

# Improvement of reliability of nuclear matrix element of double-beta decay

J. Terasaki, *Inst. of Exp. and Appl. Phys., Czech Technical Univ. in Prague*

1. Phenomenological improvement of  $0\nu\beta\beta$  nuclear matrix element (NME) of the shell model and QRPA  
J. T. and Y. Iwata, *Eur. Phys. Jour. Plus*, **136**, 908 (2021)
2. The discrepancy problem of running sum of  $2\nu\beta\beta$  NME  
J. T., *Phys. Rev. C*, **108**, 014301 (2023)
3. Vertex correction to  $0\nu\beta\beta$  NME    In progress

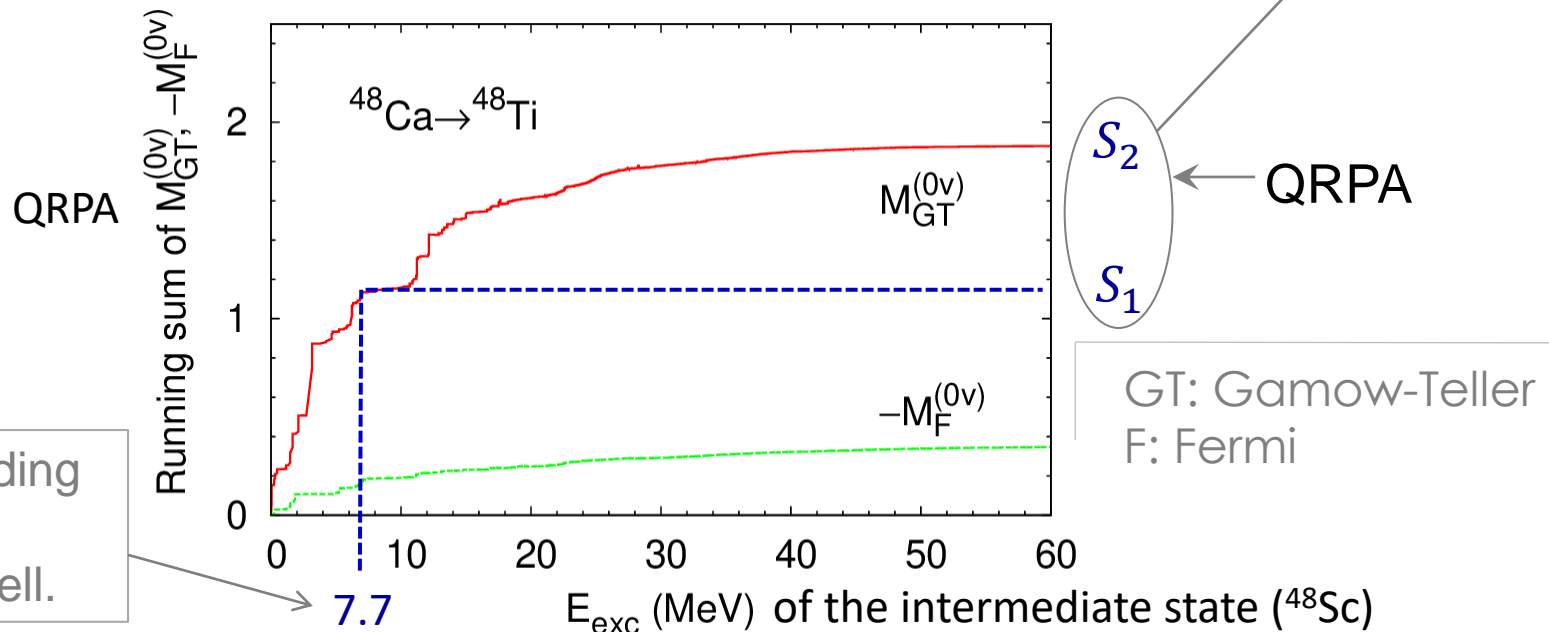
$0\nu\beta\beta$ : neutrinoless double- $\beta$ ;  $2\nu\beta\beta$ : two-neutrino double- $\beta$   
QRPA: quasiparticle random-phase approximation

# 1. Phenomenological improvement of $0\nu\beta\beta$ nuclear matrix element (NME) of the shell model and QRPA

J. T. and Y. Iwata, Eur. Phys. J. Plus, **136**, 908 (2021)

## Modification of the shell model (SM) $0\nu\beta\beta$ GT NME

$$M_{GT}^{(0\nu)}(\text{SM, modified}) = M_{GT}^{(0\nu)}(\text{SM, 1-maj. val. sh.}) \times \frac{S_2}{S_1}$$



Corresponding to 1-major valence shell.

# Justification

$$M_{GT}^{(0\nu)}(\text{SM, modified}) = M_{GT}^{(0\nu)}(\text{SM, 1- maj. val. sh.}) \times \frac{S_2}{S_1}$$

Shell	Max. 1p-1h energy (Woods-Saxon) (MeV)	Max. $E_{\text{exc}}$ from GT str. fns. of SM (MeV)
<i>pf</i>	8.8 ( $1f_{5/2}-1f_{7/2}$ )	7.7
<i>sdpf</i>	14.4 ( $1f_{5/2}-1d_{5/2}$ )	

$$\frac{\text{Run. sum } M_{GT}^{(0\nu)}(\text{QRPA, 14.4 MeV})}{\text{Run. sum } M_{GT}^{(0\nu)}(\text{QRPA, 8.8 MeV})} = 1.31, \quad \frac{M_{GT}^{(0\nu)}(\text{SM, } sdpf)}{M_{GT}^{(0\nu)}(\text{SM, } pf)} = 1.30$$

# Evaluation of possible region of modified QRPA result

Anticoherent lin

Quenching factor to GT<sup>-</sup> str. (<sup>48</sup>Ca → <sup>48</sup>Sc)

Coherent limit

0.593 (SM),  
0.5 (QRPA),  
in  $E_{\text{exc}} < 13$  MeV

$$M_{\text{GT}}^{(0\nu)}(\text{QRPA})$$

+ Reduction  
in low- $E$  rgn.

- Enhancement  
in high- $E$  rgn.

$$\leq M_{\text{GT}}^{(0\nu)}(\text{QRPA, modified}) \leq$$

$$M_{\text{GT}}^{(0\nu)}(\text{QRPA})$$

+ Reduction  
in low- $E$  rgn.

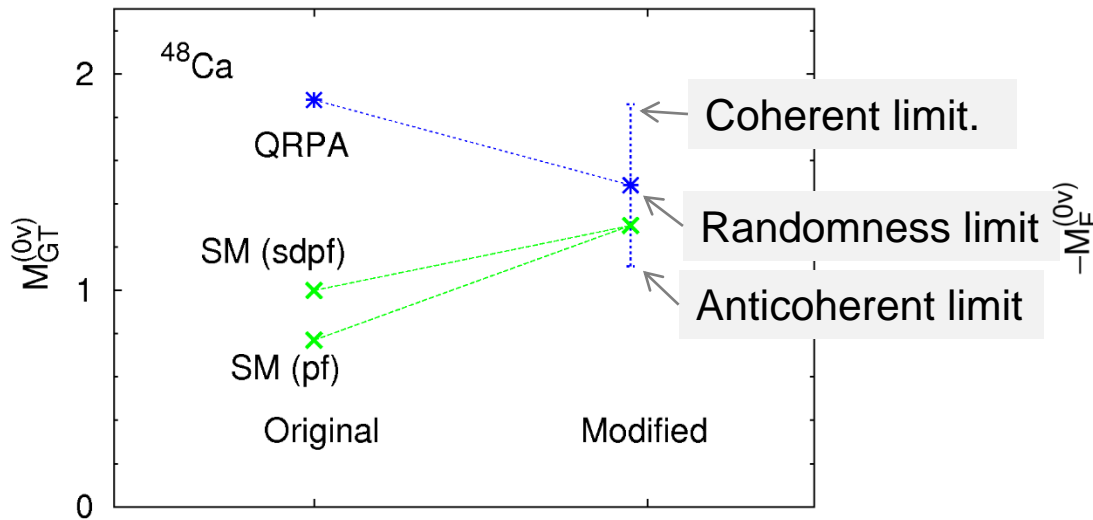
+ Enhancement  
in high- $E$  rgn.

$$\left( \sqrt{\frac{\text{shifted str.} + \text{original}}{\text{original str.}}} - 1 \right) M_{\text{GT}}^{(0\nu)}(\text{QRPA, high } E)$$

$$-(1 - R_q) M_{\text{GT}}^{(0\nu)}(\text{QRPA, low } E)$$

Each has two comp. related to  
trnsn. ME from initial and to  
final states

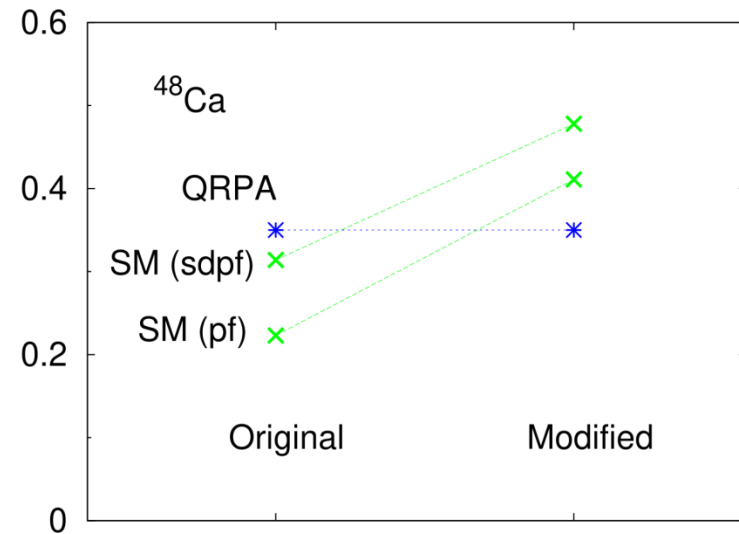
# Modified $0\nu\beta\beta$ NME of $^{48}\text{Ca}$



My speculation

The randomness limit is closer to the true value than the coherent and anticoherent limits.

Sign of correction terms unknown



No modification for QRPA

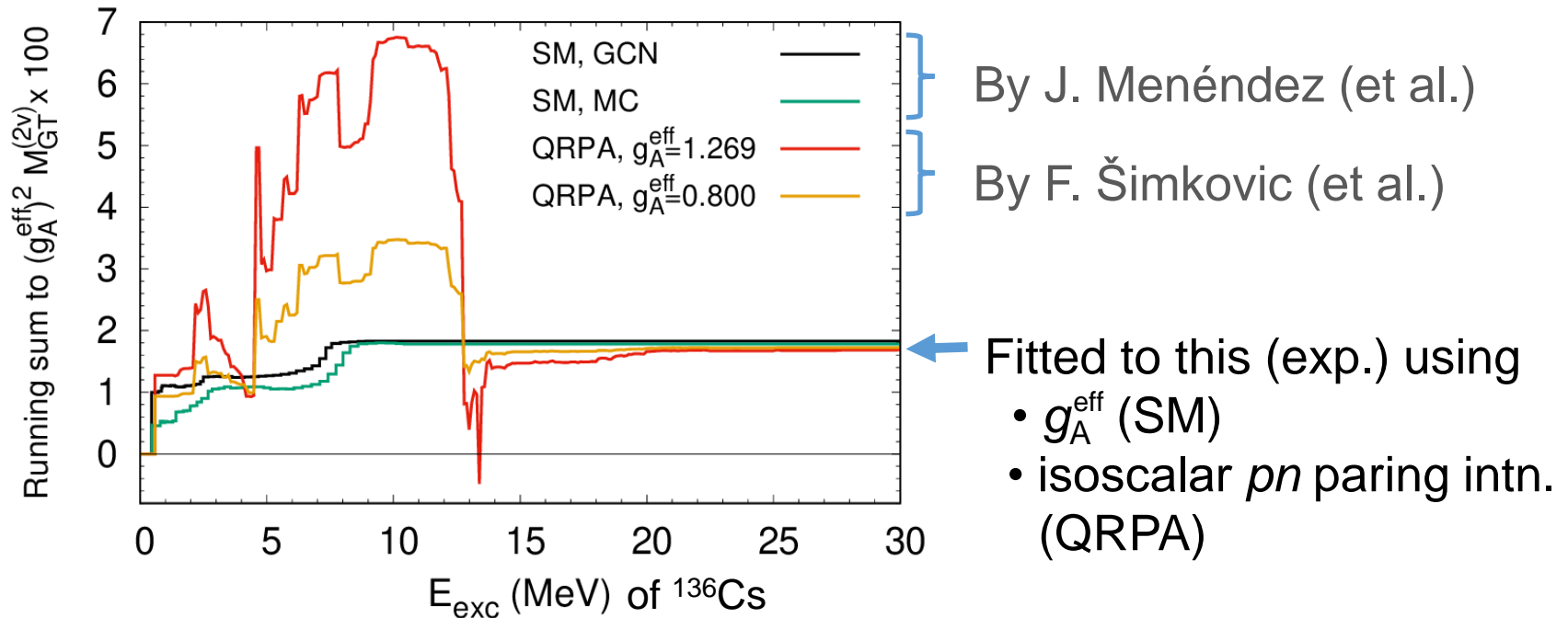
- Fermi transition
- Coordinate operators do not cause quenching

## 2. The discrepancy problem of running sum of $2\nu\beta\beta$ NME

A. Gando et al., PRL **122**, 192501 (2019)

Effective axial-vector current coupling

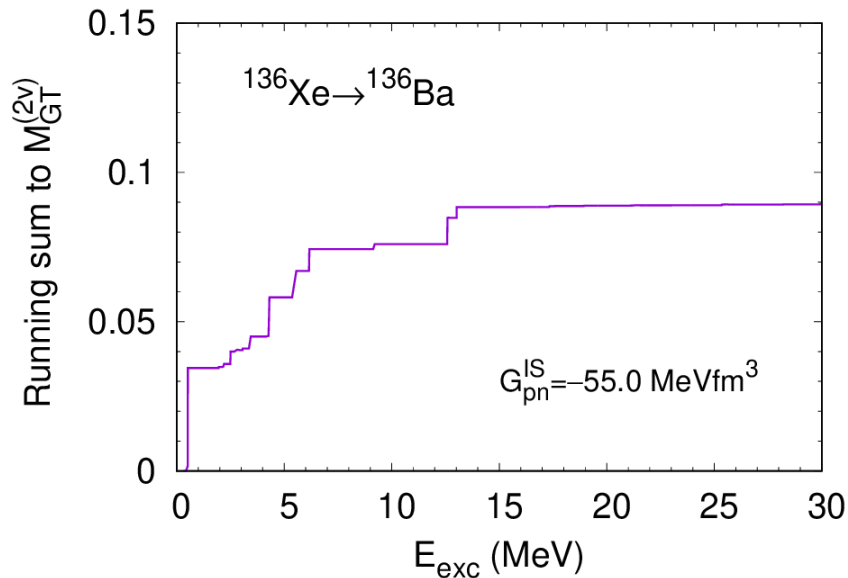
Running sum for  $2\nu\beta\beta$  NME  $\times (g_A^{\text{eff}})^2$  of  $^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$



*Why was the  $2\nu\beta\beta$  used?*

Because the main part of their paper is on a higher-order term of the  $2\nu\beta\beta$  NME, which was extracted from their exp. data. (My speculation).

# Variety of results



My QRPA calculation of  $M_{GT}^{(2v)}$

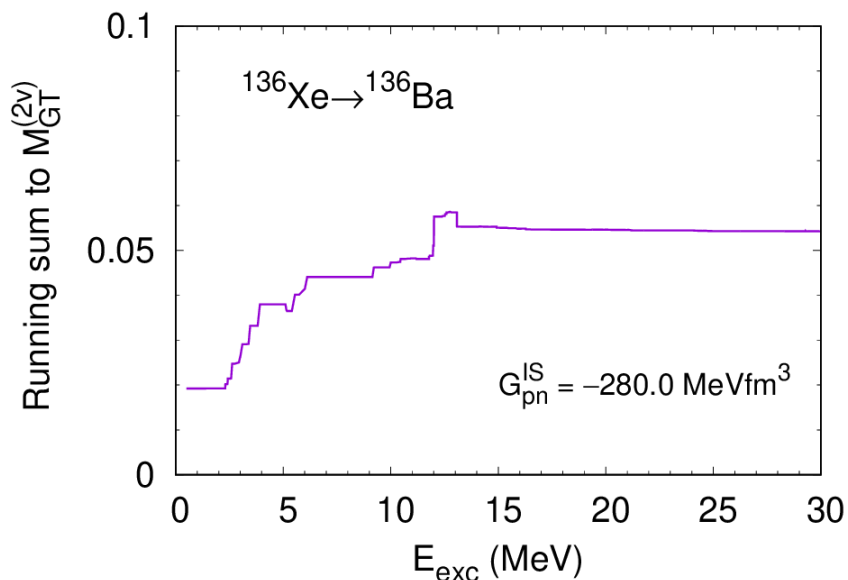
Skyrme + Coulomb + contact  
isovector and isoscalar pairing  
( $pp$ ,  $nn$ , and  $pn$ ) interactions.

Shell model cal. in M. Horoi and A. Brown,  
PRL **110**, 222502 (2013)

	Menéndez	Horoi	Šimkovic	Terasaki
Method	SM	QRPA	QRPA	QRPA
Variation of comp. of $M^{(2v)}$	Small	Large	Large	Small

The cause of the discrepancy problem is not the theoretical differences between SM and QRPA.

# Changing interaction (my QRPA cal.)



Strength of the isoscalar  $pn$  pairing intr.  $G_{pn}^{IS}$  is increased (previously  $-55.0 \text{ MeV fm}^3$ ).

Enhancing an interaction strength  $\rightarrow$  A local decrease in the running sum

Candidate of the cause of the discrepancy problem: difference in the interaction strengths

Explain this analytically.  $\Rightarrow$  confirming the cause.

$2\nu\beta\beta$  NME

$$M^{(2\nu)} \cong \sum_B \frac{m_e c^2}{E_B - \bar{M}}$$

$$\langle F | \sigma \tau^- | B \rangle \cdot \langle B | \sigma \tau^- | I \rangle$$

Mean value of  $I$  and  $F$  masses

GT operator

Final state ( $Z+2, N-2$ )

Initial state ( $Z, N$ )

Intermediate state ( $Z+1, N-1$ )



# The analytical discussion using the separable approximation

- Matrix element of two-body interaction

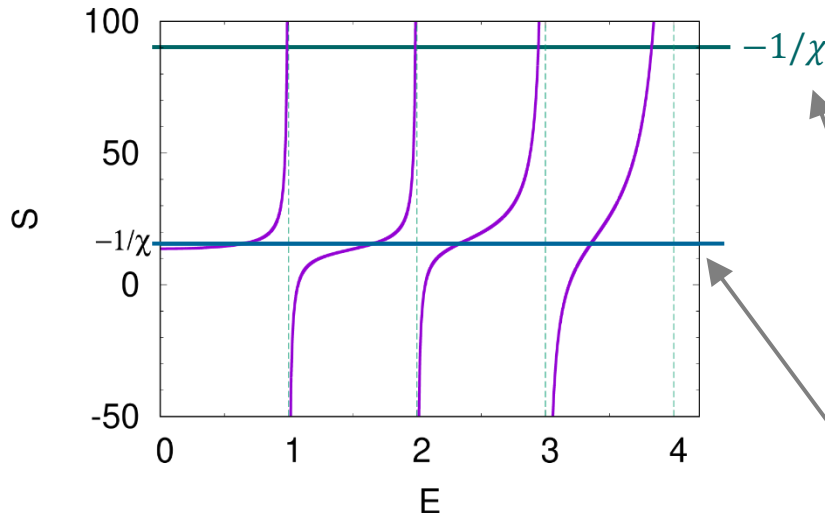
$$\frac{1}{2} \chi C_{\mu i} C_{\nu j}, \quad [\chi < 0 \text{ (attractive)}] \quad \textit{Pairing correlations ignored}$$

- Exchange terms neglected in derivation of the RPA eq.

Eq. to determine exc. state energy  $E_B$ :

$$S(E_B) \equiv 2 \sum_{\mu i} \frac{|C_{\mu i}|^2 (\epsilon_{\mu} - \epsilon_i)}{(\epsilon_{\mu} - \epsilon_i)^2 - E_B^2} = -\frac{1}{\chi}$$

Single-particle energies



Suppose that  $E = 3$  is the unperturbed energy  $\epsilon_{\mu} - \epsilon_i$  of GT-GR on  $|I\rangle$

- $\chi < 0$  and  $|\chi| \approx 0$   
 $E_B$  closest to 3 is slightly lower than that.
- $\chi < 0$  and  $|\chi|$ : large  
 $E_B$ : higher than 3 is closest to 3.

## Creation operator of the intermediate states

$$O_B^\dagger = \dots + \frac{N_B C_{\mu 3 i 3}}{3 - E_B} c_{\mu 3}^\dagger c_{i 3} + \frac{N_B C_{\mu 4 i 4}}{4 - E_B} c_{\mu 4}^\dagger c_{i 4} + \dots,$$

$$O_{B'}^\dagger = \dots + \frac{N_{B'} C_{\mu 3 i 3}}{3 - E_{B'}} c_{\mu 3}^\dagger c_{i 3} + \frac{N_{B'} C_{\mu 4 i 4}}{4 - E_{B'}} c_{\mu 4}^\dagger c_{i 4} + \dots,$$

Main comp. of GT<sup>-</sup>GR

$N_B$ : normalization  
const. of  $O_B^\dagger$

$$\langle \text{GT}^- \text{GR} | \sigma \tau^- | I \rangle = \langle I | O_B \sigma \tau^- | I \rangle \text{ or } \langle I | O_{B'} \sigma \tau^- | I \rangle$$

The NME of GT<sup>-</sup>GR changes its sign with enhancement of  $\chi$

NME from intermediate to final state

$$\langle F | \sigma \tau^- | B \rangle \cong \langle F | \sigma \tau^- | \text{others} \rangle$$

$$= \langle F | \sigma \tau^- \left( \dots + \frac{N_{BF} C_{i 2 \mu 2}}{2 - E_{BF}} c_{i 2}^\dagger c_{\mu 2} + \frac{N_{BF} C_{i 4 \mu 4}}{4 - E_{BF}} c_{i 4}^\dagger c_{\mu 4} + \dots \right) | F \rangle.$$

The GT<sup>-</sup>GR comp. is missing.

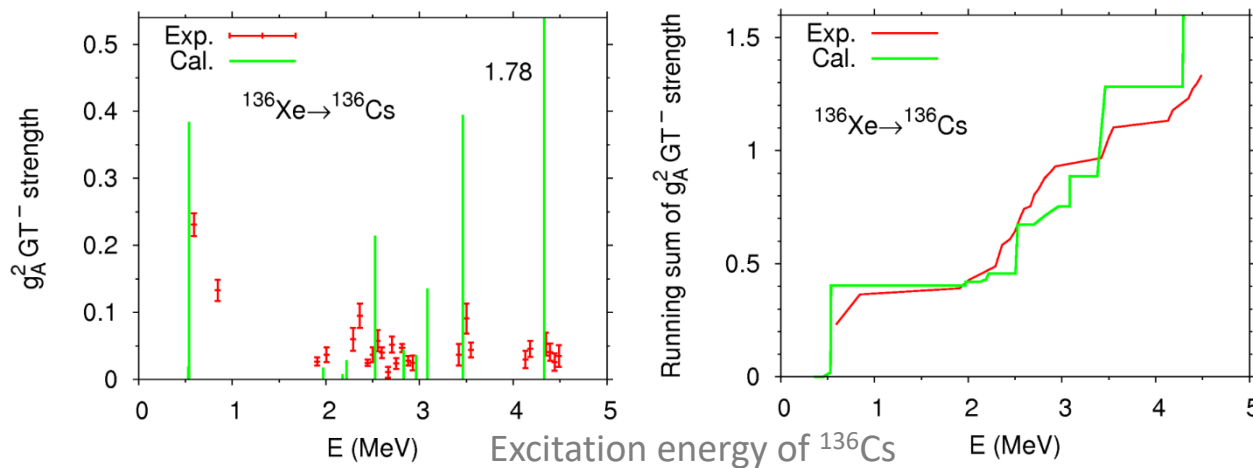
$C_{i 2 \mu 2}, \dots$  are smaller than that of the GT<sup>-</sup>GR comp.

The possibility of sign change of  $\langle F | \sigma \tau^- | B \rangle$  at the GT<sup>-</sup>GR energy is low.

The significant decrease in the running sum at the GT-GR implies that the interaction is stronger than that of calculations with less decrease.

**The cause of the problem is the difference in the interaction strength.**

## Exp. data and calculation of $g_A^2 \times \text{GT}^-$ strength



$^{136}\text{Xe}(^3\text{He}, t)^{136}\text{Cs}$  and  $e$  capture

J. T., Phys. Rev. C, **100**, 034325 (2019)

Exp. data from D. Frekers et al., Nucl. Phys. A **916**, 219 (2013);  
 J.T. used  $g_A^{\text{eff}} = 0.49$ . ← to reproduce exp. half-life of  $2\nu\beta\beta$  decay

### 3. Vertex correction to $0\nu\beta\beta$ NME

QRPA is good for  $^{136}\text{Xe}$ , but the  $g_A^{\text{eff}}$  for the  $2\nu\beta\beta$  NME needs quenching.

Many-body effects in the transition operator are indicated.

Effects not described by the lowest-order transition operator with the perturbed initial and final states – *vertex correction*.

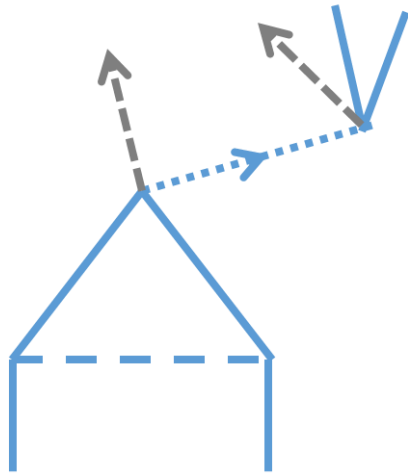
I calculate the higher-order\* terms of the effective transition operator to obtain  $g_A^{\text{eff}}$ .

\*Higher order in terms of the interaction vertexes.

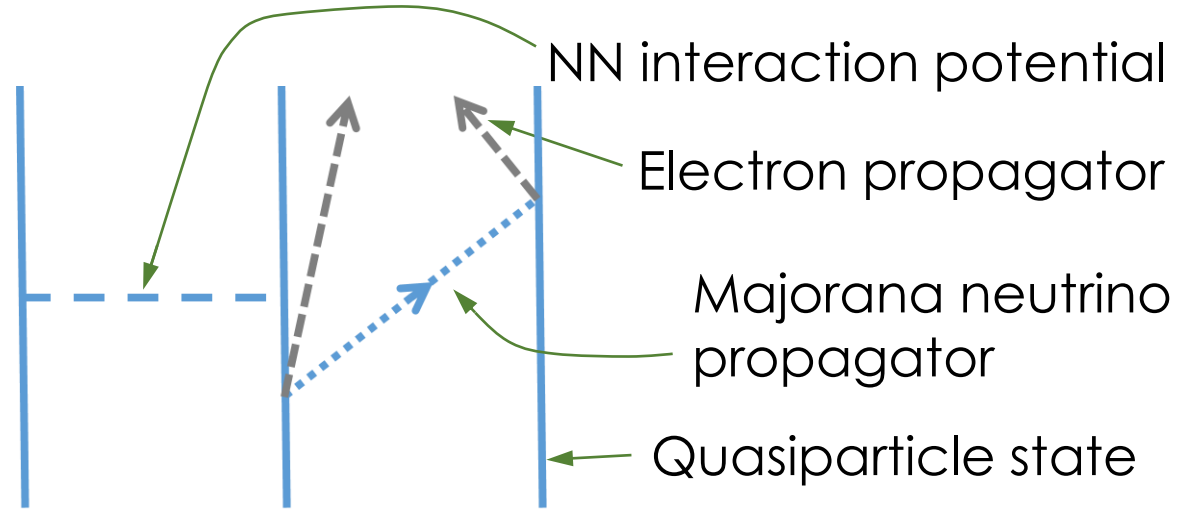
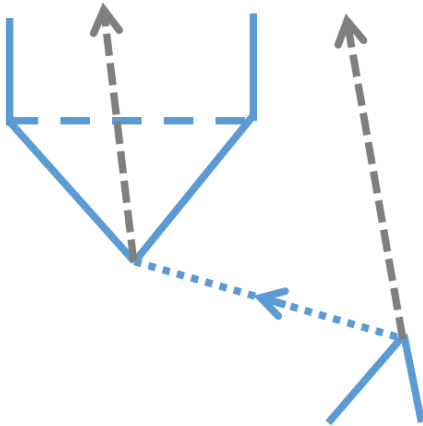
Calculation of the vertex correction and  $g_A^{\text{eff}}$  is an important step toward the solution of the uncertainty problem of the  $0\nu\beta\beta$  NME.

The  $g_A^{\text{eff}}$  is useful for comparison of the  $0\nu\beta\beta$  and  $2\nu\beta\beta$  decays.

# Goldstone diagrams for nucleons

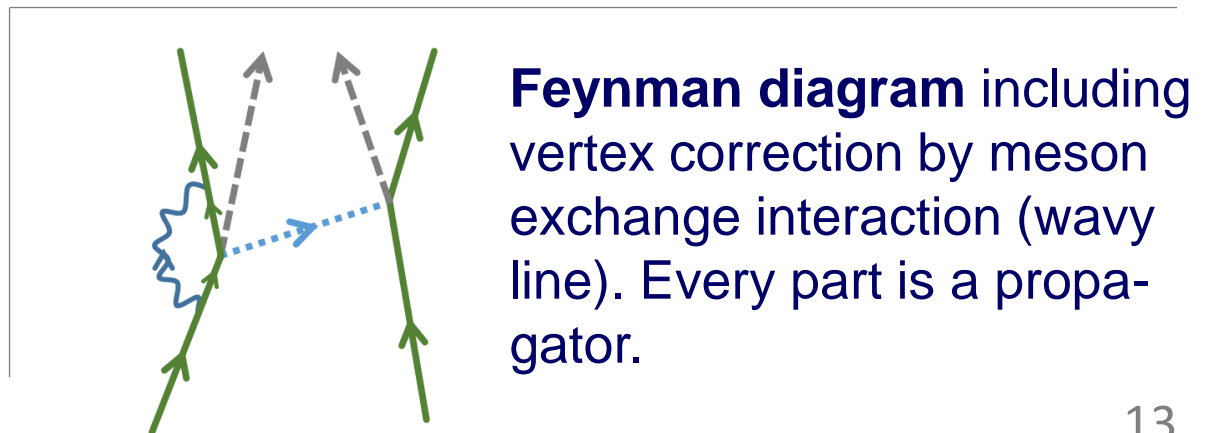


Exchange ME



Two-body current ME

The NN interaction is vertically between the two neutrino vertexes.



**Feynman diagram** including vertex correction by meson exchange interaction (wavy line). Every part is a propagator.

## Basic idea to derive equation of vertex correction to NME

Extension of the nucleon part of the usual lowest-order equation of the  $0\nu\beta\beta$  NME according to the Rayleigh-Schrödinger perturbation theory.

I pick up terms corresponding to those diagrams from the general equation of the second-order perturbation with approximations for feasibility of calculation.

## Equation of the exchange ME

$$M_{\text{ex}}^{(0\nu)} = \sum_{B_F B_I} \sum_{\kappa\nu'\mu'\lambda} \sum_{\mu\nu} \mathcal{W}_{\mu'\lambda,\mu\nu}^{(4a)} \langle F | a_{\mu}^F a_{\nu}^F | B_F \rangle \langle B_F | B_I \rangle \langle B_I | a_{\kappa}^{I\dagger} a_{\nu'}^{I\dagger} | I \rangle \mathcal{V}_{\kappa\nu',\mu'\lambda}^{(4a)I} \\ + \sum_{B_F B_I} \sum_{\nu'\mu'\lambda\kappa} \sum_{\mu\nu} \mathcal{V}_{\mu'\lambda,\kappa\nu'}^{(4b)F} \langle F | a_{\kappa}^F a_{\nu'}^F | B_F \rangle \langle B_F | B_I \rangle \langle B_I | a_{\mu}^{I\dagger} a_{\nu}^{I\dagger} | I \rangle \mathcal{W}_{\mu\nu,\mu'\lambda}^{(4a)}$$

$\mathcal{W}_{\mu'\lambda,\mu\nu}^{(4a)}$  : ME of the  $\nu$  potential. This is the usual one.

$\mathcal{V}_{\kappa\nu',\mu'\lambda}^{(4a)I}$  : ME of the perturbative interaction divided by an energy denominator.

## Equation of the exchange ME

$$\begin{aligned}
 M_{\text{ex}}^{(0\nu)} = & \sum_{B_F B_I} \sum_{\kappa\nu'\mu'\lambda} \sum_{\mu\nu} \mathcal{W}_{\mu'\lambda,\mu\nu}^{(4a)} \langle F | a_{\mu}^F a_{\nu}^F | B_F \rangle \langle B_F | B_I \rangle \langle B_I | a_{\kappa}^{I\dagger} a_{\nu'}^{I\dagger} | I \rangle \mathcal{V}_{\kappa\nu',\mu'\lambda}^{(4a)I} \\
 & + \sum_{B_F B_I} \sum_{\nu'\mu'\lambda\kappa} \sum_{\mu\nu} \mathcal{V}_{\mu'\lambda,\kappa\nu'}^{(4b)F} \langle F | a_{\kappa}^F a_{\nu'}^F | B_F \rangle \langle B_F | B_I \rangle \langle B_I | a_{\mu}^{I\dagger} a_{\nu}^{I\dagger} | I \rangle \mathcal{W}_{\mu\nu,\mu'\lambda}^{(4a)}.
 \end{aligned}$$

## Equation of the two-body current ME

$$\begin{aligned}
 M_{2b}^{(0\nu)} = & -\frac{1}{2} \sum_{B_I B_F} \sum_{\substack{abcdefg \\ \text{with conditions} \\ \langle B_I | c_c^{I\dagger} c_b^I | I \rangle}} W_{gf,cb}^{2b} \langle F | c_g^{F\dagger} c_f^F | B_F \rangle V_{de,ac}^I \langle B_F | : c_d^{I\dagger} c_e^{I\dagger} c_c^I c_a^I : | B_I \rangle
 \end{aligned}$$

Suffixes  $a$ - $g$ : single particles

$W_{gf,cb}^{2b}$  : ME of  $\nu$  potential including two energy denominators.

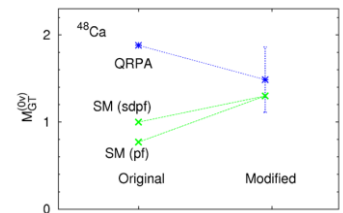
Coding is in progress.

## 4. Summary

### 1. Phenomenological improvement of $0\nu\beta\beta$ NME of the SM and the QRPA

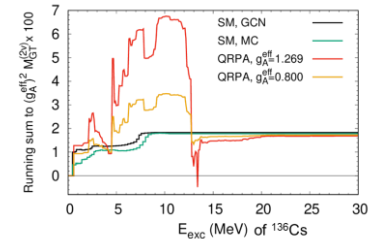
If more many-body correlations are added to the QRPA and single-particle space is enlarged in the SM, their  $0\nu\beta\beta$  NMEs approach to each other.

I look for shell model collaborators to apply this method to  $^{136}\text{Xe}$ .



### 2. The discrepancy problem of running sum of $2\nu\beta\beta$ NME

The cause of the problem is the interaction strength.



### 3. Vertex correction to $0\nu\beta\beta$ NME

The diagrams and equations of the vertex correction to the  $0\nu\beta\beta$  NME were shown.

