Quenched charmonium spectrum

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Charmonium spectroscopy on the lattice is not at all straightforward. Charm is too heavy for most current simulations – typically $ma > 1$ – but too light to blindly rely on the heavy quark approximation – for $c\bar{c}$, $v^2/c^2 \sim 0.3$. A good probe of relativistic effects is the hyperfine splitting between the $^3S_1$ and the $^1S_0$ states, which for charmonium is $\Delta M = 117$ MeV. Lattice quenched calculations underestimate ΔM by about 30−50% [1], the prediction from NRQCD being $\Delta M = 55(5)$ MeV [2]. The discrepancy could be due to the quenched approximation. However, first estimates including dynamical quarks seem to indicate that they account for a difference of only $\sim 10\%$ [3]. Our methodology here is to compute ΔM on very fine *isotropic* lattices within the quenched relativistic formalism. If the lattices used are sufficiently fine, this approach should allow for a controlled quenched continuum extrapolation, free of the systematic uncertainties which may have affected other determinations up to now. Preliminary results of this calculation have been presented in [4]. Two approximations will be made: (*i*) quenched, (*ii*) OZI, meaning that Zweig-rule forbidden diagrams, although contributing to singlet mesons like charmonium, will not be included.

Our results for the non-perturbatively improved clover Dirac operator are presented in Fig. 1. The observed lattice spacing dependence is very small. Scaling violations are expected to be $\mathcal{O}(a^2)$ and indeed the cutoff dependence is well fitted linearly in a^2. The continuum extrapolation is $\Delta M = 82(1)$ MeV. This is about 30% below the experimental value.

It has been often reported [1, 2] that the continuum extrapolated hyperfine splitting strongly depends on the choice of Dirac operator. Since the continuum limit is unique this necessarily reflects the existence of big lattice artifacts (typically $aM(\eta_c) > 1$). As shown next, even with the non-improved Wilson Dirac operator a reasonable estimate of the hyperfine splitting can be obtained iff, lattices with spacing $a \leq 0.07$ fm, i.e. $aM(\eta_c) \leq 1$, are used.

To test the approach to the continuum, we present in Fig. 1 the result of extrapolating the three largest β values: linearly in a (a^2) for the Wilson (tree-level clover) Dirac operator. We obtain as continuum extrapolations for the Wilson and tree-level clover respectively: $\Delta M = 73(5)$ and 75(1) MeV. Several remarks are in order here: (*i*) Only the non-perturbatively improved data show a weak dependence on the lattice spacing. Wilson and tree-level clover data are both significantly below their continuum extrapolations at all the simulated β values. (*ii*) Linear extrapolations in a and a^2 for Wilson and tree-level clover respectively, including $\beta = 6.0$ ($aM(\eta_c) = 1.4$) are not justified. (*iii*) If too coarse lattices are included in the fit, extrapolations become quite sensitive to the assumed dependence on the lattice spacing. A lower bound for consistent extrapolations seems to be $a \leq 0.07$ fm ($aM(\eta_c) \leq 1$) for Wilson and tree-level clover. Surprisingly, though, non-perturbative clover improvement seems to work well even on the coarser lattice where $aM(\eta_c) = 1.4$. To summarize, non-perturbative clover improvement seems crucial to remove strong scaling violations.

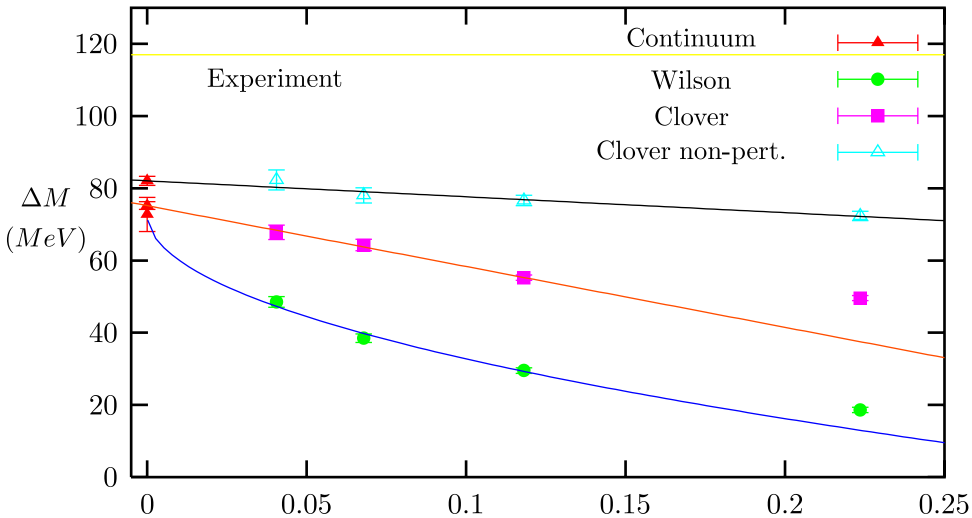

Figure 1: Continuum extrapolation of ΔM from different Dirac operators.

Conclusions: We believe our result finally closes the long debate on the magnitude of the quenched charmonium hyperfine splitting within the OZI approximation. Our final result is $M(J/\Psi - \eta_c) = 82(1)(4)$MeV. Our value remains 30% below the experimental result. Dynamical quark effects are usually expected to be O(10-20)%, which amounts for a large part but not all of the discrepancy. Actual dynamical quark effects may turn out to be larger for this quantity but the remaining discrepancy might also be due to the OZI approximation.

Our study shows the virtue of the “brute force” approach: reliable, consistent continuum extrapolations can be obtained from improved or non-improved discretizations if the lattice is fine enough, $aM_{q\bar{q}} \leq 1$. Remarkably however, the non-perturbatively improved clover Dirac operator appears to give reliable extrapolations even starting from $aM_{q\bar{q}} \sim 1.4$. We consider our Figure 1 as a spectacular advertisement for using this Dirac discretization.

Acknowledgements: We would like to acknowledge the use of the SX5 at the Research Center for Nuclear Physics at Osaka University where the simulations have been performed.

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