Unveiling Nature of the Tetraquark Candidate $Z_c(3900)$ from Coupled-Channel Scattering on the Lattice

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One of the most important subjects in hadron physics is to establish the existence of exotic hadrons different from the standard quark-antiquark mesons and three-quark baryons. Such exotic candidates include the pentaquark candidates $P_c(4380)$ and $P_c(4450)$ observed by the LHCb Collaboration [1] and the tetraquark states $Z_c(3900)$ reported by the BESIII [2] and the Belle [3] Collaborations. The $Z_c(3900)$, in particular, is observed as a peak in the $\pi J/\psi$ and the $\bar{D}D^*$ invariant mass spectra of $e^+ + e^- \to Y(4260) \to \pi + \pi + J/\psi$ and $\pi + \bar{D} + D^*$ reactions.

Various phenomenological attempts [4] have been made to understand the nature of the $Z_c(3900)$ as a compact tetraquark and a s-wave hadronic molecule as well as a threshold cusp when opening the $\bar{D}D^*$ threshold. However, no conclusive result is achieved due to the lack of the information about the coupled-channel interaction relevant to $Z_c(3900)$. (See Fig. 1 for the expected structures and Fig. 2 for the level structure.)

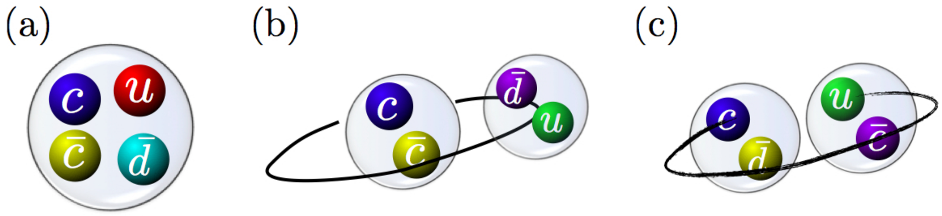

Figure 1: Expected structures of $Zc(3900)$ from phenomenological models. (a) tetraquark state, (b) $\pi + J/\psi$ atomic-like state and (c) $\bar{D} + D^*$ molecule-like state.

In this circumstance, the first principle lattice QCD calculations with explicit channel couplings is the most desirable method to reveal the nature of the $Z_c(3900)$. The HAL QCD Collaboration extracts the s-wave diagonal and off-diagonal potentials among the $\pi J/\psi$, $\rho\eta_c$ and $\bar{D}D^*$ channels by the so-called coupled-channel HAL QCD method [5]. The key quantity in the HAL QCD method is the equal-time Nambu-Bethe-Salpeter (NBS) wave functions $\psi_n^\alpha(\vec{r})$ calculated from the spacial correlation of four point hadron-hadron correlation functions $C^{\alpha\beta}(\vec{r}, t)$,

$$\begin{aligned} C^{\alpha\beta}(\vec{r},t) &\equiv \sum_{\vec{x}} \langle 0|\, \phi_1^\alpha(\vec{x}+\vec{r},t)\phi_2^\alpha(\vec{x},t)\overline{\mathcal{J}}^\beta \,|0\rangle \,/\sqrt{Z_1^\alpha Z_2^\alpha}, \\ &= \sum_n \psi_n^\alpha(\vec{r}) A_n^\beta e^{-W_n t}, \end{aligned} \tag{1}$$

where each channel is specified by $\alpha = (\pi J/\psi, \rho\eta_c, \bar{D}D^*)$, and $\phi_i^\alpha(\vec{y}, t)$ is a local Heisenberg operator at Euclidian time $t > 0$ and the spatial point $\vec{y}$ for the meson i $(= 1, 2)$ with mass m_i^α in channel α. The corresponding wave function renormalization factor is given by Z_i^α. $\overline{\mathcal{J}}^\beta$ denotes a two-meson operator in channel β with zero-momentum wall quark source located at $t = 0$. The NBS wave function $\psi_n^\alpha(\vec{r})$ for each scattering state is specified by the eigenvalue of the n th QCD eigenstate. $A_n^\beta \equiv \langle W_n|\overline{\mathcal{J}}^\beta|0\rangle$ is an overlap between the eigenstate and QCD vacuum by the insertion of $\overline{\mathcal{J}}^\beta$.

Outside hadron-hadron interactions, the NBS wave function satisfies the Helmholtz equation, so the NBS wave function is faithful to the QCD S-matrix. It can be shown that $R^{\alpha\beta}(\vec{r}, t) \equiv C^{\alpha\beta}(\vec{r}, t)e^{(m_1^\alpha + m_2^\alpha)t}$ satisfies

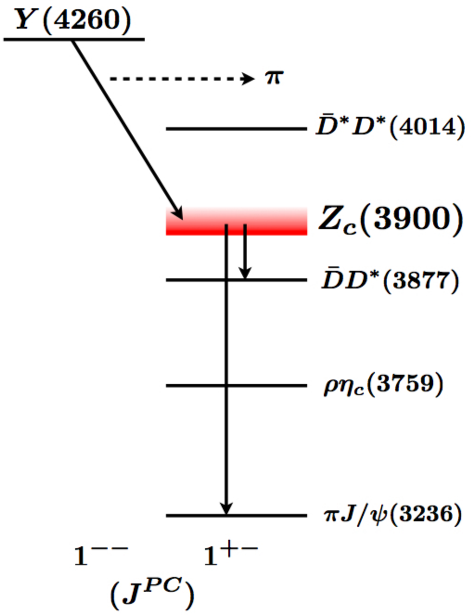

Figure 2: A possible decay scheme of the $Y(4260)$ through $Zc(3900)$. The arrows represent the observed decay modes in the experiments [2, 3].

the Schrödinger-type equation [6, 7],

$$\left(-\frac{\partial}{\partial t} - H_0^\alpha\right) R^{\alpha\beta}(\vec{r}, t) = \sum_\gamma \Delta^{\alpha\gamma} \int d\vec{r'} U^{\alpha\gamma}(\vec{r}, \vec{r'}) R^{\gamma\beta}(\vec{r'}, t), \tag{2}$$

where $H_0^\alpha = -\nabla^2/2\mu^\alpha$ with the reduced mass $\mu^\alpha = m_1^\alpha m_2^\alpha/(m_1^\alpha + m_2^\alpha)$ and $\Delta^{\alpha\gamma} = e^{(m_1^\alpha+m_2^\alpha)t}/e^{(m_1^\gamma+m_2^\gamma)t}$. Employing the lowest order of the velocity expansion, $U^{\alpha\beta}(\vec{r}, \vec{r'}) = V^{\alpha\beta}(\vec{r})\delta(\vec{r} - \vec{r'}) + O(\nabla^2)$ to extract the spherical and local potential $V^{\alpha\beta}(r)$. We are able to calculate any scattering observables directly based on QCD using the extracted coupled-channel potential from the NBS wave functions.

It is found that the resulting diagonal elements of the s-wave coupled-channel potential are all week. This indicates that the $Z_c(3900)$ is not a state associated with a hadronic molecule. Also, the off-diagonal $\pi J/\psi$-$\rho\eta_c$ potential is week. This is a consequence of the heavy quark spin symmetry. On the other hand, the off-diagonal elements of the $\pi J/\psi$-$\bar{D}D^*$ and the $\rho\eta_c$-$\bar{D}D^*$ are found to be strong.

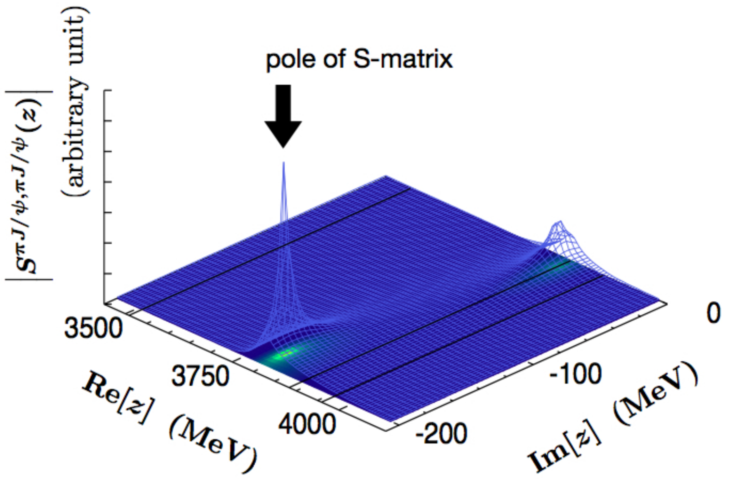

Figure 3: (b) The pole of the S-matrix on the complex energy plane. The coupled-channel S-matrix is analytically continued into the $\pi J/\psi$ second, $\rho\eta_c$ second and $\bar{D}D^*$ second Riemann sheets. The complex energy z is defined by $z = m_1^\alpha + m_2^\alpha + p_\alpha^2/2\mu^\alpha$.

In order to reveal the structure of the $Z_c(3900)$, the most ideal reaction process is the two-body $\pi J/\psi$-$\rho\eta_c$-$\bar{D}D^*$ coupled-channel scattering. With the above coupled-channel potential $V^{\alpha\beta}$, we calculate the scattering amplitudes in the two-body $\pi J/\psi$, $\rho\eta_c$ and $\bar{D}D^*$ channels by solving the Lippmann-Schwinger equation. In the imaginary part of two-body amplitudes (invariant mass spectra of two-body scatterings), the peak appears just above the $\bar{D}D^*$ threshold due to opening the threshold. We also examine the complex pole of the amplitudes to understand whether the peak structure is associated with a conventional resonance or not. The result of the pole position is shown in Fig. 3. The pole is far below the $\bar{D}D^*$ threshold and has the large imaginary part, so the pole does not contribute to the amplitudes. In addition to the above analyses of the two-body scatterings, we investigate the $Y(4260)$ decay and compare with the experiments. As shown in Fig. 4, the peak observed in the experiments is well reproduced. Therefore we conclude that the $Z_c(3900)$ is not a conventional resonance but the threshold cusp when opening the $\bar{D}D^*$ threshold.

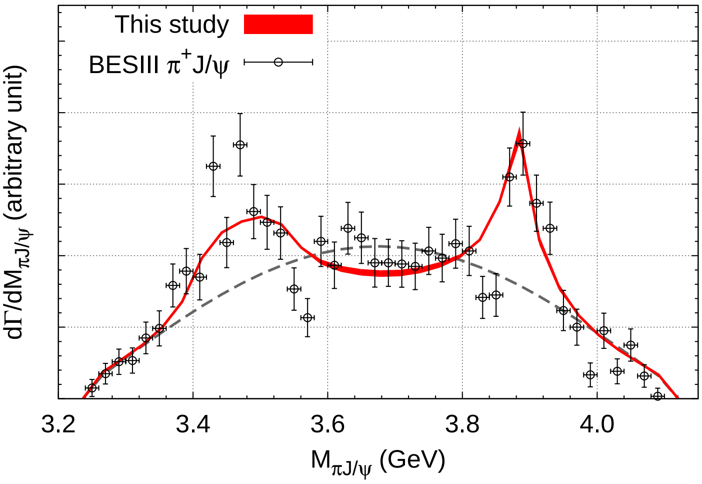

Figure 4: The invariant mass spectra of $Y(4260) \to \pi\pi J/\psi$. The shaded areas show the statistical errors. The dashed lines show the invariant mass spectra of the $Y(4260)$ decay without the off-diagonal components of $V^{\alpha\beta}$. The experimental data are taken from Ref. [2].

In summary, thanks to the coupled-channel HAL QCD method, it turns out that the $Z_c(3900)$ is not a conventional resonance but the threshold cusp just opening the $\bar{D}D^*$ threshold assisted by the strong $\pi J/\psi$-$\bar{D}D^*$ coupling: The pole position is far below the $\bar{D}D^*$ threshold, and the $Y(4260)$ decay is well reproduced (for more details, see original paper by Ikeda *et al.* [8]). The novel method developed in this study paves the way to understand the nature of exotic hadron candidates directly based on QCD: Some of interesting future targets include P_c ' s, $X(3872)$ and $Z_c(4430)$.

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