β-decay half-life as an indicator of shape-phase transition in neutron-rich nuclei

Kenichi Yoshida[1], Yifei Niu[2], and Futoshi Minato[3]
[1]*Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan*
[2]*School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China*
[3]*Department of Physics, Kyushu University, Fukuoka 819-0395, Japan*

The physics of exotic nuclei has been one of the major subjects in the field of nuclear science with the upgrading and constructing of the radioactive-ion (RI) beam accelerator facilities around the world. Recent progresses in the development of the experimental technique of spectroscopic studies have unveiled rich structures of exotic nuclei [1]. It has attracted much interest in how the shape of a nucleus changes as a function of the number of neutrons and protons. The β-decay half-life is one of the most experimentally accessible physical quantities for RI beam facilities and plays a decisive role in determining the time scale of the r-process nucleosynthesis [2].

We have initiated an international collaborative research project to theoretically tabulate the decay and reaction rates of nuclei required for r-process simulations [3], as experimental access to many unstable nuclei involved in the r-process is not feasible. Evaluation of various reaction rates through theoretical calculations is thus essential.

The Zr isotopes with $A \simeq 100$ have been of theoretical and experimental interest in nuclear structure as a region of competition between various coexisting prolate, oblate, and spherical nuclear shapes. The first-order phase transition occurs uniquely in this region, while we usually see a gradual change of deformation with an increase in the neutron and/or proton number in other regions such as in the rare-earth nuclei. The calculations predicted the shape transition from the deformed to the spherical configuration around $N = 74$. Furthermore, the observed short half-lives around $A \simeq 110$ region speed up the r-matter flow [4].

As part of the project, we investigated the effect of nuclear deformation on β decay rates. Using the Zr isotopes where multiple deformed states compete energetically as an example, we demonstrated significant changes in decay rates when shape transitions occur with changes in neutron number.

In the framework of the nuclear energy-density functional (EDF) method we employ, the ground state of a mother nucleus is described by solving the Hartee–Fock–Bogoliubov (HFB) equation [5]. The single-particle and pair potentials are given by the functional derivative of the EDF with respect to the particle density and the pair density, respectively. While, the excited states $|f\rangle$ in a daughter nucleus are described as one-phonon excitations built on the ground state $|\mathrm{RPA}\rangle$ of the mother nucleus as

$$|f\rangle = \hat{\Gamma}^\dagger_f |\mathrm{RPA}\rangle, \tag{1}$$

$$\hat{\Gamma}^\dagger_f = \sum_{\alpha\beta} \left\{ X^f_{\alpha\beta} \hat{a}^\dagger_{\alpha,\mathrm{n}} \hat{a}^\dagger_{\beta,\mathrm{p}} - Y^f_{\alpha\beta} \hat{a}_{\beta,\mathrm{p}} \hat{a}_{\alpha,\mathrm{n}} \right\}, \tag{2}$$

where $\hat{a}^\dagger_\mathrm{n}(\hat{a}^\dagger_\mathrm{p})$ and $\hat{a}_\mathrm{n}(\hat{a}_\mathrm{p})$ are the neutron (proton) quasiparticle (labeled by α and β) creation and annihilation operators that are defined in terms of the solutions of the HFB equation with the Bogoliubov transformation. The phonon states, the amplitudes X^f, Y^f and the vibrational frequency ω_f, are obtained in the pnQRPA with a cutoff at 60 MeV [6]. The residual interactions entering into the pnQRPA equation are given by the EDF self-consistently

Figure 1 shows the potential energy surfaces of the neutron-rich Zr isotopes with $N = 72$–78 calculated by using the SkM* functional, where the shape transition is predicted to occur with

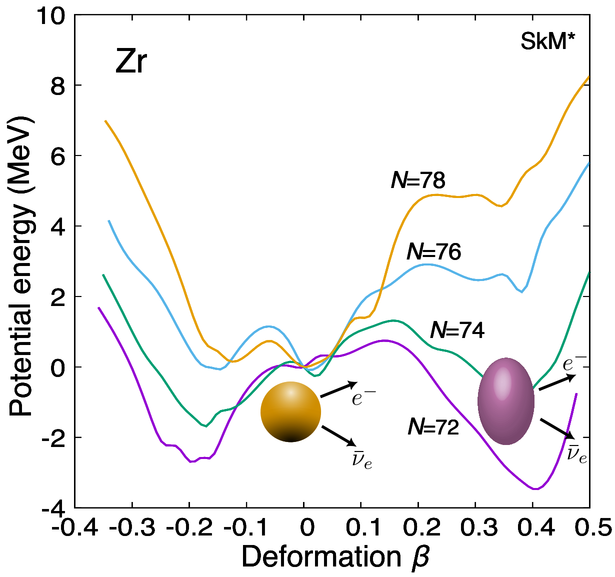

Figure 1: Potential energy surface (zero at the spherical configuration) of $^{112\text{–}118}$Zr obtained by employing the SkM* functional.

an increase in the neutron number. The prolate and oblate configurations compete in energy in ^{112,114}Zr, while the spherical and oblately deformed configurations compete in energy in ^{116,118}Zr. In heavier isotoeps, the sphrical configuration is favored due to the magic number $N = 82$. We found a similar feature in the results obtained by employing the SLy4 functional, where the spherical and oblate configurations compete in energy. A standard probe of the shape change from the prolate to oblate deformations is a sign change of the spectroscopic quadrupole moment of the 2_1^+ state. However, it is still challenging to measure the spectroscopic quadrupole moment for these neutron-rich nuclei.

The β-decay half-life $T_{1/2}$ can be calculated with the Fermi's golden rule as

$$\frac{1}{T_{1/2}} = \frac{\lambda_\beta}{\log 2} = \frac{(g_A/g_V)^2_{\text{eff}}}{D} \sum_{E_f^* < Q_\beta} f(Z, Q_\beta - E_f^*) |\langle f|\hat{F}|\text{RPA}\rangle|^2, \tag{3}$$

where $D = 6147.0$ s and we set $(g_A/g_V)_{\text{eff}} = 1$ rather than its actual value of 1.26 to account for the quenching of the spin matrix in nuclei. The transition matrix element is evaluated in the allowed approximation, and thus the oprator $\hat{F}$ is taken as the Gamow–Teller (GT) type.

The calculated half-lives together with the available experimantal data are shown in Fig. 2. The observed half-lives up to $N = 72$ are well reproduced by the calculation using the SkM* functional with the prolate configuration. We see that the calculated half-lives calculated assuming the prolate shape shorten monotonically beyond $N = 72$ as the FRDM prediction, wheres a sudden drop occurs at $N = 74$ when the nuclear shape changes to the oblate deformation.

We discuss the mechanism for the shortening of the half-lives due to the shape change from the prolate to oblate deformations. We found that the GT states appear in low energies for the oblate configuration. In ^{114}Zr, the GT strengths for the oblate configuration are larger than those for the prolate configuration. The GT strengths for the spherical configuration appear relatively higher in energy but are much larger than those for the deformed configuration, leading to the half-lives being as short as for the oblate configuration.

We show in Fig. 2 the results considering the quasiparticle-phonon coupling for the spherical configuration denoted as QPVC. Comparing with the results of half-lives for the spherical

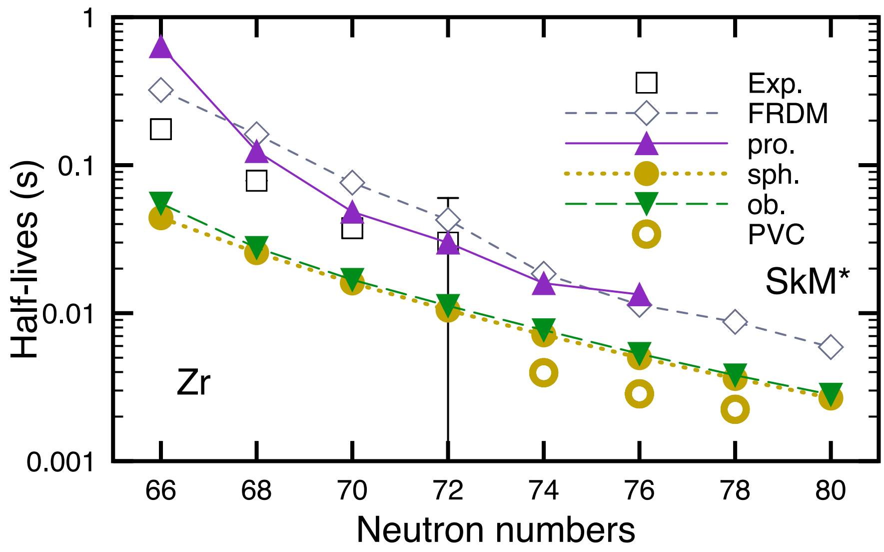

Figure 2: β-decay half-lives of the Zr isotopes obtained by employing the SkM* functional. The calculated half-lives are compared with the experimental data [7] and the FRDM+QRPA calculation [8].

configuration, those for QPVC are shortened. This is because the GT states couple with other phonon states, resulting in distributions to lower energies. It is considered that the PVC effect is weaker in deformed nuclei than in spherical nuclei because the quadrupole correlation is mostly described as a deformed mean field. In a simplified model, we showed this consideration being valid in the present case [3].

To summarize, we have investigated the β-decay half-lives in the Zr isotopes with shape changes. The GT strength distributions were evaluated in the proton–neutron QRPA and QPVC approaches. The spherical and oblate configurations give similar half-lives, and the oblate configuration is shorter than the prolate one at the RPA level. The PVC effect could further reduce the half-lives; however, the effect would be smaller for a deformed configuration than that for a spherical one. When a sudden drop of half-lives around $N = 74$ is observed experimentally, it is an indication of the shape transition. The present model does not take into account the triaxial shape and the shape mixing. Considering these effects remains a challenge in the future.

References

[1] T. Nakamura, H. Sakurai, and H. Watanabe, Prog. Part. Nucl. Phys. **97**, 53 (2017).

[2] S. Nishimura, Prog. Theor. Exp. Phys. **2012**, 03C006 (2012).

[3] K. Yoshida, Y. Niu, and F. Minato, Phys. Rev. C **108**, 034305 (2023).

[4] S. Nishimura, *et al.*, Phys. Rev. Lett. **106**, 052502 (2011).

[5] H. Kasuya and K. Yoshida, Prog. Theor. Exp. Phys. **2021**, 013D01 (2021).

[6] K. Yoshida, Prog. Theor. Exp. Phys. **2013**, 113D02 (2013).

[7] G. Lorusso, Phys. Rev. Lett. **114**, 192501 (2015).

[8] P. Möller, *et al.*, Phys. Rev. C, **67**, 055802 (2003).