Neutrinoless $\beta\beta$ decay matrix elements: overview and future directions

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Workshop for “Theoretical and experimental approaches for nuclear matrix elements of double-beta decay” (NME2023)

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Argonne NATIONAL LABORATORY	A. Lovato, B. Wiringa

Creation of matter in nuclei: $0\nu\beta\beta$ decay

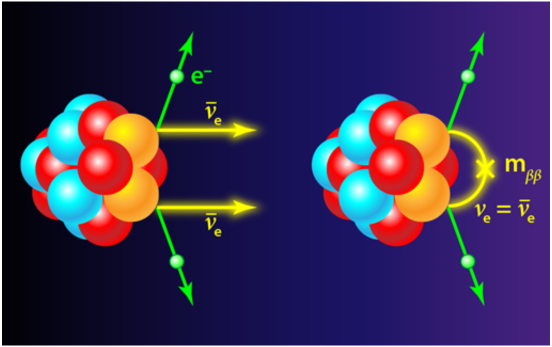
Lepton number conserved
in all processes observed:

single β decay,
$\beta\beta$ decay with ν emission...

Neutral massive particles (Majorana ν's)
allow lepton number violation:

neutrinoless $\beta\beta$ decay
creates two matter particles (electrons)

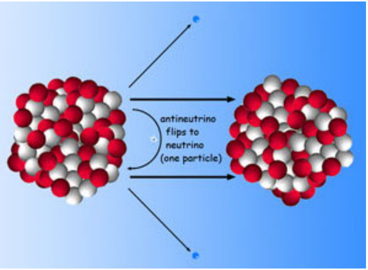
Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. 95, 025002 (2023)



Nuclear matrix elements for new-physics searches

Neutrinos, dark matter studied in experiments using nuclei

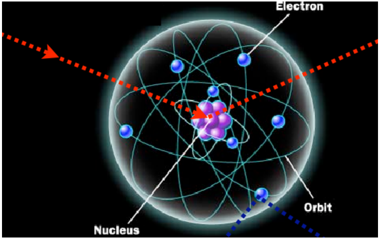
Nuclear structure physics
encoded in nuclear matrix elements
key to plan, fully exploit experiments



$0\nu\beta\beta$: $\left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} \propto g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$

Dark matter: $\dfrac{d\sigma_{\chi\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i\,\zeta_i\,\mathcal{F}_i\right|^2$

CEνNS: $\dfrac{d\sigma_{\nu\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i\,\zeta_i\,\mathcal{F}_i\right|^2$



$M^{0\nu\beta\beta}$: Nuclear matrix element
$\mathcal{F}_i$: Nuclear structure factor

Scales in new-physics searches using nuclei

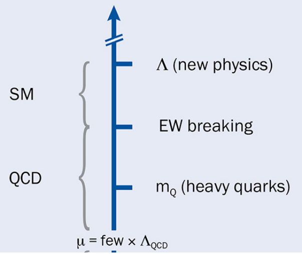
New physics scale: $\Lambda \gg 250$ GeV

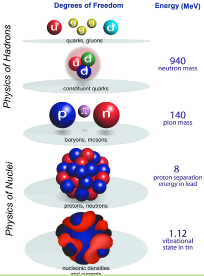
Electroweak scale:

$$v = \left(\sqrt{2}G_F\right)^{-1/2} \sim 250 \text{ GeV}$$

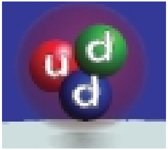
QCD (hadron) scale: $m_N \sim$ GeV

Nuclear scale: $k_F \sim m_\pi \sim 200$ MeV



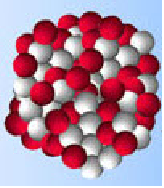


Particle, hadronic and nuclear physics



ν scattering off nuclei
interplay of particle, hadronic and nuclear physics:

ν's: interaction with quarks and gluons
Quarks and gluons: embedded in the nucleon
Nucleons: form complex, many-nucleon nuclei



General ν-nucleus scattering cross-section:

$$\frac{d\sigma_{\nu\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left| \sum_i c_i \, \zeta_i \, \mathcal{F}_i \right|^2$$

ζ: kinematics $(q^2, \cdots)$

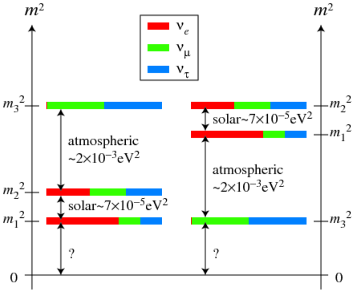
c coefficients:
ν couplings to quark, gluons (Wilson coefficients), particle physics
convoluted with hadronic matrix elements, hadronic physics

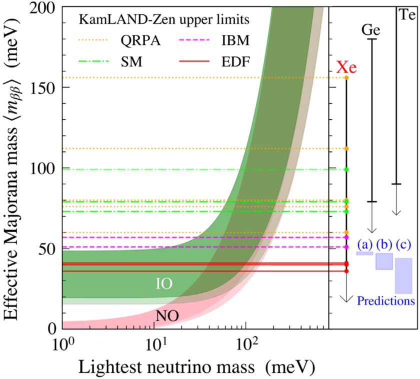
$\mathcal{F}$ functions: $\mathcal{F}^2 \sim$ structure factor, nuclear structure physics

Next generation experiments: inverted hierarchy

Decay rate sensitive to
neutrino masses, hierarchy
$m_{\beta\beta} = |\sum U_{ek}^2 m_k|$

$$\left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} = G_{0\nu}\, g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$

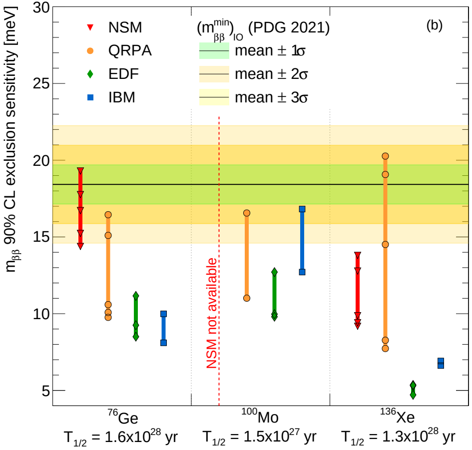




KamLAND-Zen, PRL130 051801(2023)

Matrix elements assess
if next-generation experiments
fully cover "inverted hierarchy"

Uncertainty in physics reach of $0\nu\beta\beta$ experiments



Agostini, Benato, Detwiler, JM, Vissani

Phys. Rev. C 104 L042501 (2021)

Nuclear matrix element theoretical uncertainty critical to anticipate $m_{\beta\beta}$ sensitivity of future experiments

Current uncertainty in $m_{\beta\beta}$ prevents to foresee if next-generation experiments will fully cover parameter space of “inverted” neutrino mass hierarchy

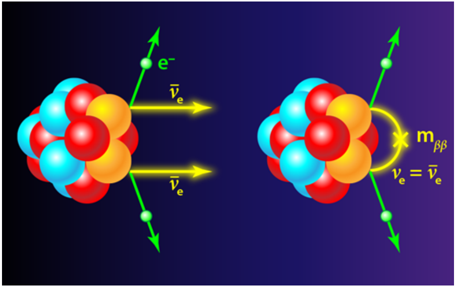
Uncertainty needs to be reduced!

Nuclear matrix elements

Nuclear matrix elements needed in low-energy new-physics searches

$$\langle\,\text{Final}\,|\mathcal{L}_{\text{leptons}-\text{nucleons}}|\,\text{Initial}\,\rangle = \langle\,\text{Final}\,|\int dx\, j^{\mu}(x) J_{\mu}(x)\,|\,\text{Initial}\,\rangle$$

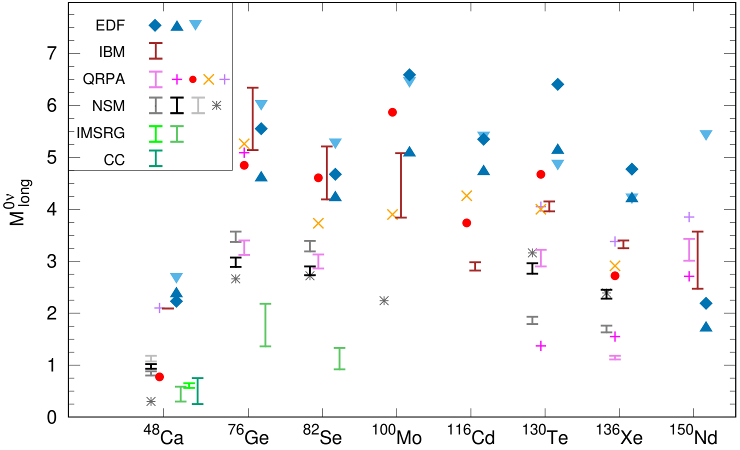
- Nuclear structure calculation of the initial and final states:
  Shell model, QRPA, IBM,
  Energy-density functional
  Ab initio many-body theory
  QMC, Coupled-cluster, IMSRG...



- Lepton-nucleus interaction:
  Hadronic current in nucleus:
  phenomenological,
  effective theory of QCD

$0\nu\beta\beta$ decay nuclear matrix elements

Large difference in nuclear matrix element calculations: factor $\sim$ 3



Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. 95, 025002 (2023)

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$0\nu\beta\beta$ decay nuclear matrix elements

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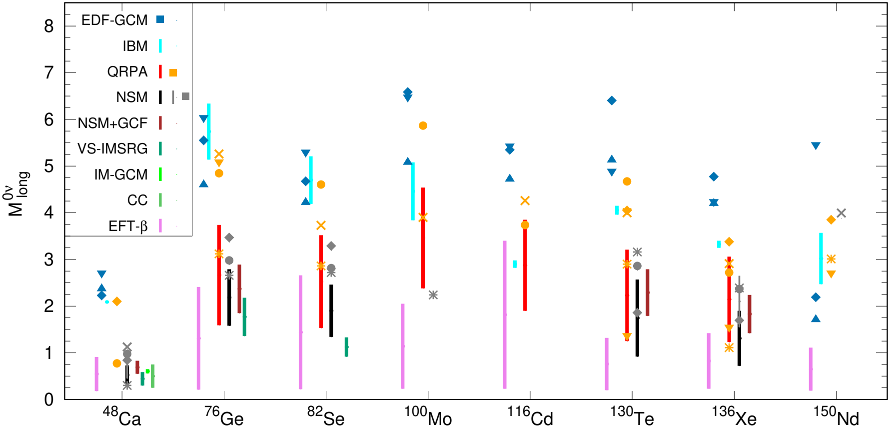


Gómez-Cadenas, Martín-Albo, JM, Mezzeto, Monrabal, Sorel, Riv. Nuov. Cim. in press

Outline

Tests of nuclear structure

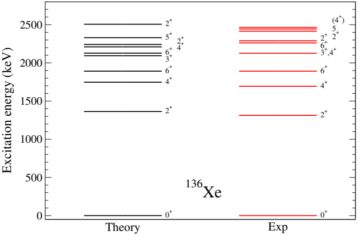
Improved nuclear matrix element calculations: correlations

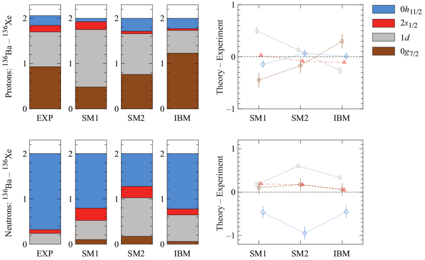
Correlation of $0\nu\beta\beta$ NMEs with other observables

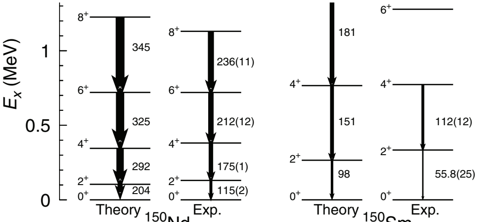
Current status of nuclear matrix elements

Tests of nuclear structure

Spectroscopy well described: masses, spectra, transitions, knockout...







Schiffer et al. PRL100 112501(2009)

Kay et al. PRC79 021301(2009)

...

Szwec et al., PRC94 054314 (2016)

Rodríguez et al. PRL105 252503 (2010)

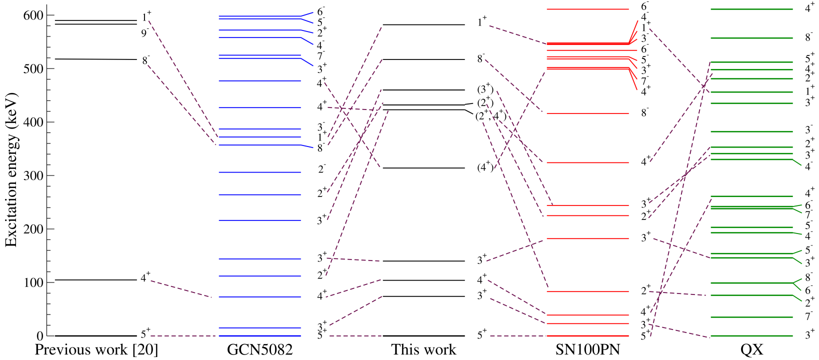
...

Vietze et al. PRD91 043520 (2015)

^{136}Cs experimental spectrum

While all these interactions are well, tested recent data on ^{136}Cs suggests GCN5082 results agree better with experiment than QX

Rebeiro, Triambak et al. PRL131 052501 (2023)



QX systematically smaller ^{136}Xe $0\nu\beta\beta$-decay nuclear matrix elements

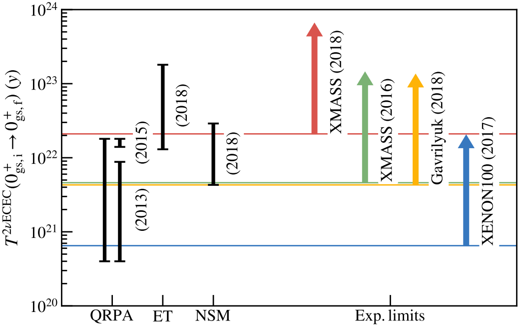
$2\nu\beta\beta$ decay, 2νECEC of ^{124}Xe

Two-neutrino $\beta\beta$ predicted for ^{48}Ca before measurement

Caurier, Poves, Zuker, PLB 252 13(1990)

Recent predictions for 2νECEC ^{124}Xe half-life:

shell model error bar largely dominated by “quenching” uncertainty



Suhonen
JPG 40 075102 (2013)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Coello Pérez, JM,
Schwenk

PLB 797 134885 (2019)

Shell model, QRPA and Effective theory (ET) predictions suggest experimental detection close to XMASS 2018 limit

$2\nu\beta\beta$ decay, 2νECEC of ^{124}Xe

Two-neutrino $\beta\beta$ predicted for ^{48}Ca before measurement

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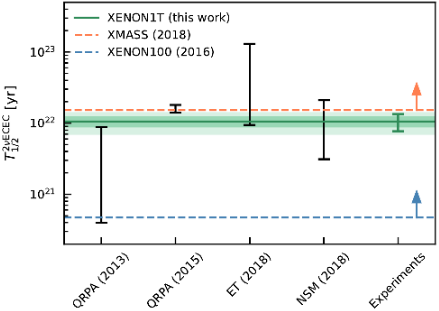


Suhonen
JPG 40 075102 (2013)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Coello Pérez, JM, Schwenk

PLB 797 134885 (2019)

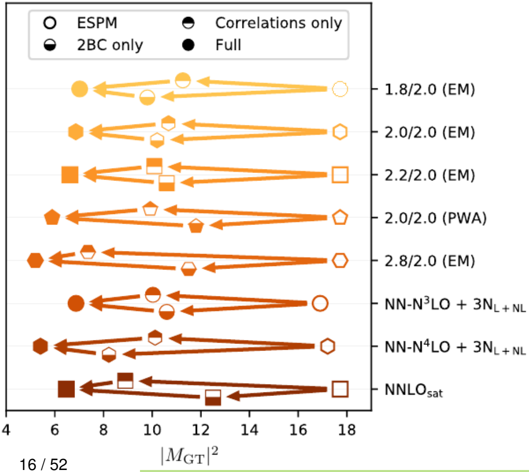
XENON1T
Nature 568 532 (2019)

PRC106, 024328 (2022)

Shell model, QRPA, Effective theory (ET)
good agreement with XENON1T measurement!

Origin of β decay "quenching"

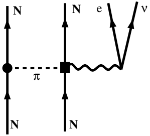
Which are main effects missing in conventional β-decay calculations?
Test case: GT decay of ^{100}Sn



Relatively similar and complementary impact of

- nuclear correlations
- meson-exchange currents

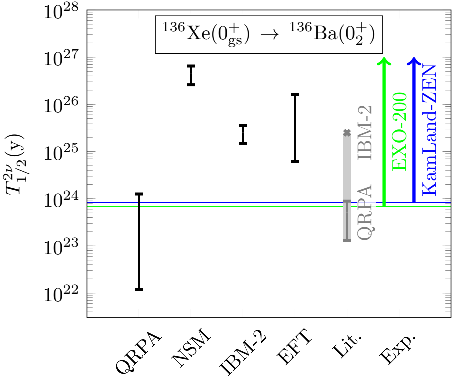
Gysbers et al.
Nature Phys. 15 428 (2019)



$2\nu\beta\beta$ decay of ^{136}Xe to ^{136}Ba 0_2^+

Current experiments sensitive to two-neutrino $\beta\beta$ of ^{136}Xe to ^{136}Ba 0_2^+

EXO-200, KamLAND-Zen



Nuclear shell model
QRPA, EFT and IBM
very different predictions!

Barea et al.
PRC 91 034304 (2015)

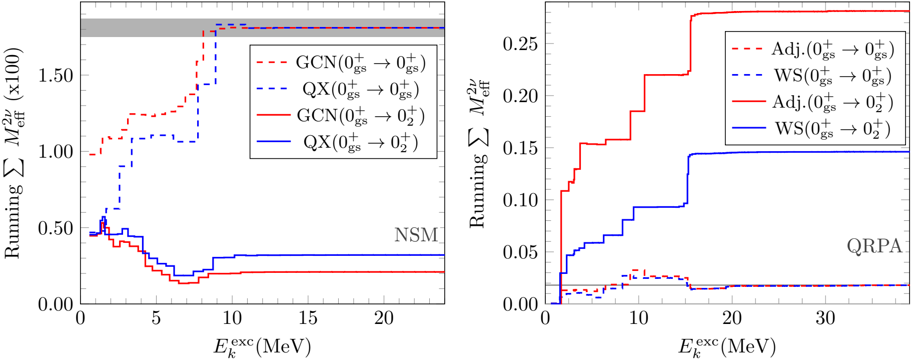
Pirinen, Suhonen
PRC 91, 054309 (2015)

Jokiniemi, Romeo, Brase, Kotila
Soriano, Schwenk, JM
PLB 838 137689 (2023)

Very good test of theoretical calculations!

^{136}Xe $\longrightarrow$ ^{136}Ba 0_2^+ running sums

Subtle cancellation NME running sum, depends on many-body method



Jokiniemi, Romeo, Brase, Kotila et al. PLB 838 137689 (2023)

Shell-model running sum shows cancellations in decay to ground state

QRPA running sum shows cancellations in decay to excited state

Outline

Tests of nuclear structure

Improved nuclear matrix element calculations: correlations

Correlation of $0\nu\beta\beta$ NMEs with other observables

Current status of nuclear matrix elements

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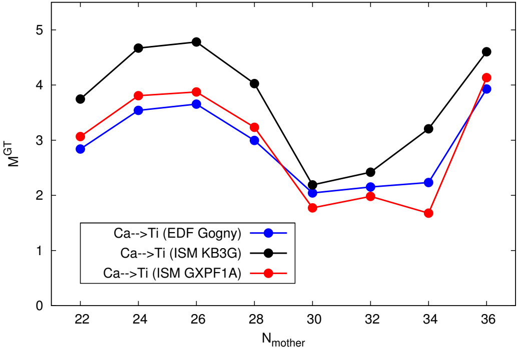
$0\nu\beta\beta$ decay without correlations

Non-realistic spherical (uncorrelated) mother and daughter nuclei:

- Shell model (SM): zero seniority, neutron and proton $J = 0$ pairs
- Energy density functional (EDF): only spherical contributions



In contrast to full (correlated) calculation SM and EDF NMEs agree!

NME scale set by pairing interaction

JM, Rodríguez, Martínez-Pinedo, Poves PRC90 024311(2014)

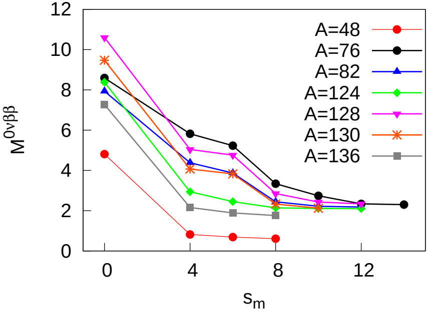
NME follows generalized seniority model:

$$M_{GT}^{0\nu\beta\beta} \simeq \alpha_\pi \alpha_\nu \sqrt{N_\pi + 1}\sqrt{\Omega_\pi - N_\pi}\sqrt{N_\nu}\sqrt{\Omega_\nu - N_\nu + 1},$$ Barea, Iachello PRC79 044301(2009)

Pairing correlations and $0\nu\beta\beta$ decay

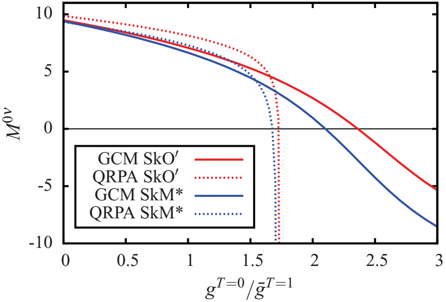
$0\nu\beta\beta$ decay favoured by proton-proton, neutron-neutron pairing, but it is disfavored by proton-neutron pairing

Ideal case: superfluid nuclei reduced with high-seniorities



Caurier et al. PRL100 052503 (2008)

Addition of isoscalar pairing reduces matrix element value



Hinohara, Engel PRC90 031301 (2014)

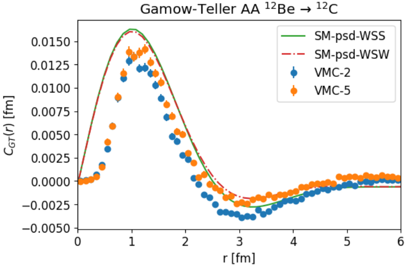
Related to approximate $SU(4)$ symmetry of the $\sum H(r)\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j\tau_i\tau_j$ operator

Shell model vs quantum Monte Carlo: correlations

Compare $\beta\beta$ transition densities in
nuclear shell model and quantum Monte Carlo calculations in light nuclei

$$4\pi r^2 \rho_{GT}(r) = \langle \Psi_f | \sum_{a<b} \delta(r - r_{ab}) \sigma_{ab} \tau_a^+ \tau_b^+ | \Psi_i \rangle \qquad M^{0\nu}_{GT} = \int_0^\infty dr\, C^{0\nu}_{GT}\,,$$

Agree at long distances, shell model misses short-range correlations



Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)

Similar findings in Wang et al. PLB 798 134974 (2019)

Generalized contact formalism (GCF)

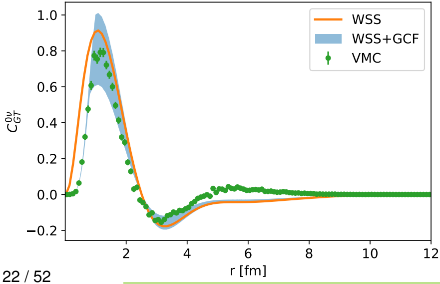
Generalized contact formalism Weiss, Bazak, Barnea PRL 114 012501 (2015)

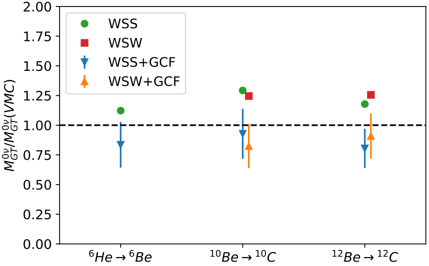
Separation of scales: wf, transition density factorize for nearby nucleons

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_{\alpha} \varphi^{\alpha}(\boldsymbol{r}_{ij}) A^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}), \quad \rho_{GT}(r) \xrightarrow[r\to 0]{} -3|\varphi^{0}(r)|^{2} C^{0}_{pp,nn}(f,i)$$

The contact $C^0(f,i) = \frac{A(A-1)}{2}\langle A^{\alpha}(f)|A^{\beta}(i)\rangle$ is model dependent

Replace shell-model by QMC contact: improve transition density





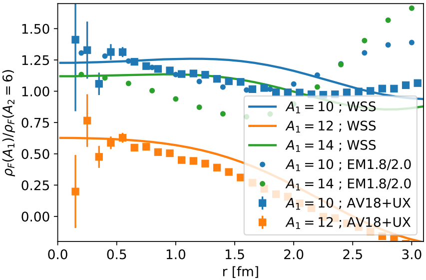
GCF: model independence of ratios

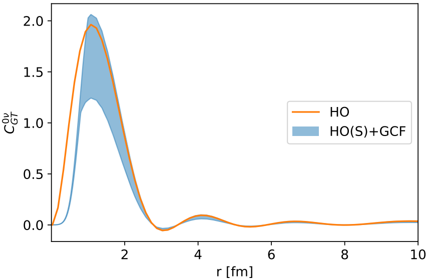
Generalized contact formalism Weiss, Bazak, Barnea PRL 114 012501 (2015)

The contact $C^0(f,i) = \frac{A(A-1)}{2}\langle A^\alpha(f)|A^\beta(i)\rangle$ is model dependent (shell model, quantum Monte Carlo, no-core shell model...)

but the ratio $C^0_{pp,nn}(X)/C^0_{pp,nn}(Y)$ relatively model independent:

combine QMC calculation in light nuclei with two shell-model ones:



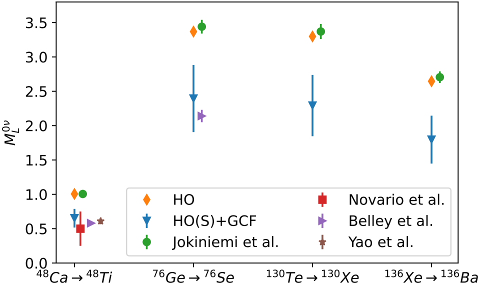


Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)

Shell model + Generalized contact formalism

GCF builds QMC correlations to shell model transition densities extended to heavy nuclei where shell model calculations are possible

Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)



Short-range correlations included by GCF reduce $0\nu\beta\beta$ NMEs mildly

$\sim$ 30% reduction in general consistent with ab initio NMEs in ^{48}Ca, ^{76}Ge

Good agreement benchmark in light nuclei with ab initio NMEs

Outline

Tests of nuclear structure

Improved nuclear matrix element calculations: correlations

Correlation of $0\nu\beta\beta$ NMEs with other observables

Current status of nuclear matrix elements

Systematic shell-model calculations

Explore systematic shell-model matrix elements
in configuration spaces relevant for $0\nu\beta\beta$ decay searches

- $^{46-58}$Ca, $^{50-58}$Ti, and $^{54-60}$Cr
  in pf-shell with KB3G and GXPF1B interactions
- $^{72-76}$Ni, $^{74-80}$Zn, $^{76-82}$Ge, and ^{82,84}Se
  in $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$, and $0g_{9/2}$ configuration space
  with GCN2850, JUN45, and JJ4BB interactions
- $^{124-132}$Sn, $^{130-134}$Te, and ^{134,136}Xe
  in $1d_{5/2}$, $0g_{7/2}$, $2s_{1/2}$, $1d_{3/2}$, and $0_{h11/2}$ configuration space
  with the GCN5082 and QX interactions

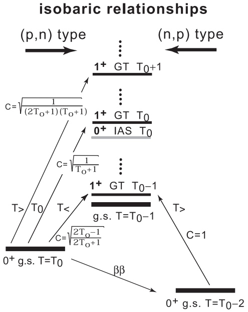
Overall, $\sim 20 - 40$ different calculations for each configuration space

Complementary approach to randomly varying nuclear interaction

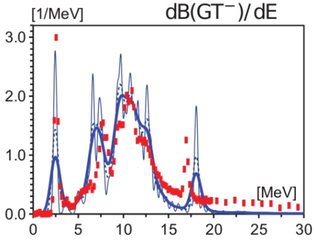
Horoi et al. PRC106 054302 (2022), PRC107 045501 (2023)

Gamow-Teller strength distributions

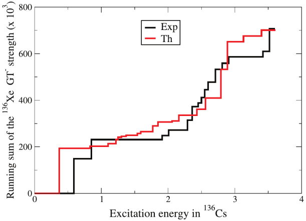
GT strength distribution complements β-decay beyond Q-value region



Frekers et al.
NPA916 219 (2013)



Iwata et al. JPSCP6 3057 (2015)



Caurier et al. PLB711 62 (2012)

$$\frac{d\sigma}{d\Omega}(\theta=0)\propto\sum\boldsymbol{\sigma}_i\tau^{\pm}, \qquad \langle 1_n^+|\sum g_A^{\rm eff}\,\boldsymbol{\sigma}_i\tau_i^{\pm}\,|0_{gs}^+\rangle$$

$\beta\beta$ decay: relative phase between two Gamow-Teller legs unknown

Double Gamow-Teller strength distribution

Measurement of Double Gamow-Teller (DGT) resonance
in double charge-exchange reactions ^{48}Ca(pp,nn)^{48}Ti proposed in 80's

Auerbach, Muto, Vogel... 1980's, 90's

Recent experimental plans in RCNP, RIKEN, INFN Catania

Takaki et al. JPS Conf. Proc. 6 020038 (2015)

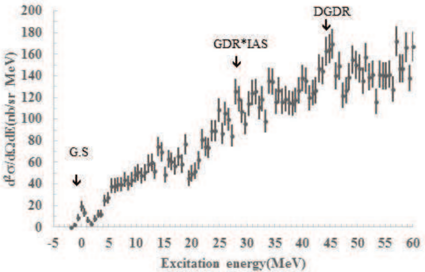
Capuzzello et al. EPJA 51 145 (2015), Takahisa, Ejiri et al. arXiv:1703.08264

Sakaue et al. DNP-JPS joint meeting 2023 talk

Promising connection to $\beta\beta$ decay,
two-particle-exchange process,
especially the (tiny) transition
to ground state of final state

Two-nucleon transfers related to
$0\nu\beta\beta$ decay matrix elements

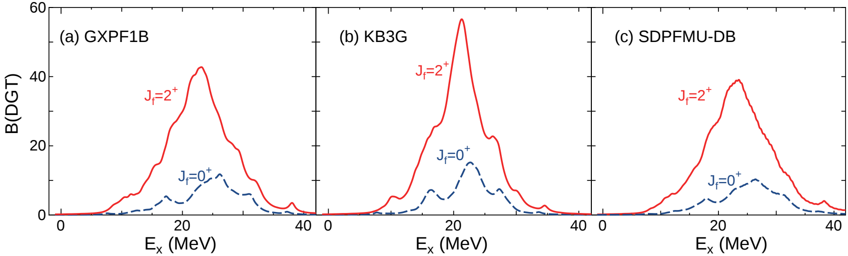
Brown et al. PRL113 262501 (2014)



^{48}Ca Double Gamow-Teller distribution

Calculate with shell model ^{48}Ca $0^+_{\rm gs}$ Double Gamow-Teller distribution

$$B(DGT^-;\lambda;i\to f)=\frac{1}{2J_i+1}\left|\left\langle {}^{48}\mathrm{Ti}\left|\left|\left[\sum_i \boldsymbol{\sigma}_i\tau_i^- \times \sum_j \boldsymbol{\sigma}_j\tau_j^-\right]^{(\lambda)}\right|\right|{}^{48}\mathrm{Ca}_{gs}\right\rangle\right|^2$$



Shell model calculation with Lanczos strength function method

Double GT resonances in one and two shells rather similar result

Shimizu, JM, Yako, PRL120 142502 (2018)

Double Gamow-Teller distribution and pairing

Study the sensitivity of Double GT distribution to pairing correlations

Add/remove pairing

$$H' = H + G^{JT} P^{JT}$$

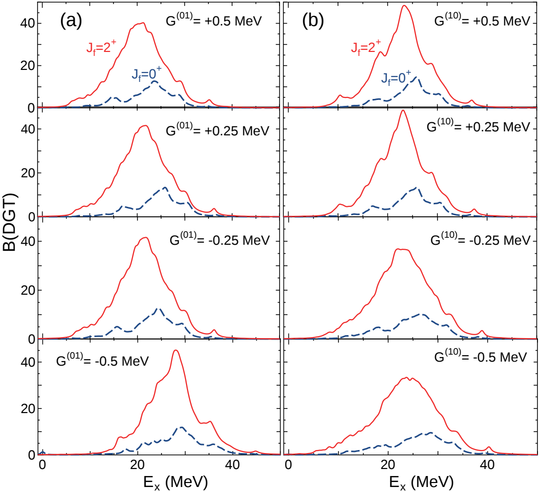
like-particle (T=1) or proton-neutron (T=0)

Position of the DGT giant resonance very sensitive to like-particle pairing

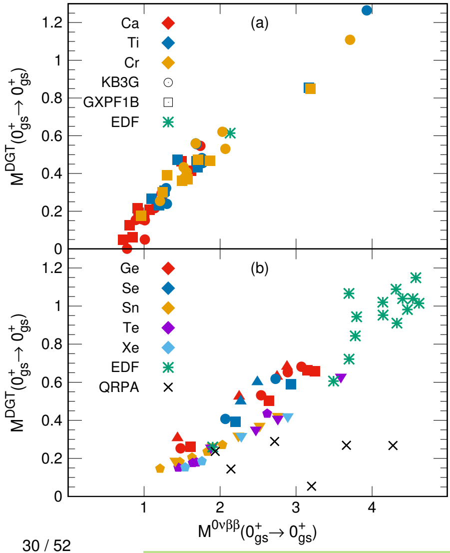
DGT resonance width probes isoscalar pairing

Shimizu, JM, Yako

PRL120 142502 (2018)



DGT and $0\nu\beta\beta$ decay: heavy nuclei



DGT transition to ground state

$$M^{\mathrm{DGT}} = \sqrt{B(DGT_-; 0; 0^+_{\mathrm{gs}} \to 0^+_{\mathrm{gs}})}$$

very good linear correlation
with $0\nu\beta\beta$ decay
nuclear matrix elements

Correlation across wide range,
nuclei from Ca to Ge and Xe

Common to shell model and
energy-density functional theory
$0 \lesssim M^{0\nu\beta\beta} \lesssim 5$
disagreement to QRPA

Shimizu, JM, Yako,
PRL120 142502 (2018)

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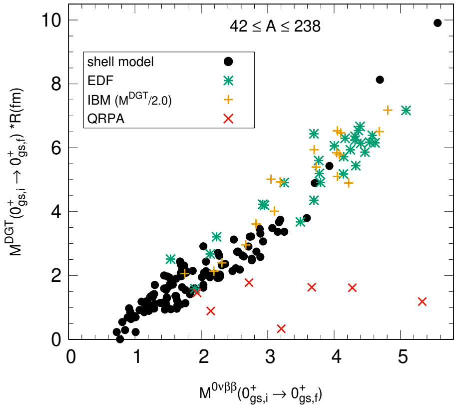

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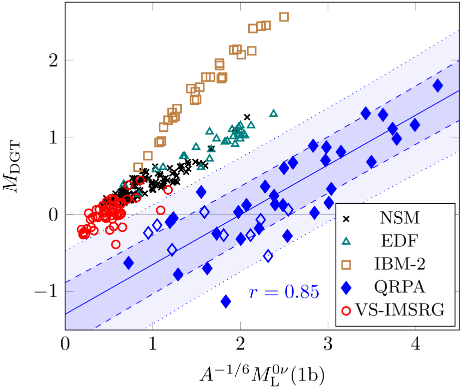
Shimizu, JM, Yako,
PRL120 142502 (2018)

Correlation of $0\nu\beta\beta$ decay to DGT in QRPA

In QRPA, g_{pp} parameter usually fitted to measured $2\nu\beta\beta$ half-lifes but actually some tension between g_{pp} values to reproduce β decays

Faessler et al., J. Phys. G 35, 075104 (2008)



Jokiniemi, JM, PRC 107 044316 (2023)

Correlation between DGT and $0\nu\beta\beta$ NMEs! for $g_{pp} = (0.6 - 0.9)$ but different to other methods

Partially caused by relevance of $J > 1$ intermediate states in QRPA wrt eg shell model

Ejiri et al. Phys. Rept. 797 1 (2019)

Horoi et al, PRC 93, 044334 (2016)

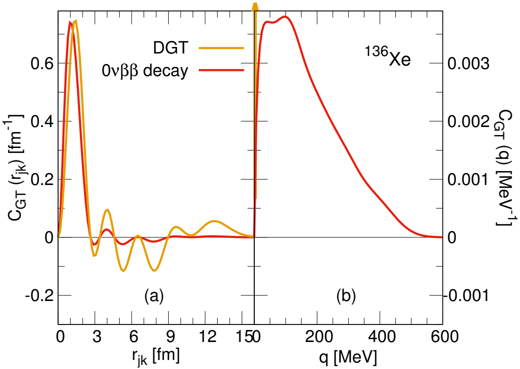
Ab initio VS-IMSRG study of DGT vs $0\nu\beta\beta$ NME also finds correlation (somewhat weaker)

Yao et al. PRC106 014315(2022)

Short-range character of DGT, $0\nu\beta\beta$ decay

Correlation between DGT and $0\nu\beta\beta$ decay matrix elements explained by transition involving low-energy states combined with dominance of short distances between exchanged/decaying neutrons

Bogner et al. PRC86 064304 (2012)



$0\nu\beta\beta$ decay matrix element limited to shorter range

Short-range part dominant in double GT matrix element due to partial cancellation of mid- and long-range parts

Long-range part dominant in QRPA DGT matrix elements

Shimizu, JM, Yako,
PRL120 142502 (2018)

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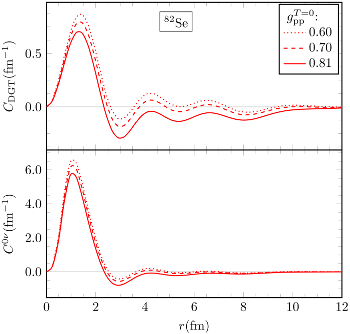

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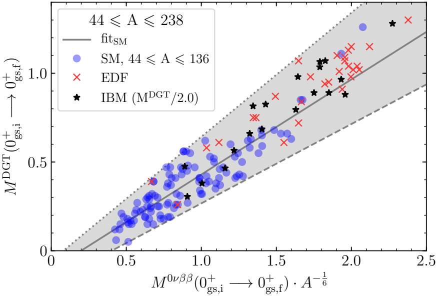
Shimizu, JM, Yako,
PRL120 142502 (2018)

$0\nu\beta\beta$ decay NMEs in EFT of β decay

Effective field theory of β decay can calculate DGT with uncertainties
(similar to calculation of $2\nu\beta\beta$, no energy denominator)
DGT vs 0nbb correlation $\Rightarrow$ predict $0\nu\beta\beta$ NMEs with uncertainties

Because EFT couplings fitted to β decay and GT strengths
shell-model DGT NMEs in correlation need “quenching”: $q = 0.42 - 0.65$



As a result, ET $0\nu\beta\beta$ NMEs
^{76}Ge: $M^{0\nu} = 0.2 - 2.4$
^{82}Se: $M^{0\nu} = 0.2 - 2.7$

small NMEs

large uncertainty:
LO in ET, fit, “quenching”

Brase, Coello Pérez, JM, Schwenk
PRC106 034309(2022)

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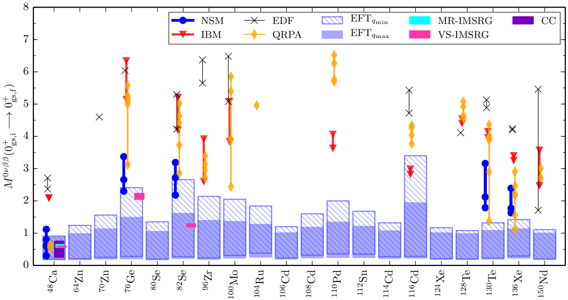
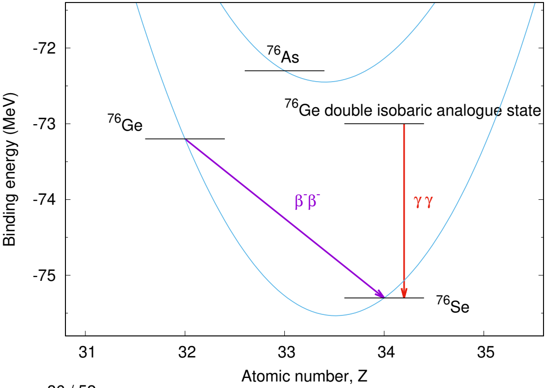


Brase, Coello Pérez, JM, Schwenk, PRC106 034309 (2022)

$\gamma\gamma$ decay of the DIAS of the initial $\beta\beta$ nucleus

Explore correlation between $0\nu\beta\beta$ and $\gamma\gamma$ decays,
focused on double-M1 transitions

$$M^{\gamma\gamma}_{M1\,M1} = \sum_k \frac{\langle 0^+_f | \sum_n (g^l_n \boldsymbol{l}_n + g^s_n \boldsymbol{\sigma}_n)^{IV} | 1^+_k(\text{IAS})\rangle\langle 1^+_k(\text{IAS}) | \sum_m (g^l_m \boldsymbol{l}_m + g^s_m \boldsymbol{\sigma}_m)^{IV} | 0^+_i(\text{DIAS})\rangle}{E_k - (E_i + E_f)/2}$$



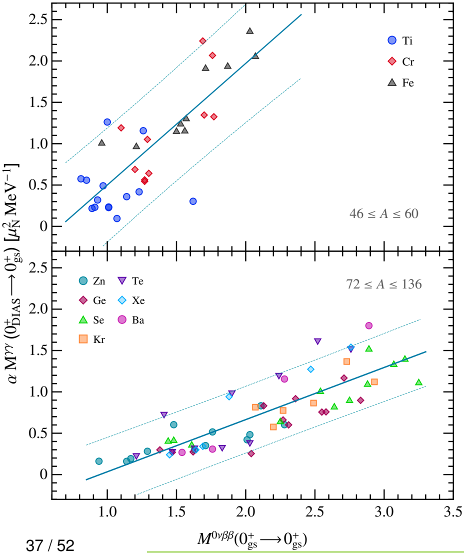
Similar initial and final states
but both in same nucleus
for electromagnetic transition

M1 and GT operators similar,
physics of spin operator
M1 also angular momentum

Different energy denominator

Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Correlation between $M1M1$ and $0\nu\beta\beta$ NMEs



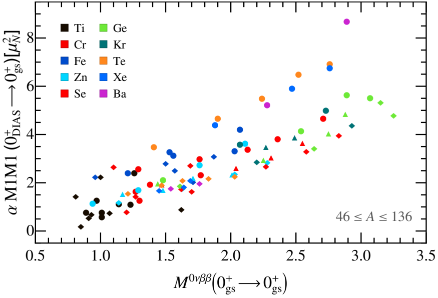
Good correlation between $M1M1$ same-energy photons and shell-model $0\nu\beta\beta$ NMEs

A dependence:
energy denominator
dominant states at higher
energy in heavier nuclei

Overall, study $\sim$ 50 transitions
several nuclear interactions
for each of them

Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Intermediate states of the *M*1*M*1 transition

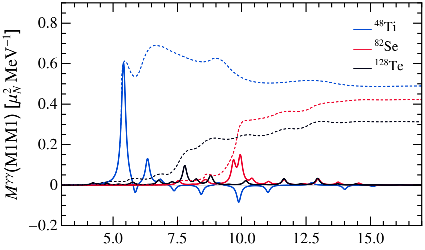


When energy denominators are (artificially) removed, same correlation across the nuclear chart

Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Dominant intermediate states lower energies for lighter nuclei, otherwise similar energies

One or few intermediate states typically dominate the transition

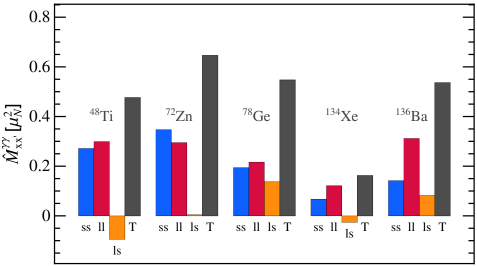


Spin, angular momentum decomposition

The numerator NME can be decomposed into

$$\hat{M}_{\gamma\gamma} = \hat{M}_{ss} + \hat{M}_{ll} + \hat{M}_{ls}$$

spin, angular momentum and interference components



Spin, angular momentum terms strikingly similar,
always carry same sign

Interference term
can cancel the other two
but always much smaller

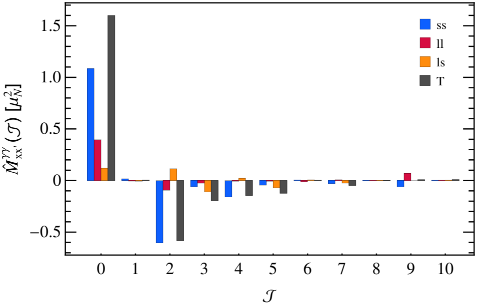
Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Total angular momentum decomposition

The numerator NME can be decomposed into

$$\hat{M}_{\gamma\gamma}(\mathcal{J}) = \hat{M}_{ss}(\mathcal{J}) + \hat{M}_{ll}(\mathcal{J}) + \hat{M}_{ls}(\mathcal{J})$$

spin, angular momentum and interference components
and total angular momentum of the nucleons involved in the transition



Dominance of $\mathcal{J} = 0$ terms
for spin and orbital contributions
just like in $0\nu\beta\beta$ decay

Cancellation from $\mathcal{J} > 0$ terms
less pronounced in orbital part

Explains similar behaviour of
spin and orbital components:

$$s_1\, s_2 = \mathcal{S}^2 - 3/2 < 0$$

$$l_1\, l_2 = \mathcal{L}^2 - l_1^2 - l_2^2 < 0$$

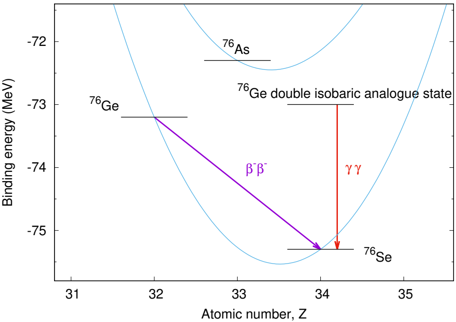
Romeo et al. PLB 827 136965 (2022)

Experimental feasibility of $\gamma\gamma$ decay?

$\gamma\gamma$ decays are very suppressed with respect to γ decays
just like $\beta\beta$ decays are much slower than β decays

$\gamma\gamma$ decays have been observed recently
in competition with γ decays

Waltz et al. Nature 526, 406 (2015), Soderstrom et al. Nat. Comm. 11, 3242 (2020)



Outlook:

Study in detail leading decay channels for $M1M1$ decay in DIAS of $\beta\beta$ nuclei

Particle emission $M1$, $E1$ decay: BR $\sim 10^{-7} - 10^{-8}$

Experimental proposal for ^{48}Ti by Valiente-Dobón et al.

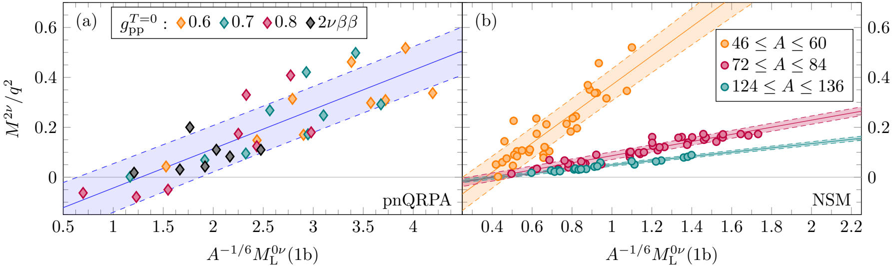
Valiente-Dobón, Romeo et al., in prep

Correlation of $0\nu\beta\beta$ decay and $2\nu\beta\beta$ decay

Good correlation between 2ν and 0ν modes of $\beta\beta$ decay
in nuclear shell model (systematic calculations of different nuclei)
and QRPA calculations (decays of $\beta\beta$ emitters with different g_{pp} values)

Similar but not common correlation, depends on mass for shell model
$0\nu\beta\beta - 2\nu\beta\beta$ correlation also observed in ^{48}Ca, ^{136}Xe

Horoi et al. PRC106 054302 (2022), PRC107 045501 (2023)



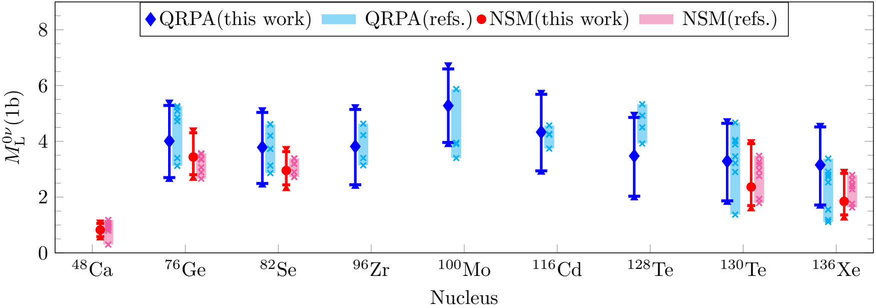
Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

$0\nu\beta\beta$ NMEs from $2\nu\beta\beta - 0\nu\beta\beta$ correlation

NMEs consistent with previous nuclear shell model, QRPA results

Theoretical uncertainty involves
systematic calculations covering dozens of nuclei and interactions
error of each calculation (eg quenching) and experimental $2\nu\beta\beta$ error

Previous theoretical uncertainty mostly ignored: collection of calculations



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

2b currents in $0\nu\beta\beta$ decay

In $0\nu\beta\beta$ decay, two weak currents lead to four-body operator when including the product of two 2b currents: computational challenge

Approximate 2b current as effective 1b current normal ordering with respect to a Fermi gas

JM, Gazit, Schwenk, PRL107 062501(2011)

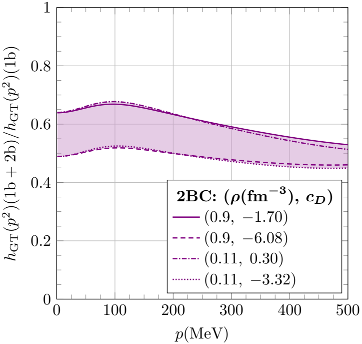
Normal-odering approximation works remarkably well for β decay ($q = 0$)

Gysbers et al. Nature Phys. 15 428 (2019)

Some reduction of quenching due to 2b currents at $p \sim m_\pi$ relevant for $0\nu\beta\beta$ decay

Hoferichter, JM, Schwenk

PRD102 074018 (2020)

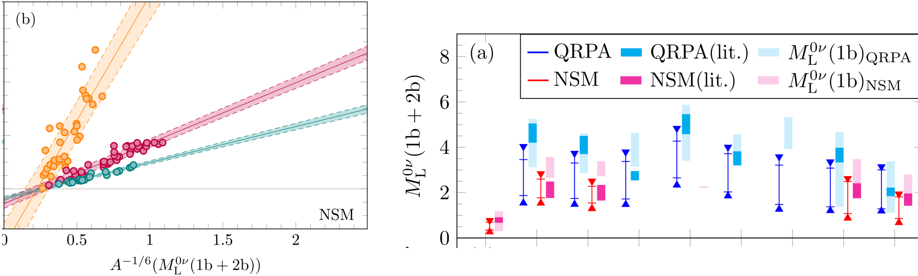


Jokiniemi, Romeo, Soriano, JM

PRC 107 044305 (2023)

Correlation of $0\nu\beta\beta$ decay to $2\nu\beta\beta$: 2b currents

Good correlation between $2\nu\beta\beta$ and $0\nu\beta\beta$
also present when including 2b currents to the $0\nu\beta\beta$ NMEs



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

Use $2\nu\beta\beta$ data to predict $0\nu\beta\beta$ NMEs with 2b currents

Outline

Tests of nuclear structure

Improved nuclear matrix element calculations: correlations

Correlation of $0\nu\beta\beta$ NMEs with other observables

Current status of nuclear matrix elements

Light-neutrino exchange: contact operator

Short-range operator contributes to light-neutrino exchange
for RG invariance of two-nucleon decay amplitude: high-energy ν's

$$T_{1/2}^{-1} = G_{01}\, g_A^4 \left(M^{0\nu}_{\text{long}} + M^{0\nu}_{\text{short}}\right)^2 m_{\beta\beta}^2,$$

Cirigliano et al. PRL120 202001(2018)

$$M^{0\nu}_{\text{short}} \equiv \frac{1.2A^{1/3}\,\text{fm}}{g_A^2} \langle 0_f^+ | \sum_{n.m} \tau_m^- \tau_n^- \, \mathbb{1} \left[\frac{2}{\pi} \int j_0(qr)\, 2g_\nu^{\text{NN}}\, g(p/\Lambda)\, p^2 dp \right] | 0_i^+ \rangle,$$

$$M^{0\nu}_{\text{GT}} \simeq \frac{1.2A^{1/3}\,\text{fm}}{g_A^2} \langle 0_f^+ | \sum_{n.m} \tau_m^- \tau_n^- \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \left[\frac{2}{\pi} \int j_0(qr)\, \frac{1}{p^2}\, g_A^2\, f^2(p/\Lambda_A)\, p^2 dp \right] | 0_i^+ \rangle$$

Unknown value (and sign) of the hadronic coupling g_ν^{NN}!

Lattice QCD calculations can obtain value of g_ν^{NN}

Davoudi, Kadam, Phys. Rev. Lett. 126, 152003 (2021), PRD105 094502('22)

match $nn \to pp + ee$ amplitude calculated with dispersion QCD methods

Cirigliano et al. PRL126 172002 (2021), JHEP 05 289 (2021)

charge-independence breaking of nuclear Hamiltonians

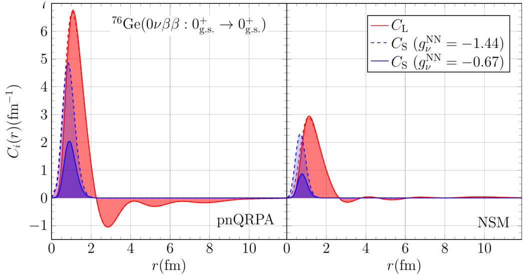
Cirigliano et al. PRC100, 055504 (2019)

Long and short-range NME in heavy nuclei

Relatively stable contribution of new term M_S/M_L:

20% – 50% impact of short-range NME in shell model
30% – 70% impact of short-range NME in QRPA

consistent with 43% effect in IM-GCM for ^{48}Ca
using calculated $nn \to pp + ee$ decay Wirth et al. PRL127 242502 (2021)

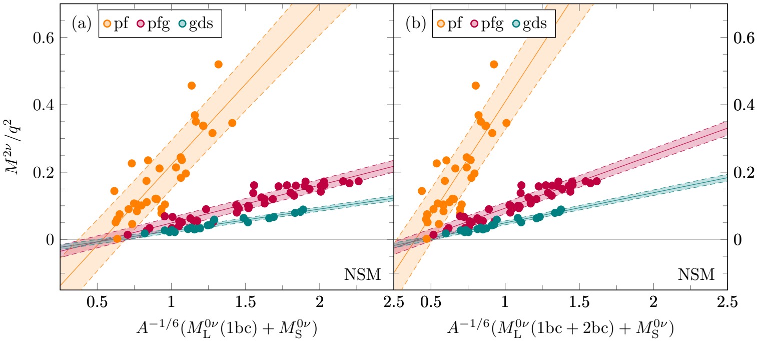


Jokiniemi, Soriano, JM, Phys. Lett. B 823 136720 (2021)

Uncertainty dominated by coupling g_ν^{NN}

Correlation of 0$\nu\beta\beta$ decay to 2$\nu\beta\beta$: general case

A good correlation between 2$\nu\beta\beta$ and 0$\nu\beta\beta$
also appears when we include to the calculation of 0$\nu\beta\beta$ NMEs
2b currents and the short-range nuclear matrix element



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

Use 2$\nu\beta\beta$ data to predict 0$\nu\beta\beta$ NMEs with 2b currents, short-range NME

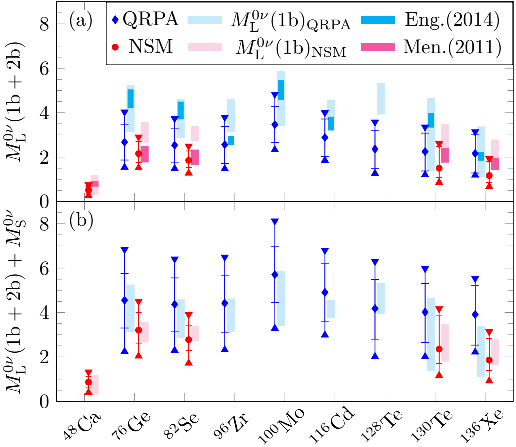
$0\nu\beta\beta$ NMEs from correlation: 2bc, short-range

$0\nu\beta\beta$ NMEs including 2b currents and short-range NME obtained from $0\nu\beta\beta - 2\nu\beta\beta$ correlation and $2\nu\beta\beta$ data

Theoretical uncertainty due to correlation, calculation uncertainties: quenching, 2bc, short-range NME coupling (dominant uncertainty)

First complete estimation of $0\nu\beta\beta$ nuclear matrix elements with theoretical uncertainties

Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

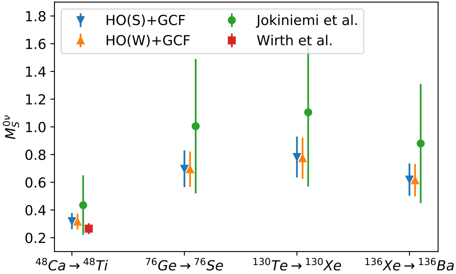


Short-range NME: GCF + shell model

Shell model with short-range correlations from QMC using the GCF give consistent contribution of new term M_S

$\sim$ 25% impact of short-range NME in GCF + shell model obtained with g_ν^{NN} from AV18 CIB term

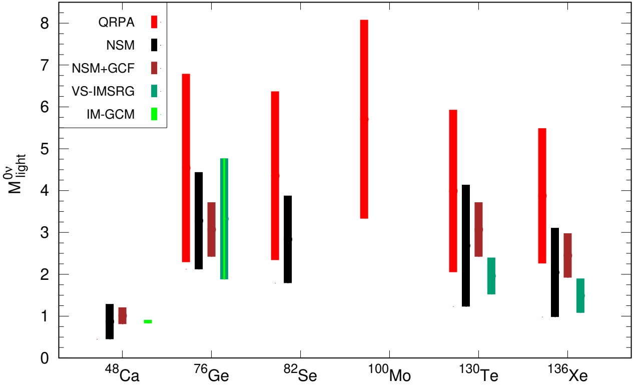
consistent with 43% effect in IM-GCM for ^{48}Ca using calculated $nn \to pp + ee$ decay Wirth et al. PRL127 242502 (2021)



Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)

$0\nu\beta\beta$ decay total (long- and short-range) NMEs

Not-so-large difference in nuclear matrix element calculations!



Gómez-Cadenas, Martín-Albo, JM, Mezzeto, Monrabal, Sorel
Rivista Nuovo Cimento, in press

Wirth et al.
PRL127 242502 (2021)

Belley et al.
arXiv:2307:15156

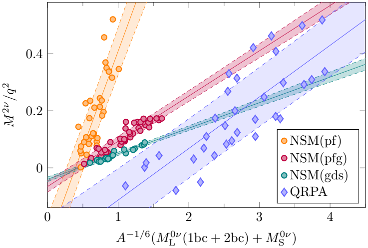
Belley et al.
arXiv:2308.15634

Jokiniemi et al.
PRC 107 044305 (2023)

Weiss et al.
PRC106 065501 (2022)

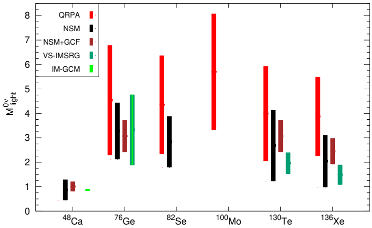
Summary

Calculations of $0\nu\beta\beta$ NMEs challenge nuclear many-body methods, searches demand reliable NMEs



Individual nuclear spectroscopy $2\nu\beta\beta$ measurements test many-body methods used to compute $0\nu\beta\beta$ NMEs

Systematic calculations of double Gamow-Teller transitions, electromagnetic $M1M1$ decay of DIAS good correlation with $0\nu\beta\beta$ NMEs



Good $0\nu\beta\beta - 2\nu\beta\beta$ correlation exploit $2\nu\beta\beta$ data to obtain $0\nu\beta\beta$ NMEs with theoretical uncertainties