



**N $\nu$ DEx**

# HIGH PRECISION DESCRIPTION OF TWO NEUTRINO DOUBLE BETA DECAY ELECTRON SPECTRA

DONG-LIANG FANG  
INSTITUTE OF MODERN PHYSICS, CAS

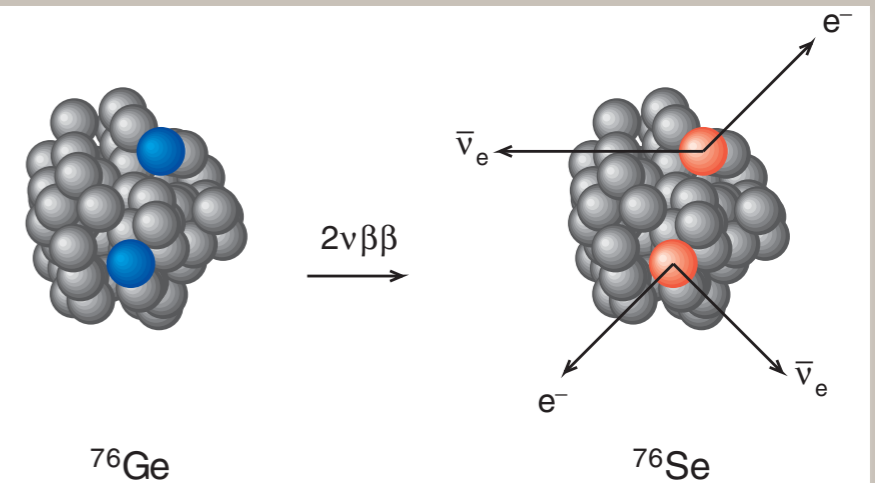
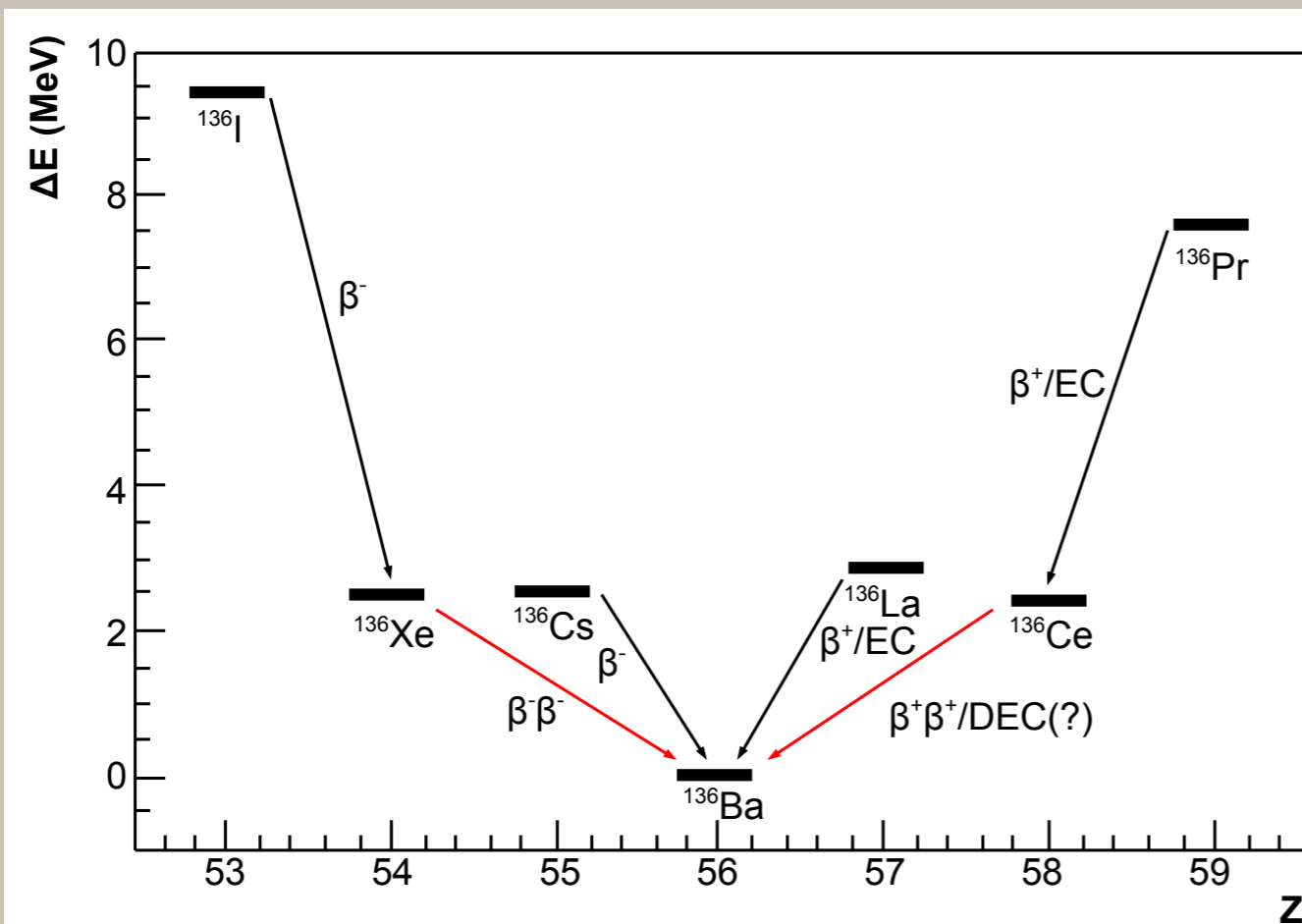
**NME2023**

RCNP Workshop for  
"Theoretical and Experimental Approaches for Nuclear Matrix Elements of Double Beta Decay"

# OUTLINE

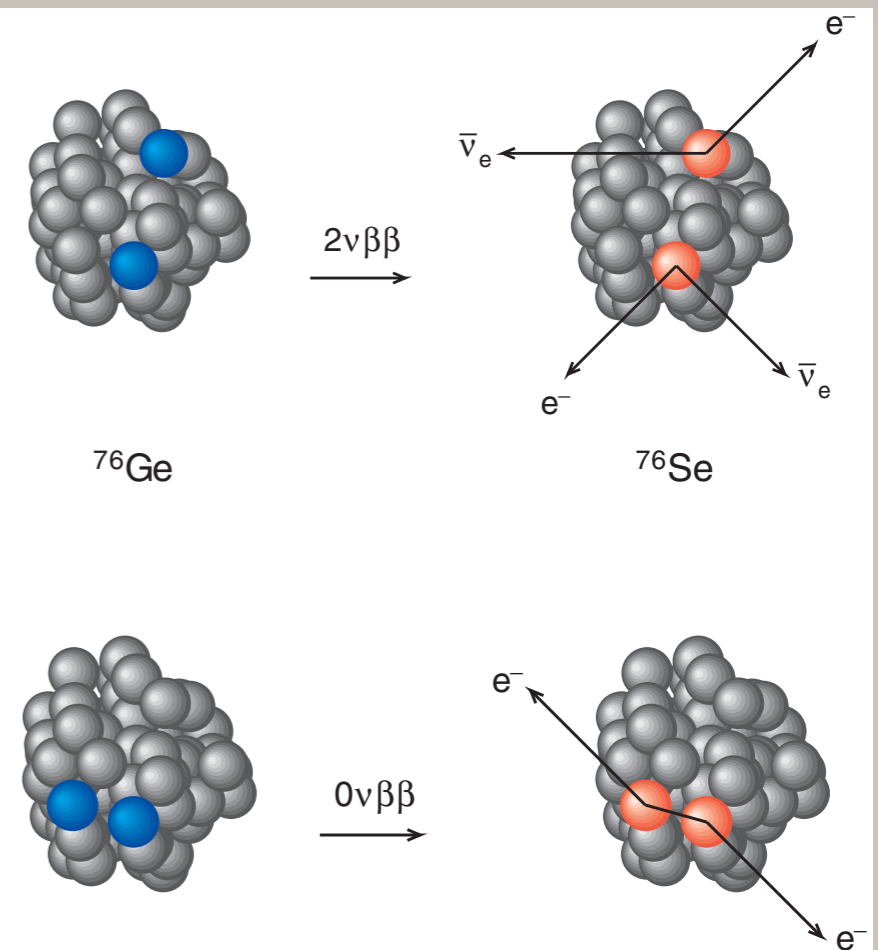
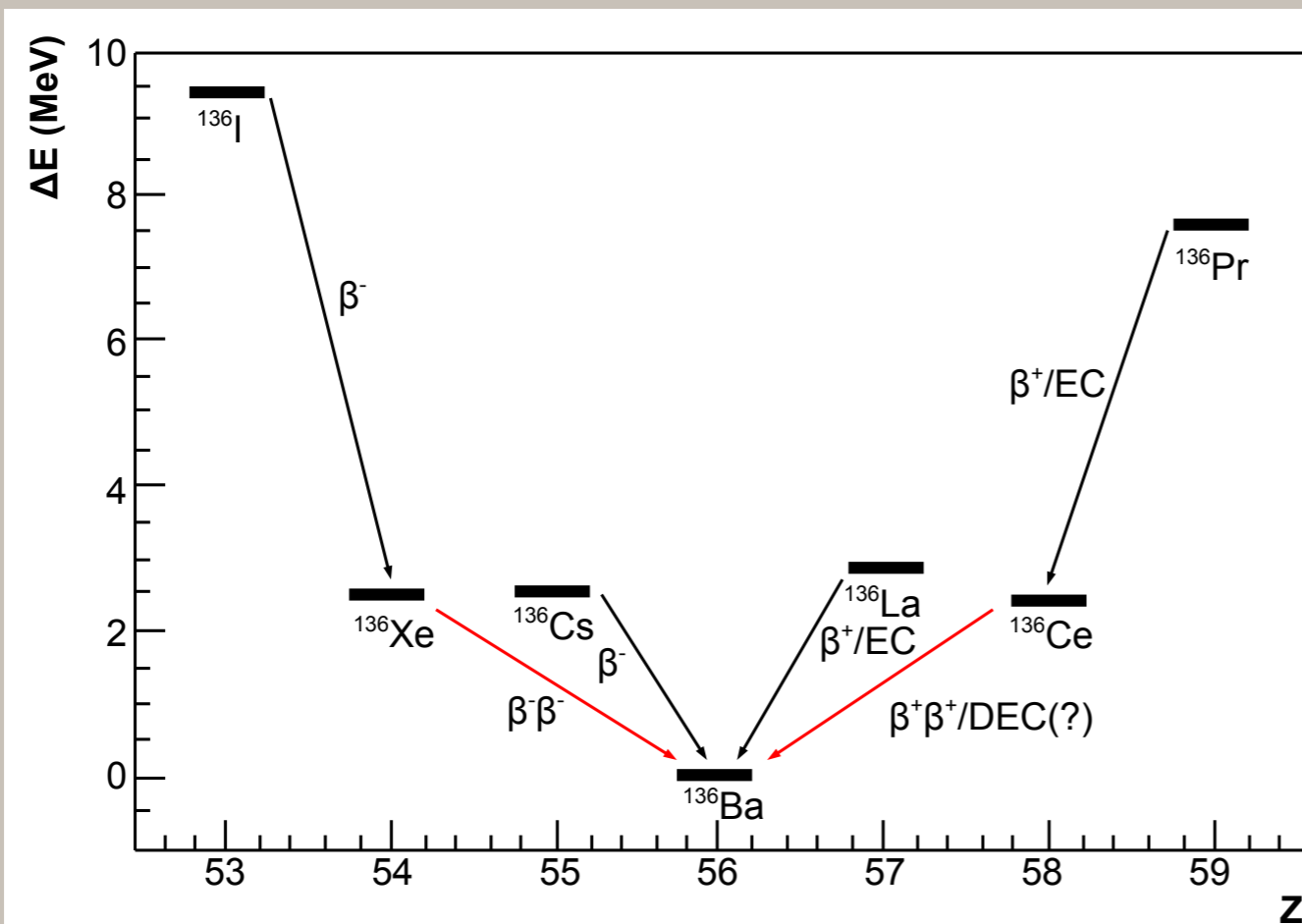
- Background
- Experimental status
- Formalisms
- Theoretical results
- Conclusions and outlooks

# BACKGROUND



- Two neutrino double beta decay is a second order weak process

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- Two neutrino double beta decay has been well studied for decades
- For theory, previously the observable is the nuclear matrix element
- This leaves too many d.o.f's

*Barabash NPA935,52(2015)*

Isotope	$T_{1/2}(2\nu)$ , yr
$^{48}\text{Ca}$	$4.4^{+0.6}_{-0.5} \cdot 10^{19}$
$^{76}\text{Ge}$	$1.65^{+0.14}_{-0.12} \cdot 10^{21}$
$^{82}\text{Se}$	$(0.92 \pm 0.07) \cdot 10^{20}$
$^{96}\text{Zr}$	$(2.3 \pm 0.2) \cdot 10^{19}$
$^{100}\text{Mo}$	$(7.1 \pm 0.4) \cdot 10^{18}$
$^{100}\text{Mo}-^{100}\text{Ru}(0_1^+)$	$6.7^{+0.5}_{-0.4} \cdot 10^{20}$
$^{116}\text{Cd}$	$(2.87 \pm 0.13) \cdot 10^{19}$
$^{128}\text{Te}$	$(2.0 \pm 0.3) \cdot 10^{24}$
$^{130}\text{Te}$	$(6.9 \pm 1.3) \cdot 10^{20}$
$^{136}\text{Xe}$	$(2.19 \pm 0.06) \cdot 10^{21}$
$^{150}\text{Nd}$	$(8.2 \pm 0.9) \cdot 10^{18}$
$^{150}\text{Nd}-^{150}\text{Sm}(0_1^+)$	$1.2^{+0.3}_{-0.2} \cdot 10^{20}$
$^{238}\text{U}$	$(2.0 \pm 0.6) \cdot 10^{21}$
$^{130}\text{Ba}$ , ECEC( $2\nu$ )	$\sim 10^{21}$

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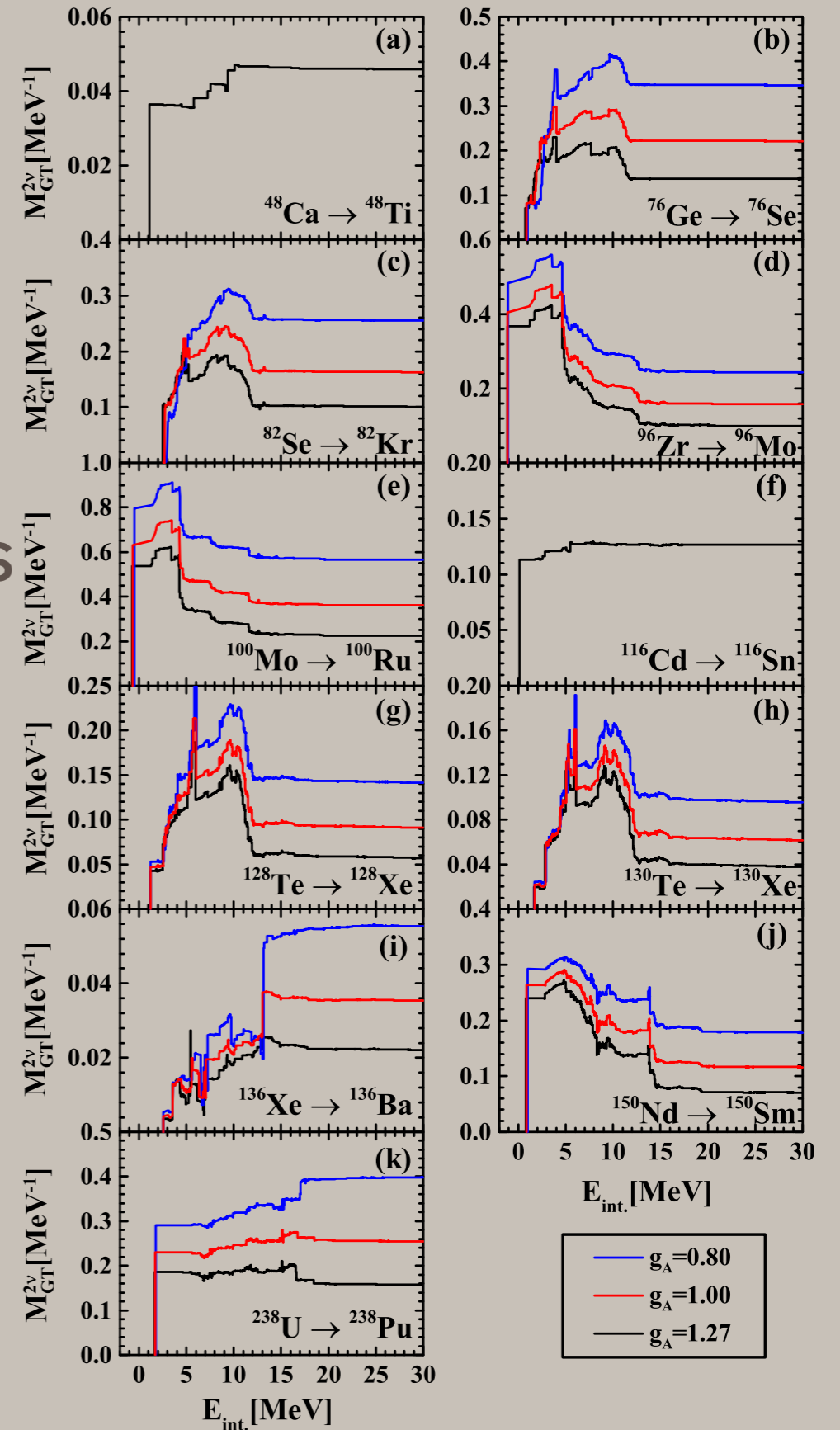
Isotope	$T_{1/2}(2\nu)$ , yr	Recommended value
$^{48}\text{Ca}$	$4.4^{+0.6}_{-0.5} \cdot 10^{19}$	$0.038 \pm 0.003$
$^{76}\text{Ge}$	$1.65^{+0.14}_{-0.12} \cdot 10^{21}$	$0.113 \pm 0.006$
$^{82}\text{Se}$	$(0.92 \pm 0.07) \cdot 10^{20}$	$0.083 \pm 0.004$
$^{96}\text{Zr}$	$(2.3 \pm 0.2) \cdot 10^{19}$	$0.080 \pm 0.004$
$^{100}\text{Mo}$	$(7.1 \pm 0.4) \cdot 10^{18}$	$0.185 \pm 0.005$
$^{100}\text{Mo}-^{100}\text{Ru}(0_1^+)$	$6.7^{+0.5}_{-0.4} \cdot 10^{20}$	$0.151 \pm 0.005$
$^{116}\text{Cd}$	$(2.87 \pm 0.13) \cdot 10^{19}$	$0.105 \pm 0.003$
$^{128}\text{Te}$	$(2.0 \pm 0.3) \cdot 10^{24}$	$0.046 \pm 0.006$
$^{130}\text{Te}$	$(6.9 \pm 1.3) \cdot 10^{20}$	$0.031 \pm 0.004$
$^{136}\text{Xe}$	$(2.19 \pm 0.06) \cdot 10^{21}$	$0.0181 \pm 0.0007$
$^{150}\text{Nd}$	$(8.2 \pm 0.9) \cdot 10^{18}$	$0.058 \pm 0.004$
$^{150}\text{Nd}-^{150}\text{Sm}(0_1^+)$	$1.2^{+0.3}_{-0.2} \cdot 10^{20}$	$0.044 \pm 0.005$
$^{238}\text{U}$	$(2.0 \pm 0.6) \cdot 10^{21}$	$0.13^{+0.09}_{-0.07}$
$^{130}\text{Ba}$ , ECEC( $2\nu$ )	$\sim 10^{21}$	$\sim 0.26$

# BACKGROUND

- Widely used many-body approaches for two neutrino double beta decay
  - Nuclear shell Model
    - Adjusts quenching factors to reproduce NME
  - QRPA
    - *Adjusts particle-particle interaction to reproduce NME*
  - IBM *etc.*
    - *Use closure approximation with proper closure energy to reproduce NME*

# BACKGROUND

- In this sense,  $2\nu\beta\beta$  spectra poses more severe constraints on theoretical studies
- And could rule out certain calculations
- Give us implications for certain calculations





# FORMALISM

Doi et al. *PTPS*83,1(1985)

- Starting from S-matrix theory, one could obtain the decay width

Constant

Kinetic term

Electron w.f.

$$[T_{2\nu}(0^+ \rightarrow 0^+)]^{-1} = \frac{a_{2\nu}}{\ln 2} \int d\Omega_{2\nu} a(\varepsilon_1, \varepsilon_2) \left| \sum_a \left[ M_{GTa}^{(2\nu)} - \left( \frac{g_V}{g_A} \right)^2 M_{Fa}^{(2\nu)} \right] \frac{1}{2} (K_a + L_a) \right|^2$$

- Where  $M_{GTa}^{(2\nu)} \equiv -\langle 0_f^+ | \sum_n \tau_n^+ \sigma_n | N_a(1^+) \rangle \langle N_a(1^+) | \sum_m \tau_m^+ \sigma_m | 0_i^+ \rangle$

- $K_a = \{ [\mu_a + (\varepsilon_1 + \omega_1 - \varepsilon_2 - \omega_2) / (2m_e)]^{-1} + [\mu_a - (\varepsilon_1 + \omega_1 - \varepsilon_2 - \omega_2) / (2m_e)]^{-1} \}$

$$L_a = K_a(\omega_1 \rightleftharpoons \omega_2)$$

$$\mu_a m_e = E_a - (M_i + M_f) / 2$$

Nuclear part

# FORMALISM

Kotila et al. **PRC85**,034316(2012)

- Equal lepton energy approximation (ELEA) :

$$\epsilon_1 - m_e = \epsilon_2 - m_e = \omega_1 = \omega_2 = Q/4 \implies K_a \simeq L_a \simeq 2/\mu_a$$

- The nuclear and lepton parts can now be separated by inserting

$$dW_{2\nu} = (a^{(0)} + a^{(1)} \cos \theta_{12}) w_{2\nu} d\omega_1 d\epsilon_1 d\epsilon_2 d(\cos \theta_{12})$$

- With  $a^{(0)} = \frac{1}{4} f_{11}^{(0)} |M_{2\nu}|^2 \tilde{A}^2 [(\langle K_N \rangle + \langle L_N \rangle)^2 + \frac{1}{3}(\langle K_N \rangle - \langle L_N \rangle)^2]$ .

- And  $M_{2\nu}^{\text{GT}} = \frac{\langle 0_F^+ || \tau^+ \vec{\sigma} || 1_1^+ \rangle \langle 1_1^+ || \tau^+ \vec{\sigma} || 0_I^+ \rangle}{\frac{1}{2}(Q_{\beta\beta} + 2m_e c^2) + E_{1_1^+} - E_I}$   $\tilde{A} = Q/2 + \langle E_N \rangle - E_I$

- Finally  $[\tau_{1/2}^{2\nu}]^{-1} = G_{2\nu}^{(0)} g_A^4 |m_e c^2 M_{2\nu}|^2$

# FORMALISM

*SSD: Single State Dominance*

*HSD: High-lying State Dominance*

*Simkovic et al. **PRC97**,034315(2018)*

- Two commonly used  $\langle E_N \rangle$ :  $E_{1\uparrow}$  (SSD) and  $E_{GTR}$  (HSD)

- Beyond ELEA:  $A^{2\nu} = \left[ \frac{1}{4} |M_{GT}^K + M_{GT}^L|^2 + \frac{1}{12} |M_{GT}^K - M_{GT}^L|^2 \right]$

- Here  $M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon_{K,L}^2}$

- With  $\varepsilon_K = (E_{e_2} + E_{v_2} - E_{e_1} - E_{v_1})/2$   $\varepsilon_L = (E_{e_1} + E_{v_2} - E_{e_2} - E_{v_1})/2$

- With Taylor expansion, one obtains the final expression

$$\left[ T_{1/2}^{2\nu\beta\beta} \right]^{-1} = (g_A^{\text{eff}})^4 |M_{GT-1}^{2\nu}|^2 \{ G_0^{2\nu} + \xi_{31}^{2\nu} G_2^{2\nu}$$

- $+ \frac{1}{3} (\xi_{31}^{2\nu})^2 G_{22}^{2\nu} + \left[ \frac{1}{3} (\xi_{31}^{2\nu})^2 + \xi_{51}^{2\nu} \right] G_4^{2\nu} \}$

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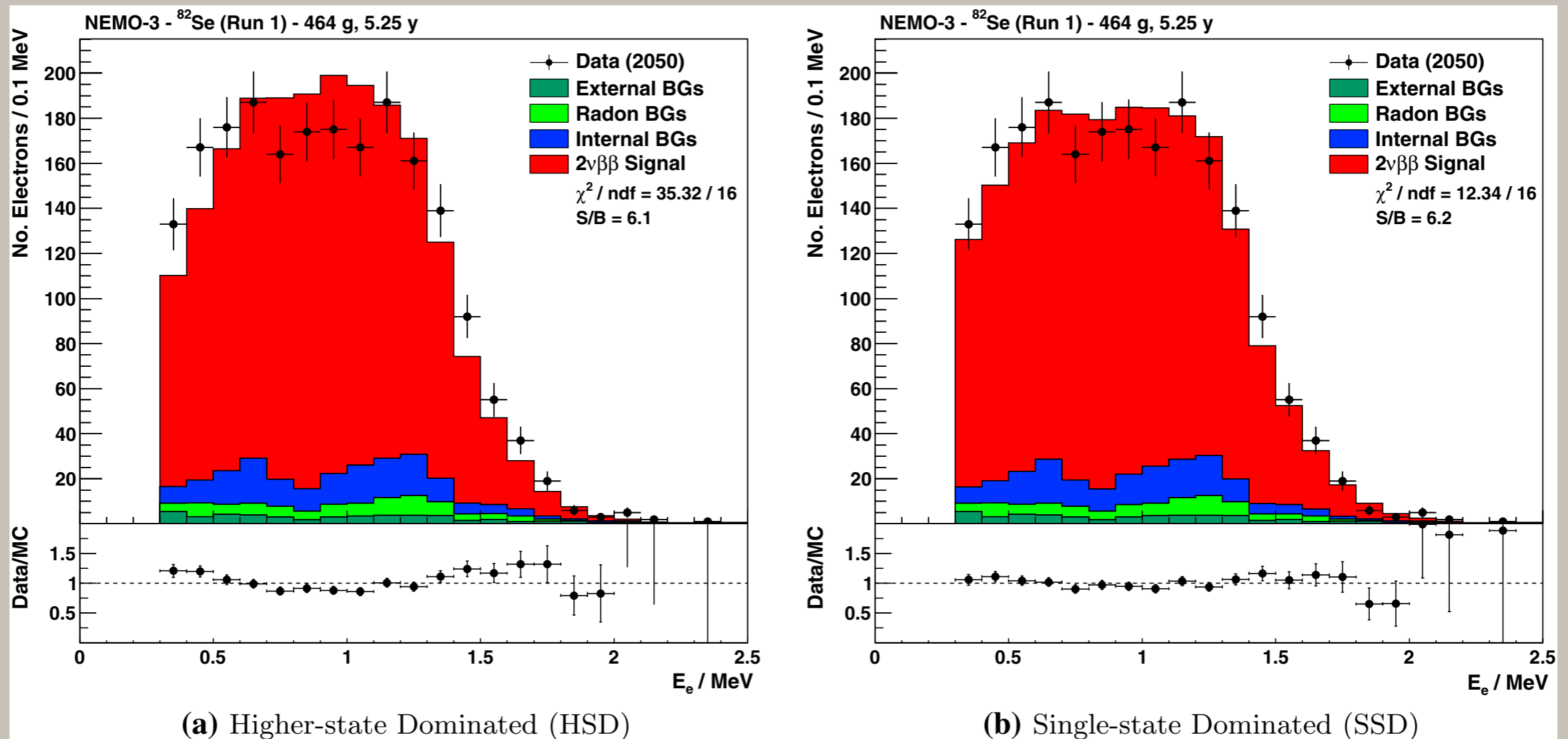
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$$M_{GT-3}^{2\nu} \equiv 4m_e^3 \sum_m [\langle 0_f^+ || \sigma\tau^+ || m \rangle \langle m || \sigma\tau^+ || 0_i^+ \rangle] / E_m^3$$

# EXPERIMENT STATUS

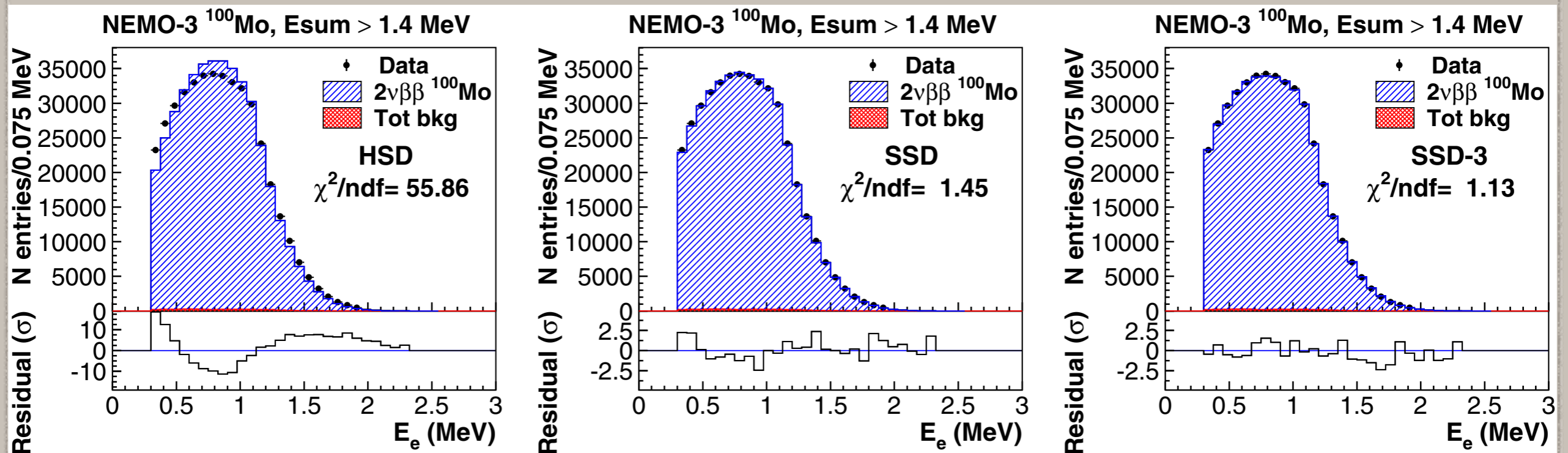
NEMO-3 *EPJC*78,821(2018)



- Results from NEMO-3 for  $^{82}\text{Se}$ , favors SSD

# EXPERIMENTAL STATUS

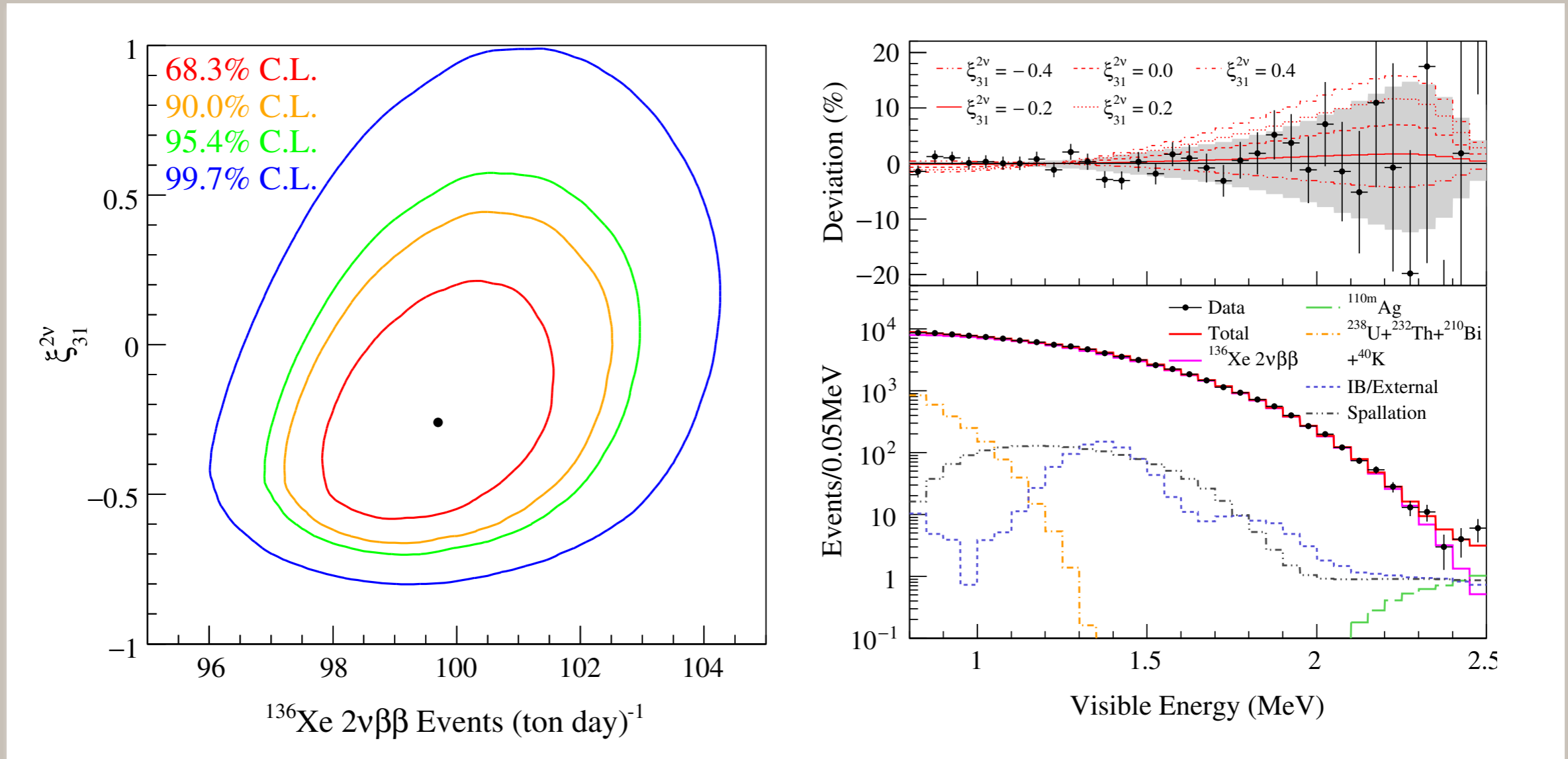
NEMO-3 *EPJC*79,440(2019)



- Results from NEMO-3 for  $^{100}\text{Mo}$
- Indication of strong SSD

# EXPERIMENTAL STATUS

KamLand-Zen *PRL*122,192501(2019)



- KamLand-Zen offers more precise spectra preferring HSD

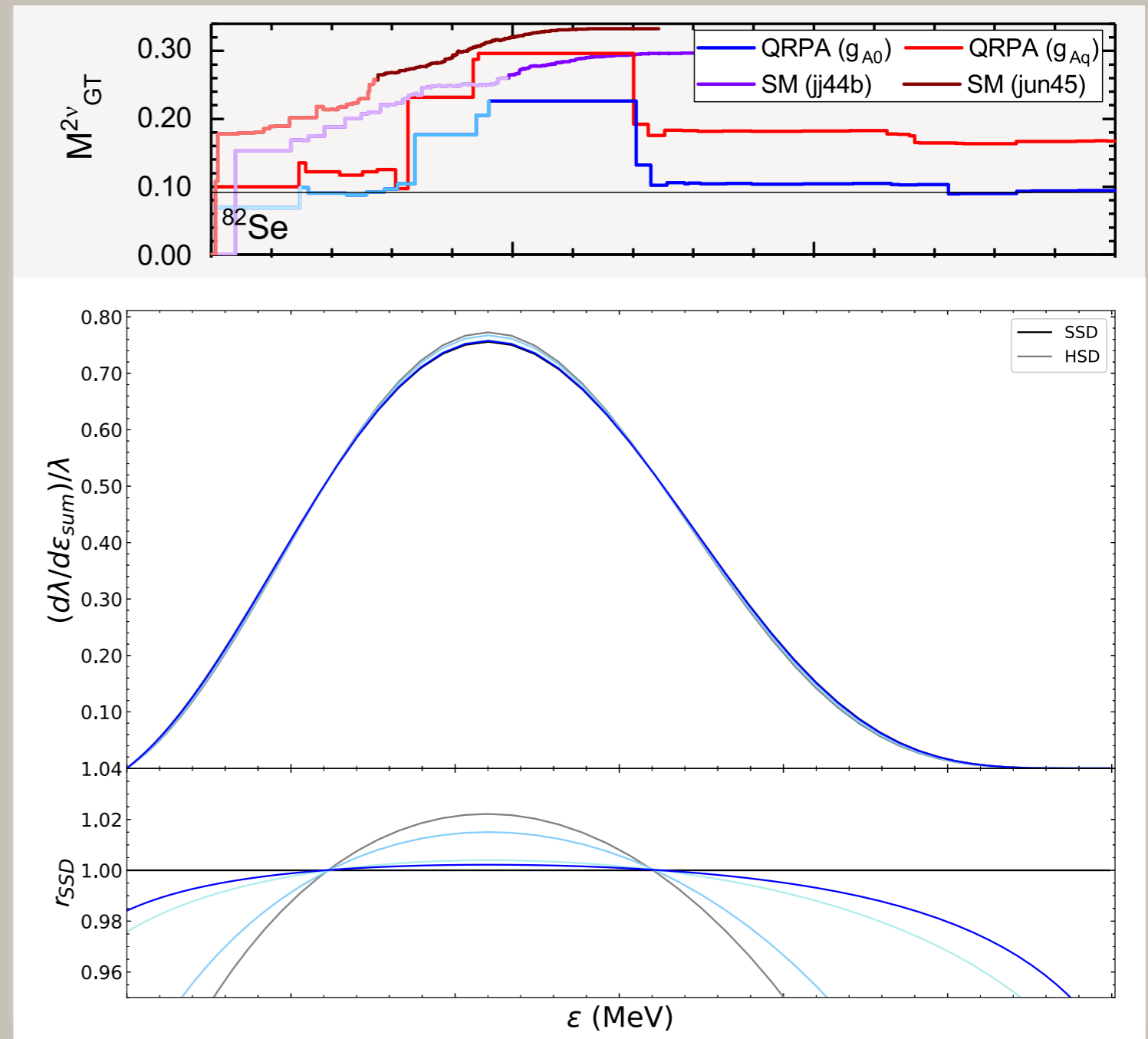
# THEORETICAL RESULTS

- We adopt two many-body approaches in our calculations:
  - pnQRPA
    - Well predicted GTR but strength not well fragmented
  - Shell Model:
    - Severely truncated model space leads to missing GTR



# THEORETICAL RESULTS

- Results for  $^{82}\text{Se}$ :
- From QRPA calculations we find that the spectra are sensitive to the low-lying states
- And the strength which cancellation at higher excitation energy will not contribute to the spectra

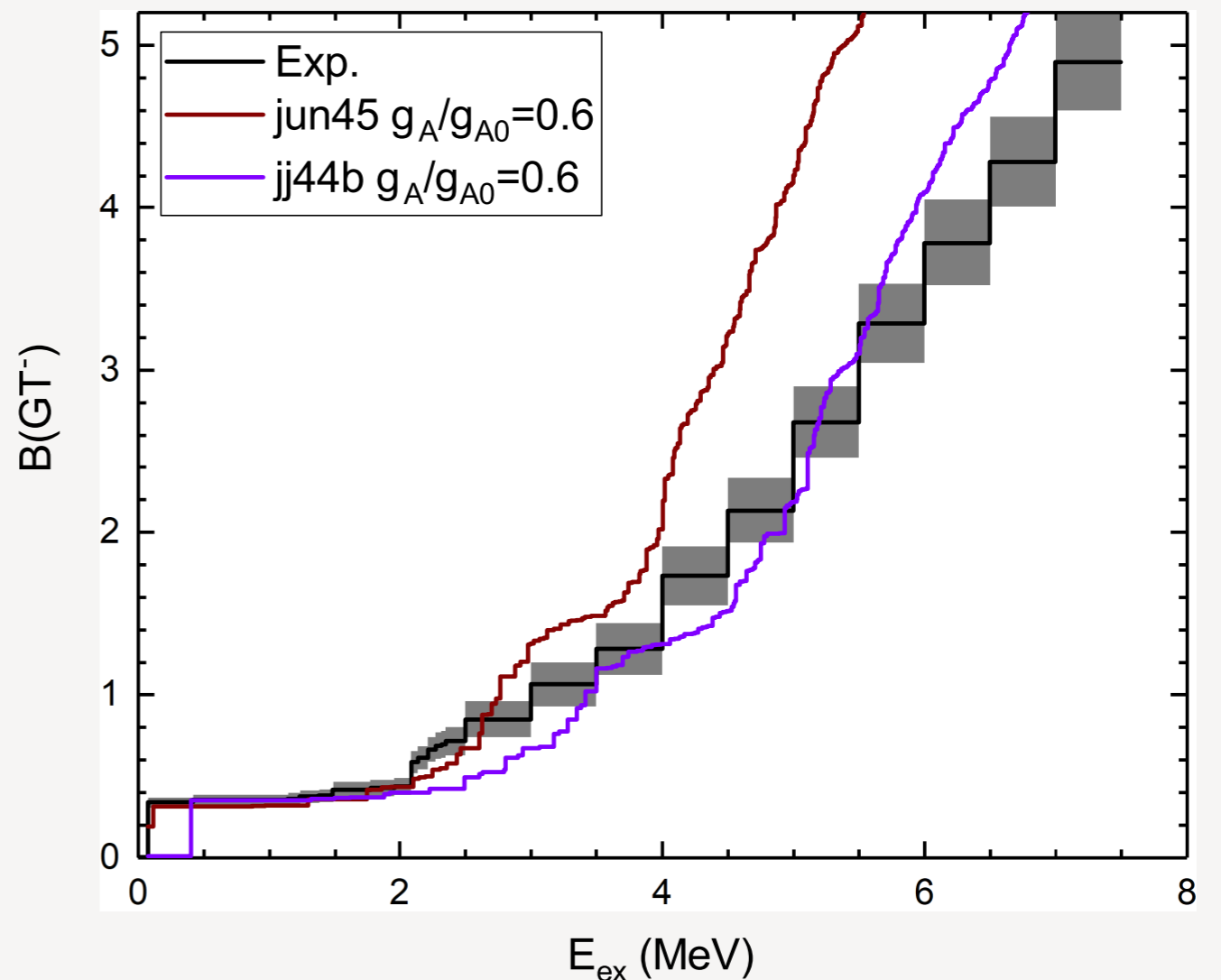
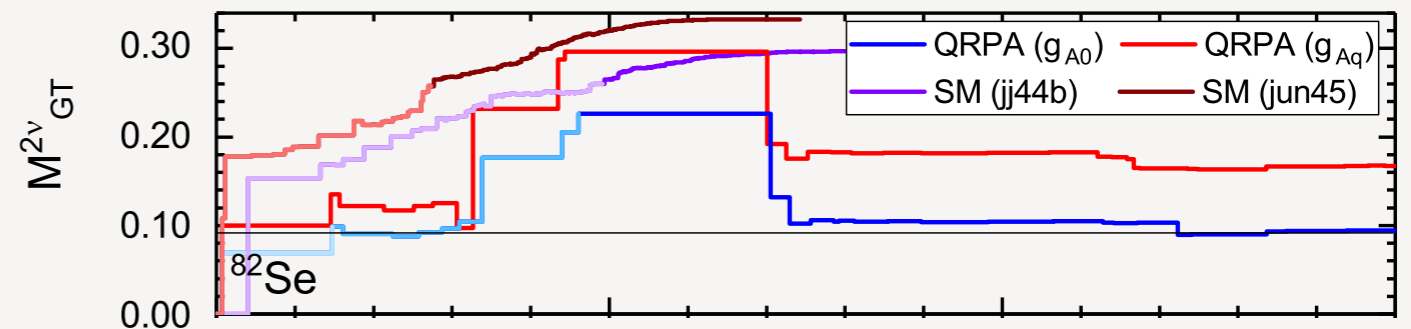


DLF arXiv:2309.13328

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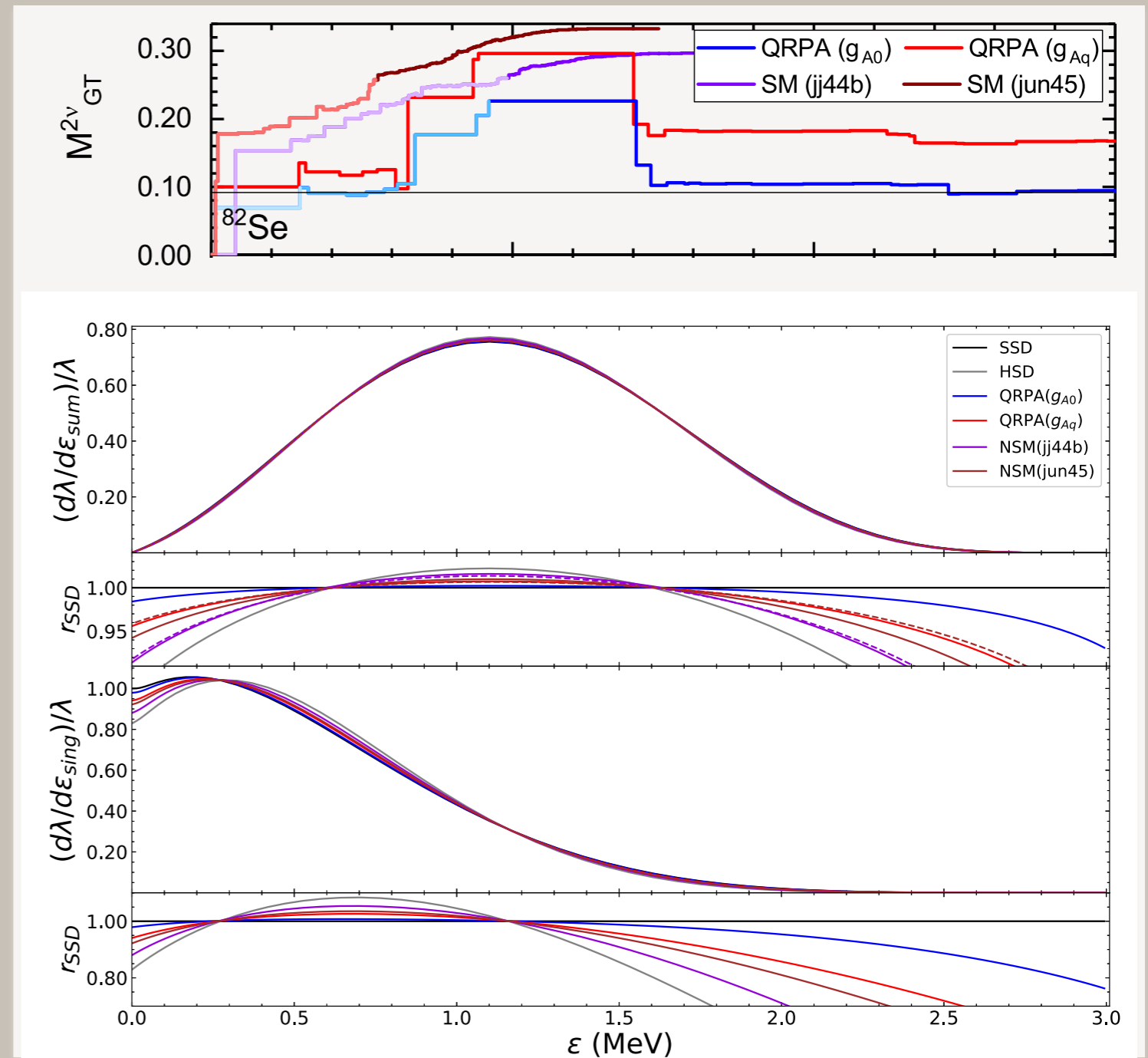
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- If we assume that for NSM the decay strength saturates at 5MeV, we will obtain a quenched  $g_A \sim 0.5$
- However, this contradicts the quenched  $g_A$  values  $\sim 0.6$  from fitting the charge exchange reaction



# THEORETICAL RESULTS

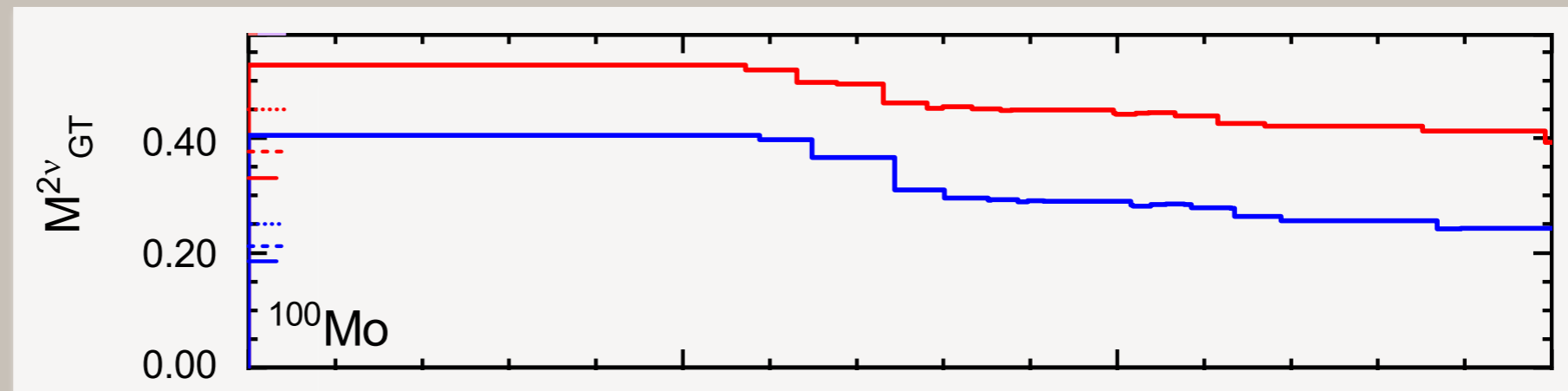
- Previous conclusion could help us to resolve the puzzle from shell model calculations
- If we assume a strong cancellation at high excitation energy, using the quenched  $g_A$  from charge exchange reaction, we obtain consistent results



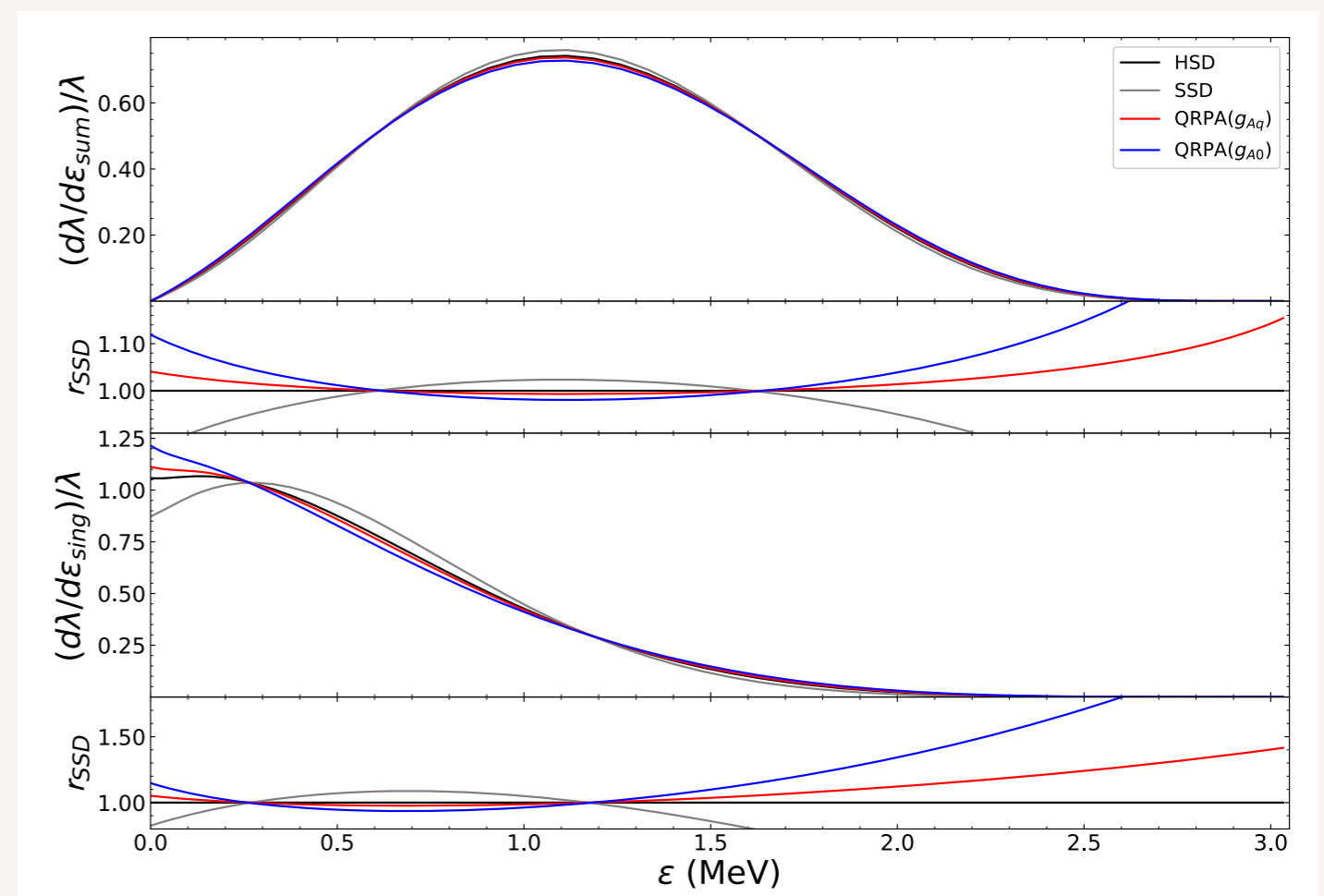
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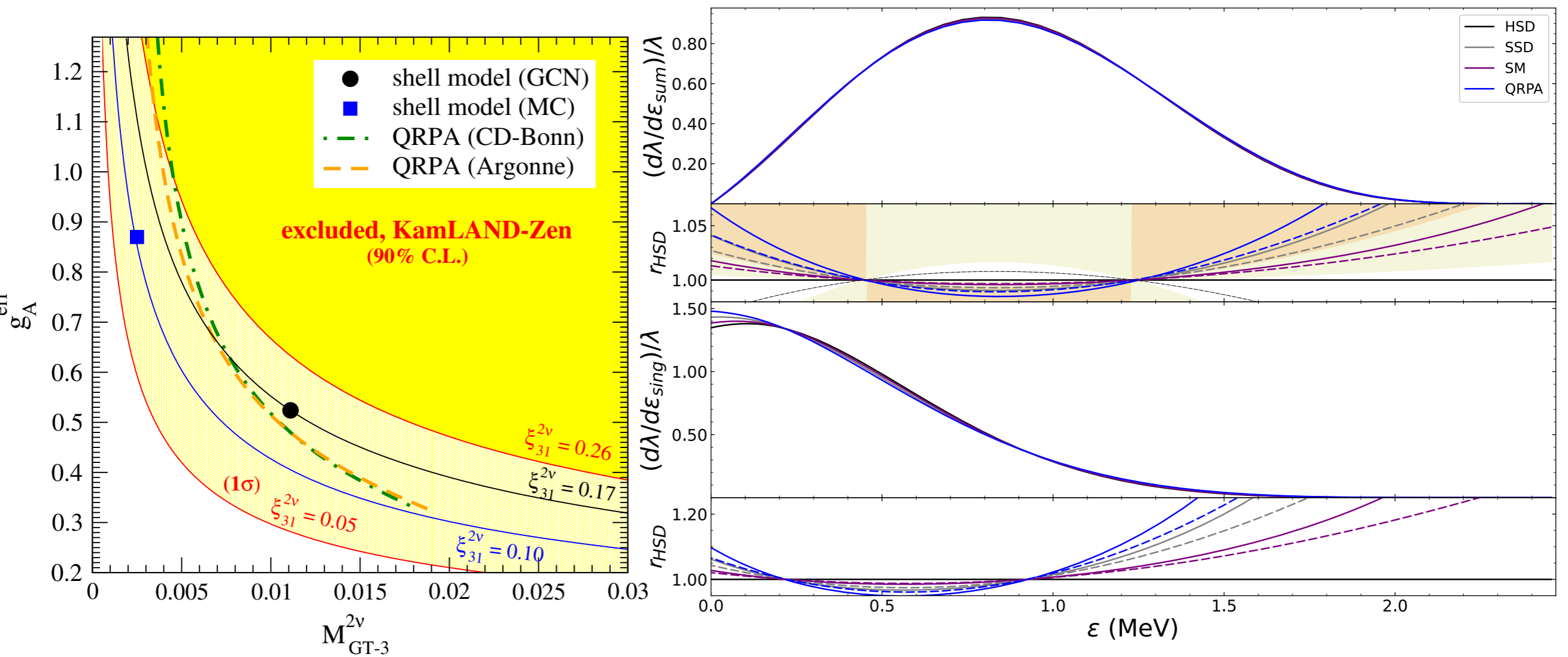


- Results for  $^{100}\text{Mo}$ , beyond SSD
- Consistent with NEMO-3 experiment



# THEORETICAL RESULTS

DLF arXiv:2309.13328

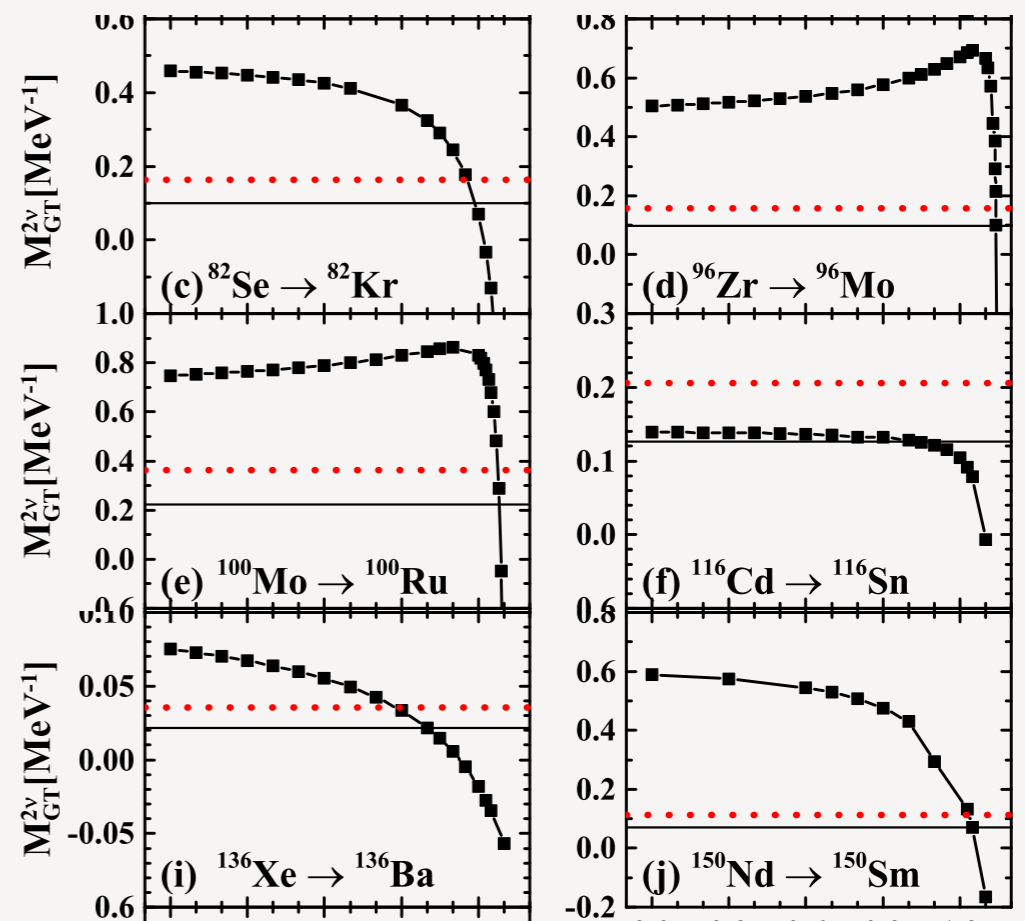


- Failures of QRPA and NSM for predicting the spectra

# THEORETICAL RESULTS

*Lv et al. PRC105,044331(2022)*

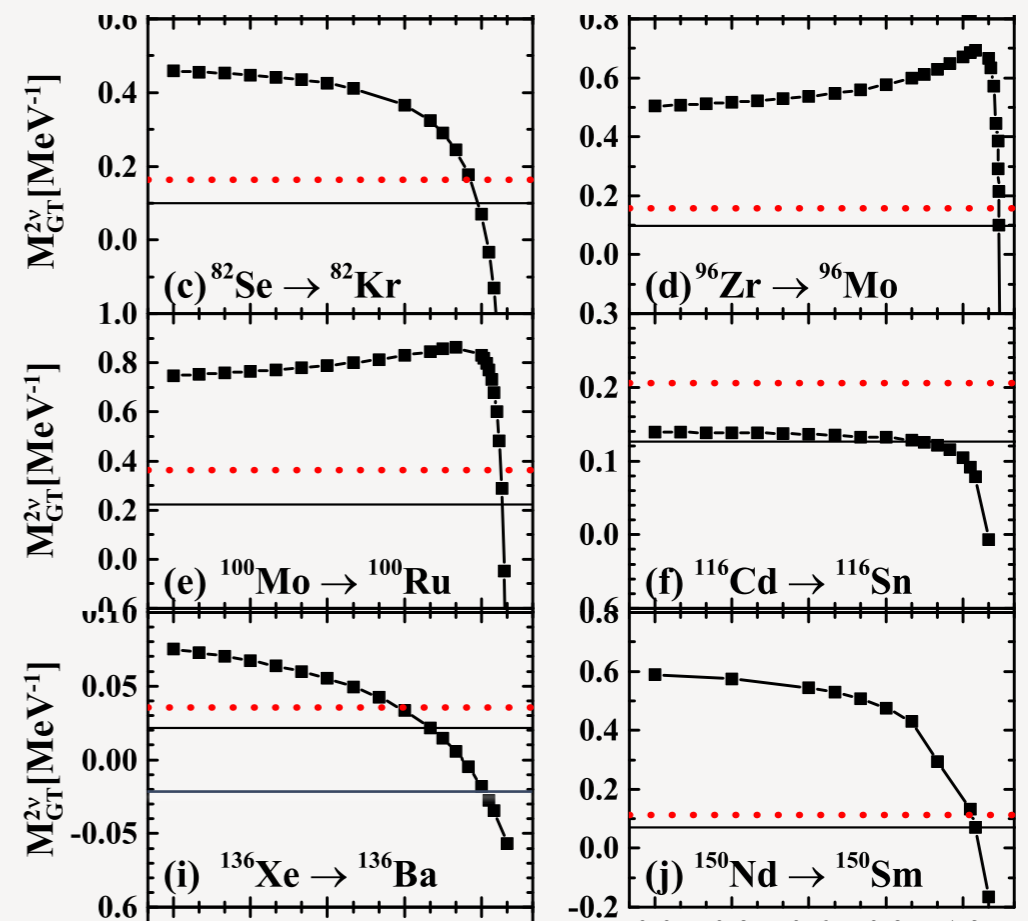
- For the fitting procedure in QRPA calculations, by default we assume the NME is "positive"
- But this has not firm physics foundation
- And  $\beta\beta$  spectra could offer the answer



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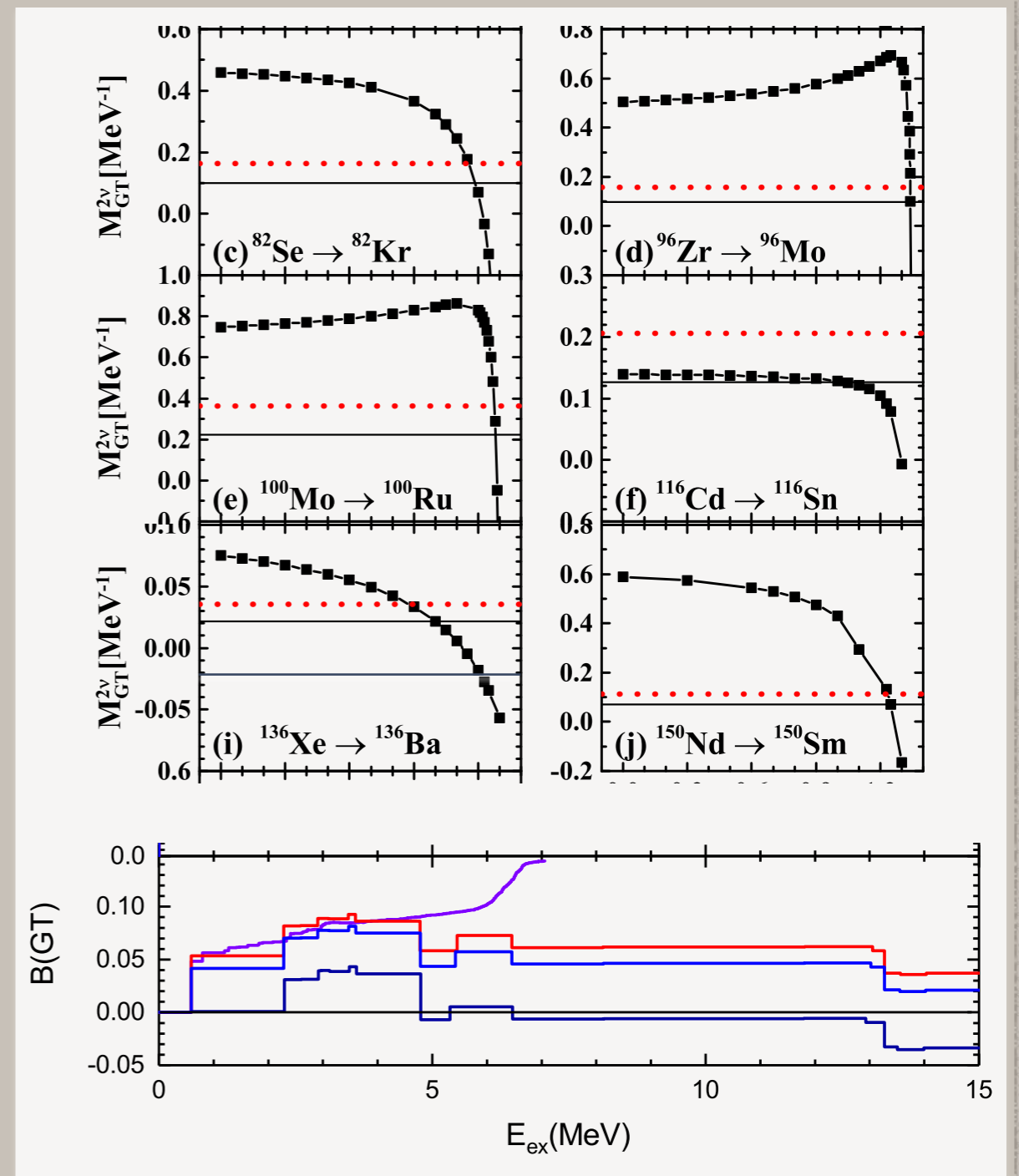
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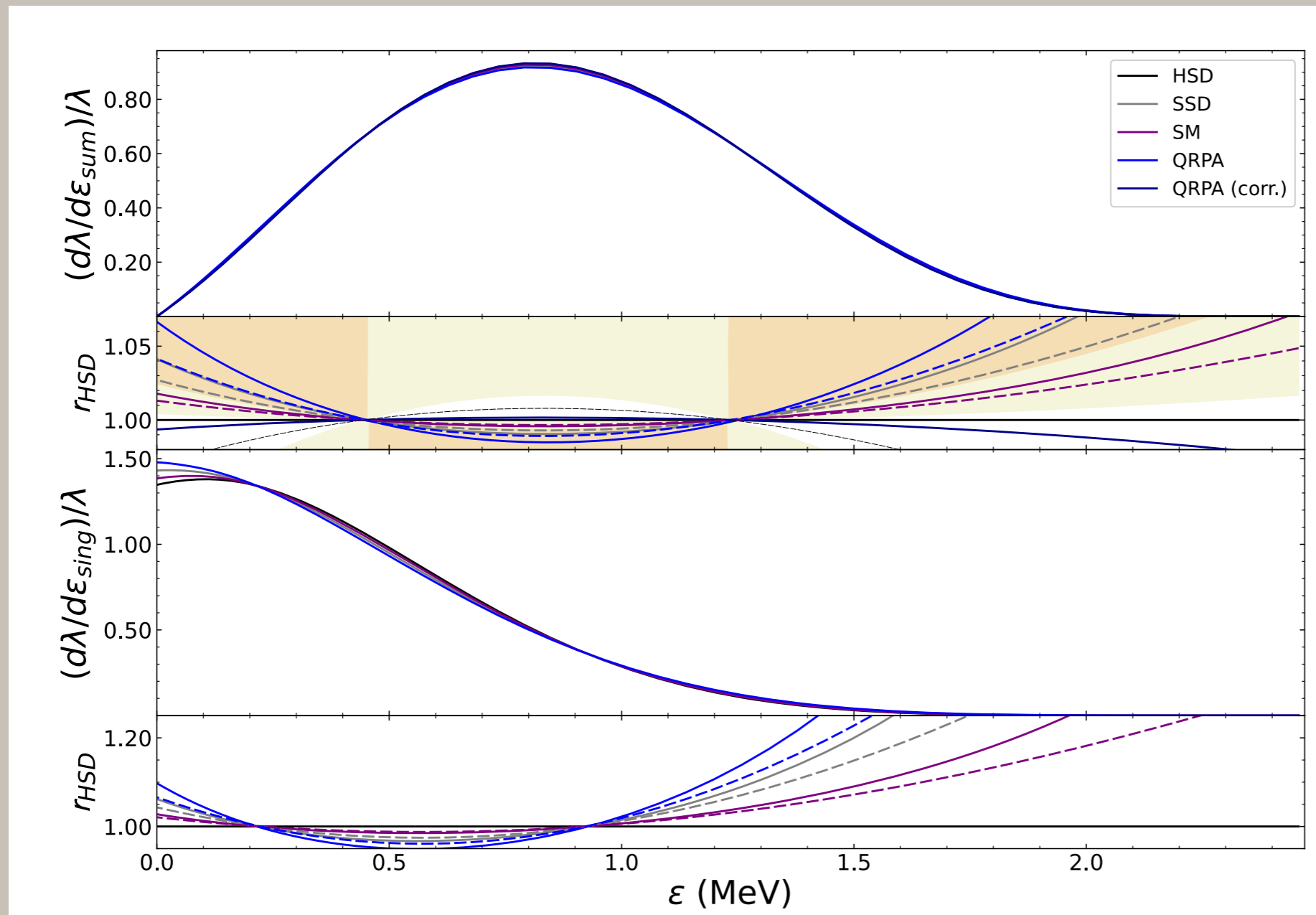
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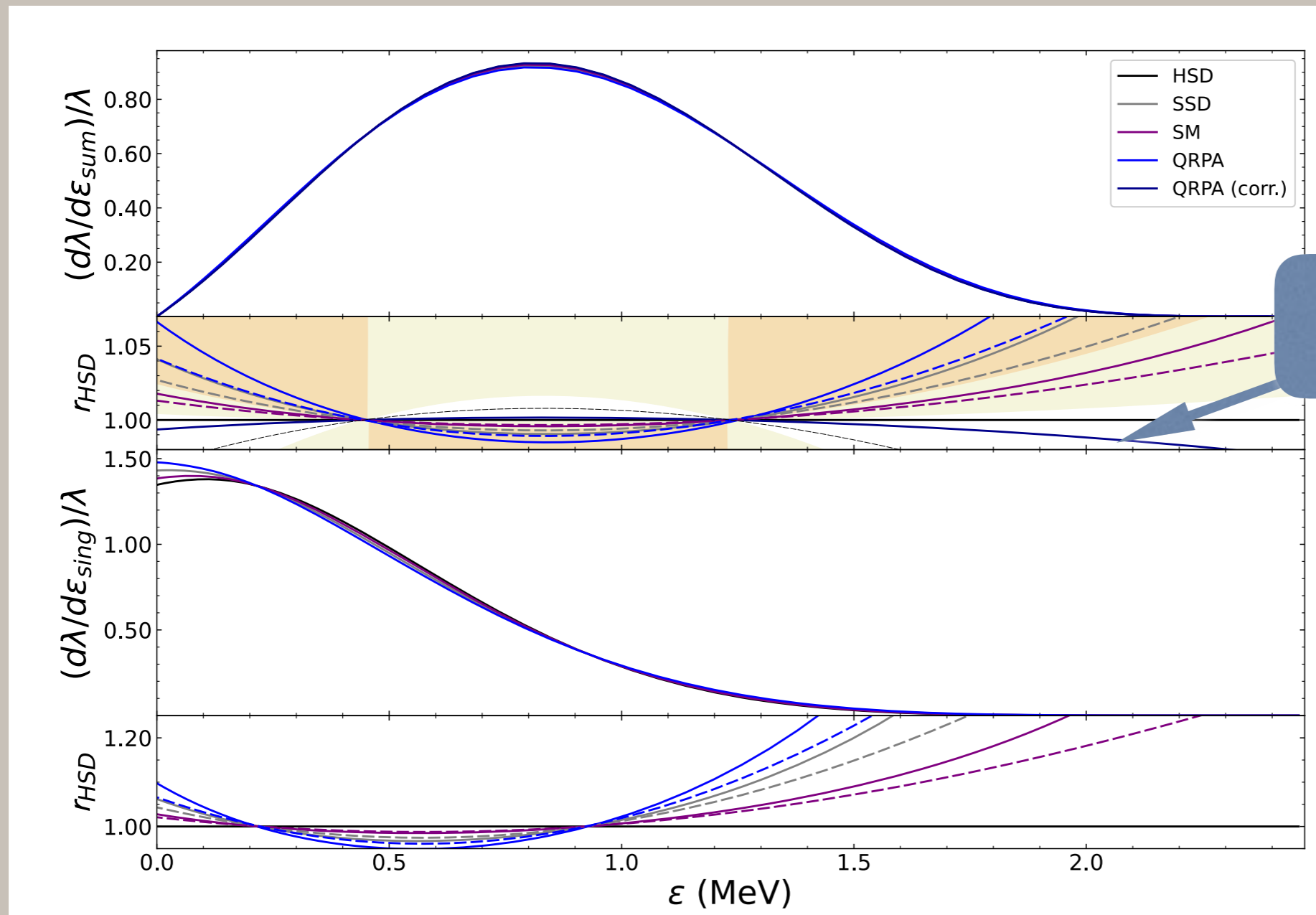
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- KamLand-Zen results offers the probability

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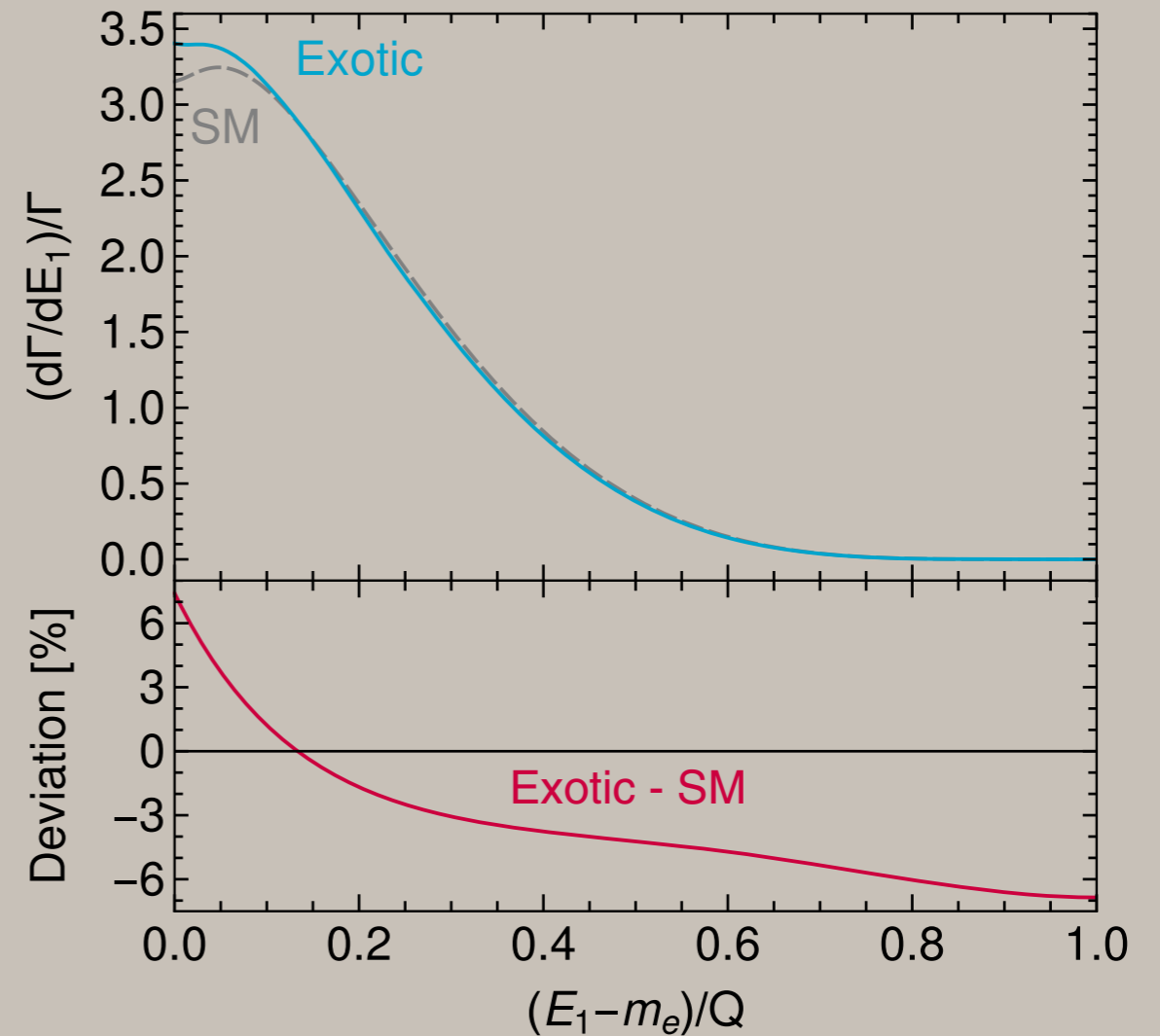
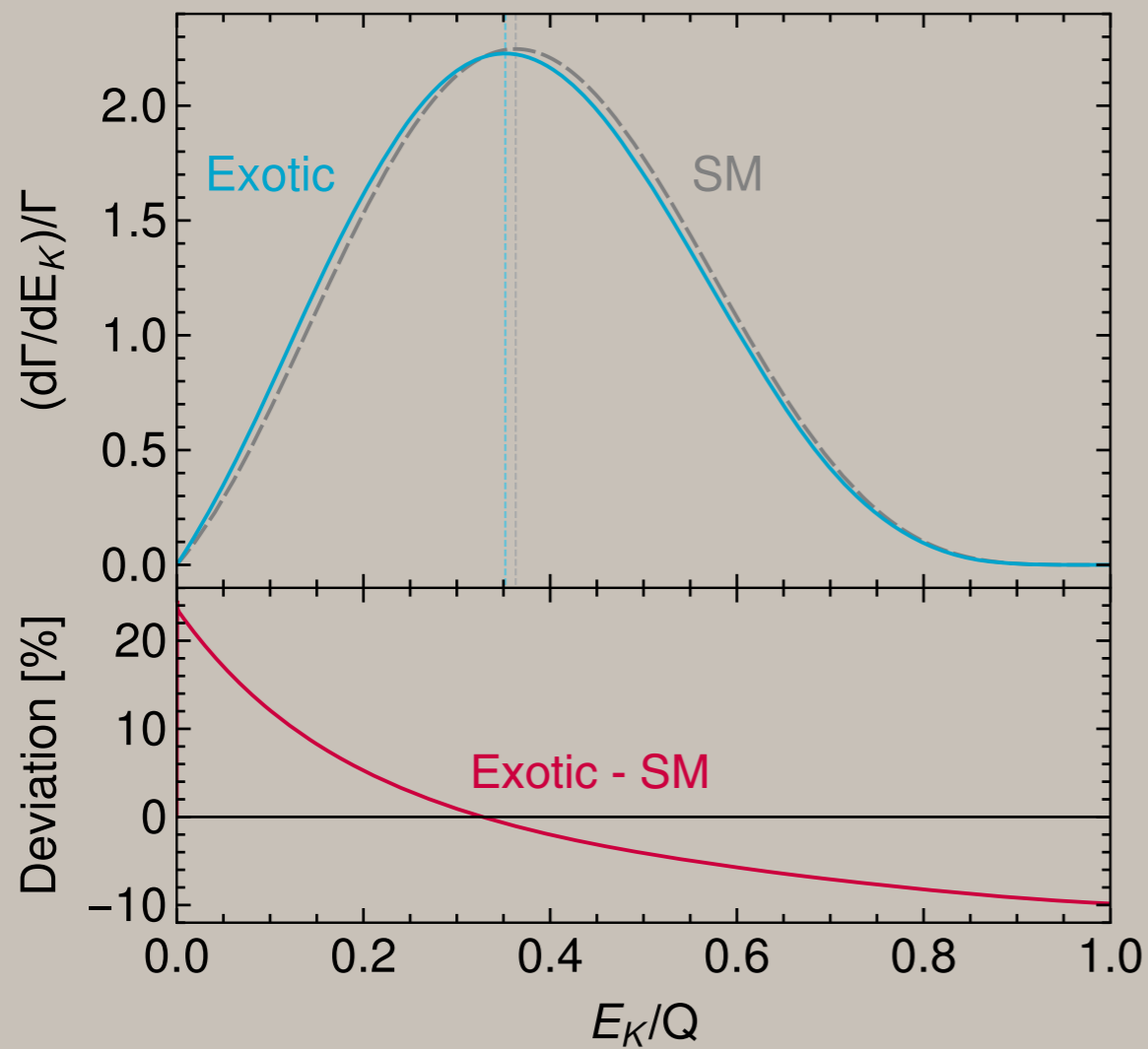
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# NEW PHYSICS

Deppisch et al. *PRL* **125**, 171801 (2023)



- Right-handed gauge boson and weak current

# CONCLUSION AND OUTLOOK

- $2\nu\beta\beta$  spectra as an addition to the half-life measurement can well constrain the nuclear theory
- Current results suggest cancellation on decay strength mediated by high-lying intermediate states for  $^{82}\text{Se}$  and  $^{136}\text{Xe}$
- These need to be verified with future charge exchange experiments
- All these analyses will shed light on neutrinoless double beta decay studies

**Thanks**