Color Superconducting Quark Droplets

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The properties of quark matter have attracted a good deal of interest since it has been suggested that strange quark matter could be a ground state of strongly interacting matter [1]. The hypothesis may be realized in ongoing experimental strangelet searches and forthcoming cosmic ray space experiments. From the theory side, on the other hand, the stability of strange quark matter has been extensively investigated by making use of the various QCD motivated models. Within the MIT bag model, there exists a reasonable window of the model parameters (i.e. the bag constant, current quark masses and strong coupling constant) for which (finite lumps of) strange quark matter is stable as compared to a gas of ^{56}Fe. In Ref. [2], Madsen finds striking effects of a phase called color-flavor locked phase (CFL phase): CFL strangelets are more stable than strangelets without the CFL. However, this conjecture depends on the phase structure of cold dense QCD. In this work, as the first attempt to tackle this problem, we investigate the two-flavor color superconductivity (2SC) in finite quark droplets.

To describe the 2SC we choose to use the Nambu–Jona-Lasinio (NJL) model. The Lagrangian is given by

$$\mathcal{L} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + G_1 \left[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \vec{\tau} \psi)^2 \right],$$

where ψ denotes a quark field with two flavors ($N_f = 2$) and three colors ($N_c = 3$), the Pauli matrices $\vec{\tau}$ act in the flavor space and the coupling constant G_1 has a dimension $[G_1] = [\mathrm{mass}]^{-2}$. Since the model is not renormalizable, we use a sharp cutoff Λ in the three dimensional momentum space.

In the case of the 2SC, the quarks prefer to form $J^P = 0^+$ Cooper pairs in the color antitriplet flavor singlet channel. Then, the interaction corresponding to the channel is given by

$$\mathcal{L}_{int} = G_2 (\bar{\psi} i \gamma_5 \tau_2 \lambda_2 C \bar{\psi}^T)(\psi C i \gamma_5 \tau_2 \lambda_2 \psi),$$

where C is a charge conjugation matrix and τ_2 $[\lambda_2]$ denotes the antisymmetric generator of $\mathrm{SU}(2)_{\mathrm{flavor}}$ $[\mathrm{SU}(3)_{\mathrm{color}}]$. The coupling constant G_2 can be obtained from the NJL Lagrangian by making use of the Fierz transformation. In this work, however, we consider it to be a free parameter.

We restrict ourselves to zero temperature and work in the mean-field approximation. In the mean-field approximation, the thermodynamic potential $\Omega = \Omega(\Delta; \mu)$ at finite chemical potential μ is obtained as

$$\begin{aligned} \Omega \;=\; & \frac{\Delta^2}{4G_2} - 2N_f \int_0^\Lambda \frac{k^2 dk}{2\pi^2} \left[k - (k-\mu)\theta(\mu - k) \right] \\ & - 2N_f \int_0^\Lambda \frac{k^2 dk}{2\pi^2} \left[|k-\mu| \sqrt{1 + \frac{\Delta^2}{(k-\mu)^2}} + (k+\mu)\sqrt{1 + \frac{\Delta^2}{(k+\mu)^2}} \right], \end{aligned}$$

where $\Delta = -2G_2\langle\bar{\psi}i\gamma_5\tau_2\lambda_2 C\bar{\psi}^T\rangle$ denotes the dynamically generated gap.

To incorporate finite size effects, we use the so-called multiple reflection expansion (MRE). In the MRE framework, the density of states for a spherical system and for *massless quarks* is written as $k^2\rho_{\rm MRE}/2\pi^2$, where $\rho_{\rm MRE} = \rho_{\rm MRE}(k, R)$ is given by

$$\rho_{\rm MRE} = 1 - \frac{1}{2k^2R^2},$$

with R being the radius of the sphere. Here, it should be noted that the second term, proportional to $1/R^2$, represents the curvature contribution to the fermionic density of states.

Using the MRE density of states, we express the thermodynamic potential for the spherical system as a function of the variational parameter Δ:

$$\begin{aligned}\Omega &= \frac{\Delta^2}{4G_2} - 2N_f\int_0^\Lambda \frac{k^2dk}{2\pi^2}\rho_{\rm MRE}\left[k - (k-\mu)\theta(\mu - k)\right] \\ &\quad -2N_f\int_0^\Lambda \frac{k^2dk}{2\pi^2}\rho_{\rm MRE}\left[|k-\mu|\sqrt{1+\frac{\Delta^2}{(k-\mu)^2}} + (k+\mu)\sqrt{1+\frac{\Delta^2}{(k+\mu)^2}}\right].\end{aligned}$$

Let us consider a droplet which has a definite baryon number A and a volume V. Then, the quark number density n has to satisfy the condition $n = 3A/V$ where n is given by

$$\begin{aligned}n &= -\frac{\partial\Omega}{\partial\mu} \\ &= \frac{N_f\mu^3}{3\pi^2} - \frac{N_f\mu}{2\pi^2R^2} - 2N_f\int_0^\Lambda \frac{k^2dk}{2\pi^2}\rho_{\rm MRE}\left\{\frac{k-\mu}{\sqrt{(k-\mu)^2+\Delta^2}} - \frac{k+\mu}{\sqrt{(k+\mu)^2+\Delta^2}}\right\}.\end{aligned}$$

Combining these equations, we can calculate the effective potential as a function of Δ and investigate the 2SC in finite quark droplets. Here, we mention an example in regard to the numerical results. Our study shows that the MRE density of states have an insignificant effect on the 2SC gap and the bulk limit is a good approximation. The result would implies that the physics in the vicinity of the Fermi surface is not influenced by the finite size effects.

Finally, we comment on the outlook for future studies. We plan to study the color-flavor unlocking phase transition of (2+1) flavor QCD and the stability of the color superconducting quark droplets, including finite isospin density which is required from chemical equilibrium and charge neutrality.

References

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