# Improvement of reliability of nuclear matrix element of double-beta decay

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1. **Phenomenological improvement of $0\nu\beta\beta$ nuclear matrix element (NME) of the shell model and QRPA**

   J. T. and Y. Iwata, Eur. Phys. Jour. Plus, **136**, 908 (2021)

2. **The discrepancy problem of running sum of $2\nu\beta\beta$ NME**

   J. T., Phys. Rev. C, **108**, 014301 (2023)

3. **Vertex correction to $0\nu\beta\beta$ NME** In progress

$0\nu\beta\beta$: neutrinoless double-β; $2\nu\beta\beta$: two-neutrino double-β
QRPA: quasiparticle random-phase approximation

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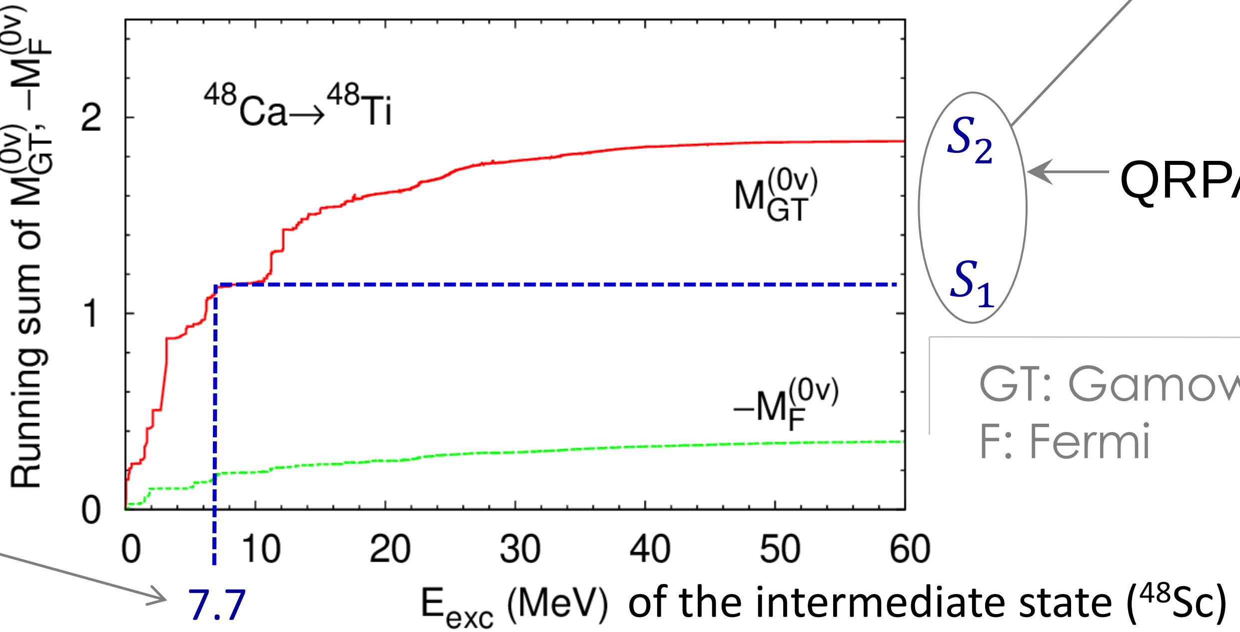
# 1. Phenomenological improvement of 0νββ nuclear matrix element (NME) of the shell model and QRPA

J. T. and Y. Iwata, Eur. Phys. J. Plus, **136**, 908 (2021)

## Modification of the shell model (SM) 0νββ GT NME

$$M_{\mathrm{GT}}^{(0\nu)}(\mathrm{SM, modified}) = M_{\mathrm{GT}}^{(0\nu)}(\mathrm{SM, 1\text{-}maj.\,val.\,sh.}) \times \frac{S_2}{S_1}$$

QRPA




Corresponding to 1-major valence shell.

GT: Gamow-Teller
F: Fermi

# Justification

$$M_{\mathrm{GT}}^{(0\nu)}(\mathrm{SM, modified}) = M_{\mathrm{GT}}^{(0\nu)}(\mathrm{SM, 1\text{-} maj. val. sh.}) \times \frac{S_2}{S_1}$$

| Shell | Max. 1p-1h energy (Woods-Saxon) (MeV) | Max. $E_{\mathrm{exc}}$ from GT str. fns. of SM (MeV) |
|---|---|---|
| *pf* | 8.8 ($1f_{5/2}$–$1f_{7/2}$) | 7.7 |
| *sdpf* | 14.4 ($1f_{5/2}$–$1d_{5/2}$) | |

$$\frac{\mathrm{Run.\,sum}\ M_{GT}^{(0\nu)}(\mathrm{QRPA, 14.4\ MeV})}{\mathrm{Run.\,sum}\ M_{GT}^{(0\nu)}(\mathrm{QRPA, 8.8\ MeV})} = 1.31, \qquad \frac{M_{GT}^{(0\nu)}(\mathrm{SM}, sdpf)}{M_{GT}^{(0\nu)}(\mathrm{SM}, pf)} = 1.30$$

## *Evaluation of possible region of modified QRPA result*

Anticoherent limit

Quenching factor to GT⁻ str. ($^{48}$Ca→$^{48}$Sc)
0.593 (SM),
0.5 (QRPA),
in $E_{\text{exc}}$ < 13 MeV

Coherent limit

$M_{\text{GT}}^{(0\nu)}$(QRPA)
+ **Reduction in low-*E* rgn.**
− **Enhancement in high-*E* rgn.**

$\leq$ $M_{\text{GT}}^{(0\nu)}$(QRPA, modified) $\leq$

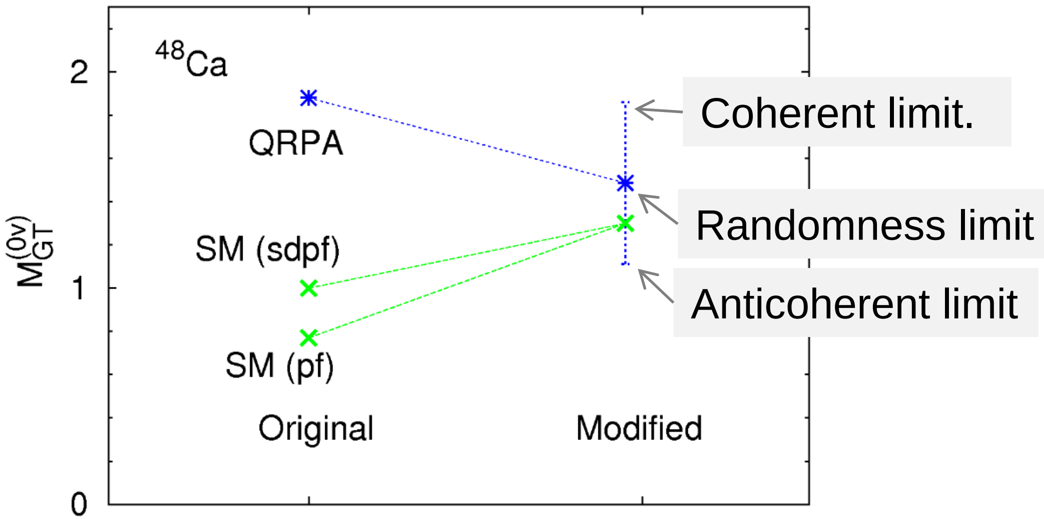
$M_{\text{GT}}^{(0\nu)}$(QRPA)
+ **Reduction in low-*E* rgn.**
+ **Enhancement in high-*E* rgn.**

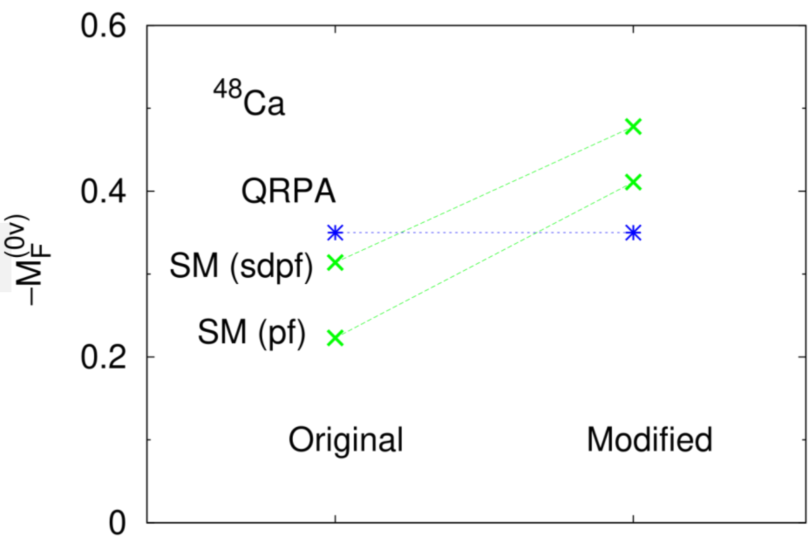
$$\left(\sqrt{\frac{\text{shifted str.} + \text{original}}{\text{original str.}}} - 1\right) M_{\text{GT}}^{(0\nu)}(\text{QRPA, high } E)$$

$$-(1 - R_q) M_{\text{GT}}^{(0\nu)}(\text{QRPA, low } E)$$

Each has two comp. related to trsn. ME from initial and to final states

# Modified 0νββ NME of $^{48}$Ca




My speculation

The randomness limit is closer to the true value than the coherent and anticoherent limits.

Sign of correction terms unknown

No modification for QRPA

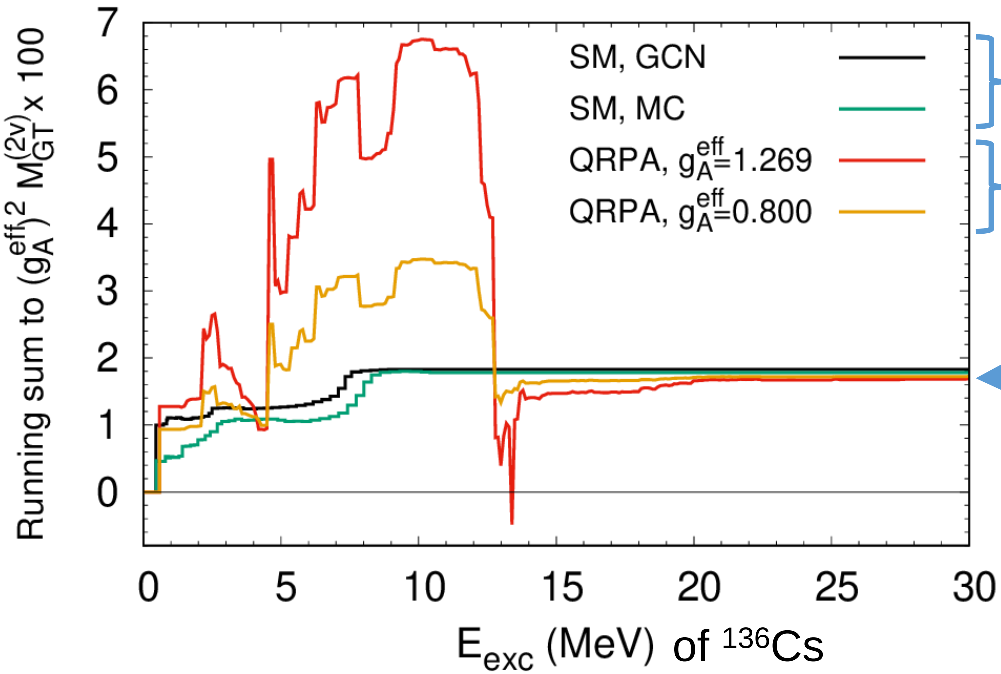
- Fermi transition
- Coordinate operators do not cause quenching

# 2. The discrepancy problem of running sum of $2\nu\beta\beta$ NME

A. Gando et al., PRL **122**, 192501 (2019)

Effective axial-vector current coupling

Running sum for $2\nu\beta\beta$ NME $\times (g_A^{\rm eff})^2$ of $^{136}$Xe→$^{136}$Ba




By J. Menéndez (et al.)

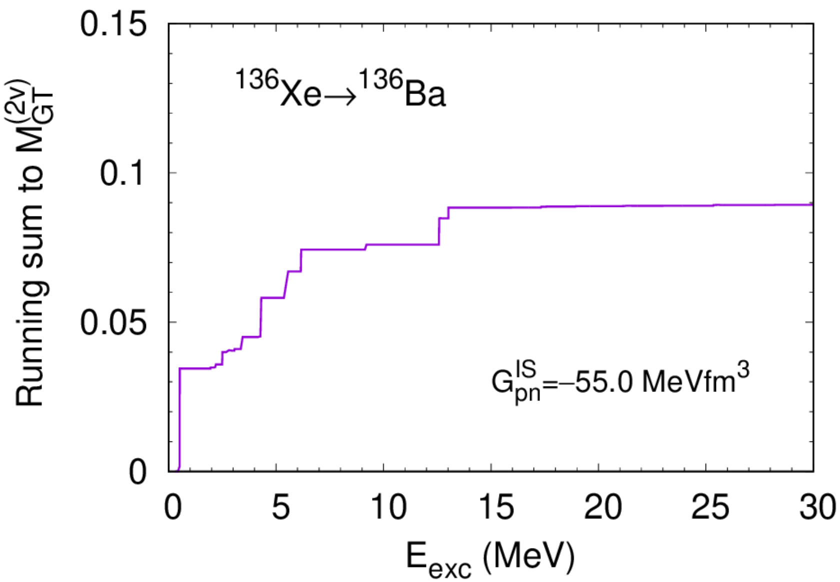
By F. Šimkovic (et al.)

Fitted to this (exp.) using
- $g_A^{\rm eff}$ (SM)
- isoscalar *pn* paring intn. (QRPA)

*Why was the $2\nu\beta\beta$ used?*

Because the main part of their paper is on a higher-order term of the $2\nu\beta\beta$ NME, which was extracted from their exp. data. (My speculation).

# Variety of results




My QRPA calculation of $M_{GT}^{(2\nu)}$

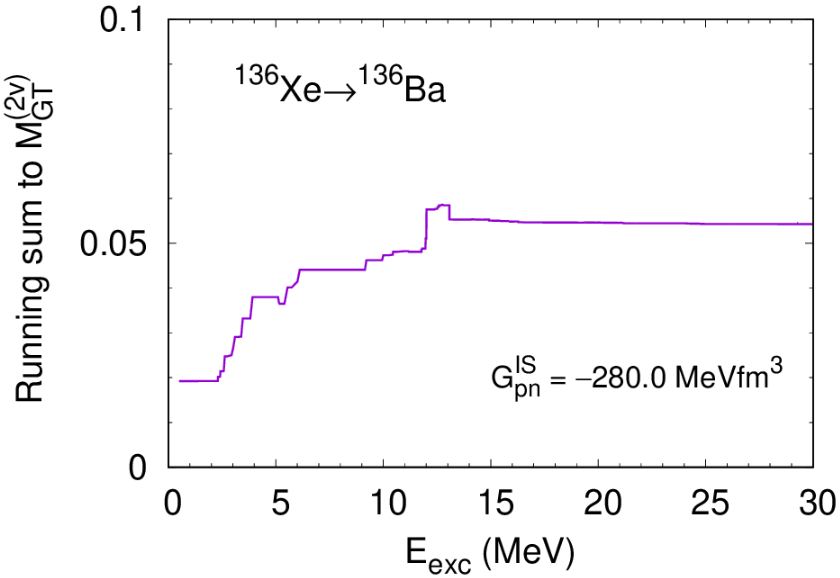
Skyrme + Coulomb + contact isovector and isoscalar pairing (*pp*, *nn*, and *pn*) interactions.

Shell model cal. in M. Horoi and A. Brown, PRL **110**, 222502 (2013)

| | Menéndez | Horoi | Šimkovic | Terasaki |
|---|---|---|---|---|
| Method | SM | | QRPA | |
| Variation of comp. of $M^{(2\nu)}$ | Small | Large | Large | Small |

The cause of the discrepancy problem is not the theoretical differences between SM and QRPA.

# Changing interaction (my QRPA cal.)




Strength of the isoscalar *pn* pairing intn. $G_{pn}^{\mathrm{IS}}$ is increased (previously –55.0 MeV fm$^3$).

Enhancing an interaction strength ⇒ A local decrease in the running sum

Candidate of the cause of the discrepancy problem: difference in the interaction strengths

Explain this analytically. ⇒ confirming the cause.

2$\nu\beta\beta$ NME

$$M^{(2\nu)} \cong \sum_B \frac{m_e c^2}{E_B - \bar{M}} \langle F|\boldsymbol{\sigma}\tau^-|B\rangle \cdot \langle B|\boldsymbol{\sigma}\tau^-|I\rangle$$

Final state ($Z$+2,$N$ –2)

Initial state ($Z$,$N$)

Intermediate state ($Z$+1,$N$ –1)

Mean value of $I$ and $F$ masses

GT operator

## The analytical discussion using the separable approximation

- Matrix element of two-body interaction

$$\frac{1}{2}\chi C_{\mu i} C_{\nu j}, \qquad [\chi < 0 \text{ (attractive)}]$$

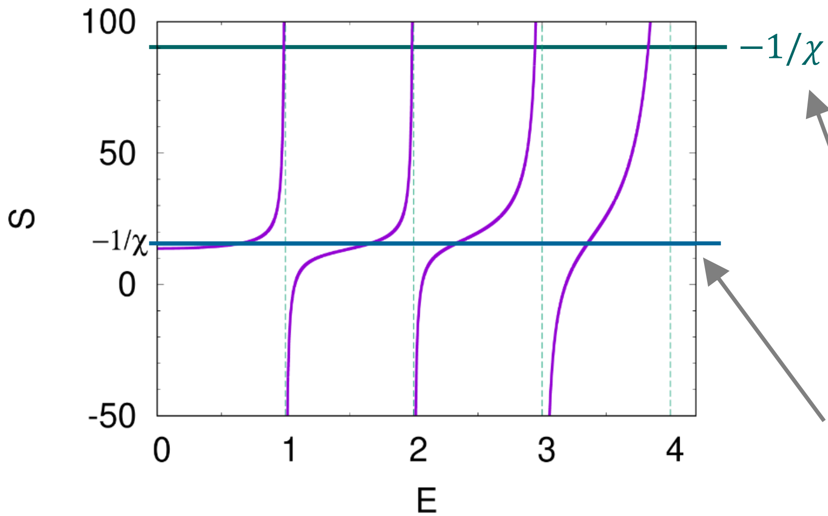
*Pairing correlations ignored*

- Exchange terms neglected in derivation of the RPA eq.

Eq. to determine exc. state energy $E_B$:

$$S(E_B) \equiv 2\sum_{\mu i} \frac{|C_{\mu i}|^2 (\epsilon_\mu - \epsilon_i)}{(\epsilon_\mu - \epsilon_i)^2 - E_B^2} = -\frac{1}{\chi}$$

Single-particle energies




Suppose that $E$ = 3 is the unperturbed energy $\epsilon_\mu - \epsilon_i$ of GT⁻GR on $|I\rangle$

i) $\chi$<0 and ≈ 0

$E_B$ closest to 3 is slightly lower than that.

ii) $\chi$<0 and $|\chi|$: large

$E_{B'}$ higher than 3 is closest to 3.

# Creation operator of the intermediate states

$$O_B^\dagger = \cdots + \frac{N_B C_{\mu 3 i 3}}{3 - E_B} c_{\mu 3}^\dagger c_{i3} + \frac{N_B C_{\mu 4 i 4}}{4 - E_B} c_{\mu 4}^\dagger c_{i4} + \cdots,$$

$$O_{B'}^\dagger = \cdots + \frac{N_{B'} C_{\mu 3 i 3}}{3 - E_{B'}} \underline{c_{\mu 3}^\dagger c_{i3}} + \frac{N_{B'} C_{\mu 4 i 4}}{4 - E_{B'}} c_{\mu 4}^\dagger c_{i4} + \cdots,$$

Main comp. of GT⁻GR

$N_B$: normalization const. of $O_B^\dagger$

$\langle \mathrm{GT^-GR} | \boldsymbol{\sigma}\tau^- | I \rangle = \langle I | O_B \boldsymbol{\sigma}\tau^- | I \rangle$ or $\langle I | O_{B'} \boldsymbol{\sigma}\tau^- | I \rangle$

The NME of GT⁻GR changes its sign with enhancement of $\chi$

NME from intermediate to final state

$$\langle F | \boldsymbol{\sigma}\tau^- | B \rangle \cong \langle F | \boldsymbol{\sigma}\tau^- | \mathrm{others} \rangle$$

$$= \langle F | \boldsymbol{\sigma}\tau^- \left( \cdots + \frac{N_{BF} C_{i2\mu 2}}{2 - E_{BF}} c_{i2}^\dagger c_{\mu 2} + \frac{N_{BF} C_{i4\mu 4}}{4 - E_{BF}} c_{i4}^\dagger c_{\mu 4} + \cdots \right) | F \rangle.$$

The GT⁻GR comp. is missing.

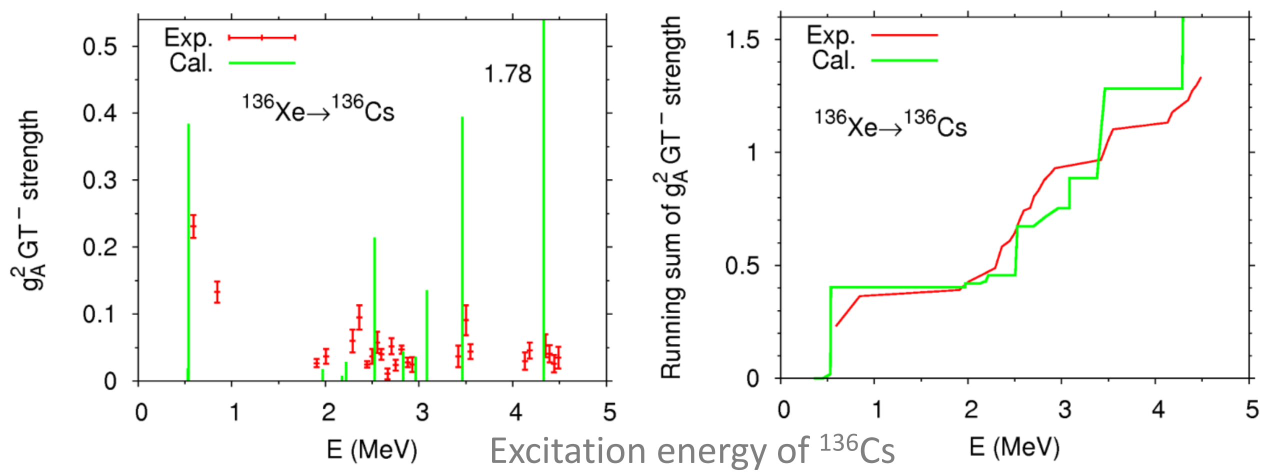
$C_{i2\mu 2}, \cdots$ are smaller than that of the GT⁻GR comp.

The possibility of sign change of $\langle F | \boldsymbol{\sigma}\tau^- | B \rangle$ at the GT⁻GR energy is low.

The significant decrease in the running sum at the GT⁻GR implies that the interaction is stronger than that of calculations with less decrease.

**The cause of the problem is the difference in the inter-action strength.**

## Exp. data and calculation of $g_A^2$ x GT⁻ strength




J. T., Phys. Rev. C, **100**, 034325 (2019)

$^{136}$Xe($^3$He,*t* )$^{136}$Cs and *e* capture

Exp. data from D. Frekers et al., Nucl. Phys. A **916**, 219 (2013);

J.T. used $g_A^{\mathrm{eff}}$ = 0.49. to reproduce exp. half-life of 2νββ decay

## 3. Vertex correction to 0νββ NME

QRPA is good for $^{136}$Xe, but the $g_A^{\mathrm{eff}}$ for the 2νββ NME needs quenching.

Many-body effects in the transition operator are indicated.

Effects not described by the lowest-order transition operator with the perturbed initial and final states – *vertex correction*.

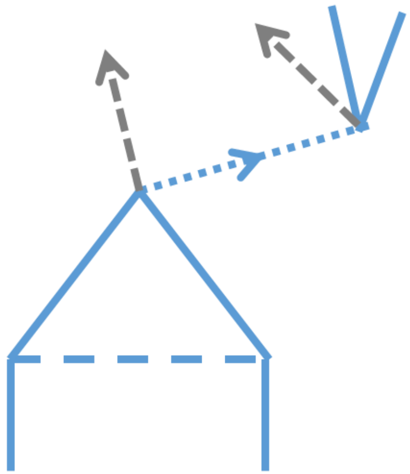
I calculate the higher-order* terms of the effective transition operator to obtain $g_A^{\mathrm{eff}}$.

*Higher order in terms of the interaction vertexes.

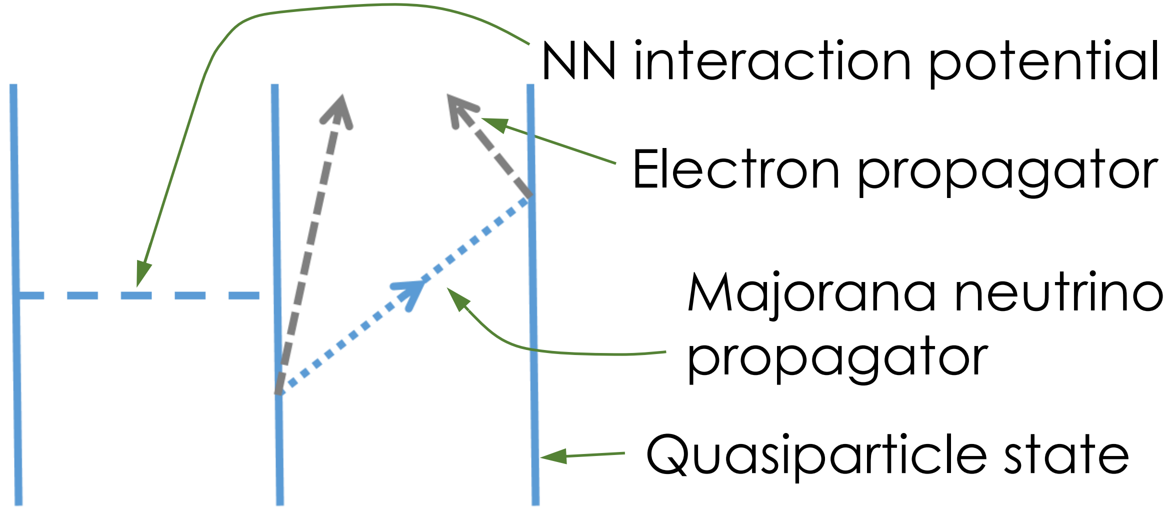
Calculation of the vertex correction and $g_A^{\mathrm{eff}}$ is an important step toward the solution of the uncertainty problem of the 0νββ NME.

The $g_A^{\mathrm{eff}}$ is useful for comparison of the 0νββ and 2νββ decays.

# **Goldstone** diagrams for nucleons

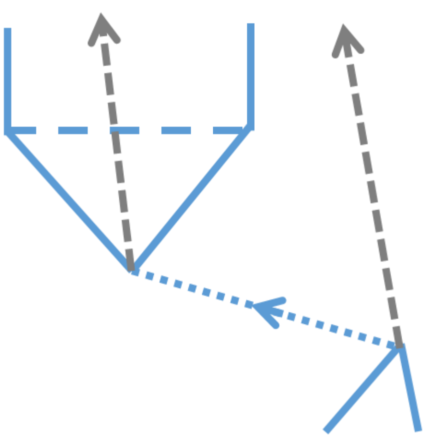


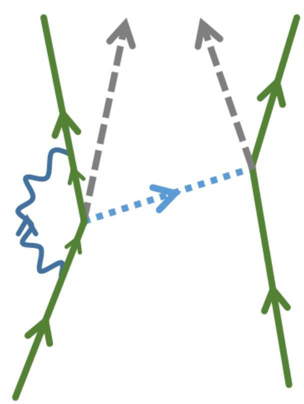
Exchange ME




Two-body current ME

The NN interaction is vertically between the two neutrino vertexes.





**Feynman diagram** including vertex correction by meson exchange interaction (wavy line). Every part is a propagator.

*Basic idea to derive equation of vertex correction to NME*

Extension of the nucleon part of the usual lowest-order equation of the 0vββ NME according to the Rayleigh-Schrödinger perturbation theory.

I pick up terms corresponding to those diagrams from the general equation of the second-order perturbation with approximations for feasibility of calculation.

Equation of the exchange ME

$$M_{\mathrm{ex}}^{(0\nu)} = \sum_{B_F B_I} \sum_{\kappa\nu'\mu'\lambda} \sum_{\mu\nu} \mathcal{W}^{(4a)}_{\mu'\lambda,\mu\nu} \langle F | a^F_\mu a^F_\nu | B_F \rangle \langle B_F | B_I \rangle \langle B_I | a^{I\dagger}_\kappa a^{I\dagger}_{\nu'} | I \rangle \mathcal{V}^{(4a)I}_{\kappa\nu',\mu'\lambda}$$

$$+ \sum_{B_F B_I} \sum_{\nu'\mu'\lambda\kappa} \sum_{\mu\nu} \mathcal{V}^{(4b)F}_{\mu'\lambda,\kappa\nu'} \langle F | a^F_\kappa a^F_{\nu'} | B_F \rangle \langle B_F | B_I \rangle \langle B_I | a^{I\dagger}_\mu a^{I\dagger}_\nu | I \rangle \mathcal{W}^{(4a)}_{\mu\nu,\mu'\lambda}.$$

$\mathcal{W}^{(4a)}_{\mu'\lambda,\mu\nu}$ : ME of the $\nu$ potential. This is the usual one.

$\mathcal{V}^{(4a)I}_{\kappa\nu',\mu'\lambda}$ : ME of the perturbative interaction divided by an energy denominator.

## Equation of the exchange ME

$$M_{\mathrm{ex}}^{(0\nu)} = \sum_{B_F B_I} \sum_{\kappa\nu'\mu'\lambda} \sum_{\mu\nu} \mathcal{W}^{(4a)}_{\mu'\lambda,\mu\nu} \langle F | a^F_\mu a^F_\nu | B_F \rangle \langle B_F | B_I \rangle \langle B_I | a^{I\dagger}_\kappa a^{I\dagger}_{\nu'} | I \rangle \mathcal{V}^{(4a)I}_{\kappa\nu',\mu'\lambda}$$

$$+ \sum_{B_F B_I} \sum_{\nu'\mu'\lambda\kappa} \sum_{\mu\nu} \mathcal{V}^{(4b)F}_{\mu'\lambda,\kappa\nu'} \langle F | a^F_\kappa a^F_{\nu'} | B_F \rangle \langle B_F | B_I \rangle \langle B_I | a^{I\dagger}_\mu a^{I\dagger}_\nu | I \rangle \mathcal{W}^{(4a)}_{\mu\nu,\mu'\lambda}.$$

## Equation of the two-body current ME

$$M_{\mathrm{2b}}^{(0\nu)} = -\frac{1}{2} \sum_{B_I B_F} \sum_{\substack{abcdefg \\ \text{with conditions}}} W^{2b}_{gf,cb} \langle F | c^{F\dagger}_g c^F_f | B_F \rangle V^I_{de,ac} \langle B_F |{:}\, c^{I\dagger}_d c^{I\dagger}_e c^I_c c^I_a {:}| B_I \rangle \langle B_I | c^{I\dagger}_c c^I_b | I \rangle.$$

Suffixes $a$-$g$: single particles

$W^{2b}_{gf,cb}$ : ME of $\nu$ potential including two energy denominators.

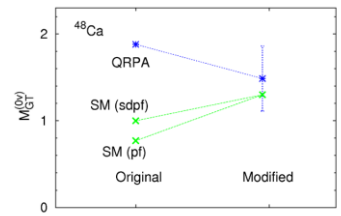
Coding is in progress.

# 4. Summary

## 1. Phenomenological improvement of 0νββ NME of the SM and the QRPA

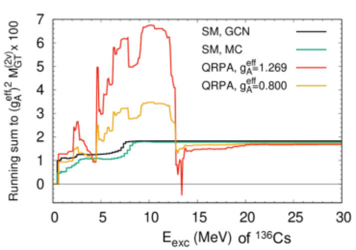
If more many-body correlations are added to the QRPA and single-particle space is enlarged in the SM, their 0*νββ* NMEs approach to each other.

**I look for shell model collaborators to apply this method to $^{136}$Xe.**




## 2. The discrepancy problem of running sum of 2*νββ* NME

The cause of the problem is the interaction strength.




## 3. Vertex correction to 0νββ NME

The diagrams and equations of the vertex correction to the 0*νββ* NME were shown.