Studies of diquarks from Lattice QCD

N. Ishii
Research Center for Nuclear Physics (RCNP), Osaka University,
10-1 Mihoga-oka, Ibaraki-shi, Osaka 567-0047, Japan

1 Introduction

The atomic nucleus, which is the core of an atom, is composed of nucleons, i.e., protons and neutrons, and the force that binds nucleons is called as the nuclear force. The nuclear force is known to be generated by exchanging mesons of various kind such as pions. Not only nucleons, but also mesons are made up of quarks, which are elementary particles with spin 1/2. Quarks have a color SU(3) quantum number. We will refer to it as (R, G, B). Composite particles made of color-neutral combination of quarks are called as hadrons. Hadrons are classified into baryons, which consist of three quarks like nucleons, and mesons, which are composed of quarks and anti-quarks like pions. For baryons, the three primary colors RGB form white color-neutral states, while for mesons, color neutrality is achieved through the provision of the complementary color by the anti-quark, for instance, $(R\bar{R})$.

While color-neutral particles (hadrons) have experimentally been found in large numbers, no color-unneutral particles have been found. This phenomenon is called "color confinement". Quark is a trivial example for a color-unneutral particle. Another example of color-unneutral particles is diquarks which consist of two quarks. Note that a quark and an anti-quark can form color-neutral mesons (such as $R\bar{R}$), but two quarks can only form color-unneutral states (such as RB).

Quarks have a quantum numbers called as "flavor" as well. We will refer to them as, in order of increasing mass, up, down, strange, charm, bottom, and top quarks. Note that nucleons are composed only of up and down quarks. By replacing them by heavier quarks, we will have different particles. For instance, by replacing them by strange quarks and charm quarks, we will have hyperons (Λ, Σ, Ξ, Ω) and superons (Λ_c, Σ_c, Ξ_c, Ω_c, etc.), respectively. By doing similar things with mesons, different mesons will be obtained. In this way, we understand that a large number of hadrons can exist, which have been experimentally observed. However, we note that these hadrons are conventional hadrons. In addition to these conventional hadrons, there are exotic hadrons, for which color neutrality is achieved in a different manner, i.e., tetra-quark $(RG\bar{G}\bar{R})$ and penta-quark $(RGBR\bar{R})$, etc. Whereas most of hadrons are classified as conventional hadron, due to the enormous experimental efforts, number of exotic hadrons (and their candidates) are gradually increasing.

The interaction between quarks is called the "strong interaction". The strong interaction is mediated by another elementary particle called the gluon. The behaviors and properties of hadrons are fully described by a non-Abelian gauge theory based on the local color SU(3) symmetry, known as the quantum chromodynamics (QCD) which describes the elementary interactions between quarks and gluons.

One of the most important aims of hadron physics is the systematic understanding of the structure and properties of these hadrons including exotic hadrons based on QCD. In this direction, diquark is expected to play an important role[1]. Although there can be diquarks of many kind, phonologically relevant diquarks are believed to be (1) the scalar diquark (=good diquark) with $J^P = 0^+$, $I = 0$, and color $\bar{\mathbf{3}}$ and (2) the axialvector diquark (=bad diquark) with $J^P = 1^+$, $I = 1$ and color $\bar{\mathbf{3}}$. In particular, the relevance of the scalar diquark is supported by the color-magnetic interaction of the constituent quark models and the instanton induced interaction. In addition, at high baryon number density, the scalar diquark is expected to condense in the vacuum to cause the spontaneous breaking of the color symmetry leading to the color superconductivity. In spite of its relevance, it is difficult to perform experimental studies of diquarks. This is due to the color unneutral nature of diquarks, i.e., because of the color confinement, color unneutral diquarks cannot be isolated experimentally so that many of the important information of diquarks are not established at all such as their mass and their existence.

The asymptotic freedom is one of the most important properties of QCD, due to which the interactions between quarks and gluons become weaker at shorter distance and the perturbation theory works at high energy. However, at long distance beyond the typical size of hadrons, interactions become stronger and the perturbation theory breaks down. Therefore, it is very difficult to handle QCD in the low-energy region to study various properties of hadrons, where various non-perturbative effects such as the color confinement and the spontaneous chiral symmetry breaking begin to manifest. Many non-perturbative methods have been developed to address this problem, among which the most powerful method is the first principle calculation based on the lattice QCD, i.e., powerful supercomputers are used to perform Monte Carlo calculations to directly solve QCD on a discretized four-dimensional space-time lattice. The first principles calculation of lattice QCD enables direct access to various physical quantities based on QCD, including the mass of hadrons.

Lattice QCD calculation has been used to study diquarks. However, due to the color unneutral nature of diquarks and the color confinement of QCD, the straightforward application of conventional lattice QCD method may be troublesome. The lattice studies of diquarks are classified in the following two types:

1. Two-point correlator of composite diquark fields are calculated with Landau gauge fixing, from which the gauge dependent diquark mass is naively extracted with the single exponential fit[2]. Note that existence of the pole in the two-point correlator of colored operators is not guaranteed in the color confinement phase so that the naive application of the single-exponential fit may not work to extract the diquark mass as a pole-mass.

2. A static quark is added to the diquark to neutralize the color charge of diquark[3]. The energy of the total system is measured and identified with the diquark mass. Note that the resulting diquark mass is contaminated with the interaction energy between the static quark and diquark leading to an uncertainty of a few hundred MeV.

In this report, we are going to explain the activity of RCNP theory group for the lattice QCD studies of diquarks, where a different strategy is employed from the above two types.

2 Diquarks and HAL QCD method for non-relativistic potentials

Recently, a method was proposed to calculate the NN potentials based on the lattice QCD calculation which is faithful to the NN scattering experiments[4]. In this method, by demanding that equal-time Nambu-Bethe-Salpeter (NBS) wave functions of two-nucleon states generated by lattice QCD should satisfy Schrödinger equation, the NN potentials are obtained by inversely solving Schrödinger equations. Note that, by using the LSZ reduction formula, it can be shown that so constructed NN potentials are faithful to the NN scattering experiments.

The HAL QCD method was attempted to be extended in many ways, among which we are interested in the extension to the $c\bar{c}$ potential by Kawanai and Sasaki[5]. We take a close look at their method. We consider the equal-time NBS wave function of $c\bar{c}$ states with Coulomb gauge fixing, i.e.,

$$\psi_n(\mathbf{x}-\mathbf{y}) \equiv \langle 0|\bar{c}_a(\mathbf{x})c_a(\mathbf{y})|n\rangle, \tag{1}$$

where $c(\mathbf{x})$ denotes the charm quark fields and a denotes the color index. $|0\rangle$ denotes the QCD vacuum state, and $|n\rangle = |\mathrm{PS}\rangle$ or $|\mathrm{V}\rangle$ denotes charmonium states of pseudo-scalar and vector states, respectively.$\mathbf{s}_1$ and $\mathbf{s}_2$ denotes the spin operators for $\bar{c}$ and c, respectively. m_c denotes the charm quark mass. Note that, unlike conventional hadrons, the existence of the pole in the two-point correlator is not guaranteed for the charm quark so that we treat m_c as an unknown parameter which will be determined later. By projecting Schrödinger equation into PS and V channels, the effective central potentials $V_{\mathrm{eff},n}(r)$ are expressed as

$$\begin{aligned} V_{\mathrm{eff,PS}}(r) &= V_0(r) - (3/4)V_s(r) \simeq \mathcal{E}_{\mathrm{PS}} + \frac{1}{m_c}\frac{\nabla^2\psi_{\mathrm{PS}}(\mathbf{r})}{\psi_{\mathrm{PS}}(\mathbf{r})} && (2)\\ V_{\mathrm{eff,V}}(r) &= V_0(r) + (1/4)V_s(r) \simeq \mathcal{E}_{\mathrm{V}} + \frac{1}{m_c}\frac{\nabla^2\psi_{\mathrm{V}}(\mathbf{r})}{\psi_{\mathrm{V}}(\mathbf{r})}. && (3)\end{aligned}$$

Hence, we have

$$\begin{aligned} V_0(r) &= \frac{1}{4}\left(3\mathcal{E}_{\mathrm{V}} + \mathcal{E}_{\mathrm{PS}}\right) + \frac{1}{4m_c}\left(3\frac{\nabla^2\psi_{\mathrm{V}}(\mathbf{r})}{\psi_{\mathrm{V}}(\mathbf{r})} + \frac{\nabla^2\psi_{\mathrm{PS}}(\mathbf{r})}{\psi_{\mathrm{PS}}(\mathbf{r})}\right) && (4)\\ V_s(r) &= E_{\mathrm{V}} - E_{\mathrm{PS}} + \frac{1}{m_c}\left(\frac{\nabla^2\psi_{\mathrm{V}}(\mathbf{r})}{\psi_{\mathrm{V}}(\mathbf{r})} - \frac{\nabla^2\psi_{\mathrm{PS}}(\mathbf{r})}{\psi_{\mathrm{PS}}(\mathbf{r})}\right). && (5)\end{aligned}$$

To determine m_c, Kawanai and Sasaki demand that the spin-dependent central potential $V_s(r)$ should vanish at spatial infinity, which leads to the following condition:

$$m_c = -\frac{1}{E_{\mathrm{V}} - E_{\mathrm{PS}}}\lim_{r\to\infty}\left(\frac{\nabla^2\psi_{\mathrm{V}}(\mathbf{r})}{\psi_{\mathrm{V}}(\mathbf{r})} - \frac{\nabla^2\psi_{\mathrm{PS}}(\mathbf{r})}{\psi_{\mathrm{PS}}(\mathbf{r})}\right). \tag{6}$$

We note that, by using their method, Kawanai and Sasaki successfully obtained the charmonium potential together with self-consistently determined charm quark mass by extending the HAL QCD method.

Now, we present our own strategy to study diquarks with lattice QCD, which is to apply the HAL QCD method with (a deformation of) Kawanai and Sasaki's extension to quark-diquark systems to obtain the quark-diquark potentials and self-consistently determined diquark masses. Note that our diquark mass is obtained as a mass parameter of a constituent quark-diquark models so that it is free from the problem of the possible non-existence of a pole in the diquark two-point correlator.

2.1 Scalar diquark in Λ_c

As our first target, we consider Λ_c baryon as a two-body problem between a charm quark and a scalar diquark[6]. Since the scalar diquark does not have spin, the prescription of Kawanai and Sasaki cannot be applied to this system. Instead, we determine the diquark mass by demanding that our potential should reproduce the energy spectrum in the negative parity sector. We first define the NBS wave function as

$$\psi_{n,\alpha}(\mathbf{x}-\mathbf{y}) \equiv \langle 0|c_{a,\alpha}(\mathbf{x})D_a(\mathbf{x})|n\rangle, \tag{7}$$

where $|n\rangle$ denotes the n-th state for Λ_c baryon. $a = 1, 2, 3$ denotes the color index. $\alpha = 0, 1$ denotes the Dirac index which is restricted to the upper component in the Dirac representation.

$D_a(y) \equiv \epsilon_{abc} u_b^T(y) C\gamma_5 d_c(y)$ denotes the composite operator for the scalar diquark. We demand that the NBS wave function should satisfy the Schrödinger equation

$$\left(-\frac{\nabla^2}{2\mu} + \hat{V}\right)\psi_n(\mathbf{r}) = \mathcal{E}_n \psi_n(\mathbf{r}), \tag{8}$$

where $\mathcal{E}_n \equiv E_n - m_c - m_{D0}$ with E_n and m_{D0} being the relativistic energy of the n-th state and the scalar diquark mass, respectively. $\mu \equiv \frac{1}{1/m_c + 1/m_{D0}}$ denotes the reduced mass. Note that we employ the charm quark mass m_c which is obtained by applying a similar procedure in the charmonium sector. $\hat{V}$ denotes the potential operator, which is derivatively expanded as

$$\hat{V} \simeq V_0(r) + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{s}_c, \tag{9}$$

where $\mathbf{L}$ and $\mathbf{s}_c$ denote the orbital angular momentum operator and the spin operator of the charm quark, respectively. We first consider $J = 1/2^+$ sector, where Schrödinger equation reduces to

$$\left(-\frac{\nabla^2}{2\mu} + V_0(r)\right)\psi_{1/2+}(\mathbf{r}) = \mathcal{E}_{1/2+}\psi_{1/2+}(\mathbf{r}), \tag{10}$$

where $\psi_{1/2+}(\mathbf{r})$ denotes the NBS wave function for the ground state of Λ_c for $J^P = 1/2^+$ sector. $\mathcal{E}_{1/2+} \equiv E_{1/2+} - m_c - m_{D0}$ denotes "binding energy" of this state with $E_{1/2+}$ being the relativistic energy of this state. $V_0(r)$ can easily be obtained as

$$V_0(r) = E_{1/2+} - m_c - m_{D0} + \frac{1}{2\mu}\frac{\nabla^2\psi_{1/2+}(\mathbf{r})}{\psi_{1/2+}(\mathbf{r})}, \tag{11}$$

where the scalar diquark mass m_{D0} is considered as an unknown parameter which will be fixed later. We next consider the negative parity sectors, i.e., $J^P = 1/2^-$ and $3/2^-$ states. We regard $V_0(r)$ as unperturbed and $V_{\mathrm{LS}}(r)$ as perturbing potentials and the first order perturbation theory is used compare the mass spectrum of $J^P = 1/2^-$ and $3/2^-$ states obtained by the Schrödinger equation and those obtained from the two-point correlators. Note that we have two unknowns m_{D0} and $\langle V_{\mathrm{LS}}(r)\rangle$, which can be completely determined from the two equations for $J^P = 1/2^-$ and $3/2^-$ states.

An explicit lattice QCD calculation is performed by employing the 2+1 flavor QCD gauge configurations generated by PACS-CS Collaboration on $32^3 \times 64$ lattice with $m_\pi \simeq 700$ MeV and the lattice spacing $a \simeq 0.09$ fm [7]. As a result, $m_{D0} = 1.220(45)$ GeV is obtained, which is roughly consistent with twice naive estimates of the constituent quark mass as $m_{\mathrm{rho}} = 1.098(5)$ GeV and $(2/3)m_{\mathrm{N}} = 1.049(12)$ GeV.

Fig. 1 shows the result of quark-diquark potential together with the $c\bar{c}$ potential obtained from the lattice QCD calculation. We see that the quark-diquark potential is of Cornell-type, i.e., Coulomb-like behavior is seen at short distance while the linear rising behavior is seen at long distance. We see that the string tension is roughly the same as that of the $c\bar{c}$ potential. In contrast, the strength of the Coulomb part is weaker than that of $c\bar{c}$ potential, which seems to be caused by the finite size of the diquark. (See Ref.[8].)

2.2 Axialvector diquark in Σ_c

As our second target, we consider Σ_c baryon as a two-body problem between a charm quark and an axialvector diquark [9]. Unlike the previous target, the potential of this system has a spin-dependent central potential so that the method of Kawanai and Sasaki can be applied with a slight modification. For this purpose, we first consider the NBS wave function as

$$\psi_{n,\alpha,\mu}(\mathbf{x}-\mathbf{y}) \equiv \langle 0|c_{a,\alpha}(\mathbf{x})A_{a,\mu}(\mathbf{y})|n\rangle, \tag{12}$$

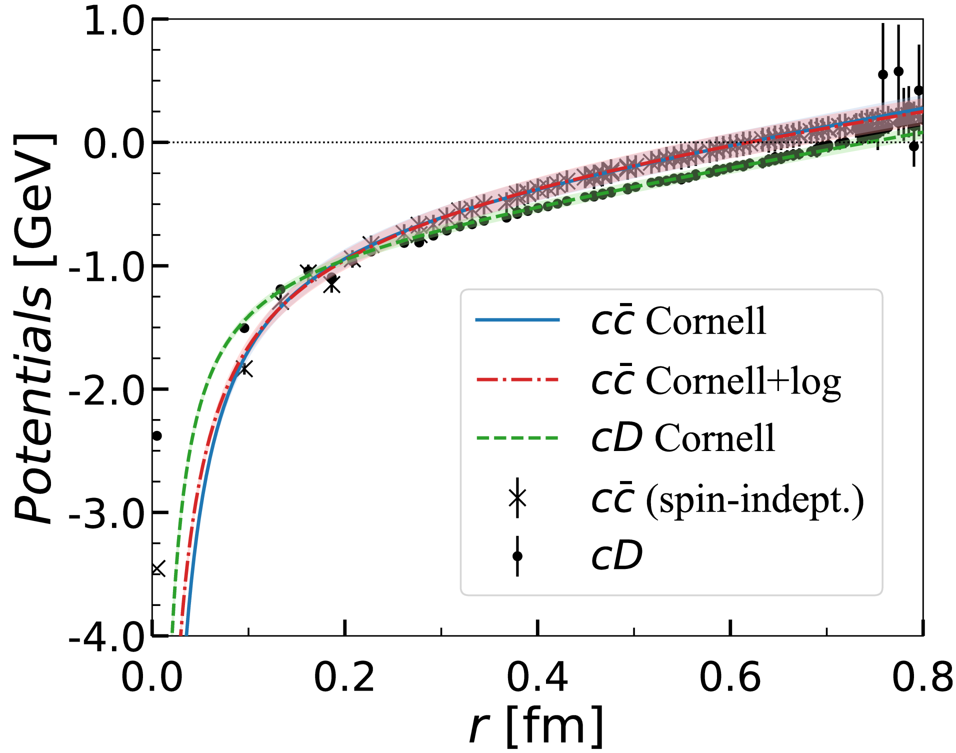

Figure 1: Cornell function fit to the central potential between a charm quark and a scalar diquark together with that to the spin-independent central $c\bar{c}$ potential. Cornell+log type fit is also added for the $c\bar{c}$ potential for a reference.

where $|n\rangle$ denotes the n-th state for Σ_c baryon. $\alpha = 0, 1$ and $\mu = 1, 2, 3$ denote Dirac index and Lorentz index, respectively, which are restricted to the non-relativistic subspaces. $a = 1, 2, 3$ denotes the color indices. $A_{a,\mu}(y) \equiv \epsilon_{abc} u_b^T(y) C\gamma_\mu u_c(y)$ denotes the composite operator for the uu axialvector diquark. We demand the NBS wave function to satisfy Schrödinger equation

$$\left(-\frac{\nabla^2}{2\mu} + \hat{V}\right)\psi_n(\mathbf{r}) = \mathcal{E}_n \psi_n(\mathbf{r}), \tag{13}$$

where $\mathcal{E}_n \equiv E_n - m_c - m_{D1}$. $\mu \equiv \frac{1}{1/m_c + 1/m_{D1}}$ denotes the reduced mass with m_{D1} being the mass of the axialvector diquark. Note that we employ m_c which is determined by applying Kawanai-Sasaki prescription for charmonium sector. $\hat{V}$ denotes the potential operator, which is derivatively expanded in the following way:

$$\hat{V} \simeq V_0(r) + V_s(r)\,\mathbf{s}_c \cdot \mathbf{s}_D, \tag{14}$$

where $\mathbf{s}_c$ and $\mathbf{s}_D$ denote the spin operators for the charm quark and the axialvector diquark, respectively. Note that there can be a D-wave contamination in the NBS wave function. However, we have explicitly examined that the D-wave contamination is negligible so that the tensor potential can be safely neglected. Using $\mathbf{s}_c \cdot \mathbf{s}_D = -1$ for total spin $S = 1/2$ while $1/2$ for $S = 3/2$, we demand the spin-dependent potential $V_s(r)$ should vanish at spatial infinity, which leads to the condition as

$$\mu = -\frac{1}{2(E_{3/2} - E_{1/2})} \lim_{r\to\infty} \left(\frac{\nabla^2 \psi_{3/2}(\mathbf{r})}{\psi_{3/2}(\mathbf{r})} - \frac{\nabla^2 \psi_{1/2}(\mathbf{r})}{\psi_{1/2}(\mathbf{r})}\right), \tag{15}$$

where $\psi_{3/2}(\mathbf{r})$, $\psi_{1/2}(\mathbf{r})$, $E_{3/2}$ and $E_{1/2}$ denote the NBS wave functions and the relativistic energies for the ground states of Λ_c in the spin 3/2 and 1/2 sectors, respectively. We give explicit expressions for the spin-independent and spin-dependent central potentials as

$$V_0(r) = \frac{2E_{3/2}+E_{1/2}}{3} - m_c - m_{D1} + \frac{1}{2\mu}\left(\frac{2}{3}\frac{\nabla^2\psi_{3/2}(\mathbf{r})}{\psi_{3/2}(\mathbf{r})} + \frac{1}{3}\frac{\nabla^2\psi_{1/2}(\mathbf{r})}{\psi_{1/2}(\mathbf{r})}\right) \tag{16}$$

$$V_s(r) = \frac{2}{3}\left(E_{3/2} - E_{1/2}\right) + \frac{1}{3\mu}\left(\frac{\nabla^2\psi_{3/2}(\mathbf{r})}{\psi_{3/2}(\mathbf{r})} - \frac{\nabla^2\psi_{1/2}(\mathbf{r})}{\psi_{1/2}(\mathbf{r})}\right). \tag{17}$$

An explicit calculation is performed by employing the 2+1 flavor QCD gauge configuration generated by PACS-CS Collaboration on the $32^3 \times 64$ lattice with $m_\pi \simeq 700$ MeV and lattice spacing $a \simeq 0.09$ fm [7]. Fig. 2 shows the preliminary results of $V_0(r)$ and $V_s(r)$. We see that the $V_s(r)$ is short ranged and that $V_0(r)$ is of Cornell-type, i.e. Coulomb-like behavior is seen at the short distance whereas the linear rising behavior is seen at the long distance. Since the statistics is poor at the moment so that we are not yet at the stage to estimate the quantitative values of the strength of the Coulomb part, the string tension and the axialvector diquark mass.

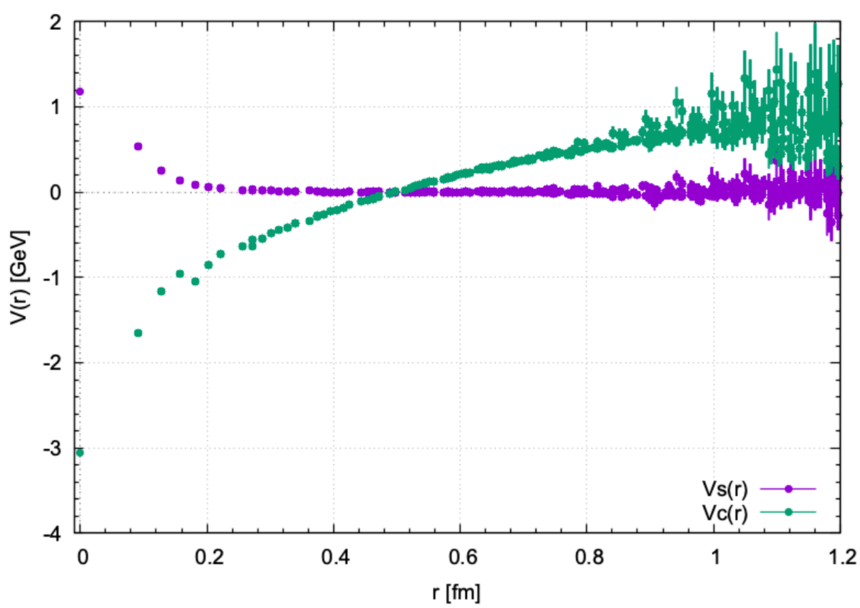

Figure 2: Spin-independent central potential $V_0(r)$ and Spin-dependent central potential $V_s(r)$ between a charm quark and an axialvector diquark in the Σ_c baryon obtained from lattice QCD.

3 Summary

We have seen that, although the diquarks play an important role in phenomenology, it is difficult to study diquark properties both in experiment and in lattice QCD calculations. We have seen two strategies for diquarks in lattice QCD. We have presented our own strategy to study the diquark by applying the HAL QCD method with (a deformation of) Kawanai and Sasaki's extension to the quark-diquark system. We have carried out two such calculations (1) Λ_c system with a charm quark and a scalar (ud) diquark and (2) Σ_c system with a charm quark and a axialvector (uu) diquark. These calculations employ the 2+1 QCD gauge configurations generated by PACS-CS Collaboration on $32^3 \times 64$ lattice which employs a heavy ud quark mass corresponding to $m_\pi \simeq 700$ MeV. It is important to go to lighter quark mass region, for which it is necessary to consider a coupled-channel extension including the meson-baryon channels since some of the employed states become a resonance state.

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References

[1] R. L. Jaffe, Phys. Rept. **409**, 1-45 (2005) doi:10.1016/j.physrep.2004.11.005 [arXiv:hep-ph/0409065 [hep-ph]].

[2] M. Hess, F. Karsch, E. Laermann and I. Wetzorke, Phys. Rev. D **58**, 111502 (1998) doi:10.1103/PhysRevD.58.111502 [arXiv:hep-lat/9804023 [hep-lat]].

[3] C. Alexandrou, P. de Forcrand and B. Lucini, Phys. Rev. Lett. **97**, 222002 (2006) doi:10.1103/PhysRevLett.97.222002 [arXiv:hep-lat/0609004 [hep-lat]].

[4] N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. **99**, 022001 (2007) doi:10.1103/PhysRevLett.99.022001 [arXiv:nucl-th/0611096 [nucl-th]].

[5] T. Kawanai and S. Sasaki, Phys. Rev. Lett. **107**, 091601 (2011) doi:10.1103/PhysRevLett.107.091601 [arXiv:1102.3246 [hep-lat]].

[6] K. Watanabe, Phys. Rev. D **105**, no.7, 074510 (2022) doi:10.1103/PhysRevD.105.074510 [arXiv:2111.15167 [hep-lat]].

[7] S. Aoki, et al. [PACS-CS], Phys. Rev. D **79**, 034503 (2009) doi:10.1103/PhysRevD.79.034503 [arXiv:0807.1661 [hep-lat]]. Copy to ClipboardDownload

[8] D. Jido and M. Sakashita, PTEP **2016**, no.8, 083D02 (2016) doi:10.1093/ptep/ptw113 [arXiv:1605.07339 [nucl-th]].

[9] S. Nishioka and N. Ishii, work in progress.

[10] International Lattice Data Grid (ILDG), http://www.lqcd.org/ildg

[11] Japan Lattice Data Grid (JLDG), http://www.jldg.org

[12] Lattice QCD code Bridge++, http://bridge.kek.jp/Lattice-code/