Determination of QCD thermodynamics from the lattice numerical calculations

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March 31, 2017

Abstract

The microscopic feature of the nuclear physics can be described by the quantum chromodynamics (QCD), which is the SU(3) gauge-invariant quantum field theory. One of the most powerful tools to investigate the QCD dynamics is "Lattice formula", since it is only known both nonperturbative and gauge-invariant regularization method. The numerical simulations based on the lattice formula have gotten several successes, for instance, it reproduces many hadron spectrum only with a few input parameter, reveals the chiral property of QCD, and explains the origin of the hadron masses.

In this report, I would like to introduce a recent progress related to the QCD in finite-temperature, namely the QCD thermodynamics, using the lattice numerical simulations.

1 Introduction

The study of QCD thermodynamics and the determination of the phase diagram of QCD in finite-temperature and finite-density are quite featured on in recent years. A transition occurs in finite-temperature, from a hadronic- and confined-matter at low-temperature to a decofined- and colorful-matter at high-temperature. The QCD in high-temperature can be produced in the laboratory, *e.g.* LHC (Large Hadron Collider) at CERN, RHIC (Relativistic Heavy Ion Collider) at BNL and SIS (SchwerIonen-Synchrotron) at GSI. Investigating the (in)consistency between these experimental data and the theoretical predictions from the lattice MonteCarlo simulation and phenomenological models leads to the understanding of the QCD properties under extreme conditions. The experimental data show that thermal QCD matters exhibit robust fluid-phenomena, which are consistent with a picture given by near-ideal relativistic hydrodynamics. The hydrodynamical models need some inputs of equation of state which are relates with the thermodynamic quantities. Therefore, a precise determination of the thermodynamic quantities (*e.g.* thermal entropy, pressure and so on) is an important task to understand the nature of QCD in the high-energy experiment as well as the early-universe.

We (Ref. [1]) have proposed the novel method to obtain the thermodynamic quantities using the lattice MonteCarlo simulations. Before our work, the "integral method" (or "differential method"), in which we basically calculate the free energy of thermodynamics, is a standard method to obtain the thermodynamic quantities. On the other hand, our method is based on the direct calculation of the energy-momentum tensor (EMT) using the "gradient flow" technique [2, 3]. In this report, I briefly review of our basic idea and the numerical results of the thermodynamic quantities shown in Ref. [1] and address recent progresses reported in Ref. [4, 5]. We found that the advantages of our method are following:

- We can define the "correctly renormalized EMT" from lattice data in the continuum limit
- Signals become much better because of the smearing effects of the gradient flow technique
- It is not necessary to calculate the wave function renormalization of the EMT operator thank to its UV finiteness (in quenched QCD)

2 Entropy and interaction measure

For simplicity, in this report, we consider the gluonic (pure Yang-Mills) theory, while our formulation can be applied to the full-QCD theory including the dynamical fermions [4].

The fundamental thermodynamic quantities in QCD are the energy density (ϵ) and the pressure (P) of the system in equilibrium states. In the integral method, the free energy (f) are numerically calculated and in the thermodynamic limit, the pressure is related to the free energy as

$$p = -\lim_{V\to\infty} f. \tag{1}$$

The energy density is given by "interaction measure (trace anomaly)" ($I = \epsilon - 3P$), and the interaction measure is a derivative of the normalized pressure: $I = T^5 \frac{\partial}{\partial T}\frac{P(T)}{T^4}$. Now, the thermal entropy (s) can be calculated by $s = \frac{\epsilon + P}{T}$. Numerically, taking a derivative is a hard task since we have to understand systematic "true functional form" of the quantities as a function of the temperature, so that it strongly depends on the systematic uncertainties.

On the other hand, in the word of EMT ($T_{\mu\nu}$) in the Euclidean co-moving frame, the energy density and pressure are directly related to a component of the EMT:

$$T_{ii} = -P \text{ (for } i = 1,2,3), \qquad T_{44} = \epsilon. \tag{2}$$

We perform the MonteCarlo simulation only at the temperature, that we want to know, and calculate the EMT. The interaction measure and entropy are given by

$$I = \sum_{\mu=1}^{4} T_{\mu\mu}, \quad sT = T_{44} - T_{11}, \tag{3}$$

from EMT.

The problem is how to formulate the proper EMT on the lattice, which is ultra-violet (UV) finite and is conserved in the continuum limit. Such a construction is not a trivial task due to the explicit breaking of the Poincaré invariance on the lattice.

In ref. [6], one possible proper EMT has been proposed on the basis of the Yang-Mills gradient flow[2]. The key idea is to represent the EMT in the continuum limit by UV-finite and local operators obtained from the gradient flow.

The EMT is given by

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)} U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}\left[E(t,x) - \langle E(t,x)\rangle_0\right]\right\}, \tag{4}$$

where the perturbative coefficients ($\alpha_U(t)$ and $\alpha_E(t)$) are calculated in Ref. [6], that are written by the running gauge coupling and the coefficient of the β-function of the Yang-Mills theory. The operators $U_{\mu\nu}(t)$ and $E(t)$ are the dimension-4, gauge-invariant, and symmetry operators at the gradient flow-time (t).

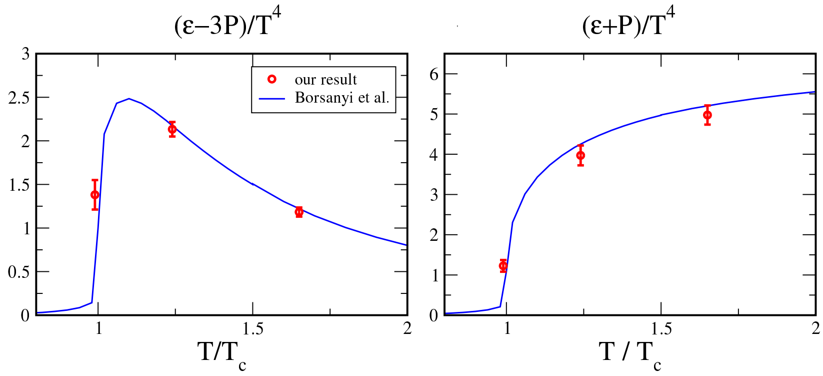

Figure 1: Continuum limit of the interaction measure and entropy density obtained by the gradient flow method for $T/T_c = 1.65$, 1.24, and 0.99 obtained in Ref. [1]. Blue solid lines are results of Ref. [7] obtained by the integral method.

In Fig. 1, $(\epsilon - 3P)/T^4 = \langle \sum_{\mu=1}^{4} T_{\mu\mu} \rangle / T^4$ and $(\epsilon + P)/T^4 = \langle T_{44} - T_{11} \rangle / T^4$ are plotted after taking the continuum limit for $T/T_c = 1.65$, 1.24, and 0.99, where ϵ, P denote energy density and pressure, respectively. For comparison, results of Ref. [7] obtained by the integral method are shown by blue solid lines in Fig. 1 (the data also has roughly 2% errors). The results of the two different approaches are consistent with each other within statistical errors.

As I explained, the integral method is based on the macroscopic picture in finite-temperature QCD, while our method is based on the microscopic picture, namely the quantum field theory. It is the first numerical confirmation of the consistency between micro- and macro-scopic pictures of the QGP phase in (quenched) QCD.

3 (Toward) determination of shear viscosity

We now move on the calculation of the two-point function of EMT. It is related to the shear and bulk viscosity, and here we focus on the former one, which is given by the correlation function of T_{12} component. There are several works [8, 9, 10], where the correlation function of EMT are calculated on lattice. In these works, we explained before, there exit at least two difficulties, the renormalization of the lattice bare EMT operator and the bad signal to noise ratio of the quantity.

In Fig. 2, we plot the correlation function of "lattice bare T_{12} operator" (U_{12}) without the gradient flow (right panel) and with the gradient flow (left panel). Here the number of the measured configurations for each color in both panels is the same. Although the data should be positive by definition and

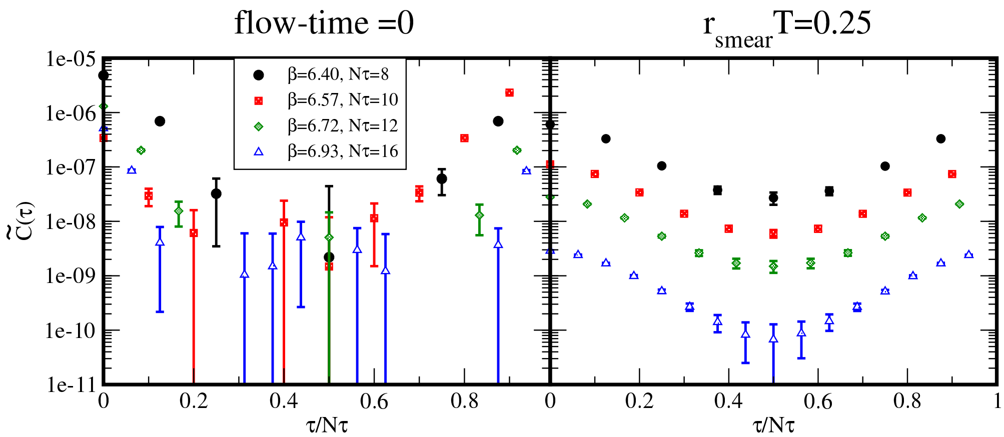

Figure 2: Correlation function of U_{12} operator, $\tilde{C}(\tau) = \langle \int d\vec{x} \langle U_{12}(0,\vec{0}) U_{12}(\tau,\vec{x}) \rangle$, without the gradient flow (left panel) and with the gradient flow (right panel), where the number of the measured configuration for each color in both panels is the same.

indeed so in Ref. [8] with high statistics, some un-flowed data in Fig. 2 become negative due to large noises in this statistics. On the other hand, at the finite flow-time (we take $r_{\rm smear}T = 0.25$ with $r_{\rm smear} = \sqrt{8t}$), the correlation function is positive at all τ despite low statistics, demonstrating that signals are highly improved.

Still, it is an ongoing project, and we will extract the spectrum function and the physical obserbles from the correlation function. We will obtain a precise prediction for the shear viscosity in QGP phase, which is very small value in experiment, using the MonteCarlo simulation, and will understand the hydrodynamic properties near future.

Acknowledgment

Numerical simulation for this study was carried out mainly on NEC SX-8, SX-9 and SX-ACE at RCNP Osaka University. I would like to thank several supports by Togawa-san, Hosaka-san, Ishii-san, and Ikeda-san.

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