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Improved neutrinoless double-beta-decay matrix elements in the pnQRPA

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22/12/2023

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Outline

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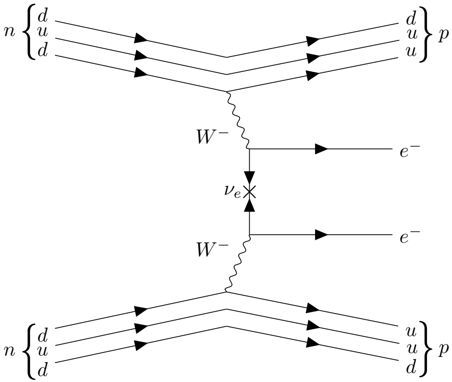
Neutrinoless double-beta decay via light neutrino exhange

$$\frac{1}{t_{1/2}^{0\nu}} = g_{\rm A}^4 G^{0\nu} |\boldsymbol{M^{0\nu}}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

- **Violates lepton-number conservation**



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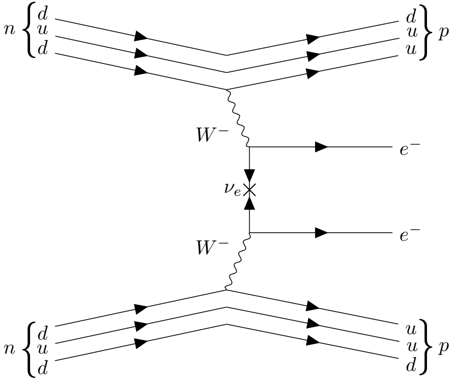

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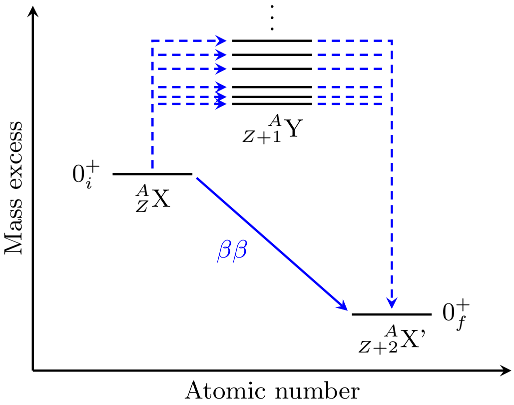

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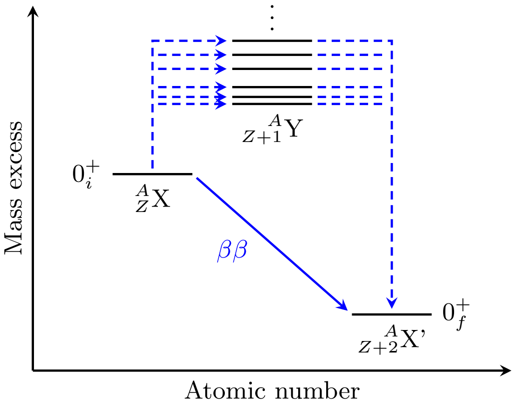

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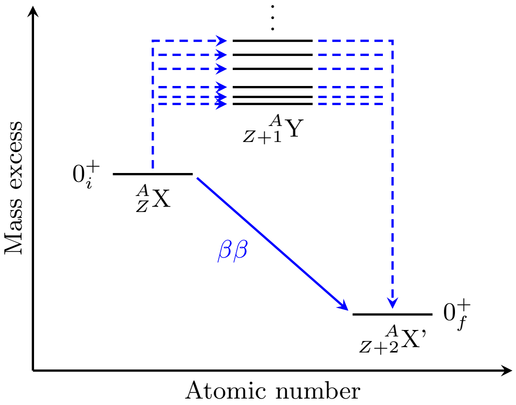

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Traditional nuclear matrix elements of neutrinoless double-beta decay

$$M^{0\nu} = \frac{R}{g_A^2} \int \frac{d\mathbf{k}}{2\pi^2} \frac{e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}}{E_\nu} \sum_n \frac{\langle f| J_\mu(\mathbf{x}) |n\rangle \langle n| J^\mu(\mathbf{y}) |i\rangle}{E_\nu + E_n - \frac{1}{2}(E_i + E_f) - \frac{1}{2}(E_1 - E_2)}$$

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Closure approximation

Without closure approximation:

$$M^{0\nu} \propto \sum_n \frac{\langle f| J_\mu(\mathbf{x}) |n\rangle \langle n| J^\mu(\mathbf{y}) |i\rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

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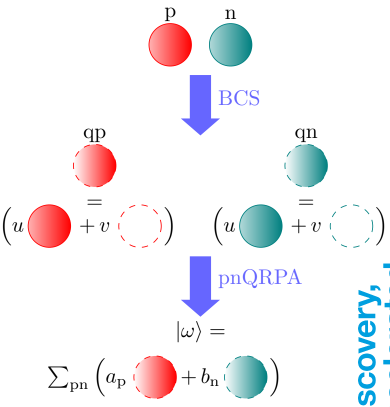
- Typically used **with other nuclear methods**

Spherical proton-neutron quasiparticle random-phase approximation (pnQRPA)

- Single-particle bases from Woods-Saxon potential



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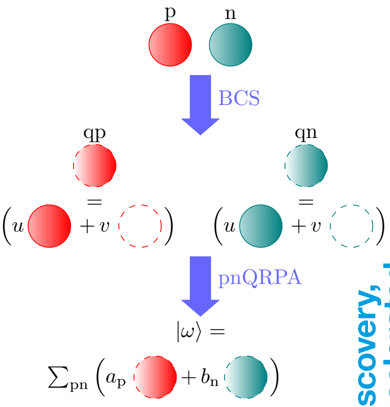

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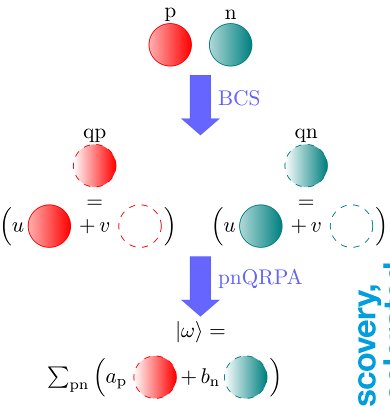
$$|J_k^\pi\rangle = \sum_{pn} \left(X_{pn}^{J_k^\pi} [a_p^\dagger a_n^\dagger]_J - Y_{pn}^{J_k^\pi} [a_p^\dagger a_n^\dagger]_J^\dagger \right) |\mathrm{QRPA}\rangle$$



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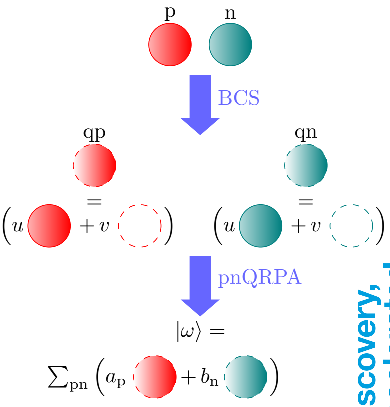
- Adjustable parameters:

$$g_{\mathrm{ph}} \langle p'n'^{-1}, J | V | pn^{-1}, J \rangle$$

$$g_{\mathrm{pp}} \langle p'n', J | V | pn, J \rangle$$



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Outline

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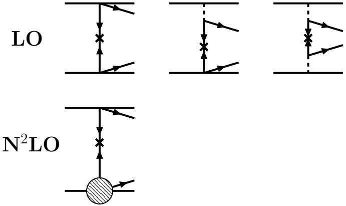
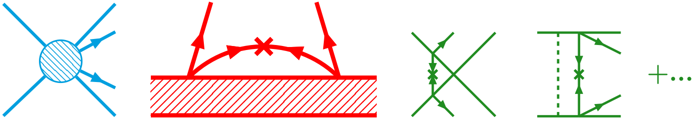
Effective field theory corrections to $0\nu\beta\beta$ decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} \left| M_L^{0\nu} + \mathbf{M_S^{0\nu}} + \mathbf{M_{usoft}^{0\nu}} + \mathbf{M_{N2LO}^{0\nu}} \right|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)





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Outline

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Contact Term in pnQRPA and NSM

$$M_{\mathrm{S}}^{0\nu} = \frac{2R}{\pi g_{\mathrm{A}}^2} \langle 0_f^+ | \sum_{m,n} \tau_m^- \tau_n^- \int j_0(qr) h_{\mathrm{S}}(q^2)\, q^2 \mathrm{d}q | 0_i^+ \rangle$$

with

$$h_{\mathrm{S}}(q^2) = 2\mathbf{g}_\nu^{\mathrm{NN}}\, e^{-q^2/(2\Lambda^2)} \; .$$

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Couplings ($g_\nu^{\rm NN}$) and scales (Λ) of the Gaussian regulator [1]*.*

$g_\nu^{\rm NN}({\rm fm}^2)$	Λ (MeV)
-0.67	450
-1.01	550
-1.44	465
-0.91	465
-1.44	349
-1.03	349

[1] V. Cirigliano *et al.*, PRC 100, 055504 (2019)

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Contact Term in pnQRPA and NSM

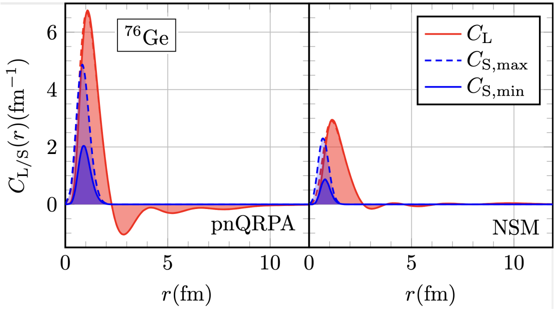
$$\int C_{\mathrm{L/S}}(r)\mathrm{d}r = M^{0\nu}_{\mathrm{L/S}}$$

In pnQRPA:

$M_\mathrm{S}/M_\mathrm{L} \approx 30\% - 80\%$

In NSM:

$M_\mathrm{S}/M_\mathrm{L} \approx 15\% - 50\%$



*LJ, P. Soriano and J. Menéndez, Phys. Lett. B **823**, 136720 (2021)*

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Effective Neutrino Masses

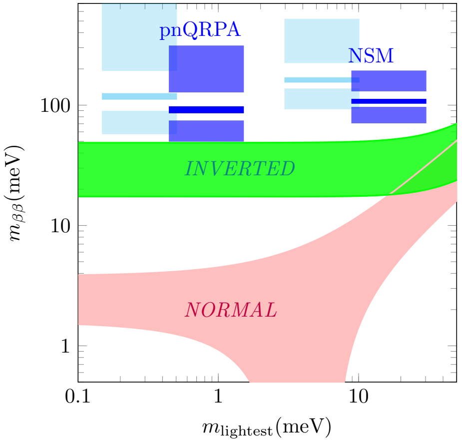
- Effective neutrino masses combining the likelihood functions of GERDA (^{76}Ge), CUORE (^{130}Te), EXO-200 (^{136}Xe) and KamLAND-Zen (^{136}Xe)

  *S. D. Biller, Phys. Rev. D **104**, 012002 (2021)*

- Middle bands: $M_{\mathbf{L}}^{(0\nu)}$

  Lower bands: $M_{\mathbf{L}}^{(0\nu)} + M_{\mathbf{S}}^{(0\nu)}$

  Upper bands: $M_{\mathrm{L}}^{(0\nu)} - M_{\mathrm{S}}^{(0\nu)}$



*LJ, P. Soriano and J. Menéndez, Phys. Lett. B **823**, 136720 (2021)*

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Outline

Introduction

Corrections to $0\nu\beta\beta$-decay nuclear matrix elements

- The contact term at the leading order
- Contribution of ultrasoft neutrinos
- N^{2}LO Corrections

Other Nuclear Observables as Probes of Neutrinoless Double-Beta Decay

Summary and Outlook

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Contribution of ultrasoft neutrinos

- Contribution of ultrasoft neutrinos ($|\mathbf{k}| << k_{\mathbf{F}}$) to $0\nu\beta\beta$ decay:

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018)

$$M^{0\nu}_{\mathrm{usoft}} = \frac{\pi R}{g_{\mathrm{A}}^2} \sum_n \frac{\mathrm{d}^{d-1}k}{(2\pi)^{d-1}} \frac{1}{|\mathbf{k}|} \left[\frac{\langle f| J_\mu |n\rangle \langle n| J^\mu |i\rangle}{|\mathbf{k}| + E_2 + E_n - E_i - i\eta} + \frac{\langle f| J_\mu |n\rangle \langle n| J^\mu |i\rangle}{|\mathbf{k}| + E_1 + E_n - E_i - i\eta} \right]$$

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- Keeping only $\mathbf{k} = \mathbf{0}$ term in the current:

$$\begin{aligned} M^{0\nu}_{\mathrm{usoft}}(\mu_{\mathrm{us}}) = & -\frac{R}{2\pi} \sum_n \langle f| \sum_a \boldsymbol{\sigma}_a \tau_a^+ |n\rangle \langle n| \sum_b \boldsymbol{\sigma}_b \tau_b^+ |i\rangle \\ & \times \left[(E_1 + E_n - E_i) \left(\ln \frac{\mu_{\mathrm{us}}}{2(E_1 + E_n - E_i)} + 1 \right) \right. \\ & \left. + (E_2 + E_n - E_i) \left(\ln \frac{\mu_{\mathrm{us}}}{2(E_2 + E_n - E_i)} + 1 \right) \right] \end{aligned}$$

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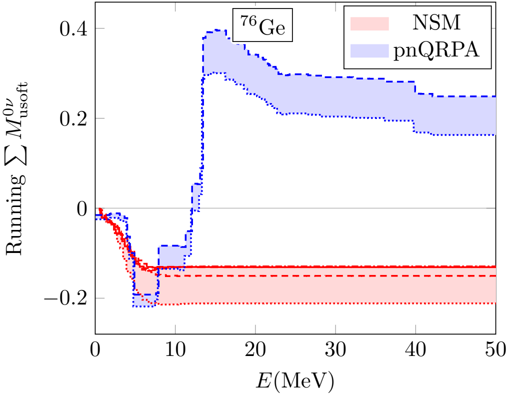
Ultrasoft neutrinos in pnQRPA and nuclear shell model

In pnQRPA:

$|M^{0\nu}_{\rm usoft}/M^{0\nu}_{\rm L}| \leq 15\%$

In NSM:

$|M^{0\nu}_{\rm usoft}/M^{0\nu}_{\rm L}| \leq 5\%$



LJ, D. Castillo, P. Soriano, J Menéndez, work in progress

Discovery, accelerated

Ultrasoft neutrinos as correction of the closure approximation

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} \left| \boldsymbol{M_L^{0\nu}} + M_S^{0\nu} + \boldsymbol{M_{usoft}^{0\nu}} + M_{N2LO}^{0\nu} \right|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

Ultrasoft neutrinos as correction of the closure approximation

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} \left| \boldsymbol{M_{\mathrm{L}}^{0\nu}} + M_{\mathrm{S}}^{0\nu} + \boldsymbol{M_{\mathrm{usoft}}^{0\nu}} + M_{\mathrm{N2LO}}^{0\nu} \right|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

In EFT:

$$M_{\mathrm{L}}^{0\nu} \propto \frac{\langle f | \, J_\mu(\mathbf{x}) J^\mu(\mathbf{y}) \, | i \rangle}{|\mathbf{k}|}$$

TRIUMF

Ultrasoft neutrinos as correction of the closure approximation

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} \left| \boldsymbol{M_{\mathrm{L}}^{0\nu}} + M_{\mathrm{S}}^{0\nu} + \boldsymbol{M_{\mathrm{usoft}}^{0\nu}} + M_{\mathrm{N2LO}}^{0\nu} \right|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

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$\rightarrow M_{\mathrm{cl}}^{0\nu}$ with $< E_n > -\frac{1}{2}(E_i - E_f) = 0$

Discovery, accelerated

Ultrasoft neutrinos as correction of the closure approximation

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |\boldsymbol{M_L^{0\nu}} + M_S^{0\nu} + \boldsymbol{M_{usoft}^{0\nu}} + M_{N2LO}^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

In EFT:

$$M_L^{0\nu} \propto \frac{\langle f|\, J_\mu(\mathbf{x}) J^\mu(\mathbf{y})\, |i\rangle}{|\mathbf{k}|}$$

$\rightarrow M_{cl}^{0\nu}$ with $< E_n > -\frac{1}{2}(E_i - E_f) = 0$

$$M_{usoft}^{0\nu} \propto \sum_n \langle f| \sum_a \boldsymbol{\sigma}_a \tau_a^+ |n\rangle \langle n| \sum_b \boldsymbol{\sigma}_b \tau_b^+ |i\rangle \times f(E_n)$$

Ultrasoft neutrinos as correction of the closure approximation

Nucleus	$M^{0\nu}$	$M^{0\nu}_{\rm cl}$	$M^{0\nu} - M^{0\nu}_{\rm cl}$	$M^{0\nu}_{\rm usoft}$
^{76}Ge	4.83	4.68	0.15	0.25
^{82}Se	4.30	4.20	0.10	0.18
^{96}Zr	4.29	4.04	0.25	0.25
^{100}Mo	3.52	2.71	0.81	0.65
^{116}Cd	4.31	4.47	-0.16	-0.03
^{124}Sn	5.12	4.88	0.24	0.29
^{128}Te	3.99	3.76	0.23	0.27
^{130}Te	3.52	3.36	0.16	0.22
^{136}Xe	2.60	2.71	-0.11	0.06

TRIUMF

Ultrasoft neutrinos as correction of the closure approximation

Nucleus	$M^{0\nu}$	$M^{0\nu}_{\rm cl}$	$M^{0\nu} - M^{0\nu}_{\rm cl}$	$\mathbf{M^{0\nu}(1^+) - M^{0\nu}_{cl}(1^+)}$	$M^{0\nu}_{\rm usoft}$
^{76}Ge	4.83	4.68	0.15	**0.26**	0.25
^{82}Se	4.30	4.20	0.10	**0.18**	0.18
^{96}Zr	4.29	4.04	0.25	**0.26**	0.25
^{100}Mo	3.52	2.71	0.81	**0.75**	0.65
^{116}Cd	4.31	4.47	-0.16	**-0.06**	-0.03
^{124}Sn	5.12	4.88	0.24	**0.31**	0.29
^{128}Te	3.99	3.76	0.23	**0.26**	0.27
^{130}Te	3.52	3.36	0.16	**0.20**	0.22
^{136}Xe	2.60	2.71	-0.11	**0.02**	0.06

Discovery, accelerated

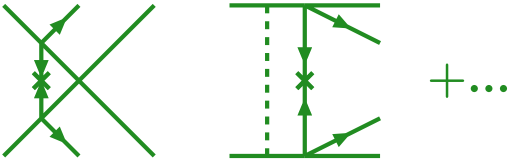
TRIUMF

Outline

Discovery, accelerated

PRELIMINARY N^2LO Corrections in pnQRPA

Nucleus	$M_{\rm L}^{0\nu}$	$M_{\rm N^2LO}^{0\nu}$	$\lvert M_{\rm N^2LO}^{0\nu}/M_{\rm L}^{0\nu}\rvert$
^{76}Ge	4.83	-0.04–0.53	$\lesssim 10\%$
^{136}Xe	2.60	-0.02–0.07	$\lesssim 3\%$

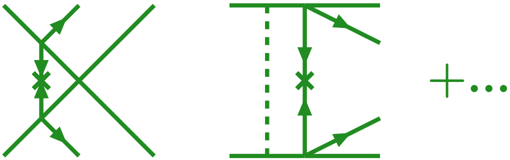


Discovery, accelerated

PRELIMINARY N^2LO Corrections in pnQRPA

Nucleus	$M^{0\nu}_{\mathrm{L}}$	$M^{0\nu}_{\mathrm{N^2LO}}$	$\lvert M^{0\nu}_{\mathrm{N^2LO}}/M^{0\nu}_{\mathrm{L}}\rvert$
^{76}Ge	4.83	-0.04–0.53	$\lesssim 10\%$
^{136}Xe	2.60	-0.02–0.07	$\lesssim 3\%$

Caveats:



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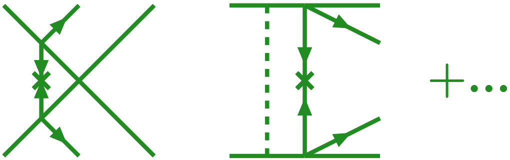

Caveats:

- Unknown parameters

TRIUMF

PRELIMINARY N^2LO Corrections in pnQRPA

Nucleus	$M_{\rm L}^{0\nu}$	$M_{\rm N^2LO}^{0\nu}$	$\lvert M_{\rm N^2LO}^{0\nu}/M_{\rm L}^{0\nu}\rvert$
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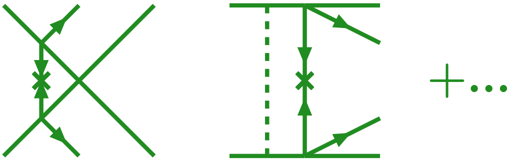

Caveats:

- Unknown parameters
- Scale dependence

Discovery, accelerated

TRIUMF

PRELIMINARY N^2LO Corrections in pnQRPA

Nucleus	$M^{0\nu}_{\mathrm{L}}$	$M^{0\nu}_{\mathrm{N^2LO}}$	$\lvert M^{0\nu}_{\mathrm{N^2LO}}/M^{0\nu}_{\mathrm{L}}\rvert$
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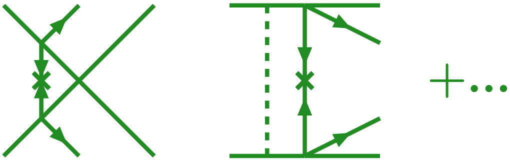


Caveats:

- Unknown parameters
- Scale dependence
- Regulator dependence

Discovery, accelerated

Outline

TRIUMF

$M^{0\nu}$ Correlated with $M_{\rm DGT}$ - or Is It?

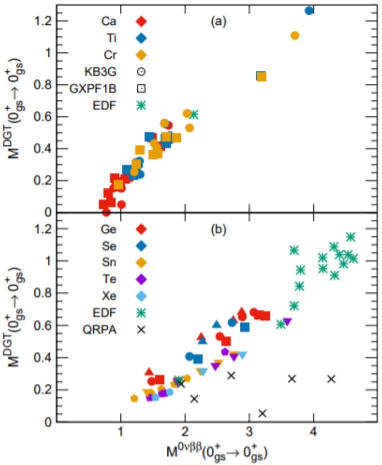
$$M_{\rm DGT} = (0^+_{\rm gs,f}||\sum_{j,k}[\boldsymbol{\sigma}_j\tau^-_j \times \boldsymbol{\sigma}_k\tau^-_k]^0||0^+_{\rm gs,i})$$

- Linear correlation between double Gamow-Teller (DGT) and $0\nu\beta\beta$ in NSM, EDF

  N. Shimizu, J. Menéndez and K. Yako, Phys. Rev. Lett. **120**, 142502 (2018),

  and IBM-2

  F. F. Deppisch *et al.*, Phys. Rev. D **102**, 095016 (2020), J. Barea *et al.*, Phys. Rev. C **91**, 034304 (2015)



N. Shimizu, J. Menéndez and K. Yako, *Phys. Rev. Lett.* ***120***, *142502 (2018)*

Discovery, accelerated

TRIUMF

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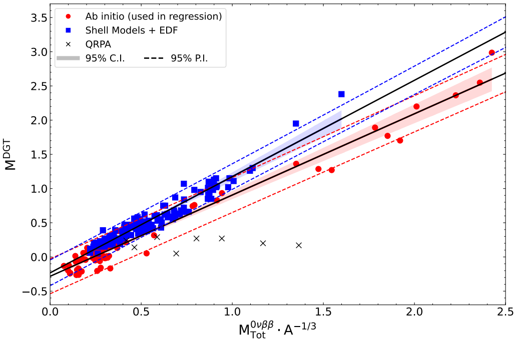
- Correlation can also be found in *ab initio* frameworks

  J. M. Yao *et al.*, Phys. Rev. C **106**, 014315 (2022)



J. M. Yao et al.,
Phys. Rev. C ***106****, 014315 (2022)*

Disco accelerated

TRIUMF

$M^{0\nu}$ Correlated with M_{DGT} - or Is It?

$$M_{\mathrm{DGT}} = (0^+_{\mathrm{gs,f}}||\sum_{j,k}[\boldsymbol{\sigma}_j\tau_j^- \times \boldsymbol{\sigma}_k\tau_k^-]^0||0^+_{\mathrm{gs,i}})$$

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- Correlation can also be found in *ab initio* frameworks

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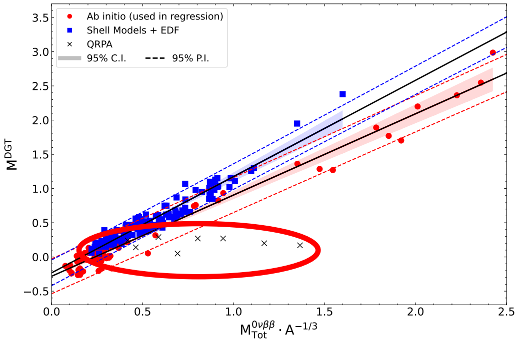
- **But not in QRPA (Why?)**



J. M. Yao et al., Phys. Rev. C ***106****, 014315 (2022)*

Discovery, accelerated

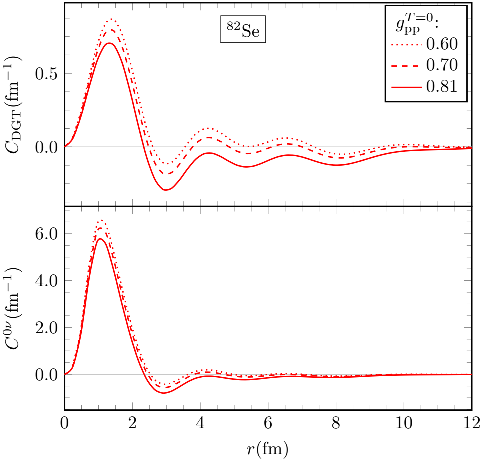
TRIUMF

Radial Densities of $M^{0\nu}$ and $M_{\rm DGT}$ in pnQRPA

$$M_{\rm L}^{0\nu} = \int_0^\infty C^{0\nu}(r)\mathrm{d}r \ ,$$

$$M_{\rm DGT} = \int_0^\infty C_{\rm DGT}(r)\mathrm{d}r$$

- $M_{\rm DGT}$ more sensitive to proton-neutron pairing ($g_{\rm pp}$) than $M^{0\nu}$



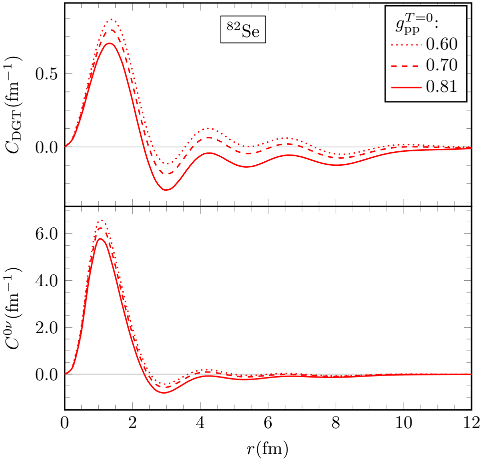
LJ, J. Menéndez, Phys. Rev. C ***107****, 044316 (2023)*

Discovery, accelerated

Radial Densities of $M^{0\nu}$ and M_{DGT} in pnQRPA

$$M_{\mathrm{L}}^{0\nu} = \int_0^{\infty} C^{0\nu}(r)\mathrm{d}r\ ,$$
$$M_{\mathrm{DGT}} = \int_0^{\infty} C_{\mathrm{DGT}}(r)\mathrm{d}r$$

- M_{DGT} more sensitive to proton-neutron pairing (g_{pp}) than $M^{0\nu}$
- Decreasing g_{pp} makes DGT more short-ranged (like $0\nu\beta\beta$ decay)



*LJ, J. Menéndez, Phys. Rev. C **107**, 044316 (2023)*

Radial Densities of $M^{0\nu}$ and M_{DGT} in pnQRPA

$$M^{0\nu}_{\mathrm{L}} = \int_0^\infty C^{0\nu}(r)\mathrm{d}r \; ,$$

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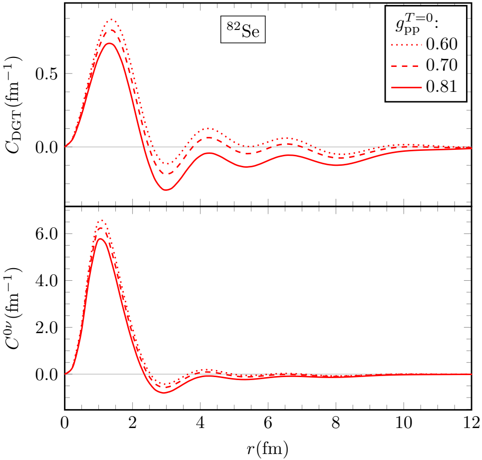
- Typically g_{pp} adjusted to $2\nu\beta\beta$ half-lives $\rightarrow$ Large g_{pp} values



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Discovery, accelerated

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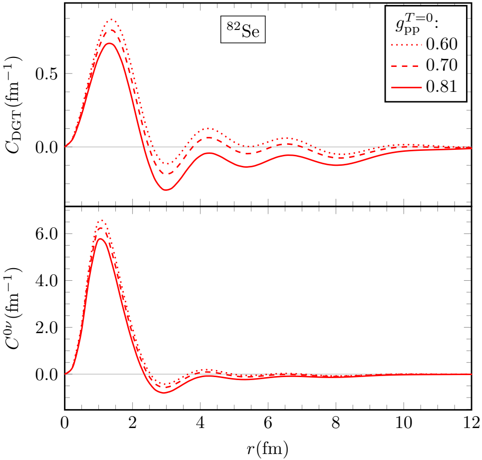
- Typically g_{pp} adjusted to $2\nu\beta\beta$ half-lives $\rightarrow$ Large g_{pp} values
- **What if we free the value of g_{pp}?**



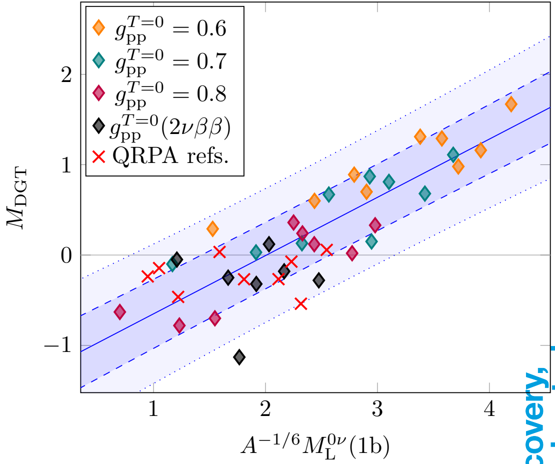
LJ, J. Menéndez, Phys. Rev. C ***107****, 044316 (2023)*

Discovery, accelerated

TRIUMF

$M^{0\nu}$ vs. M_{DGT} in pnQRPA

- By varying $g_{\mathrm{pp}}^{T=0}$ we observe a **correlation in QRPA**



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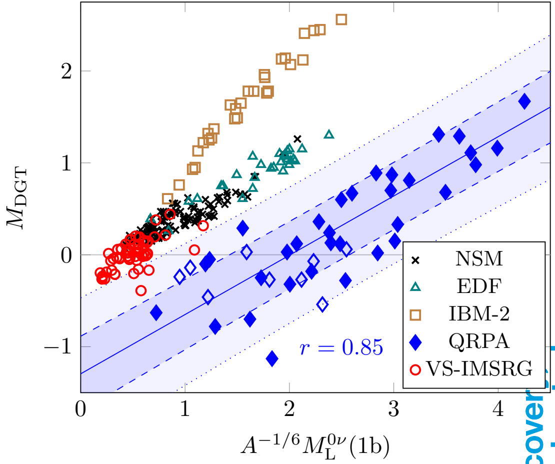
Discovery, accelerated

$M^{0\nu}$ vs. M_{DGT} in pnQRPA

- By varying $g_{\text{pp}}^{T=0}$ we observe a **correlation in QRPA**
- Correlation different from other models



LJ and J. Menéndez, Phys. Rev. C ***107****, 044316 (2023)*

$M^{0\nu}$ vs. $M_{\rm DGT}$ in pnQRPA

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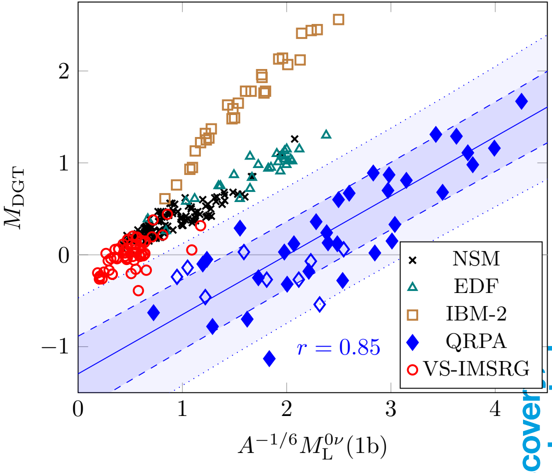
- Correlation different from other models
  - Maybe not surprising, given the dispersion of $M^{0\nu}$'s...



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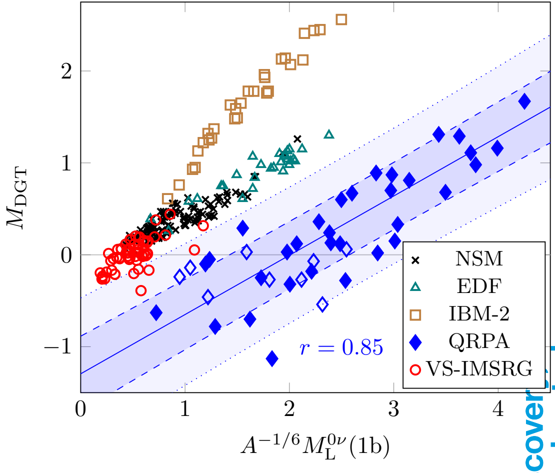
  - ...and different approaches (closure/non-closure,...)



LJ and J. Menéndez, Phys. Rev. C ***107****, 044316 (2023)*

Discovery, accelerated

TRIUMF

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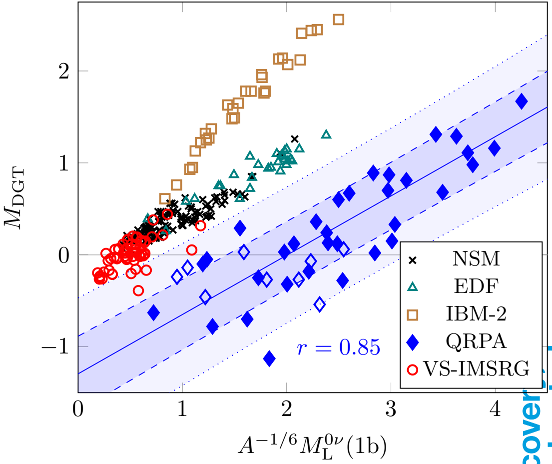
- Correlation different from other models
  - Maybe not surprising, given the dispersion of $M^{0\nu}$'s...
  - ...and different approaches (closure/non-closure,...)
- **Measuring DGT reaction could help constrain $M^{0\nu}$!**

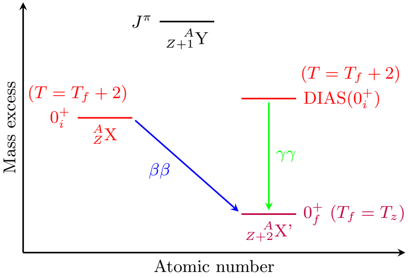


LJ and J. Menéndez, Phys. Rev. C **107**, 044316 (2023)

Discovery, accelerated

Probing $0\nu\beta\beta$ Decay by Gamma Decays

- Double magnetic dipole (M1) decay (**electromagnetic interaction**) can be related to $0\nu\beta\beta$ decay (**weak interaction**)



$$M^{\gamma\gamma}(M1M1) = \sum_n \frac{(0_\mathrm{f}^+||\mathbf{M_1}||1_n^+)(1_n^+||\mathbf{M_1}||0_\mathrm{i}^+)}{E_n - (E_i + E_f)/2},$$

$$\mathbf{M_1} = \mu_N \sqrt{\frac{3}{4\pi}} \sum_{i=1}^{A} (g_i^l \boldsymbol{\ell}_i + g_i^s \mathbf{s}_i)$$

Probing $0\nu\beta\beta$ Decay by Gamma Decays

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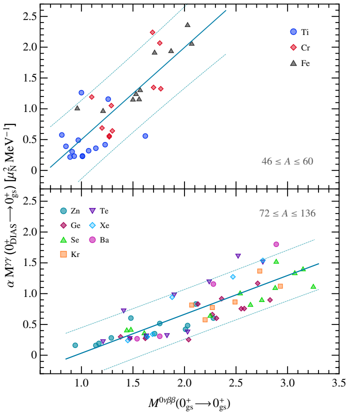
- Correlation between these processes observed in NSM

B. Romeo, J. Menéndez, C. Peña-Garay, Phys. Lett. B **827**, 136965 (2022)



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Discovery, accelerated

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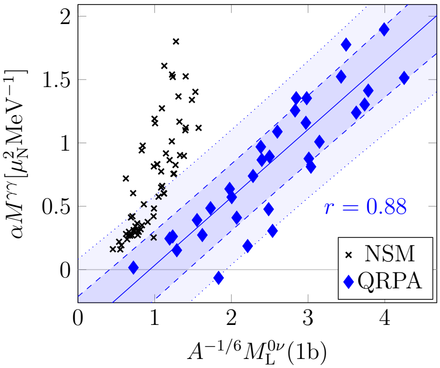
- Correlation also found in QRPA



LJ, J. Menéndez, Phys. Rev. C ***107****, 044316 (2023)*

TRIUMF Discovery, accelerated

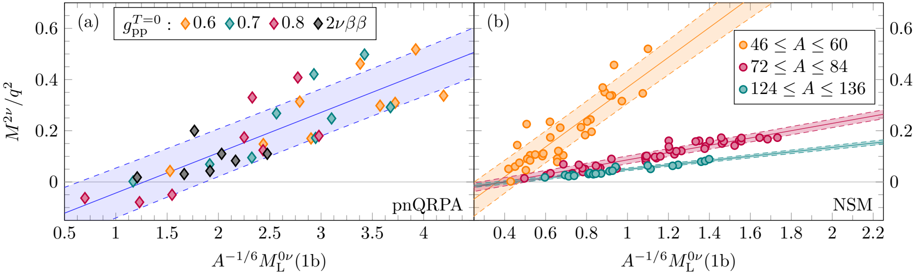
Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

- *How about $2\nu\beta\beta$ decay?*

TRIUMF

Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

- ***How about $2\nu\beta\beta$ decay?***
- $2\nu\beta\beta$-decay also correlated with $0\nu\beta\beta$-decay!



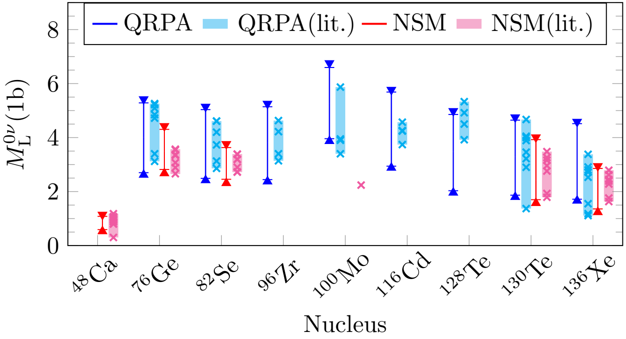
LJ, B. Romeo, P. Soriano and J. Menéndez, Phys. Rev. C ***107****, 044305 (2023)*

Discovery, accelerated

TRIUMF

Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

- ***How about $2\nu\beta\beta$ decay?***
- $2\nu\beta\beta$-decay also correlated with $0\nu\beta\beta$-decay!
- We can use the existing data to estimate $0\nu\beta\beta$-decay NMEs!



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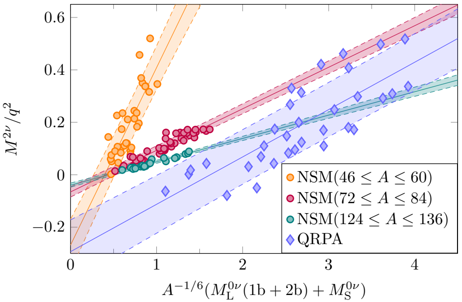
Discovery, accelerated

Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

Two-Body Currents & Contact Term

- Correlations survive when adding the 2BCs and the contact term



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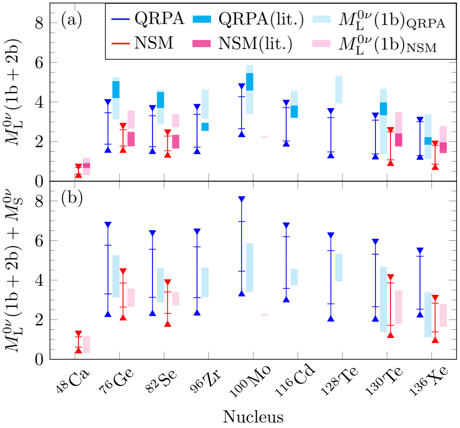
J. Menéndez, D. Gazit, A. Schwenk, Phys. Rev. Lett. 107, 062501 (2011)

J. Engel, F. Šimkovic, P. Vogel, Phys. Rev. C 89, 064308 (2014)



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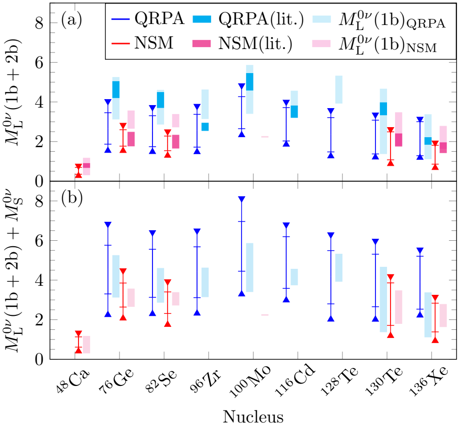
J. Engel, F. Šimkovic, P. Vogel, Phys. Rev. C 89, 064308 (2014)

- 2BCs and the contact term largely cancel each other



*LJ, B. Romeo, P. Soriano and J. Menéndez, Phys. Rev. C **107**, 044305 (2023)*

Discovery, accelerated

Outline

TRIUMF

Summary

- Newly introduced contact term significantly enhances the $0\nu\beta\beta$-decay NMEs
- Studying the contribution from ultrasoft neutrinos may help us estimate the closure correction to the $0\nu\beta\beta$-decay NMEs
- Correction from additional N^{2}LO corrections $\lesssim 5\%$
- Correlations between $0\nu\beta\beta$ decay and DGT, M1M1 and $2\nu\beta\beta$ decays may help constrain the $0\nu\beta\beta$-decay NMEs

Discovery, accelerated

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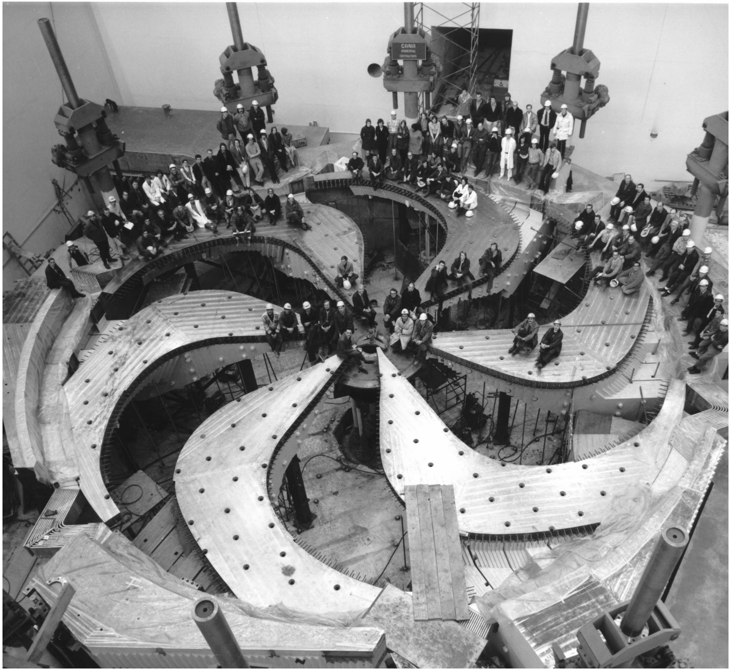
Thank you

Merci



Discovery, accelerated

Axial-Vector Two-Body Currents (2BCs)

- One-body (1b) axial-vector currents given by

$$\mathbf{J}^3_{i,1\mathrm{b}} = \frac{\tau^3_i}{2}\left(g_\mathrm{A}\boldsymbol{\sigma}_i - \frac{g_\mathrm{P}}{2m_\mathrm{N}}\mathbf{q}\cdot\boldsymbol{\sigma}_i\right) ,$$

  where $g_\mathrm{P} = (2m_\mathrm{N}q/(q^2+m_\pi^2))g_\mathrm{A}$

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- Additional **pion-exchange, pion-pole, and contact** two-body (2b) currents

Hoferichter, Klos, Schwenk *Phys. Lett. B* **746**, 410 (2015)

$$\mathbf{J}^3_{12} = -\frac{g_\mathrm{A}}{2F_\pi^2}[\tau_1\times\tau_2]^3\left[c_4\left(1-\frac{\mathbf{q}}{\mathbf{q}^2+M_\pi}\mathbf{q}\cdot\right)(\boldsymbol{\sigma}_1\times\mathbf{k}_2)+\frac{c_6}{4}(\boldsymbol{\sigma}_1\times\mathbf{q})+i\frac{\mathbf{p}_1+\mathbf{p}_1'}{4m_\mathrm{N}}\right]\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{M_\pi^2+k_2^2}$$
$$-\frac{g_\mathrm{A}}{F_\pi^2}\tau_2^3\left[c_3\left(1-\frac{\mathbf{q}}{\mathbf{q}^2+M_\pi}\mathbf{q}\cdot\right)\mathbf{k}_2+2c_1M_\pi^2\frac{\mathbf{q}}{\mathbf{q}^2+M_\pi^2}\right]\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{M_\pi^2+k_2^2}$$
$$-d_1\tau_1^3\left(1-\frac{\mathbf{q}}{\mathbf{q}^2+M_\pi^2}\mathbf{q}\cdot\right)\boldsymbol{\sigma}_1+(1\leftrightarrow 2)-d_2(\tau_1\times\tau_2)^3(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\left(1-\cdot\mathbf{q}\frac{\mathbf{q}}{\mathbf{q}^2+M_\pi^2}\right)$$

where $\mathbf{k}_i = \mathbf{p}_i' - \mathbf{p}_i$ and $\mathbf{q} = -\mathbf{k}_1 - \mathbf{k}_2$

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Axial-Vector Two-Body Currents (2BCs)

- Approximate 2BCs by normal-ordering w.r.t. spin-isospin–symmetric reference state with $\rho = 2k_{\mathrm{F}}^3/(3\pi^2)$:

Hoferichter, Menéndez, Schwenk, *Phys. Rev. D* **102**,074018 (2020)

$$\mathbf{J}_{i,2\mathrm{b}}^{\mathrm{eff}} = \sum_j (1 - P_{ij}) \mathbf{J}_{ij}^3$$

$$\rightarrow \boxed{\mathbf{J}_{i,2\mathrm{b}}^{\mathrm{eff}} = g_{\mathrm{A}} \frac{\tau_i^3}{2} \left[\delta a(\mathbf{q}^2) \boldsymbol{\sigma}_i + \frac{\delta a^P(\mathbf{q}^2)}{\mathbf{q}^2} (\mathbf{q} \cdot \boldsymbol{\sigma}_i) \mathbf{q} \right]},$$

where

$$\delta_a(\mathbf{q}^2) = -\frac{\rho}{F_\pi^2} \left[\frac{c_4}{3} [3I_2^\sigma(\rho, \mathbf{q}) - I_1^\sigma(\rho, |\mathbf{q}|)] - \frac{1}{3} \left(c_3 - \frac{1}{4m_{\mathrm{N}}} \right) I_1^\sigma(\rho, |\mathbf{q}|) - \frac{c_6}{12} I_{c6}(\rho, |\mathbf{q}|) - \frac{c_D}{4g_A \Lambda_\chi} \right],$$

$$\delta_a^P(\mathbf{q}^2) = \frac{\rho}{F_\pi^2} \left[-2(c_3 - 2c_1) \frac{m_\pi^2 \mathbf{q}^2}{(m_\pi^2 + \mathbf{q}^2)^2} + \frac{1}{3} \left(c_3 + c_4 - \frac{1}{4m_{\mathrm{N}}} \right) I^P(\rho, |\mathbf{q}|) - \left(\frac{c_6}{12} - \frac{2}{3} \frac{c_1 m_\pi^2}{m_\pi^2 + \mathbf{q}^2} \right) I_{c6}(\rho, |\mathbf{q}|) \right.$$
$$\left. - \frac{\mathbf{q}^2}{m_\pi^2 + \mathbf{q}^2} \left(\frac{c_3}{3} [I_1^\sigma(\rho, |\mathbf{q}|) + I^P(\rho, |\mathbf{q}|)] + \frac{c_4}{3} [I_1^\sigma(\rho, |\mathbf{q}|) + I^P(\rho, |\mathbf{q}|) - 3I_2^\sigma(\rho, |\mathbf{q}|)] \right) - \frac{c_D}{4g_A \Lambda_\chi} \frac{\mathbf{q}^2}{m_\pi^2 + \mathbf{q}^2} \right]$$

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