“Theoretical and numerical study of QCD phase – What do high energy nuclear experiments tell us?”

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1 Introduction

We study a phase transition in a dense matter for understanding the early Universe and the inside of neutron stars. An understanding of hadron physics is very important for the study of such phenomena. A hadron is a subatomic composite particle made of two or more quarks held together by a strong force. For study hadrons, people use collision of heavy ions which are composed of many nucleons, each of which includes quarks and gluons. After heavy ion collisions, many quarks and gluons are created. See Fig.1. In such collisions, high density QCD state is expected to be realized. But it is know that if the incident energy is very high, like RHIC (USA) and LHC (Europe), very dense state is not created. For a theoretical study, J-PARC (Japan) as well as NICA (Russia) is very appropriate to study the very high density.

For it, we are using the first-principle, i.e., lattice QCD simulation which is a great approach for studying non-perturbation physics. This was impossible for many years, because (1) it requires very heavy numerical works and (2) at the finite density simulations, the sign problem makes numerical simulation impossible.

This simulation is possible only with high performance computational environments: we use the **GPGPU** (General-purpose computing on graphics processing units) technology, which allows us to transfer mathematical calculations to a graphic card. RCNP provides the resources from the Large-Scale Computer System operated by Cyber Media Center of Osaka university, which includes **GPGPU**.

2 Project Description

Our project is dedicated to the study of dense baryonic matter at zero and finite temperatures. To simulate baryonic matter, we use lattice calculations for several types of fermion action and color number. We used two-color QCD with staggered quarks and three-color

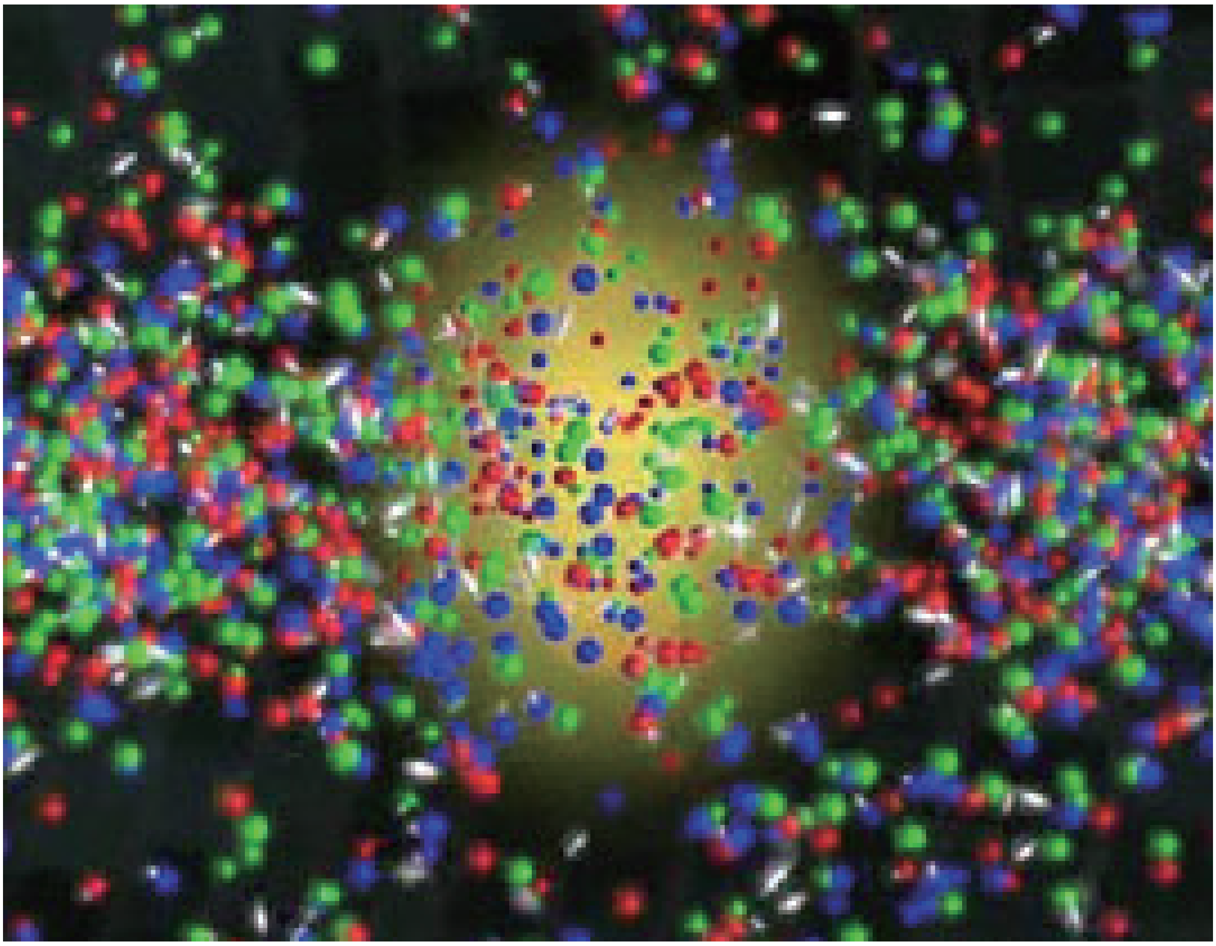

Figure 1: Conceptual rendering of the quark-gluon plasma created by RHIC. https://www.bnl.gov/science/QCD-matter.php

QCD with Wilson fermions, Y. Iwasaki gauge action and clover improvements. Two-color QCD, i.e. QCD with the SU(2) gauge group is the simplest non-Abelian gauge theory without a sign problem for a finite quark density. For staggered quark Dirac operator we also use stout smearing improvement. This project is aimed at studying the properties and calculating a large number of physical observables in this theory at nonzero temperature and high quark density, as well as developing and testing new computational methods applicable in conventional 3-color lattice QCD at large values of the baryon chemical potential. The results obtained will also allow us to create a platform for testing other non-perturbative approaches at finite density, which include additional simplifications or assumptions.

3 Highlights in 2020

As is well-known, QCD simulations in the dense system suffer from the sign problem. There is a place where no sign problem appears, i.e., two-color QCD. We can perform simulations at both imaginary and real quark chemical potentials.

First, we study two-color QCD with two flavors of staggered fermions at imaginary and real quark chemical potential μ_q and $T > T_c$. We employ various methods of an-

Figure 2: View of NICA. https://nica.jinr.ru/physics.php

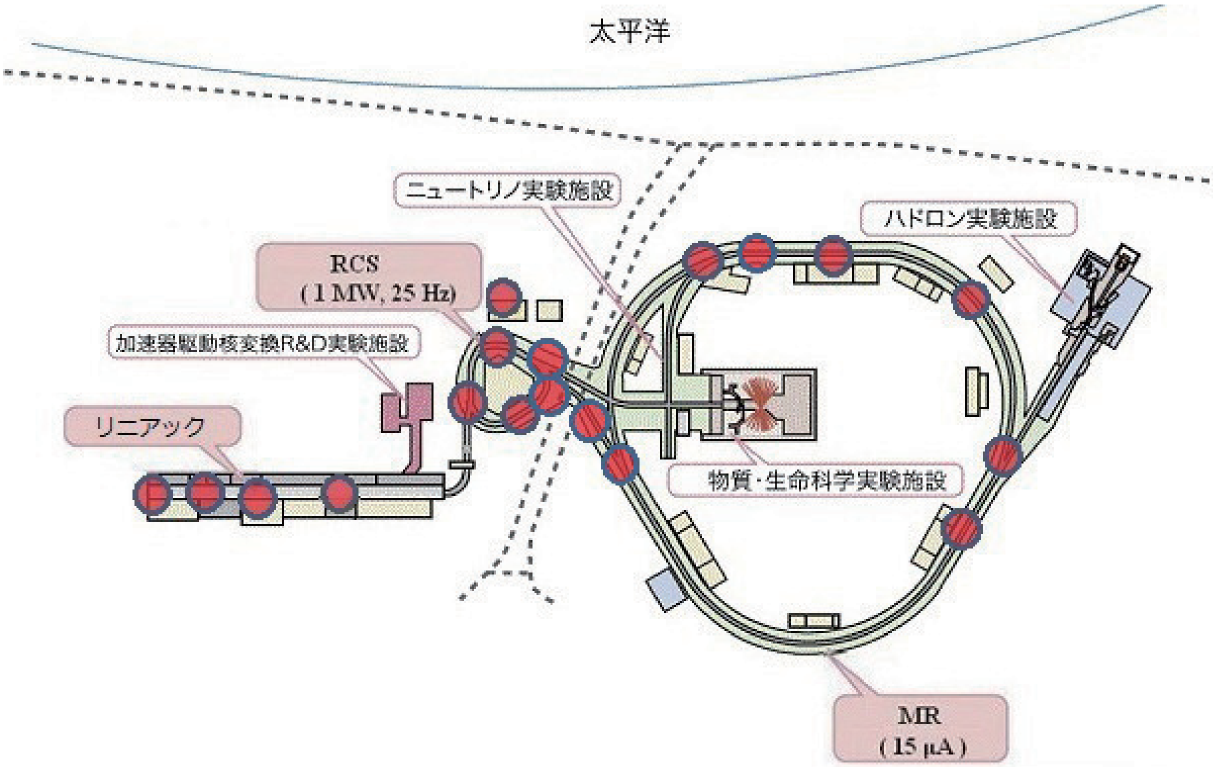

Figure 3: View of J-PARC. https://j-parc.jp/public/Acc/ja/index.html

alytic continuation of the quark number density from imaginary to real quark chemical potentials μ_q on the basis of the numerical results for imaginary $\mu_{q,I}$. Below the Roberge-Weisse temperature T_{RW}, we find that the cluster expansion model combined with the canonical formalism provides rather good analytic continuation. Above T_{RW}, we see that the analytic continuation to the real values of μ_q based on trigonometric functions works equally well with the conventional method based on the Taylor expansion in powers of μ_q.

We also computed a lot of observables at zero temperature in two-color QCD. Large chemical potential corresponds to the high density state. In the figures 4 and 5, the simulation results of the quark density and pressure are shown as a function of the quark chemical potential. In the first figure (left,top) of Fig. 4, we can see this relation between the chemical potential and the density. The results are stable when we change the lattice size.

The second figure (right,top) of Fig. 4 shows the thermodynamical quantity, the pressure P, increased rapidly when the chemical potential starts to increase, and it reached to an asymptotic value. The 3rd and 4th figures (left, bottom and right bottom) are normalized to the free quark gas values.

These results are calculated on OCTOPUS.

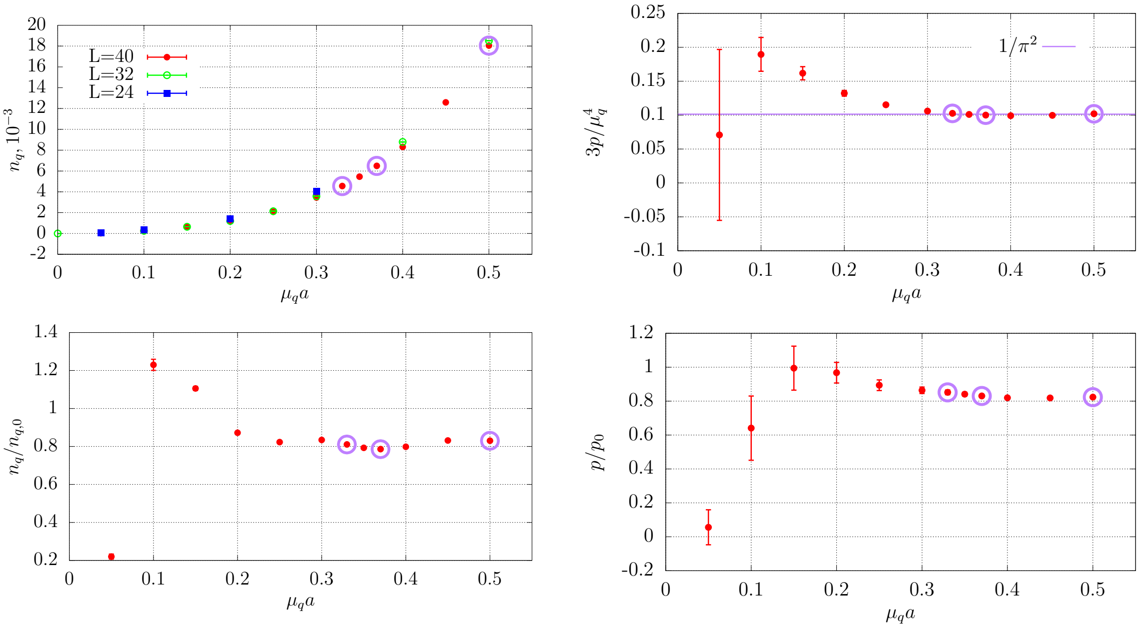

Figure 4: Quark density and pressure at finite $\mu_q a$ and zero temperature. Here $n_{q,0}$, P_0 are the quark density and pressure of free fermion gas. Selected points by purple circles are calculated on the OCTOPUS.

Then, we studied the QCD matter at finite density and temperature. The sign problem

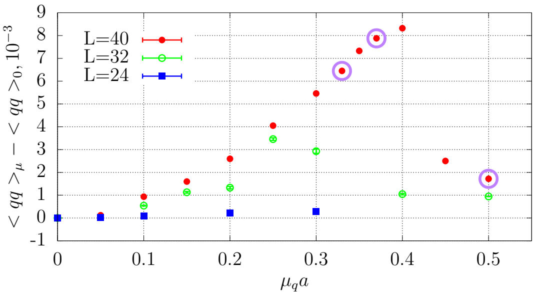

Figure 5: Diquark condensates at finite $\mu_q a$ and zero temperature. Selected points by purple circles are calculated on the OCTOPUS.

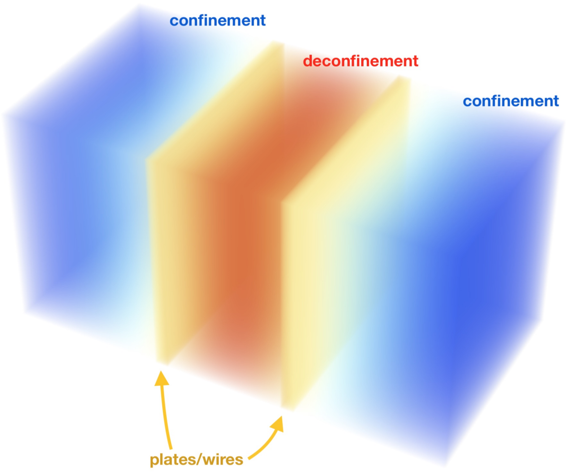

Figure 6: An illustration of the deconfinement in the space between the plates

is overcome by our new canonical approach. We compare RHIC energy scan data and lattice QCD simulations. We extracted from the RHIC data approximate value of chemical potential and $T(V)$ dependence.

We have also started to study of the MIT bag model in hadron physics. It is a phenomenological model where hadrons considered as bags embedded into a non-perturbative QCD vacuum. In this case space divided into 2 regions: interior of bag and exterior of bag. Between two regions exist boundaries. When boundaries are introduced into the theory, the Casimir force appears. In the vacuum, particles and anti-particles are pair created and annihilated. The presence of physical objects affects this vacuum fluctuations nearby. This is the Casimir effect. We studied this Casimir effect via machine-learning techniques in the third article by numerical simulations. Vacuum fluctuations of quantum fields between physical objects depend on the shapes, positions, and internal composition of the latter. For objects of arbitrary shapes, even made from idealized materials, the calculation of the associated zero-point (Casimir) energy is an analytically intractable challenge. We propose a different numerical approach to this problem based on machine-learning techniques and illustrate the effectiveness of the method in a (2+1)-dimensional scalar field theory. The Casimir energy is first calculated numerically using a Monte Carlo algorithm for a set of the Dirichlet boundaries of various shapes. Then, a neural network is trained to compute this energy given the Dirichlet domain, treating the latter as black-and-white pixelated images. We show that after the learning phase, the neural network is able to quickly predict the Casimir energy for new boundaries of general shapes with reasonable accuracy.

4 Summary and perspectives

Based on these activities, we pursue the following projects.

1. N. Gerasimenyuk, A. Korneev ”Lee-Yang zeros for determining the QCD phase transition point”.
2. V. Goy, A. Hosaka and A. Nakamura, ”Hadrons at finite baryon density”.
3. A. Molochkov, A. Hosaka and V. Goy, ”Hadrons at finite box”.

In the first paper, we demonstrate that we can construct the Lee-Yang zero distribution by the first principle calculation. The Lee-Yang zeros indicate the phase transition points in $T - \mu$ plane. This is possible only when we can calculate the finite chemical potential region reliably.

The 2nd paper shows that now we can calculate not only the phase boundary, but also hadron behavior on the phase space.

In 3rd article, we are the first who started to use machine learning (ML) techniques in the study of the Casimir effect. We have now developed two numerical approaches for solving the problem of studying the Casimir effect. Using our developments, we will continue to study this effect in real QCD at the finite box.

References

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