KEK Theory Center

Neutrino-nucleus interactions in the few GeV region

Theoretical challenges in neutrino scattering studies: weak pion production off the nucleon

E. Hernández, U. Salamanca

J. Nieves, IFIC (CSIC & UV)

Outline

1. Motivation: Neutrino oscillations, neutrino detectors, nuclear cross sections and systematic errors
2. Llewellyn-Smith: $\Delta(1232)$ & the $\nu_l\, N \longrightarrow l^- N' \pi$ reaction
3. Chiral symmetry and non-resonant contributions
4. Deuteron effects and ANL & BNL data
5. Unitarity corrections and Watson's theorem
6. The $\nu_\mu\, n \longrightarrow \mu^- n \pi^+$ channel and the crossed Δ term: spin 1/2 dof in the Δ propagator & contact terms
7. Parity-violating contributions to the pion angular differential cross section and T-odd correlations.
8. Conclusions

Bibliography:

- Alvarez-Ruso L., Hayato Y. and Nieves J.: *Progress and open questions in the physics of neutrino cross sections,* New J.Phys. 16 (2014) 075015
- Hernández E., Nieves J. and Valverde M.: *Weak Pion Production off the Nucleon,* Phys. Rev. D76 (2007) 033005
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- Hernández E., Nieves J., Valverde M. and Vicente-Vacas M.J.: $N\Delta(1232)$ *axial form factors from weak pion production,* Phys. Rev. D81 (2010) 085046
- Alvarez-Ruso L., Hernández E., Nieves J. and Vicente-Vacas M.J.: *Watson's theorem and the* $N\Delta(1232)$ *transition, Phys. Rev. D93 (2016) 014016*
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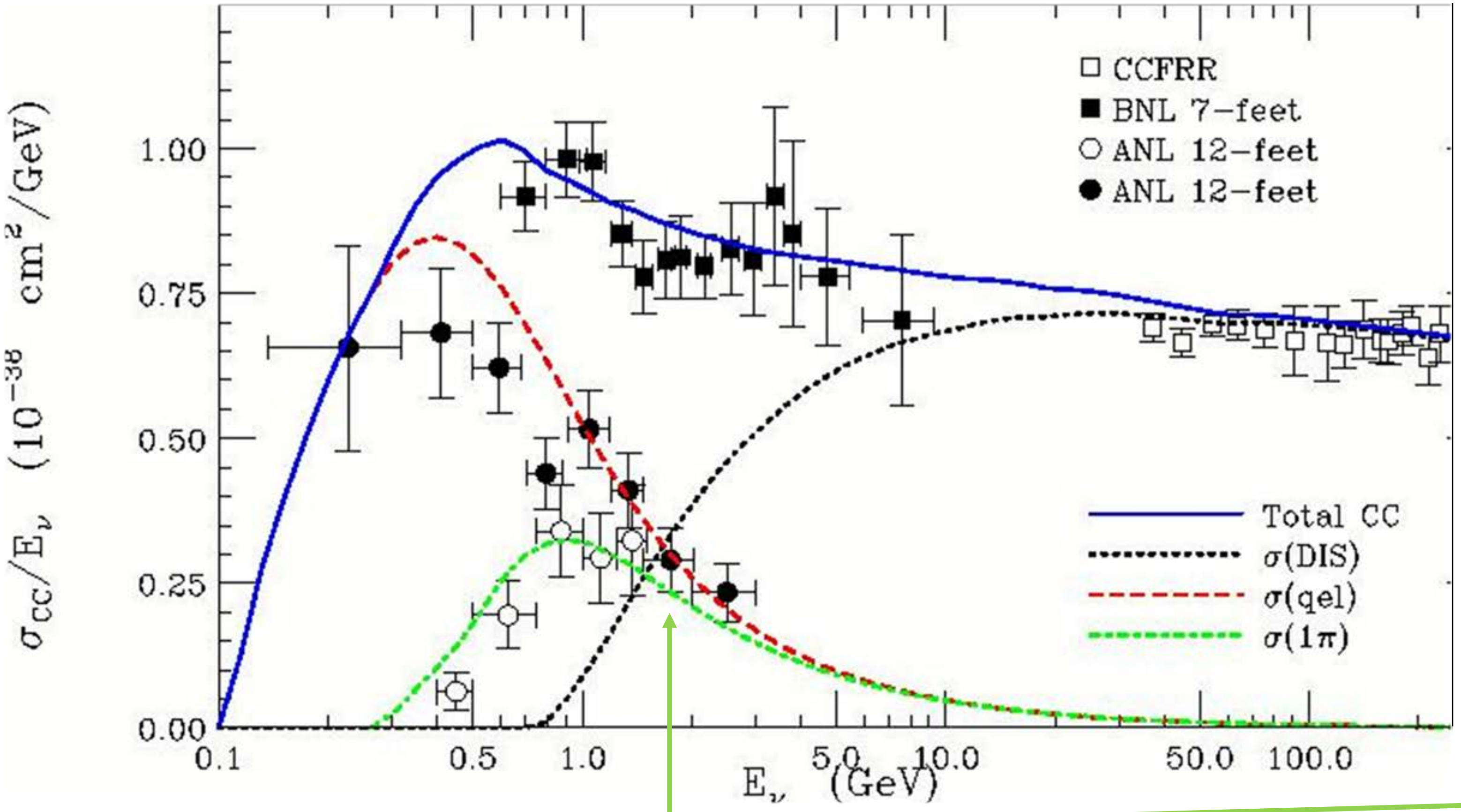
1. Motivation

- Details on the axial structure of hadrons in the free space and inside of nuclei
- Neutrinos are detected through nuclear interactions

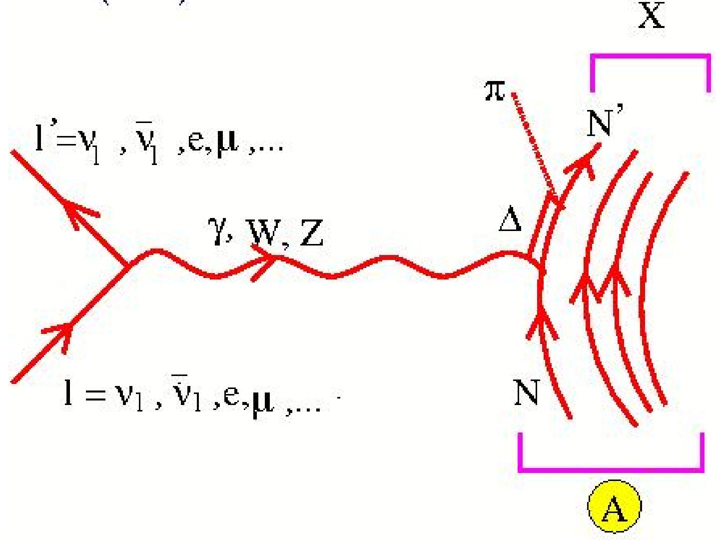
Theoretical knowledge of QE, 1π and DIS cross sections is important to carry out a precise neutrino oscillation data analysis...

$^{12}C \rightarrow$ Liquid scintillators
$^{16}O \rightarrow$ Cerenkov detectors
$^{40}A \rightarrow$ TPC's (time projection chambers) ....



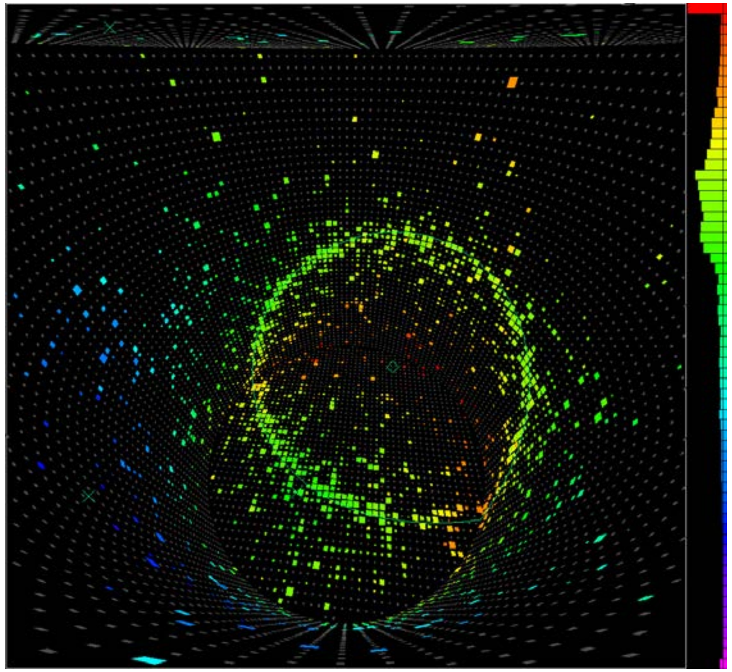
Th: NUANCE (D. Casper, 2002)

Δ(1232) RESONANCE PEAK



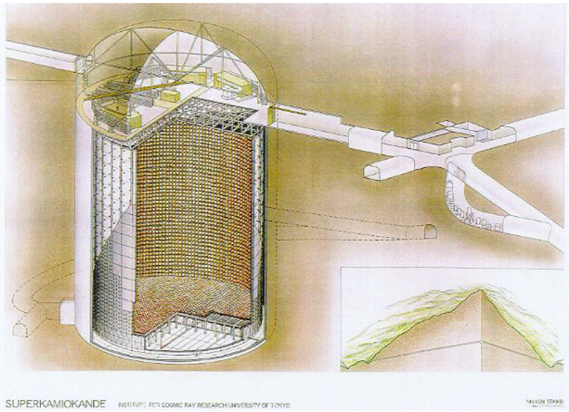
EXCITATION OF Δ (1232) DEGREES OF FREEDOM

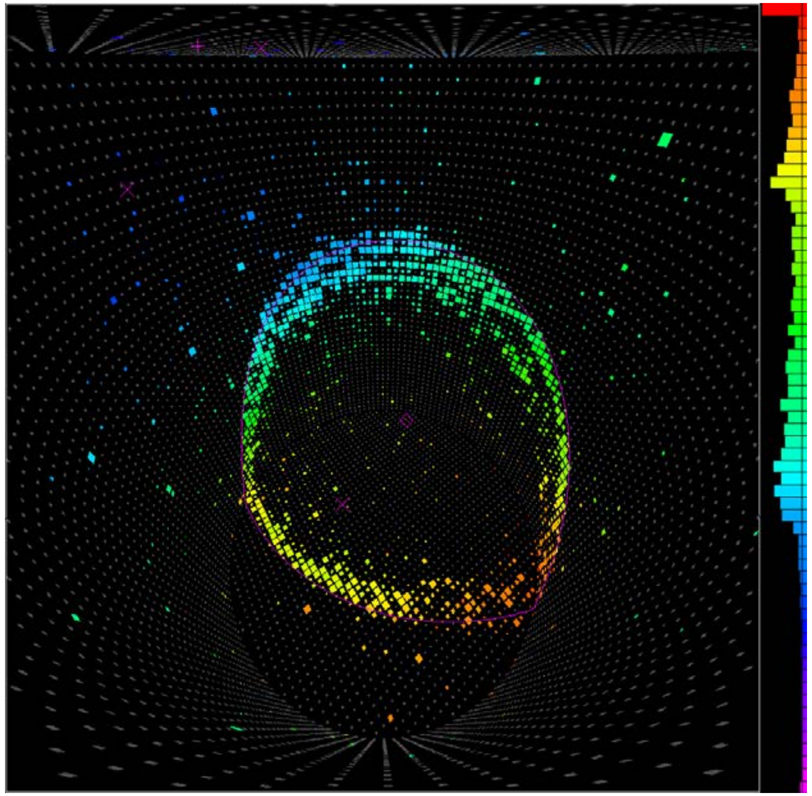
Juan Nieves, IFIC (CSIC & UV)



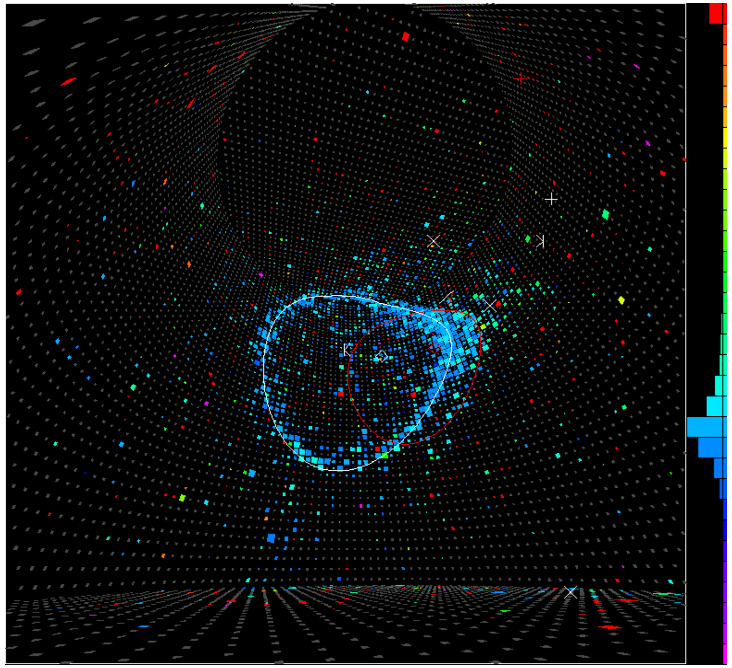
$\nwarrow$ e

SuperKamiokande





$\uparrow$ μ



$\leftarrow \pi^0$

$$\beta > \frac{1}{n} \rightarrow E_{\pi,\mu} > 200 - 300 \text{ MeV}$$

Pion production → misidentification of 1 Cherenkov ring events that are assumed to be produced by charged current (CC) QE reactions $\boldsymbol{\nu_\alpha A} \rightarrow \boldsymbol{l^\alpha}$ A'

Even distinguishing between μ- and e-like rings

- Appearance Probability $\boldsymbol{P(\nu_\mu \rightarrow \nu_e)}$: The CC QE signature $\nu_e A \rightarrow e\, A'$ used to identify ν_e can be confused with the NC 1π production $\nu_\mu A \rightarrow \nu_\mu A' \pi^0$
- Survival Probability $\boldsymbol{P(\nu_\mu \rightarrow \nu_\mu)}$: The CC QE signature $\nu_\mu A \rightarrow \mu\, A'$ used to identify ν_μ can be confused with the CC or NC $\nu_{\mu,\tau} A \rightarrow (\nu_{\mu,\tau}\ or\ \mu, \tau) A' \pi$ when only one of the particles emits Cherenkov light. For instance, processes (ν_μ, μ, π) might produce an **incorrect reconstruction of the neutrino energy** E → L/E analysis ?

Nuclear cross sections are crucial to reduce the systematic errors of oscillation analysis !

There exist dedicated experiments as MINERνA (FermiLab), which **seeks to measure low energy neutrino interactions both in support of neutrino oscillation experiments and also to study the strong dynamics of the nucleon and nucleus that affect these interactions**

Neutrino Energy Reconstruction:

QE: $\nu_\mu + n \longrightarrow p\,\mu^-$ (bound in the nucleus)

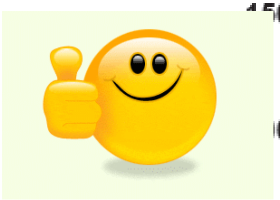
$$E_{\rm rec} = \frac{ME_\mu - m_\mu^2/2}{M - E_\mu + |\vec{p}_\mu|\cos\theta_\mu}$$

Exp

QE-like: problem absorbed or not detected pions and…

exp: only 1μ (from the lepton vertex). But, for instance if pions are produced:

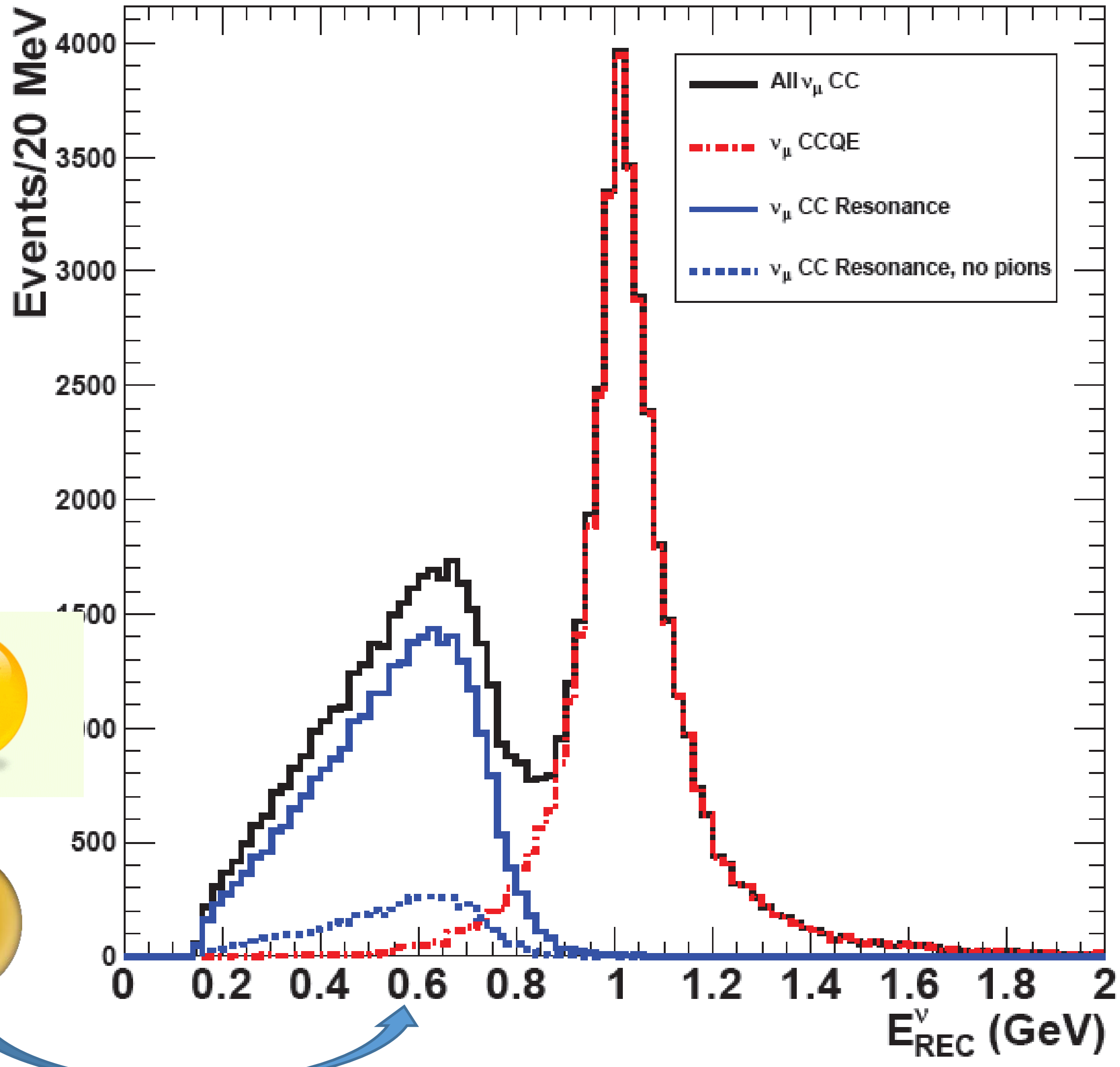
- pion decays and the extra muon is detected (2 muons in the final state)



- pion is absorbed or not detected

(MC corrected if the pion production cross section is well known…)

GENIE E_ν = 1 GeV



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Neutrino Energy Reconstruction:

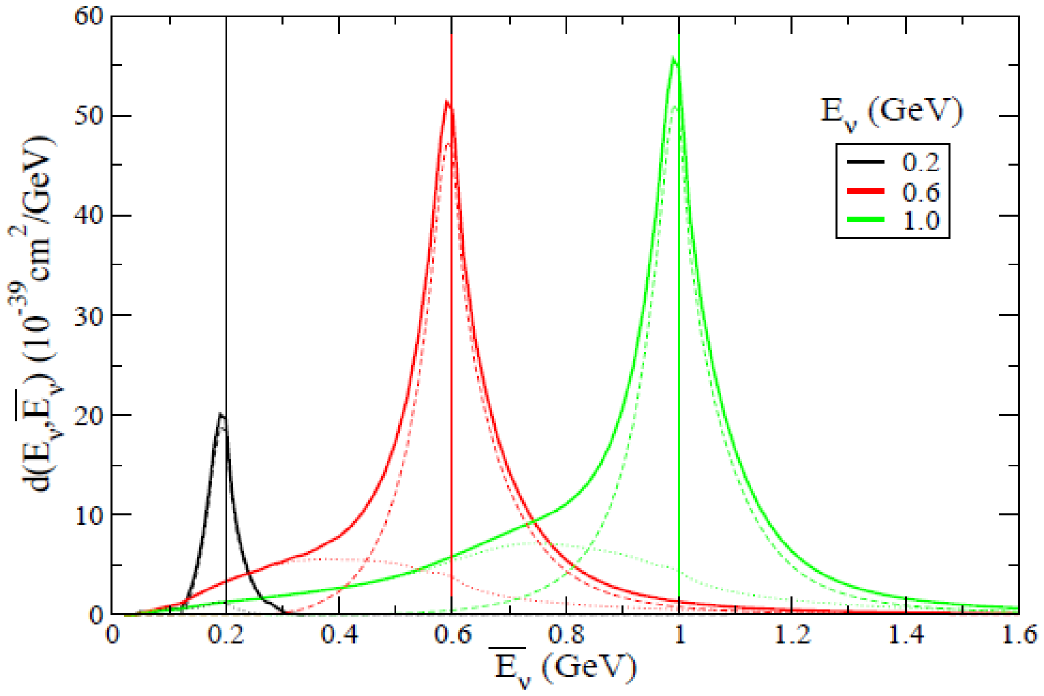
QE: $\nu_\mu + n \longrightarrow p\,\mu^-$ (bound in the nucleus)

$$E_{\rm rec} = \frac{M E_\mu - m_\mu^2/2}{M - E_\mu + |\vec{p}_\mu|\cos\theta_\mu}$$

Exp

QE-like: problem absorbed or not detected pions and 2p2h (nucl. effect)

M. Martini, M. Ericson, PRD 87 (2013)



QE Energy Reconstruction will be wrong !!

2p2h: $\nu_\mu + NN \longrightarrow N'N''\,\mu^-$

MC correct for this effect: ⟵ cross section

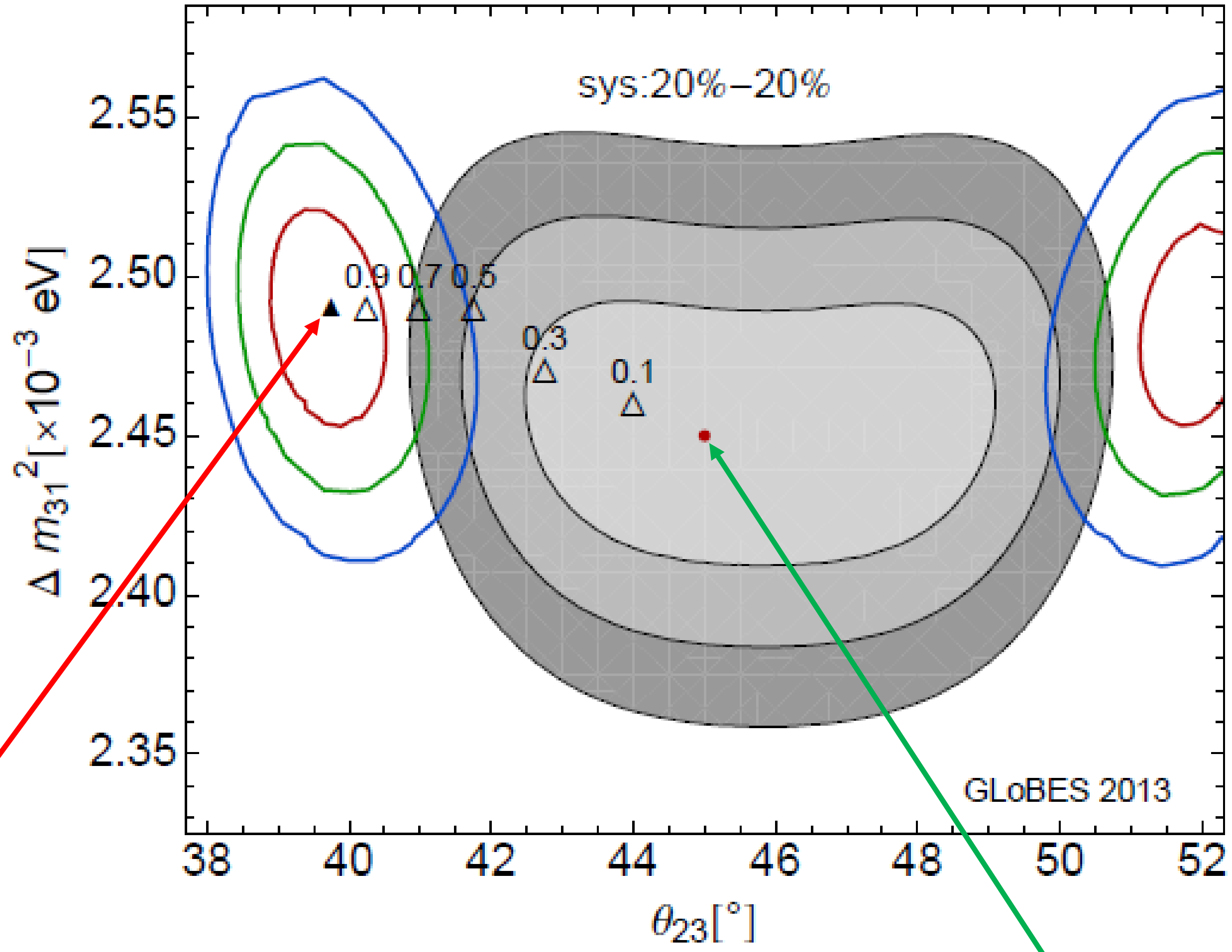
P. Coloma, P. Huber, PRL 111 (2013)

Quantitative impact in the determination of the oscillation parameters

Effects of a simple model for QE-like events ....

$$N_i^{\text{test}}(\alpha) = \alpha \times N_i^{\text{QE}} + (1-\alpha) \times N_i^{\text{QE-like}}$$

α parametrizes the fraction of two-nucleon absorption that is neglected in the fit



Reconstructed from naive QE dynamics

true

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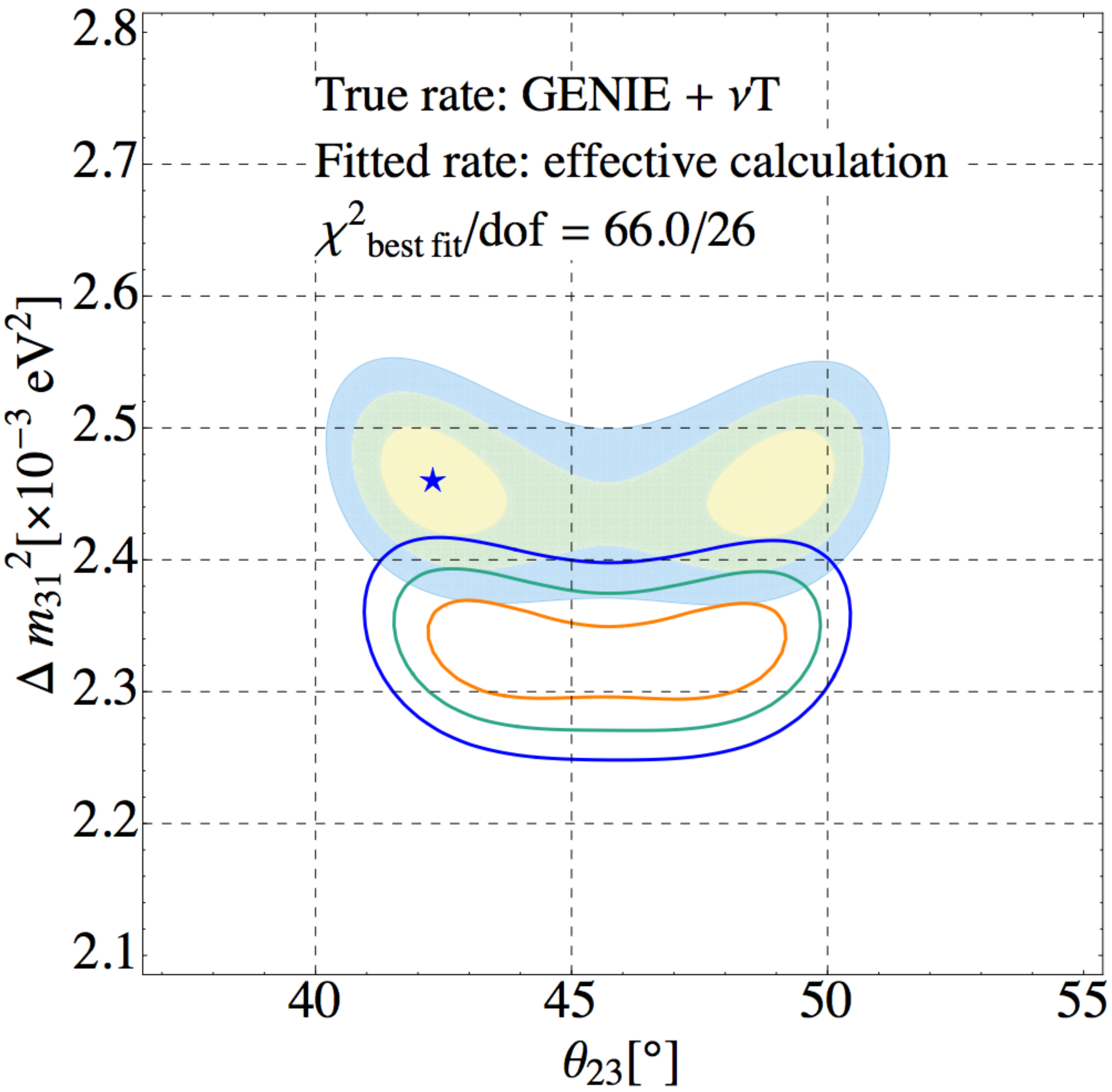
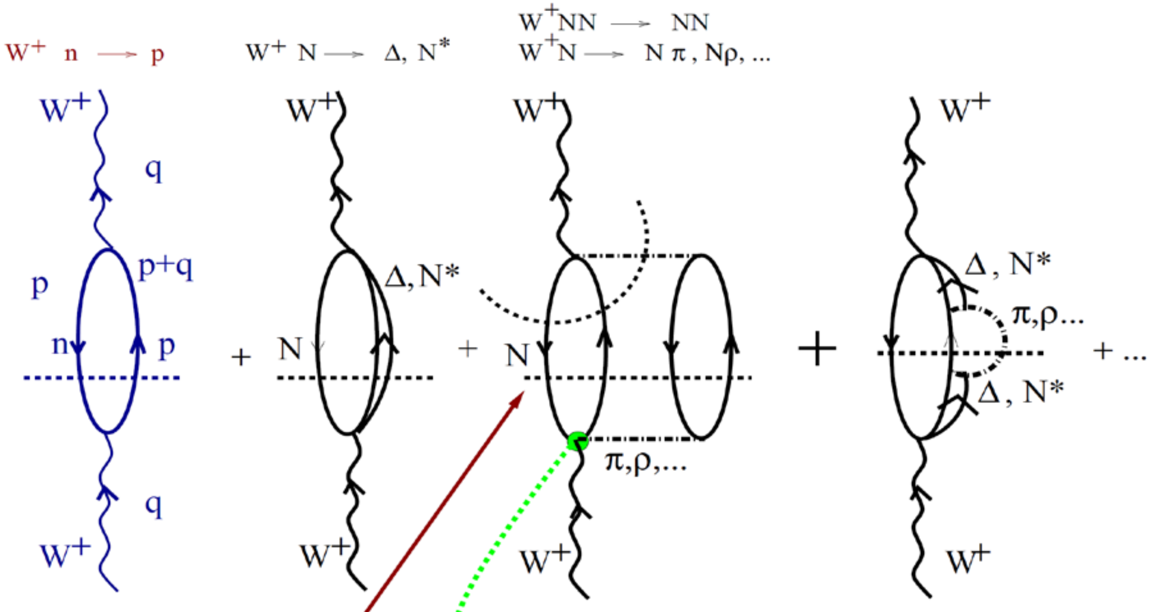
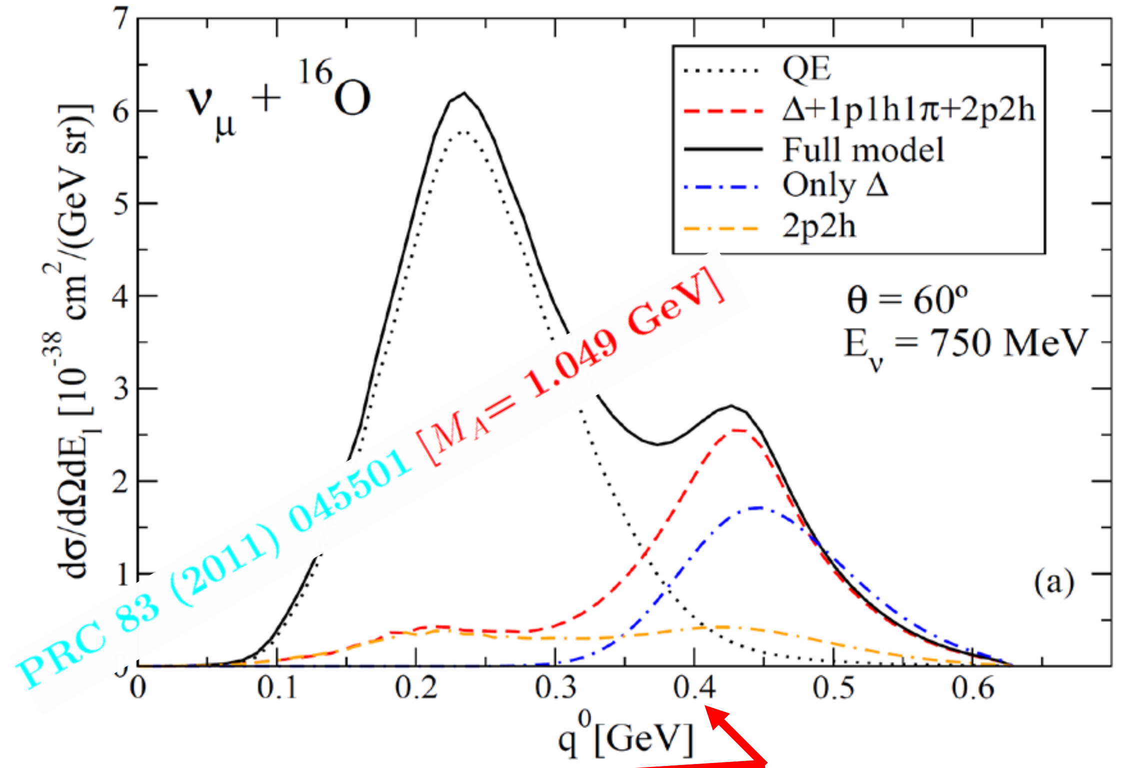


Systematic uncertainties in long-baseline neutrino-oscillation experiments,
Artur M Ankowski and Camillo Mariani,
J.Phys. G44 (2017) 054001

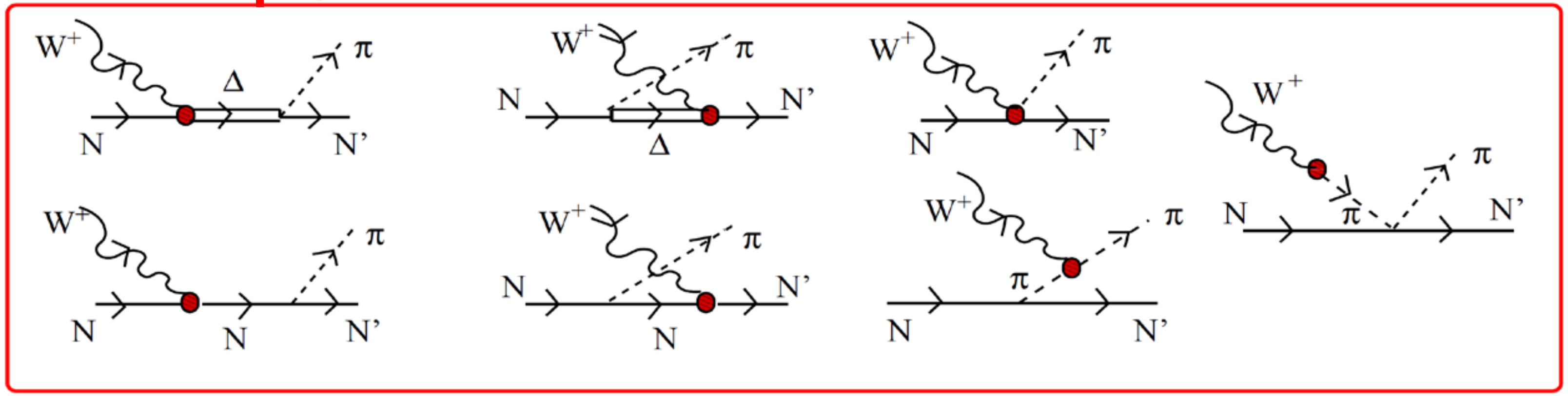
$\nu + A \to \ell + X$





MEC ⟶ QE like !

PRD D76 (2007) 033005
PRD D81 (2010) 085046 →



PRD93 (2016) 014016 (Watson's theorem)
PRD95 (2017) 053007 (1/2 dof in Δ propagator)

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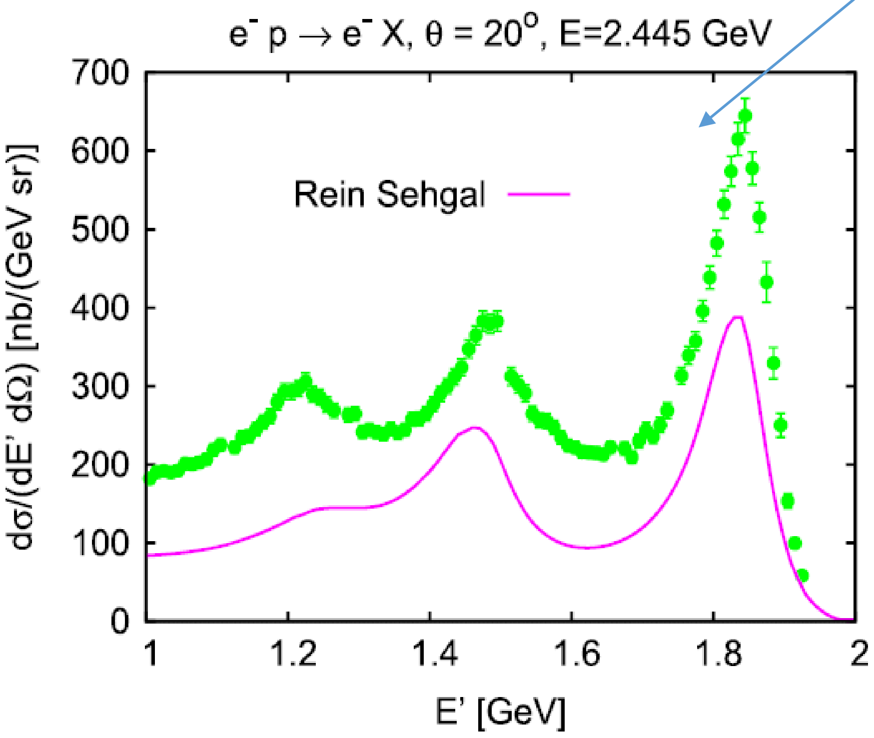
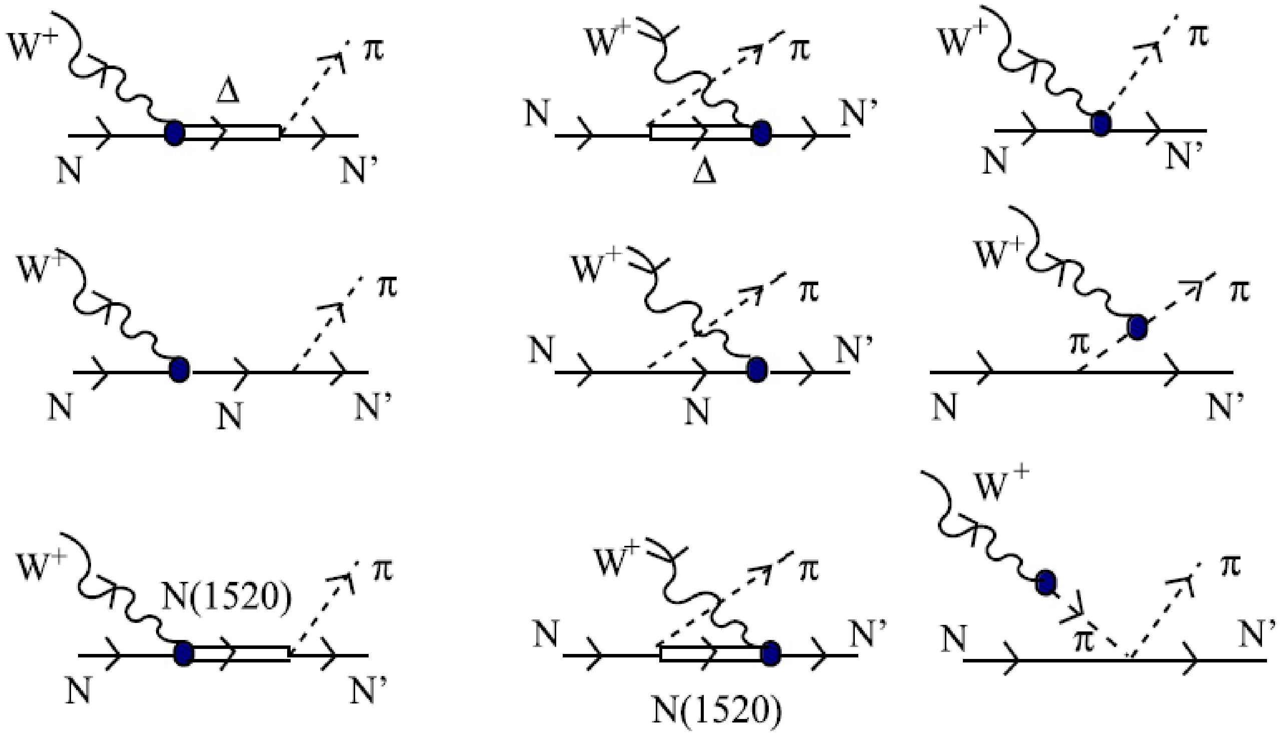
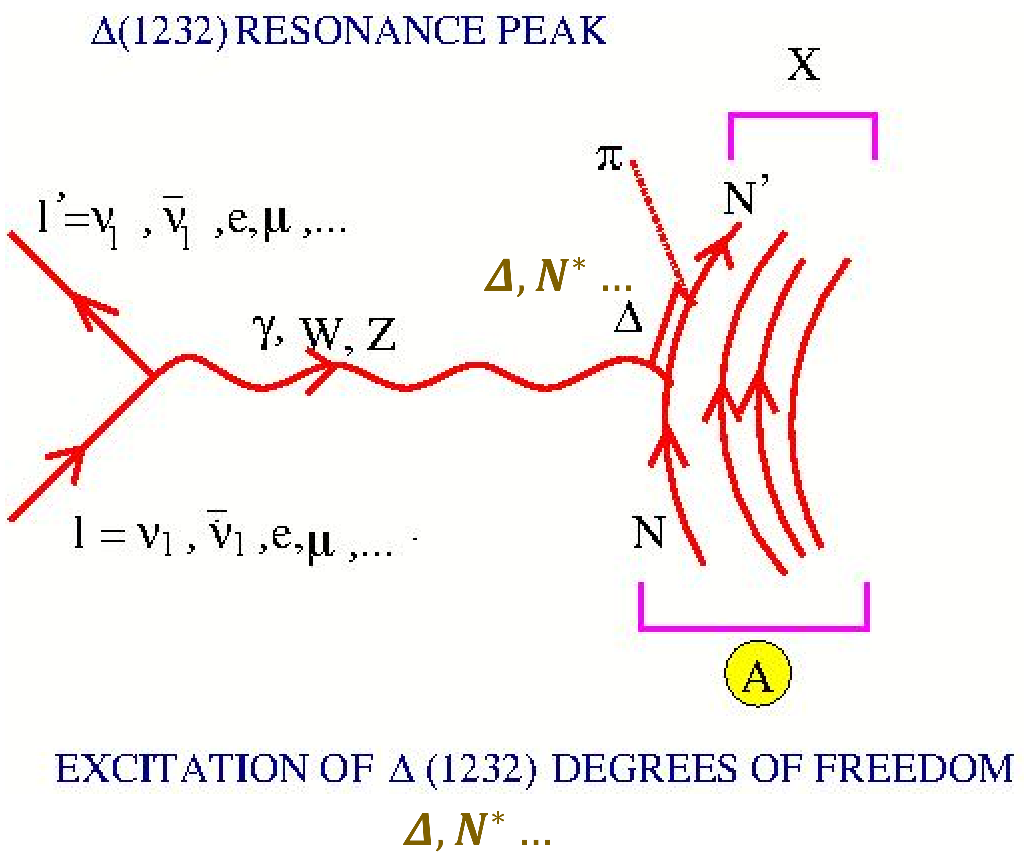
Resonance Production

Deficiencies of the Rein Sehgal model ! ⇒ Improved models





Electron data ⇒ Resonance vector form factors !
PCAC ⇒ Resonance axial form factors !
Background: chiral symmetry (when possible !)



Nuclear effects are relevant! (see talk by E. Hernández)

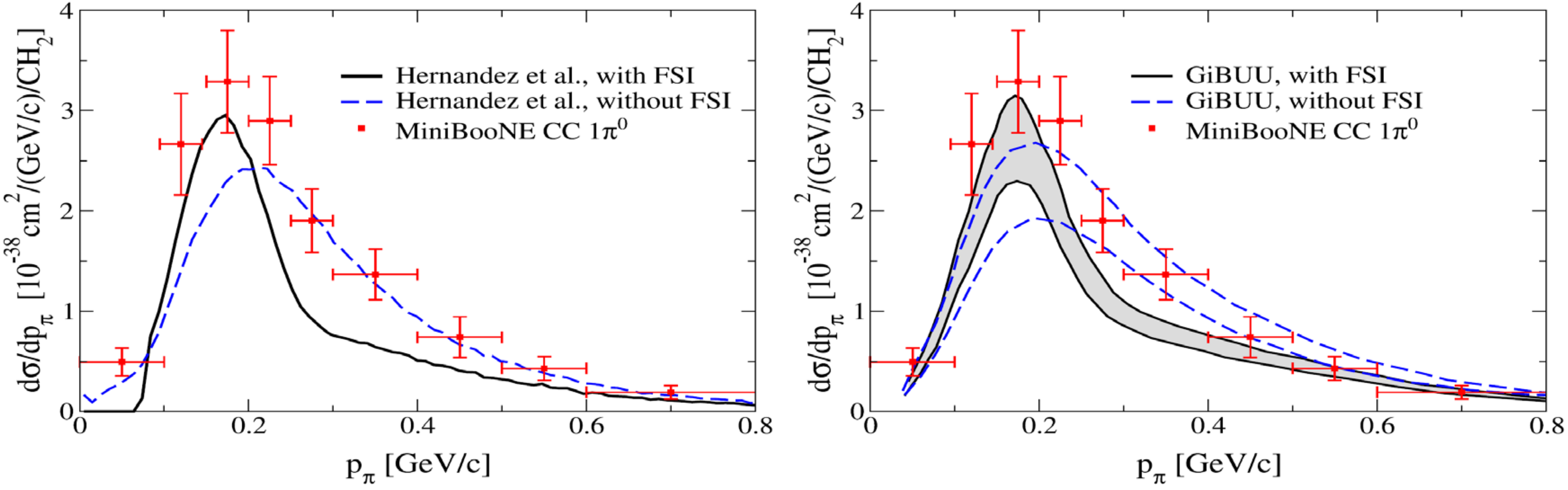


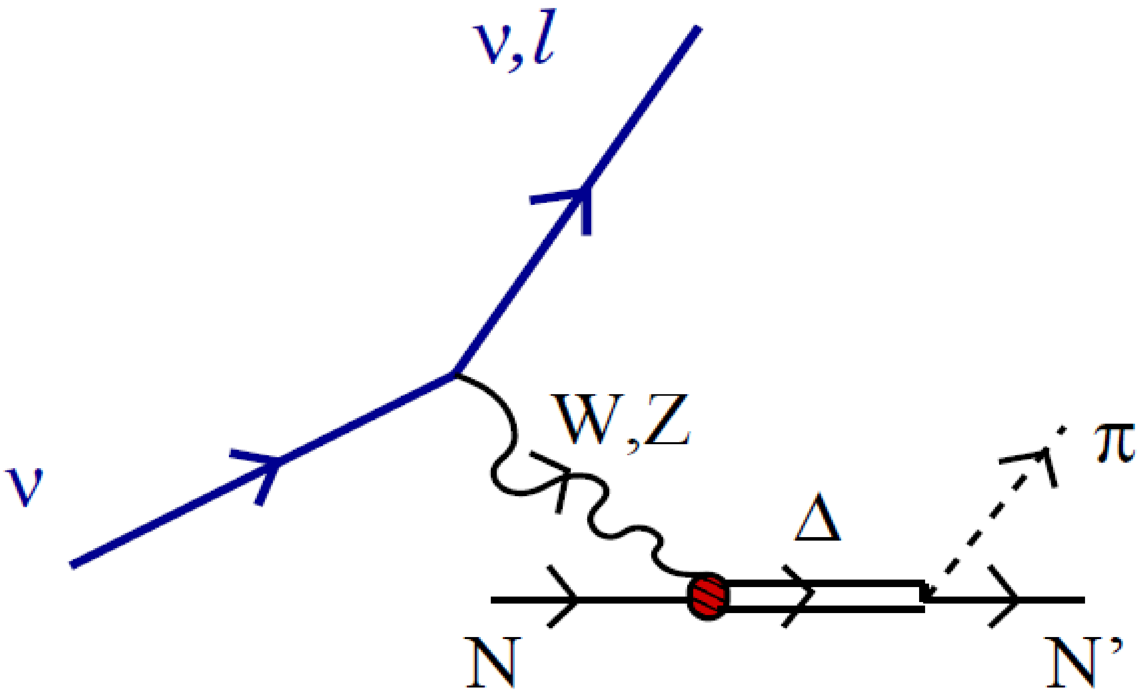
Figure 15. MiniBooNE flux-folded differential $d\sigma/dp_{\pi}$ cross section for $CC1\pi^0$ production by ν_{μ} in mineral oil. Data are from [27]. Left: predictions from the cascade approach of [184]. The solid curve corresponds to the full model and the dashed one stands for the results obtained neglecting FSI effects. Right: predictions from the GiBUU transport model of [207]. The dashed curves give the results before FSI, the solid curves those with all FSI effects included. Two different form factors $C_5^A\left(q^2\right)$, tuned to the ANL and BNL data-sets have been employed and give rise to the systematic uncertainty bands displayed in the figure.

New J.Phys. 16 (2014) 075015

There exist some discrepancies between theoretical predictions and data!

2. Llewellyn-Smith: $\Delta(1232)$ & the $\nu_l N \longrightarrow l^- N' \pi$ reaction

Theoretical Model $\nu_l N \to l N'\pi$, $\nu_l N \to \nu_l N'\pi$ **(C.H. Llewellyn Smith, 1972)**: weak excitation of the $\Delta(1232)$ resonance and its subsequent decay into $N\pi$,



$$\langle \Delta^+; p_\Delta = p + q\,| j^\mu_{cc+}(0) | n; p \rangle = \bar{u}_\alpha(\vec{p}_\Delta)\Gamma^{\alpha\mu}\,(p,q)\,u(\vec{p}\,)\cos\theta_C\,,$$

$$\Gamma^{\alpha\mu} = \left[\frac{\mathbf{C_3^A}}{M}\left(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu\right) + \frac{\mathbf{C_4^A}}{M^2}\left(g^{\alpha\mu} q\cdot p_\Delta - q^\alpha p^\mu_\Delta\right) + \mathbf{C_5^A} g^{\alpha\mu} + \frac{\mathbf{C_6^A}}{M^2} q^\mu q^\alpha\right]$$

$$+\left[\frac{\mathbf{C_3^V}}{M}\left(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu\right) + \frac{\mathbf{C_4^V}}{M^2}\left(g^{\alpha\mu} q\cdot p_\Delta - q^\alpha p^\mu_\Delta\right) + \frac{\mathbf{C_5^V}}{M^2}\left(g^{\alpha\mu} q\cdot p - q^\alpha p^\mu\right)\right.$$

$$\left. + \mathbf{C_6^V}\, g^{\mu\alpha}\right]\gamma_5, \quad \mathbf{C^A_{3,4,5,6}} \text{ axial FF's}, \ \mathbf{C^V_{3,4,5,6}} \text{ vector FF's}, \ \text{furthermore}$$

$$\mathcal{L}_{\pi N\Delta} = \frac{f^*}{m_\pi}\bar{\Psi}_\mu \vec{T}^\dagger(\partial^\mu\vec{\phi})\Psi + \text{h.c.}, \qquad f^* = 2.14$$

$$\mathbf{G}^{\mu\nu}(\mathbf{p_\Delta}) = \frac{\not{p}_\Delta + M_\Delta}{p^2_\Delta - M^2_\Delta + iM_\Delta\Gamma_\Delta}\left[-\mathbf{g}^{\mu\nu} + \frac{\mathbf{1}}{\mathbf{3}}\gamma^\mu\gamma^\nu + \frac{\mathbf{2}}{\mathbf{3}}\frac{\mathbf{p}^\mu_\Delta\mathbf{p}^\nu_\Delta}{\mathbf{M}^2_\Delta} - \frac{\mathbf{1}}{\mathbf{3}}\frac{\mathbf{p}^\mu_\Delta\gamma^\nu - \mathbf{p}^\nu_\Delta\gamma^\mu}{\mathbf{M}_\Delta}\right]$$

$eN \to e'\Delta \to e'N'\pi \Rightarrow C^V_{3,4,5,6}$ **FF's.** <u>**CVC**</u> $\Rightarrow C^V_6 = 0$ and $(M_V = 0.84$ GeV$)$

$$\frac{\mathbf{C_3^V(q^2)}}{2.13} = \frac{\mathbf{C_4^V(q^2)}}{-1.51} = \frac{1-\frac{q^2}{0.776M_V^2}}{1-\frac{q^2}{4M_V^2}}\frac{\mathbf{C_5^V(q^2)}}{0.48} = \frac{1}{(1-q^2/M_V^2)^2}\times\frac{1}{1-\frac{q^2}{4M_V^2}}$$

$C^A_{3,4,5,6}$ **Axial FF's :** Δ^{++} $(\nu_\mu p \to \mu^- p\pi^+)$ **data taken in the ANL and BNL bubble chambers (filled in with deuterium)**

<u>**Dominant form factor**</u>: $\mathbf{C_5^A(q^2)}$. $\mathbf{C_3^A(q^2)}$ and $\mathbf{C_4^A(q^2)}$ contributions are small and we have taken as (**Adler's model 1968**)

$$\mathbf{C_4^A(q^2)} = -\frac{\mathbf{C_5^A(q^2)}}{4}, \quad \mathbf{C_3^A(q^2)} = 0$$

PCAC ($\partial_\mu A^\mu \propto m_\pi^2$) and Goldberger–Treiman

$$C_5^A(0) \sim \sqrt{\frac{2}{3}}\frac{f_\pi}{m_\pi} f^* = 1.2$$

$$\mathbf{C_5^A(q^2)} = \frac{\mathbf{1.2}}{(1-q^2/\mathbf{M_{A\Delta}^2})^2} \times \frac{1}{1-\frac{q^2}{3\mathbf{M_{A\Delta}^2}}}, \quad \underbrace{\mathbf{C_6^A(q^2)} = \mathbf{C_5^A(q^2)}\frac{M^2}{m_\pi^2 - q^2}}_{\text{PCAC}}$$

$M_{A\Delta}$ **fitted to the q^2 dependence of the $\nu_\mu p \to \mu^- p \pi^+$** cross section (neutrino energy averaged) **with ($M(\pi N) < 1.4$ GeV) measured at ANL and BNL.** It varies in the range 0.95 GeV (ANL) – 1.28 GeV (BNL).

E. Paschos, J-Y. Yu and M. Sakuda (PRD69, 014013 (2004)),

$$\mathbf{M_{A\Delta} \sim 1.05\ GeV}$$

ANL

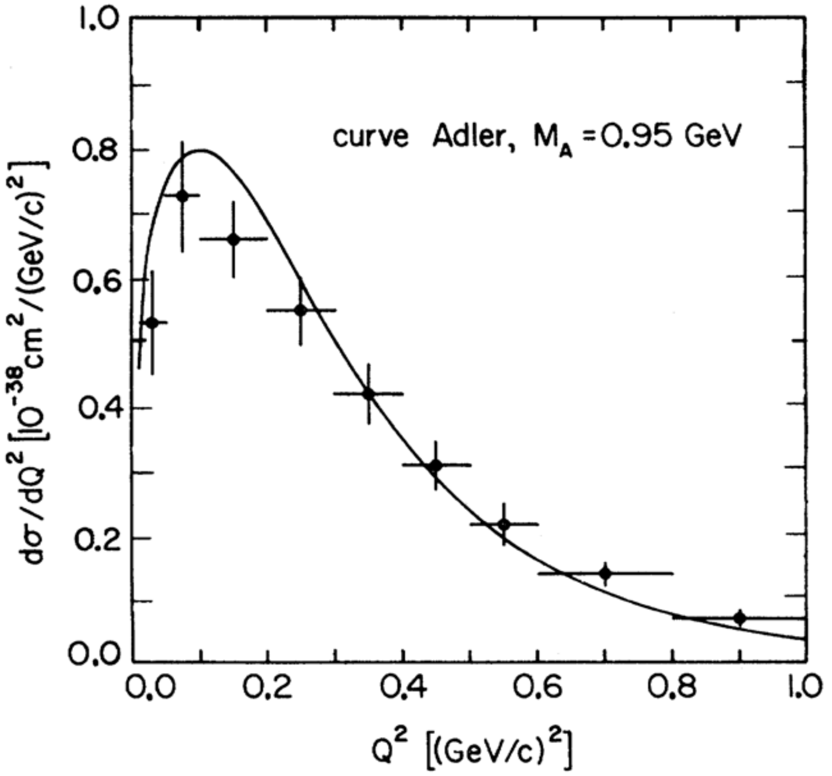


FIG. 12. Differential cross section $d\sigma/dQ^2$ evaluated with the selections $0.5 \le E_\nu < 6.0$ GeV and $M(p\pi^+) < 1.4$ GeV. The curve is the flux-averaged prediction of the Adler model with the dipole form factor and $M_A = 0.95$ GeV.

only Δ provides already a good description of data!

BNL: QE and Δ^{++}

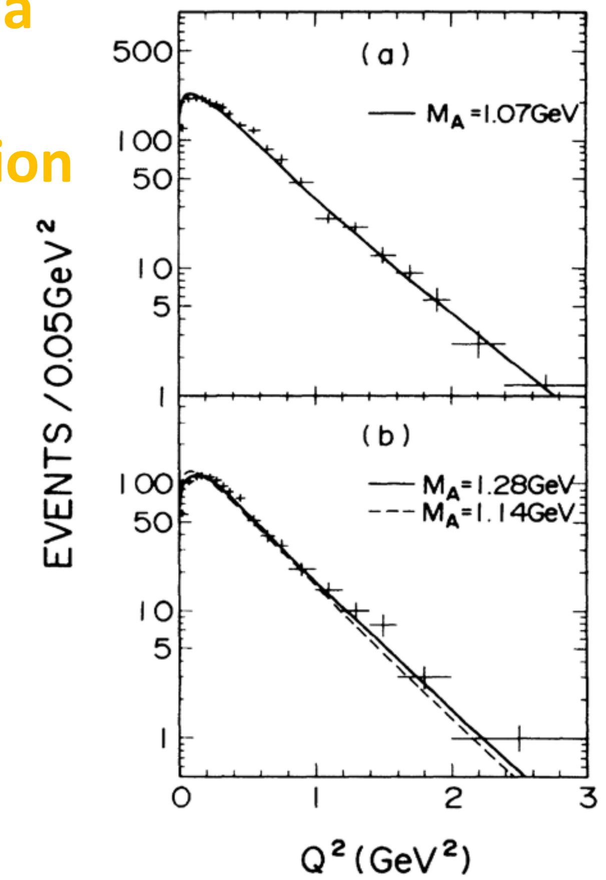
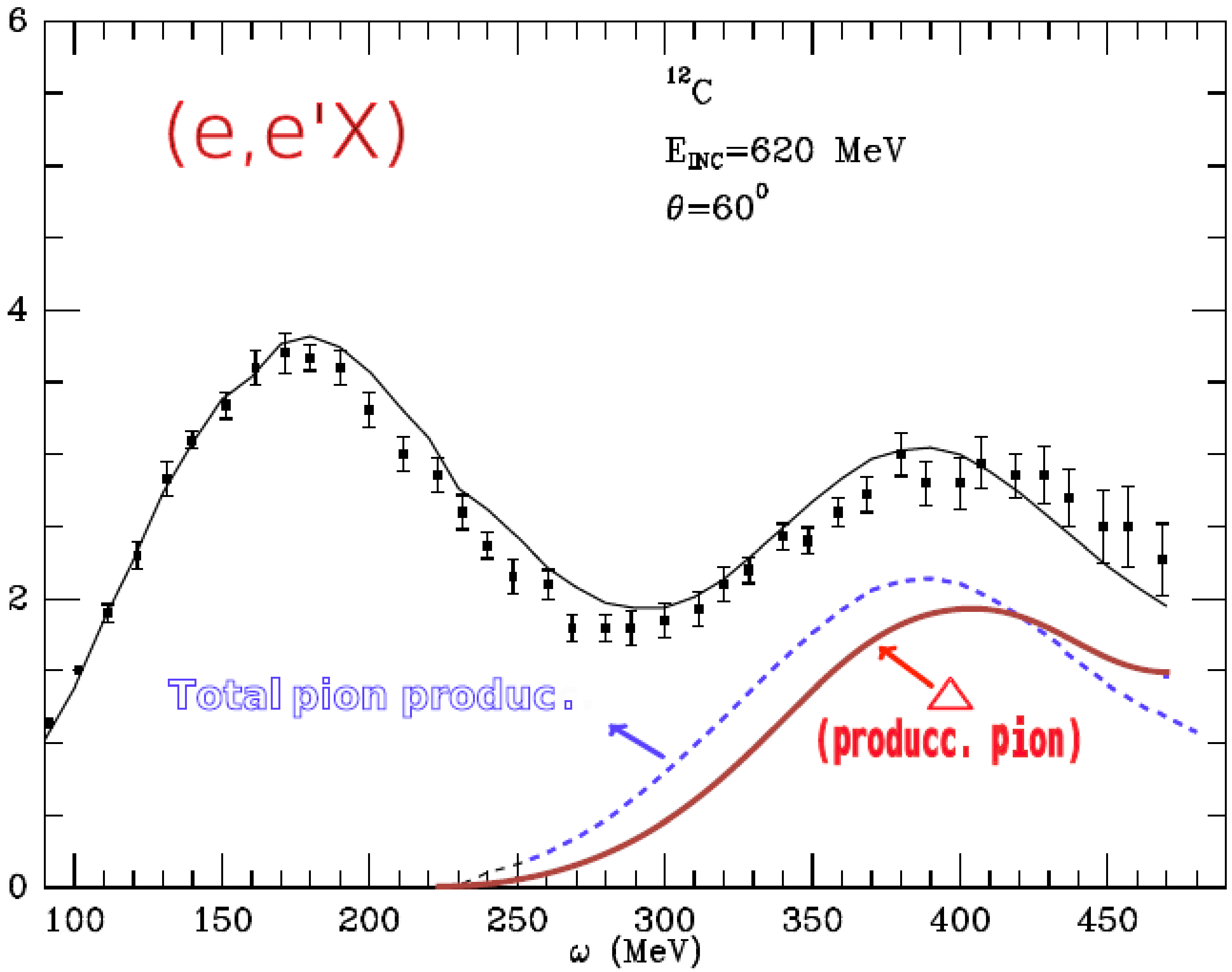


FIG. 5. The Q^2 distribution for (a) the quasielastic and (b) the Δ^{++} production reactions. The curves are the theoretical predictions obtained from least-squares fits with the fitted M_A values for the $Q^2 < 3.0$ (GeV/c)^2.



... but **only the Δ pole** contribution turns out to be **an** **insufficient** model, even at the Δ peak, and **specially close to pion threshold.** Close to pion threshold, the pion from the $(\nu_\mu, \mu\pi)$ **reaction will not radiate Čerenkov light** and thus it would be necessary an improved theoretical model to carry out a proper L/E **oscillation analysis.**

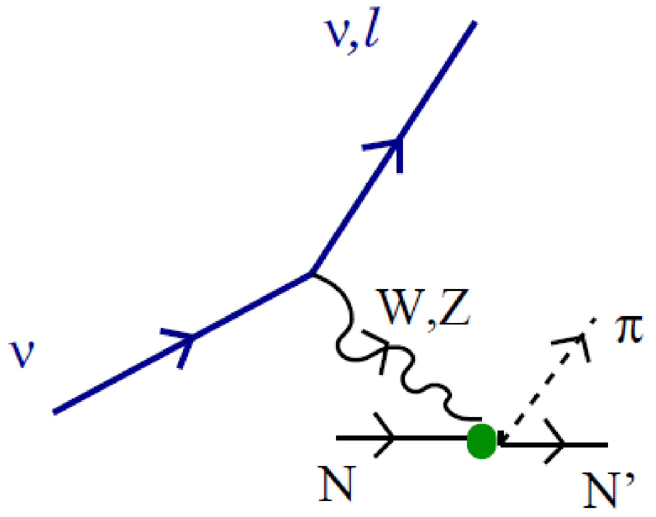
Such model for the $\nu_l N \to l N' \pi$, $\nu_l N \to \nu_l N' \pi$ should include **non resonant** terms $\Rightarrow$ **Realization of the axial and vector currents, which couple to the W, Z^0 bosons, for a system of pions and nucleons.**

3. Chiral symmetry and non-resonant contributions

PRD76 (2007) 033005

Non-linear σ−Model: EFT involving pions and nucleons which implements spontaneous chiral symmetry breaking.

If $\Psi_q = \begin{pmatrix} \Psi_u \\ \Psi_d \end{pmatrix}$, the CC and NC, which induce $W(Z^0)N \to N'\pi$



$$j^\mu_{cc\pm} = \cos\theta_C \bar{\Psi}_q \gamma^\mu (1-\gamma_5)\left(-\frac{\tau^1_{\pm 1}}{\sqrt{2}}\right)\Psi_q$$

$$j^\mu_{nc} = \bar{\Psi}_q \gamma^\mu (1 - 2\sin^2\theta_W - \gamma_5)\, \tau^1_0\, \Psi_q$$

$$- \quad 4\sin^2\theta_W \, s^\mu_{em,IS} - \bar{\Psi}_s \gamma^\mu (1-\gamma_5)\Psi_s$$

$$s^\mu_{em} = \underbrace{\frac{1}{6}\bar{\Psi}_q\gamma^\mu\Psi_q - \frac{1}{3}\bar{\Psi}_s\gamma^\mu\Psi_s}_{s^\mu_{em,IS}} + \frac{1}{\sqrt{2}}\bar{\Psi}_q\gamma^\mu \frac{\tau^1_0}{\sqrt{2}}\Psi_q$$

$$\langle N'\pi | j^\mu_{cc+}(0), j^\mu_{cc-}(0), j^\mu_{nc}(0) | N\rangle = ? \Leftarrow$$ QCD and its pattern of SχSB

Two flavor, u and d with mass m, QCD Lagrangian

$$\mathcal{L}_{QCD} = \bar{\Psi}_q(\mathrm{i}\not{D} - m)\Psi_q + \frac{1}{2g^2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu})$$

with $D^\mu = \partial^\mu - B^\mu$, $F^{\mu\nu} = -[D^\mu, D^\nu]$, $B^\mu = igT^a B^\mu_a$, **matrices in the colour space.** **Chiral symmetry** $\Rightarrow$

$$\Psi_q \to \Psi'_q = e^{-\mathrm{i}\vec{\theta}_V\cdot\vec{\tau}/2}\,\Psi_q, \quad \text{isospin rotation}$$

$$\Psi_q \to \Psi'_q = e^{-\mathrm{i}\vec{\theta}_A\cdot\vec{\tau}\gamma_5/2}\,\Psi_q, \quad \text{axial} - \text{flavor rotation}$$

$\delta\mathcal{L}_{QCD} \propto m$.... Currents **(Noether)**

$$\vec{V}^\mu = \bar{\Psi}_q\gamma^\mu\frac{\vec{\tau}}{2}\Psi_q, \qquad \partial_\mu\vec{V}^\mu = 0$$

$$\vec{A}^\mu = \bar{\Psi}_q\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi_q, \qquad \partial_\mu\vec{A}^\mu = \mathbf{m}\bar{\Psi}_q\mathrm{i}\gamma_5\vec{\tau}\Psi_q \neq 0$$

Charges

$$\vec{\mathbf{Q}}(\mathbf{t}) = \int_{R^3} d^3x\vec{V}^0(\vec{x},t), \quad \vec{\mathbf{Q}}_5(\mathbf{t}) = \int_{R^3} d^3x\vec{A}^0(\vec{x},t)$$

$\vec{Q}$ (isospin) and $\vec{Q}_5$ (neglecting m) indep. of $\mathbf{t}$ $\Rightarrow$ **conserved!!**

$$[Q^i, Q^j] = \mathrm{i}\epsilon^{ijk}Q^k, \quad [Q^i, Q^j_5] = \mathrm{i}\epsilon^{ijk}Q^k_5, \quad [Q^i_5, Q^j_5] = \mathrm{i}\epsilon^{ijk}Q^k$$

SχSB: $\vec{Q}|0\rangle = 0$ but $\vec{Q}_5|0\rangle \neq 0 \Rightarrow \pi's$ **Isotriplet Goldstone bosons** from **spontaneous chiral symmetry breaking.**

Non-linear σ–model ⇒ EFT involving pions and nucleons which implements chiral symmetry and its pattern of spontaneous breaking.

If $\Psi = \begin{pmatrix} p \\ n \end{pmatrix}$, $U = \frac{f_\pi}{\sqrt{2}} e^{\mathrm{i}\,\vec{\tau}\cdot \boxed{\vec{\phi}}/f_\pi} = \frac{f_\pi}{\sqrt{2}}\xi^2$, with $f_\pi \sim 93$ MeV,

$$\begin{aligned}\mathcal{L}_{N\pi} &= \bar{\Psi}\mathrm{i}\gamma^\mu \left[\partial_\mu + \mathcal{V}_\mu\right]\Psi - M\bar{\Psi}\Psi + g_A\bar{\Psi}\gamma^\mu\gamma_5\mathcal{A}_\mu\Psi \\ &+ \frac{1}{2}\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] \boxed{+m_\pi^2 \frac{f_\pi}{2\sqrt{2}}\mathrm{Tr}(U + U^\dagger - \sqrt{2}f_\pi)}\end{aligned}$$

$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right) \qquad \mathcal{A}_\mu = \frac{\mathrm{i}}{2}\left(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi\right)$$

Isospin rotat. $\xi \to \mathbf{T_V}\,\xi\,\mathbf{T_V^\dagger}$, $\Psi \to \mathbf{T_V}\,\Psi$, $\quad T_V = e^{-\mathrm{i}\,\frac{\vec{\tau}\cdot\vec{\theta}_V}{2}}$

Axial rotat. $\xi \to \mathbf{T_A^\dagger}\,\xi\,\mathbf{T_\Lambda^\dagger} = \mathbf{T_\Lambda}\,\xi\,\mathbf{T_A^\dagger}$, $\Psi \to \mathbf{T_\Lambda}\,\Psi$, $\quad T_{\Lambda,A} = e^{-\mathrm{i}\,\frac{\vec{\tau}\cdot\vec{\theta}_{\Lambda,A}}{2}}$

Isospin rotat. $\Rightarrow \delta\mathcal{L}_{\mathrm{N}\pi} = \mathbf{0}$, **Axial rotat.** $\Rightarrow \delta\mathcal{L}_{\mathrm{N}\pi} \boxed{\propto m_\pi^2 \neq 0}$

Up to order $\mathcal{O}(1/f_\pi^4)$, $\mathcal{L}_{N\pi}$ reads,

$$\mathcal{L}_{\mathbf{N}\pi} = \bar{\Psi}[i\partial\!\!\!/ - M]\Psi + \frac{1}{2}\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - \frac{1}{2}m_\pi^2\vec{\phi}^2 \quad \text{(kinetic)} +$$

$$\frac{\mathbf{g_A}}{\mathbf{f}_\pi}\bar{\mathbf{\Psi}}\gamma^\mu\gamma_5\frac{\vec{\tau}}{\mathbf{2}}(\partial_\mu\vec{\phi})\mathbf{\Psi} - \frac{\mathbf{1}}{\mathbf{4f}_\pi^\mathbf{2}}\bar{\mathbf{\Psi}}\gamma_\mu\vec{\tau}\left(\vec{\phi}\times\partial^\mu\vec{\phi}\right)\mathbf{\Psi} - \frac{\mathbf{g_A}}{\mathbf{6f}_\pi^\mathbf{3}}\bar{\mathbf{\Psi}}\gamma^\mu\gamma_5\left[\vec{\phi}^\mathbf{2}\frac{\vec{\tau}}{\mathbf{2}}\partial_\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})\frac{\vec{\tau}}{\mathbf{2}}\vec{\phi}\right]\mathbf{\Psi}$$

$$-\frac{\mathbf{1}}{\mathbf{6f}_\pi^\mathbf{2}}\left(\vec{\phi}^\mathbf{2}\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})(\vec{\phi}\partial^\mu\vec{\phi})\right) + \frac{\mathbf{m}_\pi^\mathbf{2}}{\mathbf{24f}_\pi^\mathbf{2}}(\vec{\phi}^\mathbf{2})^\mathbf{2} + \mathcal{O}(1/f_\pi^4)$$

Contact interactions $NN\pi$, $\underbrace{NN\pi\pi}_{\text{WT}}$, $NN\pi\pi\pi$ and $\pi\pi\pi\pi$.

Parameters: f_π and g_A . **Noether's currents**

$$j^\mu = \frac{\partial\mathcal{L}_{N\pi}}{\partial(\partial_\mu\varphi_a)}\delta\varphi_a, \quad a = 1, 2, \cdots$$

up to order $\mathcal{O}(1/f_\pi^3)$ ...

$$\vec{\mathbf{V}}^\mu = \vec\phi\times\partial^\mu\vec\phi + \frac{g_A}{2f_\pi}\bar\Psi\gamma^\mu\gamma_5(\vec\phi\times\vec\tau)\Psi + \bar\Psi\gamma^\mu\frac{\vec\tau}{2}\Psi - \frac{1}{4f_\pi^2}\bar\Psi\gamma^\mu\left[\vec\tau\vec\phi^{\,2} - \vec\phi(\vec\tau\cdot\vec\phi)\right]\Psi$$

$$- \frac{\vec\phi^{\,2}}{3f_\pi^2}(\vec\phi\times\partial^\mu\vec\phi) + \mathcal{O}(\frac{1}{f_\pi^3}), \quad \partial_\mu\vec{\mathbf{V}}^\mu = \mathbf{0}$$

$$\vec{\mathbf{A}}^\mu = f_\pi\partial^\mu\vec\phi + \frac{1}{2f_\pi}\bar\Psi\gamma^\mu(\vec\phi\times\vec\tau)\Psi + g_A\bar\Psi\gamma^\mu\gamma_5\frac{\vec\tau}{2}\Psi + \frac{2}{3f_\pi}\left[\vec\phi(\vec\phi\cdot\partial^\mu\vec\phi) - \vec\phi^{\,2}\partial^\mu\vec\phi\right]$$

$$- \frac{g_A}{4f_\pi^2}\bar\Psi\gamma^\mu\gamma_5\left[\vec\tau\vec\phi^{\,2} - \vec\phi(\vec\tau\cdot\vec\phi)\right]\Psi + \mathcal{O}(\frac{1}{f_\pi^3}), \quad \underbrace{\partial_\mu\vec{A}^\mu \propto m_\pi^2\ldots}_{\text{PCAC}}$$

+ isospin relations ⇒ evaluate **CC** $\langle N'\pi|\, j^\mu_{cc+}(0), j^\mu_{cc-}(0)|N\rangle$

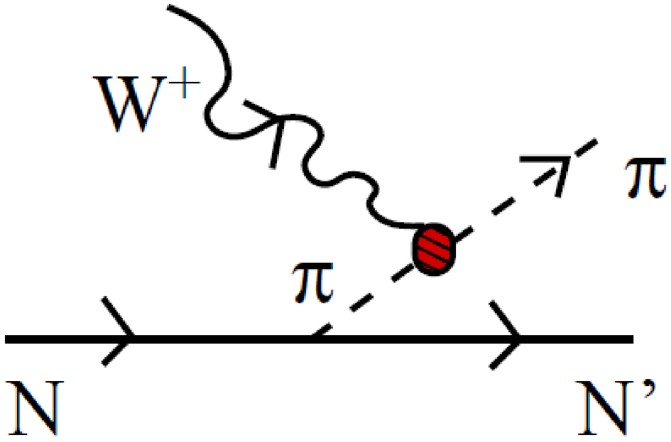
$$\langle p\pi^0|j^\mu_{cc+}(0)|n\rangle = -\frac{1}{\sqrt2}\left[\langle \mathbf{p}\pi^+|\mathbf{j}^\mu_{cc+}(\mathbf{0})|\mathbf{p}\rangle - \langle \mathbf{n}\pi^+|\mathbf{j}^\mu_{cc+}(\mathbf{0})|\mathbf{n}\rangle\right]$$

$$\langle p\pi^-|j^\mu_{cc-}(0)|p\rangle = \langle \mathbf{n}\pi^+|\mathbf{j}^\mu_{cc+}(\mathbf{0})|\mathbf{n}\rangle$$

$$\langle n\pi^-|j^\mu_{cc-}(0)|n\rangle = \langle \mathbf{p}\pi^+|\mathbf{j}^\mu_{cc+}(\mathbf{0})|\mathbf{p}\rangle$$

$$\langle n\pi^0|j^\mu_{cc-}(0)|p\rangle = -\langle p\pi^0|j^\mu_{cc+}(0)|n\rangle = \frac{1}{\sqrt2}\left[\langle \mathbf{p}\pi^+|\mathbf{j}^\mu_{cc+}(\mathbf{0})|\mathbf{p}\rangle - \langle \mathbf{n}\pi^+|\mathbf{j}^\mu_{cc+}(\mathbf{0})|\mathbf{n}\rangle\right]$$

$$\vec{\mathbf{V}}^\mu = \vec{\phi} \times \partial^\mu \vec{\phi} + \frac{g_A}{2f_\pi}\bar{\Psi}\gamma^\mu\gamma_5(\vec{\phi}\times\vec{\tau})\Psi + \bar{\Psi}\gamma^\mu\frac{\vec{\tau}}{2}\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma^\mu\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi$$
$$- \frac{\vec{\phi}^{\,2}}{3f_\pi^2}(\vec{\phi}\times\partial^\mu\vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3})$$
$$\vec{A}^\mu = f_\pi\partial^\mu\vec{\phi} + \frac{1}{2f_\pi}\bar{\Psi}\gamma^\mu(\vec{\phi}\times\vec{\tau})\Psi + g_A\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi + \frac{2}{3f_\pi}\left[\vec{\phi}(\vec{\phi}\cdot\partial^\mu\vec{\phi}) - \vec{\phi}^{\,2}\partial^\mu\vec{\phi}\right]$$
$$- \frac{g_A}{4f_\pi^2}\bar{\Psi}\gamma^\mu\gamma_5\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi + \mathcal{O}(\frac{1}{f_\pi^3})$$

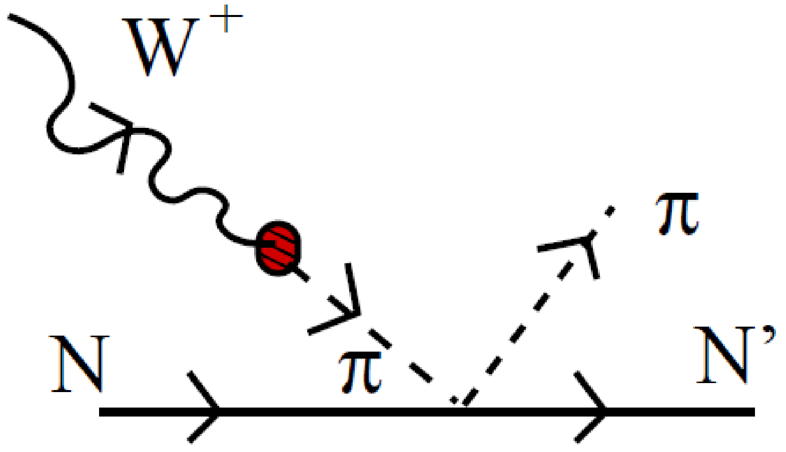


$$j^\mu_{cc+}\Big|_{PF} = \mp\, \mathrm{i}\, \mathbf{F_{PF}(q^2)}\frac{\sqrt{2}Mg_A}{f_\pi}\cos\theta_C\frac{(2k_\pi - q)^\mu}{(k_\pi - q)^2 - m_\pi^2}\bar{u}(\vec{p}\,')\gamma_5 u(\vec{p})$$

$$(- \Rightarrow W^+p \to p\pi^+,\ + \Rightarrow W^+n \to n\pi^+)$$

$$\vec{V}^\mu = \vec{\phi}\times\partial^\mu\vec{\phi} + \frac{g_A}{2f_\pi}\bar{\Psi}\gamma^\mu\gamma_5(\vec{\phi}\times\vec{\tau})\Psi + \bar{\Psi}\gamma^\mu\frac{\vec{\tau}}{2}\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma^\mu\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi - \frac{\vec{\phi}^{\,2}}{3f_\pi^2}(\vec{\phi}\times\partial^\mu\vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3})$$

$$\vec{\mathbf{A}}^\mu = \mathbf{f}_\pi\partial^\mu\vec{\phi} + \frac{1}{2f_\pi}\bar{\Psi}\gamma^\mu(\vec{\phi}\times\vec{\tau})\Psi + g_A\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi + \frac{2}{3f_\pi}\left[\vec{\phi}(\vec{\phi}\cdot\partial^\mu\vec{\phi}) - \vec{\phi}^{\,2}\partial^\mu\vec{\phi}\right] - \frac{g_A}{4f_\pi^2}\bar{\Psi}\gamma^\mu\gamma_5\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi + \mathcal{O}(\frac{1}{f_\pi^3})$$



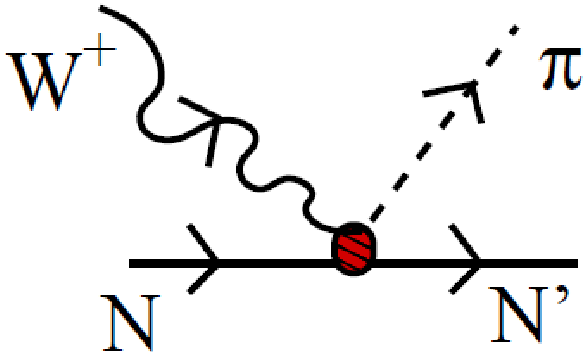
$$j^\mu_{cc+}\Big|_{PP} = \mp\, \mathrm{i}\, \mathbf{F}_\rho\left((\mathbf{q}-\mathbf{k}_\pi)^2\right)\frac{\cos\theta_C}{\sqrt{2}f_\pi}\frac{q^\mu}{q^2-m_\pi^2}\bar{u}(\vec{p}\,')\,\not{q}\,u(\vec{p})$$

$$(- \Rightarrow W^+p\to p\pi^+,\; + \Rightarrow W^+n\to n\pi^+ \;)$$

$$\mathbf{F}_\rho(\mathbf{t}) = \frac{\mathbf{1}}{\mathbf{1}-\mathbf{t}/\mathbf{m}_\rho^2}$$

$$\vec{\mathbf{V}}^\mu = \vec\phi \times \partial^\mu \vec\phi + \frac{\mathbf{g_A}}{\mathbf{2f_\pi}} \bar{\boldsymbol\Psi}\gamma^\mu\gamma_5(\vec\phi\times\vec\tau)\boldsymbol\Psi + \bar\Psi\gamma^\mu\frac{\vec\tau}{2}\Psi - \frac{1}{4f_\pi^2}\bar\Psi\gamma^\mu\left[\vec\tau\vec\phi^{\,2} - \vec\phi(\vec\tau\cdot\vec\phi)\right]\Psi$$
$$- \frac{\vec\phi^{\,2}}{3f_\pi^2}(\vec\phi\times\partial^\mu\vec\phi) + \mathcal{O}(\frac{1}{f_\pi^3})$$

$$\vec{\mathbf{A}}^\mu = f_\pi\partial^\mu\vec\phi + \frac{\mathbf{1}}{\mathbf{2f_\pi}}\bar{\boldsymbol\Psi}\gamma^\mu(\vec\phi\times\vec\tau)\boldsymbol\Psi + g_A\bar\Psi\gamma^\mu\gamma_5\frac{\vec\tau}{2}\Psi + \frac{2}{3f_\pi}\left[\vec\phi(\vec\phi\cdot\partial^\mu\vec\phi) - \vec\phi^{\,2}\partial^\mu\vec\phi\right]$$
$$- \frac{g_A}{4f_\pi^2}\bar\Psi\gamma^\mu\gamma_5\left[\vec\tau\vec\phi^{\,2} - \vec\phi(\vec\tau\cdot\vec\phi)\right]\Psi + \mathcal{O}(\frac{1}{f_\pi^3})$$

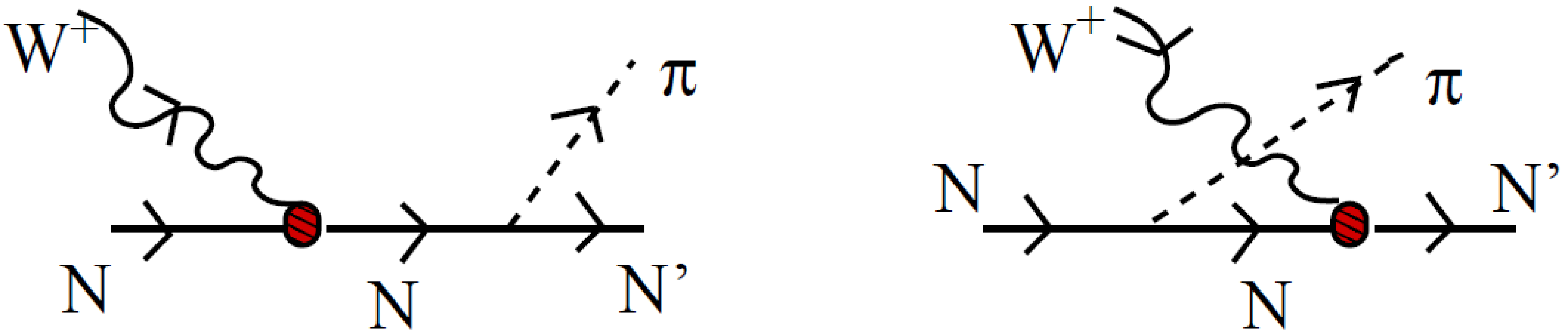


$$j^\mu_{cc+}\Big|_{CT} = \mp\, \mathrm{i}\,\frac{\cos\theta_C}{\sqrt{2}f_\pi}\bar u(\vec p\,')\gamma^\mu\left(g_A\mathbf{F^V_{CT}}(\mathbf{q^2})\gamma_5 - \mathbf{F}_\rho\left((\mathbf{q}-\mathbf{k}_\pi)^2\right)\right)u(\vec p)$$

$$(\,- \Rightarrow W^+p\to p\pi^+,\; + \Rightarrow W^+n \to n\pi^+\,)$$

$$\vec{\mathbf{V}}^\mu = \vec{\phi}\times\partial^\mu\vec{\phi} + \frac{g_A}{2f_\pi}\bar{\Psi}\gamma^\mu\gamma_5(\vec{\phi}\times\vec{\tau})\Psi + \bar{\mathbf{\Psi}}\gamma^\mu\frac{\vec{\tau}}{\mathbf{2}}\mathbf{\Psi} - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma^\mu\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi$$
$$- \frac{\vec{\phi}^{\,2}}{3f_\pi^2}(\vec{\phi}\times\partial^\mu\vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3})$$

$$\vec{\mathbf{A}}^\mu = f_\pi\partial^\mu\vec{\phi} + \frac{1}{2f_\pi}\bar{\Psi}\gamma^\mu(\vec{\phi}\times\vec{\tau})\Psi + \mathbf{g_A}\bar{\mathbf{\Psi}}\gamma^\mu\gamma_5\frac{\vec{\tau}}{\mathbf{2}}\mathbf{\Psi} + \frac{2}{3f_\pi}\left[\vec{\phi}(\vec{\phi}\cdot\partial^\mu\vec{\phi}) - \vec{\phi}^{\,2}\partial^\mu\vec{\phi}\right]$$
$$- \frac{g_A}{4f_\pi^2}\bar{\Psi}\gamma^\mu\gamma_5\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi + \mathcal{O}(\frac{1}{f_\pi^3})$$



... improve the WNN transition vertex

$$\langle p;\vec{p}\,' = \vec{p}+\vec{q}\,|\mathbf{j}^{\alpha}_{\mathbf{cc+}}(\mathbf{0})|n;\vec{p}\,\rangle = \cos\theta_C\,\bar{u}(\vec{p}\,')(\mathbf{V}^{\alpha}_{\mathbf{N}}(\mathbf{q})-\mathbf{A}^{\alpha}_{\mathbf{N}}(\mathbf{q}))u(\vec{p}\,)$$

$$\mathbf{V}^{\alpha}_{\mathbf{N}}(\mathbf{q}) = 2\times\left(\mathbf{F}^{\mathbf{V}}_{\mathbf{1}}(\mathbf{q^2})\gamma^{\alpha} + \mathrm{i}\mu_{\mathbf{V}}\frac{\mathbf{F}^{\mathbf{V}}_{\mathbf{2}}(\mathbf{q^2})}{2M}\sigma^{\alpha\nu}q_{\nu}\right)$$

$$\mathbf{A}^{\alpha}_{\mathbf{N}}(\mathbf{q}) = \underbrace{\frac{g_A}{(1-q^2/M_A^2)^2}}_{\mathbf{G_A(q^2)}}\times\left(\gamma^{\alpha}\gamma_5+\underbrace{\frac{\not{q}}{m_\pi^2-q^2}}_{\mathrm{PCAC}}q^{\alpha}\gamma_5\right),\quad \begin{cases} g_A = 1.26 \\ M_A = 1.05\ \mathrm{GeV}\end{cases}$$

$$\mathbf{F}^{\mathbf{V}}_{\mathbf{1}}(\mathbf{q^2}) = \frac{1}{2}\left(\mathbf{F}^{\mathbf{p}}_{\mathbf{1}}(\mathbf{q^2}) - \mathbf{F}^{\mathbf{n}}_{\mathbf{1}}(\mathbf{q^2})\right), \qquad \mu_{\mathbf{V}}\mathbf{F}^{\mathbf{V}}_{\mathbf{2}}(\mathbf{q^2}) = \frac{1}{2}\left(\mu_{\mathbf{p}}\mathbf{F}^{\mathbf{p}}_{\mathbf{2}}(\mathbf{q^2}) - \mu_{\mathbf{n}}\mathbf{F}^{\mathbf{n}}_{\mathbf{2}}(\mathbf{q^2})\right),$$

furthermore **CVC** $\Rightarrow$ $\boxed{F_{PF}(q^2) = F^V_{CT}(q^2) = 2F^V_1(q^2) = F^p_1 - F^n_1}$

$$\vec{V}^\mu = \vec{\phi}\times\partial^\mu\vec{\phi} + \frac{g_A}{2f_\pi}\bar{\Psi}\gamma^\mu\gamma_5(\vec{\phi}\times\vec{\tau})\Psi + \bar{\Psi}\gamma^\mu\frac{\vec{\tau}}{2}\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma^\mu\left[\vec{\tau}\vec{\phi}^2 - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi$$

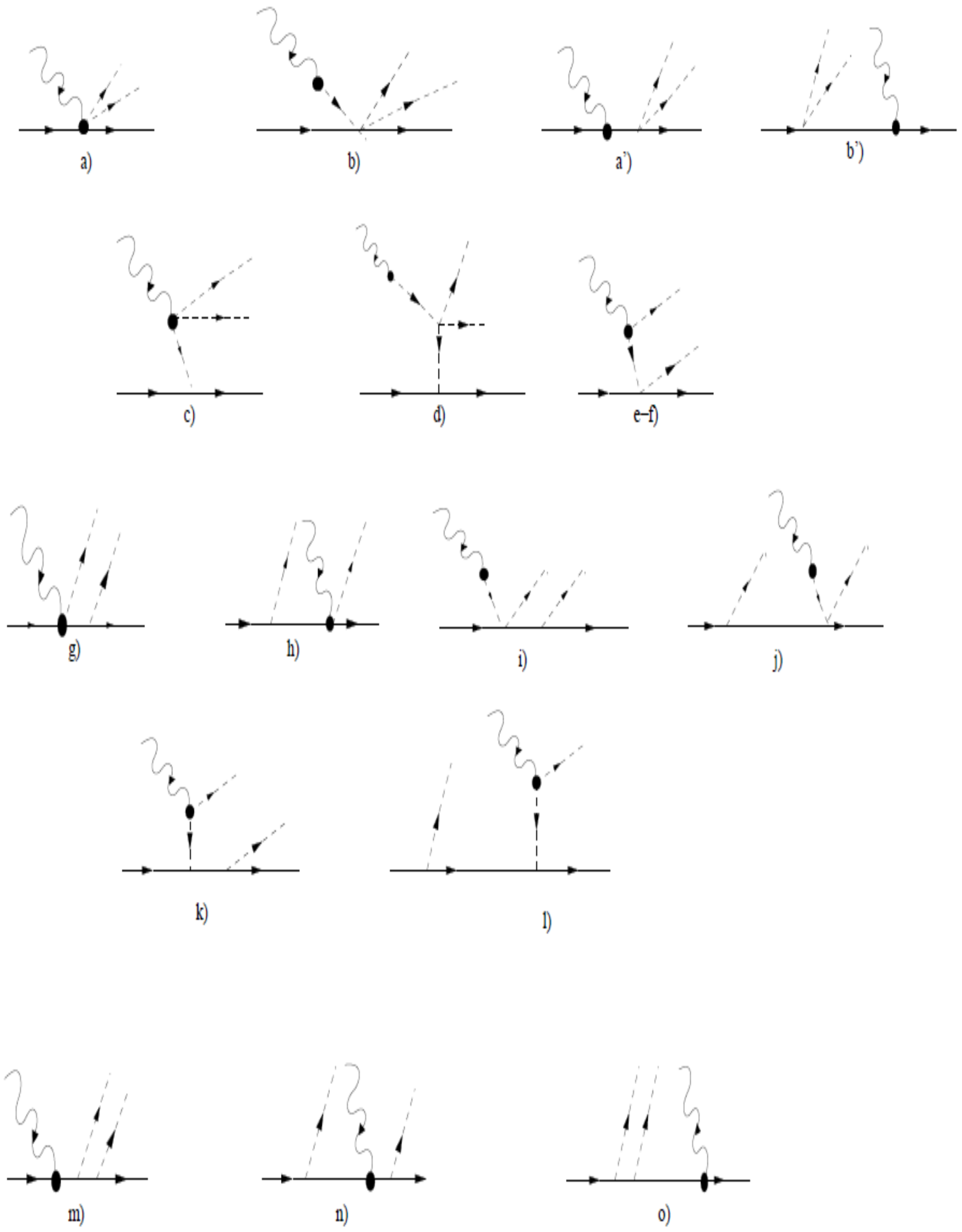
$$- \frac{\vec{\phi}^2}{3f_\pi^2}(\vec{\phi}\times\partial^\mu\vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3})$$

$$\vec{A}^\mu = f_\pi\partial^\mu\vec{\phi} + \frac{1}{2f_\pi}\bar{\Psi}\gamma^\mu(\vec{\phi}\times\vec{\tau})\Psi + g_A\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi + \frac{2}{3f_\pi}\left[\vec{\phi}(\vec{\phi}\cdot\partial^\mu\vec{\phi}) - \vec{\phi}^2\partial^\mu\vec{\phi}\right]$$

$$- \frac{g_A}{4f_\pi^2}\bar{\Psi}\gamma^\mu\gamma_5\left[\vec{\tau}\vec{\phi}^2 - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi + \mathcal{O}(\frac{1}{f_\pi^3})$$

$\nu_l N \to l N'\pi\pi$, $\nu_l N \to \nu_l N'\pi\pi$ close to threshold. $N^*(1440)$

degrees of freedom (PRD77 (2008) 053009)



Evaluation of NC $\langle N'\pi |\, j^\mu_{\rm nc}(0)|N\rangle$:

$$j^\mu_{\rm nc} = \bar\Psi_q \gamma^\mu (1 - 2\sin^2\theta_W - \gamma_5)\boxed{\tau^1_0}\Psi_q - \boxed{4\sin^2\theta_W \mathbf{s}^\mu_{\rm em,IS} - \bar{\mathbf{\Psi}}_{\rm s}\gamma^\mu(\mathbf{1}-\gamma_5)\mathbf{\Psi}_{\rm s}}$$

$$s^\mu_{\rm em} = \underbrace{\frac{1}{6}\bar\Psi_q\gamma^\mu\Psi_q - \frac{1}{3}\bar\Psi_s\gamma^\mu\Psi_s}_{\mathbf{s}^\mu_{\rm em,IS}} + \frac{1}{\sqrt{2}}\bar\Psi_q\,\gamma^\mu\frac{\boxed{\tau^1_0}}{\sqrt{2}}\Psi_q$$

- ME's j^μ_{cc+} $\Rightarrow$ ME's **isovector** (τ^1_0) $j^\mu_{\rm nc}$ **contribution**
- Δ **does not contribute to the isoscalar $j^\mu_{\rm nc}$ part**
- $\langle n\pi^+|s^\mu_{{\rm em},IS}|p\rangle = \langle p\pi^-|s^\mu_{{\rm em},IS}|n\rangle = \sqrt{2}\langle \mathbf{p}\pi^0|\mathbf{s}^\mu_{\rm em,IS}|\mathbf{p}\rangle = -\sqrt{2}\langle n\pi^0|s^\mu_{{\rm em},IS}|n\rangle$

$$\langle p\pi^0|s^\mu_{{\rm em},IS}|p\rangle = -\frac{\langle n\pi^0|s^\mu_{\rm em}(0)|n\rangle - \langle p\pi^0|s^\mu_{\rm em}(0)|p\rangle}{2}$$

$$\mathbf{s}^\mu_{\rm em} = \underbrace{\bar\Psi\gamma^\mu\left(\frac{1+\tau_z}{2}\right)\Psi}_{\rm PN,PNC} + \underbrace{\frac{ig_A}{2f_\pi}\bar\Psi\gamma^\mu\gamma_5\left(\tau^1_{-1}\phi^\dagger + \tau^1_{+1}\phi\right)\Psi}_{\rm CT} + \underbrace{i\left(\phi^\dagger\partial^\mu\phi - \phi\partial^\mu\phi^\dagger\right)}_{PF} + \cdots$$

CT, PF do not contribute $\Rightarrow$ PN and PNC $\Rightarrow$ ME's of $s^\mu_{{\rm em},IS}$

- ME's $\mathbf{j}^\mu_{\rm nc,str} = \bar{\mathbf{\Psi}}_{\mathbf{s}}\gamma^\mu(\mathbf{1}-\gamma_5)\mathbf{\Psi}_{\mathbf{s}}$: *nucleon strange content*

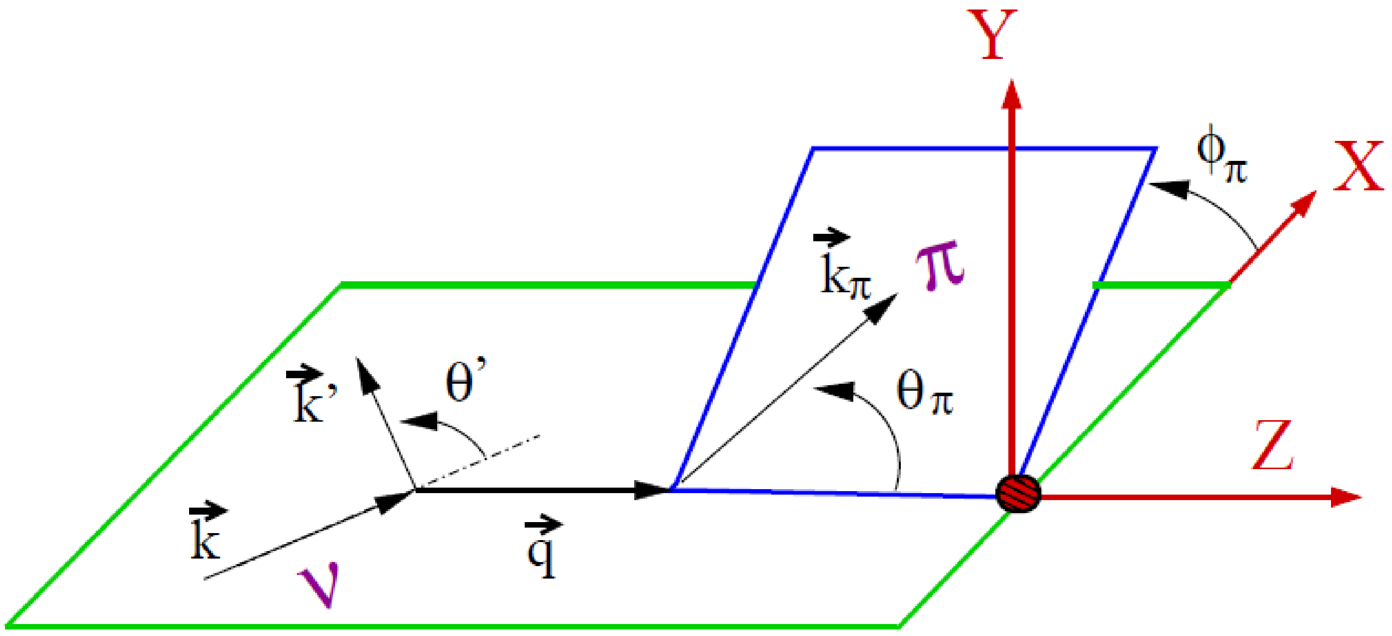
$$\langle p\pi^0|\mathbf{j}^\mu_{\rm nc,str}(\mathbf{0})|p\rangle = -i\,\frac{g_A}{2f_\pi}\bar u(\vec p\,')\left\{\not k_\pi\gamma_5\frac{\not p+\not q+M}{(p+q)^2-M^2+i\epsilon}\left[\mathbf{V}^\mu_{\rm N,s}(\mathbf{q})-\mathbf{A}^\mu_{\rm N,s}(\mathbf{q})\right]\right.$$

$$\left.+\left[\mathbf{V}^\mu_{\rm N,s}(\mathbf{q})-\mathbf{A}^\mu_{\rm N,s}(\mathbf{q})\right]\frac{\not p'-\not q+M}{(p'-q)^2-M^2+i\epsilon}\not k_\pi\gamma_5\right\}u(\vec p) \quad \Leftarrow \mathbf{PN}+\mathbf{PNC}$$

$$\mathbf{V}^\mu_{\rm N,s}(\mathbf{q}) = \underbrace{F_1^s(q^2)}_{\approx 0}\gamma^\mu + i\mu_s\frac{\overbrace{F_2^s(q^2)}^{\approx 0}}{2M}\sigma^{\mu\nu}q_\nu, \qquad \mathbf{A}^\mu_{\rm N,s}(\mathbf{q}) = \overbrace{G_A^s(q^2)}^{\boxed{g_S}\,(\mathbf{1-q^2/M_A^2})^{-2}}\gamma^\mu\gamma_5 + \underbrace{G_P^s}_{\rm don't\ contr.}q^\mu\gamma_5$$

Results :

$$\Rightarrow \text{CC}: \nu_l(k) + N(p) \rightarrow l^-(k') + N(p') + \pi(k_\pi)$$



$$\frac{\mathrm{d}^5\sigma_{\nu_l l}}{\mathrm{d}\Omega(\hat{\mathrm{k}}')\mathrm{d}E'\mathrm{d}\Omega(\hat{\mathrm{k}}_\pi)} = \frac{|\vec{k}'|}{|\vec{k}\,|}\frac{G^2}{4\pi^2}\int_0^{+\infty}\frac{d|\vec{k}_\pi||\vec{k}_\pi|^2}{E_\pi}\boxed{L^{(\nu)}_{\mu\sigma}\,(W^{\mu\sigma}_{\mathrm{CC}\pi})^{(\nu)}}$$

$$(W^{\mu\sigma}_{CC\pi})^{(\nu)} = \frac{1}{4M}\overline{\sum_{\rm spins}}\int \frac{d^3p'}{(2\pi)^3}\frac{1}{2E'_N}\delta^4(p'+k_\pi-q-p)\langle \mathbf{N}'\pi|\mathbf{j}^\mu_{cc+}(\mathbf{0})|\mathbf{N}\rangle\langle \mathbf{N}'\pi|\mathbf{j}^\sigma_{cc+}(\mathbf{0})|\mathbf{N}\rangle^*$$

$$\mathbf{L}^{(\nu)}_{\mu\sigma} = (\mathbf{L}^{(\nu)}_{\mathbf{s}})_{\mu\sigma} + {\rm i}(\mathbf{L}^{(\nu)}_{\mathbf{a}})_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma} k\cdot k' + {\rm i}\epsilon_{\mu\sigma\alpha\beta}k'^\alpha k^\beta$$

$$\Rightarrow \mathrm{CC}: \bar\nu_l(k) + N(p) \rightarrow l^+(k') + N(p') + \pi(k_\pi)$$

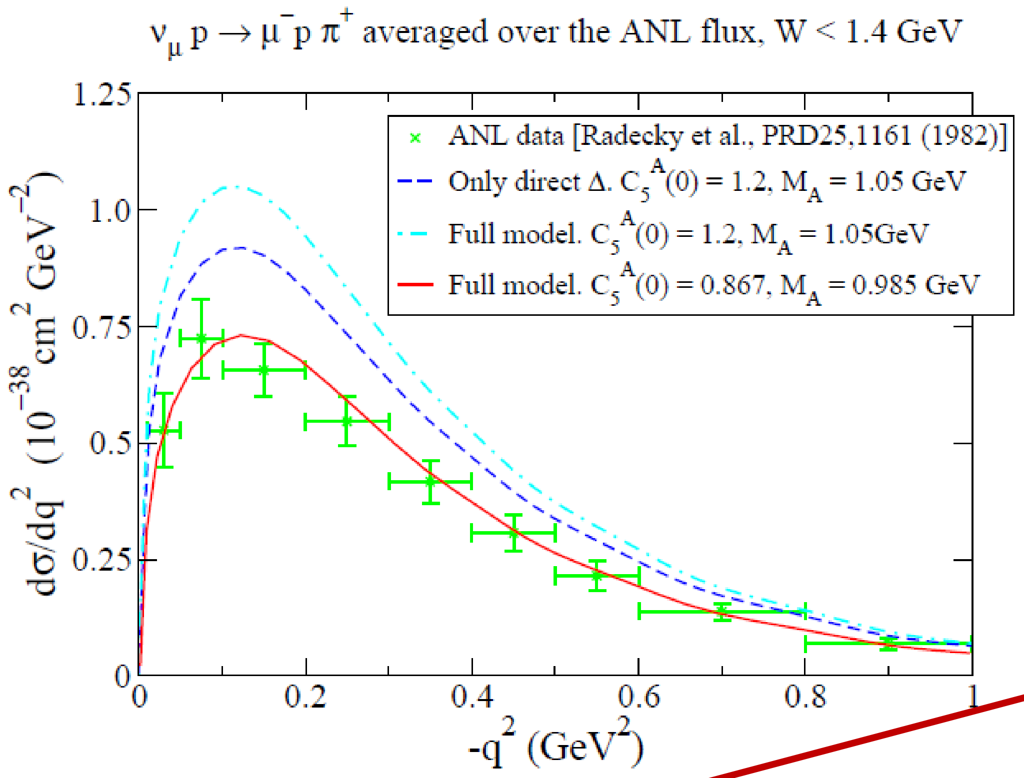
$$\mathbf{L}^{(\bar\nu)}_{\mu\sigma} = \mathbf{L}^{(\nu)}_{\sigma\mu}, \quad \mathbf{j}^\sigma_{cc+} \leftrightarrow \mathbf{j}^\sigma_{cc-}$$

$$\Rightarrow \mathrm{NC}: \nu(k) + N(p) \rightarrow \nu(k') + N(p') + \pi(k_\pi)$$

$$\mathbf{j}^\sigma_{cc+} \leftrightarrow \frac{1}{2}\mathbf{j}^\sigma_{nc}, \quad (\mathbf{W}^{\mu\sigma}_{NC\pi})^{(\nu)} = (\mathbf{W}^{\mu\sigma}_{NC\pi})^{(\bar\nu)}$$

Note $\underbrace{(E',\theta')}_{\text{outgoing lepton}} \leftrightarrow q^2, \underbrace{W^2 = (p+q)^2}_{\pi\text{N inv. mass}}$

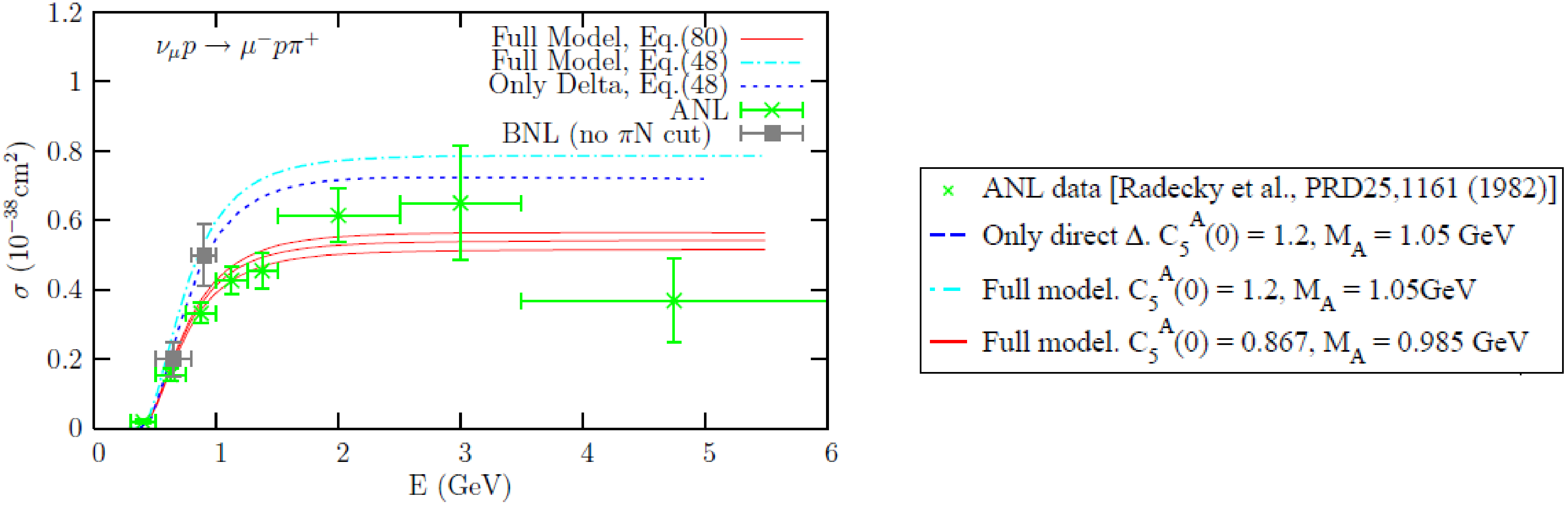
$$\int_{M+m_\pi}^{\boxed{1.4\,\mathrm{GeV}}} dW \frac{d\overline{\sigma}_{\nu_\mu\mu^-}}{dq^2 dW}, \qquad \nu_\mu \mathbf{p} \to \mu^- \mathbf{p}\pi^+$$

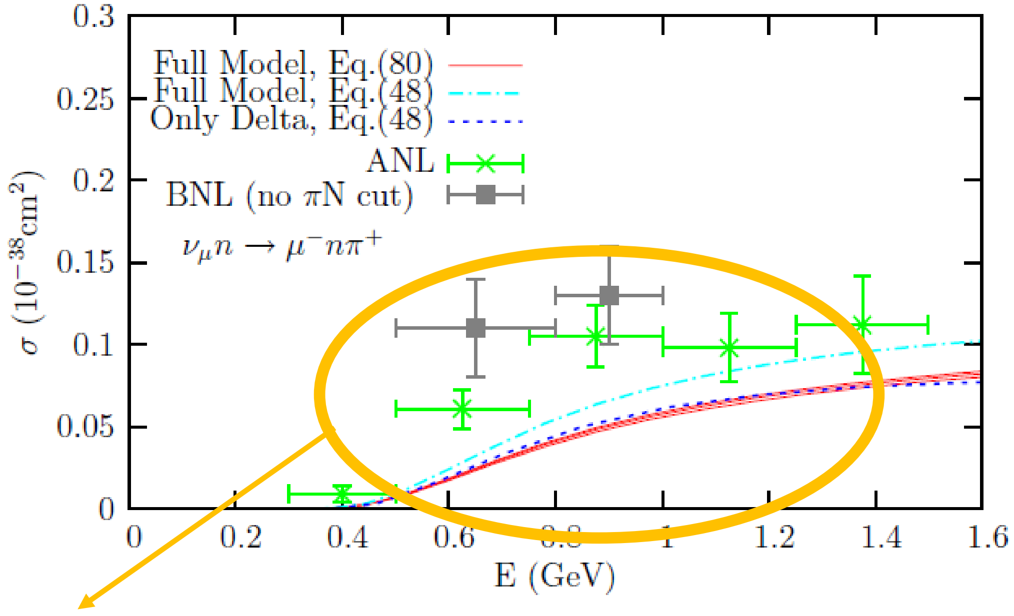


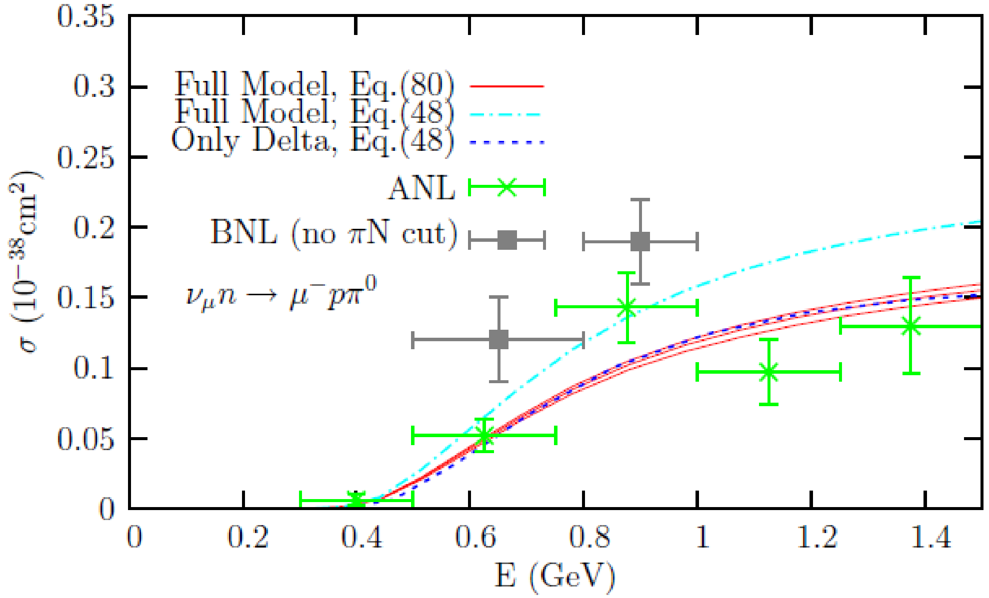
$$C_5^A(0) \sim \sqrt{\frac{2}{3}}\frac{f_\pi}{m_\pi} f^* = 1.2$$

PCAC predicts ~ 1. 2

Fit to ANL : $\boxed{C_5^A(0) = 0.867 \pm 0.075,}$ $\quad M_{A\Delta} = 0.985 \pm 0.082\,\mathrm{GeV}$

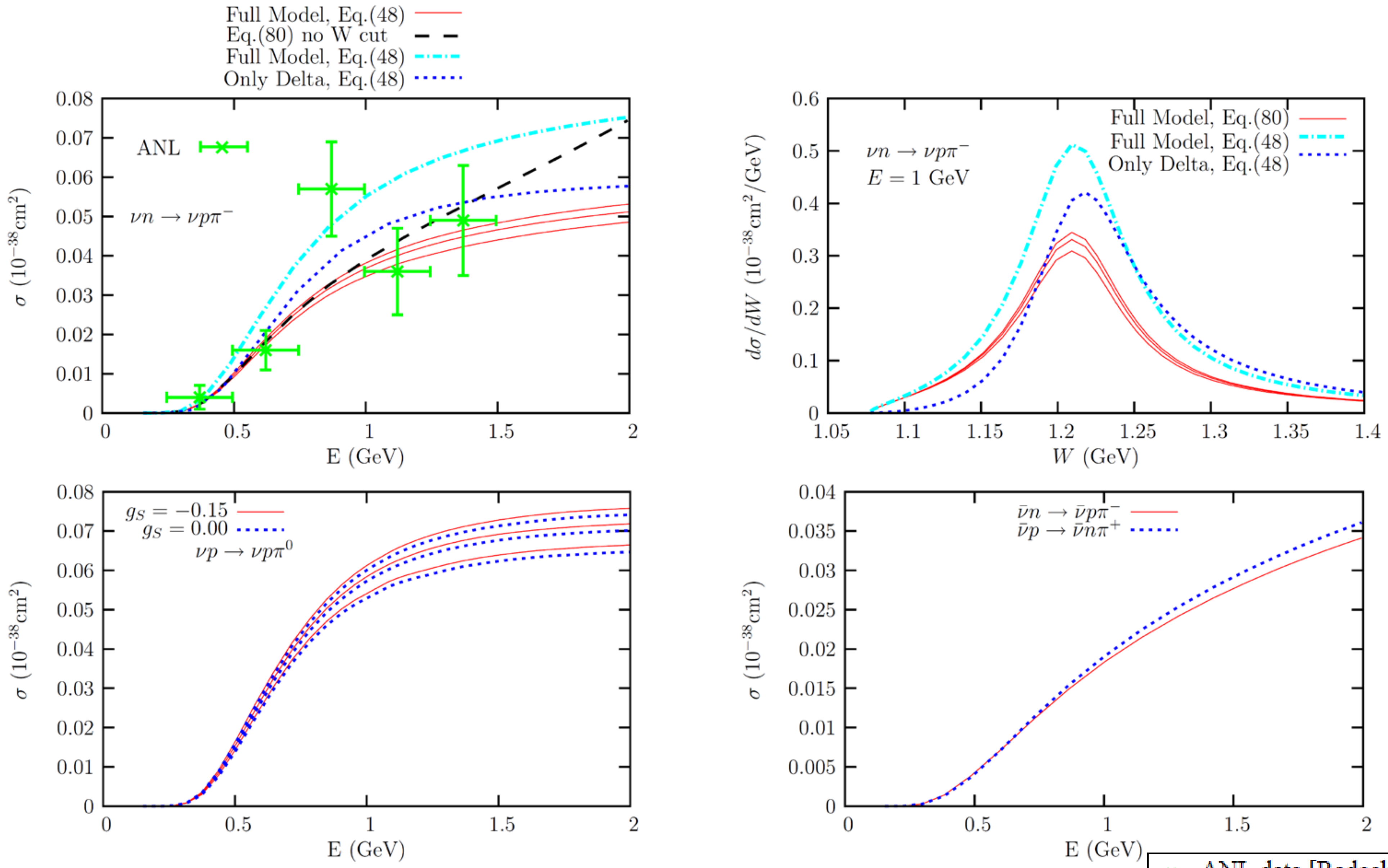

$\nu_\mu p \to \mu^- p \pi^+$
Full Model, Eq.(80)
Full Model, Eq.(48)
Only Delta, Eq.(48)
ANL
BNL (no πN cut)
σ (10^{-38}cm²)
E (GeV)
ANL data [Radecky et al., PRD25,1161 (1982)]
Only direct Δ. $C_5^A(0)$ = 1.2, M_A = 1.05 GeV
Full model. $C_5^A(0)$ = 1.2, M_A = 1.05GeV
Full model. $C_5^A(0)$ = 0.867, M_A = 0.985 GeV


Full Model, Eq.(80)
Full Model, Eq.(48)
Only Delta, Eq.(48)
ANL
BNL (no πN cut)
$\nu_\mu n \to \mu^- n \pi^+$
σ (10^{-38}cm²)
E (GeV)


Full Model, Eq.(80)
Full Model, Eq.(48)
Only Delta, Eq.(48)
ANL
BNL (no πN cut)
$\nu_\mu n \to \mu^- p \pi^0$
σ (10^{-38}cm²)
E (GeV)

problems ?

NC



- × ANL data [Radecky et al., PRD25,1161 (1982)]
- Only direct Δ. $C_5^A(0) = 1.2$, $M_A = 1.05$ GeV
- Full model. $C_5^A(0) = 1.2$, $M_A = 1.05$GeV
- Full model. $C_5^A(0) = 0.867$, $M_A = 0.985$ GeV

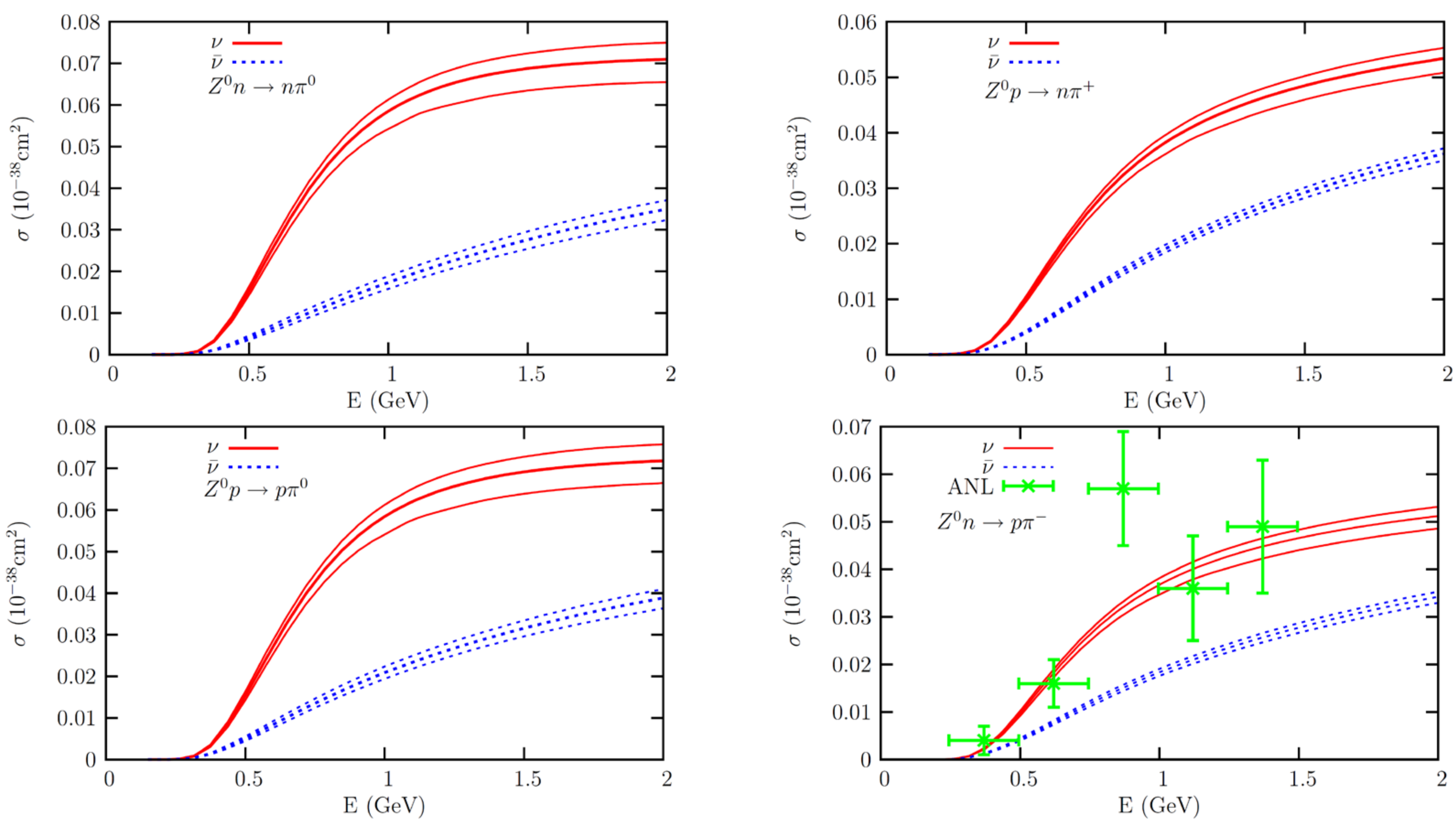
Juan Nieves, IFIC (CSIC & UV)

$\sigma_{\rm NC}/\sigma_{\rm CC}$ ANL cross sections at $E = 0.6 - 1.2$ GeV

	ANL	Our results
$R_+ = \sigma(\nu p \to \nu n \pi^+)/\sigma(\nu p \to \mu^- p \pi^+)$	0.12 ± 0.04	$0.12 - 0.10$
$R_0 = \sigma(\nu p \to \nu p \pi^0)/\sigma(\nu p \to \mu^- p \pi^+)$	0.09 ± 0.05	$0.18 - 0.14$
$R_- = \sigma(\nu n \to \nu p \pi^-)/\sigma(\nu p \to \mu^- p \pi^+)$	0.11 ± 0.022	$0.12 - 0.09$

NC: Cross sections (10^{-38}cm^2) for $\langle E \rangle = 2.2$ GeV (no cut in W)

	CERN	Our results
$\sigma(\nu p \to \nu p \pi^0)$	0.130 ± 0.020	0.105 ± 0.006
$\sigma(\nu p \to \nu n \pi^+)$	0.080 ± 0.020	0.091 ± 0.003
$\sigma(\nu n \to \nu n \pi^0)$	0.080 ± 0.020	0.104 ± 0.006
$\sigma(\nu n \to \nu p \pi^-)$	0.110 ± 0.030	0.082 ± 0.003



Below the τ prod. threshold, Distinguish ν_τ from $\bar{\nu}_\tau$?

PLB 647 (2007) 452

How to reconcile ANL & BNL data and still have $C_5^A(0) \sim 1.2$

K.M. Graczyk et al. [Phys. Rev. D 80, 093001 (2009)]

4. Deuteron effects and ANL & BNL data

- ANL and BNL data were measured in deuterium
  - Deuteron effects were estimated by L. Alvarez-Ruso et al [Phys. Rev. C 59, 3386 (1999)] to reduce the cross section by 5-10%.
- Large uncertainties in the neutrino flux normalization, 10% for BNL data and 20% for ANL data.

K.M. Graczyk et al. made a combined fit to both ANL&BNL data, assuming that only the Δ mechanism contributed, including deuteron effects, and treating flux uncertainties as systematic errors. They found

$$C_5^A(0) = 1.19 \pm 0.08, \qquad M_{A\Delta} = 0.94 \pm 0.03\,\text{GeV}$$

for a pure dipole parameterization for $C_5^A(q^2)$. Good agreement with the off-diagonal GTR is found! **No background terms included !**

Background terms included

PRD 81 085046 (2010): We included **background terms in a combined fit to ANL & BNL data that took into account deuteron effects and flux normalization uncertainties.**

We used a simpler dipole parameterization for $C_5^A(q^2)$

$$C_5^A(q^2) = \frac{C_5^A(0)}{(1 - q^2/M_{A\Delta}^2)^2}$$

Using Adler's constraints we obtained

$$C_5^A(0) = 1.00 \pm 0.11, \qquad M_{A\Delta} = 0.93 \pm 0.07\,\text{GeV}$$

$C_5^A(0)$ compatible with its GTR value (~ 1.2) at the 2σ level.

data do not discriminate Adler's constraints

PRD 81 (2010) 085046

In some of the fits we relaxed Adler's constraints allowing

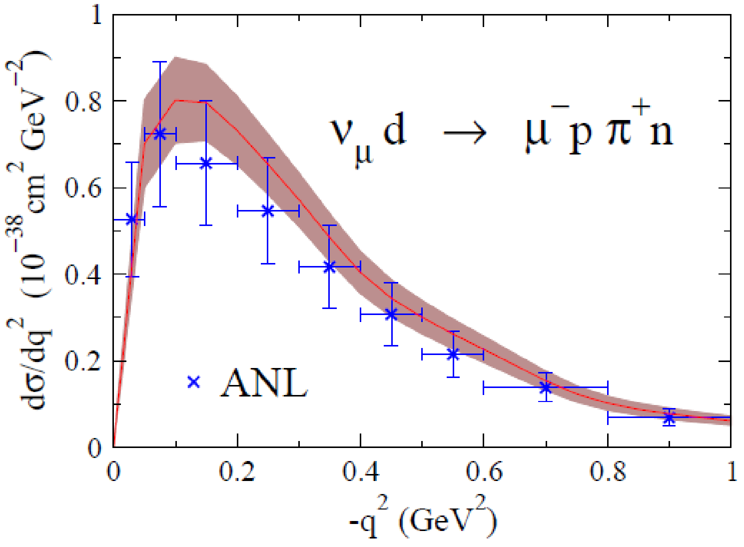
$$C^A_{3,4}(q^2) = C^A_{3,4}(0)\,\frac{C^A_5(q^2)}{C^A_5(0))}$$

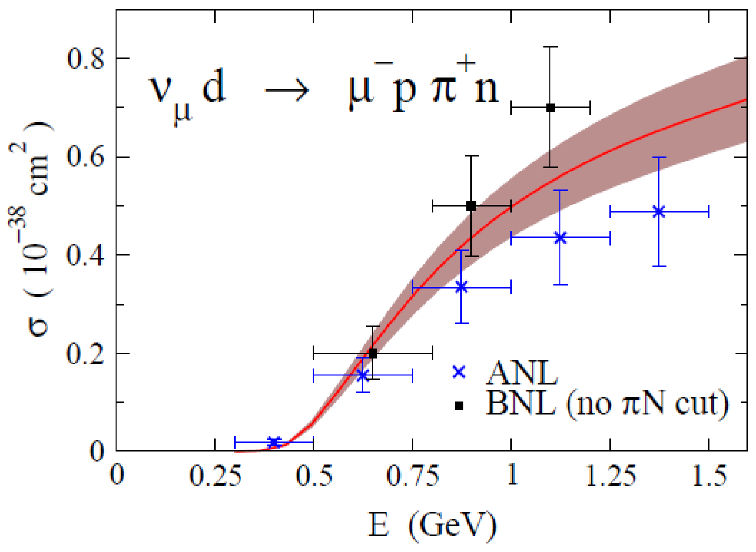
exploring the possibility of extracting some direct information on $C^A_{3,4}(0)$

	$C^A_5(0)$	$M_{A\Delta}$/GeV	$C^A_3(0)$	$C^A_4(0)$	χ^2/dof
I* (only ΔP)	1.08 ± 0.10	0.92 ± 0.06	Ad	Ad	0.36
II*	0.95 ± 0.11	0.92 ± 0.08	Ad	Ad	0.49
III (only ΔP)	1.13 ± 0.10	0.93 ± 0.06	Ad	Ad	0.32
IV	1.00 ± 0.11	0.93 ± 0.07	Ad	Ad	0.42
V	1.08 ± 0.14	0.91 ± 0.10	-1.0 ± 1.4	Ad	0.40
VI	1.08 ± 0.14	0.86 ± 0.15	Ad	-1.0 ± 1.3	0.40
VII	1.07 ± 0.15	1.0 ± 0.3	1 ± 4	-2 ± 4	0.44

* No deuteron effects included.

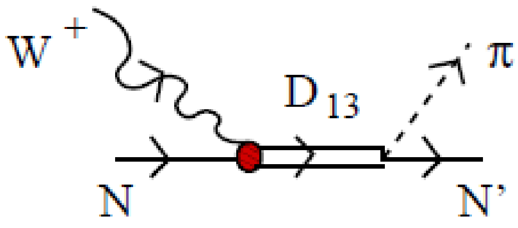
Comparison with ANL & BNL data

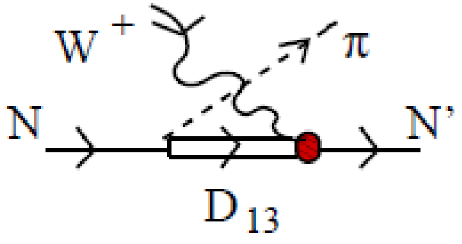




68% confidence level bands are shown. The total experimental errors shown contain flux uncertainties that are considered as systematic errors and have been added in quadratures to the statistical ones.

Later we included the D13(1520) resonance [E.Hernández., J. Nieves and M.J. Vicente-Vacas, PRD 87 (2013) 113009]

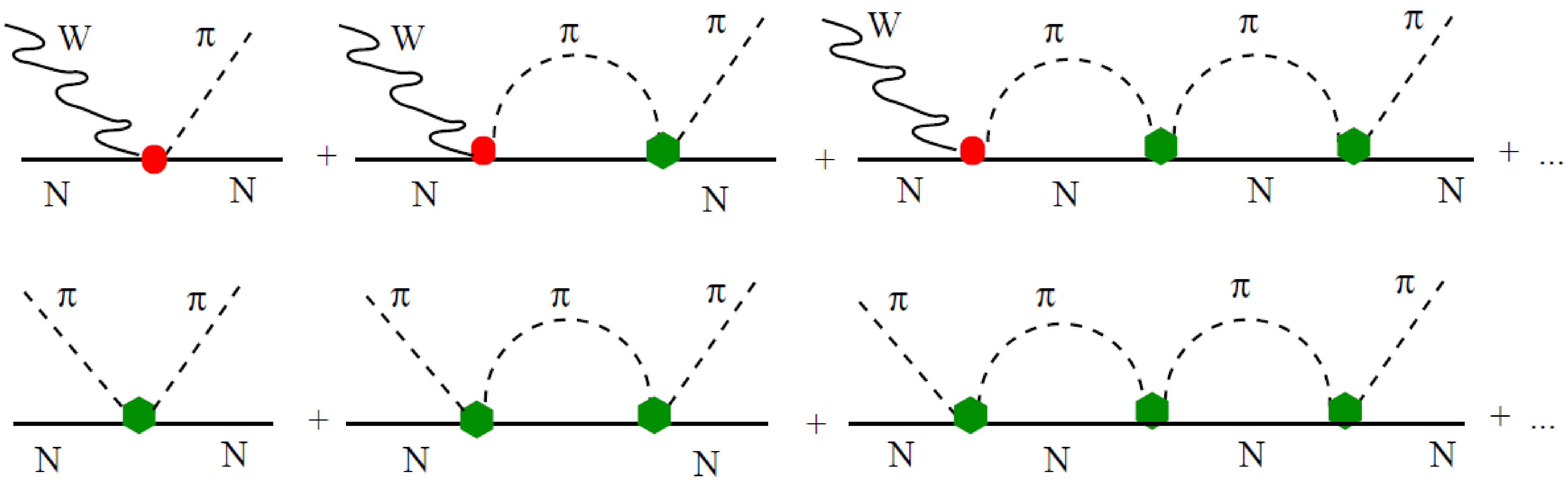




5. Unitarity corrections and Watson's theorem

PRD 93 (2016) 014016

Watson's final-state-interaction theorem (unitarity and time–reversal invariance): The phase of an amplitude leading to a final state with two strongly interacting particles in a given partial wave is the same as the scattering phase of that pair, $\boxed{\delta}$. **[PRD 88 (1952) 1163]**



Optical theorem in partial waves

$$SS^\dagger = 1 \quad \Leftrightarrow \quad i\left(T - T^\dagger\right) = T^\dagger T$$

$$a + b \quad \rightarrow \quad 1 + 2$$

$$i\left[\langle\lambda_1\lambda_2|T_J|\lambda_a\lambda_b\rangle - \langle\lambda_a\lambda_b|T_J|\lambda_1\lambda_2\rangle^*\right] \quad \sim \quad \sum_{\lambda_1'\lambda_2'}\langle\lambda_1\lambda_2|T_J^\dagger|\lambda_1'\lambda_2'\rangle\langle\lambda_1'\lambda_2'|T_J|\lambda_a\lambda_b\rangle$$

Optical theorem in partial waves

$$SS^\dagger = 1 \quad \Leftrightarrow \quad i\left(T - T^\dagger\right) = T^\dagger T$$

$$a + b \quad \rightarrow \quad 1 + 2$$

$$i[\langle\lambda_1\lambda_2|T_J|\lambda_a\lambda_b\rangle - \langle\lambda_a\lambda_b|T_J|\lambda_1\lambda_2\rangle^*] \quad \sim \quad \sum_{\lambda_1'\lambda_2'} \langle\lambda_1\lambda_2|T_J^\dagger|\lambda_1'\lambda_2'\rangle\langle\lambda_1'\lambda_2'|T_J|\lambda_a\lambda_b\rangle$$

Using CM helicity states $|p; JM\lambda_1\lambda_2\rangle$ and **invariance under time reversal**,

$$\underbrace{\langle\lambda_1\lambda_2|T_J|\lambda_a\lambda_b\rangle}_{\mathbf{a+b\to1+2}} = \underbrace{\langle\lambda_a\lambda_b|T_J|\lambda_1\lambda_2\rangle}_{\mathbf{1+2\to a+b}}$$

$$\mathbb{R} \ni \mathrm{Im}\langle\lambda_1\lambda_2|T_J|\lambda_a\lambda_b\rangle \sim \sum_{\lambda_1'\lambda_2'}\langle\lambda_1\lambda_2|T_J^\dagger|\lambda_1'\lambda_2'\rangle\langle\lambda_1'\lambda_2'|T_J|\lambda_a\lambda_b\rangle \in \mathbb{R}$$

Considering **intermediate strong interacting πN states**, Watson's theorem for the **weak $WN \to N\pi$ process** implies,

$$\sum_{\lambda''_N} \underbrace{\langle \lambda'_N}_{N\pi} | T_J^\dagger(s) | \underbrace{\lambda''_N \rangle \langle \lambda''_N}_{N\pi} | T_J(s) | \underbrace{\lambda_N \lambda_W \rangle}_{NW} \in \mathbb{R}$$

In terms πN $|p; LSJM\rangle$ states

PRD 93 (2016) 014016

$$\sum_L \sqrt{\frac{2L+1}{2J+1}} (L\tfrac{1}{2}J|0\lambda'_N\lambda'_N) \underbrace{\langle L\tfrac{1}{2}J|T_J|L\tfrac{1}{2}J\rangle^*}_{\pi N\to\pi N} \underbrace{\langle L\tfrac{1}{2}J|T_J|\lambda_N\lambda_W\rangle}_{WN\to N\pi} \in \mathbb{R}$$

For $J=3/2, T=3/2$ and neglecting the $L=2$ multipole,

$$\left\langle \mathbf{P_{33}} \left| \mathbf{T^{WN\to N\pi}_{J=\frac{3}{2},T=\frac{3}{2}}} \right| \mathbf{J} = \frac{\mathbf{3}}{\mathbf{2}}, \mathbf{M} = \lambda_\mathbf{N} - \lambda_\mathbf{W}, \lambda_\mathbf{N}\lambda_\mathbf{W} \right\rangle \times \underbrace{e^{-i\delta_{P_{33}}(s)}}_{\mathbf{L_{2J2T}}\ \mathbf{N}\pi\ \text{phase shift}} \in \mathbb{R}$$

There is a total of 6 $[(\lambda_N = \pm\frac{1}{2}) \times (\lambda_W = 0, \pm 1)]$ amplitudes which should have the same phase $(\delta_{P_{33}}(s), s = (p_N + p_\pi)^2)$.

Using CM three momentum helicity states $|p;\theta\phi\lambda_1\lambda_2\rangle$

$$|P_{33}M\rangle = \int d\Omega \sum_\lambda \sqrt{\frac{3}{4\pi}}\mathcal{D}^{\frac{3}{2}*}_{M\lambda}(\phi,\theta,-\phi)\left(1\frac{1}{2}\frac{3}{2}|0\lambda\lambda\right)|p;\theta\phi\lambda\rangle$$

$$|p;\theta=0\,\phi=0\lambda_N\lambda_W\rangle = \sum_J \sqrt{\frac{2J+1}{4\pi}}|p;JM=\lambda_N-\lambda_W,\lambda_N\lambda_W\rangle$$

$$\int \mathbf{d\Omega} \sum_\lambda \mathcal{D}^{\frac{3}{2}}_{\lambda_N-\lambda_W\lambda}(\phi,\theta,-\phi)\left(\mathbf{1}\frac{\mathbf{1}}{\mathbf{2}}\frac{\mathbf{3}}{\mathbf{2}}|\mathbf{0}\lambda\lambda\right)\underbrace{\left\langle p';\theta\phi\lambda\left|T^{W N\to N\pi}_{J=\frac{3}{2},T=\frac{3}{2}}\right|p;00\lambda_N\lambda_W\right\rangle}_{\text{related to }\bar{\mathbf{u}}(\mathbf{p}',\lambda)(\mathbf{O}_\mu\epsilon^\mu_{\lambda_\mathbf{W}})\mathbf{u}(\mathbf{p},\lambda_\mathbf{N})}\mathbf{e}^{-\mathbf{i}\delta_{\mathbf{P_{33}}}} \in \mathbb{R}$$

There is a total of 6 $[(\lambda_N = \pm\frac{1}{2})\times(\lambda_W = 0,\pm 1)]$ **amplitudes which should have the same phase** $(\delta_{P_{33}}(s), s=(p_N+p_\pi)^2)$**.**

We force the correct phase for two different linear combinations of these amplitudes that correspond to the two multipoles where the Δ mechanism (vector and axial contributions) is dominant. For instance, in the case of the vector Δ contribution, this is the M_{1+} multipole. We denote the corresponding axial multipole as $\mathcal{A}_\Delta$.

We follow a generalization of **M.G. Olsson's procedure [NPB 78 (1974) 55]** introducing two small phases $\phi_{\mathbf{V},\mathbf{A}}(\mathbf{s},\mathbf{q^2})$ which correct the vector and axial Δ contributions such that

$$\mathrm{Im}\left[\left(T_\Delta^{V,A}(s,q^2)\mathbf{e}^{\mathbf{i}\phi_{\mathbf{V},\mathbf{A}}(\mathbf{s},\mathbf{q^2})} + T_B^{V,A}(s,q^2)\right)^{M_{1+};\mathcal{A}_\Delta} \mathbf{e}^{-\mathbf{i}\delta_{\mathbf{P_{33}}}(\mathbf{s})}\right] = 0$$

$$\Gamma^{\alpha\mu} = \left[\frac{C_3^A}{M}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu) + \frac{C_4^A}{M^2}(g^{\alpha\mu} q\cdot p_\Delta - q^\alpha p_\Delta^\mu) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M^2} q^\mu q^\alpha\right] e^{i\phi_A(s,q^2)}$$

$$+ \left[\frac{C_3^V}{M}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu) + \frac{C_4^V}{M^2}(g^{\alpha\mu} q\cdot p_\Delta - q^\alpha p_\Delta^\mu) + \frac{C_5^V}{M^2}(g^{\alpha\mu} q\cdot p - q^\alpha p^\mu)\right] e^{i\phi_V(s,q^2)}\gamma_5$$

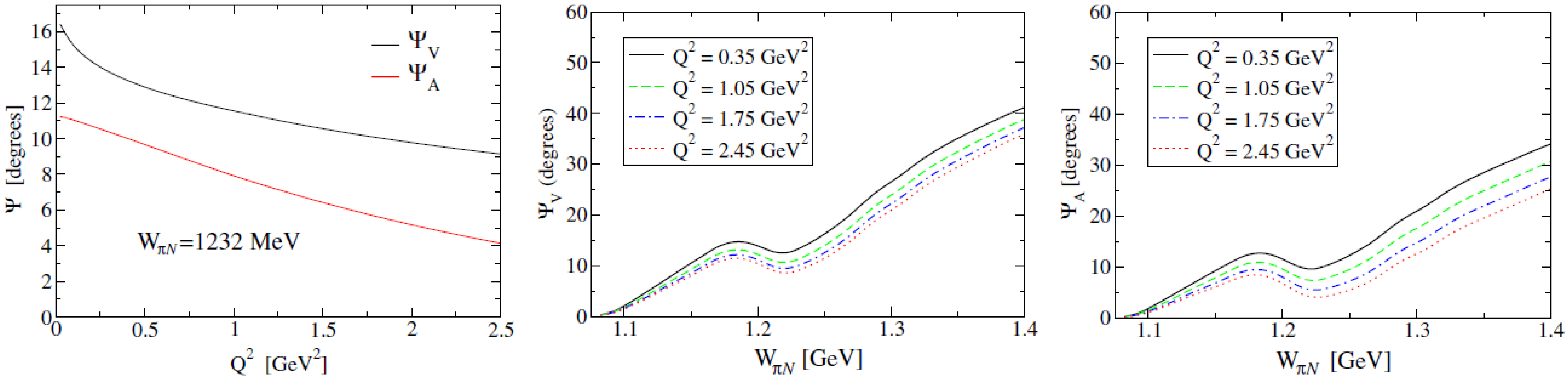
We include chiral background terms in a combined fit to ANL & BNL data that takes into account deuteron effects, flux normalization uncertainties and unitarity corrections (Watson's theorem) **PRD 93 (2016) 014016**

TABLE I. Results from different fits to the ANL and BNL data. All fits include the ANL [46] flux-averaged $d\sigma/dQ^2$ differential cross section, with a $W_{\pi N} = \sqrt{s} < 1.4$ GeV cut, and the integrated cross sections for the three lowest neutrino energies (0.65, 0.9, and 1.1 GeV) of the BNL data set [47]. Fits I*, II*, and IV are taken from Ref. [36]. In all cases, Adler's constraints ($C_3^A = 0$, $C_4^A = -C_5^A/4$) [13,14] are imposed. Deuteron effects [36] are included in fit IV and in those carried out in this work. The nonresonant chiral background contributions are included in all cases, with the exception of fit I*. For $C_5^A(q^2)$, a dipole form, $C_5^A(q^2) = C_5^A(0)/(1 - q^2/M_{A\Delta}^2)^2$, has been used in all fits except in the one carried out in Ref. [31], where an extra factor $1/(1 - q^2/3M_{A\Delta})$ was included [see Eq. (48) of that reference]. Finally, r is the Gaussian correlation coefficient between $C_5^A(0)$ and $M_{A\Delta}$. For reference, the prediction of the GTR is $C_5^A(0) = 1.15 - 1.2$.

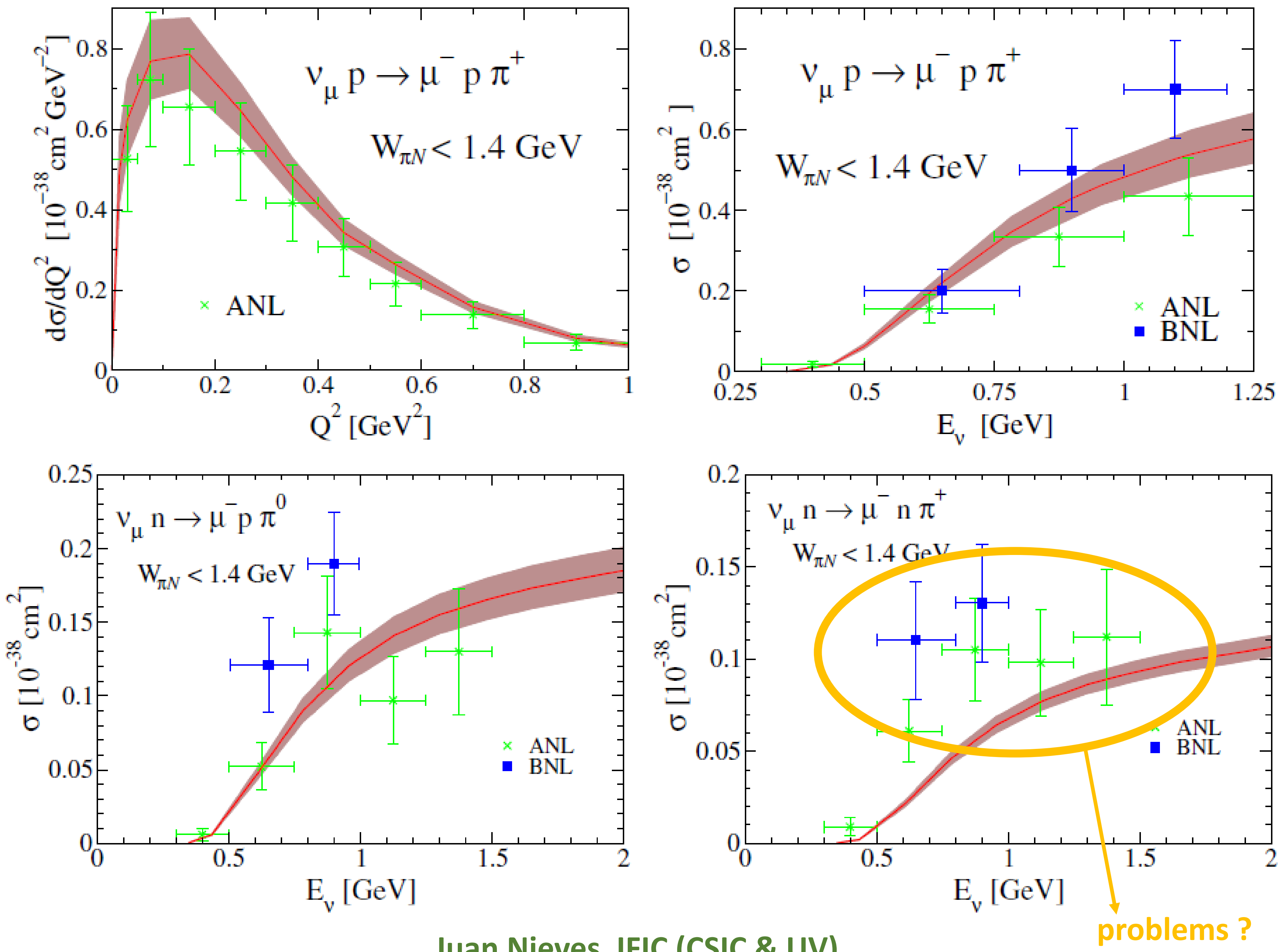
	$C_5^A(0)$	$M_{A\Delta}$/GeV	Data	r	χ^2/dof
PRD 76 (2007) 033005	0.867 ± 0.075	0.985 ± 0.082	ANL	−0.85	0.40
Fit I* (only Δ pole) no deuteron effects included.	1.08 ± 0.10	0.92 ± 0.06	ANL & BNL	−0.06	0.36
Fit II* no deuteron effects included.	0.95 ± 0.11	0.92 ± 0.08	ANL & BNL	−0.08	0.49
Fit IV (with deuteron effects) PRD 81 (2010) 085046	1.00 ± 0.11	0.93 ± 0.07	ANL & BNL	−0.08	0.42
WATSON (unitarized + deuteron effects) fit A	1.12 ± 0.11	0.954 ± 0.063	ANL & BNL	−0.08	0.46

Juan Nieves, IFIC (CSIC & UV)

$C_5^A(0)$ compatible with its GTR value (~ 1.2) at the 1σ level.

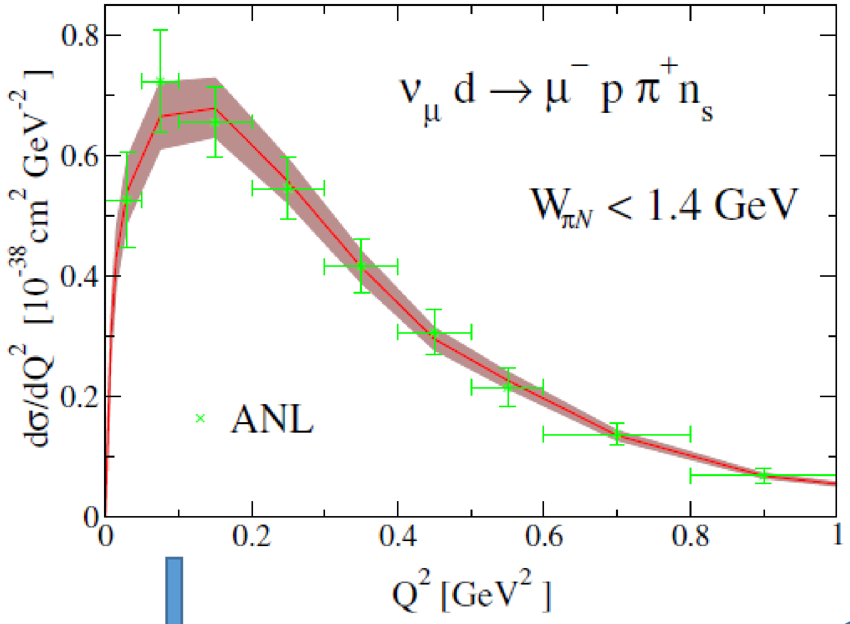


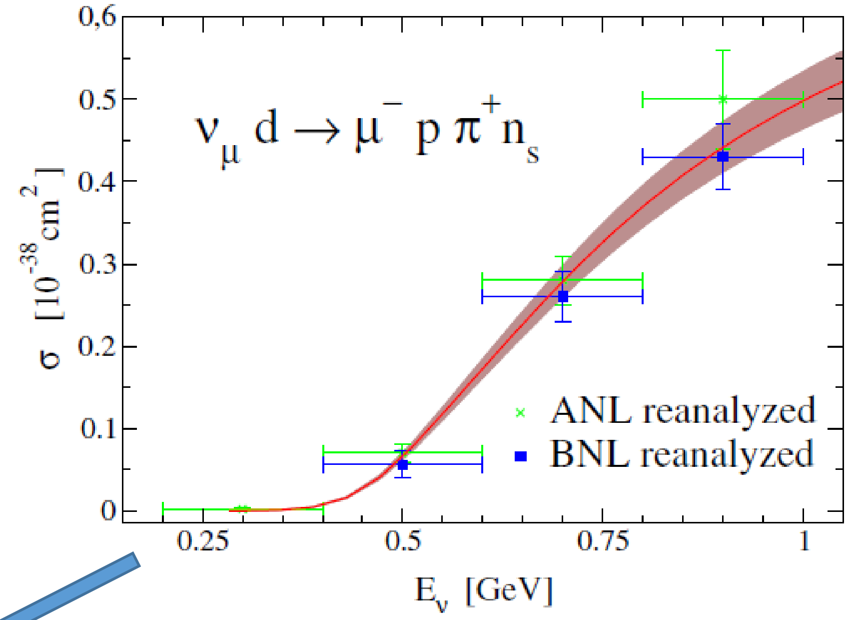
moderately small vector and axial Olsson phases!



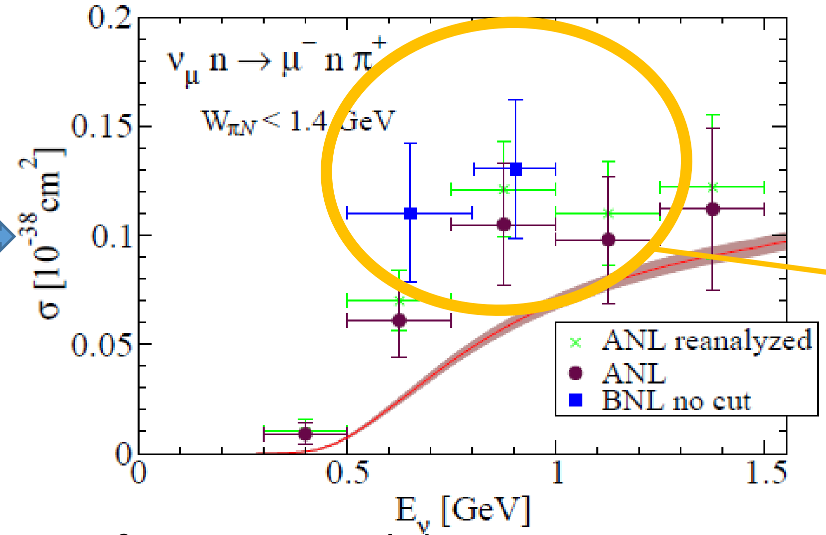
$C_5^A(0)$ compatible with its GTR value (~ 1.2) at the 1σ level.

ANL and BNL reanalyzed data: C. Wilkinson, P. Rodrigues, S. Cartwright, L. Thompson, and K. McFarland, PRD 90 (2014) 112017 (similar results)





$$C_5^A(0) = 1.14 \pm 0.07,$$
$$M_{A\Delta} = (959.4 \pm 66.9) \text{ MeV},$$



ANL reanalyzed data: P.Rodrigues, C. Wilkinson and K. McFarland, Eur. Phys. J. C 76, 474 (2016).

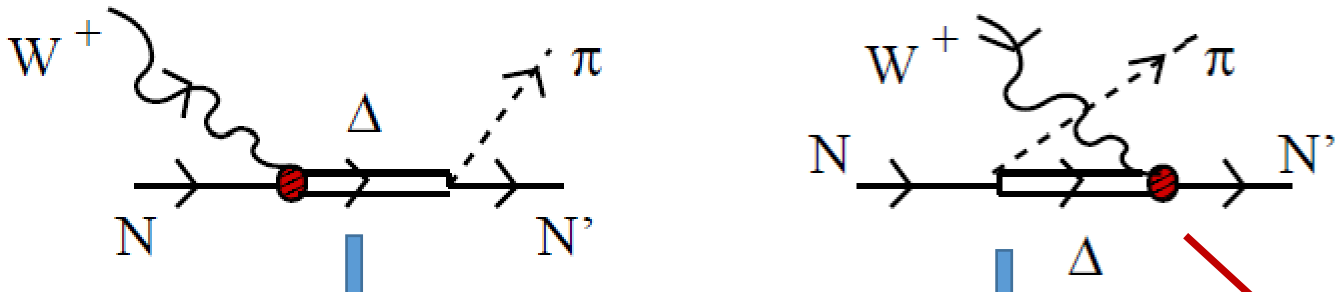
$\nu_\mu n \longrightarrow \mu^- n \pi^+$

problems ?

This underprediction of experimental data is a common problem to other models.

6. The $\nu_\mu\, n \longrightarrow \mu^- n \pi^+$ channel …

PRD95 (2017) 053007



reaction	spin-isospin factor direct term	spin-isospin factor crossed term
$\nu_\mu\, p \longrightarrow \mu^- p \pi^+$	$\sqrt{3}$	$1/\sqrt{3}$
$\nu_\mu\, n \longrightarrow \mu^- p \pi^0$	$2/\sqrt{3}$	$-2/\sqrt{3}$
$\nu_\mu\, n \longrightarrow \mu^- n \pi^+$	$1/\sqrt{3}$	$\sqrt{3}$

- sensitive to the propagation of spin 1/2 dof in the Δ propagator
- large contribution to the $\nu_\mu\, n \longrightarrow \mu^- n \pi^+$ channel

In the zero width limit, the Δ propagator is given by

$$G_{\mu\nu}(p_\Delta) = \frac{P_{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + i\epsilon}$$

with

$$P^{\mu\nu}(p_\Delta) = -(\not p_\Delta + M_\Delta)\left[g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{2}{3}\frac{p_\Delta^\mu p_\Delta^\nu}{M_\Delta^2} + \frac{1}{3}\frac{p_\Delta^\mu\gamma^\nu - p_\Delta^\nu\gamma^\mu}{M_\Delta}\right]$$

$$P_{\mu\nu}(p) = P^{\frac{3}{2}}_{\mu\nu}(p) + \underbrace{(p^2 - M_\Delta^2)\left[\frac{2}{3M_\Delta^2}(\not p + M_\Delta)\frac{p_\mu p_\nu}{p^2} - \frac{1}{3M_\Delta}\left(\frac{p^\rho p_\nu \gamma_{\mu\rho}}{p^2} + \frac{p^\rho p_\mu \gamma_{\rho\nu}}{p^2}\right)\right]}_{\text{spin}-1/2},$$

with

$$P^{\frac{3}{2}}_{\mu\nu}(p) = -(\not p + M_\Delta)\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}(\not p \gamma_\mu p_\nu + p_\mu \gamma_\nu \not p)\right].$$

$P^{\frac{3}{2}}_{\mu\nu}(p)$ satisfies the relations

$$0 = [\not p, P^{\frac{3}{2}}_{\mu\nu}(p)] = p^\mu P^{\frac{3}{2}}_{\mu\nu}(p) = P^{\frac{3}{2}}_{\mu\nu}(p)p^\nu = \gamma^\mu P^{\frac{3}{2}}_{\mu\nu}(p) = P^{\frac{3}{2}}_{\mu\nu}(p)\gamma^\nu,$$

$$P^{\frac{3}{2}}_{\mu\nu}(p)[P^{\frac{3}{2}}(p)]^{\nu\rho} = -(\not p + M_\Delta)[P^{\frac{3}{2}}(p)]^\rho_\mu$$

being the true spin-3/2 projection operator

see also discussion of consistent couplings to select spin-3/2 dof [V. Pascalutsa, Phys. Lett. B 503 (2001) 85]

the spin-1/2 component does not propagate giving rise to contact interactions

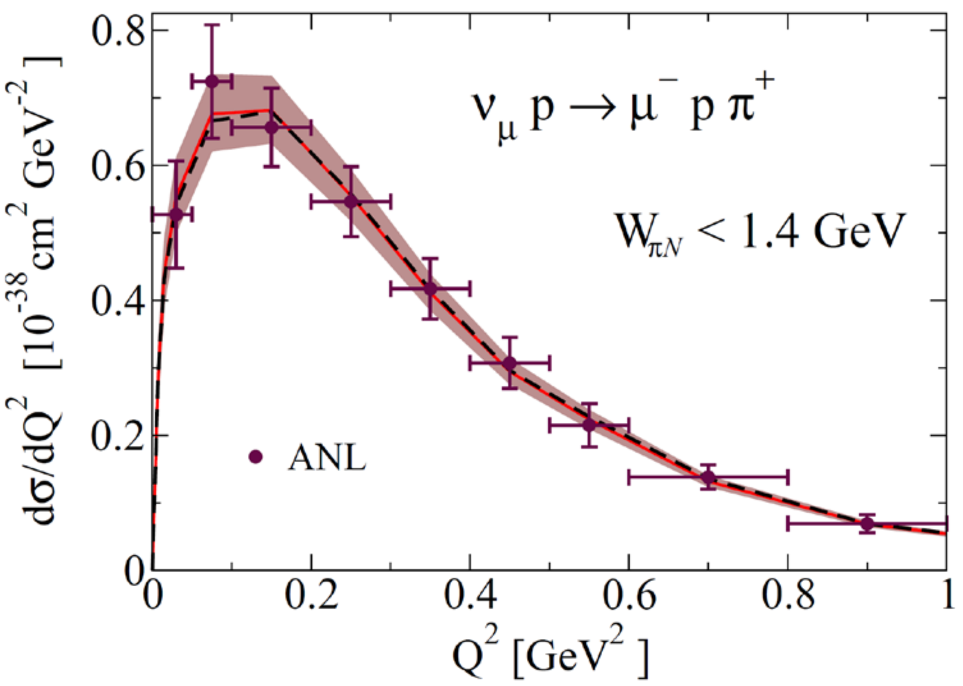
In an EFT the strength of the contact terms have to be fitted to experiment. According to this, we propose a minimal modification of our model, in which the contact terms that derive from the spin 1/2 part of the Δ propagator are multiplied by an extra parameter (low energy constant), that will be fitted to data.

$$\frac{P_{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + i\epsilon} \rightarrow \frac{P_{\mu\nu}(p_\Delta) + c\left(P_{\mu\nu}(p_\Delta) - \frac{p_\Delta^2}{M_\Delta^2} P_{\mu\nu}^{\frac{3}{2}}(p_\Delta)\right)}{p_\Delta^2 - M_\Delta^2 + i\epsilon} = \frac{P_{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + i\epsilon} + c\,\delta P_{\mu\nu}(p_\Delta)$$

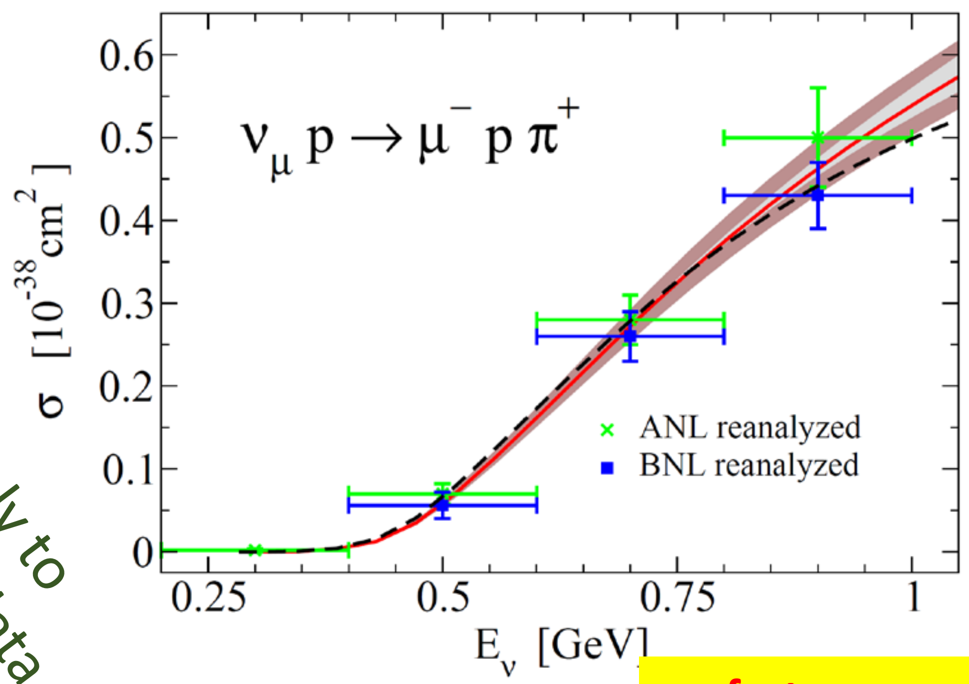
$$\rightarrow \frac{P_{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} + c\,\delta P_{\mu\nu}(p_\Delta)$$

$$= \frac{p_\Delta^2}{M_\Delta^2} \frac{P_{\mu\nu}^{\frac{3}{2}}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} + \frac{(1+c)(p_\Delta^2 - M_\Delta^2) + icM_\Delta\Gamma_\Delta}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} \delta P_{\mu\nu}(p_\Delta)$$

- the LEC c is a free parameter that will be fitted to data
- $c = 0$ original model
- $c = -1$ only propagation of spin-3/2 dof (consistent $\pi N \Delta$ coupling, see V. Pascalutsa) in the Δ propagator, up to finite Δ width corrections.



We fit only to reanalyzed data

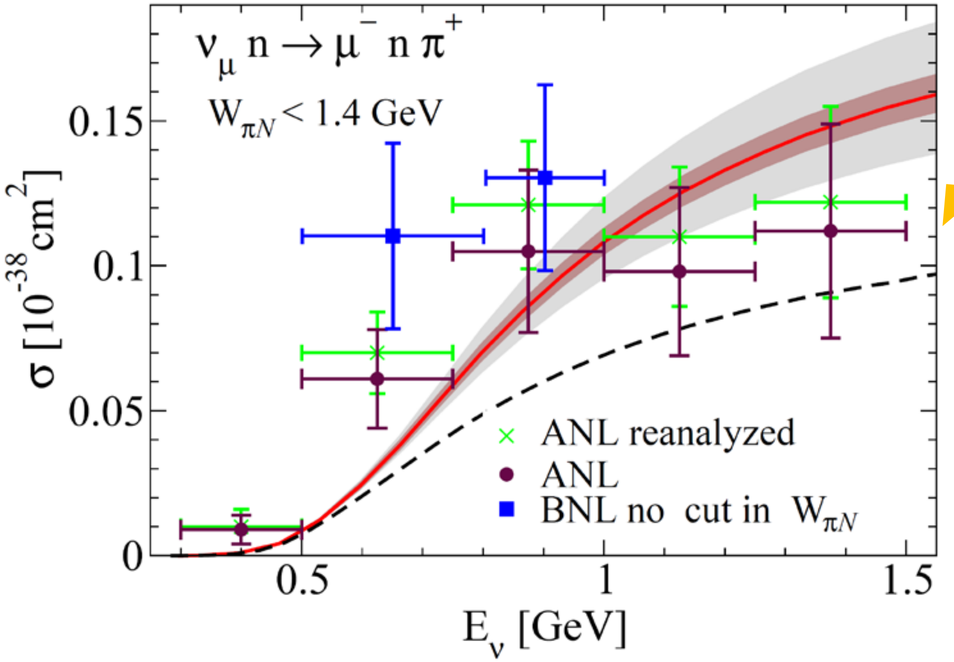


perfect agreement with the PCAC prediction ~1.2

$$C_5^A(q^2) = \underbrace{C_5^A(0)/(1-q^2/M_{A\Delta}^2)^2}$$

$C_5^A(0)=1.18\pm0.07$ $M_{A\Delta}=950\pm60$ MeV

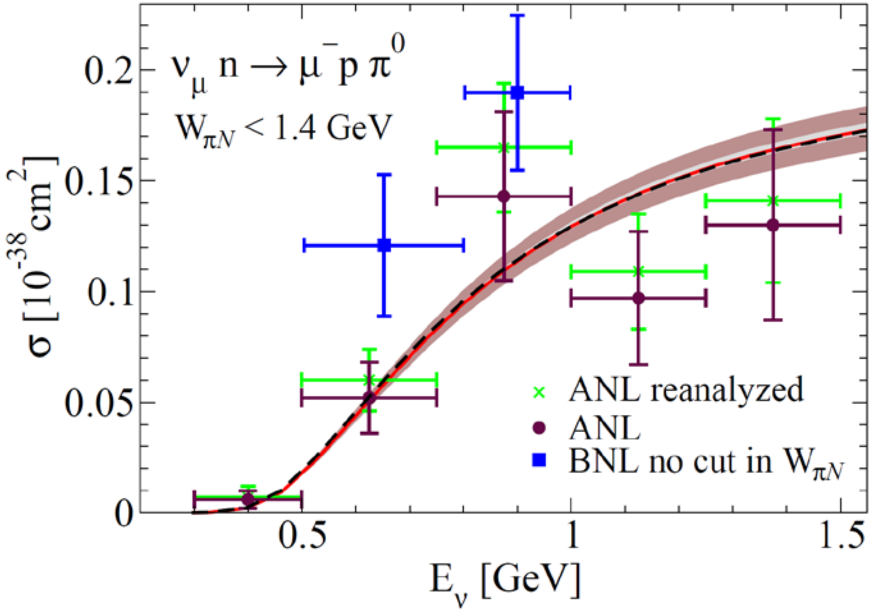
$c = -1.11 \pm 0.21$
close to −1 (large reduction of propagation of spin 1/2 dof)

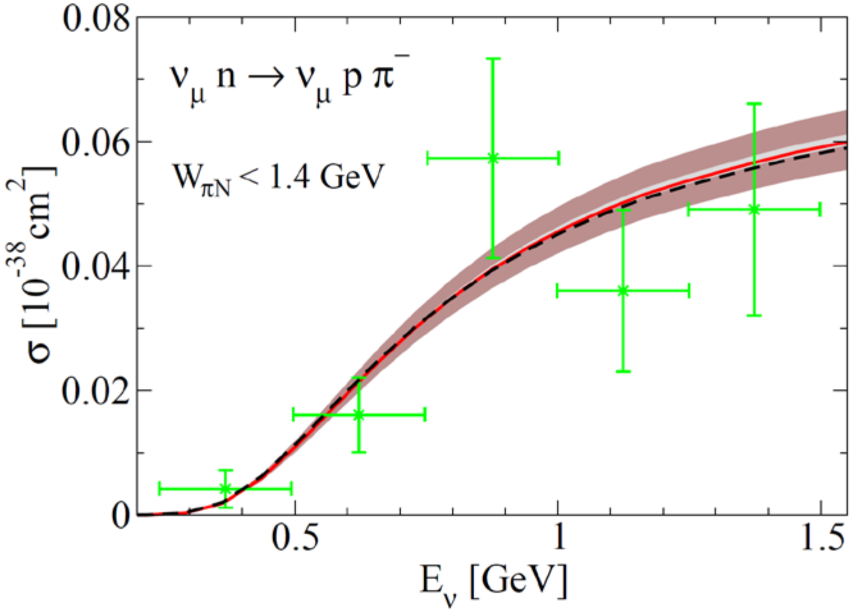


better description!

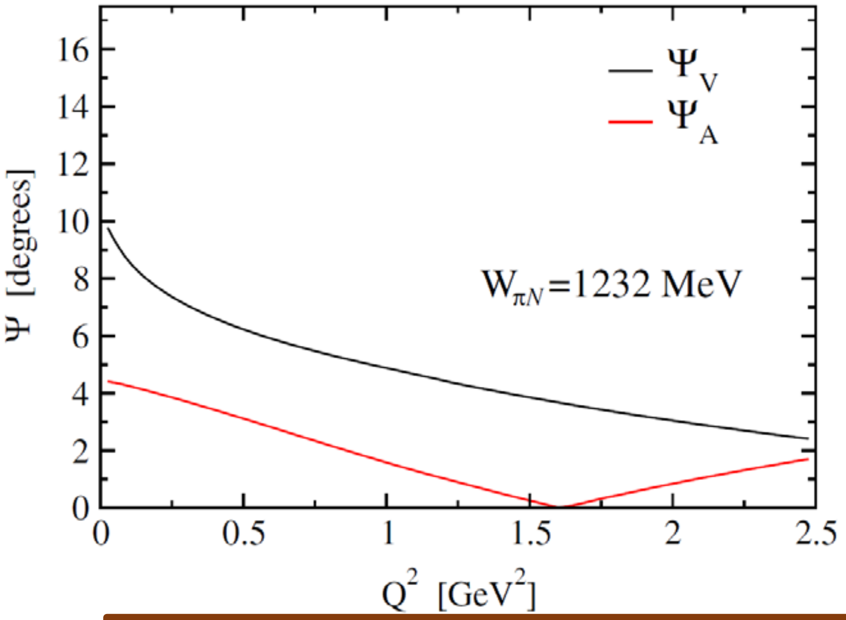
$\nu_\mu\, n \longrightarrow \mu^- n \pi^+$ data included in the fit!

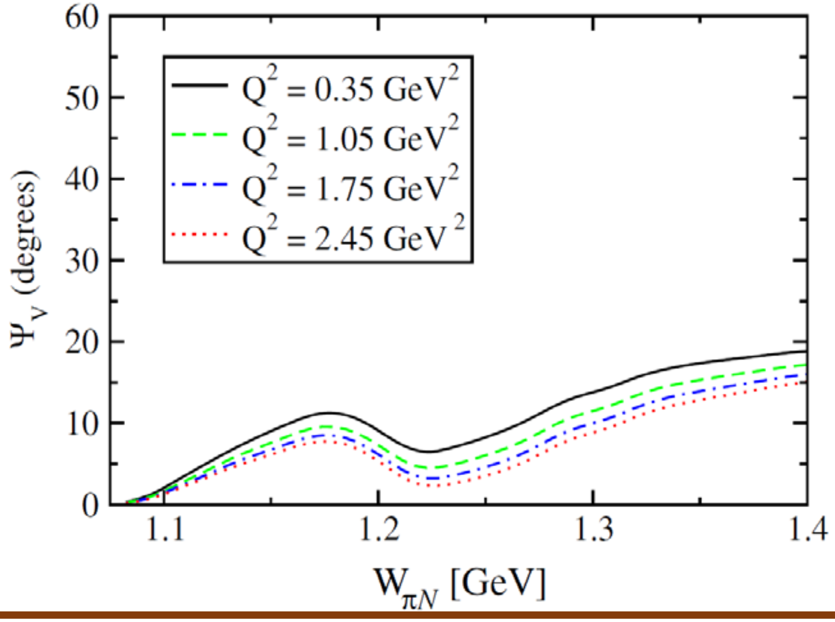
Juan Nieves, IFIC (CSIC & UV)

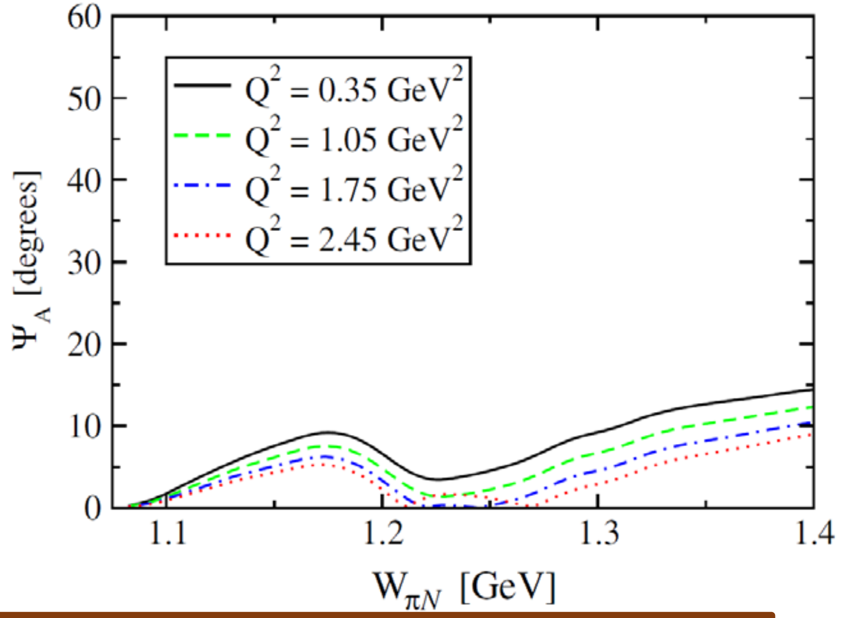




Juan Nieves, IFIC (CSIC & UV)

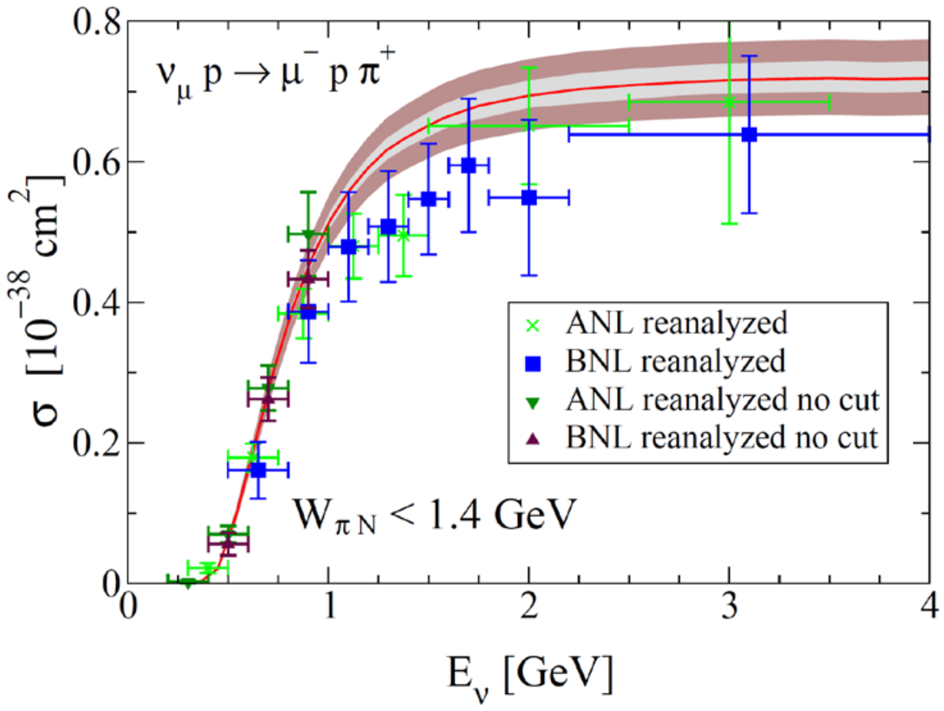






Olsson phases are significantly smaller than in the previous model. This means the present model without the phases is closer to satisfying Watson theorem!

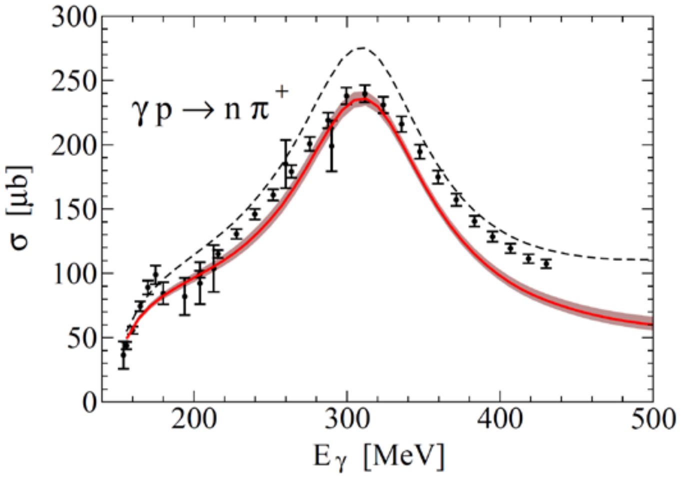
The $\nu_\mu p \to \mu^- p \pi^+$ at higher energies for $W_{\pi N} < 1.4$ GeV

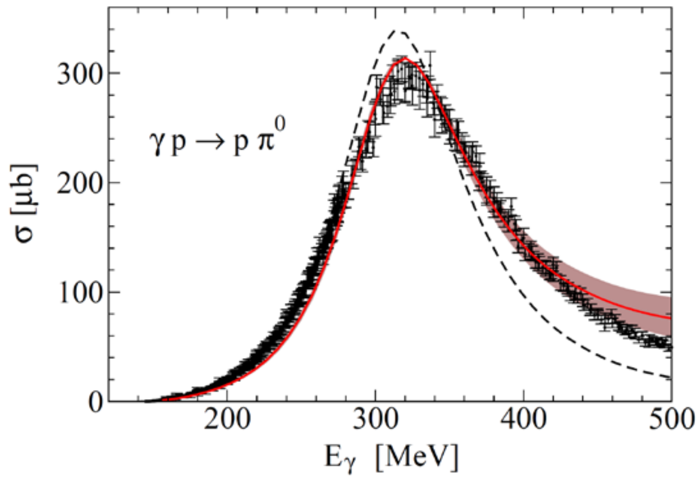


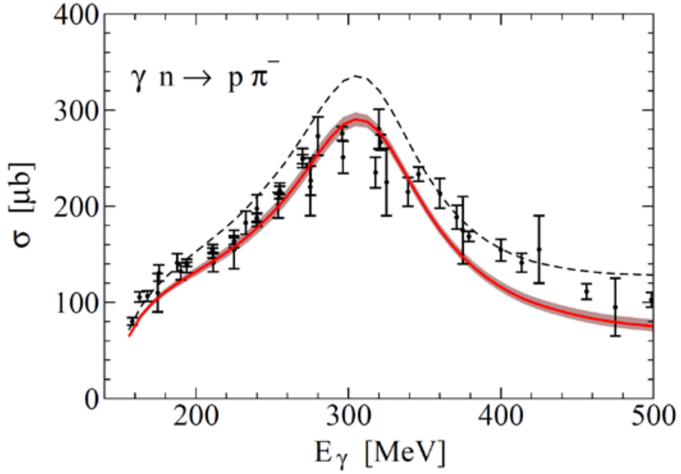
Besides, the terms that come with the C_3^A and C_4^A nucleon-to-Delta axial form factors become more relevant at higher energies, since larger q^2 values are allowed. Deviations from Adler's constraints ($C_3^A(q^2) = 0, C_4^A(q^2) = -C_5^A(q^2)/4$), that we implement so far, might play a role in describing the data at higher energies.

Effect of the new terms in pion photoproduction

The model for pion photoproduction is constructed from the vector part of our weak pion production model, including the implementation of Watson theorem

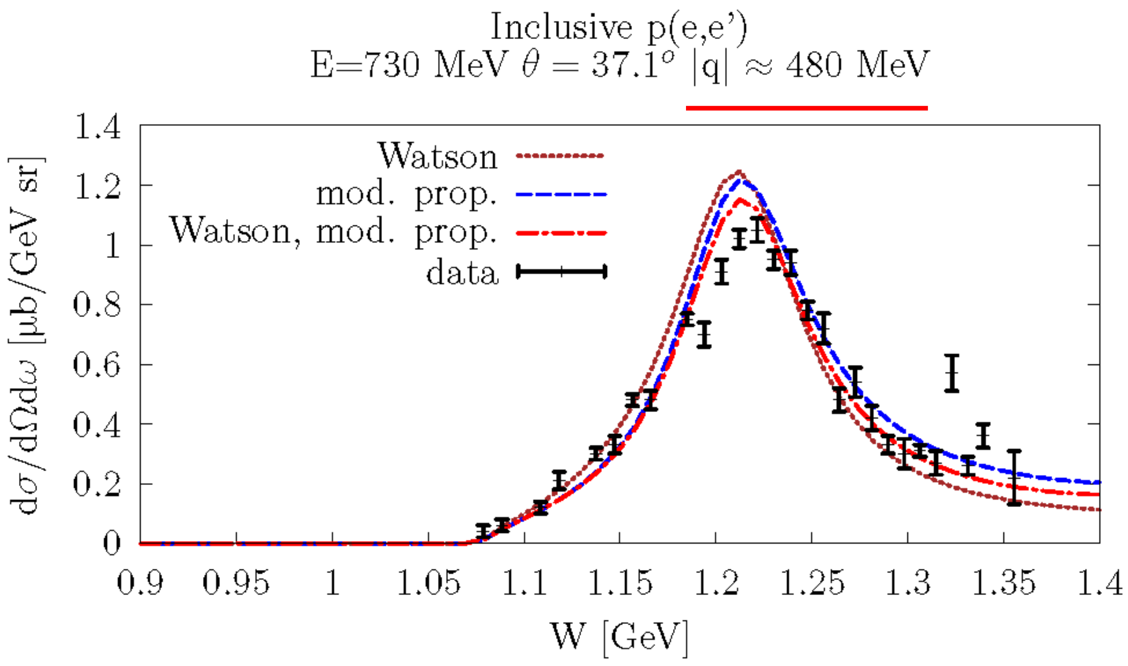


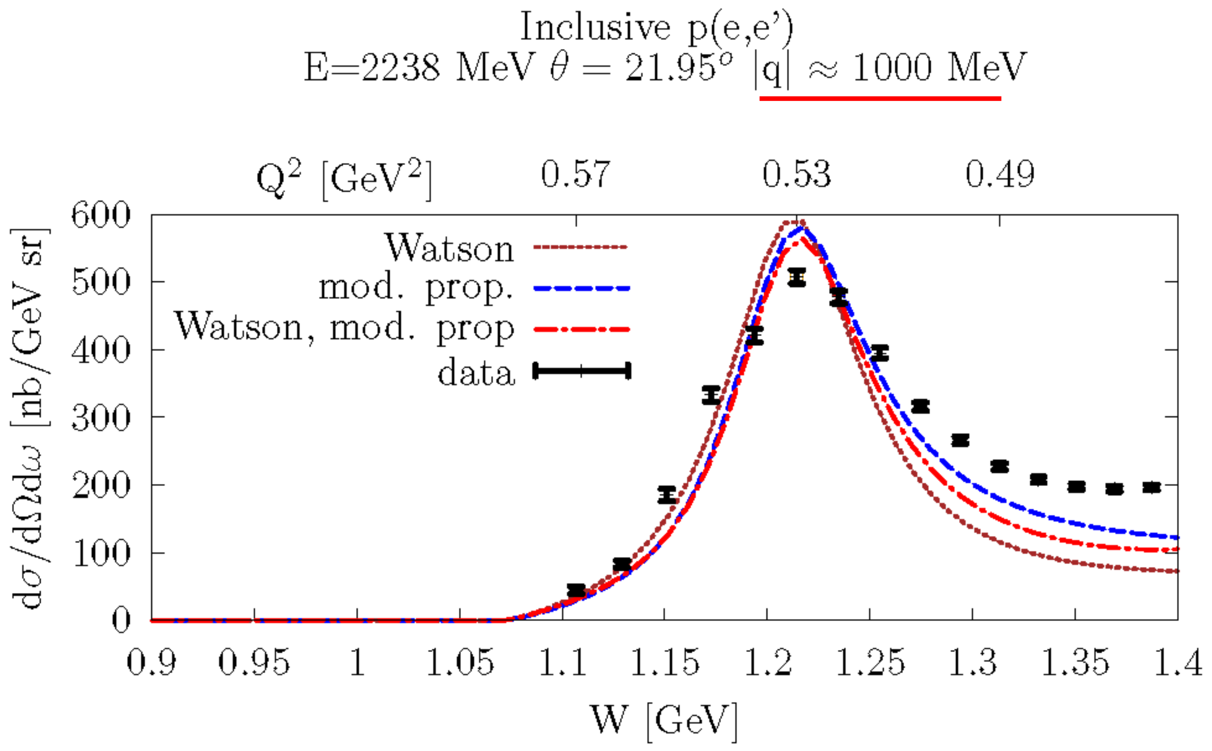


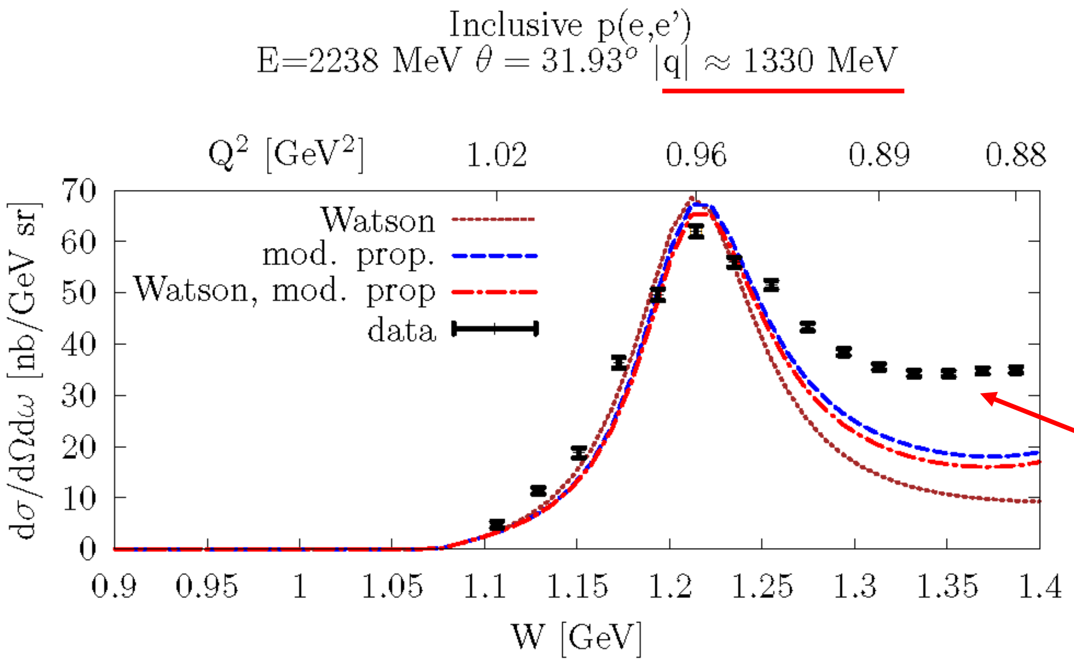


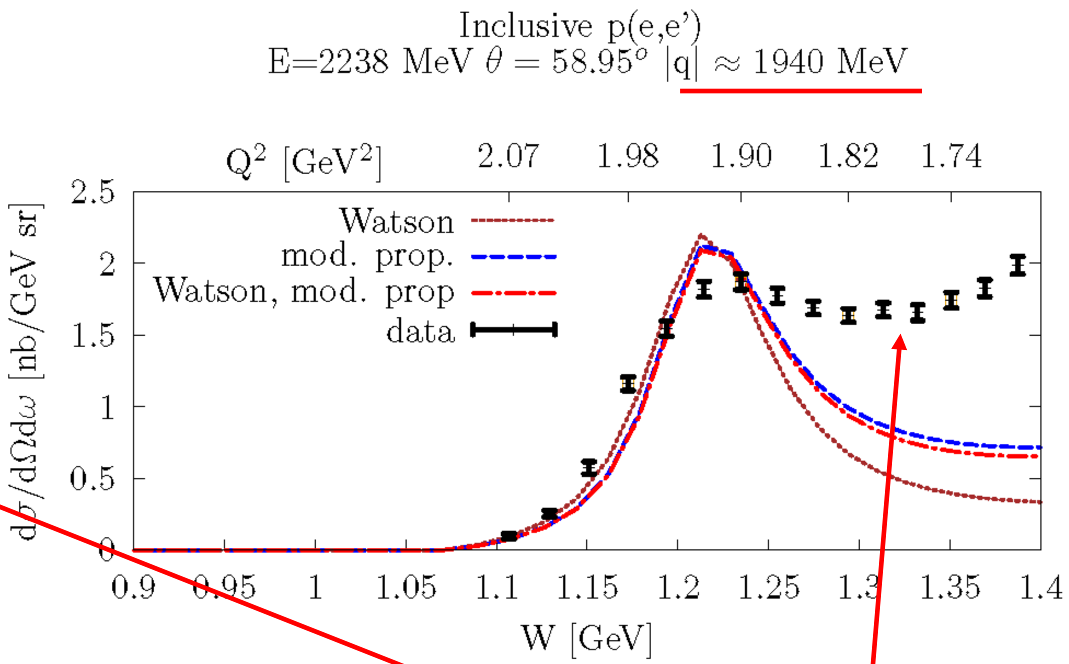
Effect of the new terms in pion eletroproduction

in collaboration with J.E. Sobczyk

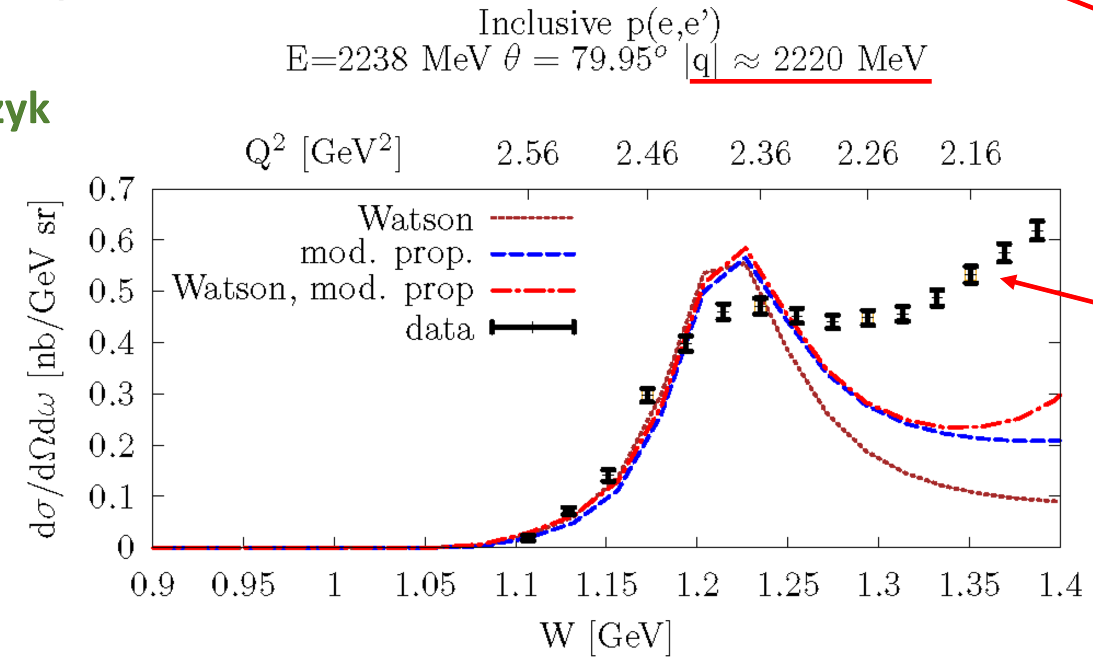






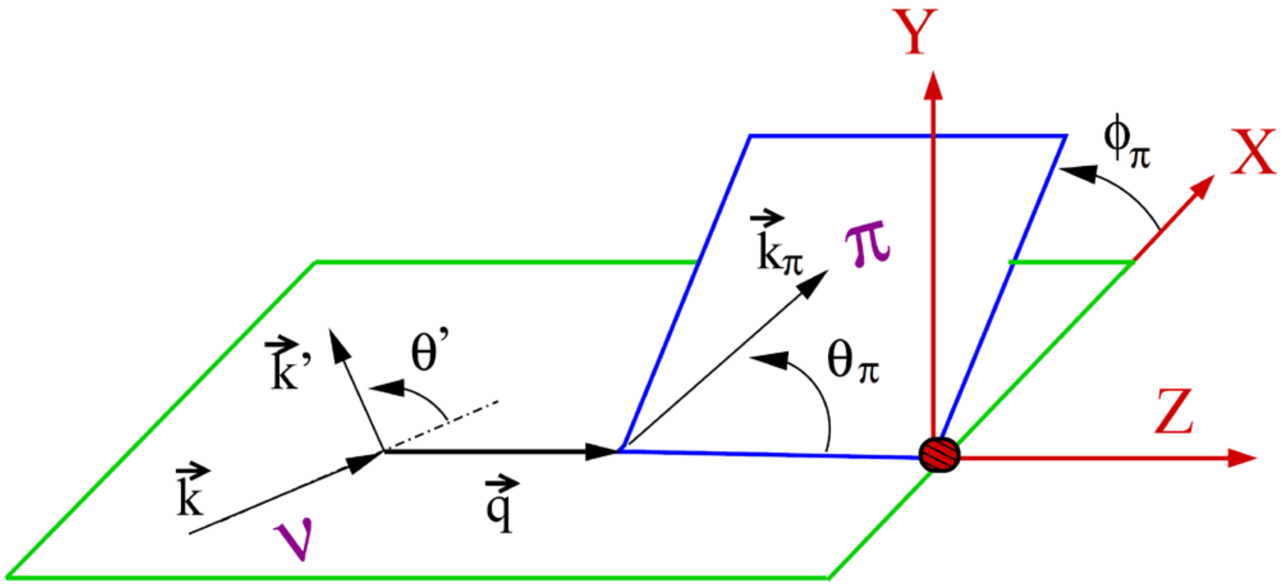


in collaboration with J.E. Sobczyk



discrepancies for momentum transfers above 1 GeV, but cross sections are much smaller!

Parity violation



- Explicit dependence on ϕ_π^*
- A^*, B^*, C^*, D^* and E^* are functions of $E, q^2, W_{\pi N}$ and θ_π^*
- For NC, C^* and E^* are the same for ν and $\bar{\nu}$

$$\frac{d^5\sigma_{\nu_l l}}{d\Omega(\hat{k}')dE'd\Omega^*(\hat{k}_\pi)} = \frac{|\vec{k}'|}{|\vec{k}\,|}\frac{G^2}{4\pi^2}\Bigg\{\overbrace{A^* + B^*\cos\phi_\pi^* + C^*\cos 2\phi_\pi^*}^{\text{Similar to } eN\to e'N\pi} + \underbrace{D^*\sin\phi_\pi^* + E^*\sin 2\phi_\pi^*}_{\text{parity violating}}\Bigg\}$$

πN CM frame

these are also T-odd terms

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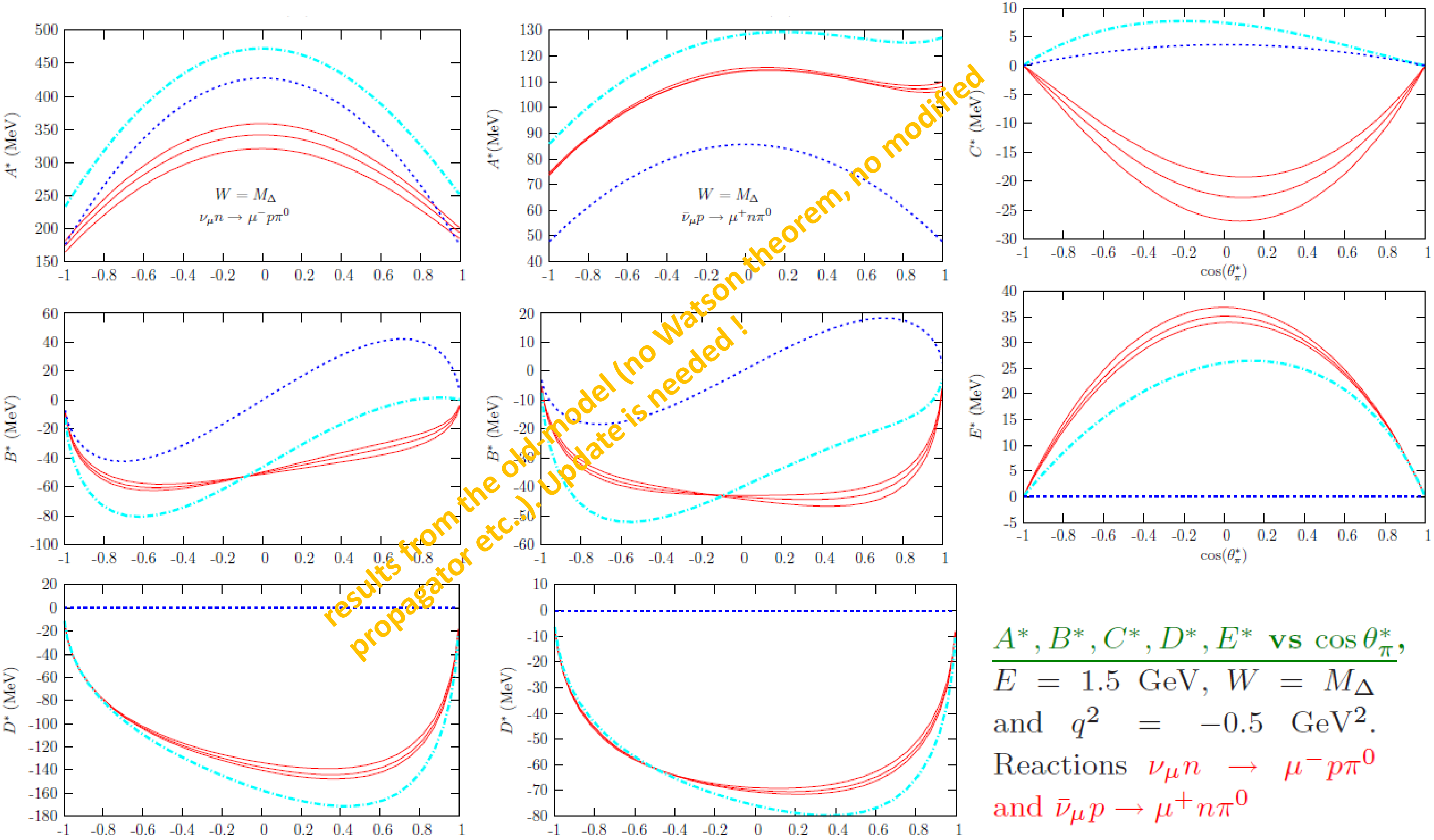

A^*, B^*, C^*, D^*, E^* **vs** $\cos\theta^*_\pi$,
$E = 1.5$ GeV, $W = M_\Delta$
and $q^2 = -0.5$ GeV^2.
Reactions $\nu_\mu n \to \mu^- p\pi^0$
and $\bar\nu_\mu p \to \mu^+ n\pi^0$

- Only direct Δ. $C_5^A(0) = 1.2$, $M_A = 1.05$ GeV
- Full model. $C_5^A(0) = 1.2$, $M_A = 1.05$GeV
- Full model. $C_5^A(0) = 0.867$, $M_A = 0.985$ GeV

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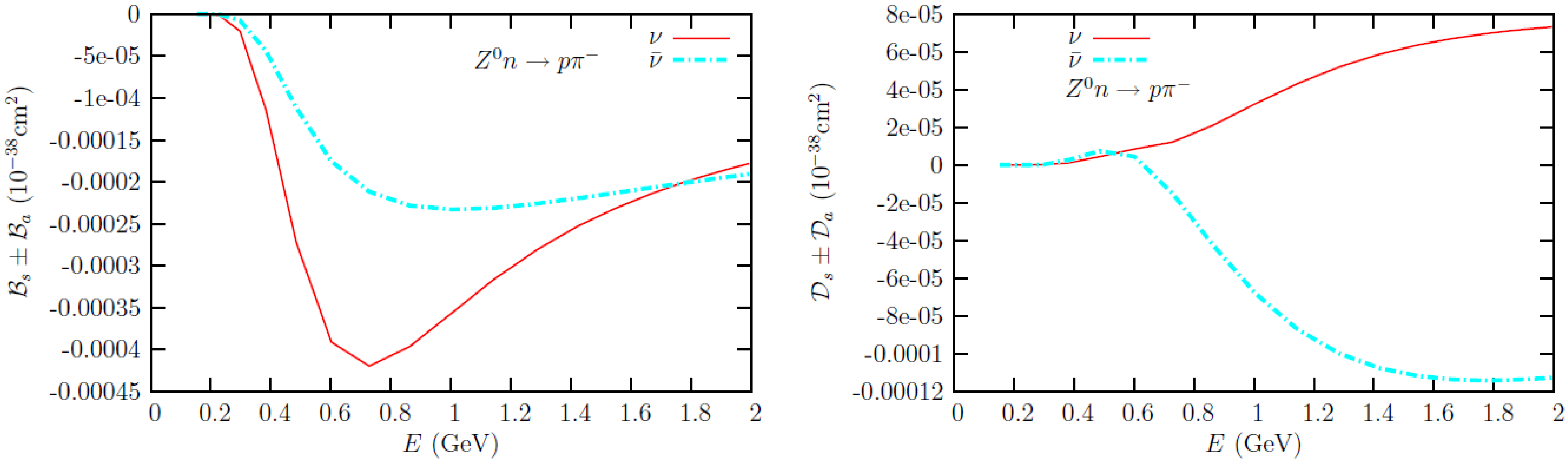
... new NC neutrino–antineutrino asymmetries

$$\frac{1}{2}\left(\frac{d\sigma(\phi_\pi)}{d\phi_\pi} - \frac{d\sigma(\phi_\pi+\pi)}{d\phi_\pi}\right)\Big|_{\nu} = (\mathcal{B}_s + \mathcal{B}_a)\cos\phi_\pi + (\mathcal{D}_s + \mathcal{D}_a)\sin\phi_\pi$$

$$\frac{1}{2}\left(\frac{d\sigma(\phi_\pi)}{d\phi_\pi} - \frac{d\sigma(\phi_\pi+\pi)}{d\phi_\pi}\right)\Big|_{\bar\nu} = (\mathcal{B}_s - \mathcal{B}_a)\cos\phi_\pi + (\mathcal{D}_s - \mathcal{D}_a)\sin\phi_\pi$$



results from the old-model (no Watson theorem, no modified propagator etc..). Update is needed !

Juan Nieves, IFIC (CSIC & UV)

$$L^{(\nu)}_{\mu\sigma} = (\mathbf{L}^{(\nu)}_{\mathbf{s}})_{\mu\sigma} + \mathrm{i}(\mathbf{L}^{(\nu)}_{\mathbf{a}})_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma} k\cdot k' + \mathrm{i}\epsilon_{\mu\sigma\alpha\beta} k'^\alpha k^\beta$$

By construction (similar for both CC and NC),

$$W^{\mu\sigma} = \mathbf{W}^{\mu\sigma}_{\mathbf{s}} + \mathrm{i}\mathbf{W}^{\mu\sigma}_{\mathbf{a}}, \quad W^{\mu\nu}_{s,a} = \left(W^{\mu\nu}_{s,a}\right)^{\mathrm{PC}} + \left(\mathbf{W}^{\mu\nu}_{\mathbf{s,a}}\right)^{\mathrm{PV}}$$

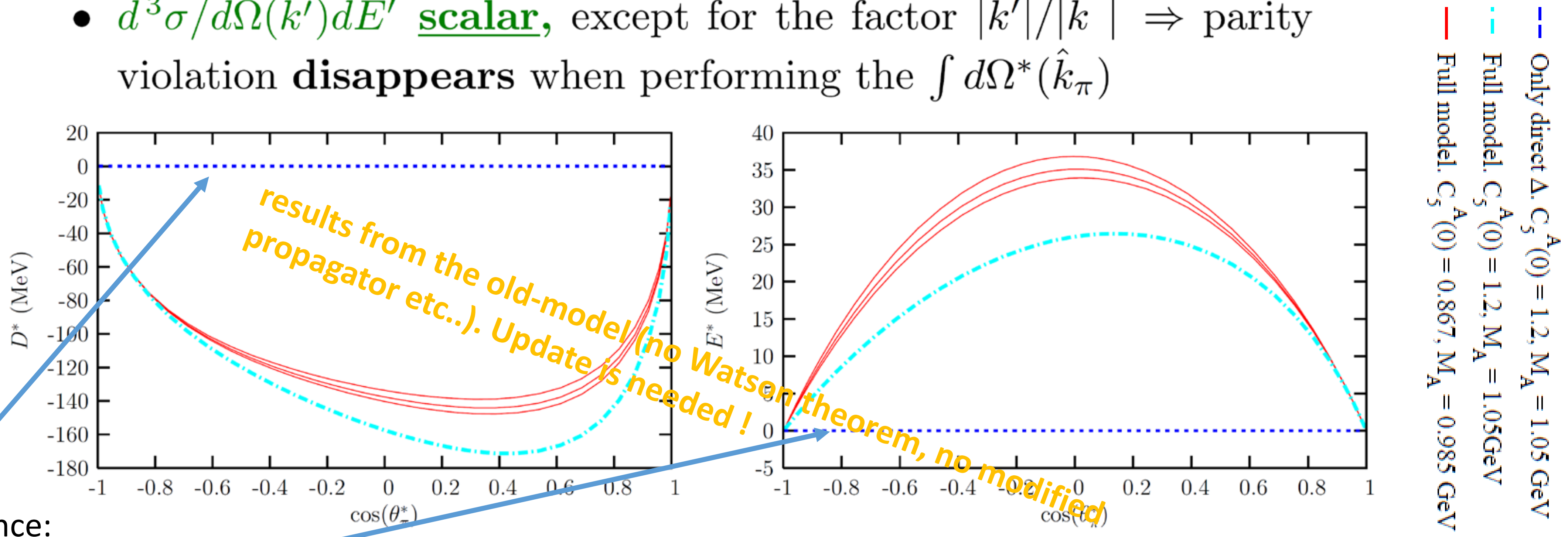
$$\begin{aligned}
(W^{\mu\nu}_s)^{\mathrm{PC}} &= W_1 g^{\mu\nu} + W_2 p^\mu p^\nu + W_3 q^\mu q^\nu + W_4 k^\mu_\pi k^\nu_\pi + \cdots \\
(W^{\mu\nu}_a)^{\mathrm{PC}} &= W_{14}\epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta + W_{15}\epsilon^{\mu\nu\alpha\beta} p_\alpha k_{\pi\beta} + W_{16}\epsilon^{\mu\nu\alpha\beta} q_\alpha k_{\pi\beta} + \cdots \\
(\mathbf{W}^{\mu\nu}_{\mathbf{s}})^{\mathrm{PV}} &= \mathbf{W_8}\left(\mathbf{q}^\mu \epsilon^\nu_{.\alpha\beta\gamma} \mathbf{k}^\alpha_\pi \mathbf{p}^\beta \mathbf{q}^\gamma + \mathbf{q}^\nu \epsilon^\mu_{.\alpha\beta\gamma} \mathbf{k}^\alpha_\pi \mathbf{p}^\beta \mathbf{q}^\gamma\right) + \cdots \\
(\mathbf{W}^{\mu\nu}_{\mathbf{a}})^{\mathrm{PV}} &= \mathbf{W_{11}}(\mathbf{q}^\mu \mathbf{p}^\nu - \mathbf{q}^\nu \mathbf{p}^\mu) + \mathbf{W_{12}}(\mathbf{q}^\mu \mathbf{k}^\nu_\pi - \mathbf{q}^\nu \mathbf{k}^\mu_\pi) + \cdots
\end{aligned}$$

Under Parity

$$L^{(\nu)}_{\mu\nu} \to (L^{\nu\mu})^{(\nu)}, \quad (W_{\mu\nu})^{\mathrm{PC}} \to (W^{\nu\mu})^{\mathrm{PC}}, \quad (\mathbf{W}_{\mu\nu})^{\mathrm{PV}} \to -(\mathbf{W}^{\nu\mu})^{\mathrm{PV}}$$

- $d^5\sigma/d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)$ **is not inv. under parity,** since the pseudovector $\vec{k}\times\vec{k}'$ is used to define the Y axis.

- $d^3\sigma/d\Omega(\hat{k}')dE'$ **scalar,** except for the factor $|\vec{k}'|/|\vec{k}\,|$ $\Rightarrow$ parity violation **disappears** when performing the $\int d\Omega^*(\hat{k}_\pi)$



only resonance: parity-violating terms vanish

$\nu_\mu n \to \mu^- p \pi^0$ $(E = 1.5\text{ GeV},\ W = M_\Delta,\ q^2 = -0.5\text{ GeV}^2)$

- **Non-resonant terms are needed to produce non-vanishing parity violating structure functions**

8. Conclusions: Model for CC and NC weak pion production off the nucleon,

- In addition to the Δ resonance, we include **non-resonant contributions** $\Longleftarrow$ QCD SχSB.
- Non resonant contributions are important $\Rightarrow$ re-adjust of $C_5^A(q^2)$. GTR prediction $C_5^A(0) \sim 1.2$.
    - Fit to ANL $\Longrightarrow C_5^A(0) = 0.867 \pm 0.075$
    - Fit to ANL & BNL + normalization uncertainties + deuteron effects $\Longrightarrow C_5^A(0) = 1.00 \pm 0.11$
    - Fit to ANL & BNL + normalization uncertainties + deuteron effects + **unitarity corrections (Watson's theorem)** $\Longrightarrow C_5^A(0) = 1.12 \pm 0.11$, **but poor description of $\nu_\mu\, p \longrightarrow n\, \pi^+$ reaction**
    - Addition of extra **contact interaction terms that mostly cancel the propagation of spin-1/2 dof in the Δ propagator** (related to the use of a consistent $\pi N \Delta$ coupling, see V. Pascalutsa) $\Longrightarrow$ $\boldsymbol{C_5^A(0) = 1.18 \pm 0.07}$ and much better **description of data, including the $\nu_\mu\, p \longrightarrow n\, \pi^+$ reaction and pion photo- and electro-production. Olsson phases become also much smaller. Nevertheless, FSI effects on single pion production off the deuteron might induce corrections on the nucleon spectator approximation, and they might be of special relevance precisely in the $\nu_\mu\, p \longrightarrow n\, \pi^+$ channel (T. Sato et al.)**

- There exist parity violation (T-odd correlations) effects due to the interferences between the non resonant and Δ contributions.
- $\boldsymbol{\nu - \bar{\nu}}$ asymmetries might be used to distinguish $\boldsymbol{\nu_\tau}$ from $\boldsymbol{\bar{\nu}_\tau}$ below the $\boldsymbol{\tau}$ −lepton production threshold.