Interglueball potential in $SU(N)$ lattice gauge theory

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Abstract

Various observations and theories have shown that the current universe is made up of 27% of its total energy from unidentified dark matter. In this study, we investigate the interaction of dark matter in hidden Yang-Mills theory (YMT), which is a good candidate for dark matter. The dark matter in YMT is a glueball, but it is necessary to perform lattice gauge theory simulation to investigate the interglueball interaction. As a method for calculating the interhadron interaction on lattice, the Namubu-Bethe-Salpeter amplitude method recently developed by HAL QCD Collaboration is used. The calculations were carried out on SX-ACE at RCNP/CMC of Osaka University. In the research project of 2019, we used 90000 node hours for calculation. We calculated for the first time the scattering cross section between lightest glueballs in $SU(2)$ pure Yang-Mills theory.

Various observations and theories have shown that the current Universe is made up of 27% of its total energy from unidentified dark matter (DM). Many studies have also revealed that DM is indispensable for the formation of the structure of the present Universe and galaxies. Recent research suggests that DM is likely to be elementary particles, but it is known that there is no such candidate in the standard model of particle physics. Some new physics beyond the standard model that may explain it is therefore required. Simulation studies assuming only gravity as the interaction between DM particles have succeeded in explaining the formation of the large-scale structure of the Universe. It is also known that it is difficult to form structures below the galactic scale, strongly suggesting that DM interacts with each other other than gravity [1]. This means that information on the interaction between DMs can be obtained from their distribution, and it is possible to discuss the elementary particle physics origin of DM.

Supersymmetric models have been studied as potential candidates for explaining the DM, but many have been denied in the direct search and accelerator experiments. Therefore, we have to investigate other DM candidates. Here we quantitatively examine the hidden Yang-Mills theory (YMT). The YMT is one of the field theories without serious hierarchy problem, and it is particularly preferable for explaining particle physics DM. It may naturally be embedded in a more ultraviolet complete frameworks such as the grand unification or string theory. The DM appearing in the hidden YMT is a glueball, a composite state of gluons (of he hidden YMT) generated by the confinement which is a characteristic of gauge theory, and is a promising candidate for DM in recent years [2]. The properties of the glueballs are still obscure. In QCD, the observation of glueballs is not yet conclusive, although there are several candidates such as $f_0(1500)$ and $f_0(1710)$, and they are currently actively searched experimentally. The study of the properties of hadrons, including glueballs, requires the calculation in the strong coupling region of gauge theory, and such nonperturbative calculations are extremely difficult analytically, so that its examination has not so far been done actively. Lattice gauge theory is currently the only way to perform reliable calculations in nonperturbative domains. Here we use the lattice gauge theory to study the properties of glueballs in hidden YMT, explore unknown DM through numerical calculations, and finally determine the hidden YMT from comparison with observational data.

In considering the particle DM scenario, an inevitable discussion is the self-interaction (or scattering) among DM. This feature is especially important since the DM self-interaction is given an upper bound from observations such as the density profile of the galactic halo or the collision between galaxies. As for the scale smaller than kpc, several phenomena, which were thought to be "problems" in relation to the density profile, are known to be explained with finite DM self-interaction, although these topics are still under discussion. The two-body potential working between $SU(N_c)$ glueballs should have a finite range due to the mass gap of the YMT, so it is in principle possible to set constraints on the scale parameter from observations, but the relation between the latter and the scattering cross section is almost totally unknown, although it was challenged in model calculations. To completely determine the dynamics of the YMT at low energy and to identify it as the theory explaining the DM, the scattering among the lightest glueballs has to be elucidated. It is also theoretically interesting to discuss it in the context of the low energy effective field theory, since the lightest 0^{++} glueball is believed to have become massive due to the trace anomaly, the anomalous violation of the scale invariance.

The lattice gauge theory simulation, which is currently almost the only way to systematically calculate observables related to glueballs, could so far only quantify their masses. In recent years, HAL QCD Collaboration has developed a powerful method for numerically calculating interhadron interaction in lattice gauge theory [3]. Therefore, we calculated the interglueball potential in the HAL QCD method. In lattice gauge theory, the path integral with large number of degrees of freedom is performed by the Monte Carlo method, which requires large computational resources. Further, it is known that quantities associated with the glueball have noisy signal. In particular, in this study, more samples are expected to be needed due to the glueball three-point function, which has not been calculated before. In fact, our previous calculations required one million gauge configurations to obtain signals (in the case of SU(2) gauge theory), confirming this expectation. The hidden gauge theory has a parameter called the number of colors (N_c), and in this study, it is necessary to perform the calculation with multiple numbers of colors. The higher the number of colors is, the higher is the computational cost of generating gauge ensembles. Furthermore, in order to improve the accuracy of the potential between the glueballs in the short distance region, it is necessary to calculate at several lattice intervals. In order to perform such a large-scale calculation, it was essential to use a large-scale calculation resource such as SX-ACE of Osaka University CMC as a joint research project for this base. The physical information of the scattering between two hadrons can be extracted from the Nambu-Bethe-Salpeter (NBS) amplitude [3]. To improve the signal, we employed the cluster-decomposition error reduction technique (CDERT) [4] to remove the fluctuation of the vacuum insertion. Thanks to the CDERT, we could reduce the error bar by more than twice, which demonstrates its efficacy.

Let us now describe the formalism of the calculation of the interglueball potential on lattice. The nonlocal potential $U(\vec{r}, \vec{r}')$ is extracted according to the following time-dependent Schrödinger-like equation

$$\left[\frac{1}{4m_\phi}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{1}{m_\phi}\nabla^2\right] R(t, \vec{r}) = \int d^3r' U(\vec{r}, \vec{r}') R(t, \vec{r}'), \qquad (1)$$

where $R(t, \vec{r}) \equiv \frac{\Psi_{\phi\phi}(t,\vec{r})}{e^{-2m_\phi t}}$. Here t should be chosen so that $1/t$ is less than the inelastic threshold $m_\phi(= 3m_\phi - 2m_\phi)$. In our calculation with $\beta = 2.5$, it is enough to take the data from $t = 2$. The physics addressed is then nonrelativistic, and the potential should be to a good approximation local and central $U(\vec{r}, \vec{r}') \approx V_{\phi\phi}(\vec{r})\delta(\vec{r} - \vec{r}')$. Nonlocal and angular momentum dependent terms appear in the derivative expansion but are neglected in this work.

In the research in 2017, the interaction between glueballs in SU(3) YMT was performed through numerical simulation of lattice gauge theory. As a result, it was suggested that the interaction between glueballs was repulsive in a short distance region. In the research project adopted as the 2018 JHPCN research project (Project ID: jh180058-NAH), we calculated the potential between glueballs in SU(2) and SU(4) YMT. The program for analyzing the potential between glueballs was made more efficient, and the computational cost was reduced to a level that was almost negligible compared to the generation of gauge configurations. In the SU(2) YMT calculation, one million gauge configuration was generated at one lattice interval and analyzed. In particular, the glueball potential in SU(2) YMT was found to be repulsive in the short range region, suggested by the results of the previous SU(3) and SU(4) YMT analyses.

The result of our work is plotted in Fig. 1. The important feature of our result is that the intergluball potential is repulsive at short range ($r \sim 0.2\,\Lambda^{-1}$). In the context of the DM, we are interested in the (s-wave) low energy limit of the cross section $\sigma_{\phi\phi} = \lim_{k\to 0} \frac{4\pi}{k^2} \sin^2[\delta(k)]$. From the two fitting forms, we obtain $\sigma_{\phi\phi} = (3.5–3.8)\Lambda^{-2}$ (Yukawa), and $\sigma_{\phi\phi} = (7.5–8.0)\Lambda^{-2}$

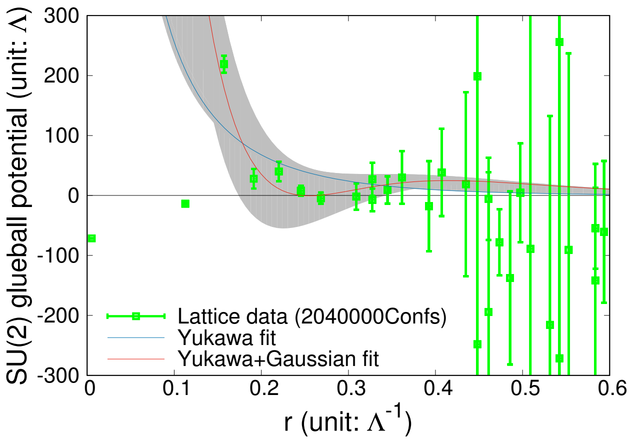

Figure 1: Interglueball potential calculated on lattice in the $SU(2)$ YMT. The fits with two fitting forms are also displayed. The grey band shows the uncertainty. The first two data points ($r = 0$ and $r = 0.1\Lambda^{-1}$) are lattice artifacts.

(Yukawa+Gaussian), with the band denoting the statistical error. By considering the difference between them as the systematic error, the interglueball scattering cross section for the $SU(2)$ YMT is

$$\sigma_{\phi\phi} = (3.5 - 8.0)\Lambda^{-2} \quad (\text{stat.} + \text{sys.}). \tag{2}$$

Let us now try to derive the constraint on Λ from observational data. The most robust bound on the DM cross section is given by the observation of the shape of the galactic halo and galactic collisions. Here we adopt that of Ref. [5]: $\sigma_{\text{DM}}/m_{\text{DM}} < 0.47\,\text{cm}^2/\text{g}$. By substituting our result (2), we obtain

$$\Lambda > 60\,\text{MeV}. \tag{3}$$

This is the first constraint on the YMT as the theory of DM derived from first principle. The results have been submitted in three papers [6, 7, 8].

It would also be interesting to compare the interglueball cross section at low energy with that derived from the glueball effective field theory which can be constructed according to the conformal Ward identity. Determining the glueball EFT and its low energy constants may give us an important insight into other fields such as the conformal field theory or hadron physics. If again the large N_c expansion holds, the determination of the cross section at a sufficiently large N_c would probably mean the quantification of the conformal physics.

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