Full-relativistic calculation of nuclear polarization in muonic atom

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In the precise evaluation of the energy levels in muonic atom, we have to consider the effect that electromagnetic field of the muon polarizes the nucleus. The calculations of the nuclear polarization (NP) correction have been presented in electronic and muonic ^{208}Pb with the nonrelativistic random-phase approximation (RPA) [1, 2]. The NP corrections in ^{208}Pb and ^{238}U have been also calculated with the nuclear collective model[3]. The common features of the results of these analyses are found that, without the inclusion of the seagull diagrams in Fig. 1, there exists a large violation of the gauge invariance in the NP results. The seagull diagram comes from the minimal prescription of the electro-magnetic coupling for the nonrelativistic Hamiltonian of nucleon, in which the anti-nucleon degree of freedom is eliminated. The NP calculation in the relativistic field theoretical nuclear model, therefore, is interesting in the view point that the negative-energy states of nucleus contribute instead of the seagull diagram.

The NP energy shifts due to the ladder and cross diagrams depicted in Fig. 1 are obtained as follows:

$$\Delta E^L_{NP} = -i(4\pi\alpha)^2 \int \frac{d\omega}{2\pi} \int \frac{d\vec{q}}{(2\pi)^3} \int \frac{d\vec{q}'}{(2\pi)^3} D_{\mu\xi}(\omega,\vec{q}) D_{\zeta\nu}(\omega,\vec{q}') \times \sum_{i'} \frac{j^\mu_m(-\vec{q})_{ii'} j^\nu_m(\vec{q}')_{i'i}}{\omega+\omega_m - iE_{i'}\epsilon} \sum_{I'} \frac{J^\xi_N(\vec{q})_{II'} J^\zeta_N(-\vec{q}')_{I'I}}{\omega-\omega_N+iE_{I'}\epsilon} \tag{1}$$

and

$$\Delta E^X_{NP} = i(4\pi\alpha)^2 \int \frac{d\omega}{2\pi} \int \frac{d\vec{q}}{(2\pi)^3} \int \frac{d\vec{q}'}{(2\pi)^3} D_{\mu\xi}(\omega,\vec{q}) D_{\zeta\nu}(\omega,\vec{q}') \times \sum_{i'} \frac{j^\mu_m(-\vec{q})_{ii'} j^\nu_m(\vec{q}')_{i'i}}{\omega+\omega_m - iE_{i'}\epsilon} \sum_{I'} \frac{J^\zeta_N(-\vec{q}')_{II'} J^\xi_N(\vec{q})_{I'I}}{\omega+\omega_N-iE_{I'}\epsilon}, \tag{2}$$

respectively. In order to carry out the full-relativistic calculation, the nuclear transition densities $J^\xi_N(\vec{x})_{II'}$ are constructed by the relativistic RPA. Then, we need not take into account the seagull diagram in the NP corrections. Thus, the lowest-order NP correction can be given by the sum only $\Delta E^L_{NP} + \Delta E^X_{NP}$.

The NP correction of $1s_{1/2}$ state for muonic ^{16}O are shown in Table I. Comparison between the results with Feynman gauge and those of the Coulomb gauge shows that the gauge invariance of NP energy is very well satisfied. In particular, the contribution from the negative-energy states of nucleus to the NP effect is significant and also essential to achieve gauge invariance in spite of its large excitation energy more that 1 GeV.

By relativistic treatment of the nucleus, the negative-energy nucleon states, as well as the negative-energy muon states, both of which represent the blocking effect of the response of

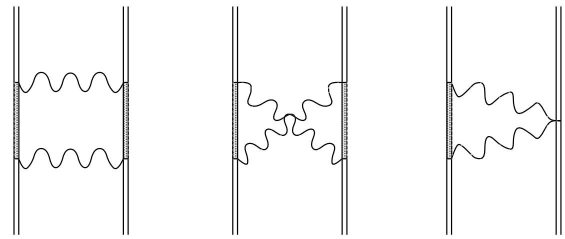

Figure 1: Diagrams contributing to the nuclear polarization in the lowest order; (a) ladder, (b) cross, and (c) seagull diagrams. The wavy line denotes photon, while the double straight lines denote the muon and the nucleus.

Table 1: Nuclear-polarization correction (eV) to the $1s_{1/2}$ state of muonic ^{16}O with the parameter set NL-HS. In the column $\omega_N\,(\omega_\mu)$, + denotes contribution from ordinary positive-energy states of ^{16}O(muon), while − denotes contribution from negative-energy nucleon(muon) states.

ω_N	ω_μ	Feynman[a]	Coulomb[b]	CNP[c]
+	+	−10.222	−10.262	−9.803
+	−	+1.002	+0.662	+0.251
−	+	+0.387	+0.213	+0.007
−	−	−1.171	−0.623	−0.001
Total		**−10.005**	**−10.008**	**−9.546**

[a] The NP correction in the Feynman gauge.

[b] The NP correction in the Coulomb gauge.

[c] The NP correction in the Coulomb gauge without transverse part.

vacuum, appear in the intermediate states and contribute to the NP correction. The transverse form factor with the negative-energy bound states of nucleus has a large overlap with that of muon. Therefore, the contribution of the negative-energy states is non-negligible in the NP correction. As found from the last column in Table I where only Coulomb interaction are taken into account, one can confirm that almost all of the contribution from the negative-energy states of nucleus comes from the transverse contributions.

It remains to be studied for heavy muonic and electronic atoms, for which the NP effects can be measured experimentally. The NP effects in these atoms may depend rather sensitively on the details of the relativistic nuclear models. The quantitative estimate of NP effects with the QHD type models may provide information on the strong binding of the anti-nucleon.

References

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