Color Van der Waals force in the color dependent confining potential

Naoko Ikezi[1], Hiroshi Toki[2], Veljko Dmitrasinovic[3]
[1] *Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan*
[2] *Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan*
[3] *Vinca Institute of Nuclear Sciences P.O.Box 522, 11001 Belgrad, Serbia*

The primary motivation of this research is to clarify the role of hidden color states for the structure of exotic hadrons by means of a non-perturbative microscopic calculation. In order to do this, we work on tetra-quark system in the non-relativistic constituent quark model by using the Gaussian expansion method [1]. It is often said that this model artificially induces large Van der Waals force due to the strong color dependent confining potential. We show that large Van der Waals force does not appear in this model even if we fully take into account hidden color states for the inner cluster in the tetra-quark system.

The model Hamiltonian is given by : $H = \sum_{i=1}^{4}\left[m_i + \frac{p_i^2}{2m_i}\right] - \sum_{i=1}^{4}\sum_{j<i}^{4}\left[e_0 + \frac{1}{2}k(r_i - r_j)^2\right] F_i \cdot F_j$, where r_i, m_i, and F_i are the position, mass, and color operators of the ith particle. We deal with the system composed of light flavor quarks and anti-quarks, namely u and d, $\bar{u}$ and $\bar{d}$ in order to avoid the complexity of anti-symmetrization of wave function. In the Gaussian expansion method, one can introduce the basis set with alternative coordinate termed "Jacobian coordinate" in accordance with possible cluster rearrangement. As for the trial wave function, we take the basis set with direct channel Fig. 1, which we expect to be most appropriate to see the dynamics caused by the color dependent confinement force. The trial wave function that we use here is

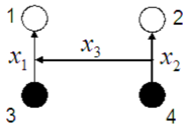

Figure 1:

$$\begin{aligned}
\Phi &= \sum_{\mu} A_\mu \Phi_\mu, \\
\Phi_\mu &= \Phi[l_1 l_2 (l_{12}) l_3 L = 0, M = 0](a_1, a_2, a_3)\zeta_{\text{c.s.}} \\
&= \sum_{m_3, m_{12}} \sum_{m_2, m_1} \langle l_1 m_1 l_2 m_2 | l_{12} m_{12} \rangle \langle l_{12} m_{12} l_3 m_3 | 0, 0 \rangle \phi_{l_1}^{m_1}(a_1, \vec{x}_1) \phi_{l_2}^{m_2}(a_2, \vec{x}_2) \chi_{l_3}^{m_3}(a_3, \vec{x}_3) \zeta_{\mathbf{c.s.}}
\end{aligned}$$

where $\zeta_{c.s.}$ represents the color states states by using the labels of Fig. 1: combination of color siglet $(\bar{q}q)$ clusters $|1_{13}1_{24}\rangle$ or combination of color octet $(\bar{q}q)$ clusters $|8_{13}8_{24}\rangle$ in the $\bar{q}\bar{q}qq$ system. The basis function is characterized by a set of the angular momentum labels l_1, l_2, l_{12}, l_3 and the extension parameters in the Gaussian wave function a_1, a_2, a_3 and the color states $\zeta_{c.s.}$. The index μ stands for possible combination of all these angular momemtum and the color labels and the Gaussian extension parameters. Then we can get the energy and the wave function of the tetra-quark system by solving the following generalized eigenvalue problem

$$\sum_{\nu} \left(\langle \Phi_\mu | H | \Phi_\nu \rangle - E \langle \Phi_\mu | \Phi_\nu \rangle\right) A_\nu = 0. \tag{1}$$

First, we analysed the tetra-quark system constructed with s-wave inner clusters. In this case, the inner color singlet states $|1_{13}1_{24}\rangle$ and the inner color octet states $|8_{13}8_{24}\rangle$ are decoupled each other. So we can solve the problem separately for the inner color singlet states $|1_{13}1_{24}\rangle$ and the inner color octet states $|8_{13}8_{24}\rangle$. We found that the energy level associated with the inner color octet states $|8_{13}8_{24}\rangle$ is just above the threshold of two meson systems.

Now we deal with the case which include the angular momentum states of the inner clusters. In this case, the inner color singlet states $|1_{13}1_{24}\rangle$ and the octet states $|8_{13}8_{24}\rangle$ are mixed through the color interaction which is proportional to the Casimir operators $F_i \cdot F_j$. We are getting first result of this case and finding how much the energy spectra will be affected by the hidden color octet states. We also carry out analysis of the exchange channel.

References

[1] E. Hiyama, M. Kamimura, A. Hosaka, H. Toki and M. Yahiro, Phys. Lett. B **633**, 237 (2006).