Two-neutrino double-β decay within the mapped IBM

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Study of $\beta\beta$ decay

- Nature of neutrinos, test of fundamental symmetries, …
- Experiments: GERDA, NEMO, KamLAND …
- Variety of theoretical predictions (QRPA, LSSM, IBM, EDF, ab initio, etc.)

Avignone et al., RMP (2008), Agostini et al., RMP (2023), Engel, Menendez RPP (2017), etc.

This work

- Framework: EDF-to-IBM mapping
- $2\nu\beta\beta$ decay of 26 even-even nuclei
- No closure approximation: calculations for odd-odd nuclei

PHYSICAL REVIEW C **105**, 044301 (2022)

Two-neutrino double-β decay in the mapped interacting boson model

Kosuke Nomura *

Contents

· Introduction, interacting boson model

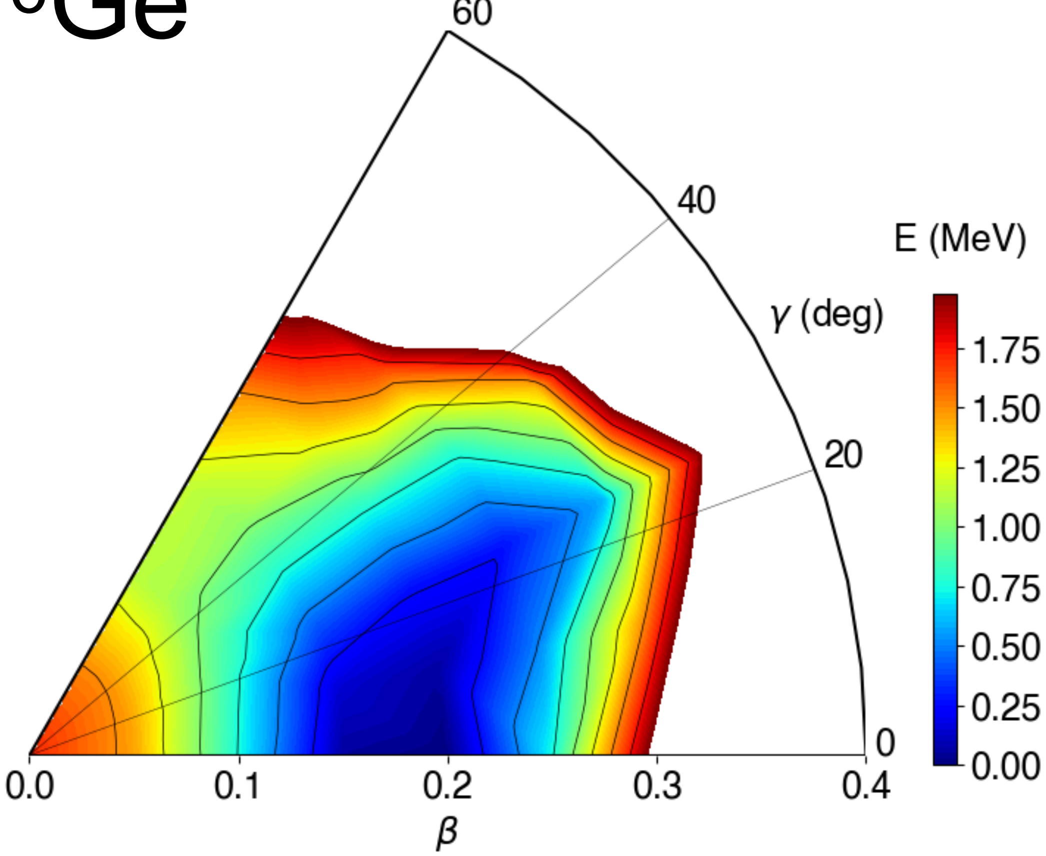
· Low-lying structure of even-even and odd-odd nuclei

· $2\nu\beta\beta$ decay

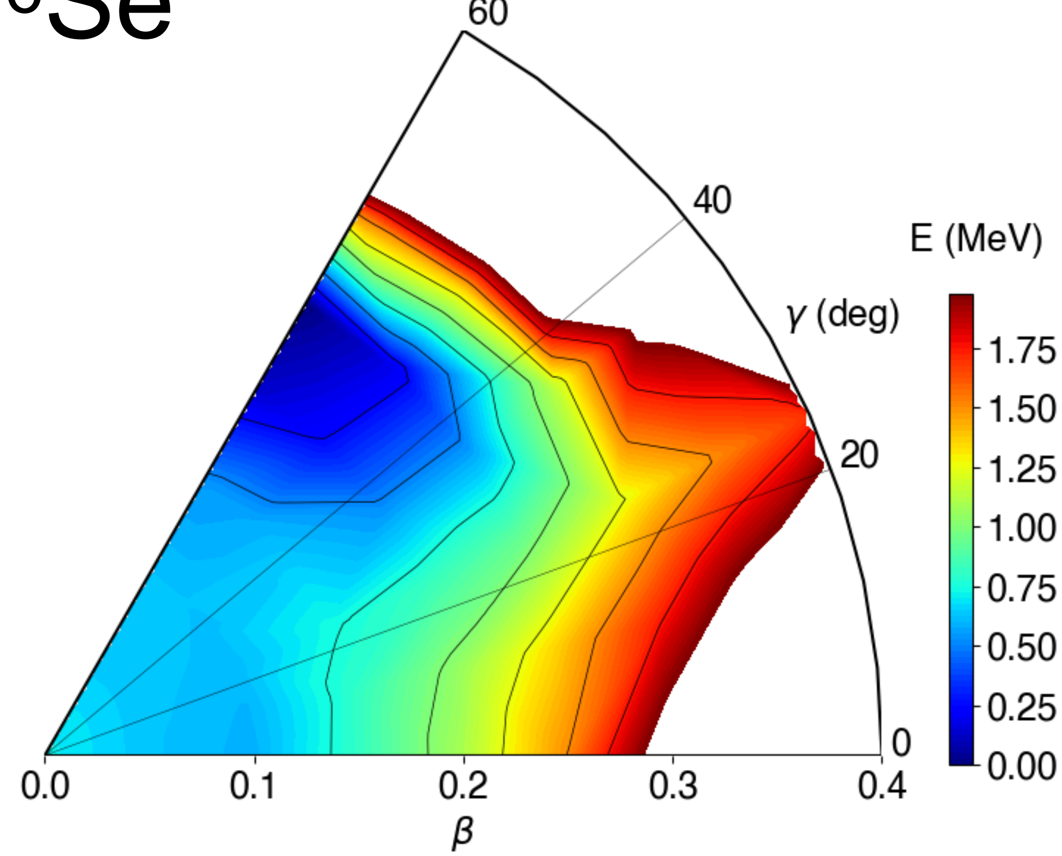
Mean-field calculation

Potential Energy Surface (PES)

^{76}Ge



^{76}Se



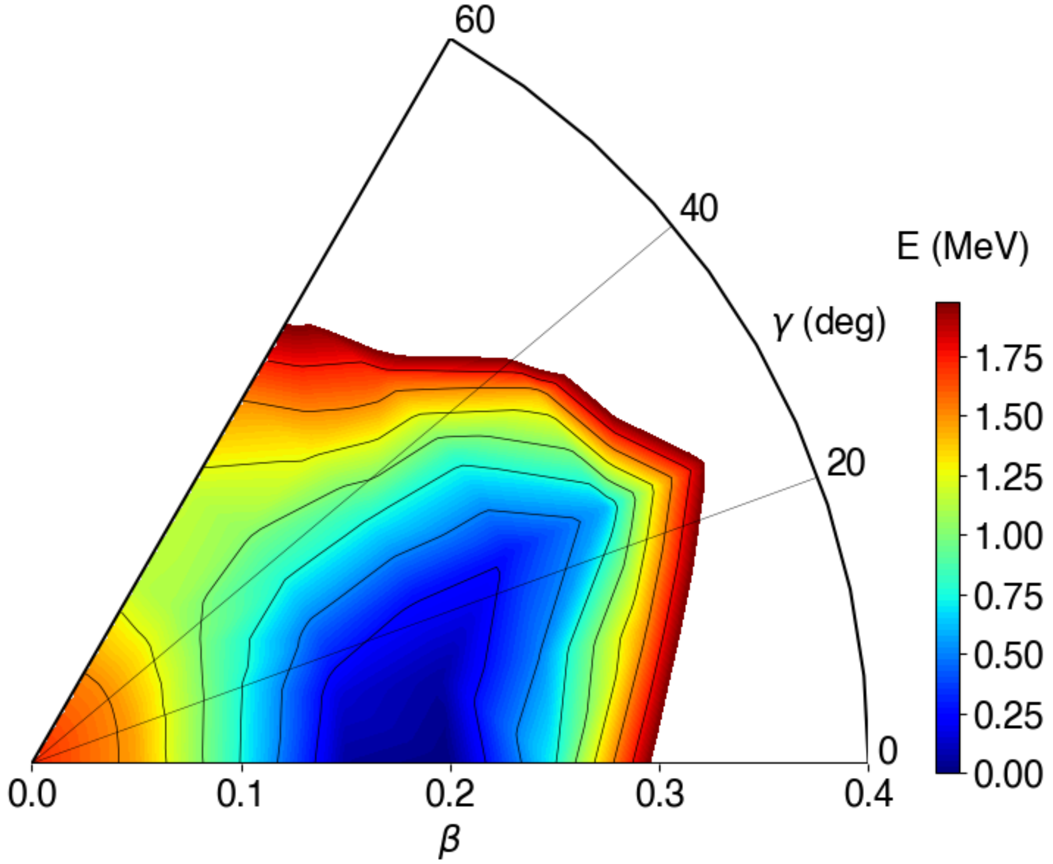
prolate deformation

Oblate deformation

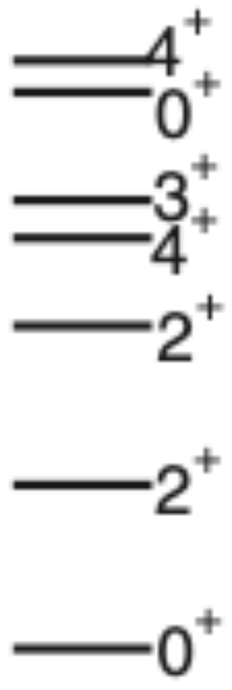
… obtained from the relativistic energy density functional

Computing energy spectra

Intrinsic frame

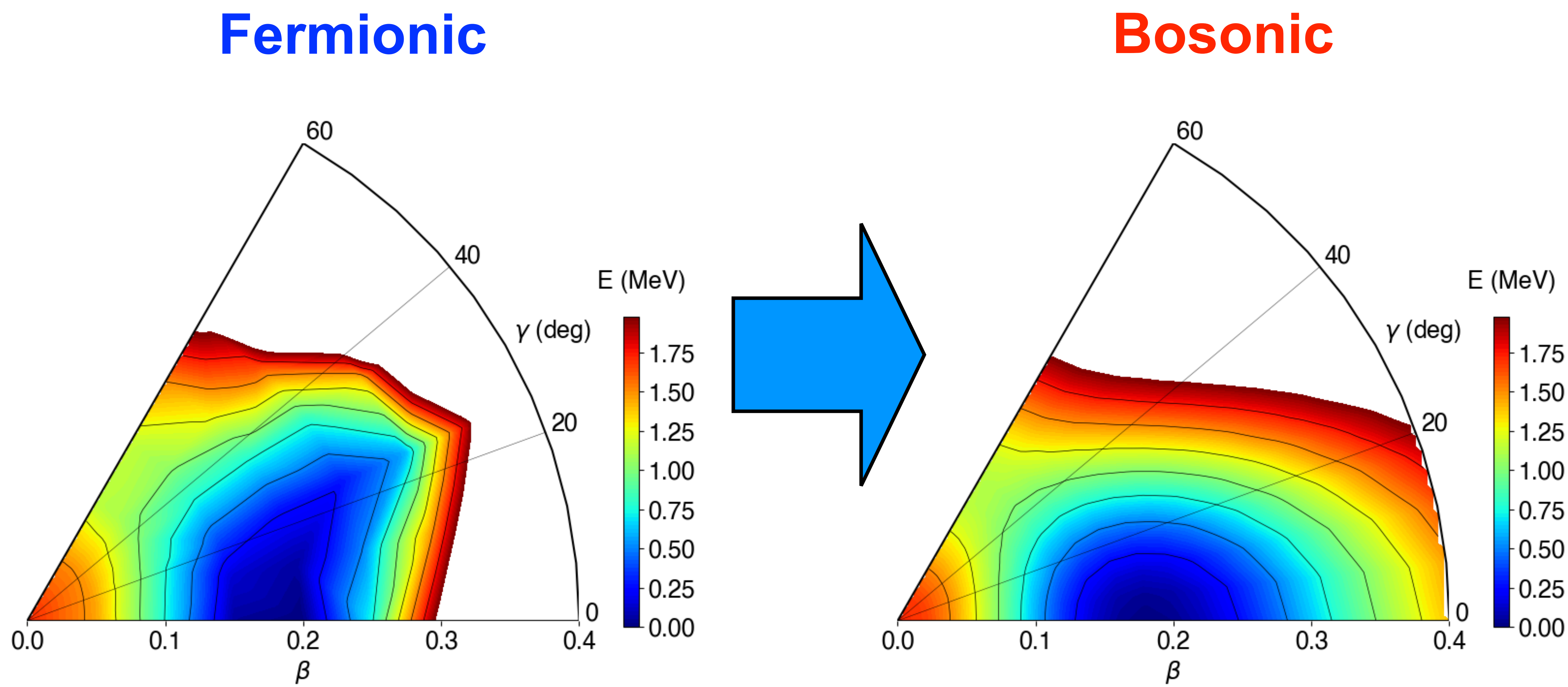


lab. frame



Observables:
Excitation spectra, EM properties, β, $\beta\beta$ decay?

Mean-field to IBM



- SCMF energy surface is mapped onto the IBM Hamiltonian
- Diagonalization of the mapped Hamiltonian yields energy spectra

KN et al. PRL101 (2008) 142501; PRC81 (2010) 044307

Interacting Boson Model

Arima, Iachello (1975)

Building blocks:

s, d bosons ~ $J = 0^+, 2^+$ collective pairs of valence nucleons

Hamiltonian:

$$\hat{H}_{\text{IBM}} = \epsilon_d (\hat{n}_{d_\nu} + \hat{n}_{d_\pi}) + \kappa \hat{Q}_\nu \cdot \hat{Q}_\pi$$

pairing-like (spherical driving)

quadrupole-quadrupole (deformation driving)

Geometry of the IBM

Energy surface:

$$E_{\mathrm{IBM}}(\beta,\gamma) = \langle \phi \,|\, \hat{H}_{\mathrm{IBM}} \,|\, \phi \rangle$$

… with boson coherent state

$$|\,\phi\rangle \propto \Pi_{\rho=\nu,\pi}\left[s_\rho^\dagger + \beta\cos\gamma d_{\rho,0}^\dagger + \frac{1}{\sqrt{2}}\beta\sin\gamma\left(d_{\rho,+2}^\dagger + d_{\rho,-2}^\dagger\right)\right]^{N_\rho}|\,0\rangle$$

Ginocchio-Kirson (1980)

IBM Hamiltonian is determined by

$$E_{\mathrm{SCMF}}(\beta,\gamma) \approx E_{\mathrm{IBM}}(\beta,\gamma)$$

KN et al. PRL101 (2008) 142501

Interacting Boson-Fermion-Fermion Model

$$\boxed{\hat{H}_{\mathrm{IBFFM}} = \hat{H}_{\mathrm{IBM}} + \hat{H}_{\mathrm{F}} + \hat{V}_{\mathrm{BF}} + \hat{V}_{\nu\pi}}$$

$$\hat{H}_{\mathrm{F}} = \sum_{j_\rho} \epsilon_{j_\rho} \hat{n}_{j_\rho}$$

$$\hat{V}_{\mathrm{BF}} = \hat{V}_{\mathrm{dyn}} + \hat{V}_{\mathrm{exc}} + \hat{V}_{\mathrm{mon}}$$

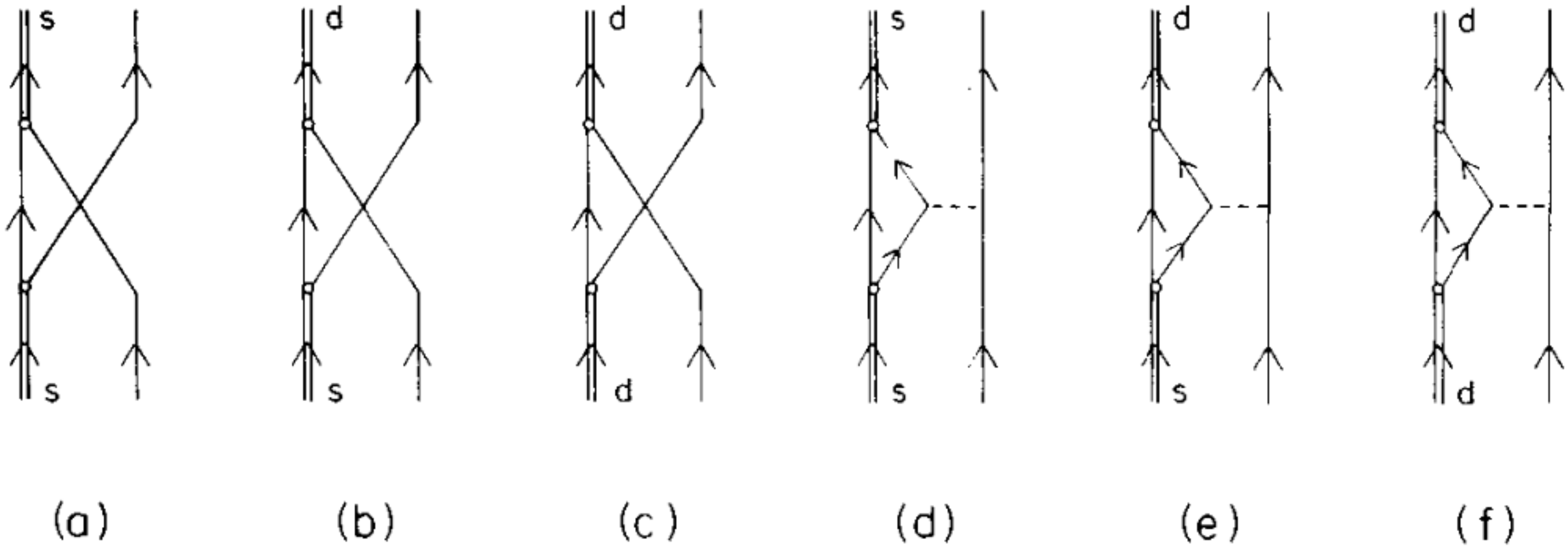
$\hat{V}_{\mathrm{dyn}}$: quadrupole dynamical; $\hat{V}_{\mathrm{exc}}$: exchange; $\hat{V}_{\mathrm{mon}}$: monopole

neutron-proton interaction

$$\hat{V}_{\nu\pi} = 4\pi[v_{\mathrm{d}} + v_{\mathrm{ssd}}\boldsymbol{\sigma}_\nu \cdot \boldsymbol{\sigma}_\pi]\delta(\boldsymbol{r})\delta(\boldsymbol{r}_\nu - r_0)\delta(\boldsymbol{r}_\pi - r_0) - \frac{1}{\sqrt{3}} v_{\mathrm{ss}}\boldsymbol{\sigma}_\nu \cdot \boldsymbol{\sigma}_\pi + v_{\mathrm{t}}\left[\frac{3(\boldsymbol{\sigma}_\nu \cdot \mathbf{r})(\boldsymbol{\sigma}_\pi \cdot \mathbf{r})}{r^2} - \boldsymbol{\sigma}_\nu \cdot \boldsymbol{\sigma}_\pi\right]$$

- Iachello, Van Isacker, “The interacting boson-fermion model” (1991)

Boson-fermion interactions



exchange terms

direct terms

Building the IBFFM Hamiltonian

Microscopic input from DFT:

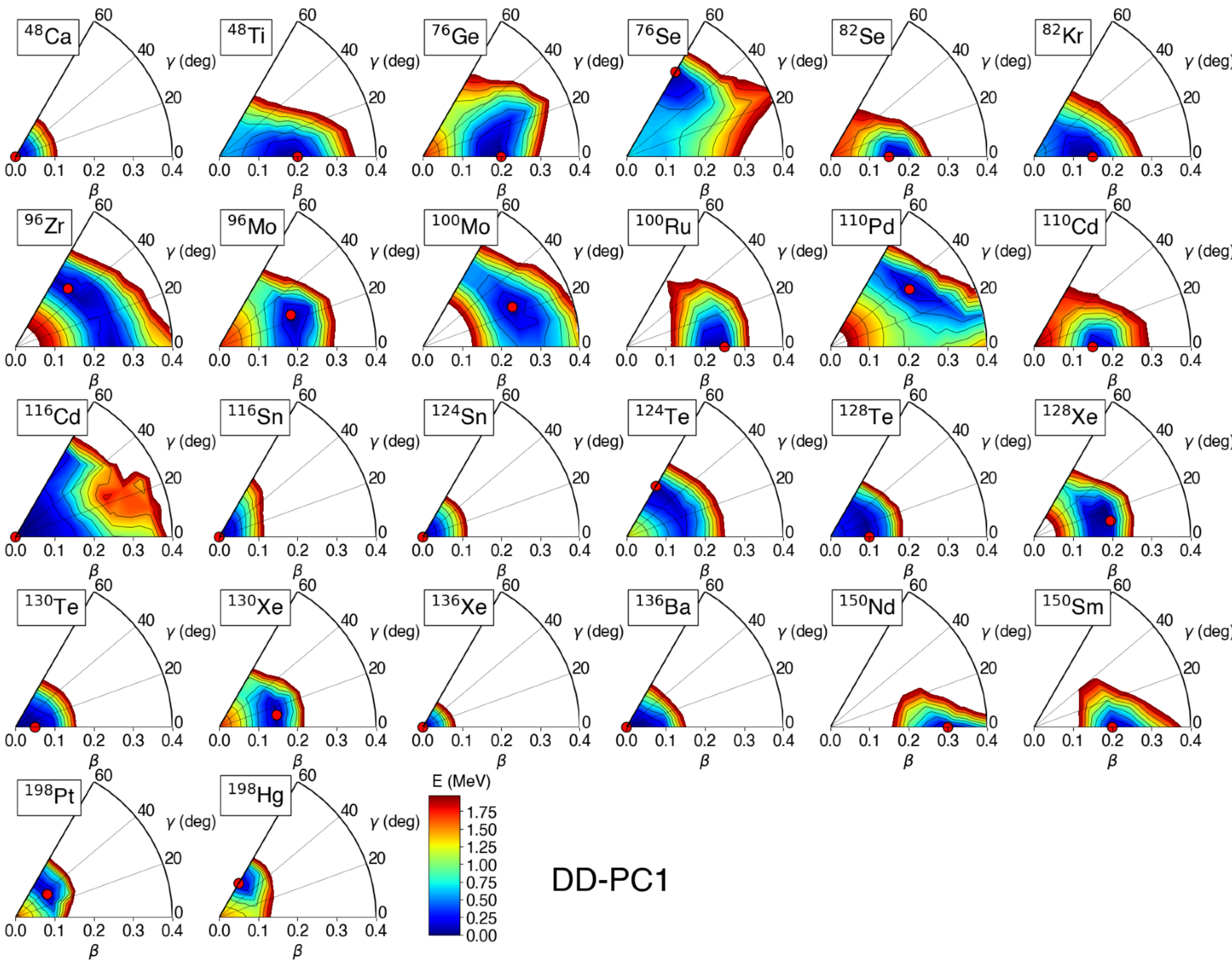
- PES : $\hat{H}_{\mathrm{IBM}}$
- Spherical single (quasi) particle energies : $\hat{H}_{\mathrm{F}}$
- Particle occupation probabilities : $\hat{V}_{\mathrm{BF}}$ and $\hat{V}_{\nu\pi}$

… remaining coupling constants of $\hat{V}_{\mathrm{BF}}$ and $\hat{V}_{\nu\pi}$ are fitted to data

KN et al. PRC93 (2016) 054305; PRC99 (2019) 034308

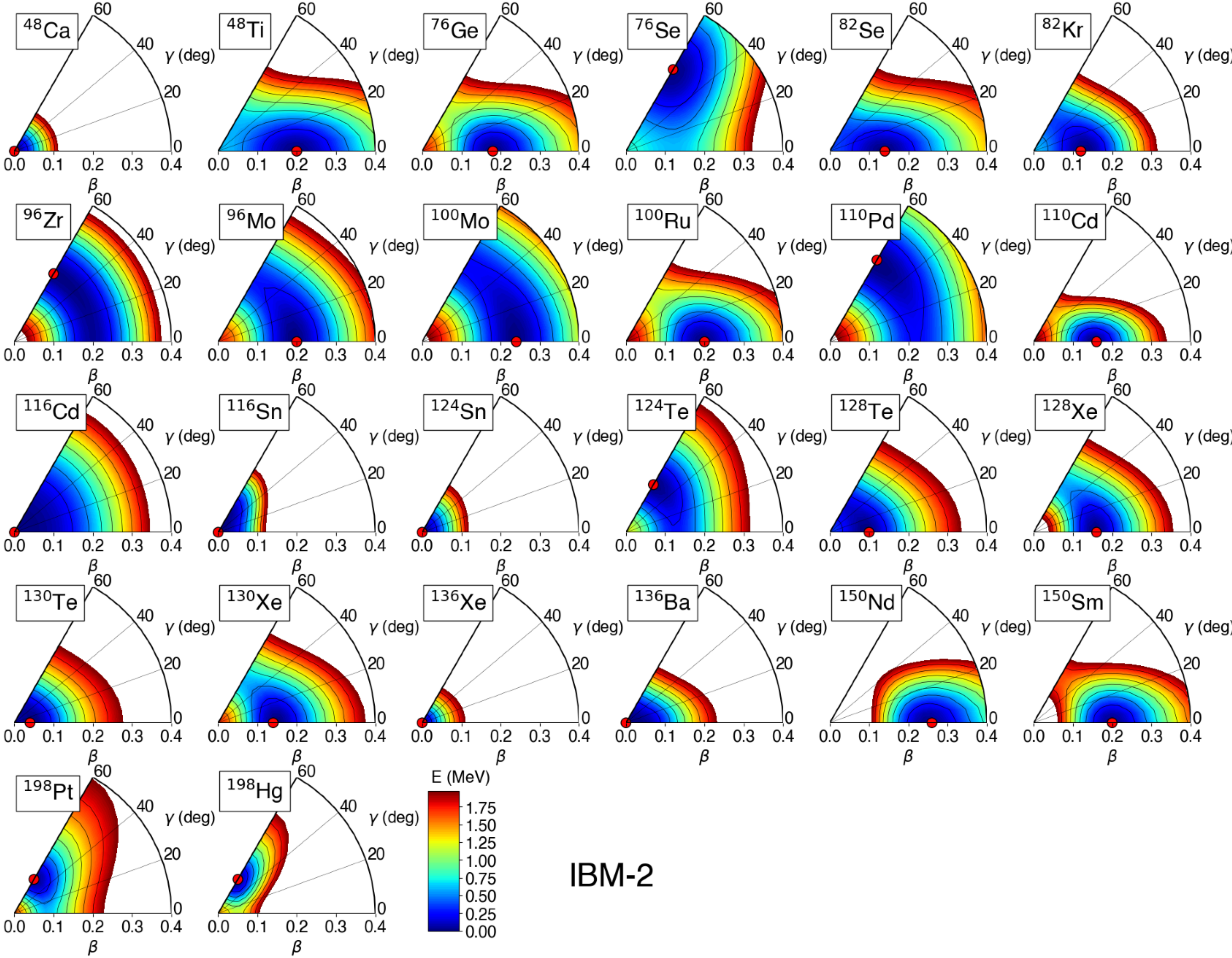
Low-lying structure

Energy surfaces of even-even nuclei

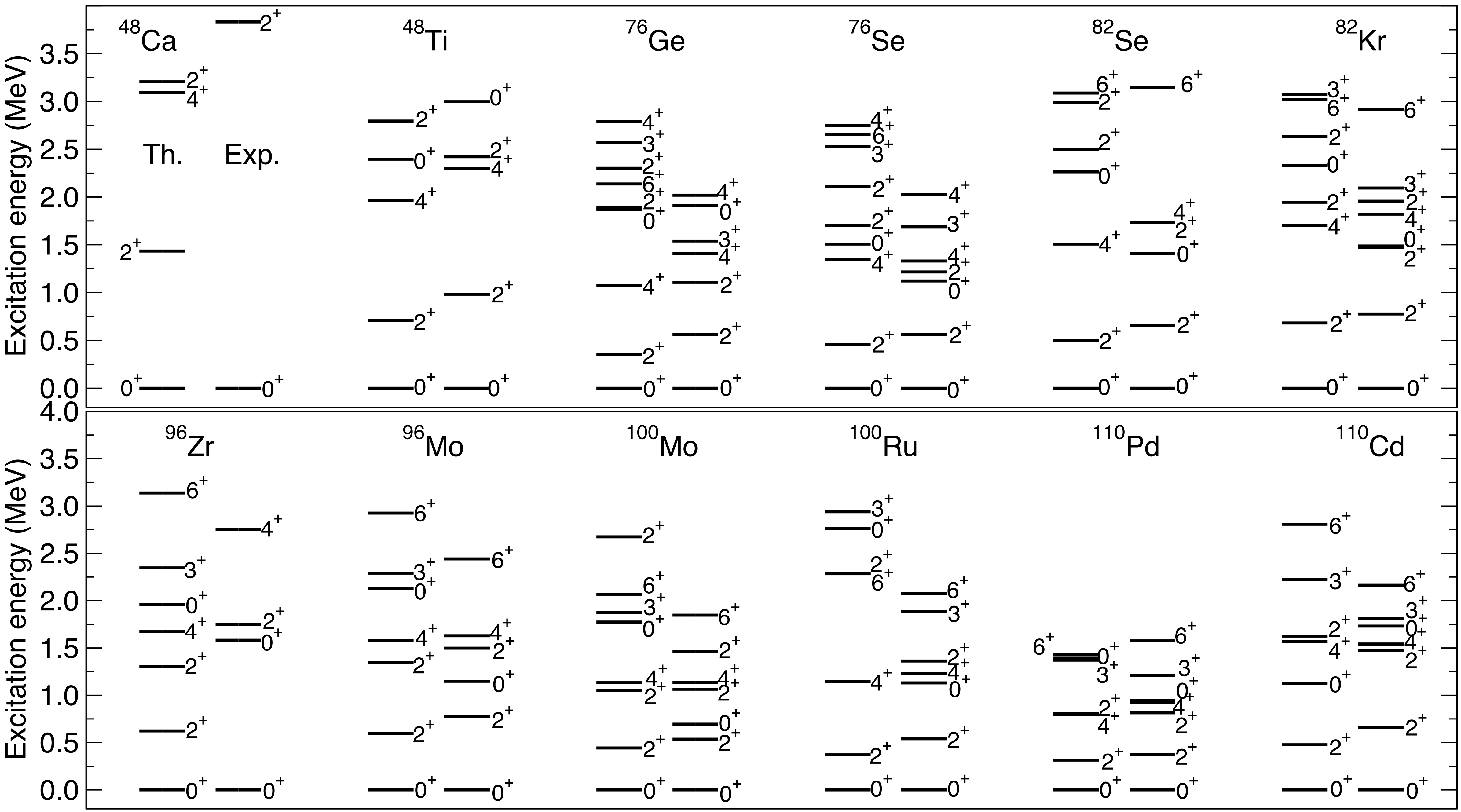


DD-PC1

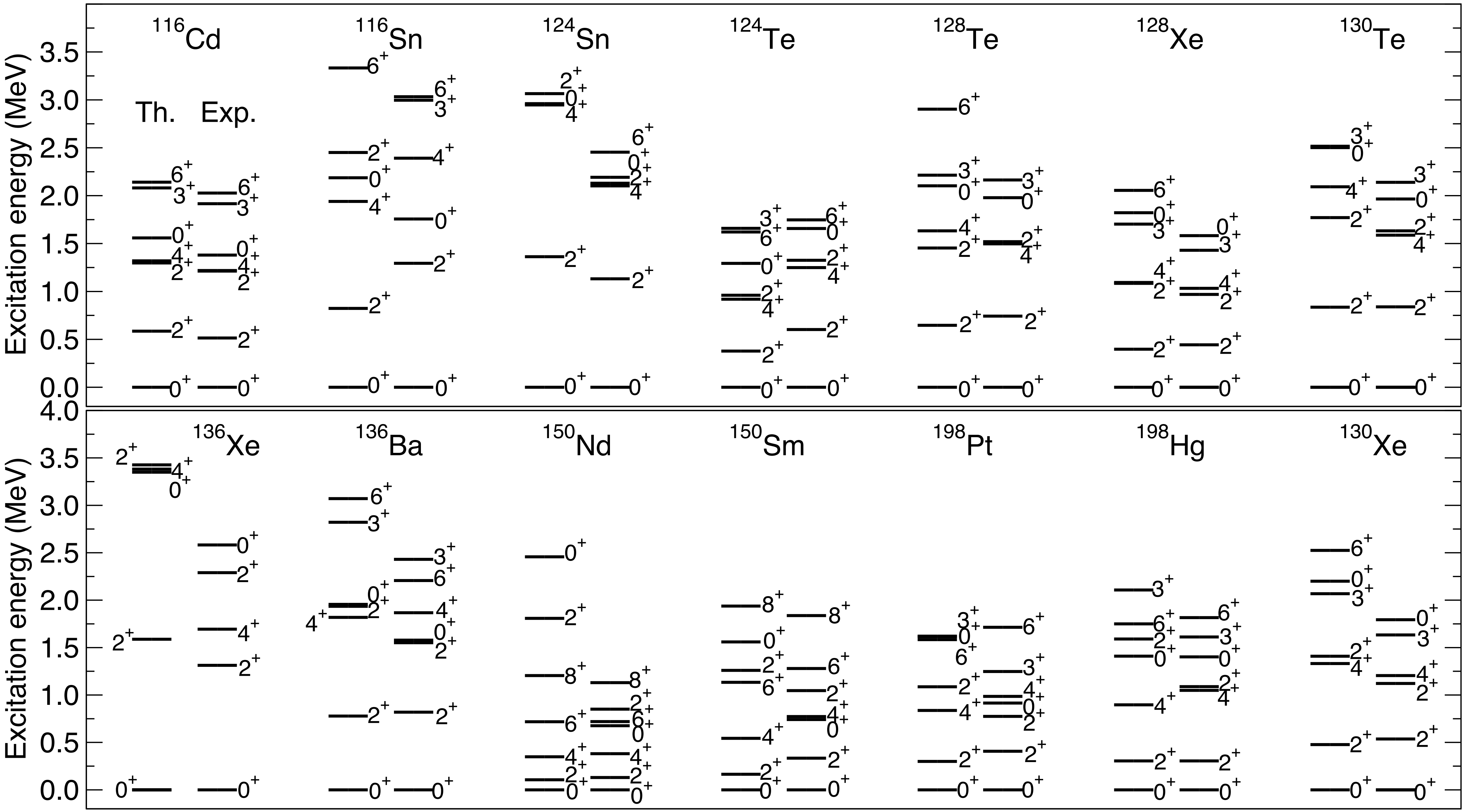
Mapped IBM energy surfaces



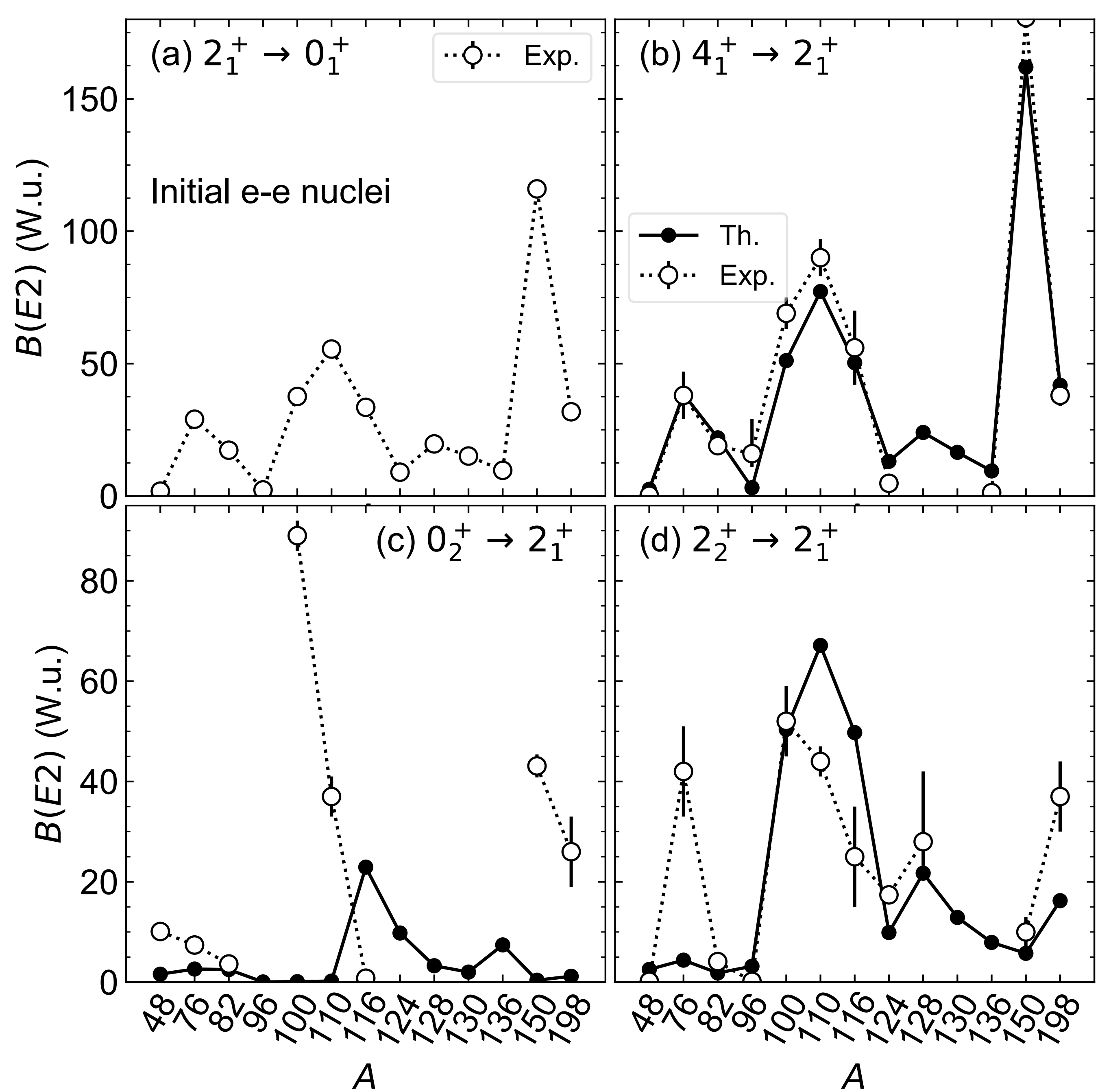
Energy spectra of even-even nuclei



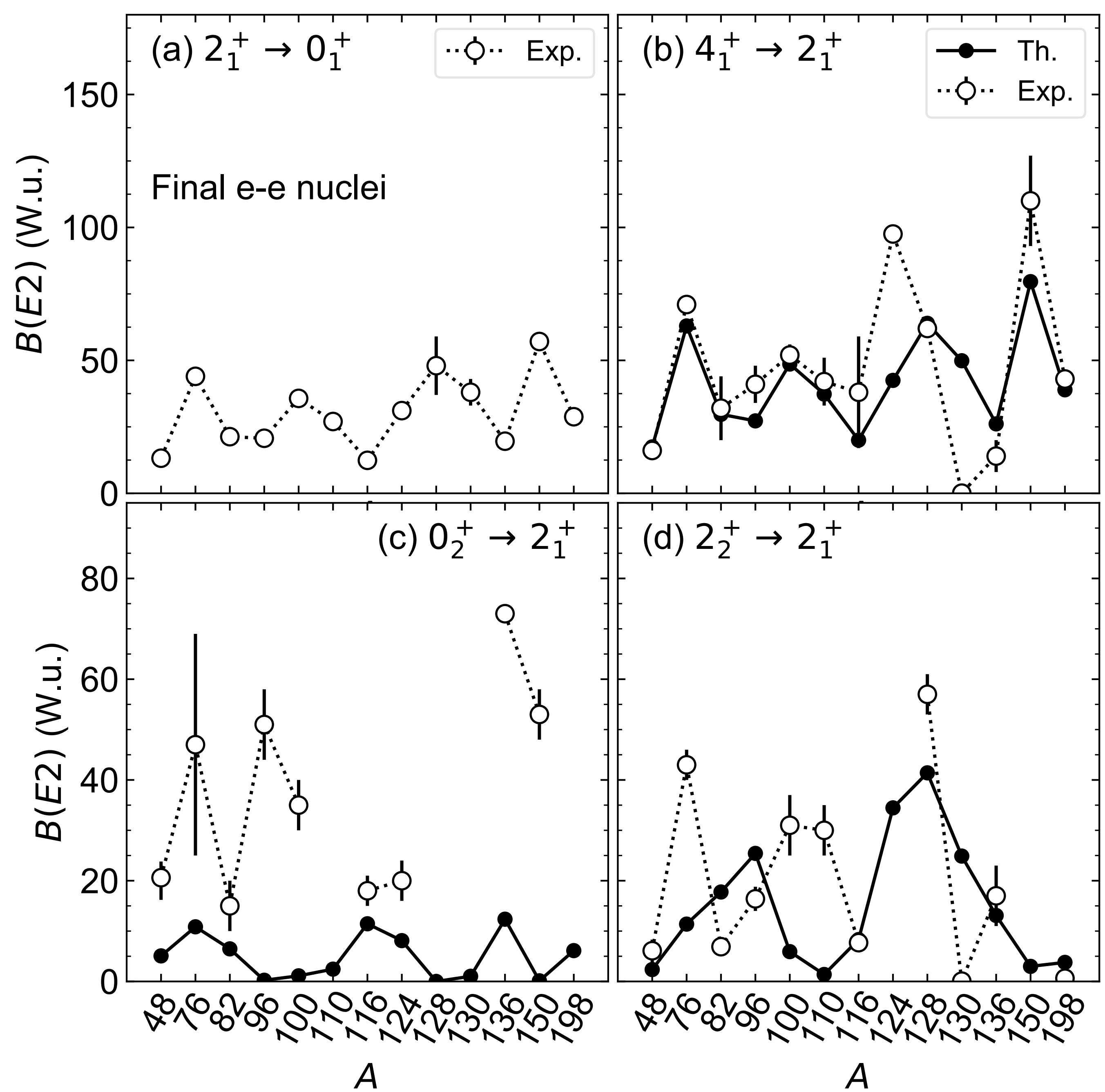
Energy spectra of even-even nuclei



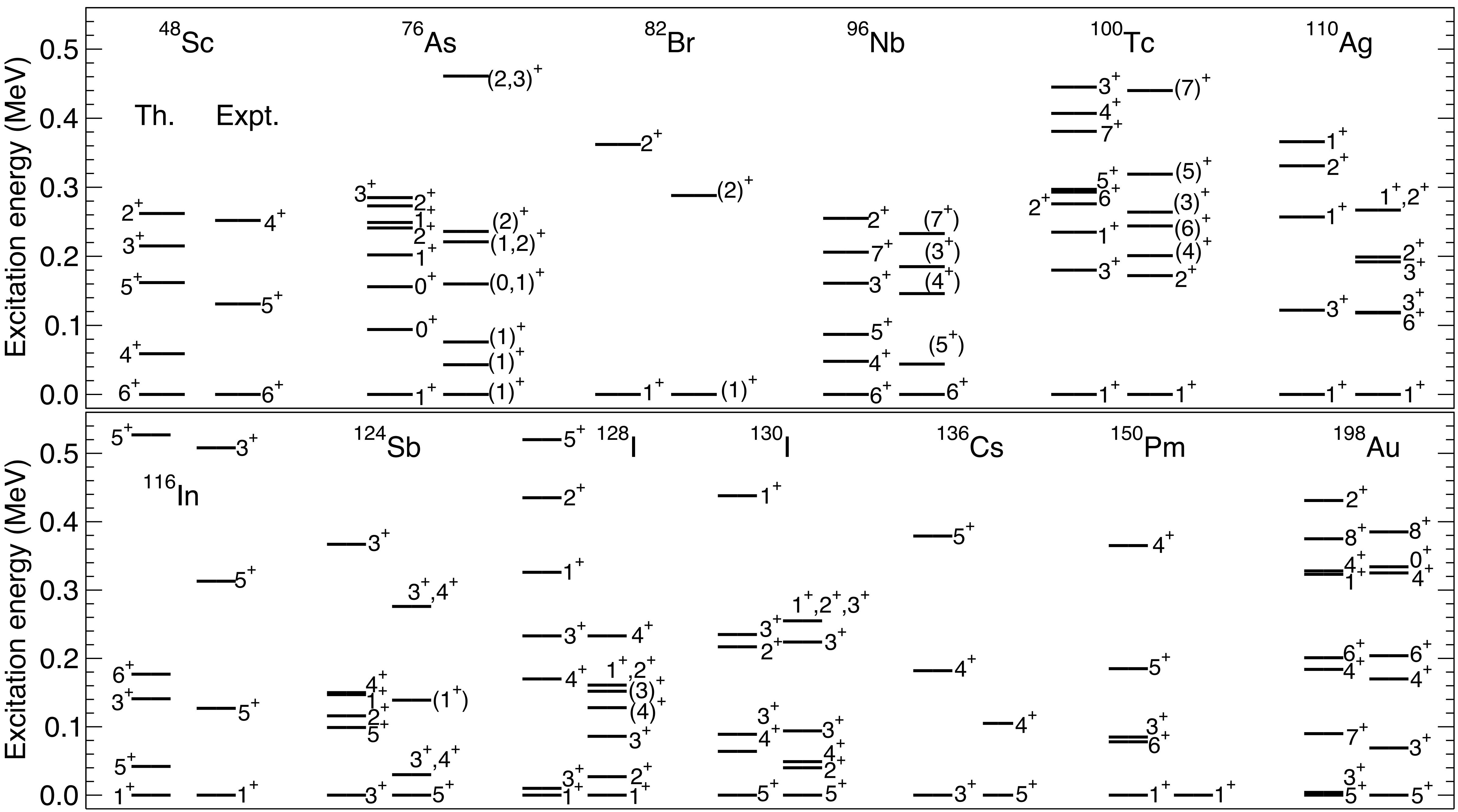
B(E2)s of parent nuclei



B(E2)s of daughter nuclei



Energy spectra of odd-odd nuclei



$2\nu\beta\beta$ decay

Calculation of NME

$$M_{2\nu} = g_A^2 \cdot m_e c^2 \left[M_{2\nu}^{\mathrm{GT}} - \left(\frac{g_V}{g_A} \right)^2 M_{2\nu}^{\mathrm{F}} \right],$$

$$M_{2\nu}^{\mathrm{GT}} = \sum_N \frac{\langle 0_F^+ \| t^+ \sigma \| 1_N^+ \rangle \langle 1_N^+ \| t^+ \sigma \| 0_{1,I}^+ \rangle}{E_N - E_I + \frac{1}{2}(Q_{\beta\beta} + 2m_e c^2)},$$

$$M_{2\nu}^{\mathrm{F}} = \sum_N \frac{\langle 0_F^+ \| t^+ \| 0_N^+ \rangle \langle 0_N^+ \| t^+ \| 0_{1,I}^+ \rangle}{E_N - E_I + \frac{1}{2}(Q_{\beta\beta} + 2m_e c^2)},$$

GT and Fermi operators

$$t^{\pm} \longmapsto \hat{T}^{\mathrm{F}} = \sum_{j_\nu j_\pi} \eta_{j_\nu j_\pi}^{\mathrm{F}} \left(\hat{P}_{j_\nu} \times \hat{P}_{j_\pi} \right)^{(0)},$$

$$t^{\pm}\sigma \longmapsto \hat{T}^{\mathrm{GT}} = \sum_{j_\nu j_\pi} \eta_{j_\nu j_\pi}^{\mathrm{GT}} \left(\hat{P}_{j_\nu} \times \hat{P}_{j_\pi} \right)^{(1)},$$

with $\hat{P}_j$ given as one of

$$A_{j_\rho m_\rho}^{\dagger} = \zeta_{j_\rho} a_{j_\rho m_\rho}^{\dagger} + \sum_{j'_\rho} \zeta_{j_\rho j'_\rho} s_\rho^{\dagger} \left(\tilde{d}_\rho \times a_{j'_\rho}^{\dagger} \right)_{m_\rho}^{(j_\rho)}$$

$$B_{j_\rho m_\rho}^{\dagger} = \theta_{j_\rho} s_\rho^{\dagger} \tilde{a}_{j_\rho m_\rho} + \sum_{j'_\rho} \theta_{j_\rho j'_\rho} \left(d_\rho^{\dagger} \times \tilde{a}_{j'_\rho} \right)_{m_\rho}^{(j_\rho)}$$

and their H.C.

$Q_{\beta\beta}$ values

$$Q_{\beta\beta} = 2(m_n - m_p - m_e)c^2 + E^I_{\rm gs} - E^F_{\rm gs}$$

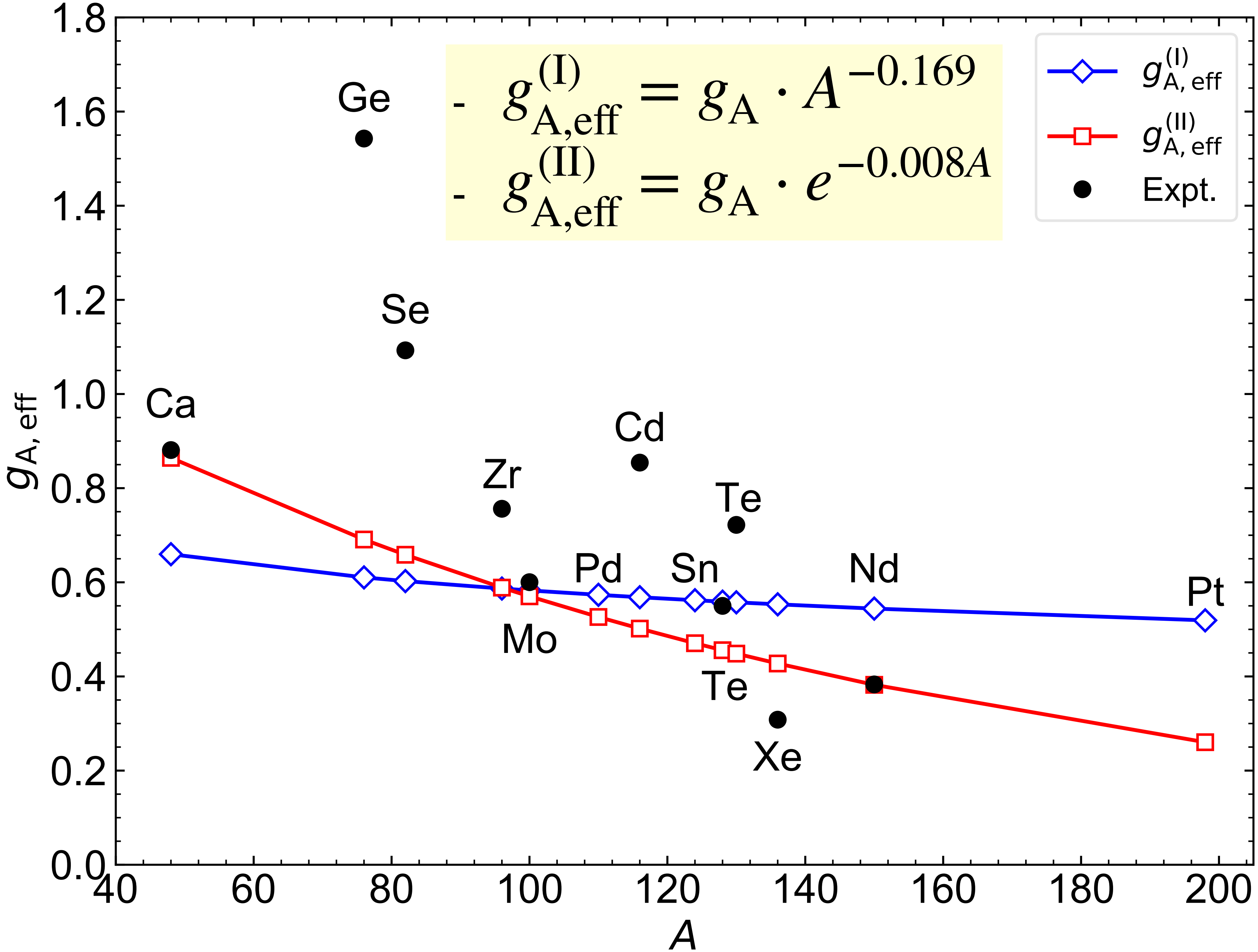
- $Q_{\beta\beta,\rm th}$: calculated by using the IBM eigen energy

$$E_{\rm gs} = E_{\rm IBM}(0^+_1) + E_0$$

- $Q_{\beta\beta,\rm ex}$: experimental value

Nucleus	$Q_{\beta\beta,\rm th}$ (MeV)	$Q_{\beta\beta,\rm ex}$ (MeV)
^{48}Ca	1.8479	4.2681
^{76}Ge	0.8831	2.0391
^{82}Se	1.6356	2.9979
^{96}Zr	4.1285	3.3560
^{100}Mo	2.8338	3.0344
^{110}Pd	2.9081	2.0171
^{116}Cd	6.1166	2.8135
^{124}Sn	−0.3795	2.2927
^{128}Te	−0.1784	0.8680
^{130}Te	1.4466	2.5290
^{136}Xe	0.0989	2.4579
^{150}Nd	3.3123	3.3714
^{198}Pt	1.2895	1.0503

Effective g_A factors



$2\nu\beta\beta$ NME

	0^+_1				0^+_2			
	$M^{GT}_{2\nu}$		$M^{F}_{2\nu}$		$M^{GT}_{2\nu}$		$M^{F}_{2\nu}$	
Decay	$Q_{\beta\beta,th}$	$Q_{\beta\beta,ex}$	$Q_{\beta\beta,th}$	$Q_{\beta\beta,ex}$	$Q_{\beta\beta,th}$	$Q_{\beta\beta,ex}$	$Q_{\beta\beta,th}$	$Q_{\beta\beta,ex}$
$^{48}Ca \rightarrow {}^{48}Ti$	0.060	0.042	0.024	0.016	0.325	0.066	−0.142	−0.075
$^{76}Ge \rightarrow {}^{76}Se$	0.040	0.034	−0.007	−0.007	0.097	0.078	−0.085	−0.069
$^{82}Se \rightarrow {}^{82}Kr$	−0.060	-0.045	0.017	0.015	0.124	0.070	−0.081	−0.064
$^{96}Zr \rightarrow {}^{96}Mo$	0.139	0.154	−0.001	−0.001	0.053	0.063	−0.000	−0.000
$^{100}Mo \rightarrow {}^{100}Ru$	0.513	0.483	−0.000	−0.000	−0.007	−0.020	0.000	0.000
$^{110}Pd \rightarrow {}^{110}Cd$	0.071	0.080	0.000	0.000	−0.052	−0.062	0.000	0.000
$^{116}Cd \rightarrow {}^{116}Sn$	0.148	0.275	0.001	0.001	0.032	−0.037	−0.001	−0.001
$^{124}Sn \rightarrow {}^{124}Te$	0.123	0.074	−0.054	−0.045	0.345	−0.066	0.016	0.012
$^{128}Te \rightarrow {}^{128}Xe$	−0.139	−0.102	0.006	0.005	0.108	0.032	0.002	0.002
$^{130}Te \rightarrow {}^{130}Xe$	−0.041	−0.037	0.025	0.022	0.043	0.037	−0.019	−0.017
$^{136}Xe \rightarrow {}^{136}Ba$	−0.173	−0.102	0.028	0.028	−2.807	0.010	0.009	0.001
$^{150}Nd \rightarrow {}^{150}Sm$	−0.375	−0.369	0.000	0.000	−0.414	−0.390	−0.000	−0.000
$^{198}Pt \rightarrow {}^{198}Hg$	−0.016	−0.016	0.001	0.001	−0.008	−0.010	−0.000	−0.000

Isospin symmetry breaking

$$\chi_F(0^+) = M_{2\nu}^{\mathrm{F}} / M_{2\nu}^{\mathrm{GT}}$$

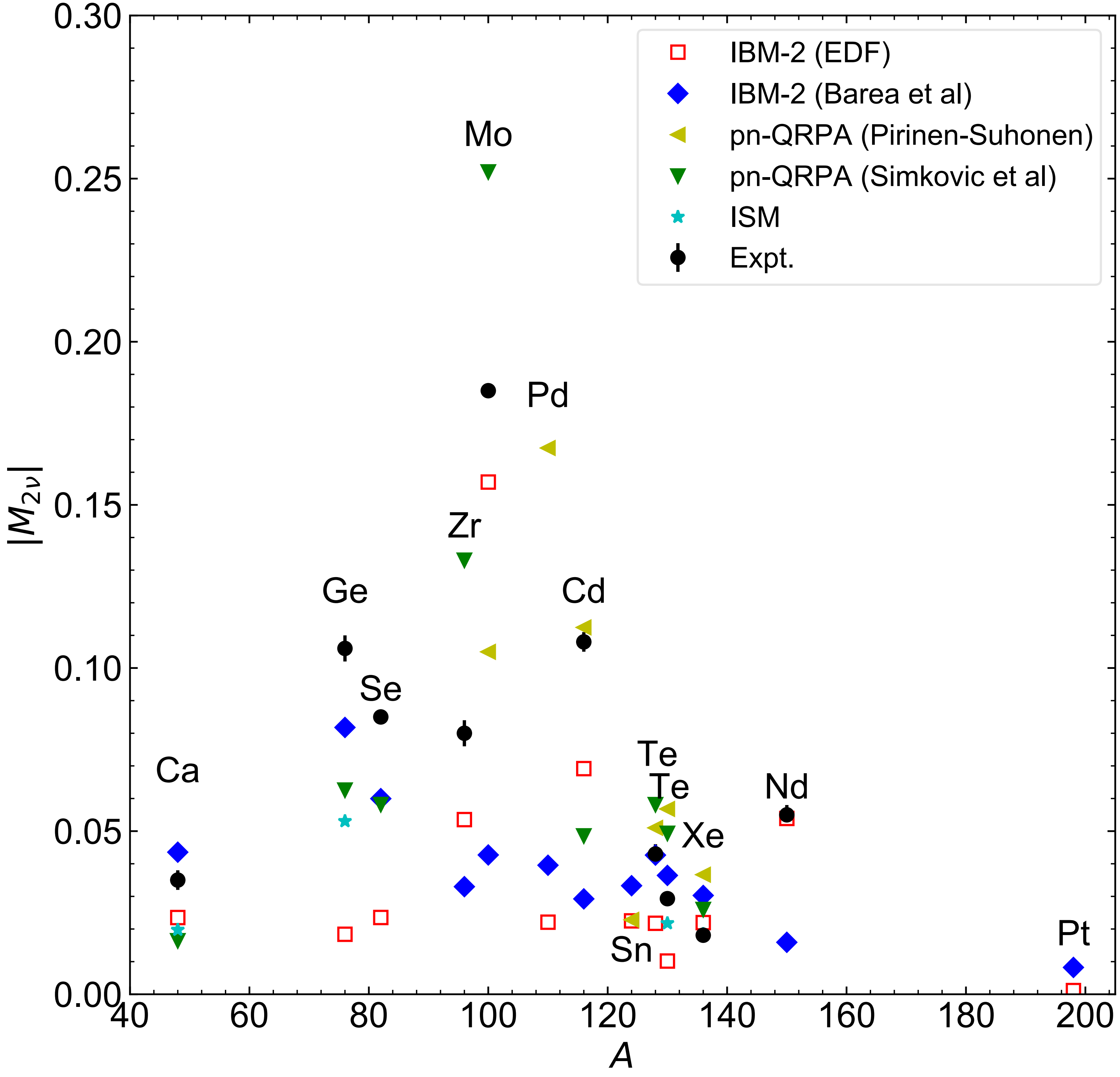
	$Q_{\beta\beta,\mathrm{th}}$		$Q_{\beta\beta,\mathrm{ex}}$	
Nucleus	$\chi_F(0_1^+)$	$\chi_F(0_2^+)$	$\chi_F(0_1^+)$	$\chi_F(0_2^+)$
^{48}Ca	0.401	−0.435	0.390	−1.147
^{76}Ge	−0.179	−0.874	−0.204	−0.888
^{82}Se	−0.287	−0.650	−0.328	−0.923
^{96}Zr	−0.006	−0.003	−0.005	−0.002
^{100}Mo	−0.000	−0.005	−0.000	−0.002
^{110}Pd	0.000	−0.000	0.000	−0.000
^{116}Cd	0.004	−0.021	0.002	0.022
^{124}Sn	−0.443	0.046	−0.608	−0.182
^{128}Te	−0.040	0.016	−0.051	0.050
^{130}Te	−0.104	−0.093	−0.601	−0.460
^{136}Xe	−0.163	−0.003	−0.278	0.061
^{150}Nd	−0.001	0.000	−0.001	0.000
^{198}Pt	−0.060	0.036	−0.059	0.030

Comparison with experiment

Decay	$Q_{\beta\beta,\mathrm{th}}$			$Q_{\beta\beta,\mathrm{ex}}$			
	$\lvert M_{2\nu}\rvert$	$\lvert M_{2\nu}^{(\mathrm{I})}\rvert$	$\lvert M_{2\nu}^{(\mathrm{II})}\rvert$	$\lvert M_{2\nu}\rvert$	$\lvert M_{2\nu}^{(\mathrm{I})}\rvert$	$\lvert M_{2\nu}^{(\mathrm{II})}\rvert$	$\lvert M_{2\nu}^{\mathrm{eff}}\rvert$ [12]
^{48}Ca → ^{48}Ti	0.073	0.020	0.034	0.051	0.014	0.024	0.035 ± 0.003
^{76}Ge → ^{76}Se	0.072	0.017	0.021	0.062	0.014	0.018	0.106 ± 0.004
^{82}Se → ^{82}Kr	0.115	0.026	0.031	0.087	0.020	0.024	0.085 ± 0.001
^{96}Zr → ^{96}Mo	0.225	0.048	0.048	0.249	0.053	0.054	0.088 ± 0.004
^{100}Mo → ^{100}Ru	0.827	0.174	0.167	0.778	0.164	0.157	0.185 ± 0.002
^{100}Mo → ^{100}Ru(0_2^+)	0.011	0.002	0.002	0.032	0.007	0.007	0.151 ± 0.004
^{110}Pd → ^{110}Cd	0.115	0.023	0.020	0.128	0.026	0.022	
^{116}Cd → ^{116}Sn	0.238	0.048	0.037	0.443	0.089	0.069	0.108 ± 0.003
^{124}Sn → ^{124}Te	0.253	0.050	0.035	0.164	0.032	0.022	
^{128}Te → ^{128}Xe	0.229	0.044	0.030	0.169	0.033	0.022	0.043 ± 0.003
^{130}Te → ^{130}Xe	0.091	0.017	0.011	0.081	0.016	0.010	0.0293 ± 0.0009
^{136}Xe → ^{136}Ba	0.307	0.058	0.035	0.194	0.037	0.022	0.0181 ± 0.0006
^{150}Nd → ^{150}Sm	0.604	0.111	0.055	0.594	0.109	0.054	0.055 ± 0.003
^{150}Nd → ^{150}Sm(0_2^+)	0.666	0.122	0.060	0.629	0.116	0.057	0.044 ± 0.005
^{198}Pt → ^{198}Hg	0.026	0.004	0.001	0.027	0.005	0.001	

data: Barabash, Universe (2020)

Comparison with other predictions

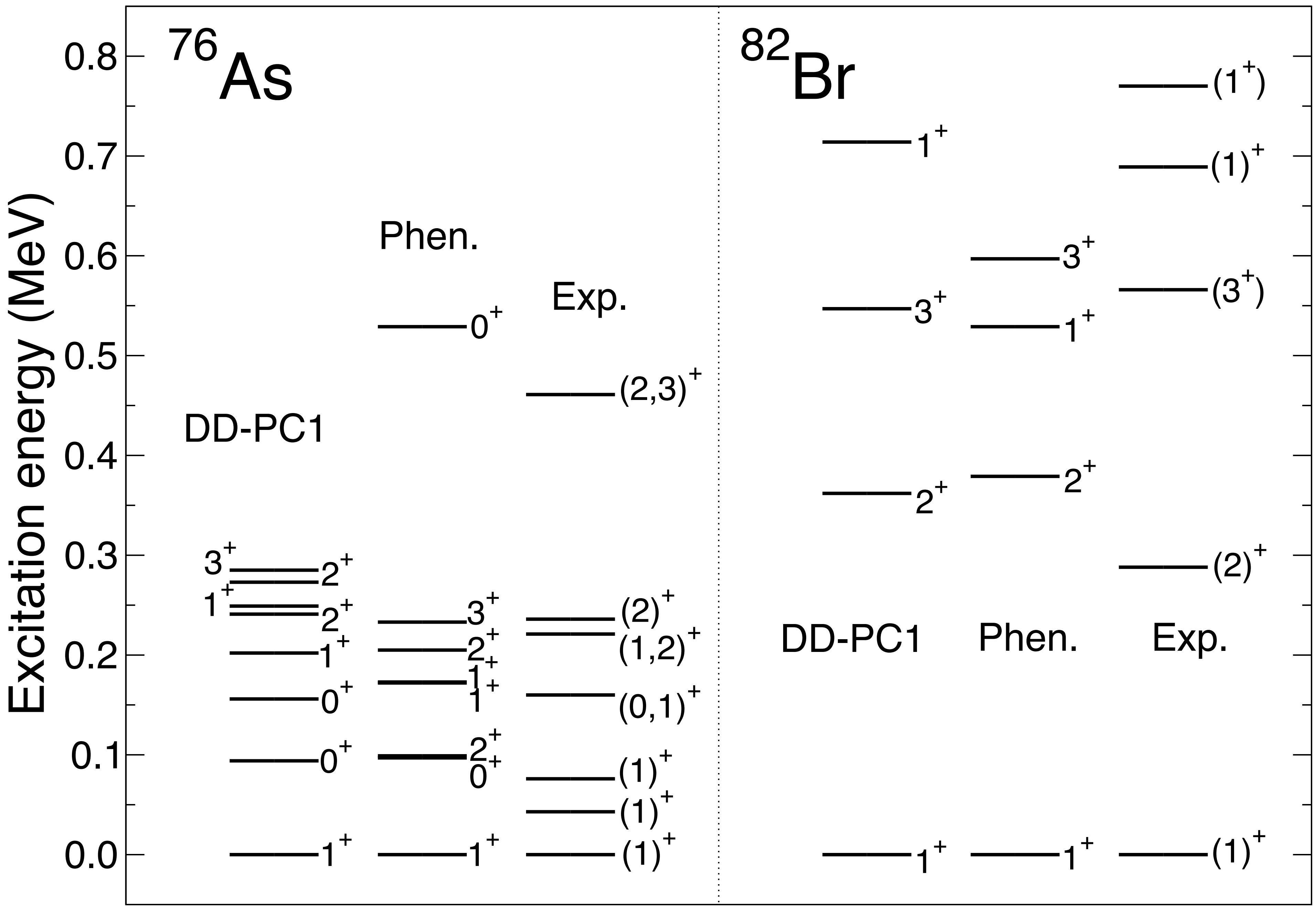


Half-lives

Decay	$\tau_{1/2}^{(2\nu)}$ (yr), with $Q_{\beta\beta,\mathrm{th}}$			$\tau_{1/2}^{(2\nu)}$ (yr), with $Q_{\beta\beta,\mathrm{ex}}$			Expt. [12]
	g_A	$g_{A,\mathrm{eff}}^{(I)}$	$g_{A,\mathrm{eff}}^{(II)}$	g_A	$g_{A,\mathrm{eff}}^{(I)}$	$g_{A,\mathrm{eff}}^{(II)}$	
^{48}Ca → ^{48}Ti	1.22×10^{19}	1.67×10^{20}	5.66×10^{19}	2.50×10^{19}	3.43×10^{20}	1.16×10^{20}	$5.3^{+1.2}_{-0.8} \times 10^{19}$
^{76}Ge → ^{76}Se	4.04×10^{21}	7.54×10^{22}	4.59×10^{22}	5.39×10^{21}	1.01×10^{23}	6.14×10^{22}	$(1.88 \pm 0.08) \times 10^{21}$
^{82}Se → ^{82}Kr	4.77×10^{19}	9.38×10^{20}	6.58×10^{20}	8.20×10^{19}	1.61×10^{21}	1.13×10^{21}	$(0.87^{+0.02}_{-0.01}) \times 10^{20}$
^{96}Zr → ^{96}Mo	2.89×10^{18}	6.32×10^{19}	6.24×10^{19}	2.37×10^{18}	5.19×10^{19}	5.12×10^{19}	$(2.3 \pm 0.2) \times 10^{19}$
^{100}Mo → ^{100}Ru	4.42×10^{17}	9.95×10^{18}	1.09×10^{19}	5.00×10^{17}	1.12×10^{19}	1.23×10^{19}	$(7.06^{+0.15}_{-0.13}) \times 10^{18}$
^{100}Mo → ^{100}Ru(0_2^+)	1.29×10^{23}	2.91×10^{24}	3.17×10^{24}	1.59×10^{22}	3.57×10^{23}	3.90×10^{23}	$6.7^{+0.5}_{-0.4} \times 10^{20}$
^{110}Pd → ^{110}Cd	5.51×10^{20}	1.32×10^{22}	1.86×10^{22}	4.40×10^{20}	1.06×10^{22}	1.49×10^{22}	
^{116}Cd → ^{116}Sn	6.37×10^{18}	1.58×10^{20}	2.61×10^{20}	1.85×10^{18}	4.59×10^{19}	7.56×10^{19}	$(2.69 \pm 0.09) \times 10^{19}$
^{124}Sn → ^{124}Te	2.83×10^{19}	7.37×10^{20}	1.50×10^{21}	6.76×10^{19}	1.76×10^{21}	3.57×10^{21}	
^{128}Te → ^{128}Xe	7.09×10^{22}	1.89×10^{24}	4.26×10^{24}	1.31×10^{23}	3.48×10^{24}	7.86×10^{24}	$(2.25 \pm 0.09) \times 10^{24}$
^{130}Te → ^{130}Xe	7.98×10^{19}	2.14×10^{21}	5.12×10^{21}	9.85×10^{19}	2.65×10^{21}	6.31×10^{21}	$(7.91 \pm 0.21) \times 10^{20}$
^{136}Xe → ^{136}Ba	7.41×10^{18}	2.05×10^{20}	5.75×10^{20}	1.86×10^{19}	5.16×10^{20}	1.45×10^{21}	$(2.18 \pm 0.05) \times 10^{21}$
^{150}Nd → ^{150}Sm	7.54×10^{16}	2.23×10^{18}	9.16×10^{18}	7.78×10^{16}	2.30×10^{18}	9.45×10^{18}	$(9.34 \pm 0.65) \times 10^{18}$
^{150}Nd → ^{150}Sm(0_2^+)	5.21×10^{17}	1.54×10^{19}	6.33×10^{19}	5.84×10^{17}	1.73×10^{19}	7.10×10^{19}	$1.2^{+0.3}_{-0.2} \times 10^{20}$
^{198}Pt → ^{198}Hg	9.59×10^{22}	3.42×10^{24}	5.43×10^{25}	8.95×10^{22}	3.20×10^{24}	5.09×10^{25}	

data: Barabash, Universe (2020)

Sensitivity to single-particle energies



… compared with the phenomenological SPEs used for LSSM

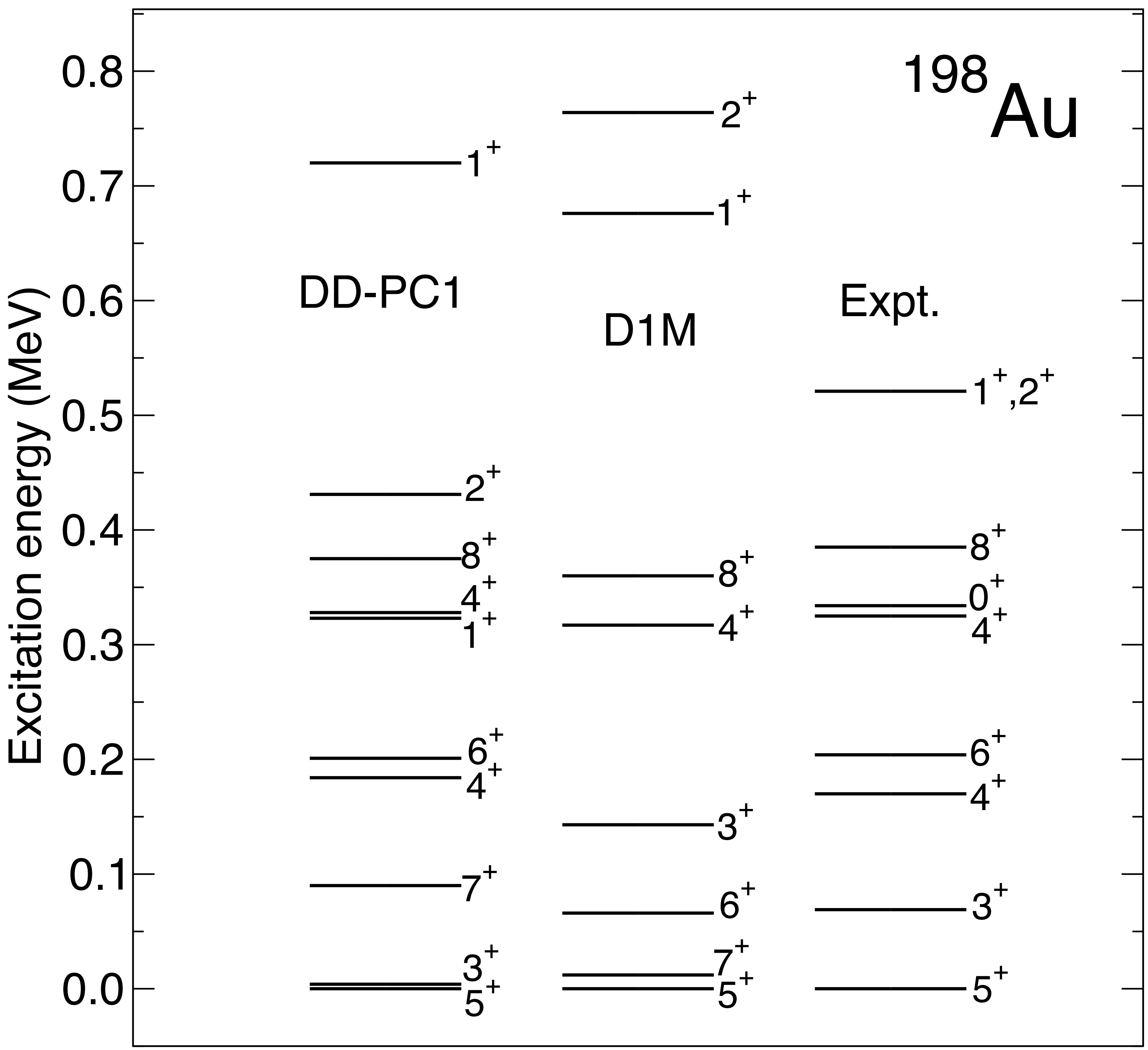
Sensitivity to single-particle energies

		^{76}Ge		^{82}Se	
		DD-PC1	Phen.	DD-PC1	Phen.
	$M_{2\nu}^{\mathrm{GT}}$	0.034	0.069	−0.045	−0.103
0_1^+	$M_{2\nu}^{\mathrm{F}}$	−0.007	−0.022	0.015	0.037
	$\|M_{2\nu}\|$	0.062	0.134	0.087	0.203
	$M_{2\nu}^{\mathrm{GT}}$	0.078	0.118	0.070	0.073
0_2^+	$M_{2\nu}^{\mathrm{F}}$	−0.069	−0.101	−0.064	−0.108
	$\|M_{2\nu}\|$	0.194	0.292	0.177	0.225

Data (for 0_1^+): ^{76}Ge: 0.106(4)
^{82}Se: 0.085(1)

… experimental $Q_{\beta\beta}$ used for both calculations

Sensitivity to the EDFs



… from the mapped IBM using the Gogny-D1M EDF.

Sensitivity to the EDFs

	0_1^+			0_2^+		
EDF	$M_{2\nu}^{\rm GT}$	$M_{2\nu}^{\rm F}$	$\lvert M_{2\nu}\rvert$	$M_{2\nu}^{\rm GT}$	$M_{2\nu}^{\rm F}$	$\lvert M_{2\nu}\rvert$
DD-PC1	−0.016	0.001	0.026	−0.008	−0.000	0.012
D1M	−0.074	0.000	0.120	−0.034	−0.000	0.054

… $Q_{\beta\beta}$ calculated for each model

Related studies

PHYSICAL REVIEW C **105**, 044306 (2022)

β decay and evolution of low-lying structure in Ge and As nuclei

Kosuke Nomura[*]

PHYSICAL REVIEW C **101**, 044318 (2020)

β decay of even-A nuclei within the interacting boson model with input based on nuclear density functional theory

K. Nomura,[1,*] R. Rodríguez-Guzmán,[2] and L. M. Robledo[3,4]

PHYSICAL REVIEW C **101**, 024311 (2020)

β decay of odd-A nuclei with the interacting boson-fermion model based on the Gogny energy density functional

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Concluding remarks

- Simultaneous description of low-lying states and $2\nu\beta\beta$ NME
- Further improvements by assessing model deficiency, coupling to higher-order deformations, shape coexistence…?
- Extension to the neutrinoless mode in progress

Thank you