Faddeev-Yakubovsky Calculation Above the 4-body Break-Up Threshold

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The Schödinger and Lippmann-Schwinger (LS) equations are equivalent within the non-relativistic quantum mechanics. The former has a difficulty to consider the boundary condition in a continuous state on three- or more-particle system. The latter has a difficulty to treat a singularity of the Green's function. Both difficulties are equivalent.

We employ the 4-body Faddeev-Yakubovsky (FY) equation based on the LS equation, due to an advantage that the same formalism is available in both of the discrete and continuous states. In the *ppnn* 4-nucleon system, as an example, there are $3N + N$, *dd*, *dpn*, and 4-body break-up thresholds, neglecting the Coulomb force. The nature of the singularities from the $3N + N$ and *dd* thresholds are the same. At lower energies than the *dpn* threshold, we can solve the FY equation using the technique of principal value and residue[1, 2]. In the case between the *dpn* and 4-body break-up thresholds, contour deformation method is applicable[3, 4]. The Green's function is expressed as $G_0 = \dfrac{1}{E + i\varepsilon - p^2/2\mu}$, where E is energy of the system, p is the momentum, and μ is the reduced mass. In both technique mentioned above, first one takes the limiting value of $\varepsilon \to 0$ analytically, next one avoids the integration pass on the complex plane. It is difficult to apply it at energies above the 4-body break-up threshold, then nobody have succeeded to solve the FY equation.

Recently two of the authors (HK and YK) found the Complex Energy Method (CEM)[5] as follows. First we solve the FY equation with finite ε. There are no singularities on the real momentum axis, then it is easy to solve. After obtaining several solutions with various ε's, we takes the limiting value of $\varepsilon \to 0$ numerically with an analytical continuation method.

No singularities does not mean no effects from them. For instance, there is a pole in the 2-body Green's function. In the finite ε case, the function changes from large positive to large negative values around the pole. The domain of the mesh points are $-1 \le x \le 1$ in the Gauss-Legendre method, and one usually convert it to $0 \le p < \infty$ using $p = A\dfrac{x+1}{x-1}$, where A is a parameter defined empirically for quicker convergence. If $f(x)$ is an increasing function and its range is $-1 \le f(x) \le 1$, we may use $p = A\dfrac{f(x)+1}{f(x)-1}$ for a converting function. In this work we employ the function $f(x) = \dfrac{x + Bx^3}{1+B}$, define A as p to be the pole when $x = 0$, and define B empirically for quicker convergence.

Present work aims to demonstrate that we can obtain enough converged solutions of the FY equation above the 4-body break-up threshold. We apply CEM to the 4-nucleon system as the first attempt and employ the Yamaguchi potential[6, 7, 8] as the N-N interaction for $^1S_0$ and $^3S_1$-$^3D_1$ states. Merely the total J and parity $1/2^+$ state is inputted in the [3+1] subsystem, and all arrowed channel which both of the two pairs to be $^1S_0$ or $^3S_1$-$^3D_1$ states in the [2+2] subsystem. These subamplitudes are expanded in a separable form using the energy dependent pole expansion[9] method and we take 4 ranks. As for the 4-body system we calculate only for 0^+ of total J and parity and total isospin 0 state. The Coulomb force is ignored. As for the analytical continuation, we employ the point method[10].

We checked the convergence of present calculations at 1MeV above from the dd threshold, 1MeV below from the 4-body break-up threshold, and 12MeV above from 4-body break-up threshold. Table 1 is a demonstration of convergence behavior in the last case. In all cases we obtain converged solutions by 4 or 5 digits.

Next schedule is to check convergence by the separable expansion in 3-body and [2+2] subsystems, or to improve the calculation code to apply some technique without separable expansion. After doing this, we'd like to increase the state channels for discussing Physics.

Table 1: Scattering amplitudes at 12MeV above from 4-body break-up threshold. We set $\varepsilon = 0.75$, 1.0, 1.25, ..., 4.25MeV. Column [A] shows numerical results for p^3H elastic channel, [B] for dd to p^3H or reversed reaction channel, and [C] for dd elastic channel. All of the initial and final channels are in S-state. The columns expressed as "Re" and "Im" are the real and imaginary parts of the numerical on-shell amplitudes with unit fm^{-1}, and the column "n" indicates the number of solutions inputted into the point method, in order of ε from the smallest. The bottom row expressed as "conv." indicates the converged value.

	[A]			[B]			[C]	
n	Re	Im	n	Re	Im	n	Re	Im
1	-0.342769	0.694761	1	-0.507264	0.327950	1	0.105518	0.459786
2	-0.328530	0.654652	2	-0.548204	0.360900	2	0.191365	0.367360
3	-0.332216	0.643763	3	-0.535917	0.359346	3	0.223403	0.347832
4	-0.332067	0.644817	4	-0.534384	0.359050	4	0.223384	0.348151
5	-0.332134	0.644706	5	-0.534492	0.358726	5	0.222945	0.347691
6	-0.332112	0.644977	6	-0.534727	0.358841	6	0.223425	0.348507
7	-0.331885	0.644964	7	-0.533468	0.359478	7	0.223490	0.348477
8	-0.331893	0.644958	8	-0.534110	0.358765	8	0.223499	0.348237
9	-0.331891	0.644963	9	-0.534213	0.358803	9	0.223543	0.348190
10	-0.331888	0.644945	10	-0.534188	0.358847	10	0.223520	0.348229
11	-0.331885	0.644943	11	-0.533989	0.358687	11	0.223303	0.347957
12	-0.331888	0.644947	12	-0.534219	0.358676	12	0.223232	0.348764
13	-0.331879	0.644946	13	-0.534201	0.358710	13	0.223325	0.348618
14	-0.331881	0.644948	14	-0.534200	0.358696	14	0.223323	0.348633
15	-0.331867	0.644948	15	-0.534198	0.358726	15	0.223325	0.348616
conv.	-0.3319	0.64495	conv.	-0.53420	0.3587	conv.	0.2233	0.3486

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