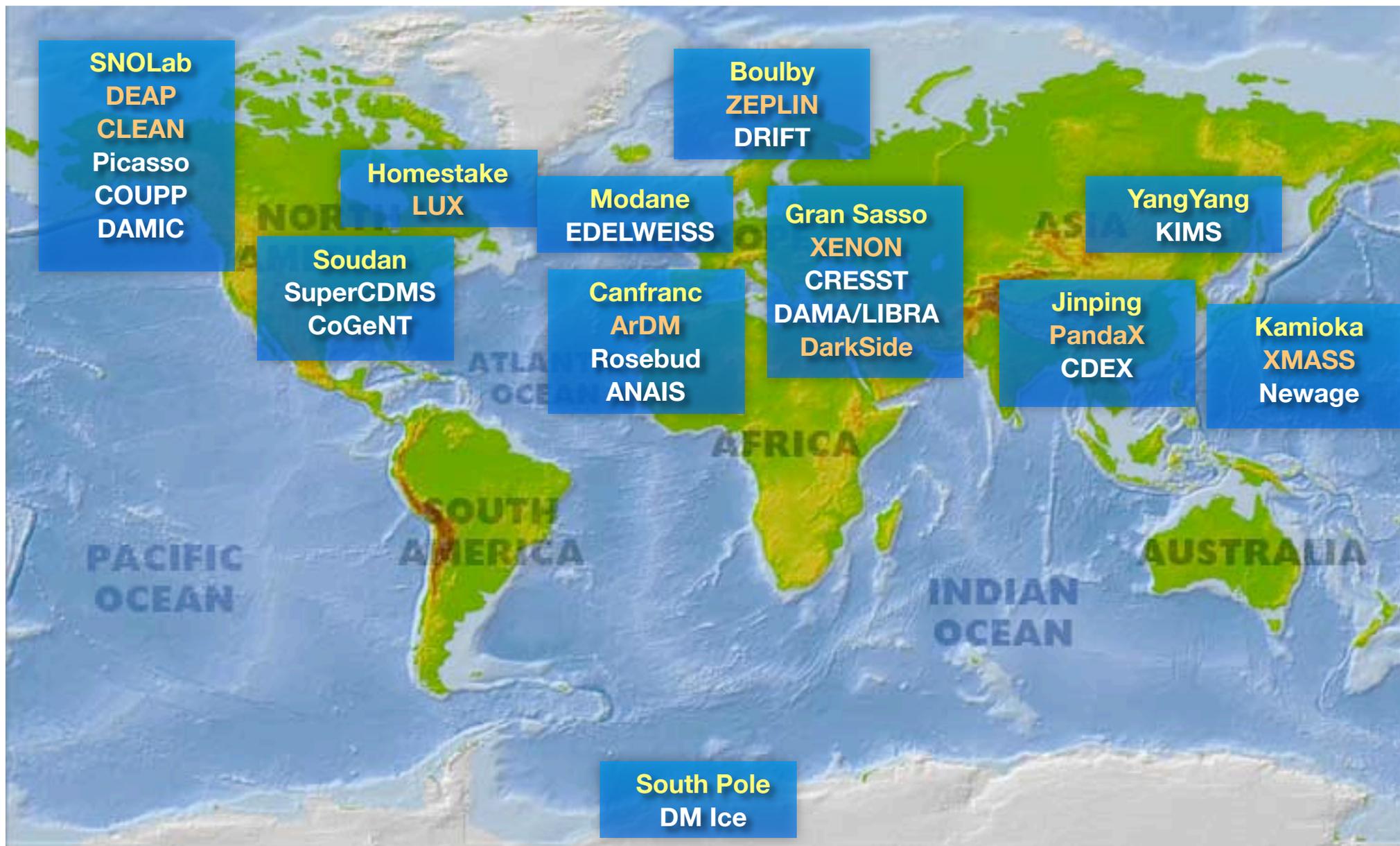
Apart from the minor neutrino-mass component, outside the SM

Detection: their non-gravitational detection channels include

- collider production SM particles \rightarrow WIMPs
- indirect detection: astrophysical signals WIMPs \rightarrow SM particles
- **direct detection** SM particle + WIMP \rightarrow SM particle + WIMP



Laura Baudis's WWW Search Map

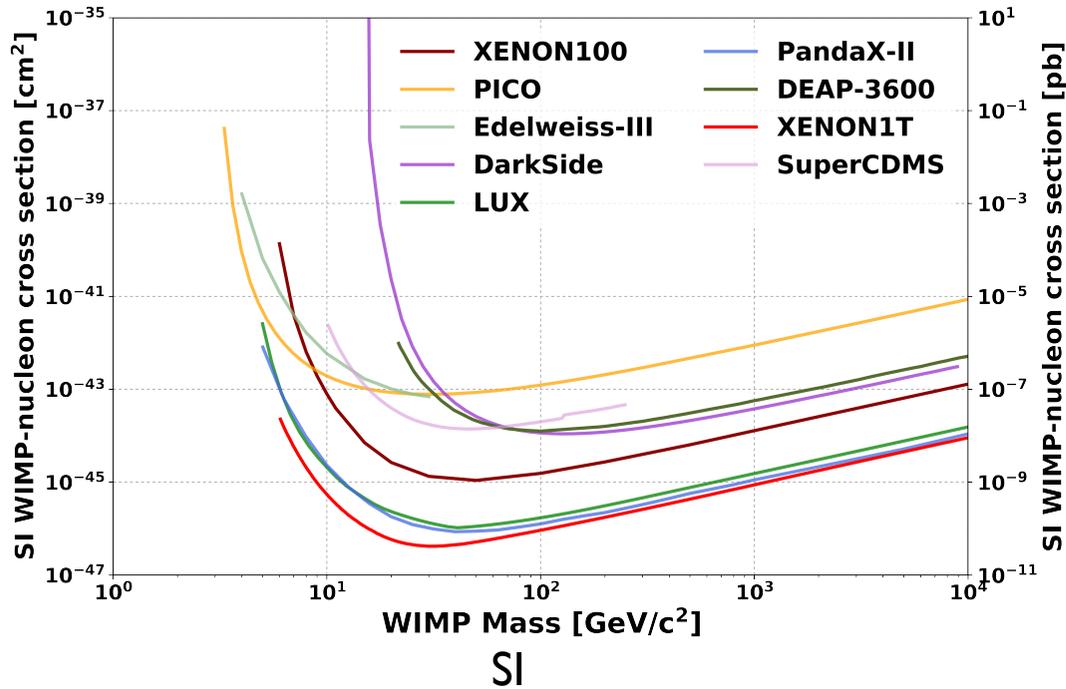


Xe:	Xenon 100/IT; LUX/LZ; XMASS; Zeplin; NEXT
Si:	CDMS; DAMIC
Ge:	COGENT; Edelweiss; SuperCDMS; TEXONO; CDEX; GERDA; Majorana
Nal:	DAMA/LIBRA; ANAIS; DM-ice; SABRE; KamLAND-PICO
Csl:	KIMS
Ar:	DEAP/CLEAN; ArDM; Darkside
Ne:	CLEAN
C/F-based:	PICO; DRIFT; DM-TPC
CF ₃ I:	COUP
Cs ₂ :	DRIFT
TeO ₂ :	CUORE
CaWO ₄ :	CRESST

A large variety of nuclei with different spins, isospin, masses

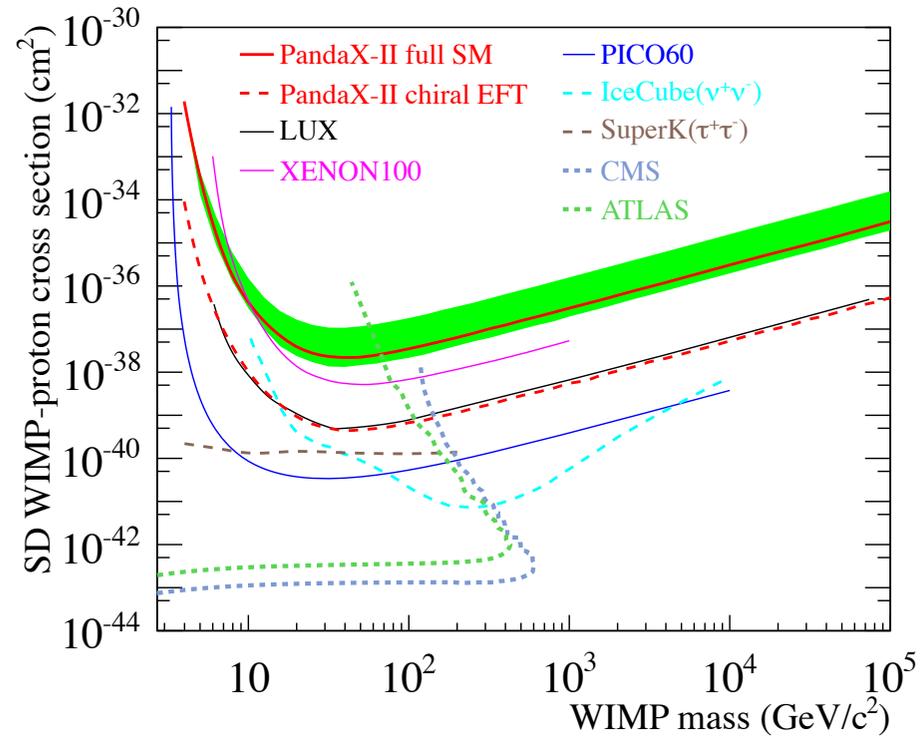
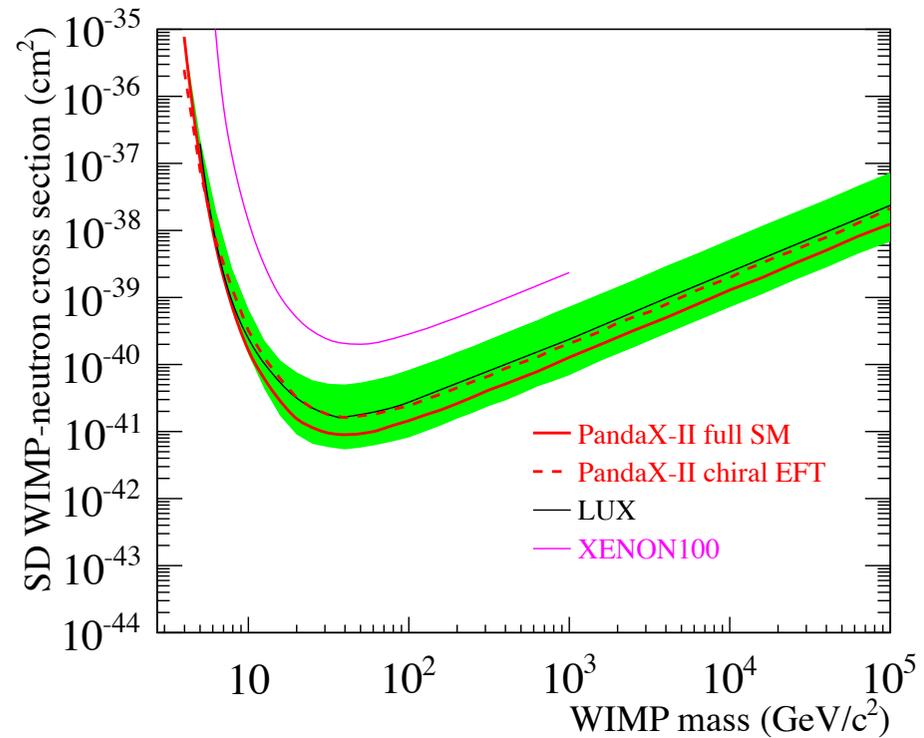
unpaired valence nucleons carrying a variety of values of the orbital angular momentum

$$\vec{j} = \vec{\ell} + \vec{s}$$



SI
from Kaixuan Ni
CIPANPI8

SD neutron (top)
& proton (bottom)
from PandaX-II



Are such comparisons of experiments reliable as a sensitivity measure?

Basic parameters of direct detection

- WIMP velocity relative to our rest frame $\sim 10^{-3}$
- if mass is on the weak scale, WIMP momentum transfers in elastic scattering can range to $q_{\max} \sim 2v_{\text{WIMP}}\mu_T \sim 200 \text{ MeV}/c$
- WIMP kinetic energy $\sim 30 \text{ keV}$: nuclear excitation (in most cases) not possible
- $R_{\text{NUC}} \sim 1.2 A^{1/3} \text{ f} \Rightarrow q_{\max} R \sim 3.2 \Leftrightarrow 6.0$ for F \Leftrightarrow Xe: the WIMP can “see” the structure of the nucleus

An expression can be written for the rate as a function of nuclear recoil energy E_R

$$\frac{dR}{dE_R} = N_N \frac{\rho_0}{m_W} \int_{v_{min}} d\mathbf{v} f(\mathbf{v}) v \frac{d\sigma}{dE_R}$$

Astrophysics

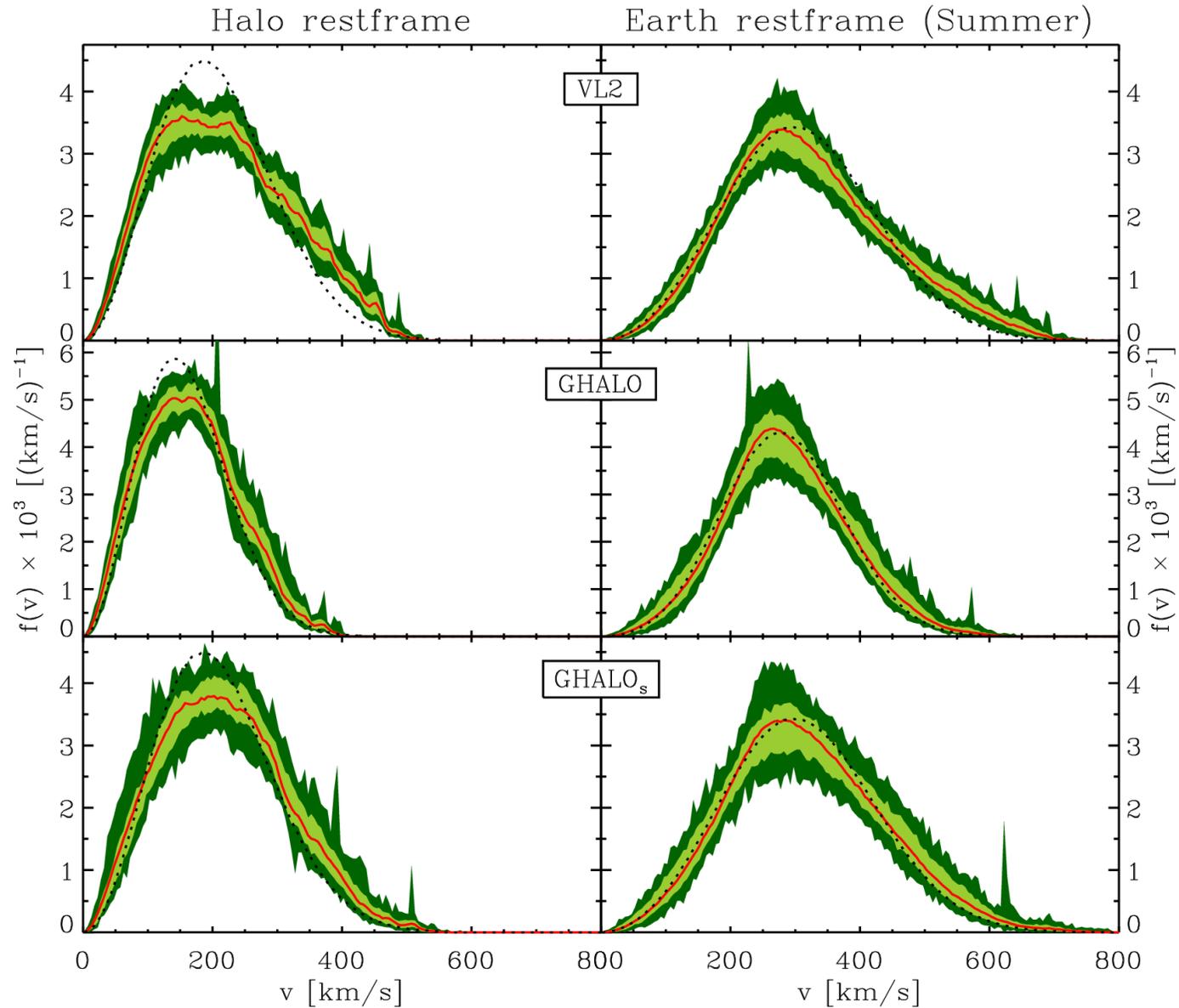
Particle+nuclear physics

- $N_N =$ number of target nuclei in detector
- $\rho_0 =$ Milky Way dark matter density
- $f(\mathbf{v}) =$ WIMP velocity distribution, Earth frame
- $m_W =$ WIMP mass
- $\sigma =$ WIMP – nucleus elastic scattering cross section

$$v_{min} = \sqrt{\frac{m_N E_{th}}{2\mu^2}}$$

Our motion through
the WIMP “wind”
can be modeled,
reasonably

$$\rho_{\text{local}} \sim 0.3 \text{ GeV/cm}^3 \Rightarrow$$
$$\phi_{\text{WIMP}} \sim 10^5 / \text{cm}^2\text{s}$$



M. Kuhlen et al, JCAP02 (2010) 030

In contrast, the particle/nuclear physics involves major uncertainties

The cross section — the WIMP-nucleus interaction — what is its form?

How do we compare results from different experiments, as spins, charges, magnetic moments, and isospins vary?

How many experiments should we do?

More generally, what can and cannot be learned about the WIMP-matter interaction from these low-energy elastic scattering experiments?

Hard to address these questions in the SI/SD framework — but answers can be found through the effective theory approach

Consider for example the SD interaction: can arise from many DM mediators or “portals.” Mimicking the SM, one possibility is an effective coupling of a mediator Z to SM matter of the form

$$\sum_q g_{\text{SM}}^q Z''_\mu \bar{q} \gamma^\mu \gamma^5 q$$

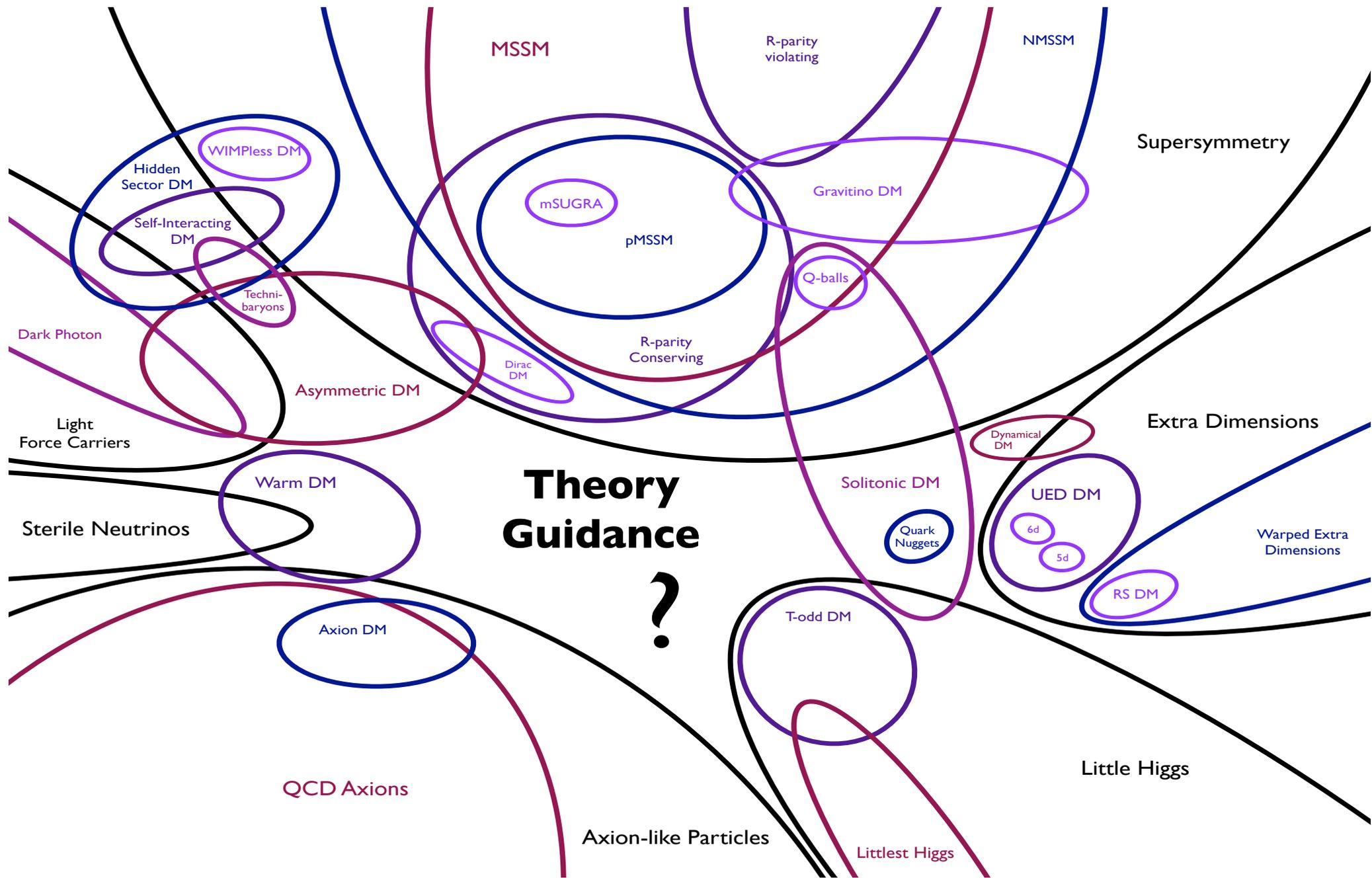
which generates a nucleon level operator of the form

$$\langle N(k') | A_\mu^{(q)} | N(k) \rangle = \bar{u}^{(N)}(k') \left[F_A^{(N,q)}(q^2) \gamma_\mu \gamma^5 + \frac{q_\mu}{2m_N} F_P^{(N,q)} \gamma^5 \right] u^{(N)}(k)$$

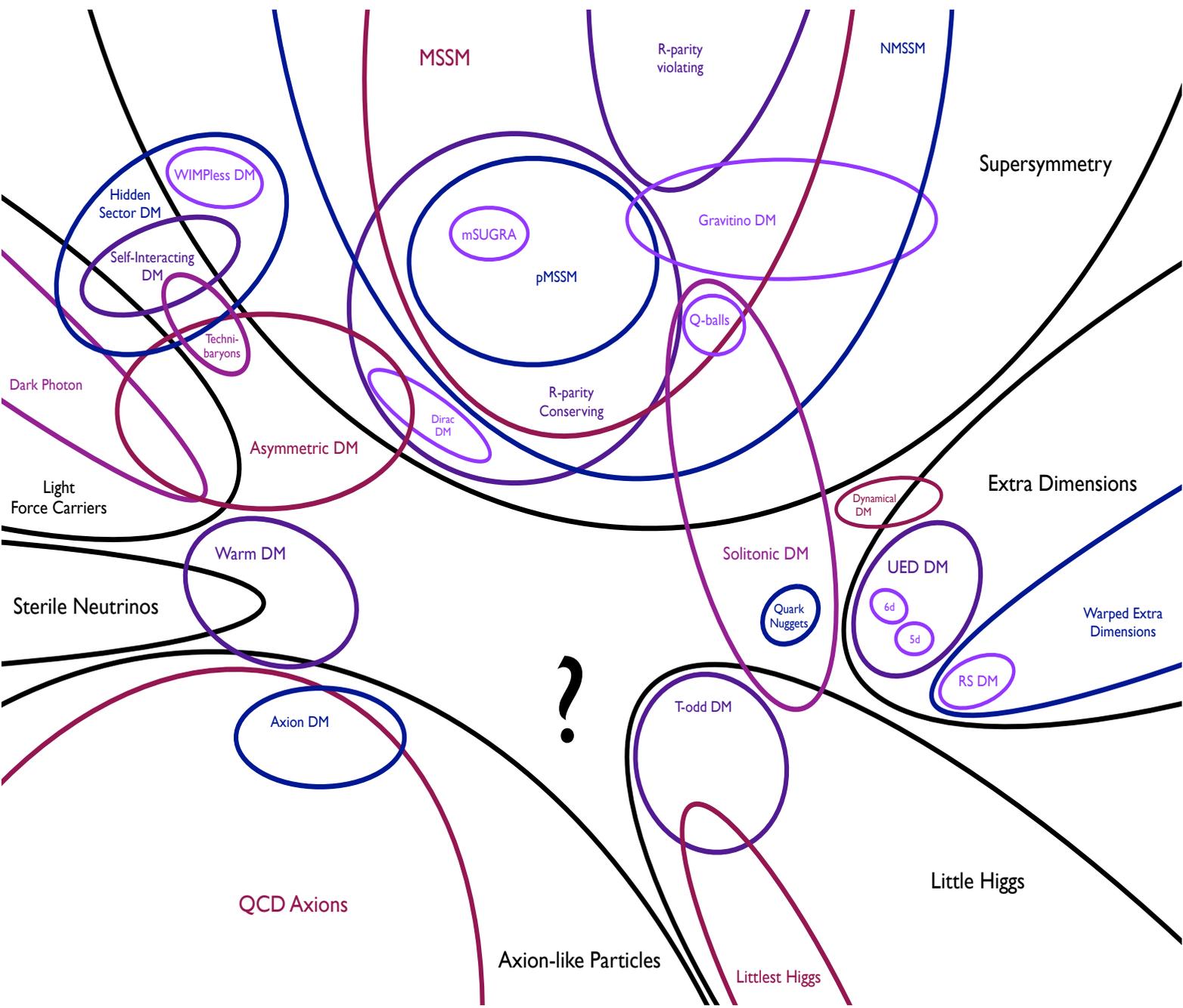
equivalent Galilean formulation : $\vec{\sigma} \cdot \frac{\vec{q}}{m_N}$

Standard SD treatments then generally replace these couplings with their SM values. But even in this simple example, we have made implicit assumptions that cannot be justified experimentally

- 1) equivalent transverse and longitudinal DM couplings to spin
- 2) flavor couplings of DM like those of the SM



from Tim Tait



Nuclear theorist



DM experimentalist

- The standard “point nucleus” approach motivating the SI/SD starting point underestimates the power of current experiments. It not only limits responses to the macroscopic nuclear quantum nos

$$\text{S.I.} \quad \Rightarrow \quad \langle g.s. | \sum_{i=1}^A (a_0^F + a_1^F \tau_3(i)) | g.s. \rangle$$

$$\text{S.D.} \quad \Rightarrow \quad \langle g.s. | \sum_{i=1}^A \vec{\sigma}(i) (a_0^{GT} + a_1^{GT} \tau_3(i)) | g.s. \rangle$$

but allows only one velocity, $v_{\text{WIMP}} \sim 10^{-3}$. Rates for any theory of DM involving derivatives are then tiny, unmeasurable: but this conclusion is not correct.

- The effective theory approach to DM elastic scattering returns us to to early days of weak interactions, in which without prejudice the most general short-range interaction was constructed (S,P,T,V,A) and the requisite number of experiments was done to determine V-A

Galilean invariant effective theory (nucleon level)

- The most general Hermitian WIMP-nucleon interaction can be constructed from the four variables

$$\vec{S}_\chi \quad \vec{S}_N \quad \vec{v}^\perp \quad i \vec{q}$$

Here $\vec{v}^\perp \equiv \vec{v} + \frac{\vec{q}}{2\mu_N} \equiv (\vec{v}_{\chi,in} - \vec{v}_{N,in}) + \frac{\vec{q}}{2\mu_N}$ is dimensionless

(units of c)

But what about $i \vec{q}$? Carries dimensions: need $\frac{i \vec{q}}{\Lambda}$

Confusing ... nucleon level, so maybe m_N ?

But our SI/SD point nucleus picture would argue $\Lambda = m_T$

There is some important physics here, again relevant to derivative coupled theories...

Let's do an example by taking a velocity-dependent DM interaction

$$\sum_{i=1}^A \vec{S}_\chi \cdot \vec{v}^\perp(i) \quad \vec{v}^\perp(i) = \vec{v}_\chi - \vec{v}_N(i)$$

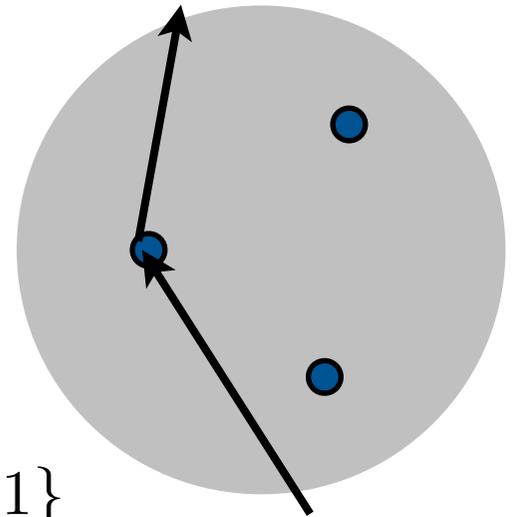
□ In the point-nucleus limit $\vec{S}_\chi \cdot \vec{v}_{\text{WIMP}} \sum_{i=1}^A 1(i)$

where $\vec{v}_{\text{WIMP}} \sim 10^{-3}$.

□ But in reality

$\{\vec{v}^\perp(i), i = 1, \dots, A\} \rightarrow \{\vec{v}_{\text{WIMP}}; \vec{v}(i), i = 1, \dots, A - 1\}$

and $\vec{v}(i) \sim 10^{-1}$: point nucleus limit tosses out the large terms!



Kinematics and the connection between velocities and Λ

- These velocities hide: the $\vec{v}(i)$ carry odd parity and cannot contribute by themselves to elastic nuclear matrix elements.
- But in elastic scattering, momentum transfers are significant. The full velocity operator is

$$e^{i\vec{q}\cdot\vec{r}(i)}\vec{v}(i) \quad \text{where} \quad \vec{q}\cdot\vec{r}(i) \sim 1$$

- We can combine the two vector nuclear operators $\vec{r}(i)$, \vec{v} to form a scalar, vector, and tensor. To first order in \vec{q} for this new “SD” case

$$-\frac{1}{i}q\vec{r} \times \vec{v} = -\frac{1}{i}\frac{q}{m_N}\vec{r} \times \vec{p} = -\frac{q}{m_N}\vec{\ell}(i)$$

$\vec{\ell}(i)$ is a new dimensionless operator. And we deduce an instruction for the ET that is not obvious. Internal nucleon velocities are encoded

$$\dot{v} \sim 10^{-1} \sim \frac{q}{m_N}$$

Galilean invariant effective theory is now defined

- The most general Hermitian WIMP-nucleon interaction can be constructed from the four variables

$$\vec{S}_\chi \quad \vec{S}_N \quad \vec{v}^\perp \quad \frac{q}{m_N} \quad H_{ET} = \sum_i a_i \mathcal{O}_i(\vec{S}_\chi, \vec{S}_N, \vec{v}^\perp, \frac{\vec{q}}{m_N})$$

- This interaction constructed to 2nd order in velocities

$$\begin{aligned}
 H_{ET} = & \left[a_1 + a_2 \vec{v}^\perp \cdot \vec{v}^\perp + a_5 i \vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp \right) \right] + \vec{S}_N \cdot \left[a_3 i \frac{\vec{q}}{m_N} \times \vec{v}^\perp + a_4 \vec{S}_\chi + a_6 \frac{\vec{q}}{m_N} \vec{S}_\chi \cdot \frac{\vec{q}}{m_N} \right] \\
 + & \left[a_8 \vec{S}_\chi \cdot \vec{v}^\perp \right] + \vec{S}_N \cdot \left[a_7 \vec{v}^\perp + a_9 i \frac{\vec{q}}{m_N} \times \vec{S}_\chi \right] \quad (\text{parity odd}) \\
 + & \left[a_{11} i \vec{S}_\chi \cdot \frac{\vec{q}}{m_N} \right] + \vec{S}_N \cdot \left[a_{10} i \frac{\vec{q}}{m_N} + a_{12} \vec{v}^\perp \times \vec{S}_\chi \right] \quad (\text{time and parity odd}) \\
 + & \vec{S}_N \cdot \left[a_{13} i \frac{\vec{q}}{m_N} \vec{S}_\chi \cdot \vec{v}^\perp + a_{14} i \vec{v}^\perp \vec{S}_\chi \cdot \frac{\vec{q}}{m_N} \right] \quad (\text{time odd})
 \end{aligned}$$

The coefficients (low energy constants) represent the information that survives at GeV energies from a semi-infinite set of UV theories

- If 14 independent NN experiments were done - controlling the WIMP and nucleon spin, the relative velocity, the momentum transfer - 14 constraints on the underlying UV theory would be obtained
- Were we so able!
- In fact, our experiments are done on nuclei, and limited to elastic scattering: more information is integrated out, lost to us
- How much information is then available at the very-low-energy scale of WIMP-nucleus scattering?

We can (and did) answer this by direct computation, but as is usual in a proper ET, the answer is dictated just by symmetries

- A familiar electroweak interactions problem: What is the form of the elastic response for a nonrelativistic theory with vector and axial-vector interactions?

		even	odd	
charges:	vector	C_0	C_1	...
	axial	C_0^5	C_1^5	

	even	odd	even	odd	even	odd	
axial spin	L_0^5	L_1^5	T_2^{5el}	T_1^{5el}	T_2^{5mag}	T_1^{5mag}	
vector velocity	L_0	L_1	T_2^{el}	T_1^{el}	T_2^{mag}	T_1^{mag}
vector spin – velocity	L_0	L_1	T_2^{el}	T_1^{el}	T_2^{mag}	T_1^{mag}	

(where we list only the leading multipoles in J above)

Response constrained by good **parity** and time reversal of nuclear g.s.

	even	odd
vector	C_0	
axial		C_1^5

	even	odd	even	odd	even	odd
axial spin		L_1^5		T_1^{5el}	T_2^{5mag}	-
vector velocity	L_0		T_2^{el}			T_1^{mag}
vector spin – velocity	L_0		T_2^{el}			T_1^{mag}

Response constrained by good **parity** and **time reversal** of nuclear g.s.

	even	odd
vector	C_0	
axial		

	even	odd	even	odd	even	odd
axial spin		L_1^5		T_1^{5el}	-	-
vector velocity						T_1^{mag}
vector spin – velocity	L_0		T_2^{el}			

The resulting table of allowed responses has **six** entries (not two)

... the direct computation verifies this, identifying the response functions, their connections to quantities we can measure in the SM

Six constraints (8 if interferences are included) can in principle be derived from the right set of DM elastic scattering experiments

This is the information that survives at low energy, starting from an infinity of UV possibilities

This defines ultimately the job that experimentalists should finish...

- When we embed our nucleon-level operator into the nucleus more detailed information is obtained...

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) W_i(q^2 b^2)$$

- When we embed our nucleon-level operator into the nucleus more detailed information is obtained...

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) W_i(q^2 b^2)$$



WIMP tensor:
contains all of the DM particle physics

depends on two “velocities”

$$\vec{v}^{\perp 2} \sim 10^{-6} \qquad \frac{\vec{q}^2}{m_N^2} \sim \langle v_{\text{internucleon}} \rangle^2 \sim 10^{-2}$$

- We can then embed this in the nucleus (filter #2) to find what information survives, accessible to experiment.

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) W_i(q^2 b^2)$$



Nuclear tensor:

“nuclear knob” that can be turned
by the experimentalists to deconstruct
dark matter

Game - vary the W_i to determine the R_i :
change the nuclear charge, spin, isospin,
and any other relevant nuclear
properties that can help

- What does the effective theory say about these responses?

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) W_i(q^2 b^2)$$



$$W_1 \sim \langle J | \sum_{i=1}^A 1(i) | J \rangle^2$$

take $q \rightarrow 0$
 suppress isospin

the S.I. response

contributes for $J=0$ nuclear targets

- What does the effective theory say about these responses?

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) W_i(q^2 b^2)$$



take $q \rightarrow 0$
 suppress isospin

$$W_2 \sim \langle J | \sum_{i=1}^A \hat{q} \cdot \vec{\sigma}(i) | J \rangle^2$$

$$W_3 \sim \langle J | \sum_{i=1}^A \hat{q} \times \vec{\sigma}(i) | J \rangle^2$$

the S.D. response ($J>0$)
 but split into two components, as the longitudinal and transverse responses are independent, coupled to different particle physics

- What does the effective theory say about these responses?

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) W_i(q^2 b^2)$$



take $q \rightarrow 0$
 suppress isospin

$$W_4 \sim \langle J | \sum_{i=1}^A \vec{\ell}(i) | J \rangle^2$$

A second type of vector (requires $J > 0$) response, with selection rules very different from the spin response

- What does the effective theory say about these responses?

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) W_i(q^2 b^2)$$



$$W_5 \sim \langle J | \sum_{i=1}^A \vec{\sigma}(i) \cdot \vec{\ell}(i) | J \rangle^2$$

take $q \rightarrow 0$
 suppress isospin

A second type of scalar response, with coherence properties very different from the simple charge operator

- What does the effective theory say about these responses?

$$\frac{d\sigma}{dE_R} \sim G_F^2 \sum_i R_i(\vec{v}^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) W_i(q^2 b^2)$$



take $q \rightarrow 0$
 suppress isospin

$$W_6 \sim \langle J | \sum_{i=1}^A \left[\vec{r}(i) \otimes \left(\vec{\sigma}(i) \times \frac{1}{i} \vec{\nabla}(i) \right) \right]_1 \rangle_2 | J \rangle^2$$

A exotic tensor response: in principle interactions can be constructed where no elastic scattering occurs unless J is at least 1

We can now allow isospin to appear explicitly

$$H_{ET} = \sum_i a_i \mathcal{O}_i \rightarrow \sum_i (c_i^0 + c_i^1 \tau_3) \mathcal{O}_i$$

$$c_i^0 = \frac{1}{2}(c_i^p + c_i^n) \quad c_i^1 = \frac{1}{2}(c_i^p - c_i^n)$$

One then measures the cross section and calculate the nuclear matrix element. Then...

The coefficients are what one then derives. They define the particle physics that can be determined, the direct detection constraints on models

$$\begin{aligned}
 R_M^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi + 1)}{3} \left[\frac{\vec{q}^2}{m_N^2} \vec{v}_T^{\perp 2} c_5^\tau c_5^{\tau'} + \vec{v}_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{\vec{q}^2}{m_N^2} c_{11}^\tau c_{11}^{\tau'} \right] \\
 R_{\Phi''}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{\vec{q}^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi + 1)}{12} \left(c_{12}^\tau - \frac{\vec{q}^2}{m_N^2} c_{15}^\tau \right) \left(c_{12}^{\tau'} - \frac{\vec{q}^2}{m_N^2} c_{15}^{\tau'} \right) \\
 R_{\Phi''M}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi + 1)}{3} \left(c_{12}^\tau - \frac{\vec{q}^2}{m_N^2} c_{15}^\tau \right) c_{11}^{\tau'} \\
 R_{\tilde{\Phi}'}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{j_\chi(j_\chi + 1)}{12} \left[c_{12}^\tau c_{12}^{\tau'} + \frac{\vec{q}^2}{m_N^2} c_{13}^\tau c_{13}^{\tau'} \right] \\
 R_{\Sigma''}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{\vec{q}^2}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi + 1)}{12} \left[c_4^\tau c_4^{\tau'} + \right. \\
 &\quad \left. \frac{\vec{q}^2}{m_N^2} (c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{\vec{q}^4}{m_N^4} c_6^\tau c_6^{\tau'} + \vec{v}_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{\vec{q}^2}{m_N^2} \vec{v}_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'} \right] \\
 R_{\Sigma'}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{1}{8} \left[\frac{\vec{q}^2}{m_N^2} \vec{v}_T^{\perp 2} c_3^\tau c_3^{\tau'} + \vec{v}_T^{\perp 2} c_7^\tau c_7^{\tau'} \right] + \frac{j_\chi(j_\chi + 1)}{12} \left[c_4^\tau c_4^{\tau'} + \right. \\
 &\quad \left. \frac{\vec{q}^2}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{\vec{v}_T^{\perp 2}}{2} \left(c_{12}^\tau - \frac{\vec{q}^2}{m_N^2} c_{15}^\tau \right) \left(c_{12}^{\tau'} - \frac{\vec{q}^2}{m_N^2} c_{15}^{\tau'} \right) + \frac{\vec{q}^2}{2m_N^2} \vec{v}_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'} \right] \\
 R_{\Delta}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{j_\chi(j_\chi + 1)}{3} \left[\frac{\vec{q}^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'} \right] \\
 R_{\Delta\Sigma'}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{j_\chi(j_\chi + 1)}{3} \left[c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'} \right].
 \end{aligned}$$

Nuclear world information

nucleon world information

The **point-nucleus world** is what we thought we could probe
 But the **derivative coupling world** is easy to see, with the right target

$$\begin{aligned}
 R_M^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi + 1)}{3} \left[\frac{\vec{q}^2}{m_N^2} \vec{v}_T^{\perp 2} c_5^\tau c_5^{\tau'} + \vec{v}_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{\vec{q}^2}{m_N^2} c_{11}^\tau c_{11}^{\tau'} \right] \\
 R_{\Phi''}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{\vec{q}^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi + 1)}{12} \left(c_{12}^\tau - \frac{\vec{q}^2}{m_N^2} c_{15}^\tau \right) \left(c_{12}^{\tau'} - \frac{\vec{q}^2}{m_N^2} c_{15}^{\tau'} \right) \\
 R_{\Phi''M}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi + 1)}{3} \left(c_{12}^\tau - \frac{\vec{q}^2}{m_N^2} c_{15}^\tau \right) c_{11}^{\tau'} \\
 R_{\tilde{\Phi}'}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{j_\chi(j_\chi + 1)}{12} \left[c_{12}^\tau c_{12}^{\tau'} + \frac{\vec{q}^2}{m_N^2} c_{13}^\tau c_{13}^{\tau'} \right] \\
 R_{\Sigma''}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{\vec{q}^2}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi + 1)}{12} \left[c_4^\tau c_4^{\tau'} + \right. \\
 &\quad \left. \frac{\vec{q}^2}{m_N^2} (c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{\vec{q}^4}{m_N^4} c_6^\tau c_6^{\tau'} + \vec{v}_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{\vec{q}^2}{m_N^2} \vec{v}_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'} \right] \\
 R_{\Sigma'}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{1}{8} \left[\frac{\vec{q}^2}{m_N^2} \vec{v}_T^{\perp 2} c_3^\tau c_3^{\tau'} + \vec{v}_T^{\perp 2} c_7^\tau c_7^{\tau'} \right] + \frac{j_\chi(j_\chi + 1)}{12} \left[c_4^\tau c_4^{\tau'} + \right. \\
 &\quad \left. \frac{\vec{q}^2}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{\vec{v}_T^{\perp 2}}{2} \left(c_{12}^\tau - \frac{\vec{q}^2}{m_N^2} c_{15}^\tau \right) \left(c_{12}^{\tau'} - \frac{\vec{q}^2}{m_N^2} c_{15}^{\tau'} \right) + \frac{\vec{q}^2}{2m_N^2} \vec{v}_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'} \right] \\
 R_{\Delta}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{j_\chi(j_\chi + 1)}{3} \left[\frac{\vec{q}^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'} \right] \\
 R_{\Delta\Sigma'}^{\tau\tau'}(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2}) &= \frac{j_\chi(j_\chi + 1)}{3} \left[c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'} \right].
 \end{aligned}$$

Observations:

- The set of operators found here map on to the ones necessary in describing *known* SM electroweak interactions
- ES can in principle give us 6+2 constraints on DM interactions
- This argues for a variety of detectors - or at least, continued development of a variety of detector technologies
- None of the dimensionless couplings in front of operators are known
- $1 : v^2 : v^4$ hierarchy extends the range of cross sections that one might consider weak-scale

For illustration purposes only!

DAMA/LIBRA:

NaI

Majorana/GERDA/CoGENT:

Ge

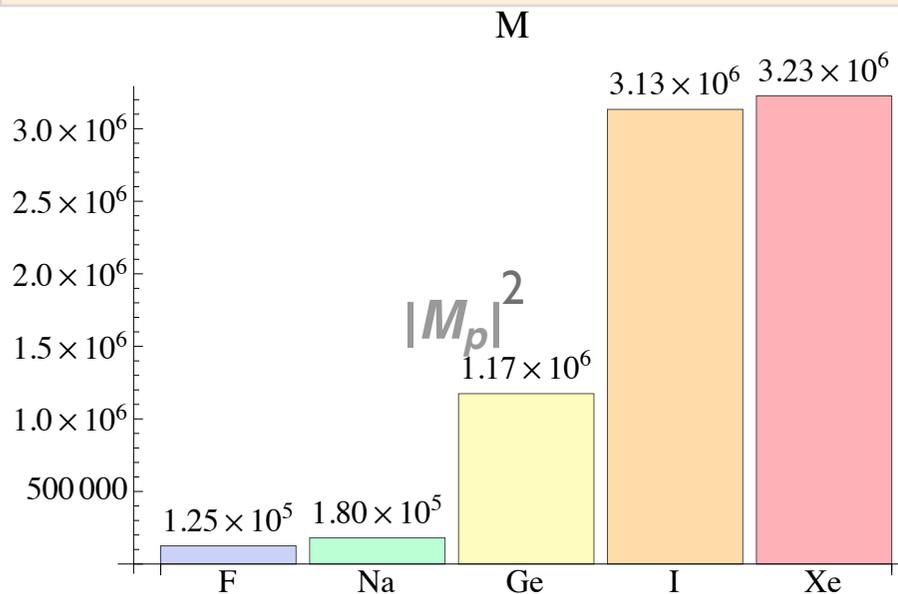
Xenon IT/PandaX/LZ/LUX:

Xe

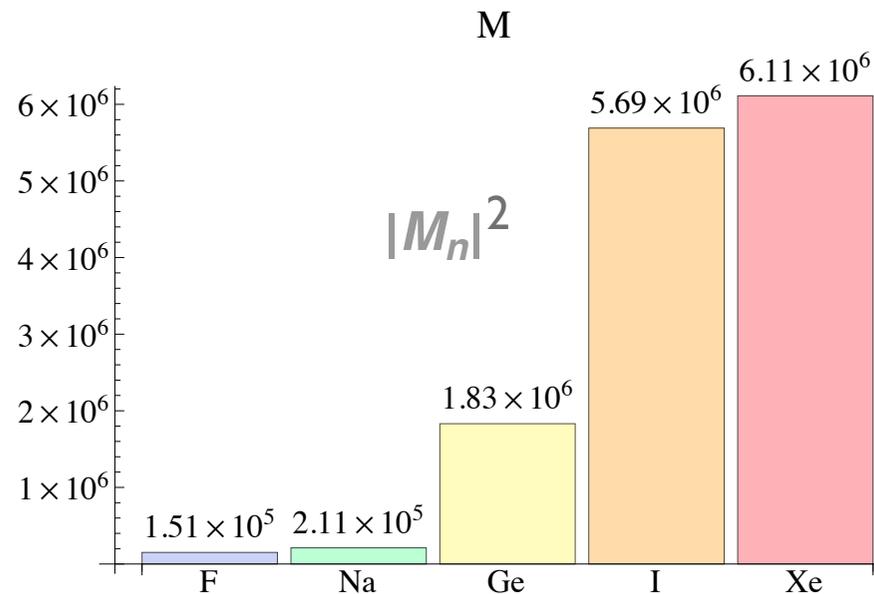
PICASSO/COUPP/PICO:

F

scalar charge responses: p vs. n S.I.



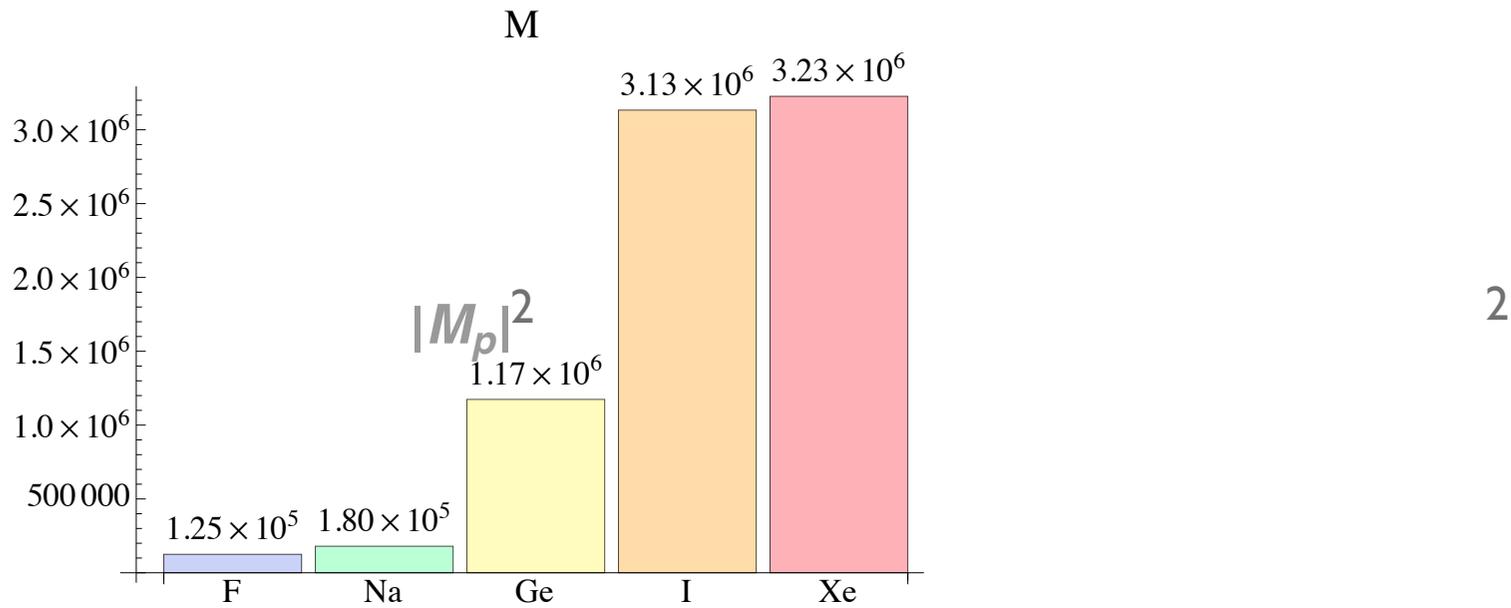
(normalized to natural abundance)



Standard SI sensitivity: $Xe \sim NaI > Ge$

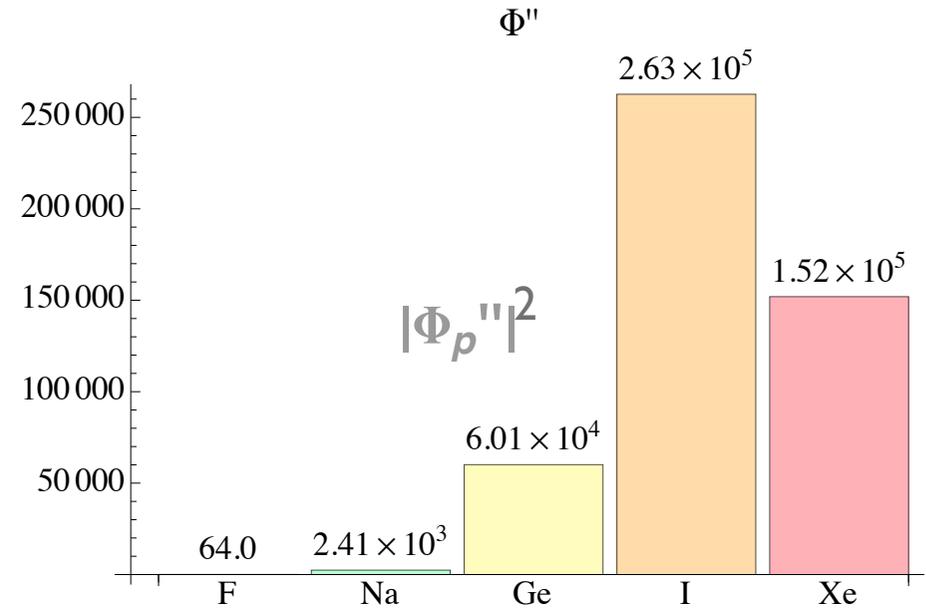
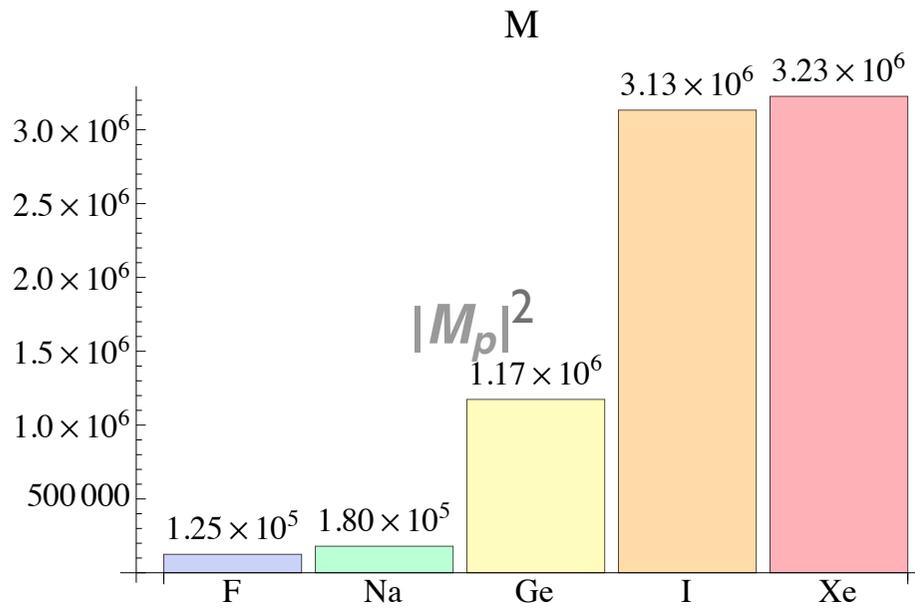
Little sensitivity to isospin (unless tuned)

Scalar operators coupled to protons: $1(i)$ vs $\vec{\sigma}(i) \cdot \vec{\ell}(i)$



Xe ~ NaI

Scalar operators coupled to protons: $1(i)$ vs $\vec{\sigma}(i) \cdot \vec{\ell}(i)$



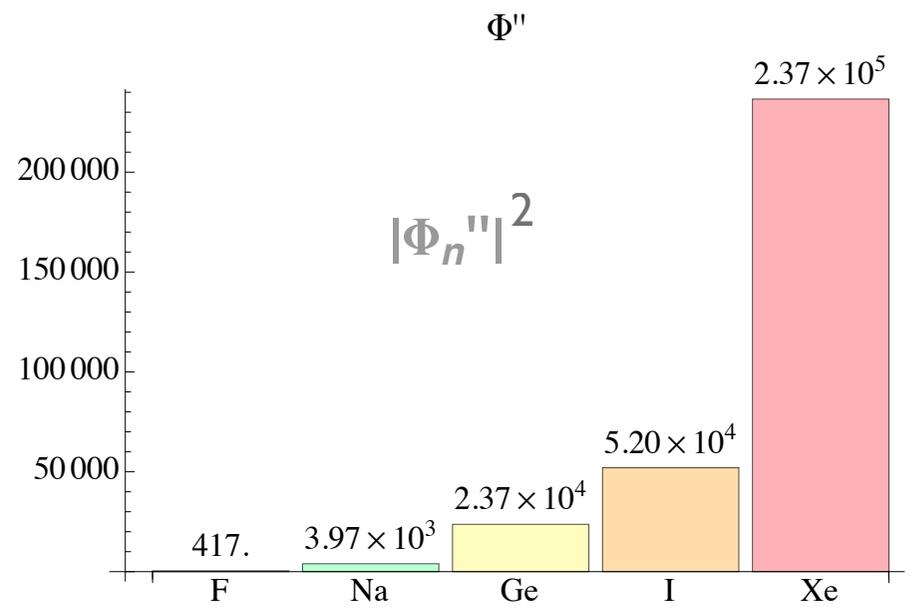
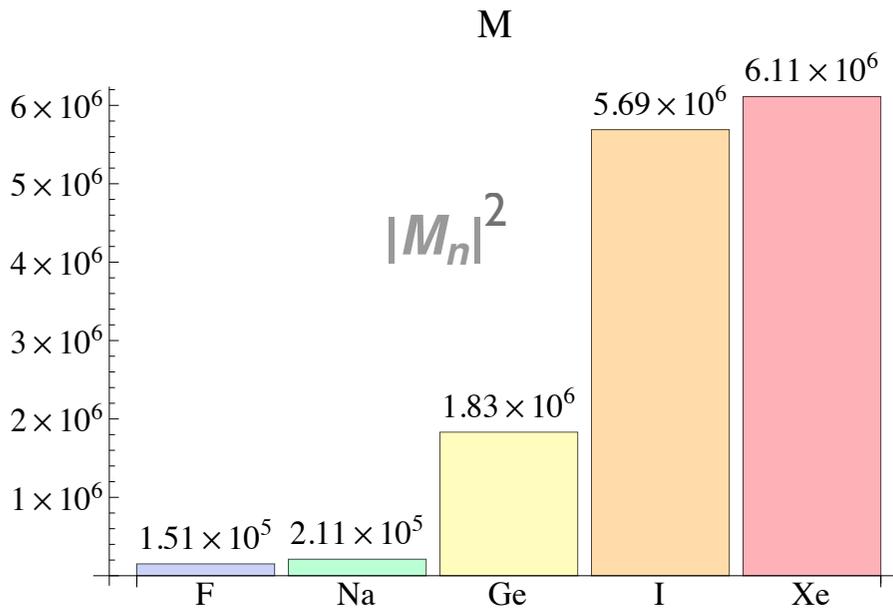
Xe ~ NaI

\Rightarrow

NaI > Xe

(new EFT velocity-dependent operator)

Scalar operators coupled to neutrons: $1(i)$ vs $\vec{\sigma}(i) \cdot \vec{\ell}(i)$



Xe ~ NaI

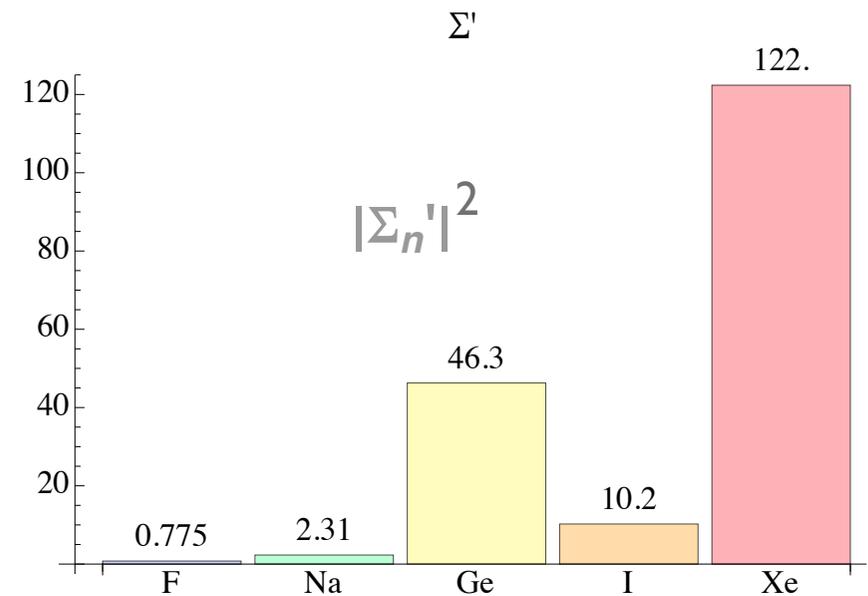
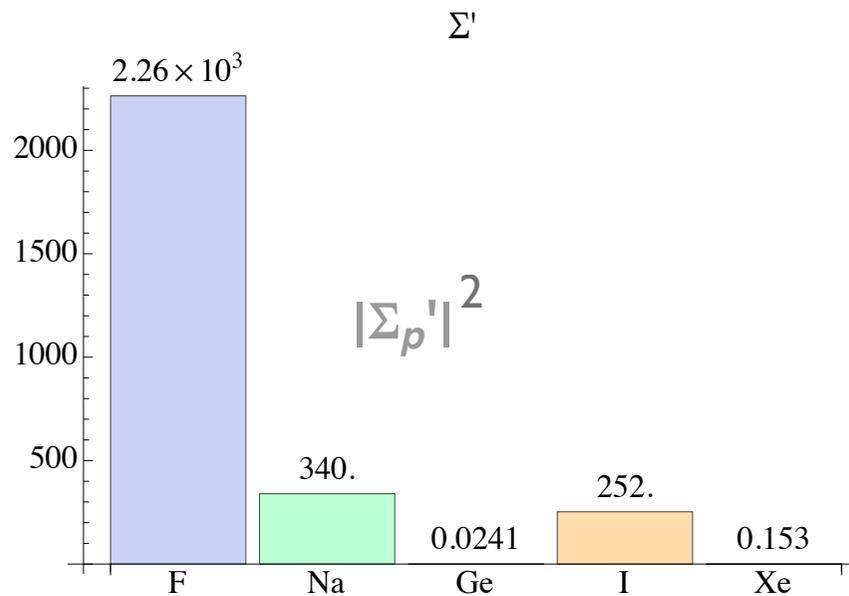
\Rightarrow

NaI < Xe

(new EFT velocity-dependent operator)

vector (transverse) spin response

(normalized to natural abundance)

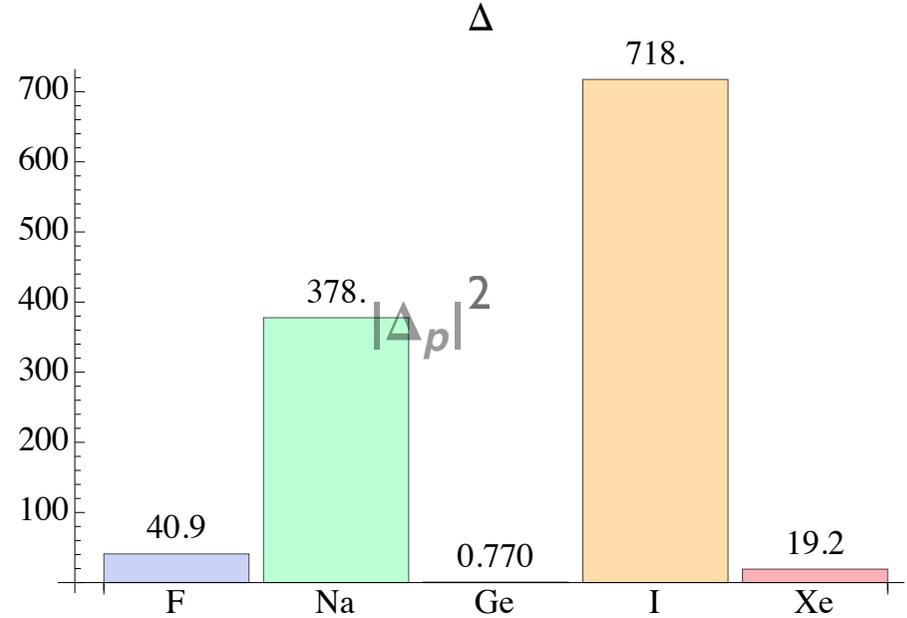
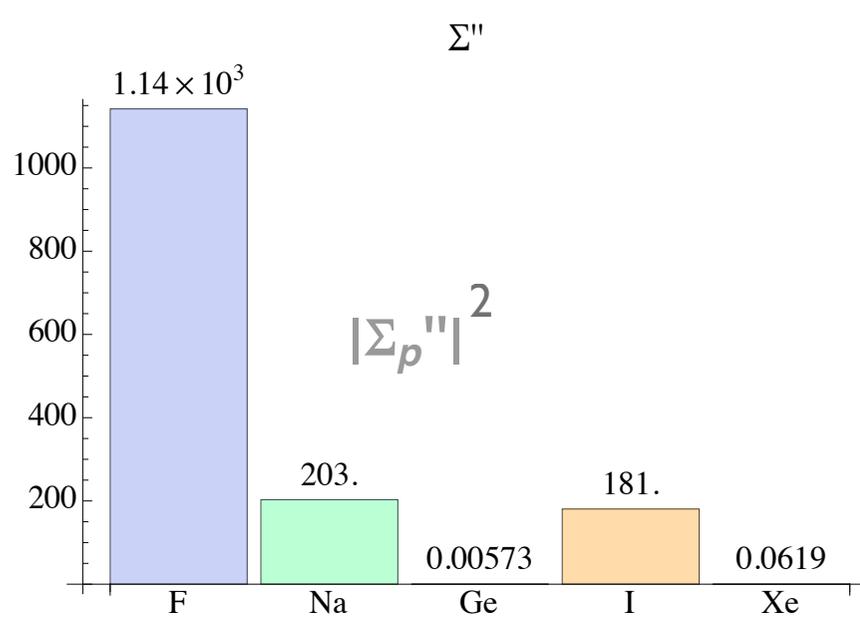


proton coupled: F > Na > I \gg Ge & Xe

neutron coupled: Ge & Xe \gg Na > I \gg F

standard SD sensitivity: only isospin has been changed

Vector, proton coupled: $\vec{\sigma}(i)$ vs. $\vec{\ell}(i)$



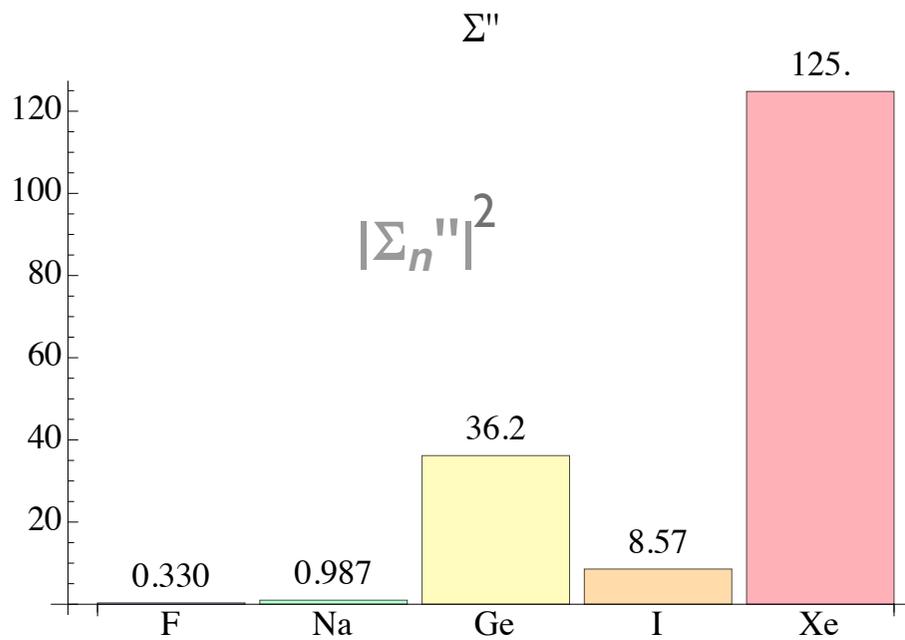
SD: F > NaI

NaI \gg F

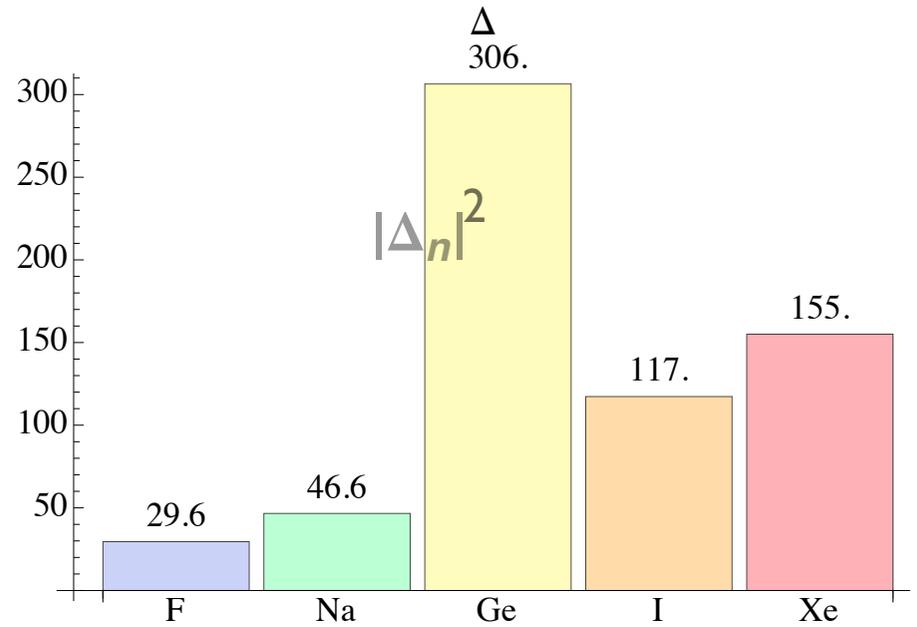
(new EFT velocity-dependent operator)

orbital vs. spin ambiguity

Vector, neutron coupled: $\vec{\sigma}(i)$ vs. $\vec{\ell}(i)$



$Xe > Ge \gg NaI$



$Ge > Xe \sim NaI$

(new EFT velocity-dependent operator)

orbital vs. spin ambiguity

Mathematica script v2:

N.Anand, A. Liam Fitzpatrick, WCH v1; +Johnson, McElvain

$$\sigma(v) \quad \frac{d\sigma(v, E_R)}{dE_R} \quad \frac{dR_D}{dE_R} = N_T \langle n_\chi v \frac{d\sigma(v, E_R)}{dE_R} \rangle$$

1) Targets (all isotopes, separately or summed) ^{19}F , ^{23}Na , Ge , ^{127}I , $\text{Xe} \rightarrow$
 C , O , Ne , Si , S , Ar , Ca , Cs , Te

State-of-the-art full-shell response functions: $\sim 10^{10}$ basis states

2) Dropbox library of structure functions

3) v2 “beta” version used by PandaX-II (generalized SD interactions)

e.g.,

$$\begin{aligned} \mathcal{L}_{\text{int}}^9 &\equiv \bar{\chi} i \sigma^{\mu\nu} \frac{q_\nu}{m_N} \chi \bar{N} \gamma_\mu N \\ \mathcal{L}_{\text{int}}^{17} &\equiv i \bar{\chi} i \sigma^{\mu\nu} \frac{q_\nu}{m_N} \gamma^5 \chi \bar{N} \gamma_\mu N \\ \mathcal{L}_{\text{int}}^{10} &\equiv \bar{\chi} i \sigma^{\mu\nu} \frac{q_\nu}{m_N} \gamma^5 \chi \bar{N} i \sigma_{\mu\alpha} \frac{q^\alpha}{m_N} N \quad \dots \end{aligned}$$

Summary

- There is a lot of variability that can be introduced between detector responses by altering operators (and their isospins)
- Pairwise exclusion of experiments in general difficult
- More can be learned from elastic scattering experiments than is apparent in simplified analyses: nuclei have personalities!
- This suggests we should do more experiments, not fewer
- When the first signals are seen, things will get very interesting: those nuclei that do not show a signal may be as important as those that do

Thanks to my collaborators: Liam Fitzpatrick, Nikhil Anand, Ami Katz
Calvin Johnson, Ken McElvain