# EM $(\gamma)$ transitions from and to IAS and axialvector and vector NMEs associated with DBDs 

Hiro Ejiri
RCNP Osaka


Thanks the organizers for invitation to NME-23

## Subjects to be discussed:

I. Experimental DBD rates and DBD NMEs
II. Charge exchange nuclear reactions and axial-vector NMEs associated with DBD NMEs
III. Electro-magnetic (EM $\gamma$ ) NMEs for IAS

M1 /E1 EM- $\gamma$ NMEs for axial-vector /vector DBD NMEs
IV. Photo nuclear reactions to IASs to study DBD NMEs

## I. Exp. rate for neutrino-less $\beta \beta$ decays



## $\mathrm{M}=\mathrm{M}^{0 \mathrm{v}} \mathrm{K}_{\mathrm{vR}}$ are keys for new physics in DBD

$$
1 / \mathrm{t}_{1 / 2}=\mathrm{T}^{0 \mathrm{v}} / \ln 2=\mathrm{G}^{0 \mathrm{v}} \mathrm{~B}(\mathrm{NP})
$$

$1 /\left(\mathrm{t}_{1 / 2} \mathrm{G}^{0 \mathrm{v}}\right)=\mathrm{B}(\mathrm{NP}) \quad \mathrm{B}(\mathbf{N P})=|\mathbf{M}|^{2} \quad \mathrm{M}=\mathrm{M}^{0 \mathrm{v}} \mathbf{K}_{\mathrm{vR}}$
(Corresponds to $1 / \mathrm{ft}=\mathbf{B}(\mathbf{G T} / \mathrm{F})$ derived experimentally) $\mathbf{G}^{0 v}=$ include $\left(\mathrm{g}_{\mathrm{A}}=1.27\right)^{4}, \mathrm{M}^{0 \mathrm{v}}$ includes $\mathrm{g}_{\mathrm{A}}$ eff and $\mathrm{g}_{\mathrm{v}}{ }^{\text {eff }}$.
$K_{v R}=\left[\left\langle m_{v}>^{2}+C_{\lambda}\langle\lambda\rangle^{2}+C_{\eta}\langle\eta\rangle^{2}+\right.\right.$ interference terms $]$
$\langle\mathrm{m}\rangle=\left|\Sigma \mathrm{m}_{\mathrm{j}} \mathrm{U}_{\mathrm{ej}}\right| \quad\langle\lambda\rangle=\left(\mathrm{M}_{\mathrm{L}} / \mathrm{M}_{\mathrm{R}}\right)^{2}\left|\Sigma \mathrm{U}_{\mathrm{ej}} \mathrm{V}_{\mathrm{ej}}\right| \quad\langle\eta\rangle=\tan \theta_{\mathrm{LR}}\left|\Sigma \mathrm{U}_{\mathrm{ej}} \mathrm{V}_{\mathrm{ej}}\right|$
$C_{\lambda}$ and $C_{\eta}$ are phase spaces and NMEs in units of those for $m_{v}$

$$
1 / t_{1 / 2}=T^{0 v} / \ln 2 \text { gives product of } M^{0 v} \text { and } v \text {-mass } / \lambda / \eta \text {. }
$$

## Current limits

Current limits (GERDA PRL 125 252502, ,KamLAND PRL 130 051801)


DBD EXPs : $\mathrm{M}^{0 \mathrm{v}}=5.2-0.023 \mathrm{~A}=2 \sim 3.5$ ((Ejiri Jokiniemi,
Suhonen PR C Lett. 2023) smooth function of A. may reach IH mass, by a factor $\sim 3$ in $v$-mass and $>10^{2}$ in $\mathrm{N}^{\mathrm{F}} / /$

## $\left(\mathrm{t}_{1 / 2}\right)^{-1}=\mathrm{G}_{01} \mathrm{M}_{\mathrm{m}}{ }^{2} \mathrm{~m}_{v}{ }^{2}+\mathrm{G}_{02} \mathrm{M}_{\lambda}{ }_{\lambda} \lambda^{2}+\mathrm{G}_{09} \mathrm{M}_{\eta}{ }^{2} \eta^{2}+$ Cross terms

Phase space $\mathrm{G}_{01}: \mathrm{G}_{02}: \mathrm{G}_{09}=1: 5: 0.510^{5}$ (recoil term)
$\mathrm{M}_{\mathrm{m}}=\mathrm{M}(\mathrm{GT})(1-\mathrm{F}+\mathrm{T}) \mathrm{M}$
$\mathbf{M}_{\lambda}=$ Effective NME~ M(GT)
$\mathrm{M}_{\mathrm{n}}=$ Effective NME~ M(GT)
$\mathrm{M}_{\lambda} \sim \mathrm{M}_{\mathrm{h}} \sim \mathrm{M}_{\mathrm{m}}$ in case of QRPA

Thus the $v$-mass sensitivity in units of $\mathrm{m}_{\mathrm{e}}$ corresponds to the $\lambda$ and $\eta$ sensitivities smaller by the factors, SQRT of
$\mathrm{G}_{02}$ and $\mathrm{G}_{09}=2$ and 200.


Muto et al., Z. Phys. A 334187

## Single and double CERs at RCNP



DBD $\mathbf{M}_{1}, \mathbf{M}_{2}$ via neutrino potential by single $\beta, \nu, \mu$. CER NMEs

Nuclear DCER

${ }^{A} \mathrm{Z} X+{ }^{11} \mathrm{~B} \rightarrow{ }_{\mathrm{A}+2} \mathrm{X}+{ }^{11} \mathrm{Li}$

$$
A_{Z} X \rightarrow A_{Z+2} X+2 e
$$

$\mathrm{M}\left(\alpha, \beta^{ \pm}\right)=\left(\mathrm{g}_{\mathrm{A}}{ }^{\mathrm{eff}}\right)^{ \pm} \mathrm{M}\left(\right.$ QRPA $\left.\alpha \beta^{ \pm}\right) \quad \alpha=\mathrm{GT}, \mathrm{SD} . \mathrm{SQ}, \cdot \cdot$
CERs: H. Ejiri, Universe 6, 225 (2020); Frontiers in Physics 9, 650421 (1921).


## Double Charge Exchange Reaction H. Ejiri Universe 2022, 8, 457, Takahisa

RCNP ${ }^{56} \mathrm{Fe}\left({ }^{11} \mathrm{~B},{ }^{11} \mathrm{Li}\right){ }^{56} \mathrm{Ni}$ at $\mathrm{E}=0.88 \mathrm{GeV}$.

1. $\left(\mathbf{V}_{\tau \sigma} / \mathbf{V}_{\tau}\right)^{2} \sim 3.4$ enhance $\tau \sigma$ GT SD excitation
2. Q value $=-50 \mathrm{MeV}$. Thus p-transfer $100 \mathrm{MeV} / \mathrm{c}$ is same as DBD , and $\mathrm{L}=1$ enhances SD

${ }^{56} \mathrm{Fe}\left({ }^{11} \mathrm{~B},{ }^{11} \mathrm{Li}\right){ }^{56} \mathrm{Ni}$

GT SD NMEs derived by referirng to IAS DIAS cross sections with known B(F) SCER ${ }^{76} \mathrm{Ge}\left({ }^{3} \mathrm{He}, \mathrm{t}\right){ }^{76} \mathrm{As}$ excites SD states with strength of 0.1 of QP strength. Then we get the quenching of sqrt of $0.1=k_{\tau \sigma} \sim 0.3$ with respect to QP. DCER ${ }^{56} \mathrm{Fe}\left({ }^{11} \mathrm{~B},{ }^{11} \mathrm{Li}\right){ }^{56} \mathrm{Ni}$ excites very little low-QP GT-SD states with strength of 0.01 of QP strength. Then the quenching of sqrt of $0.01=\left(\mathrm{k}_{\tau \sigma}\right)^{2} \sim 0.1$

## Limits of nuclear CERs

1. Nuclear interactions:

Strong interactions, large cross sections, easy experiments. Complex/multi interaction operators, hard analyses of the data. Absolute NMEs are uncertain because of the absorptions and distortion, and complex interaction strength.
2. Limited to relative $\tau \sigma$ (GT) NME. $\mathrm{E}=200 \mathrm{MeV}$ 0. deg. q~0
3. Tensor operator

Mainly axial vector $\mathbf{T}(\mathrm{J}=\mathrm{i}+)=\tau \sigma+\delta \tau\left[\sigma \mathbf{x} \mathbf{Y}_{2}\right]_{1}$

No vector NMEs


The Nucleon-Nucleon Interaction and Nucleon-Nucleus Scattering WG Love, MA Franey, F Petrovich - Spin Excitations in Nuclei, 1984 - Springer

## EM-IAS transition schemes and DBD Axial-vector NMEs



IAS- $\gamma$ H. Ejiri, Phys. Rev. C. Lett, 108 L011302 (2023). $\gamma$-IAS H. Ejiri, A. Titov, Phys. Rev. C 88054010 (2013.)

## EM-IAS studies : 4 merits

1. EM couplings of e and $\mu$ and the interaction operators are well defined compared with nuclear ones in nuclear CERs.
No distortion effect. No tensor interaction. No multipole mixing.
2. IAS is a sharp strong $\tau$-GR with small non-resonant effects.
 Then $\gamma$-branch versus n -decay gets as large as $10^{-4}$ or so.
3. BG free exp. is possible by $\gamma$-detection in coincidence with CER to IAS.
4. Experiments are feasible by using a $\gamma$-detector array and a high energy-resolution spectrometer,

DIAS $\gamma$ Romeo, Menendez Pena PL B 8271369682022.

## Comparison of NMEs pnQRPA, ISM, IBM2

$$
M^{0 \nu}=\left(\frac{g_{\mathrm{A}}^{\text {eff }}}{g_{\mathrm{A}}}\right)^{2}\left[M_{\mathrm{GT}}^{0 \nu}+\left(g_{\mathrm{V}} / g_{\mathrm{A}}^{\text {eff }}\right)^{2} M_{\mathrm{F}}^{0 \nu}+M_{\mathrm{T}}^{0 \nu}\right]
$$

QRPA P.R. C98 0246082018 Jokiniemi, Ejiri Suhonen
ISM M. Horoi S. Stoica PR C 810243022010 .
IBM J. Barea, . Kotila F. Iachello PR C 870143152013


Axial-vector \& vector NMEs depend much on the models used. QRPA Large and smooth as A ISM Small and smooth as A.
IBM Fluctuate much as A
Axial vector NMEs depend on models. Thus require model-dependent $\mathrm{g}_{\mathrm{A}}$ eff to incorporate the effects beyond models.



QRPA Large and smooth as A ISM Small and smooth as A. IBM Fluctuate very much as A
Vector NMEs do depend on models. Thus require model-dependent $\mathrm{g}_{\mathrm{v}}$ eff to incorporate the effects beyond models. CVC $0^{+-} 0^{+}$IAS : constant, but not for non-IAS states.

In case of M1 $\gamma$ from IAS of ${ }^{100} \mathrm{Mo}$.

$\beta$ NMEs with $\alpha=$ GT, V1 are given by $\gamma$ NMEs with $\alpha^{\prime}=$ M1,E1 $\gamma$ from IAS (Ejiri et al, PRL 1968) as $\quad M^{-}(\alpha) \approx \sqrt{2 T} M^{1 \mathrm{~A}}\left(\alpha^{\prime}\right)$,

IAS- $\gamma$ cross section (1000-100 nb) is given by product of IAS cross section ( 10 mb ) and $\gamma$ branching ratio ( $10^{-(4-5)}$ ) as

$$
\frac{d \sigma^{\mathrm{IA}}\left(\alpha^{\prime}\right)}{d \Omega}=\frac{d \sigma^{\mathrm{IA}}}{d \Omega} \frac{\Gamma^{\mathrm{IA}}\left(\alpha^{\prime}\right)}{\Gamma(T)},
$$

$\gamma$-width $(1-0.1 \mathrm{eV})$ is given by product of $\left(\mathrm{E}_{\gamma}\right)^{3}=(10 \mathrm{MeV})^{3} \sim 1000$ and the reduced width $\mathrm{B}^{\text {IA }}$

$$
\Gamma^{\mathrm{IA}}\left(\alpha^{\prime}\right)=K_{\alpha^{\prime}} E_{\alpha^{\prime}}^{3} B^{\mathrm{IA}}\left(\alpha^{\prime}\right),
$$

The reduced width is given by the square of the IAS $\gamma$ NME

$$
B^{\mathrm{IA}}\left(\alpha^{\prime}\right)=g_{\alpha^{\prime}}^{2}\left|M^{\mathrm{IA}}\left(\alpha^{\prime}\right)\right|^{2} S^{-1},
$$

$\mathbf{g}_{\mathrm{e}}{ }^{\text {eff }}$ derived from $\operatorname{Exp} \mathbf{M}(\mathbf{E} 1) /$ QRPA M (E1) is used to get $\mathrm{g}_{\mathrm{v}}{ }^{\text {eff }}$ $\mathrm{g}_{\mathrm{m}}{ }^{\text {eff }}$ derived from $\operatorname{Exp} \mathbf{M}(\mathrm{M} 1) /$ QRPA M(M1) is used to get $\mathrm{g}_{\mathrm{A}}{ }^{\text {eff }}$

## IAS total widths are known experimentally as

 given below.

# Dark blue ; CER Exp. Present work 

Light blue H.L.Harney et al, RMP 58 6071986

It is given as a function of the isospin $z$ component as

$$
\Gamma(T) \approx 3.5 T_{z} \mathrm{keV}, \quad T_{z}=(N-Z) / 2
$$

It is a factor 30 smaller than a typical n-decay width because of the isospin forbidden for n decay.

## Evaluated IAS $\gamma$ widths and the cross-sections

TABLE I. M1 $\gamma$ widths and the IAS $\gamma$ cross sections for DBD nuclei and ${ }^{71} \mathrm{Ga}$ for the solar $\nu \mathrm{s}$. Shown are $E$ (IA) in units of $\mathrm{MeV}, E(\mathrm{GT})$ in units of $\mathrm{MeV}, B(\mathrm{GT}), B^{\mathrm{IA}}(\mathrm{M} 1)$ in units of $10^{-2}, \quad \Gamma^{\mathrm{IA}}(\mathrm{M} 1)$ in units of $10^{-2} \mathrm{eV}$, and $\sigma^{\mathrm{IA}}(\mathrm{M} 1)=\mathrm{d} \sigma^{\mathrm{IA}}(\mathrm{M} 1) / d \Omega$ in units of $\mathrm{nb}\left(10^{-9} \mathrm{~b}\right) /$ str.

| A | $E(\mathrm{IA})$ | $E(\mathrm{GT})$ | $B(\mathrm{GT})$ | $B^{\text {IA }}(\mathrm{M} 1)$ | $\Gamma^{\text {IA }}(\mathrm{M} 1)$ | $\sigma^{\text {IA }}(\mathrm{M} 1)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{76} \mathrm{Ge}$ | 8.31 | 1.07 | 0.14 | 1.45 | 6.4 | 41 |
| ${ }^{82} \mathrm{Se}$ | 9.58 | 0.075 | 0.34 | 3.0 | 30.0 | 150 |
| ${ }^{96} \mathrm{Zr}$ | 10.9 | 0.69 | 0.16 | 1.25 | 15.3 | 76 |
| ${ }^{100} \mathrm{Mo}$ | 11.1 | 0 | 0.35 | 2.7 | 43.4 | 170 |
| ${ }^{116} \mathrm{Cd}$ | 12.1 | 0 | 0.14 | 0.88 | 18.0 | 51 |
| ${ }^{128} \mathrm{Te}$ | 12.0 | 0 | 0.079 | 0.41 | 8.2 | 17 |
| ${ }^{130} \mathrm{Te}$ | 12.7 | 0 | 0.072 | 0.35 | 8.2 | 17 |
| ${ }^{136} \mathrm{Xe}$ | 13.4 | 0.59 | 0.23 | 1.03 | 25 | 45 |
| ${ }^{150} \mathrm{Nd}$ | 14.4 | 0.11 | 0.13 | 0.54 | 18.0 | 35 |
| ${ }^{71} \mathrm{Ga}$ | 8.91 | 0 | 0.085 | 1.2 | 9.8 | 51 |
| A | $E(\mathrm{IAS}) E(\mathrm{~V} 1)$ |  | 1) $B(\mathrm{~V} 1)$ |  | 1) $\Gamma(\mathrm{E} 1)$ | $\sigma^{1 \mathrm{~A}}(\mathrm{E} 1)$ |
| ${ }^{96} \mathrm{Zr}$ | 10.9 | 9 | 6. | 43 | 220 | 1080 |
| ${ }^{100} \mathrm{Mo}$ | 11.1 | 13 | 7. | 47 | 260 | 1020 |
| ${ }^{130} \mathrm{Te}$ | 12.7 | 73 | 1. | O 3.8 | 36 | 75 |

The M1 and E1 widths are 10-50 $10^{-2} \mathrm{eV}$ and $30-300 \quad 10^{-2} \mathrm{eV}$ The M1 and E1 cross sections are 50-200 nb and 100-1000 nb.

Technical Information on the Scintillation Gamma-Ray Detector Array Coupled with the Grand Raiden Spectrometer
A. Tamii ${ }^{\text {a }}$, N. Kobayashi ${ }^{\text {a }}$



## Photo nuclear reaction

Isovector component by IAS isospin T from Ground state with T-1



Ejiri H, et al. Phys. Rev. Lett. 21191968373
Ejiri H, Titov. A, et al., Phys. Rev, C 882013054610

$$
\begin{equation*}
\sigma(\gamma, n)=\frac{S(2 J+1) \pi}{k_{\gamma}^{2}} \frac{\Gamma_{\gamma} \Gamma_{n}}{\left(E-E_{R}\right)^{2}+\Gamma_{t}^{2} / 4}, \tag{10}
\end{equation*}
$$

where $\Gamma_{\gamma}, \Gamma_{t}$, and $\Gamma_{n}$ are the $\gamma$ capture width, the total width, and the neutron decay width, $S$ is the spin factor, and $k_{\gamma}$ is the incident photon momentum.

The integrated photonuclear cross section is given by

$$
\begin{equation*}
\int \sigma(\gamma, n) d E=\frac{S(2 J+1) 2 \pi^{2}}{k_{\gamma}^{2}} \frac{\Gamma_{\gamma} \Gamma_{n}}{\Gamma_{t}} \tag{11}
\end{equation*}
$$

$\Gamma_{t} \approx \Gamma_{n}$, where $\Gamma_{n}$ is the sum of the neutron decay widths

$$
\int \sigma(\gamma, n) d E=\pi^{2} k_{\gamma}^{-2} \Gamma_{\gamma}
$$

$$
\begin{align*}
& \int \sigma(\gamma, n) d E=2.9 \times 10^{-3} \mathrm{MeV} \mathrm{fm}^{2} \quad(E 1)  \tag{14}\\
& \int \sigma(\gamma, n) d E=2.7 \times 10^{-3} \mathrm{MeV} \mathrm{fm}^{2} \tag{15}
\end{align*}
$$

Then the counting rates with a typical target of $10 \mathrm{~g} / \mathrm{cm}^{2}$ are $Y(E 1)=1.7 \times 10^{-6} \epsilon N_{\gamma} / \mathrm{s}$ and $Y(M 1)=1.6 \times 10^{-6} \epsilon N_{\gamma} / \mathrm{s}$,
$N_{\gamma} \approx 10^{8-9} /(\mathrm{MeV} \mathrm{s})$.

## 4. Concluding remarks

1. DBD experiments give $1 /\left(\mathrm{t}_{1 / 2} \mathrm{G}^{0 \mathrm{v}}\right)=\mathrm{B}(\mathrm{NP}), \quad \mathrm{B}(\mathrm{NP})=\left|\mathrm{M}^{0 \mathrm{v}} \mathrm{K}_{\mathrm{vR}}\right|^{2}$ with $K_{v R}=v$-mass, $\lambda \eta$ in LR model. NMEs are crucial for them.
2. CE $\left({ }^{3} \mathrm{He}, \mathrm{t}\right)$ and $\mathrm{DCE}\left({ }^{11} \mathrm{~B},{ }^{11} \mathrm{Li}\right)$ reactions with $\mathrm{E} / \mathrm{A} \sim 0.1 \mathrm{GeV}$ give axial-vector GT-SD. Cross section ratios to the IAS and DIAS are used. The tensor term interfere with the central ones.
3. E1 /M1 $\gamma$ NMEs for IAS and photo-nuclear excitations of IAS provide absolute GT / V1 $\beta$ NMEs since the EM couplings and transition operators are well known.
4. Small $\gamma$ width of $\mathrm{eV}-$ sub-eV is overcome by using sharp IAS with small width around 30 keV to get some $10^{-4} \gamma$ branch.
5. Measurements of $\gamma$ rays in coincidence with CER to IAS makes BG-free measurement. The event rates are around 50 per day, showing that the experiments are quite feasible .


Thank you for your attention

## PHYSICAL REVIEW C 108, L011302 (2023)

# Electromagnetic transitions from isobaric analog states to study nuclear matrix elements for neutrinoless $\beta \boldsymbol{\beta}$ decays and astro-neutrino inverse $\boldsymbol{\beta}$ decays 

Hiroyasu Ejiri ${ }^{*}{ }^{*}$<br>Research Center for Nuclear Physics, Osaka University, Osaka 567-0047, Japan(Received 26 February 2023; revised 7 April 2023; accepted 22 June 2023; published 10 July 2023)

Experimental studies of nuclear matrix elements (NMEs) for neutrinoless double- $\beta$ decays (DBDs) and astroneutrino $(v)$ inverse $\beta$ decays (IBDs) are crucial for $v$ studies beyond the standard model and the astro- $v$ studies since accurate theoretical calculations of the NMEs are hard due to the high sensitivity of the NMEs to the nuclear models and the nuclear parameters used for the models. Some of the important NMEs of electromagnetic transition operators associated with DBD and IBD, including the effective weak couplings, are found to be experimentally obtained by measuring the corresponding electromagnetic gamma (EM: $\gamma$ ) transitions from the isobaric analog states (IASs) of the DBD and IBD nuclei. Then the experimental NMEs and the couplings are used for evaluating the DBD and IBD NMEs and for checking the model calculations. The EM-NMEs, the cross sections, and the event rates for the IAS- $\gamma$ transitions are estimated for DBD and IBD nuclei to show the feasibility of the experiments.


Figure 3. Reduction coefficients for xial-vextor NMEs. Light blue triangles: $k_{\tau \sigma}(\mathrm{SD}, \mathrm{QP})$ for the QP SD states by SCERs on DBD nuclei. Blye diamords: $k_{\tau \sigma}^{\prime}(\mathrm{SD}, \mathrm{L})$ for low-ling SD states by SCERs on DBD nuclei. Light blue square: $\left(k_{\tau \sigma}(\mathrm{GTSD}, \mathrm{QP})\right)^{1 / 2}$ for the QP GT-SD states by DCER on ${ }^{56} \mathrm{Fe}$. Solid line: the reduction coefficient of 0.3 to guide eye

## RCNP Ring cyclotron provided ${ }^{11} \mathrm{~B}$ with $\mathrm{E} / \mathrm{A}=0.08 \mathrm{GeV}$.

Grand RAIDEN high resolution spectrometer for ${ }^{11} \mathrm{Li}$ momentum analysis, and identification by TOF and energy loss.

## ${ }^{13} \mathrm{C}\left({ }^{(11} \mathrm{B},{ }^{11} \mathrm{Li}\right){ }^{13} \mathrm{O}$ ground state is well identified.




Figure 1. The Grand Raiden spectrometer. Q1, Q2: quadrupole D1: DSR: for DCER ( $\left.{ }^{11} \mathrm{~B},{ }^{11} \mathrm{Li}\right)$


## Experimental inputs on DBD NMES to help Theories

1. They depend on the models the parameters (weak couplings etc), $\mathrm{H}_{\mathrm{ij},}$ etc.
2. The region of NMEs do not mean the possible region of the NMEs
3. Adjusted $g_{A}$, $g_{\mathrm{pp}}$ etc for $2 \nu \beta \beta$ do not guarantee the right 0 v NMEs.
4. Shell model interactions are not adjusted to fit to $\beta-\gamma$ NMEs. ${ }^{71} \mathrm{Ga},{ }^{69} \mathrm{Ga}$, $\mathrm{M}(\mathrm{SM}) / \mathrm{M}(E X P)=\mathrm{g}_{\mathrm{A}}=0.75$ for $3 / 2-1 / 2$, 6.1 for $3 / 2-5 / 2$.
5. $\mathrm{M}^{0 \mathrm{v}}, \mathrm{M}(\mathrm{GT}), \mathrm{M}(\mathrm{F})$ in pnQRPA depends $30 \%$ on SP levels and $\mathrm{g}_{\mathrm{A}}$ parameters. for given 2nbb NME (Ejiri Jokiniemi, Suhonen PR C Lett. 2023
geff from $2 v \beta \beta$
M(EXP)/M(Model)


Fig. 29. Effective values of $g_{A}$ in different theoretical $\beta$ and $2 \nu \beta \beta$ analyses for the nuclear mass range $A=41-136$. The quoted references are Suhonen2017 [216], Caurier2012 [233], Faessler2007 [242], Suhonen2014 [243] and Horoi2016 [235]. These studies are contrasted with the ISM $\beta$-decay studies of M-P1996 [229], Iwata2016 [230], Kumar2016 [231] and Siiskonen2001 [228]. For more information see the text and Table 3 in Section 3.1.2 and the text in Section 3.1.3.

## . Ejiri H, Suhonen J and Zuber Z 2019 Phys. Rep. 7971

## Schematic diagrams of SCER, DCER, and DBD



Figure 1. Schematic diagram for the $0 \nu \beta \beta$ DBD transition of ${ }_{Z}^{A} X \rightarrow{ }_{Z+2}^{A} X$ with a neutrino exchange. SCER: ${ }_{Z}^{A} X \rightarrow{ }_{Z+1}^{A} X$. DCER: ${ }_{Z}^{A} X \rightarrow{ }_{Z+2}^{A} X$.. QP: Quasi particle-hole state. GR: Giant resonance. DGR: Double giant resonance. $\mathrm{M}^{-}\left(\mathrm{M}^{+}\right): \tau^{-}\left(\tau^{+}\right)$single- $\beta$ response associate with DBD.

Axial-vector $\tau \sigma$ NMEs for low-lying (Q) states are reduced by nucleonic and non-nucleonic $\tau \sigma$ correlations, some of them are in model, others are incorporated by axial-vector coupling, ( $\mathrm{g}_{\mathrm{A}}^{\text {eff }} / \mathrm{g}_{\mathrm{A}}$ )
Double $\tau \sigma \tau \sigma$ NMEs for low-lying (Q) states are doubly reduced by nucleonic and non-nucleonic $\tau \sigma \tau \sigma$ correlations, some of them are in model, others are incorporated by axial-vector coupling, $\left(g_{A} \text { eif } \mathrm{ef}_{\mathrm{A}}\right)^{2}$

