Study of Neutrinoless Double Beta Decay in Nuclear Shell Model

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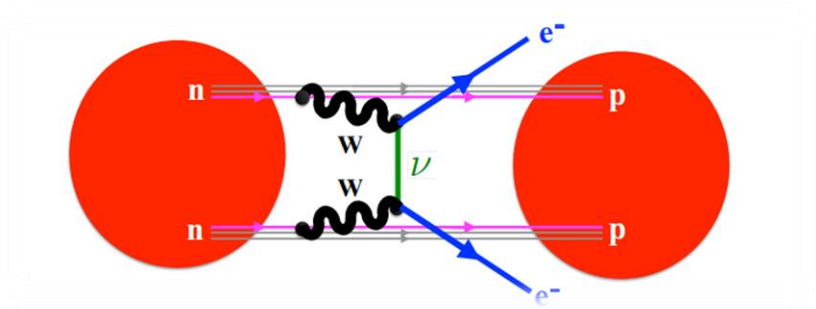

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Outlines

Neutrinoless double beta decay ($0\nu\beta\beta$) and its importance in neutrino physics

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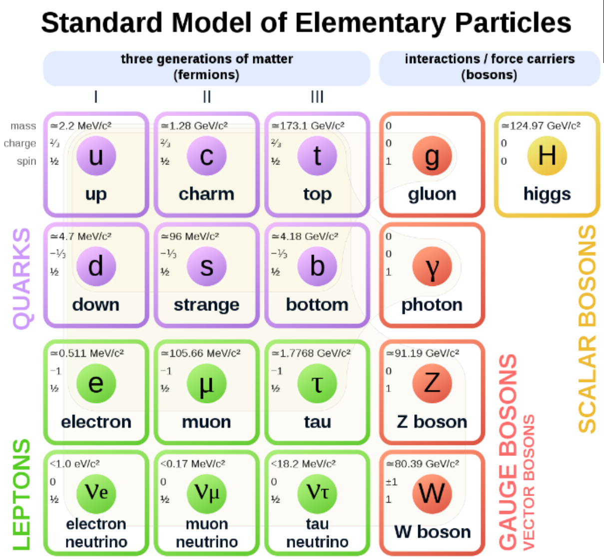
Interacting Nuclear Shell Model Approach to Study $0\nu\beta\beta$ Decay

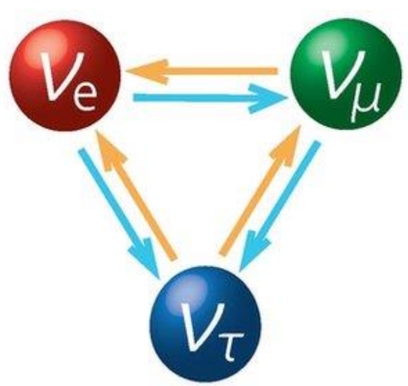
↓

Results for Nuclear Matrix Elements of ^{48}Ca, ^{124}Sn Using Nuclear Shell Model in Both Closure and Nonclosure Approximations

What we know and do not know about neutrinos (ν)?



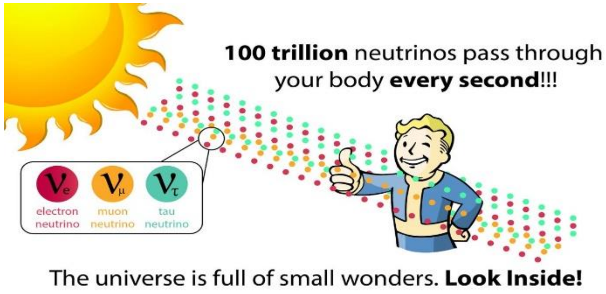
Neutrino oscillations



First predicted by Wolfgang Pauli in 1930

Neutrino mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



neutrinos.fnal.gov

What we know?

- Spin ½ neutral fundamental lepton
- They are of three types(?)
- Weakly interacting: 1 out 10^{14} only interact.
- They are 2nd most abundant particles in the universe
- In Standard Model they kept massless but in actual they have tiny masses ($\lesssim 1$ eV)

What We don't know?

Q1. Are neutrinos their own anti-particle?
No: Dirac particle
Yes: Majorana particle

Q2. What are the absolute neutrino masses?

Neutrinoless Double Beta Decay ($0\nu\beta\beta$)

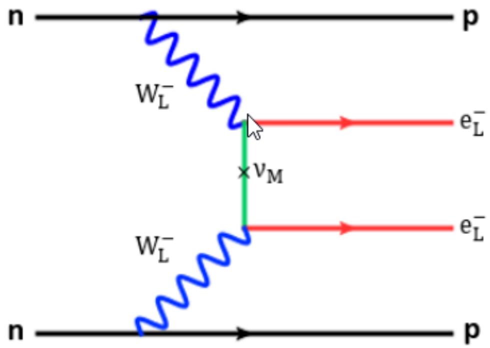


Fig1: Feynman diagram for light neutrino exchange $0\nu\beta\beta$

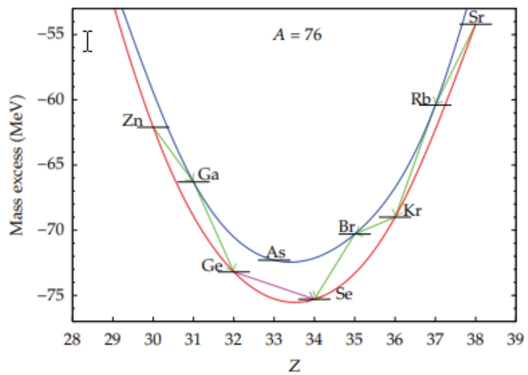
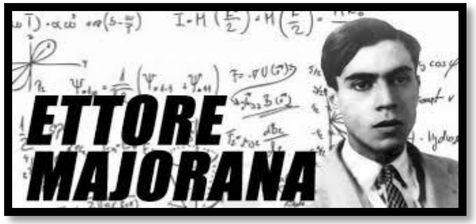


Fig2: Representation of the energies of the A 76 isobars

If this process is observed…

- Neutrinos must be their own antiparticle (Majorana particle), which has important implications in BSM physics theories.
- Put some light on absolute neutrino mass scale.
- Lepton number violation ($\Delta L = 2$) will be observed.

History



1937: **Ettore Majorana** predicted a that neutrino can be its own antiparticle

1939: **Wolfgang Furry** first predicted Neutrinoless double beta decay based on Majorana neutrino

Possible Decaying Isotopes

^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{124}Sn , ^{130}Te ,^{136}Xe, ^{150}Nd

Rarity of the process:

Half life can be in the range $> 10^{26}$ Years

But….$0\nu\beta\beta$ is still unobserved even after 80 years.

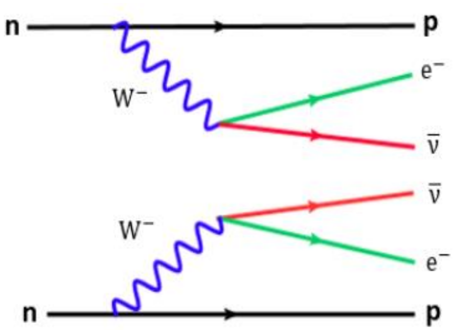


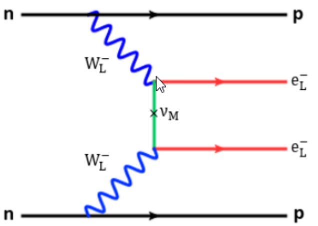
Fig 3: Feynman diagram for two-neutrino double beta decay

Two-Neutrino Double Beta decay

Observed in the experiment

Decay rate of $0\nu\beta\beta$

$$\Gamma^{0\nu} = \frac{1}{T^{0\nu}_{\frac{1}{2}}} = G^{0\nu}(Q,Z)\left|m_{\beta\beta}\right|^2 |M^{0\nu}|^2$$



Phase Space Factor($G^{0\nu}$):

$$G^{0\nu}(Q,Z) = \frac{1}{2(2\pi)^5} G_F^4 \frac{1}{R^2} g_A^4 \int_0^Q dT_1 \int_0^\pi \sin\theta d\theta \, (E_1E_2 - p_1p_2\cos\theta)p_1p_2F(E_1, Z+2)F(E_2, Z+2)$$

$$T_1 = E_1 - m_e, \; Q = M_i - M_f - 2m_e$$

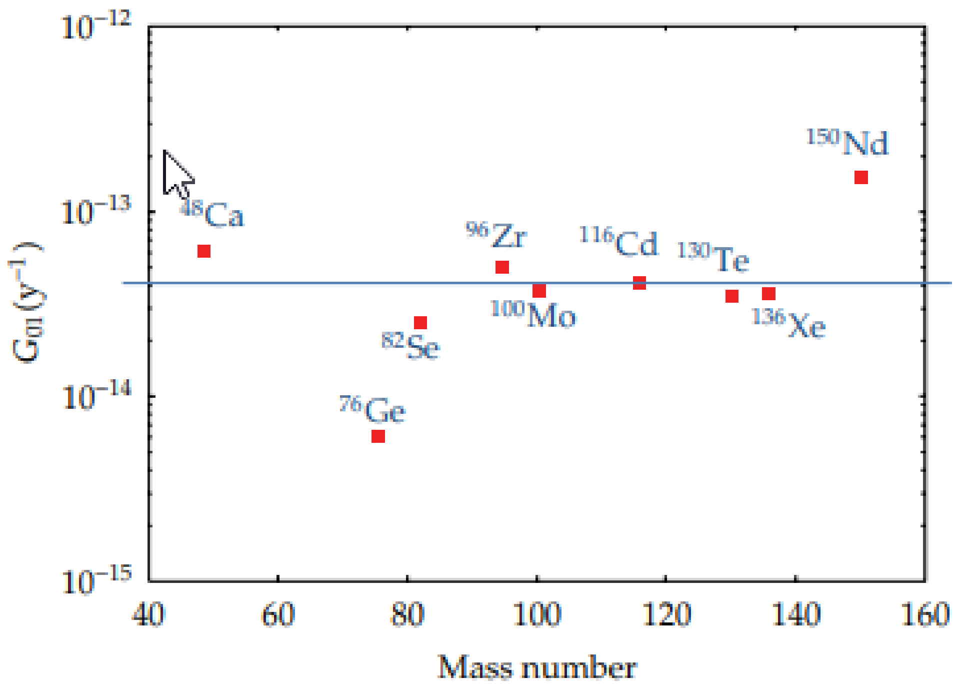


Fig 4. Phase space of the nine more favorable double-beta decay isotopes.
https://doi.org/10.1155/2012/857016

Absolute Majorana Neutrino Masses

Decay rate of $0\nu\beta\beta$

$$\Gamma^{0\nu} = \frac{1}{T^{0\nu}_{\frac{1}{2}}} = G^{0\nu}(Q,Z)\left|m_{\beta\beta}\right|^2 |M^{0\nu}|^2$$

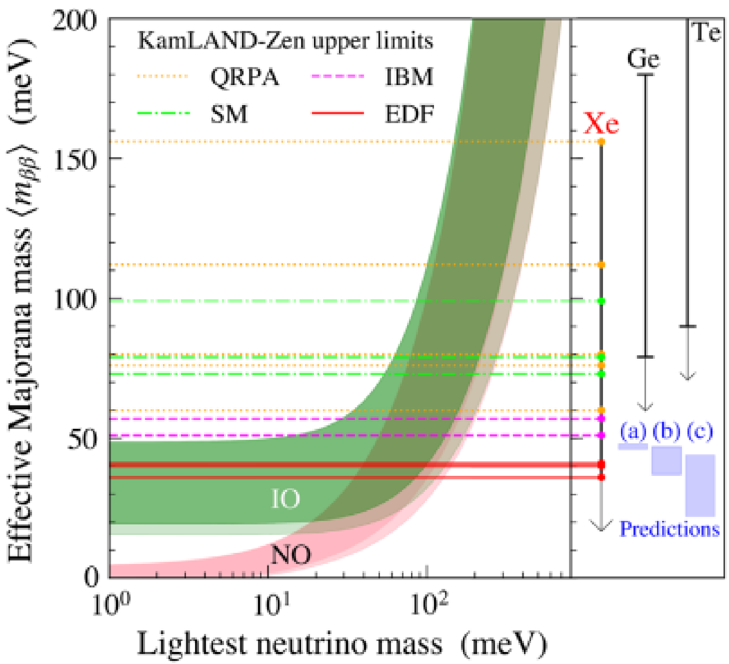
Majorana neutrino mass($m_{\beta\beta}$):

$$m_{\beta\beta} = \sum_i U_{ei}^2\, m_i$$

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

$$U = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix} \times \begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta_{13}}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta_{13}} & 0 & c_{13}\end{pmatrix}$$

$$\times \begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix} \times \begin{pmatrix}e^{1/2i\alpha_1} & 0 & 0\\ 0 & e^{1/2i\alpha_2} & 0\\ 0 & 0 & 1\end{pmatrix}$$



Where, $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$,
$0 \le \theta_{ij} \le \pi/2, 0 \le \delta_{13} \le 2\pi$, α_1 and α_2 are Majorana CP-violating phases.

Upper limit for $m_{\beta\beta}$ of 36-156 meV has been determined from $0\nu\beta\beta$ decay experiment of ^{136}Xe at KamLAND-Zen (PHYSICAL REVIEW LETTERS 130, 051801 (2023)) with lower limit of $T^{0\nu}_{1/2}$ 2.3×10^{26} yr using different nuclear matrix elements

Nuclear Matrix Element(NME) $\mathbf{M^{0\nu}}$: our research interest

$$M^{0\nu} = M^{0\nu}_{GT} - \frac{g_V^2}{g_A^2} M^{0\nu}_F + M^{0\nu}_T$$

- F- Fermi
- GT- Gamow-Teller
- T- Tensor
- g_V and g_A are vector and axial vector constant

$$M^{0\nu}_\alpha = \langle f | \tau_{-1}\tau_{-2}\, O^\alpha_{12} | i \rangle \quad \alpha = (F, GT, T)$$

0νββ transition operators

- Fermi Type: $O^F_{12} = S_F H_F(r) = H_F(r)$
- Gamow Teller type:

$$O^{GT}_{12} = S_{GT} H_{GT}(r) = \vec{\sigma}_1 . \vec{\sigma}_2 H_{GT}(r)$$

- Tensor Type:

$$O^T_{12} = S_T H_T(r) = [3(\vec{\sigma}_1 . \hat{r})(\vec{\sigma}_2 . \hat{r}) - \vec{\sigma}_1 . \vec{\sigma}_2] H_T(r)$$

Models of Nuclear Matrix Element calculations

- **Nuclear Shell Model (NSM) (We use this)**
- Quasiparticle Random Phase Approximation (QRPA)
- Projected Hartree Fock Bogliovob Method (PHFB)
- Interacting Boson Model 2 (IBM2)
- Energy Density Functional Theory (EDF)

Shell structure of nucleus

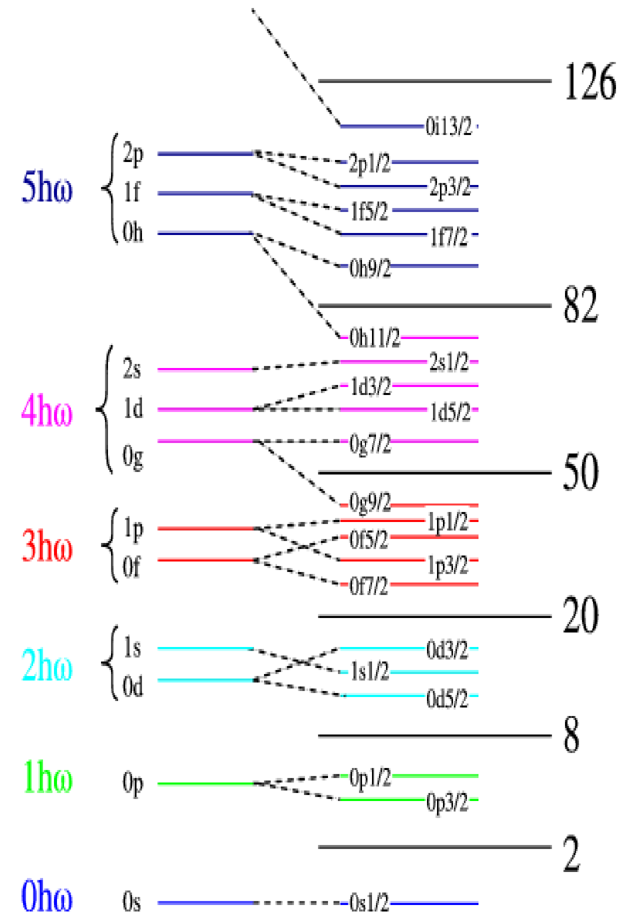


Fig 3: Nuclear shell structure

https://oer.physics.manchester.ac.uk/NP/Notes/shells.png

Approximations of NME calculations: closure vs nonclosure approximation

Nonclosure Approximation

$^{48}Ca \rightarrow {}^{48}Sc \rightarrow {}^{48}Ti$

$$H_\alpha(r, E_k^*) = \frac{2R}{\pi} * \int_0^\infty \frac{1}{q + E_0 + E_k^*} j_p(qr) g_\alpha(q) q dq$$

Where, $E_0 = \frac{Q_{\beta\beta}}{2} + \Delta M$

Closure Approximation

One replaces $E_k^* + E_0 = \langle E \rangle$

$$H_\alpha(r) = \frac{2R}{\pi} * \int_0^\infty \frac{1}{q + \langle E \rangle} j_p(qr) g_\alpha(q) q dq$$

Method: Running nonclosure and closure

Here….Neutrino potential are calculated explicitly in terms of excitation energy of ^{48}Sc

$$H_\alpha(r, E_k^*) = \frac{2R}{\pi} * \int_0^\infty \frac{1}{q + E_0 + E_k^*} j_p(qr) g_\alpha(q) q dq$$

$$\langle n'l'|H_\alpha(r, E_k^*)|nl\rangle = \int_0^\infty R_{n'l'} R_{nl} r^2 dr * H_\alpha(r, E_k^*)$$

Closure approximation

$$E_0 + E_k^* = \langle E\rangle$$

Nonclosure Closure approximation

$$E_0 + E_k^* \rightarrow 1.9\ \text{MeV} + E_k^*$$

Explicit Form of NME in running nonclosure method

$$M^{0\nu}_{\alpha-\text{running nonclosure}}(E) = \sum_{k1'k_2'k_1k_2JJ_k} \sum_{E_k^* \le E_C} \sqrt{(2J_k+1)(2J_k+1)(2J+1)} \times (-1)^{j_{k1}+j_{k2}+J} \begin{Bmatrix} j_1 & j_2 & J_k \\ j_4 & j_3 & J \end{Bmatrix} \times \text{OBTD}(k, f, k_1', k_2', J_k) \times \text{OBTD}(k, i, k_1, k_2, J_k) \langle k_1' k_2' : J \left|\left| \tau_{-1}\tau_{-2}\mathcal{O}_{12}^{0\nu} \right|\right| k_1 k_2 \rangle$$

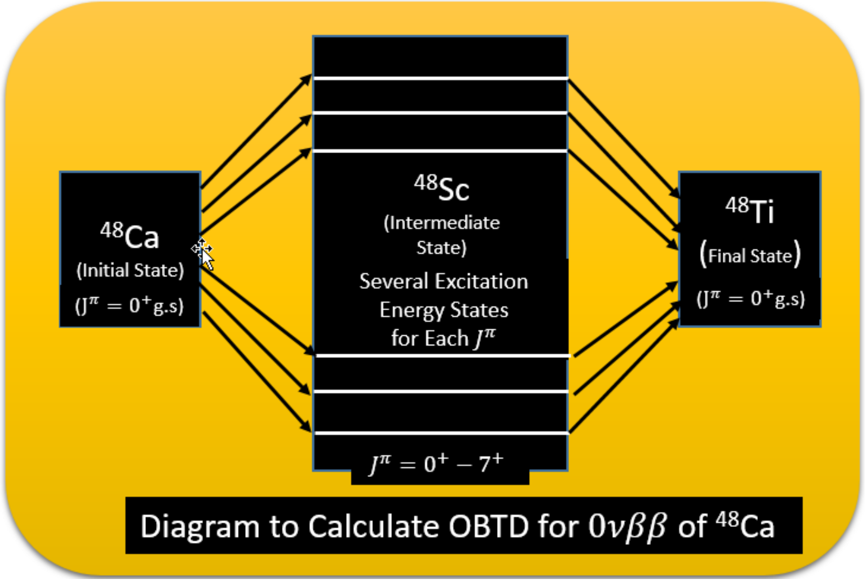


Fig 20: Schematic diagram to calculate OBTD

$$\text{OBTD}(k, f, k_1', k_2', J_k) = \frac{\langle k \left|\left| \left[a_{k_1}^+ \otimes \tilde{a}_{k_1'}\right]_{J_k} \right|\right| f\rangle}{\sqrt{2J_k+1}}$$

Two Body Matrix Elements of $0\nu\beta\beta$ decay

$0\nu\beta\beta$ transition operators

- Fermi Type:
  $$O_{12}^{F} = S_F H_F(r) = H_F(r)$$
- Gamow Teller type:
  $$O_{12}^{GT} = S_{GT} H_{GT}(r) = \vec{\sigma}_1.\vec{\sigma}_2 H_{GT}(r)$$
- Tensor Type:
  $$O_{12}^{T} = S_T H_T(r) = [3(\vec{\sigma}_1.\hat{r})(\vec{\sigma}_2.\hat{r}) - \vec{\sigma}_1.\vec{\sigma}_2] H_T(r)$$

Explicit Form of Two Body Matrix Elements

$$\langle n_{p1} l_{p1} j_{p1}, n_{p2} l_{p2} j_{p2}, J_m^{\pi} | \tau_{-1}\tau_{-2} O_{12}^{\alpha} | n_{n1} l_{n1} j_{n1}, n_{n2} l_{n2} j_{n2} J_m^{\pi} \rangle$$

$$= \sum_{S',\lambda',S,\lambda} \begin{Bmatrix} l_{p1} & \frac{1}{2} & j_{p1} \\ l_{p2} & \frac{1}{2} & j_{p2} \\ \lambda' & S' & J_m \end{Bmatrix} * \times \begin{Bmatrix} l_{n2} & \frac{1}{2} & j_{n2} \\ l_{n1} & \frac{1}{2} & j_{n1} \\ \lambda & S & J_m \end{Bmatrix}$$

$$\times \frac{1}{\sqrt{2S+1}} \langle l_{p1} l_{p2} \lambda' \frac{1}{2}\frac{1}{2} S'; J_m | S_{12}^{\alpha} | l_{n2} l_{n1} \lambda \frac{1}{2}\frac{1}{2} S; J_m \rangle$$

SPIN PART

$$\times \langle n_{p1} l_{p1} n_{p2} l_{p2} | H_\alpha(r) | n_{n1} l_{n1} n_{n2} l_{n2} \rangle$$

Radial Part

$$\langle n_{p1} l_{p1}, n_{p2} l_{p2} | H_\alpha(r) | n_{n1} l_{n1}, n_{n2} l_{n2} \rangle$$ → Individual Coordinate

$$= \sum_{n',l',N',L'} \sum_{n,l,N,L} < n'l', N'L' | n_{p1} l_{p1}, n_{p2} l_{p2} >_{\lambda'}$$

$$\times < n'l', N'L' | n_{p1} l_{p1}, n_{p2} l_{p2} >_{\lambda'} \times \langle n'l' | H_\alpha(r) | nl \rangle$$

Harmonic Oscillator Bracket | Relative and COM coordinate

Neutrino Potential Integral $\langle n'l'|H_\alpha(r)|nl\rangle$

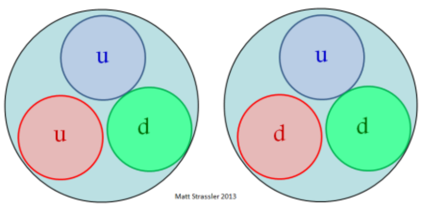
$$< n'l'|H_{Type}(r)|nl >= \int_0^\infty R_{n'l'}R_{nl}r^2dr * H_\alpha(r)$$

Where, $H_\alpha(r) = \frac{2R}{\pi} * \int_0^\infty \frac{1}{q+\langle E\rangle} j_p(qr)g_\alpha(q)qdq$

Effects of Finite nucleon size (FNS) and higher order current(HOC)

FNS + HOC

proton neutron



https://profmattstrassler.com/

\+

Weak interaction is not pure V-A type at nucleon level

$$J_L^\mu = \overline{\psi}\tau_-\left(g_V(q^2)\gamma^\mu - g_A(q^2)\gamma^\mu\gamma_5 - ig_M(q^2)\frac{\sigma^{\mu\nu}}{2m_p} + g_P(q^2)q^\mu\gamma_5\right)\psi$$

$$g_F(q) = g_V^2(q)$$

$$g_{GT}(q) = \frac{g_A^2(q)}{g_A^2}\left(1 - \frac{2}{3}\frac{q^2}{q^2+m_\pi^2} + \frac{1}{3}\left(\frac{q^2}{q^2+m_\pi^2}\right)^2\right) + \frac{2}{3}\frac{g_M^2(q)}{g_A^2} * \frac{q^2}{4m_p^2}$$

$$g_T(q) = \frac{g_A^2(q)}{g_A^2}\left(\frac{2}{3}\frac{q^2}{q^2+m_\pi^2} - \frac{1}{3}\left(\frac{q^2}{q^2+m_\pi^2}\right)^2\right) + \frac{1}{3}\frac{g_M^2(q)}{g_A^2} * \frac{q^2}{4m_p^2}$$

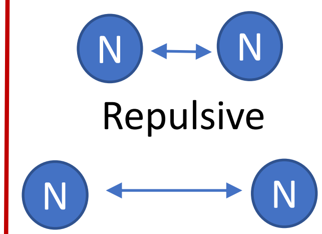
$$g_A(q^2) = g_A/(1 + q^2/\Lambda_A^2)$$

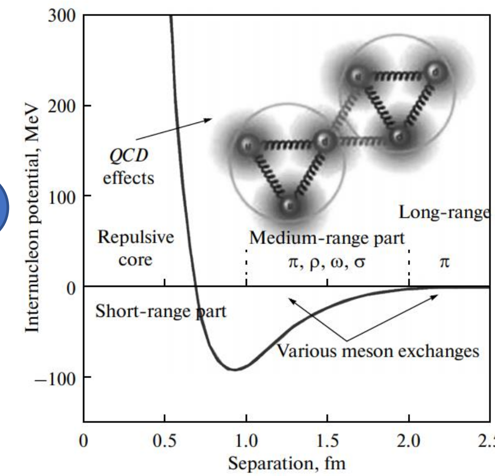
$$g_M(q^2) = (\mu_p - \mu_n)g_V(q^2)$$

$$g_P(q^2) = 2m_p g_A(q^2)/(q^2 + m_\pi^2)$$

$$g_V(q^2) = g_V/(1 + q^2/\Lambda_V^2)$$

Short Range Correlation Effect (SRC)





Physics of Particles and Nuclei, 2014, Vol. 45, No. 5, pp. 924–971.

Fig4: A general scheme for nucleon-nucleon potential

$$H_\alpha(r) \rightarrow H_\alpha(r)(1 + f(r))$$

$$f(r) = -ce^{-ar^2}(1 - br^2)$$

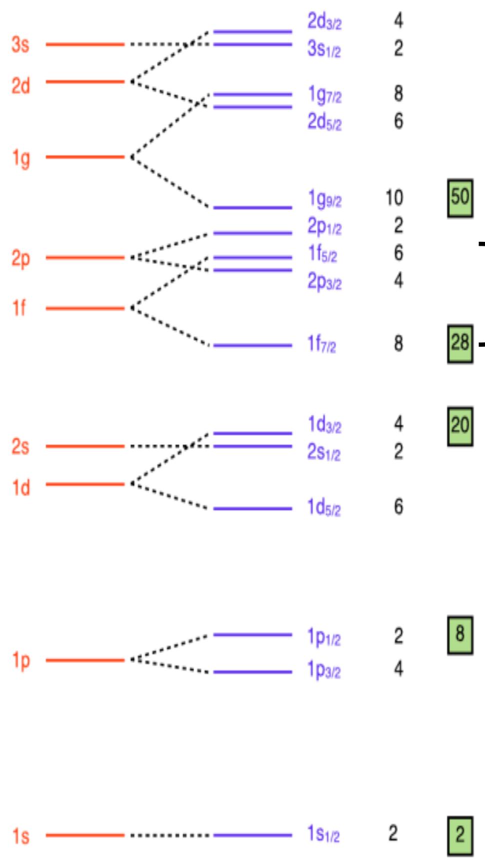
a, b, c are SRC parameters

Interacting Nuclear Shell Model and Effective Interactions

$^{48}Ca \rightarrow {}^{48}Ti + e^{-} + e^{-}$

$^{48}Ca \rightarrow {}^{40}Ca$ (Core)+8 valence neutron

$^{48}Ti \rightarrow {}^{40}Ca$ (Core)+6 valence neutron+2 valence proton



fp-valence space

$2p_{1/2}$

$1f_{5/2}$

$2p_{3/2}$

$1f_{7/2}$

ORBITALS

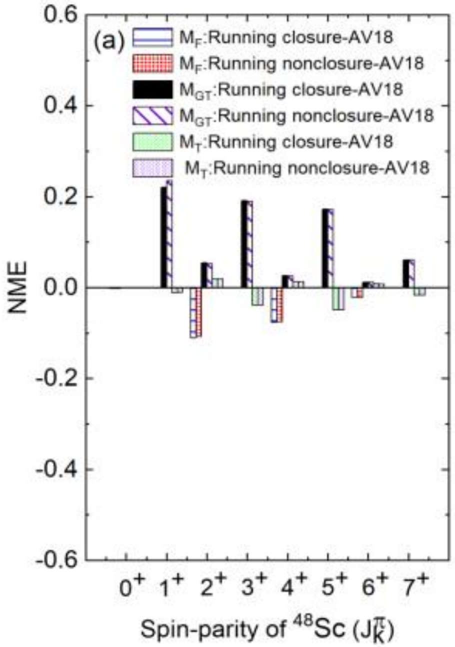
Valence protons and neutrons occupy the above orbits of fp-model space

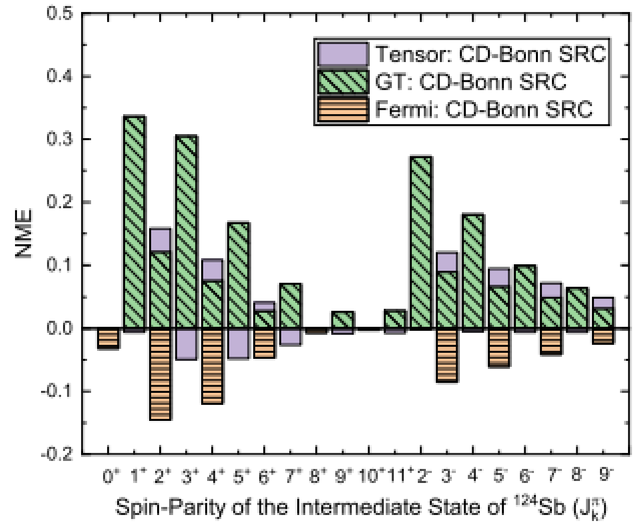
Fig 6: Nuclear Shell structure

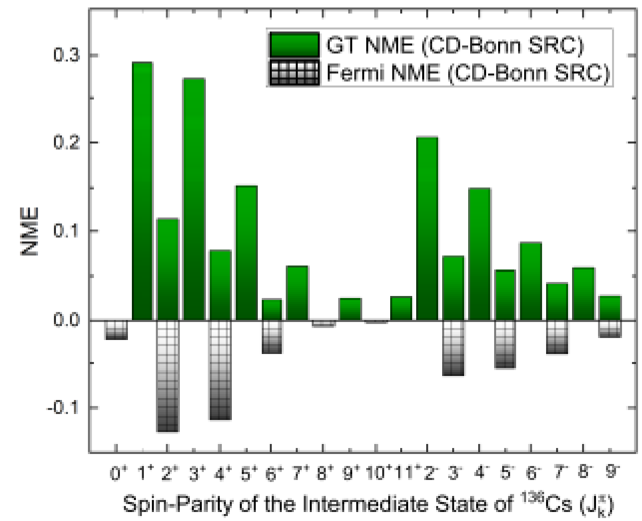
For 48Ca, shell model diagonalization is performed with gxpf1a effective interactions to calculate initial, intermediate, and final nucleus

Calculated Wavefunctions are further used to calculate the OBTD for nonclosure approach

Contribution of allowed spin-parity (J_k^{π}) of the intermediate nucleus for nonclosure approach



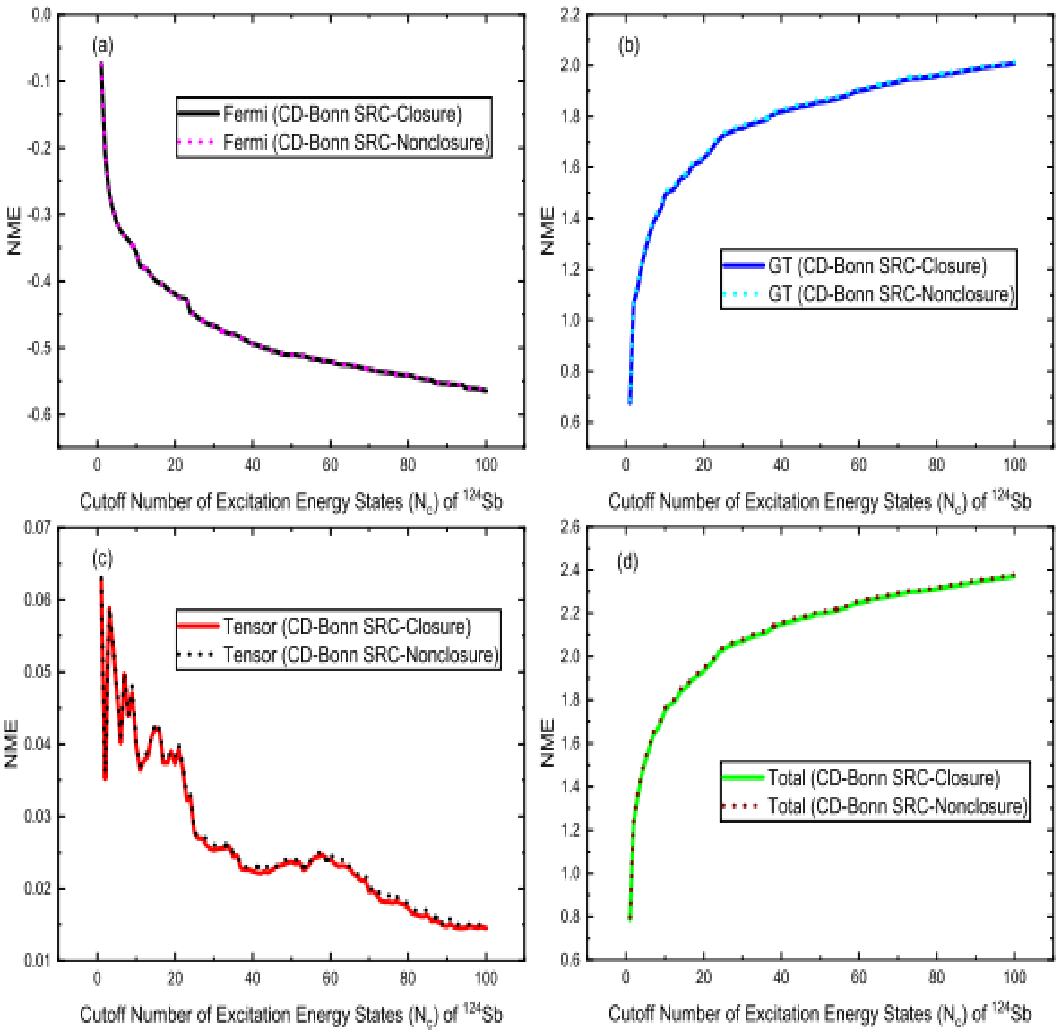


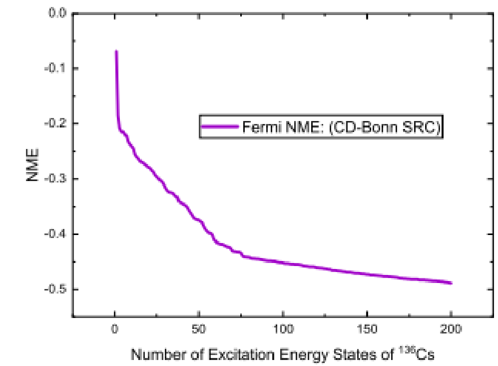


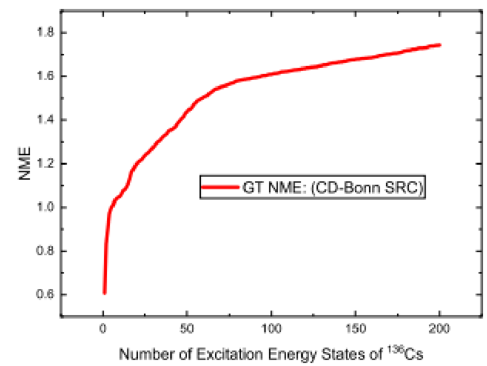
Observations

- Fermi NMEs are all negative and GT NMEs are all positive
- 1^+ contributes the most for GT type NME

Dependence of NME on number of states for each spin-parity (J_k^π) of the intermediate nucleus for nonclosure approach



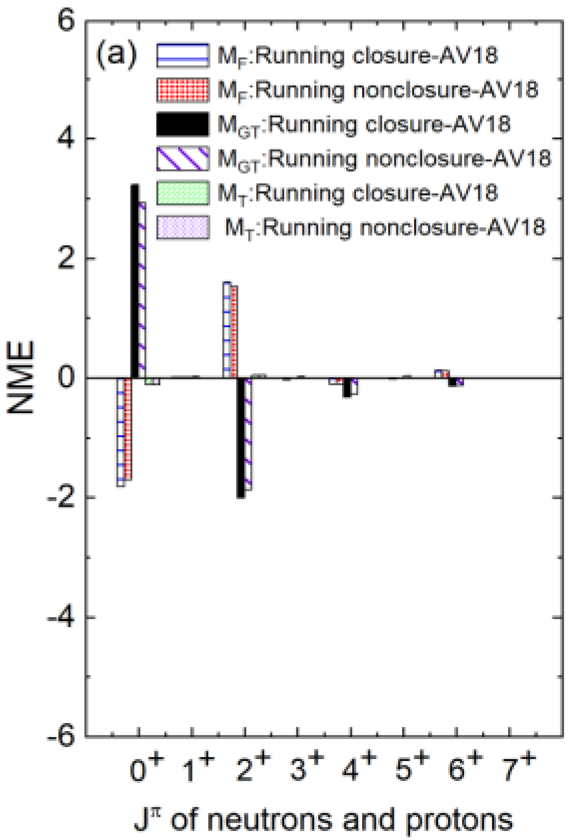


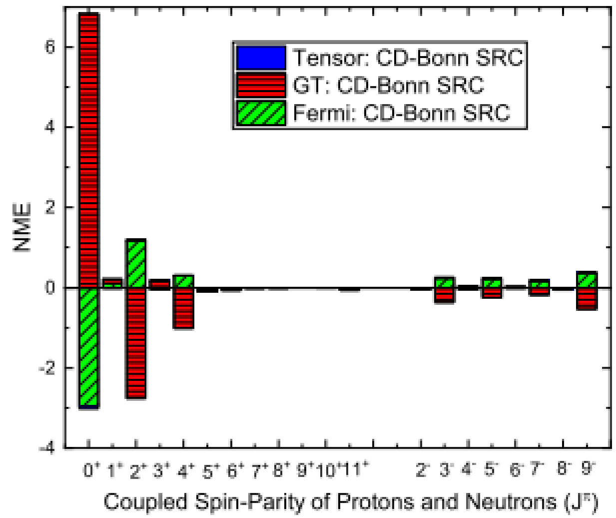


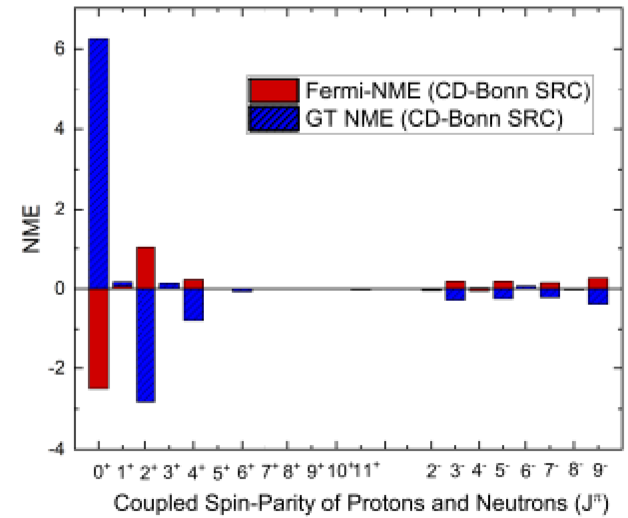
Observations

- For ^{124}Sn, NMEs are not completely converged with 100 intermediate states for each spin-parity of ^{124}Sb, but contributions are mostly small after 100th state
- For ^{136}Xe, we could calculate 200 states for each spin-parity of ^{136}Cs

Contribution of coupled spin-parity (J^{π}) of two initial neutrons and two final protons



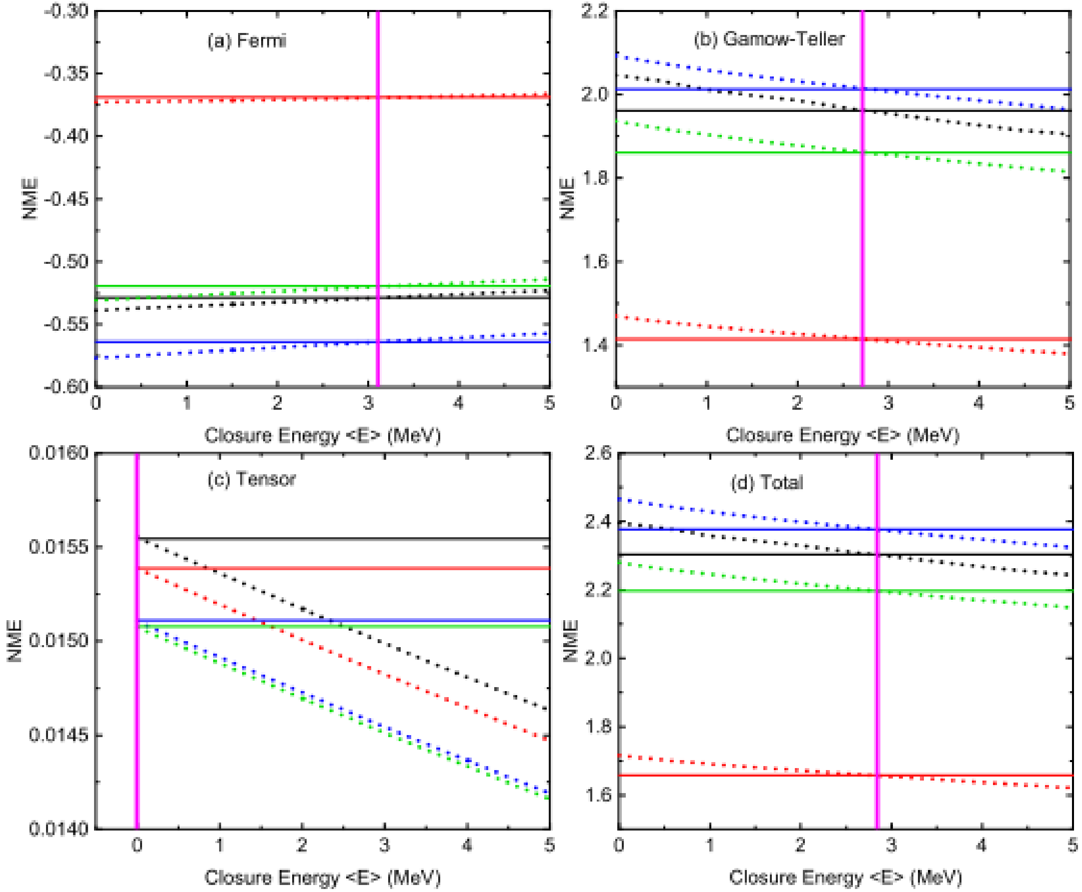




Observations

- 0+ and 2+ contributes the most having opposite phase
- Odd spin-parity contributions are negligible due to pairing effect

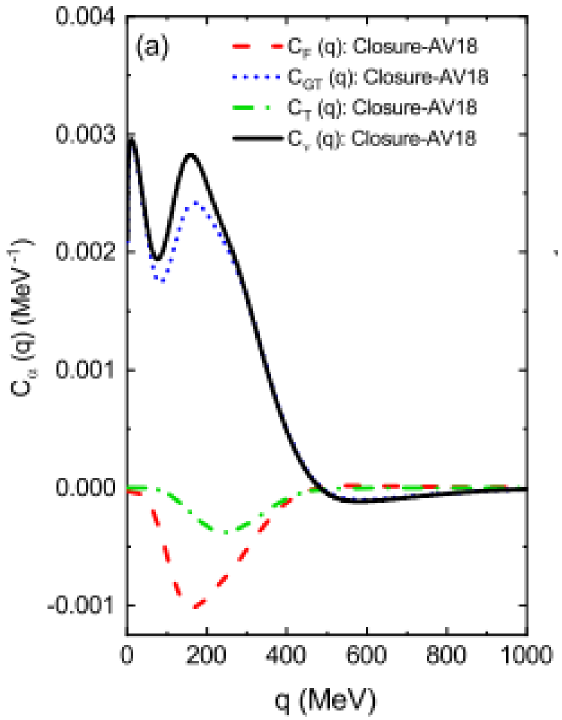
Finding Optimal Closure Energy Where NME in closure and nonclosure method overlaps

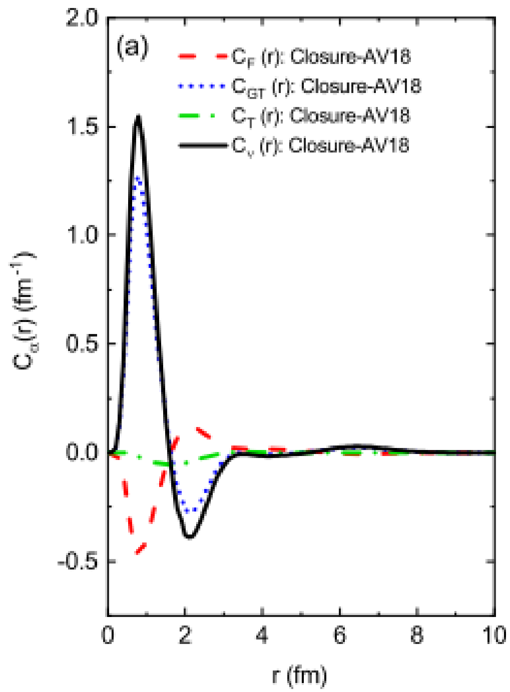


Observations

- For 124Sn, we determined that at optimal closure energy <E>~3.0 MeV, one can reproduce the nonclosure NME using closure approach across all intermediate states and using smaller computational resources

Dependence of NME on neutrino momentum (q) and inter nucleon distance (r)





$$M_\alpha = \int_0^\infty C_\alpha(q)dq.$$

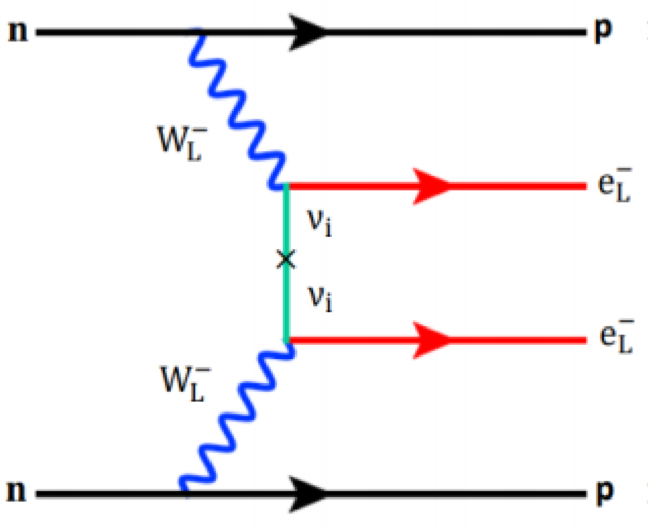
$$M_\alpha = \int_0^\infty C_\alpha(r)dr.$$

$$\alpha = \mathrm{F, GT, T}$$

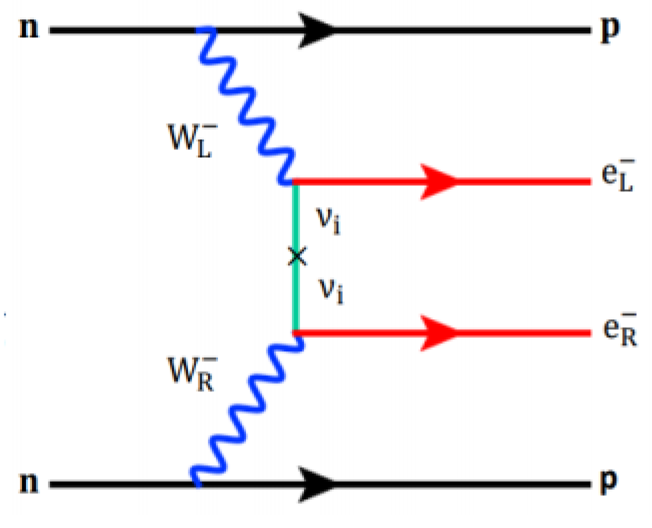
Observations

- Most of the contribution come from q below 500 MeV
- For r of about 1fm, NME contributions are largest

Study of λ mechanism of $0\nu\beta\beta$ in Nuclear Shell Model



The Feynman diagrams for light $m_{\beta\beta}$ mechanism



The Feynman diagrams for λ mechanism

Motivation of Studying λ mechanism

PHYSICAL REVIEW C **98**, 035502 (2018)

Shell model study of using an effective field theory for disentangling several contributions to neutrinoless double-β decay

Mihai Horoi* and Andrei Neacsu†

(I) Shell Model was used in paper for closure approximation to study λ mechanism of $0\nu\beta\beta$

PHYSICAL REVIEW C **92**, 055502 (2015)

Reexamining the light neutrino exchange mechanism of the $0\nu\beta\beta$ decay with left- and right-handed leptonic and hadronic currents

Dušan Štefánik,[1] Rastislav Dvornický,[1,2] Fedor Šimkovic,[1,3,4] and Petr Vogel[5]

(II) Exploited the revised formalism for λ mechanism

frontiers in Physics **The λ Mechanism of the $0\nu\beta\beta$-Decay**

Fedor Šimkovic[1,2,3], Dušan Štefánik[1] and Rastislav Dvornický[1,4]*

(III) QRPA calculations with revised formalism for λ mechanism

Decay rate and NME for λ mechanism

Decay Rate

$$\left[T^{0\nu}_{\frac{1}{2}}\right]^{-1} = \eta_\nu^2 C_{mm} + \eta_\lambda^2 C_{\lambda\lambda} + \eta_\nu \eta_\lambda \cos\psi \, C_{m\lambda}$$

$$C_{mm} = g_A^4 M_\nu^2 G_{01},$$

$$C_{m\lambda} = -g_A^4 M_\nu (M_{2-} G_{03} - M_{1+} G_{04}),$$

$$C_{\lambda\lambda} = g_A^4 \left(M_{2-}^2 G_{02} + \frac{1}{9} M_{1+}^2 G_{011} - \frac{2}{9} M_{1+} M_{2-} G_{010}\right)$$

NMEs

$$M_\nu = M_{GT} - \frac{1}{g_A^2} M_F + M_T$$

$$M_{\nu\omega} = M_{\omega GT} - \frac{1}{g_A^2} M_{\omega F} + M_{\omega T}$$

$$M_{1+} = M_{qGT} + 3\frac{1}{g_A^2} M_{qF} - 6 M_{qT}$$

$$M_{2-} = M_{\nu\omega} - \frac{1}{9} M_{1+}$$

$$M_\alpha = \langle f | \tau_{1-}\tau_{2-} \mathcal{O}_{12}^{\alpha} | i \rangle$$

Transition Operator

$$\mathcal{O}_{12}^{GT,\omega GT,qGT} = \tau_{1-}\tau_{2-}(\sigma_1 . \sigma_2) H_{GT,\omega GT,qGT}(r, E_k)$$

$$\mathcal{O}_{12}^{F,\omega F,qF} = \tau_{1-}\tau_{2-} H_{F,\omega F,qF}(r, E_k)$$

$$\mathcal{O}_{12}^{T,\omega T,qT} = \tau_{1-}\tau_{2-}(S_{12}) H_{T,\omega T,qT}(r, E_k)$$

Radial neutrino potentials

Nonclosure approximation

$$H_\alpha(r, E_k) = \frac{2R}{\pi} \int_0^\infty \frac{f_\alpha(q, r)\,dq}{q + E_k + (E_i + E_f)/2}$$

Closure approximation

$$[E_k + (E_i + E_f)/2] \rightarrow \langle E \rangle$$

$$H_\alpha(r, E_k) = \frac{2R}{\pi} \int_0^\infty \frac{f_\alpha(q, r)\,dq}{q + \langle E \rangle}$$

Revised Approach

PHYSICAL REVIEW C **92**, 055502 (2015)

Reexamining the light neutrino exchange mechanism of the $0\nu\beta\beta$ decay with left- and right-handed leptonic and hadronic currents

Dušan Štefánik,[1] Rastislav Dvornický,[1,2] Fedor Šimkovic,[1,3,4] and Petr Vogel[5]

$$f_{qGT}(q, r) = \frac{1}{\left(1 + \frac{q^2}{\Lambda_A^2}\right)^4} q j_1(qr) \qquad \text{(Old)}$$

$$f_{qGT}(q, r) = \left(\frac{g_A^2(q^2)}{g_A^2} q + 3 \frac{g_P^2(q^2)}{g_A^2} \frac{q^5}{4 m_N^2} + \frac{g_A^2(q^2) g_P^2(q^2)}{g_A^2} \frac{q^3}{m_N}\right) r j_1(qr) \qquad \text{(Revised)}$$

Summary and Outlooks

We calculated NME for 0νββ decay based on ISM calculations

Nuclear matrix elements calculation for 0νββ decay of 124Sn using nonclosure approach in nuclear shell model
Shahariar Sarkar, P. K. Rath, V. Nanal, R. G. Pillay, Pushpendra P. Singh, Y. Iwata, K. Jha, P. K. Raina
submitted; arXiv:2308.08877

Submitted to PRC

Effect of Spin-Dependent Short-Range Correlations on Nuclear Matrix Elements for Neutrinoless Double Beta Decay of 48Ca
S. Sarkar, Y. Iwata
Universe 2023, 9(10), 444

Interacting shell model calculations for neutrinoless double beta decay of 82Se with left-right weak boson exchange
Yoritaka Iwata, Shahariar Sarkar
Front. Astron. Space Sci., 8: 727880 (2021)

Nuclear matrix elements for λ mechanism of 0νββ of 48Ca in nuclear shell-model: Closure versus nonclosure approach
S. Sarkar, Y. Iwata, P. K. Raina
Phys. Rev. C 102 (2020) 034317

Estimation of nuclear matrix elements of double-β decay from shell model and quasi particle random-phase approximation
Jun Terasaki, Yoritaka Iwata
Eur. Phys. J. Plus 136, 908 (2021)

Isoscalar pairing interaction for quasiparticle random-phase approximation approach to double-β and β decays
J. Terasaki, Y. Iwata
Phys. Rev. C 100 (2019) 034325

Large-scale shell-model analysis of the neutrinoless double-beta decay of Ca48
Y. Iwata, N. Shimizu, T. Otsuka, Y. Utsuno, J. Menendez, M. Honma, T. Abe
Phys. Rev. Lett. 116 (2016) 112502