Parametrization of F_2 structure functions for inclusive γ^*p and γ^*A reactions in low Q^2 region

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(KEK)

Collaborator: Shunzo Kumano (KEK)

Workshop on “neutrino-nucleus interaction in a few GeV region”
KEK Tokai Campus, November 18-19, 2017

Outline

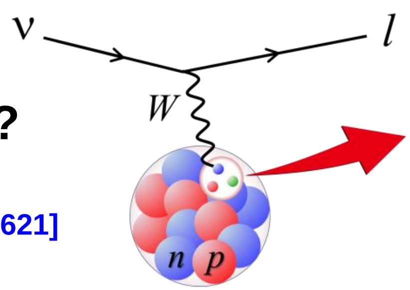
1. Introduction: Neutrino collaboration at the J-PARC Branch of KEK Theory Center

2. Approach to describing F_2 structure functions in high-W & low-Q^2 region ("Regge" region)

3. Summary and future works

1. Introduction:
Neutrino collaboration at
the J-PARC Branch of KEK Theory Center

Background and motivation



Why are neutrino-nucleon/nucleus reactions so important ?

[see e.g., recent review by Alvarez-Ruso, Hayato, and Nieves, New J. Phys. 16(2014)075015; Katori, Maltini, arXiv:1611.07770; Alvarez-Ruso et al, arXiv:1706.03621]

✓ **Accurate knowledge of neutrino reaction cross sections is necessary for precise determination of neutrino parameters via neutrino-oscillation experiments !!**

- θ_{13}, θ_{23} with high precision
- Mass hierarchy problem
- Leptonic CP violating phase (δ_{CP})

... and more !!

Initial neutrino flux produced at accelerator

Neutrino oscillation probability (neutrino parameters contained)

Neutrino event (e.g., long baseline exp.)

$$N = \phi \otimes P \otimes \sigma \otimes \text{(detector efficency etc.)}$$

Neutrino-nucleus cross section (input from reaction models)

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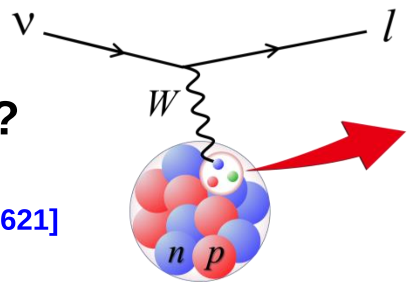

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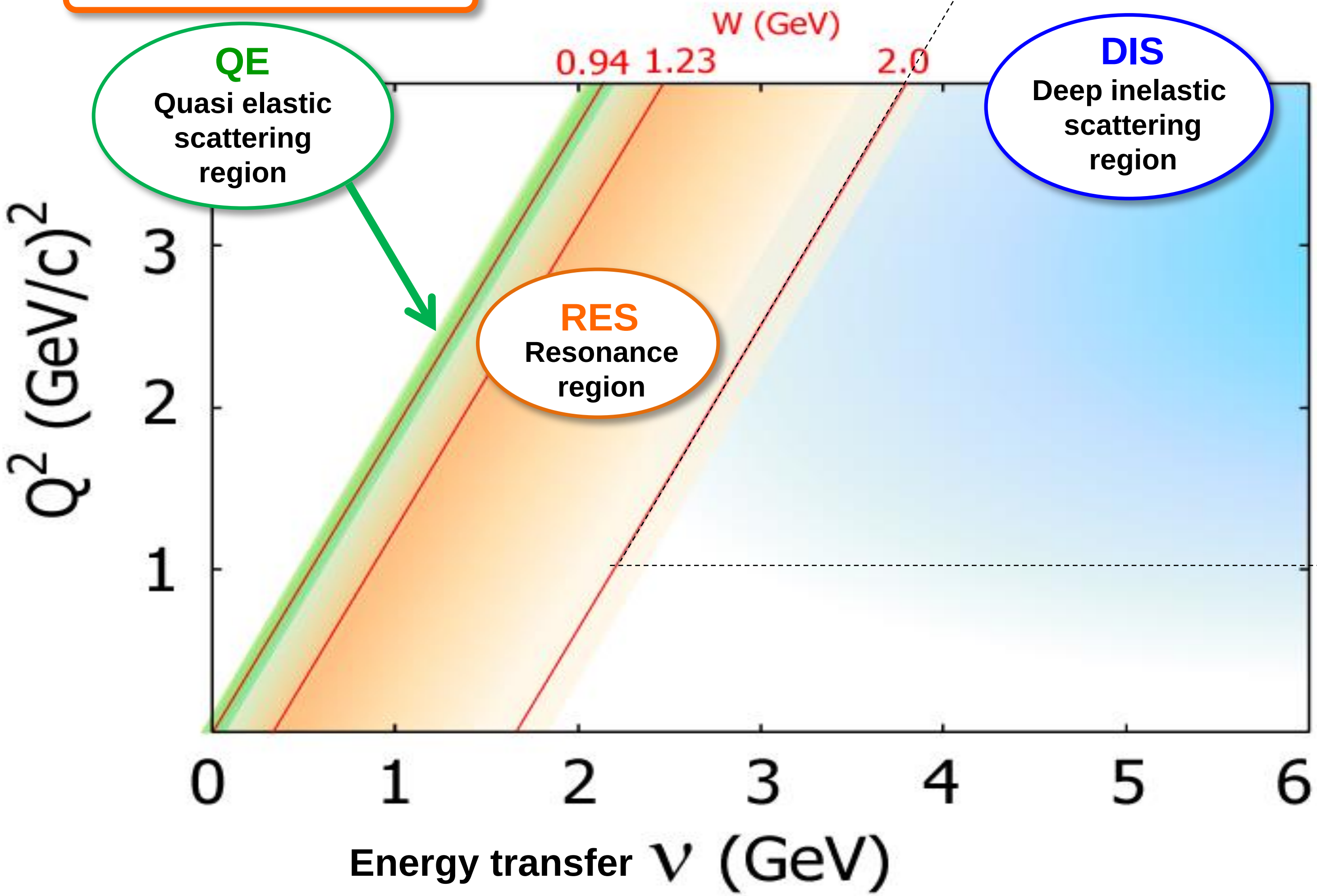
Neutrino-nucleus cross section (input from reaction models)

Need a reliable neutrino reaction model that describes various neutrino-nucleus reactions at the level of a few percent accuracy !!

Kinematical regions of neutrino reactions relevant to oscillation experiments

Relevant kinematical region extends over QE, RES, and DIS !!

$E_\nu = O(10^{-1} - 10^{1})$ GeV



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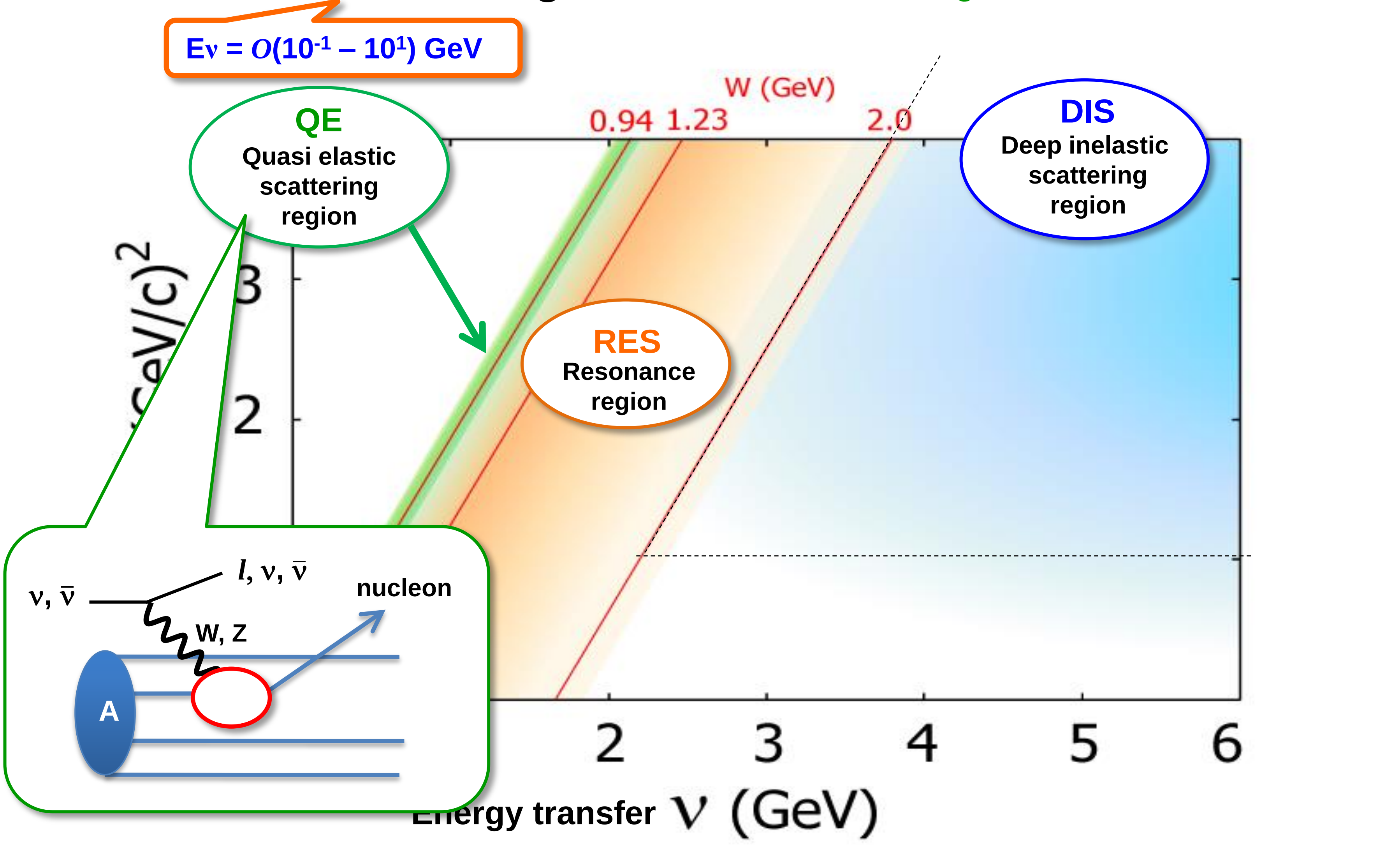

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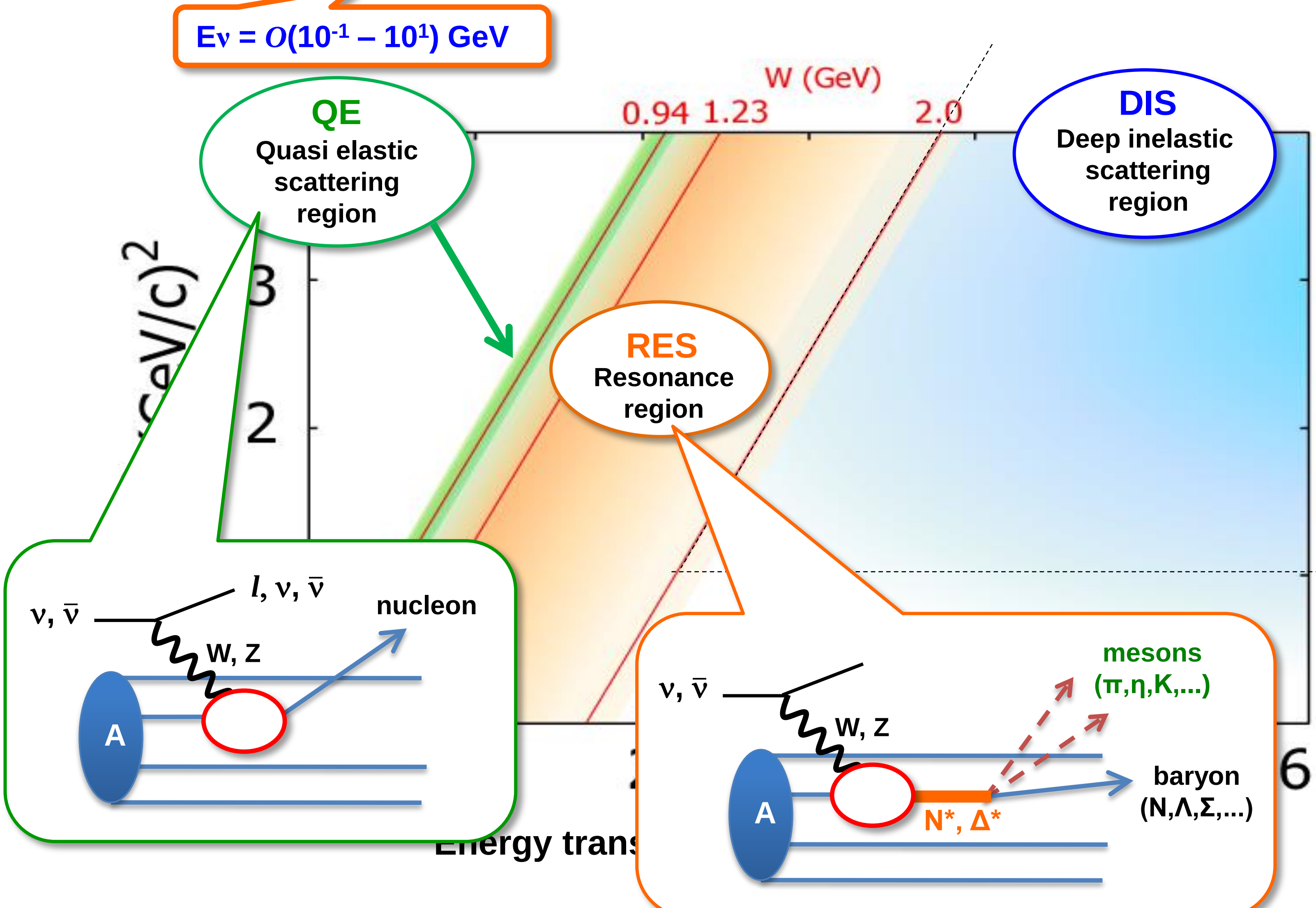

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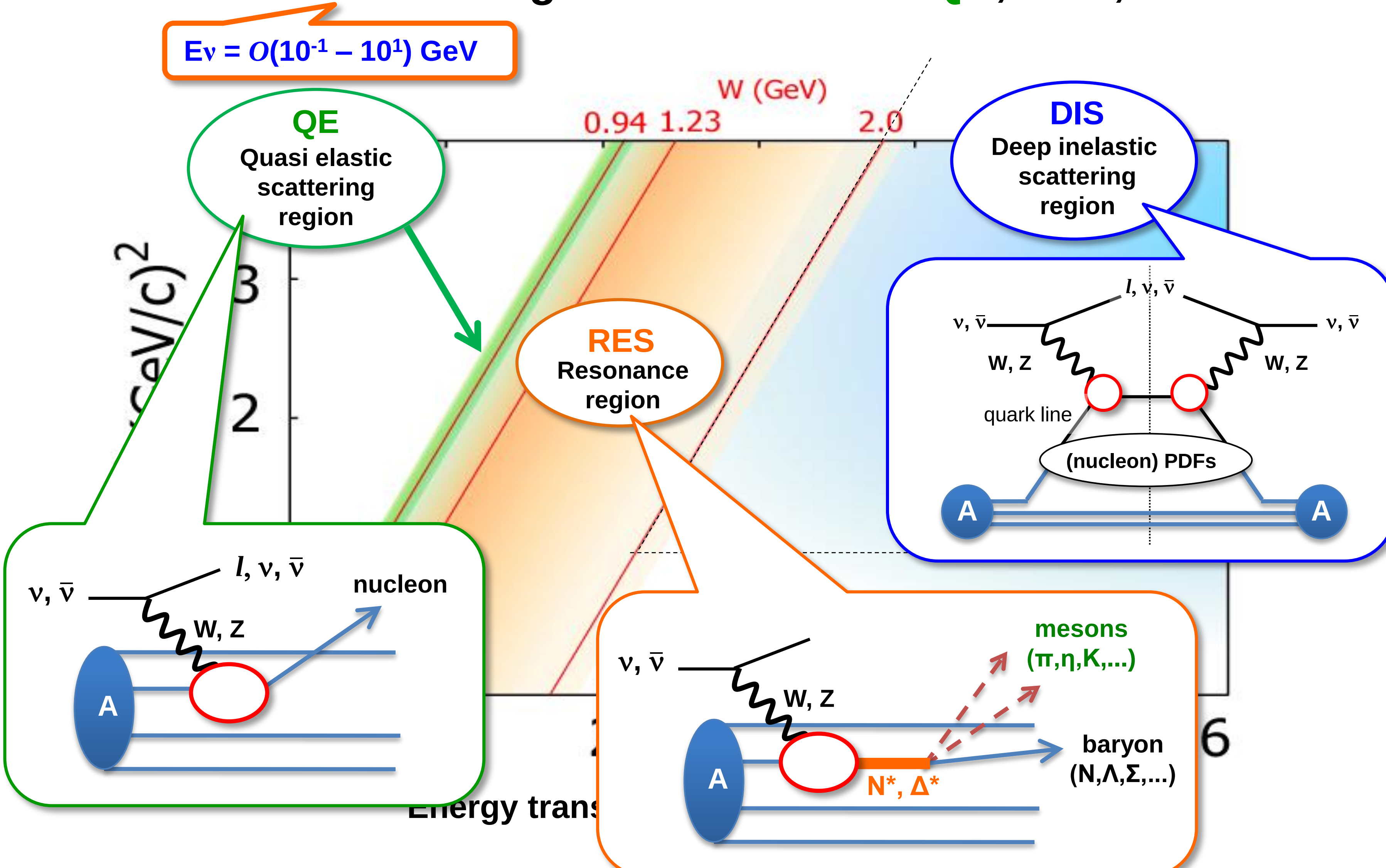

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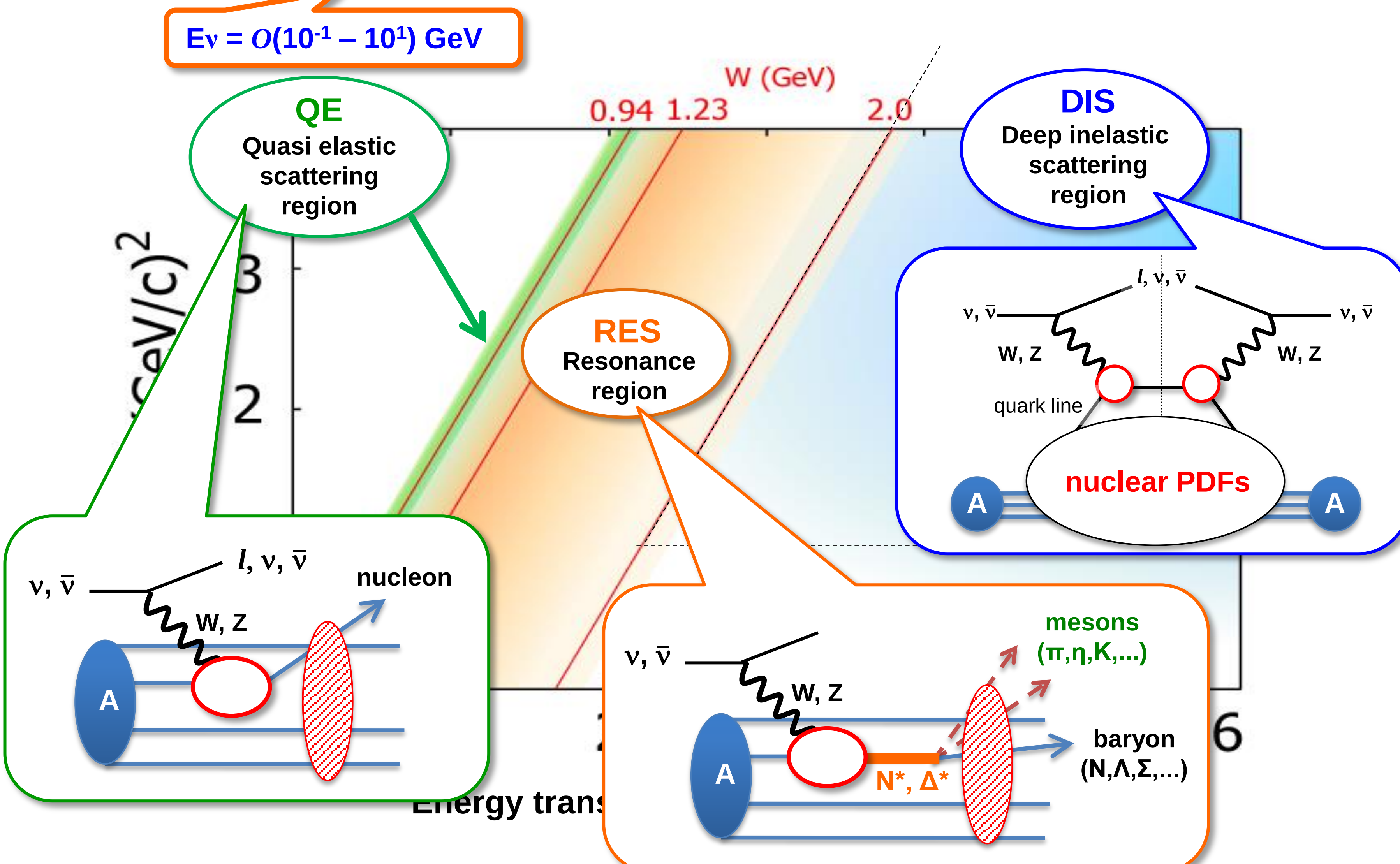

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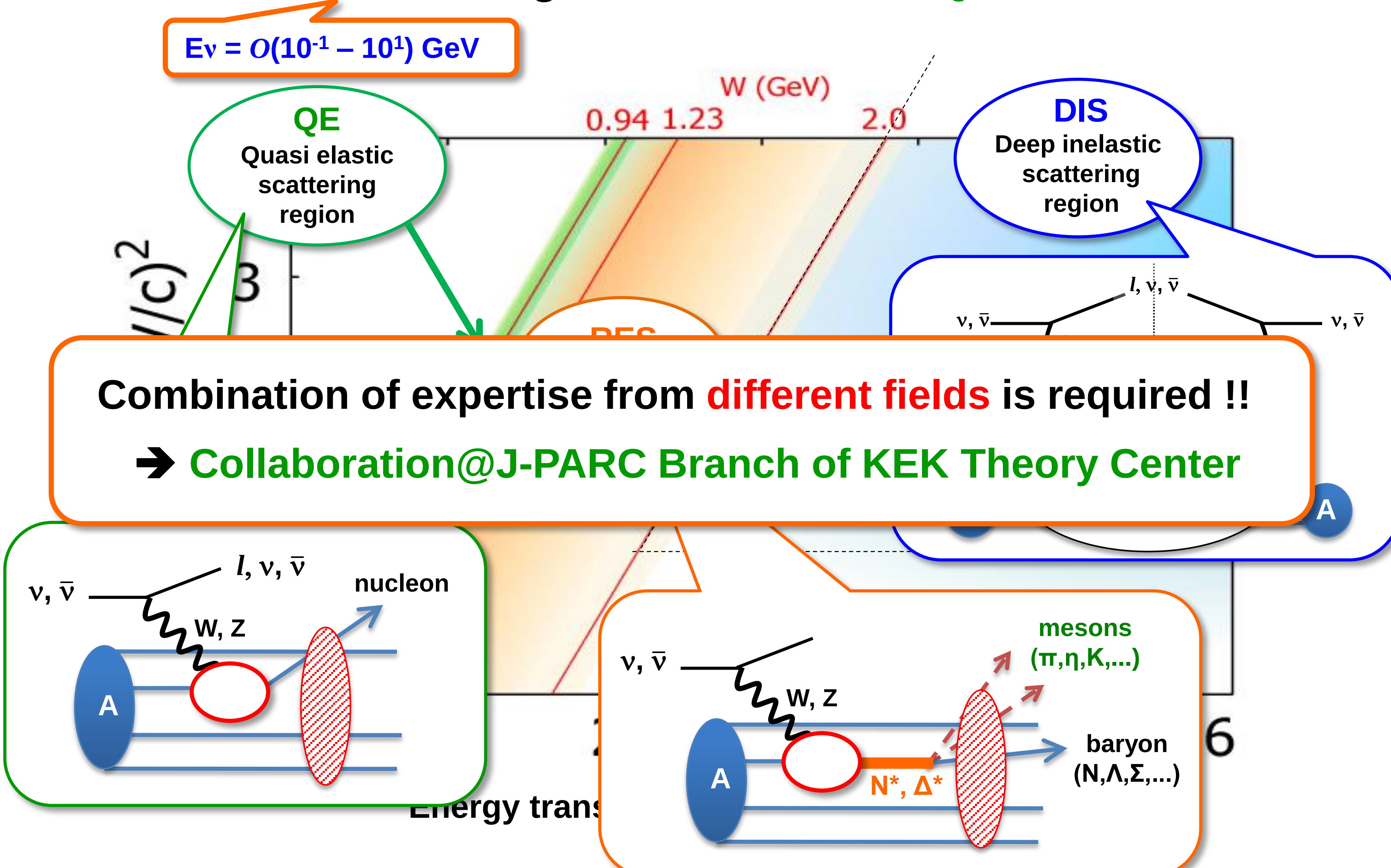


Combination of expertise from different fields is required !!

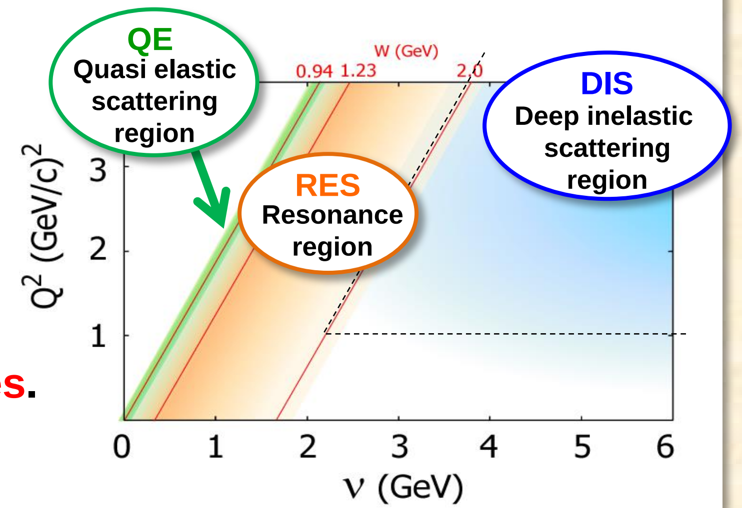
➔ **Collaboration@J-PARC Branch of KEK Theory Center**

Neutrino collaboration at J-PARC Branch of KEK Theory Center

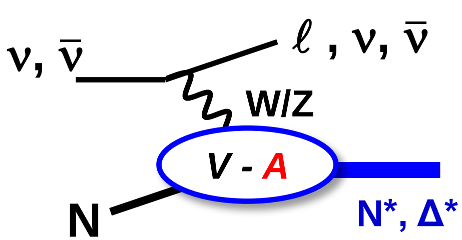
References:
Rept. Prog. Phys. 80, 056301 (2017)
http://nuint.kek.jp/index_e.html

GOAL: Construct a 'unified' model that comprehensively describes neutrino-nucleon/nucleus reactions over QE, RES, and DIS !!

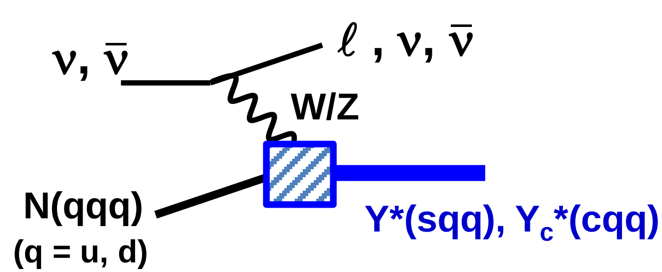
- **Deepen our knowledge on complicated neutrino interactions with nucleus**
- **Help improve neutrino event generators.**
- **Investigate the internal structure of nucleon, baryon resonances, and nuclei with weak probes. (transition form factors, nuclear PDFs, ...)**



✓ Axial transition form factors



✓ |ΔS|=1(u→s), |ΔC|=1(d→c) transition form factors



Neutrino collaboration at J-PARC Branch of KEK Theory Center

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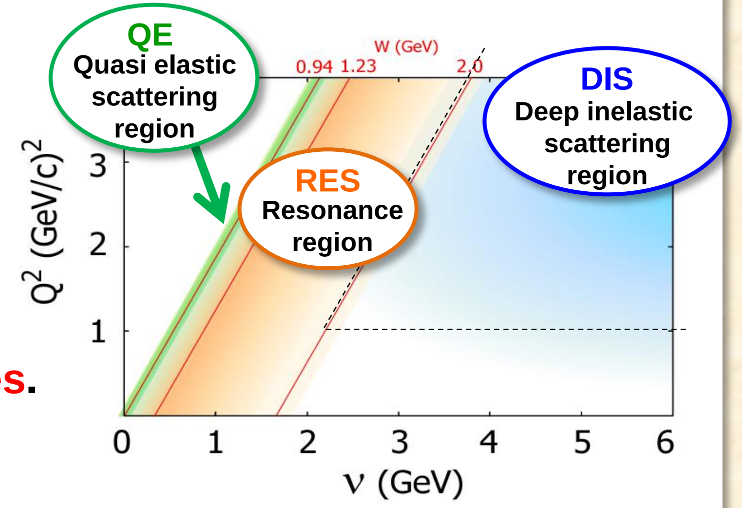

Participants:

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S. Nakamura (Cruzeiro do Sul U.) [RES,QE], K. Saito (Tokyo U. of Sci) [DIS],

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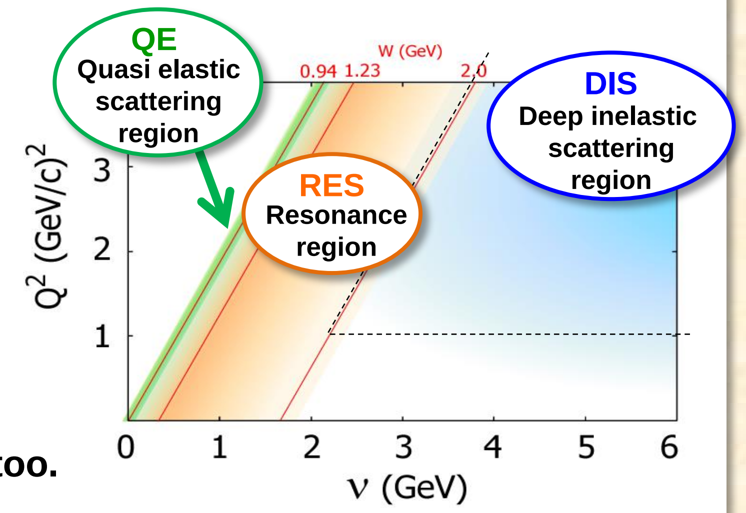
GOAL: Construct a 'unified' model that comprehensively describes neutrino-nucleon/nucleus reactions over QE, RES, and DIS !!

✓ <u>Strategy</u>

1. Construct 'baseline' models describing each of the kinematical regions individually.
   ➔ constructed with appropriate effective d.o.f.s in each region

2. Connect the 'baseline' models
   ➔ accomplished by matching observed quantities
   ➔ describes transition regions (QE⇔RES, RES⇔DIS), too.



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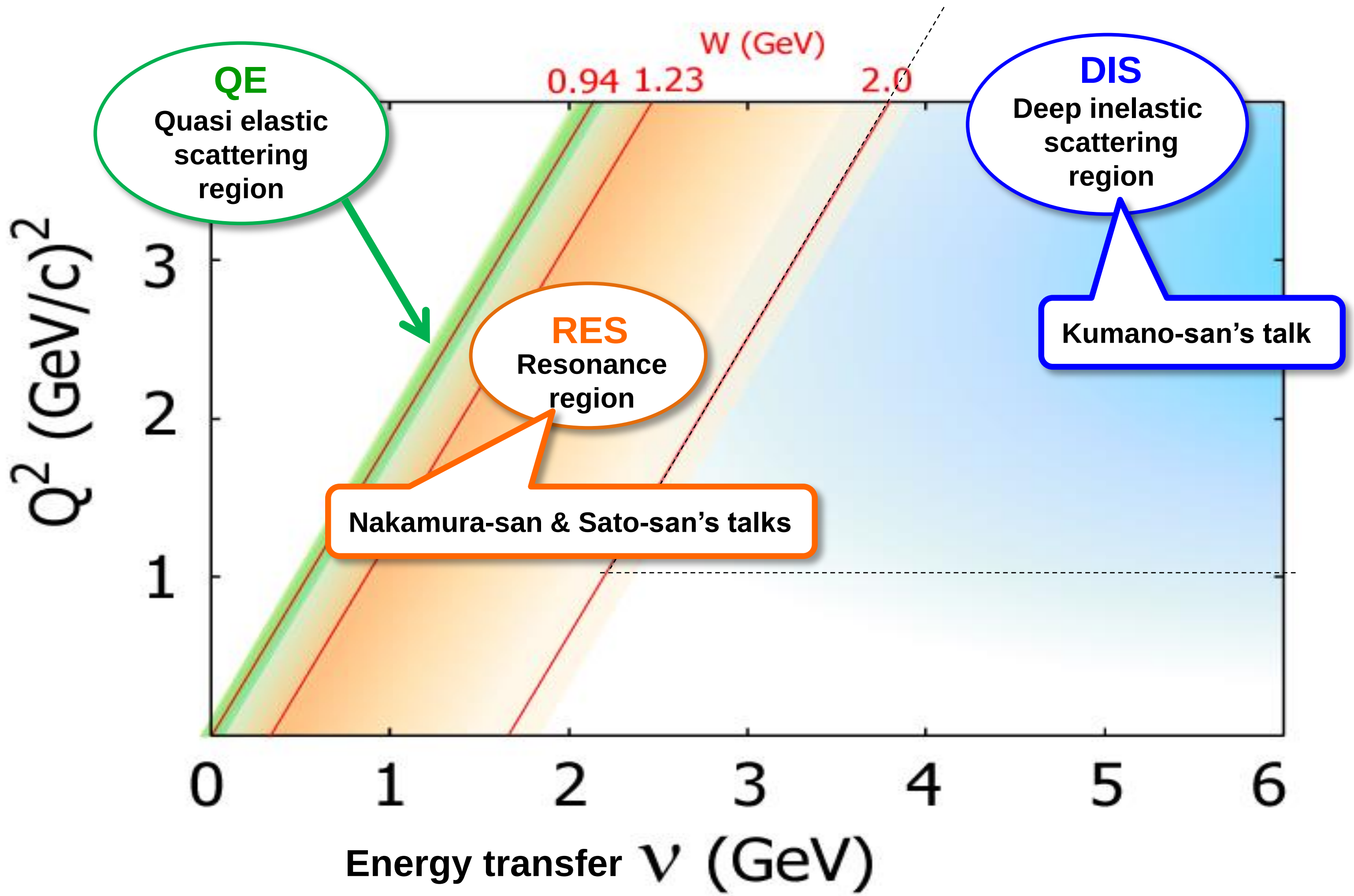

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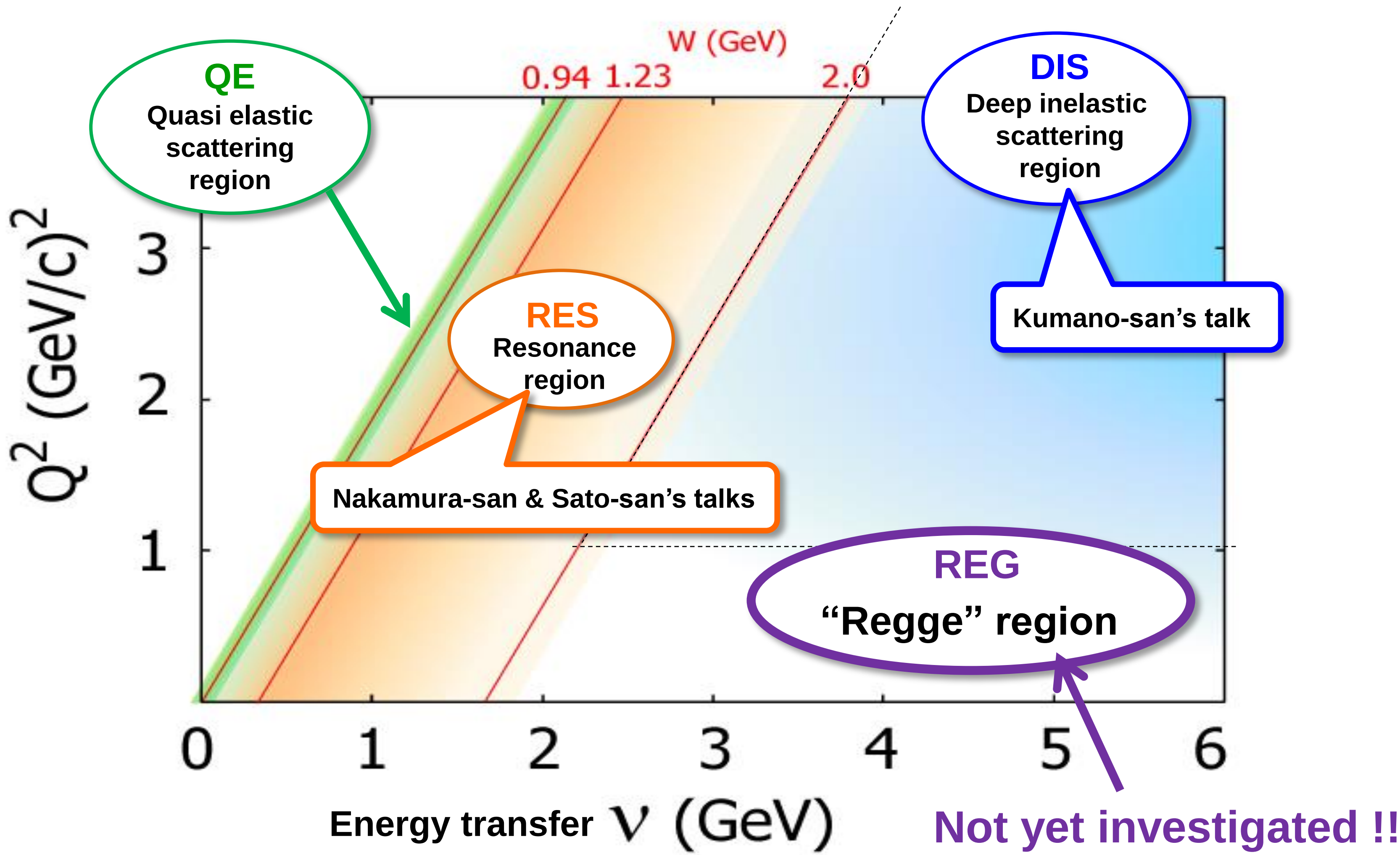

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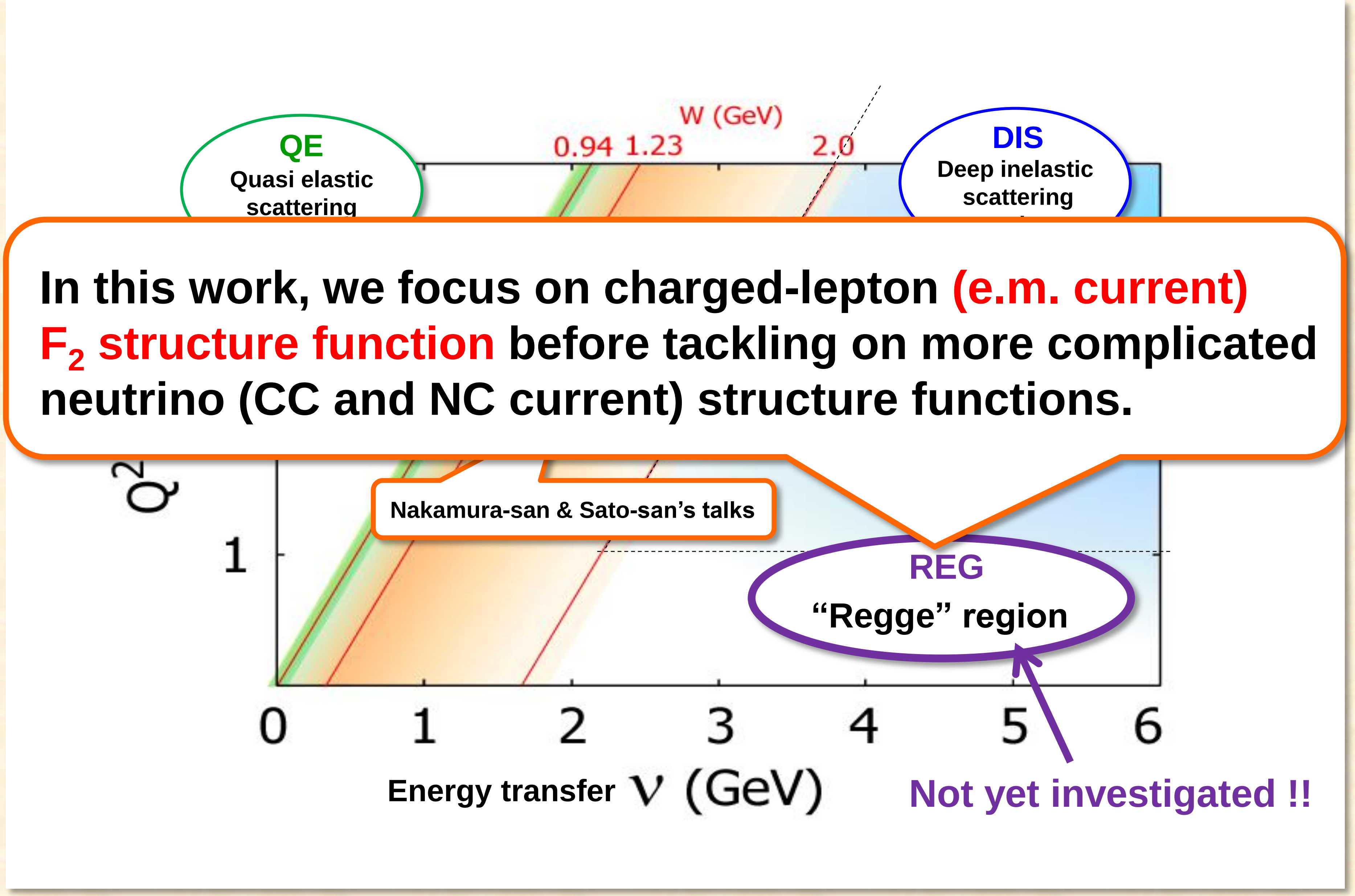

2. Approach to parametrizing F_2 structure functions in high-W & low-Q^2 region ("Regge" region)

Basic idea of our approach

- ✓ **Purpose of the present work:**
  - ➢ **Develop a model/parametrization for e.m. F_2 structure functions in high-W and low-Q^2 region (= “Regge” region).**
- ✓ **(Practical) “requirement”:**
  - ➢ **The model/parametrization should be as simple as possible for experimental use.**

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- A direct extension of the hadron-exchange model used in the RES region to the high-W region does not meet the above "requirement".
(➔ Treatment of multi-meson productions are very complicated.)
- However, an accurate structure-function model is available in the DIS region (applicable at $Q^2 > 1$ GeV^2; W > 2 GeV).
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"Extrapolate" the DIS model to the low-Q^2 region !!

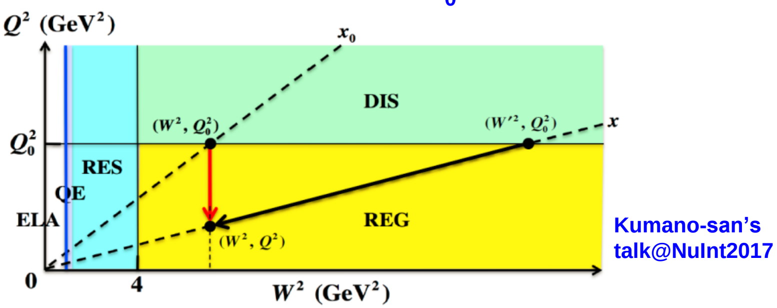
Basic idea of our approach

✓ **Parametrization of F_2 structure function at low Q^2 values:**

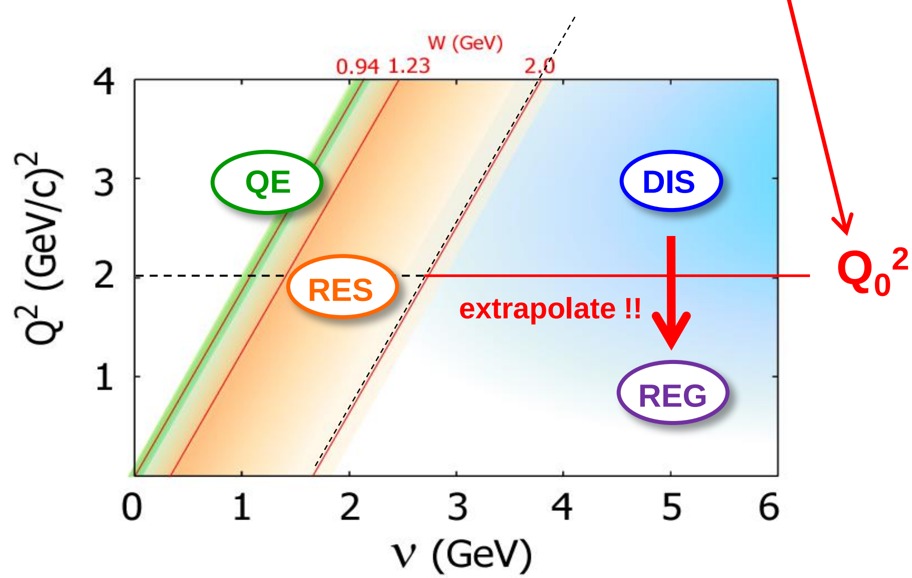
$$F_2(x, Q^2) = w(x, Q^2; x_0, Q_0^2) \times F_2^{\mathrm{DIS}}(x_0, Q_0^2) \qquad (Q^2 \leq Q_0^2)$$

- weight function [extrapolation to low $Q^2 < Q_0^2$]
- F_2 at $Q^2 = Q_0^2$ [from the DIS model]
- "matching" point

definition of x_0



Kumano-san's talk@NuInt2017



➢ F_2 is smooth w.r.t. Q^2:

$$w(x_0, Q_0^2; x_0, Q_0^2) = 1$$

$$\left.\frac{\partial F_2(x, Q^2)}{\partial Q^2}\right|_{Q^2=Q_0^2} = \left.\frac{\partial F_2^{\mathrm{DIS}}(x, Q^2)}{\partial Q^2}\right|_{Q^2=Q_0^2}$$

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✓ **Parametrization of F_2 structure function at low Q^2 values:**

$$F_2(x, Q^2) = \underbrace{w(x, Q^2; x_0, Q_0^2)}_{\text{weight function}} \times \underbrace{F_2^{\mathrm{DIS}}(x_0, Q_0^2)}_{F_2 \text{ at } Q^2=Q_0^2} \qquad (Q^2 \le \underline{Q_0^2})$$

weight function [extrapolation to low $Q^2 < Q_0^2$]

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As a first attempt, we parametrize the weight function as:

$$w(x, Q^2; x_0, Q_0^2) \equiv \frac{F_2^{\mathrm{ALLM}}(x, Q^2)}{F_2^{\mathrm{ALLM}}(x_0, Q_0^2)}$$

➔ **automatically satisfies**

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$F_2^{\mathrm{ALLM}}(x,Q^2)$: parametrization of F_2 based on the Regge + Pomeron phenomenology

➔ **proposed by Abramowicz, Levin, Levy, Maor, PLB 269 (1991) 465**

$$F_2^{\mathrm{ALLM}}(x, Q^2) = \frac{Q^2}{m_0^2 + Q^2}\left[F_2^{\mathrm{Reg.}}(x, Q^2) + F_2^{\mathrm{Pom.}}(x, Q^2)\right]$$

$$F_2^V(x, Q^2) = c_V(Q^2) x_V^{a_V(Q^2)} (1-x)^{b_V(Q^2)} \quad (V = \mathrm{Reg.},\ \mathrm{Pom.}) \qquad \frac{1}{x_V} = 1 + \frac{W^2 - M_N^2}{Q^2 + m_V^2}$$

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V = Reg. case

$$\begin{aligned} a_V(t) &= a_{V1} + a_{V2} \times t^{a_{V3}} \\ b_V(t) &= b_{V1} + b_{V2} \times t^{b_{V3}} \\ c_V(t) &= c_{V1} + c_{V2} \times t^{c_{V3}} \end{aligned}$$

V = Pom. case

$$\begin{aligned} a_V(t) &= a_{V1} + (a_{V1} - a_{V2})\left(\frac{1}{1 + t^{a_{V3}}} - 1\right) \\ b_V(t) &= b_{V1} + b_{V2} \times t^{b_{V3}} \\ c_V(t) &= c_{V1} + (c_{V1} - c_{V2})\left(\frac{1}{1 + t^{c_{V3}}} - 1\right) \end{aligned}$$

$$t = \ln\left(\frac{\ln \frac{Q^2 + \bar{Q}^2}{\Lambda^2}}{\ln \frac{\bar{Q}^2}{\Lambda^2}}\right)$$

As a

$$F_2^{\text{ALLM}}(x_0, Q_0^2) \qquad w(x_0, Q_0^2; x_0, Q_0^2) = 1$$

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Strategy (first attempt)

1. Determines parameters contained in $w(x,Q^2; x_0,Q_0^2)$ by fitting to the existing data for the proton target:

   - Electromagnetic F_2 structure functions (for low $Q^2 < Q_0^2$)
   - inclusive $\gamma + p \rightarrow X$ total cross section (for $Q^2 = 0$)
   - imposing "smoothness" of F_2 function at $Q^2 = Q_0^2$

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   - inclusive $\gamma + p \rightarrow X$ total cross section (for $Q^2 = 0$)
   - imposing "smoothness" of F_2 function at $Q^2 = Q_0^2$

2. Estimates the *nuclear* F_2 functions by replacing the DIS part:

$$F_2^{\text{proton}}(x, Q^2) = w(x, Q^2; x_0, Q_0^2) \times F_2^{\text{proton, DIS}}(x_0, Q_0^2)$$

$$\Downarrow$$

$$F_2^{\text{nucleus}}(x, Q^2) = w(x, Q^2; x_0, Q_0^2) \times F_2^{\text{nucleus, DIS}}(x_0, Q_0^2)$$

Database for fit

✓ Available F2 data for $Q^2 < 2\ \text{GeV}^2$ ($Q_0^2 = 2\ \text{GeV}^2$) and $W > 1.8$ GeV taken from the following groups.

[Data are taken from http://hepdata.cedar.ac.uk/review/f2/ unless otherwise stated below.]

- E665 (38 points)
- NMC (23 points)
- H1 (31 points)
- H1ZEUS (101 points) [d09-158.nce+p.dat, d09-158nce-p.dat in https://www.desy.de/h1zeus/combined_results/]
- JLab-HallC(2010) (52 points) [PRC81(1020)055207 https://hallcweb.jlab.org/disdata/]
- JLab-HallC(2015) (63 points) [private comm. (Kumano-san)]
- SLAC (117 points)
- ZEUS (121 points)

✓ $\gamma p \rightarrow X$ total cross section data for $W > 1.8$ GeV taken from PDG.

(229 points) http://pdg.lbl.gov/2016/hadronic-xsections/rpp2014-gammap_total.dat

✓ Impose smoothness condition via χ^2 :

$$\chi^2 = \left\{ 1 - \left(\left[\frac{\partial F_2(x,Q^2)}{\partial Q^2} \right]_{Q^2=Q_0^2} \Big/ \left[\frac{\partial F_2^{\mathrm{DIS}}(x,Q^2)}{\partial Q^2} \right]_{Q^2=Q_0^2} \right) \right\}^2 \Big/ (\text{"Error"})^2$$

Computed by taking difference

Computed with DGLAP eq.

x = 1e-5, 0.025, 0.50,..., 0.45, 0.475, 0.5 (21 points)

5% error is assigned for each point

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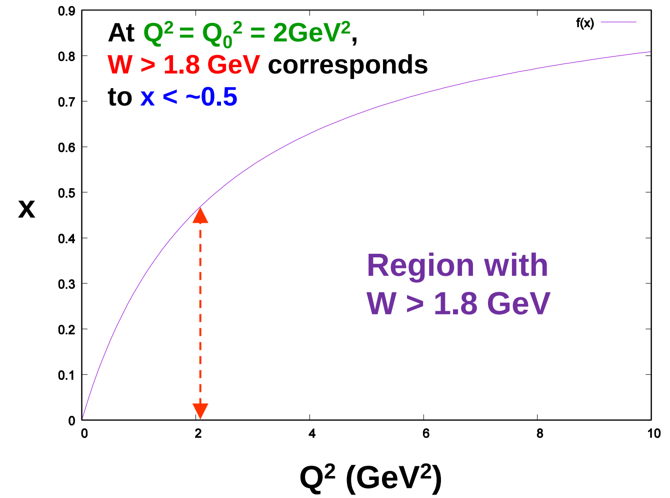

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Computed by taking difference — Computed with DGLAP eq.

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Computed by taking difference

Computed with DGLAP eq.

Total 796 points

x = 1e-5, 0.025, 0.50,..., 0.45, 0.475, 0.5 (21 points)
5% error is assigned for each point

Database for fit

- ✓ Available F2 data for $Q^2 < 2$ GeV^2 ($Q_0^2 = 2$ GeV^2) and $W > 1.8$ GeV taken from the following groups.
  [Data are taken from http://hepdata.cedar.ac.uk/review/f2/ unless otherwise stated below.]
  - E665 (38 points)
  - NMC (23 points)
  - H1 (31 points)
  - H1ZEUS (101 points) [d09-158.nce+p.dat, d09-158nce-p.dat in https://www.desy.de/h1zeus/combined_results/]
  - JLab-HallC(2010) (52 points) [PRC81(1020)055207 https://hallcweb.jlab.org/disdata/]
  - JLab-HallC(2015) (63 points) [private comm. (Kumano-san)]
  - SLAC (117 points)
  - ZEUS (121 points)
- ✓ $\gamma p \rightarrow X$ total cross section data for $W > 1.8$ GeV taken from PDG.
  (229 points) http://pdg.lbl.gov/2016/hadronic-xsections/rpp2014-gammap_total.dat
- ✓ Impose smoothness condition via χ^2 :

$$\chi^2 = \left\{ 1 - \left(\left[\frac{\partial F_2(x, Q^2)}{\partial Q^2} \right]_{Q^2=Q_0^2} \Bigg/ \left[\frac{\partial F_2^{\mathrm{DIS}}(x, Q^2)}{\partial Q^2} \right]_{Q^2=Q_0^2} \right) \right\}^2 \Big/ (\text{"Error"})^2$$

Computed by taking difference

Computed with DGLAP eq.

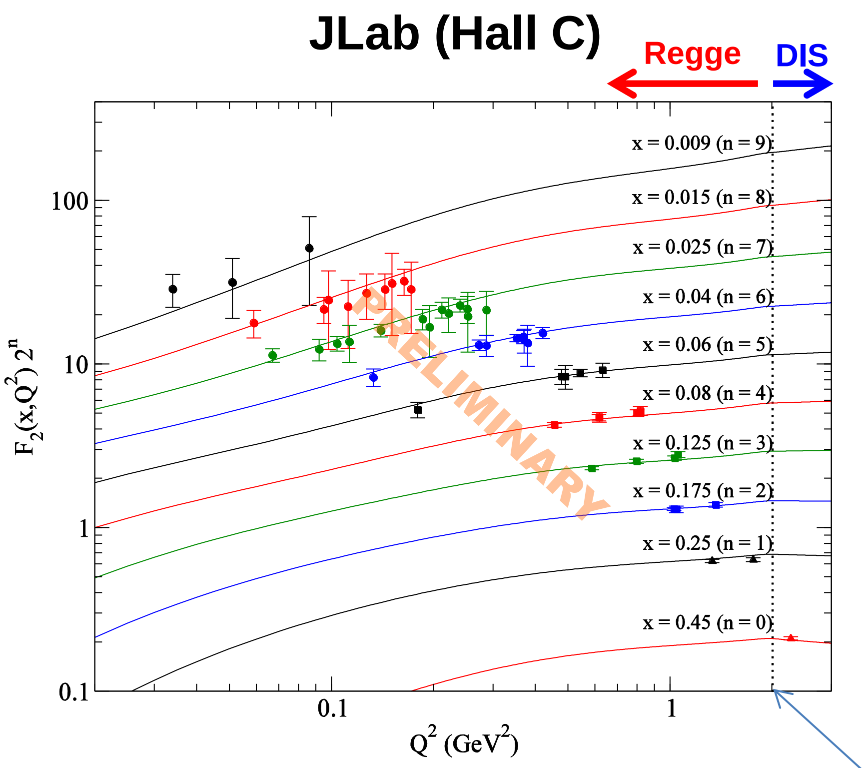
x = 1e-5, 0.025, 0.50,..., 0.45, 0.475, 0.5 (21 points)
5% error is assigned for each point

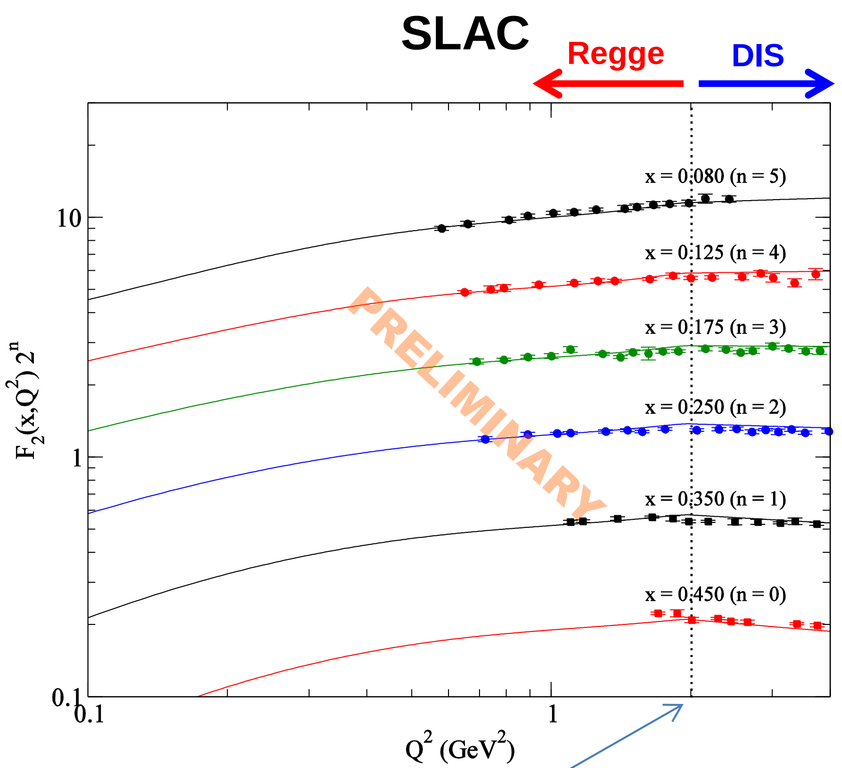
Total 796 points

Resulting χ^2/d.o.f = 1.39

Comparison with data at low Q^2

E.M. current F_2 structure function for the proton

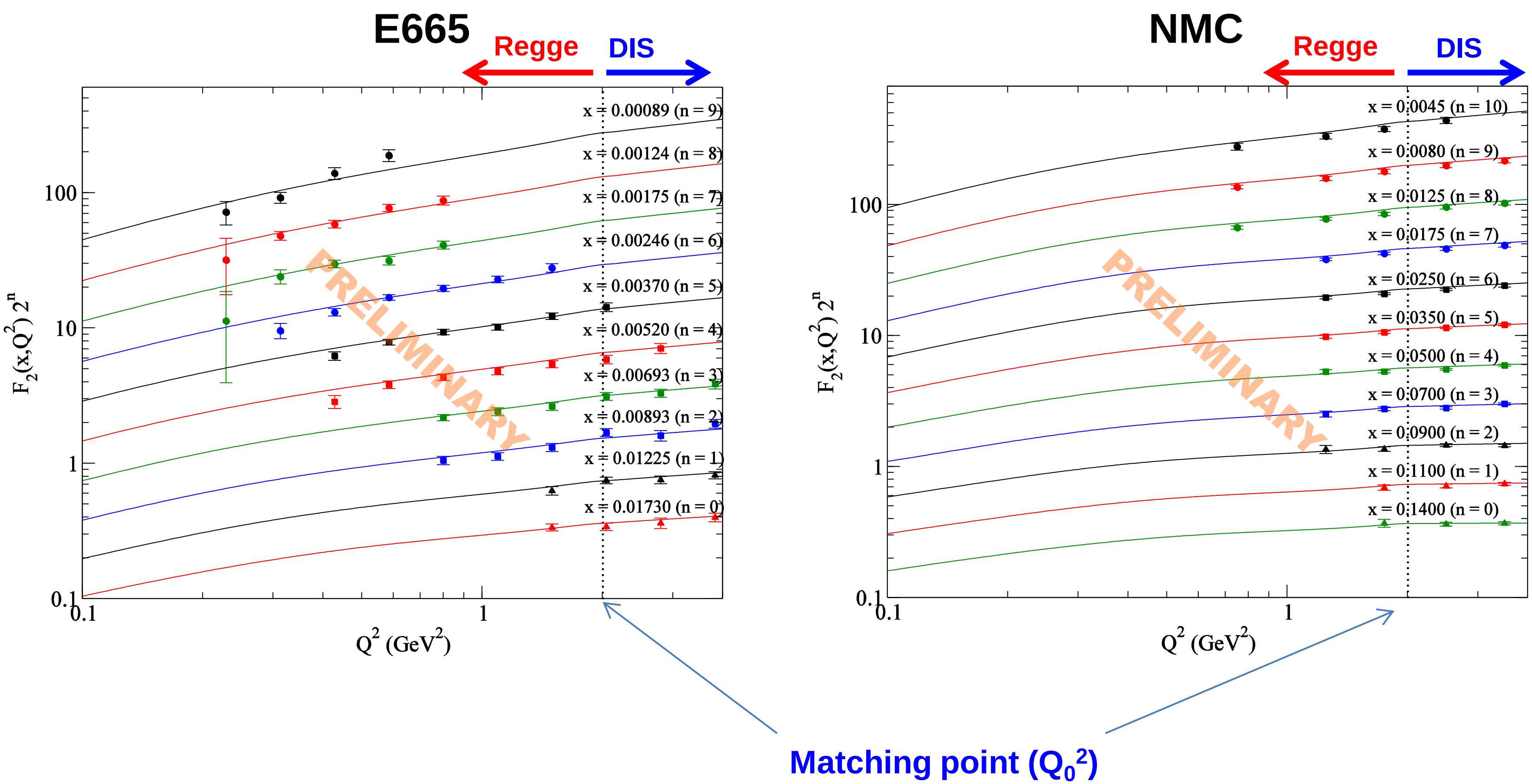




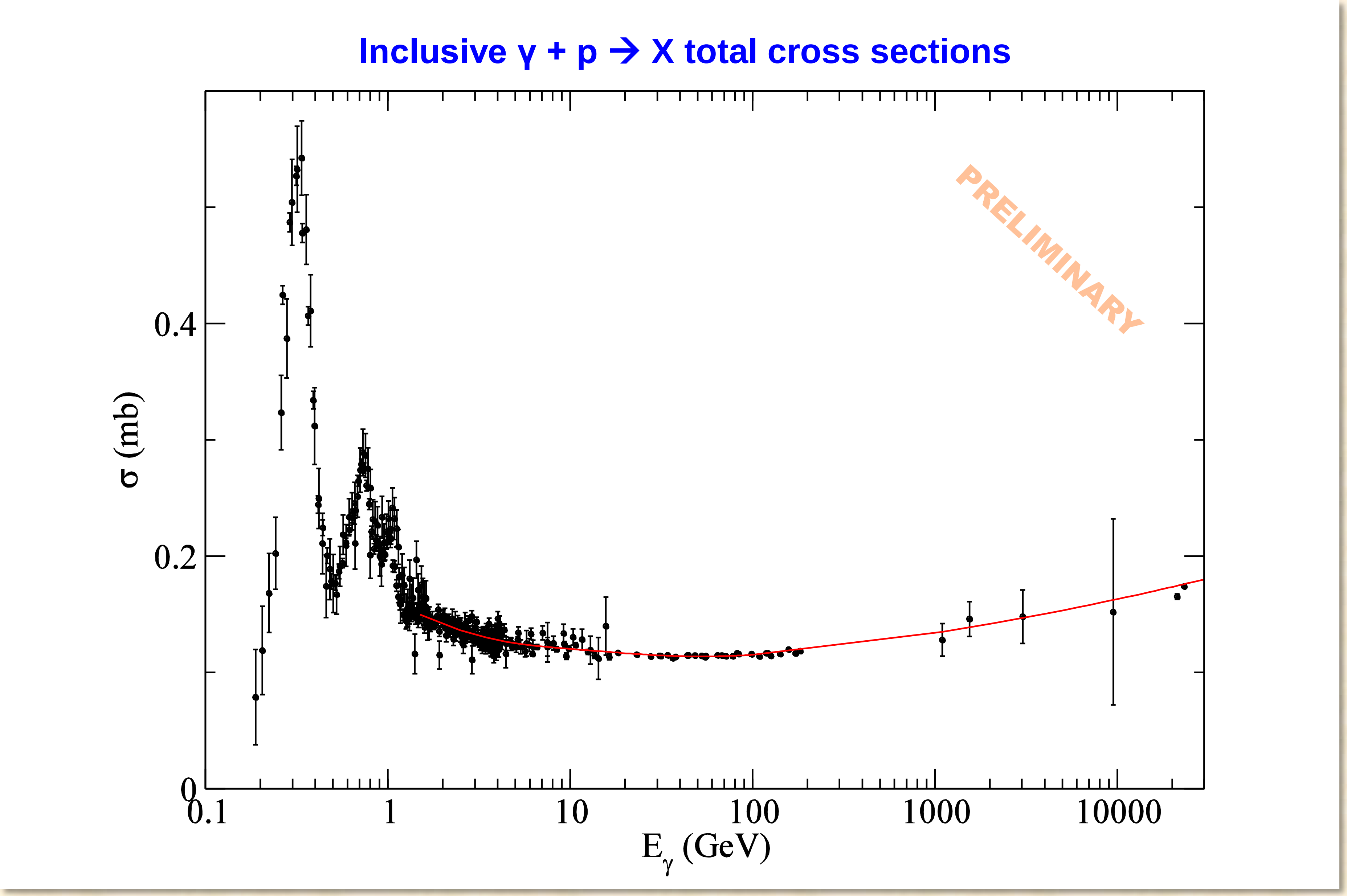
Matching point (Q_0^2)

Comparison with data at low Q^2

E.M. current F_2 structure function for the proton



Comparison with data at $Q^2 = 0$

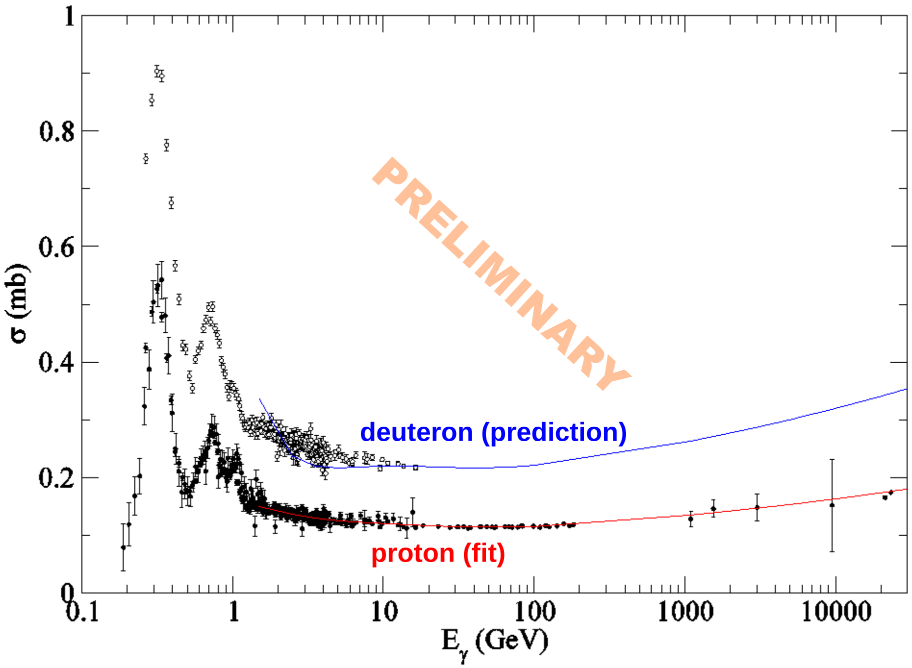


Prediction for deuteron target

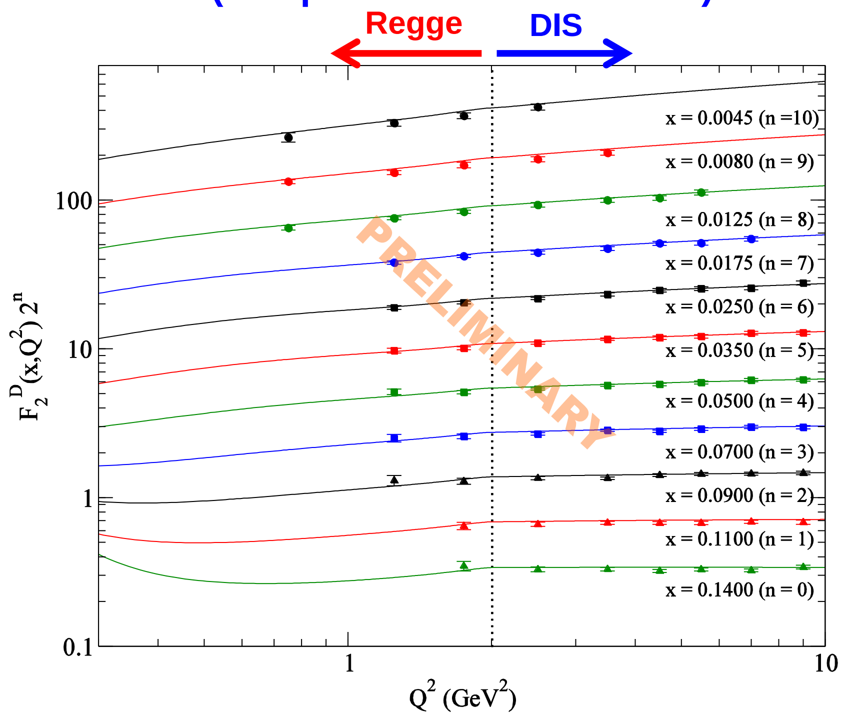
As a first attempt, try to estimate $F_2^{deuteron}$ by replacing $F_2^{proton,\ DIS}$ with $F_2^{deuteron,\ DIS}$:

$$F_2^{\text{deuteron}}(x, Q^2) = w(x, Q^2; x_0, Q_0^2) \times F_2^{\text{deuteron, DIS}}(x_0, Q_0^2)$$

Inclusive γ + p, γ + d → X total cross sections

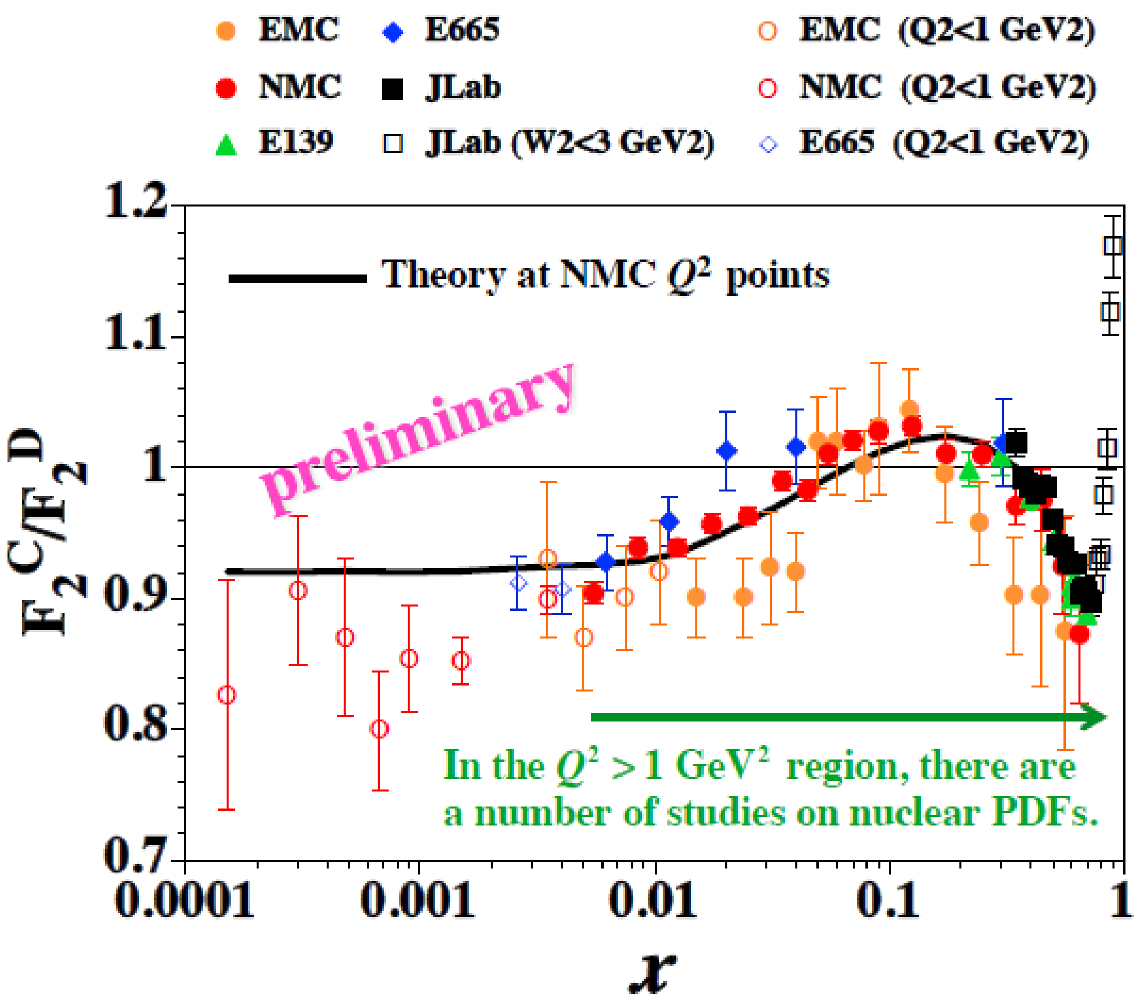


F_2 for the deuteron
(comparison with NMC data)



Prediction for nuclear target

- ✓ Ratio for Carbon/Deuteron F_2 structure functions



From Kumano-san's talk@NuInt2017

Summary & future works

Summary

✓ Investigated a possible, simple parametrization of the nucleon/nuclear structure functions in the Regge region (high-energy & low-Q^2 region).

➔ important in analyzing the data from neutrino-oscillation experiments

✓ Accomplished by extrapolating a well-established model in the DIS (high energy & high Q^2) region to the low Q^2 region:

$$F_2(x, Q^2) = w(x, Q^2; x_0, Q_0^2) \times F_2^{\mathrm{DIS}}(x_0, Q_0^2) \qquad (Q^2 \leq Q_0^2)$$

➔ the DIS part is taken from Hirai, Kumano, Nagai PRC76(2007)065207

➔ the weight function is parametrized by making use of the ALLM model [PLB269(1991)465]

Based on Reggeon, Pomeron phenomenology

Future works

✓ Better parameterization for the weight function $w(x,Q^2;x_0,Q_0^2)$??

✓ Further improvements of nuclear modification.

✓ Application to the neutrino-nucleus structure functions

Back up

The ALLM parametrization of F_2 for $\gamma^{(*)}p$ inclusive reactions

Abramowicz, Levin, Levy, Maor, PLB269(1991)4656

$$F_2(x,Q^2) = \frac{Q^2}{Q^2+m_0^2}\left(F_2^{\mathcal{P}}(x,Q^2) + F_2^{\mathcal{R}}(x,Q^2)\right)$$

$\mathcal{P}$ = Pomeron-exchange contribution, $\mathcal{R}$ = Regge-exchange contribution

$$\begin{aligned} F_2^{\mathcal{P}}(x,Q^2) &= c_{\mathcal{P}}(t) x_{\mathcal{P}}^{a_{\mathcal{P}}(t)} (1-x)^{b_{\mathcal{P}}(t)}, \\ F_2^{\mathcal{R}}(x,Q^2) &= c_{\mathcal{R}}(t) x_{\mathcal{R}}^{a_{\mathcal{R}}(t)} (1-x)^{b_{\mathcal{R}}(t)}. \end{aligned}$$

$$t = \ln\left(\frac{\ln\frac{Q^2+Q_0^2}{\Lambda^2}}{\ln\frac{Q_0^2}{\Lambda^2}}\right)$$

Monotonically increasing in Q^2.
At $Q^2 = 0$, $t = 0$.

$$\begin{aligned} \frac{1}{x_{\mathcal{P}}} &= 1 + \frac{W^2-M^2}{Q^2+m_{\mathcal{P}}^2}, \\ \frac{1}{x_{\mathcal{R}}} &= 1 + \frac{W^2-M^2}{Q^2+m_{\mathcal{R}}^2}. \end{aligned}$$

If $m_P = m_R = 0$, x_P and x_R reduces to x.
x_P and x_R are nonzero even at $Q^2 = 0$.
(cf. x = 0 at $Q^2 = 0$)

The ALLM parametrization of F_2 for $\gamma^{(*)}p$ inclusive reactions

Abramowicz, Levin, Levy, Maor, PLB269(1991)4656

$$
\begin{aligned}
F_2^{\mathcal{P}}(x,Q^2) &= c_{\mathcal{P}}(t)\, x_{\mathcal{P}}^{a_{\mathcal{P}}(t)}(1-x)^{b_{\mathcal{P}}(t)},\\
F_2^{\mathcal{R}}(x,Q^2) &= c_{\mathcal{R}}(t)\, x_{\mathcal{R}}^{a_{\mathcal{R}}(t)}(1-x)^{b_{\mathcal{R}}(t)}.
\end{aligned}
$$

$$
\begin{aligned}
a_{\mathcal{R}}(t) &= a_{\mathcal{R}1} + a_{\mathcal{R}2} \times t^{a_{\mathcal{R}3}}\\
b_{\mathcal{R}}(t) &= b_{\mathcal{R}1} + b_{\mathcal{R}2} \times t^{b_{\mathcal{R}3}}\\
c_{\mathcal{R}}(t) &= c_{\mathcal{R}1} + c_{\mathcal{R}2} \times t^{c_{\mathcal{R}3}}
\end{aligned}
$$

$$
\begin{aligned}
a_{\mathcal{P}}(t) &= a_{\mathcal{P}1} + (a_{\mathcal{P}1} - a_{\mathcal{P}2})\left(\frac{1}{1+t^{a_{\mathcal{P}3}}} - 1\right)\\
b_{\mathcal{P}}(t) &= b_{\mathcal{P}1} + b_{\mathcal{P}2} \times t^{b_{\mathcal{P}3}}\\
c_{\mathcal{P}}(t) &= c_{\mathcal{P}1} + (c_{\mathcal{P}1} - c_{\mathcal{P}2})\left(\frac{1}{1+t^{c_{\mathcal{P}3}}} - 1\right)
\end{aligned}
$$

At t = 0 (Q^2 = 0), only the first term survives.

a_R, b_R, c_R, b_P ➔ Supposed to be monotonically increasing function of t.

If a_{R3}, b_{R3}, c_{R3}, b_{P3} positive (negative), then a_{R2}, b_{R2}, c_{R2}, b_{P2}, must be positive (negative).

a_P, c_P ➔ Supposed to be monotonically decreasing function of t.

$a_{P1} > a_{P2}$, $a_{P3} > 0$ or $a_{P1} < a_{P2}$, $a_{P3} < 0$; $c_{P1} > c_{P2}$, $c_{P3} > 0$ or $c_{P1} < c_{P2}$, $c_{P3} < 0$

c_R, c_P ➔ Must be positive for all t.

c_{R1}, c_{P1} must be positive.